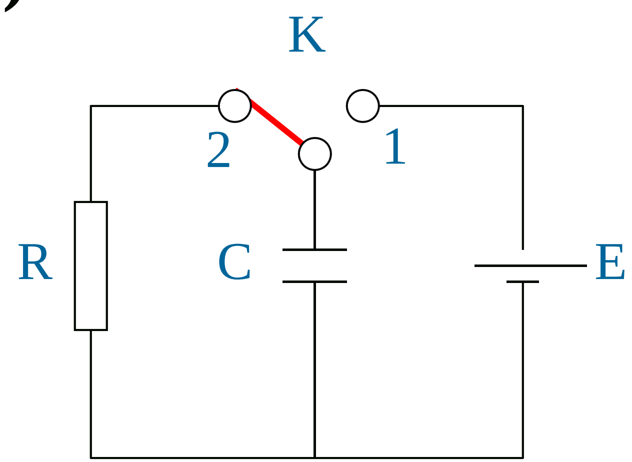
常微分方程的数值解法

RC回路

------常微分方程的数值解法

RC回路放电问题(一阶常微分方程):

$$IR+\frac{Q}{C}=0 \qquad \frac{dQ}{dt}=-\frac{Q}{\tau};\ \tau=RC$$



数值求解: $\begin{cases} \dfrac{dQ}{dt}=-\dfrac{Q}{\tau} \\ Q(t_0)=Q_0 \end{cases}$

初值问题

一般情况：

$$\begin{cases} \dfrac{dy}{dt} = f(y,t) \\ y(0) = y_0 \end{cases} \Longleftarrow \begin{cases} \dfrac{dQ}{dt} = -\dfrac{Q}{\tau} \\ Q(t_0) = Q_0 \end{cases}$$

常用算法：

$$\begin{cases} \text{欧拉法} \\ \text{改进的欧拉法} \\ Runge-Kutta\text{法} \\ Verlet\text{算法} \end{cases}$$

欧拉法

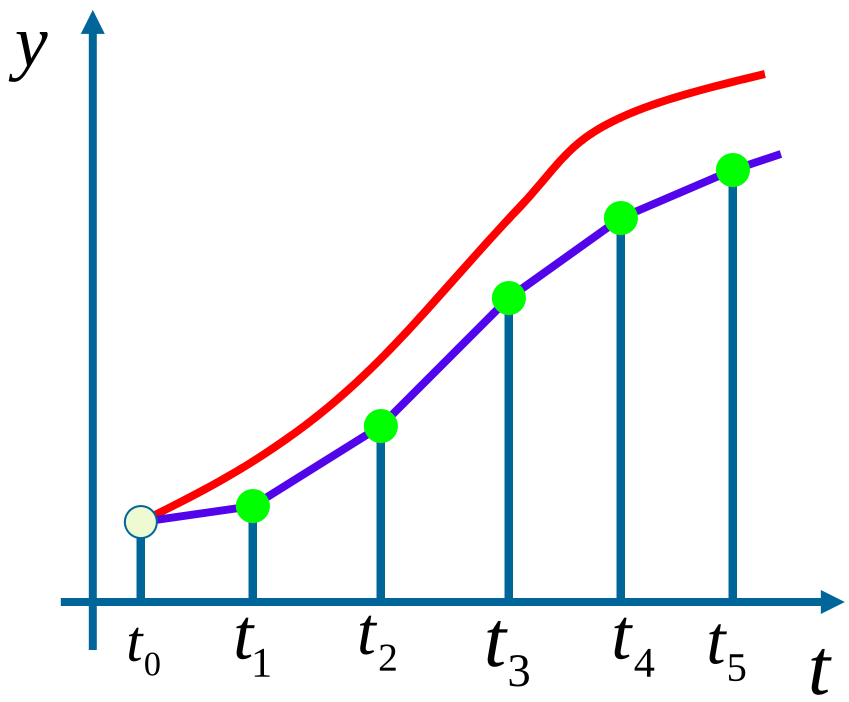
思路：向前差分代替微分，$f(y,t)$用前端点的值$f(y_n,t_n)$代替

$$\begin{cases} \dfrac{dy}{dt} = f(y,t) \\ y(0) = y_0 \end{cases} \longrightarrow \begin{cases} \dfrac{y_{n+1} - y_n}{\Delta t} = f(y_n, t_n) \\ y(t_0) = y_0 \end{cases}$$

计算公式：

$$\begin{cases} y_{n+1} = y_n + \Delta t f(y_n, t_n) \\ n = 0,1,2\cdots \end{cases}$$

欧拉法
（折线法）



RC回路放电问题：

$$\begin{cases} y_{n+1} = y_n + \Delta t f(y_n, t_n) \\ n = 0,1,2\cdots \end{cases}$$

欧拉法：

$$\begin{cases} \dfrac{dQ}{dt} = -\dfrac{Q}{\tau} \\ Q(t_0) = Q_0 \end{cases} \longrightarrow \begin{cases} Q_{n+1} = Q_n - \dfrac{Q_n}{\tau}\Delta t \\ Q(t_0) = Q_0 \end{cases}$$

设 $Q_0 = 1.0,\ \tau = RC = 10,\ \Delta t = 1$

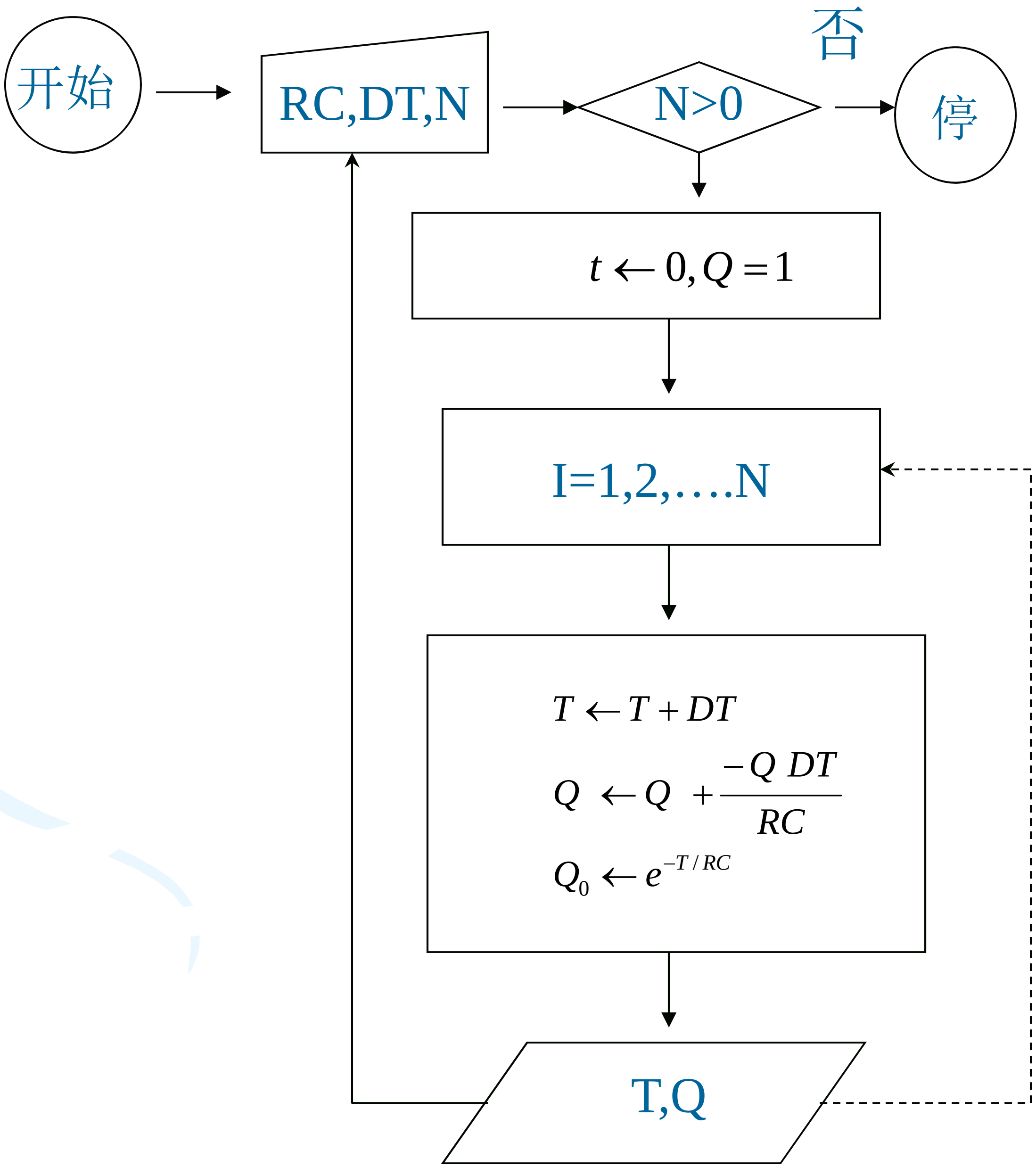
$$Q_1 = Q_0 - \frac{Q_0}{\tau}\Delta t = 1.0 - \frac{1.0}{10}\times 1 = 0.9$$

$$Q_2 = Q_1 - \frac{Q_1}{\tau}\Delta t = 0.9 - \frac{0.9}{10}\times 1 = 0.81$$

$$Q_3 = Q_2 - \frac{Q_2}{\tau}\Delta t = 0.81 - \frac{0.81}{10}\times 1 = 0.729$$

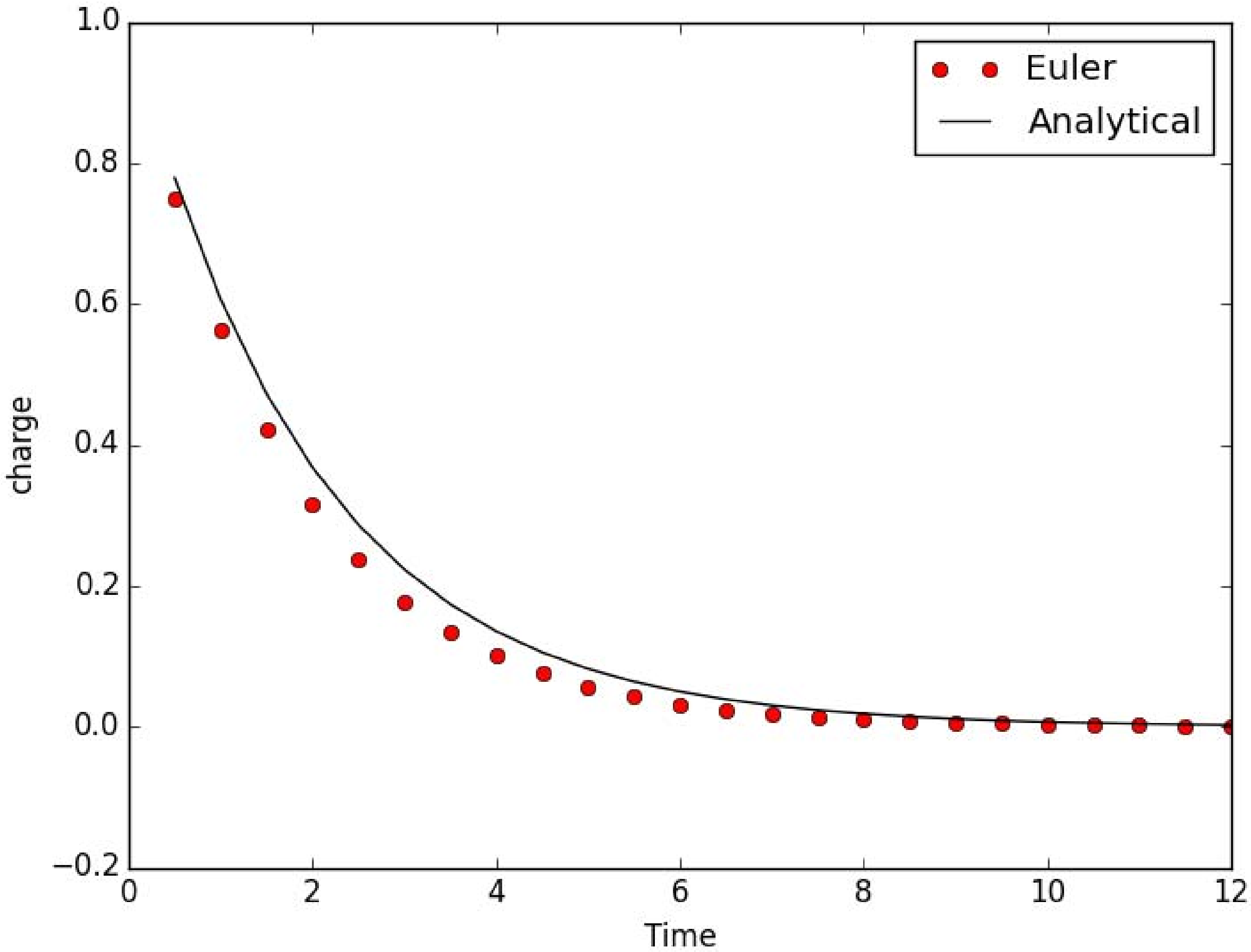
$\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet$

计算过程:



开始
RC,DT,N
N>0
否
停
$t \leftarrow 0, Q = 1$
I=1,2,….N
$T \leftarrow T + DT$
$Q \leftarrow Q + \frac{-Q\ DT}{RC}$
$Q_0 \leftarrow e^{-T/RC}$
T,Q

RC=2.0, dt=0.5



解析解: $Q = Q_0 e^{-\frac{t}{\tau}}$

误差估计

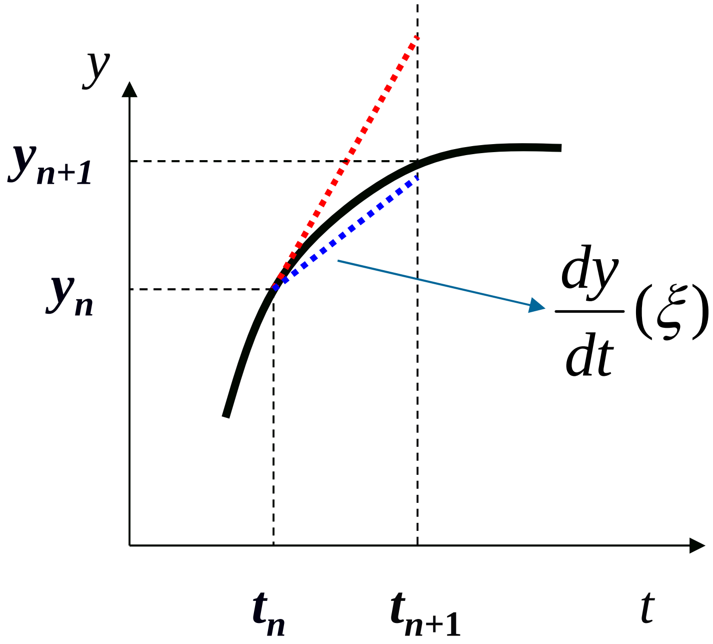
局部截断误差：

泰勒展开： $$y_{n+1} = y_n + \Delta t \frac{dy}{dt}(t_n) + \frac{\Delta t^2}{2}\frac{d^2 y}{dt^2}(t_n) + \cdots$$

欧拉公式： $$y_{n+1} = y_n + \Delta t f(y_n, t_n) = y_n + \Delta t \frac{dy}{dt}(t_n)$$

局部截断误差 $\propto \Delta t^2$

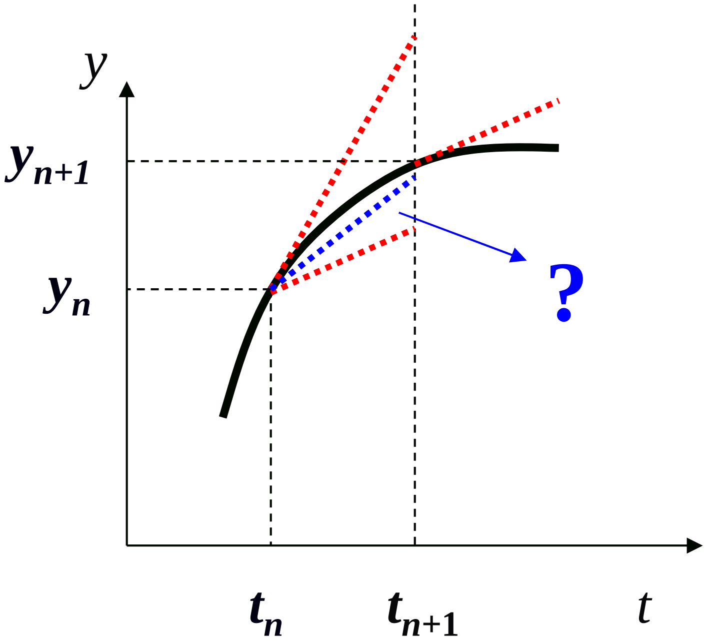
欧拉法:



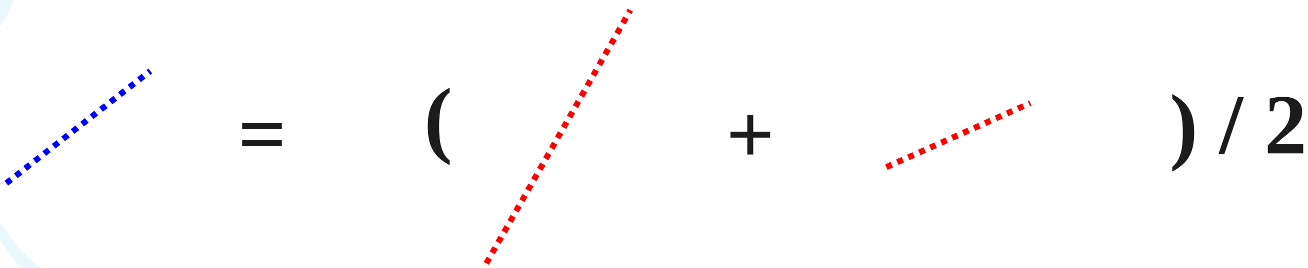
$$y_{n+1} = y_n + \frac{dy}{dt}(t_n)\Delta t$$

$$y_{n+1} = y_n + \frac{dy}{dt}(\xi)\Delta t \quad t_n < \xi < t_{n+1}$$

如何提高 $\frac{dy}{dt}(\xi)$ 计算精度------高阶算法



$$\begin{cases} \dfrac{dy}{dt} = f(y,t) \\ y(0) = y_0 \end{cases}$$



$$\frac{dy}{dt}(\xi) = \frac{1}{2}\left(\frac{dy}{dt}(t_n) + \frac{dy}{dt}(t_{n+1})\right) = \frac{1}{2}\left(f(y_n, t_n) + f(y_{n+1}, t_{n+1})\right)$$

改进的欧拉算法！

改进的欧拉法

思路：向前差分代替微分，*f* (*y*,*t*)用前后端点的平均值代替

$$\begin{cases} \dfrac{dy}{dt} = f(y,t) \\ y(0) = y_0 \end{cases} \rightarrow \begin{cases} \dfrac{y_{n+1} - y_n}{\Delta t} = \dfrac{1}{2}(f(y_n,t_n) + f(y_{n+1},t_{n+1})) \\ y(t_0) = y_0 \end{cases}$$

$$\begin{cases} y_{n+1} = y_n + \dfrac{\Delta t}{2}(f(y_n,t_n) + f(\underline{y_{n+1}},t_{n+1})) \\ n = 0,1,2\cdots \end{cases}$$

用欧拉法预测

改进的欧拉法（梯形法）

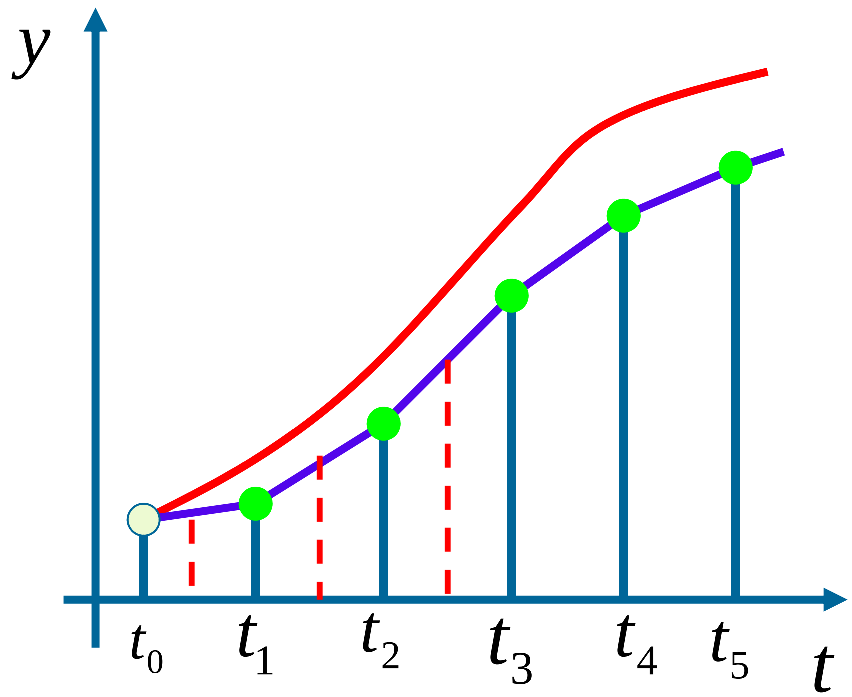
计算步骤：

1. 用欧拉公式预测 y_{n+1}

$$y_{n+1} = y_n + \Delta t f(y_n, t_n)$$

2. 用改进的欧拉公式校正 y_{n+1}

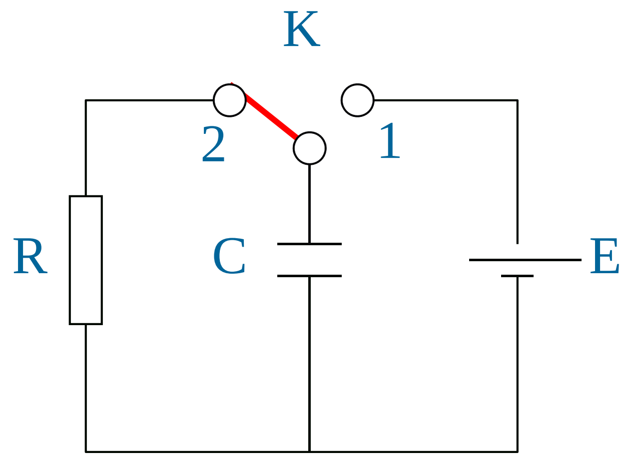
$$\begin{cases} y_{n+1} = y_n + \dfrac{\Delta t}{2}\left(f(y_n, t_n) + f(y_{n+1}, t_{n+1})\right) \\ n = 0, 1, 2 \cdots \end{cases}$$



局部截断误差： $\propto \Delta t^3$

RC回路放电问题：

$$\begin{cases} \dfrac{dQ}{dt} = -\dfrac{Q}{\tau} \\ Q(t_0) = Q_0 \end{cases}$$

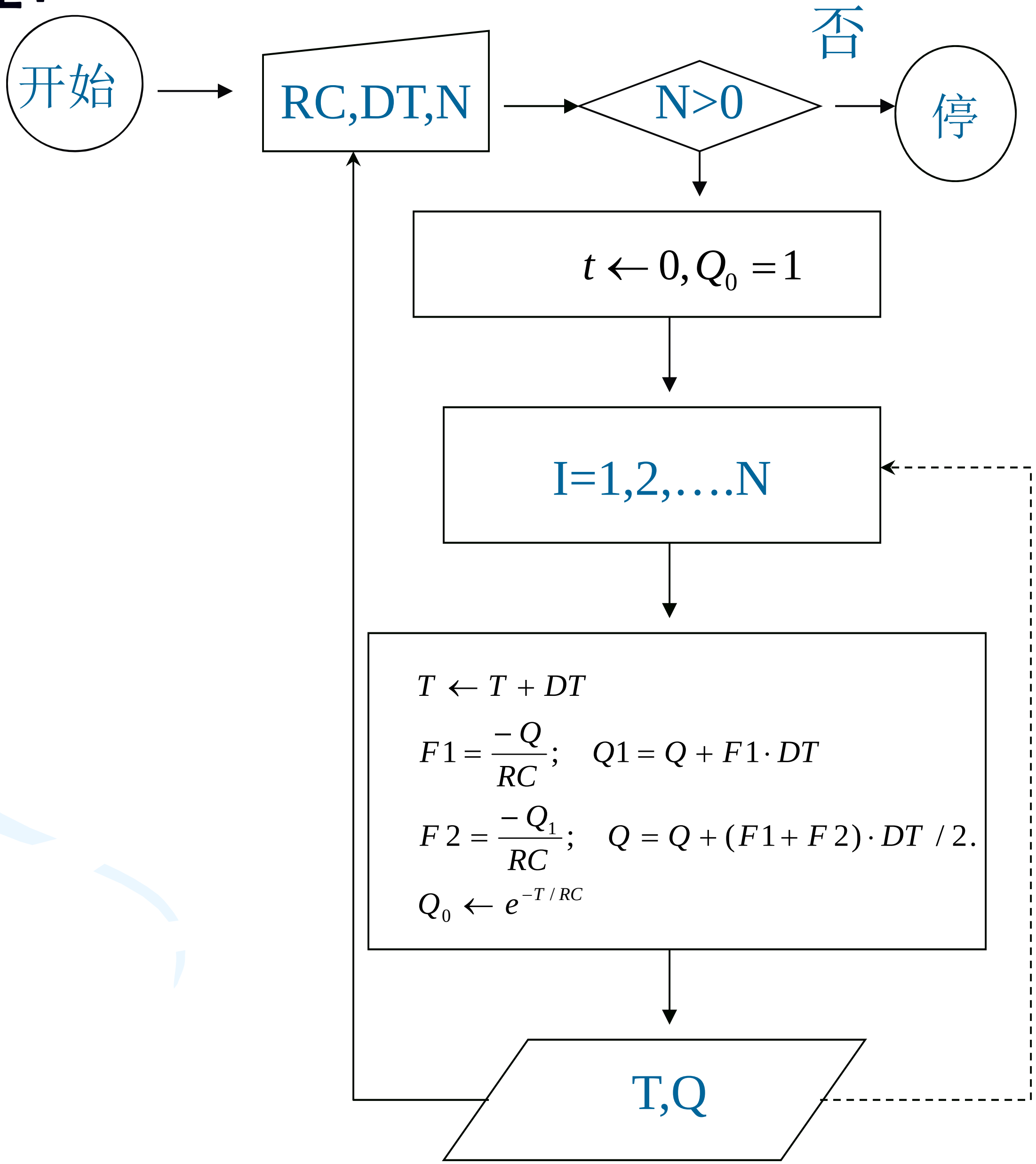


预测：$$Q_{n+1} = Q_n - \frac{Q_n}{\tau}\Delta t$$

校正：$$Q_{n+1} = Q_n - \frac{1}{2}\left(\frac{Q_n}{\tau} + \frac{Q_{n+1}}{\tau}\right)\Delta t$$

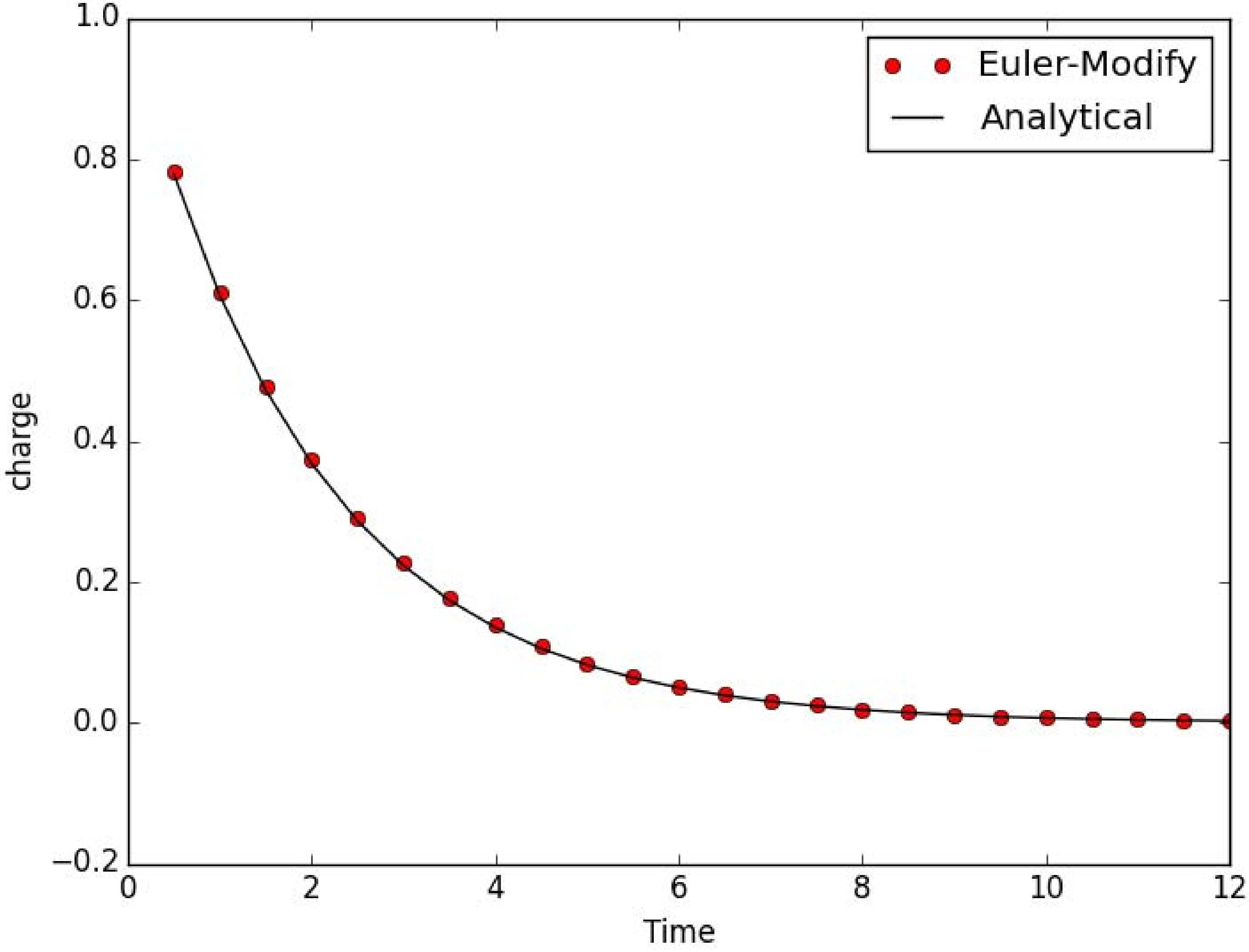
$$n = 0, 1, 2 \cdots$$

计算过程:



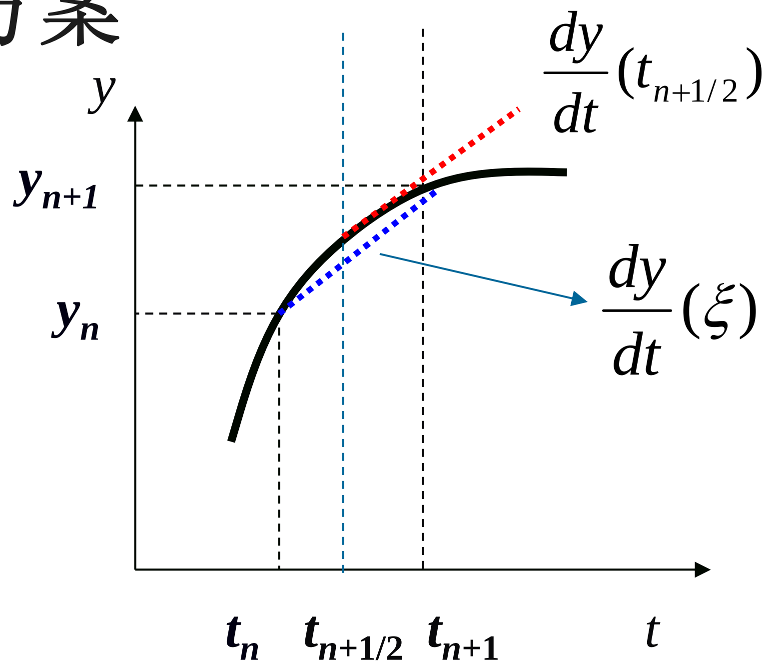
开始
RC,DT,N
N>0
否
停
$t \leftarrow 0, Q_0 = 1$
I=1,2,….N
$T \leftarrow T + DT$
$F1 = \frac{-Q}{RC}; \quad Q1 = Q + F1 \cdot DT$
$F2 = \frac{-Q_1}{RC}; \quad Q = Q + (F1 + F2) \cdot DT / 2.$
$Q_0 \leftarrow e^{-T/RC}$
T,Q

RC=2.0, dt=0.5



Euler-Modify
Analytical
charge
Time
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
2
4
6
8
10
12

另一种改进方案



$$\frac{dy}{dt}(\xi)=\frac{dy}{dt}(t_{n+1/2})=f(y_{n+1/2},t_{n+1/2})$$

二阶Runge-Kutta！

二阶Runge-Kutta法

思路：向前差分代替微分，*f* (*y*,*t*)用中点的值代替

$$\begin{cases} \dfrac{dy}{dt} = f(y,t) \\ y(0) = y_0 \end{cases} \rightarrow \begin{cases} \dfrac{y_{n+1} - y_n}{\Delta t} = f(y_{n+1/2}, t_{n+1/2}) \\ y(t_0) = y_0 \end{cases}$$

$$\begin{cases} y_{n+1} = y_n + \Delta t f(\underline{y_{n+1/2}}, t_{n+1/2}) \\ n = 0,1,2\cdots \end{cases}$$

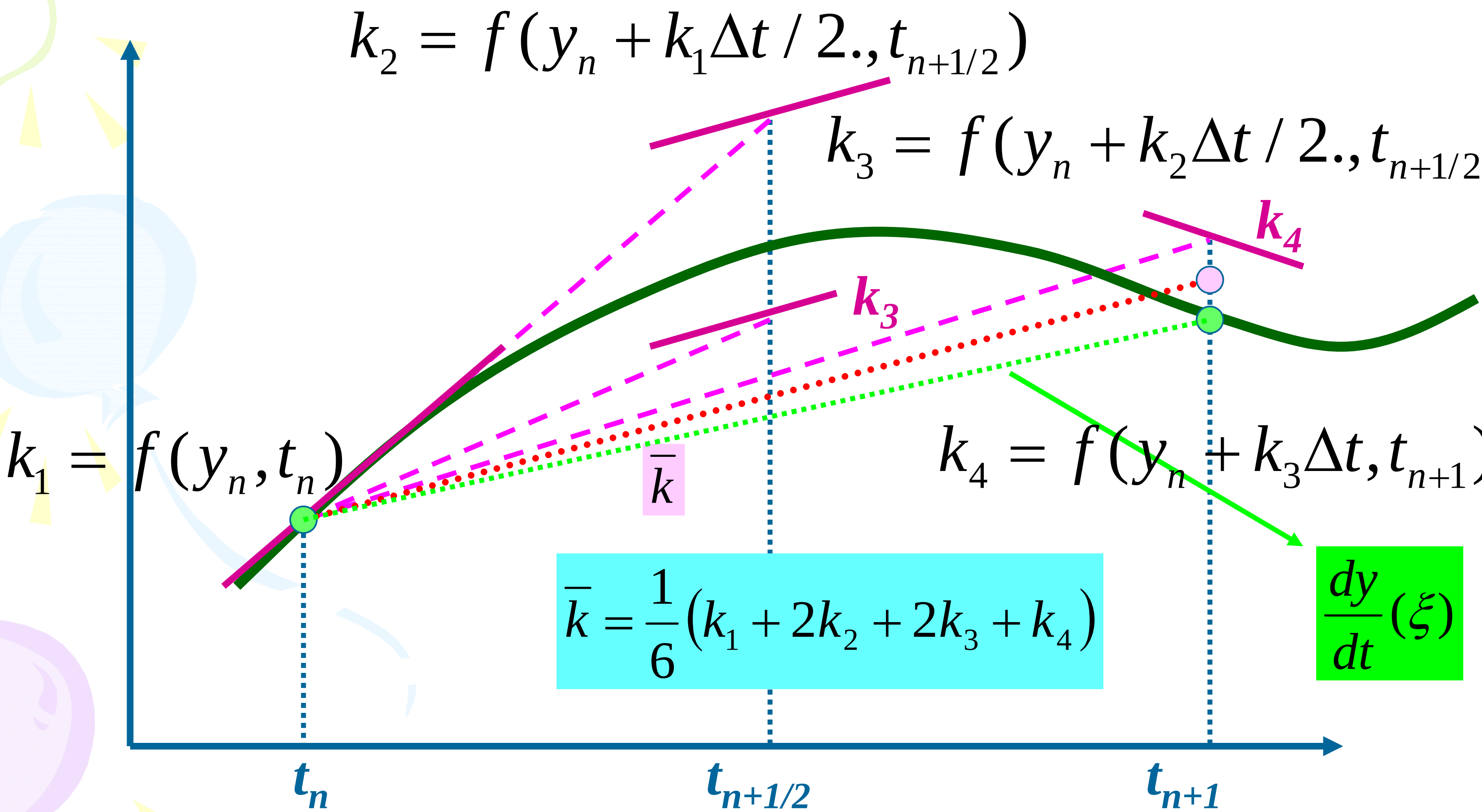
$$y_{n+1/2} = y_n + \frac{\Delta t}{2} f(y_n, t_n)$$

用欧拉法预测

二阶
Runge-Kutta法

局部截断误差： $\propto \Delta t^3$

四阶Runge-Kutta法



$$y_{n+1} = y_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

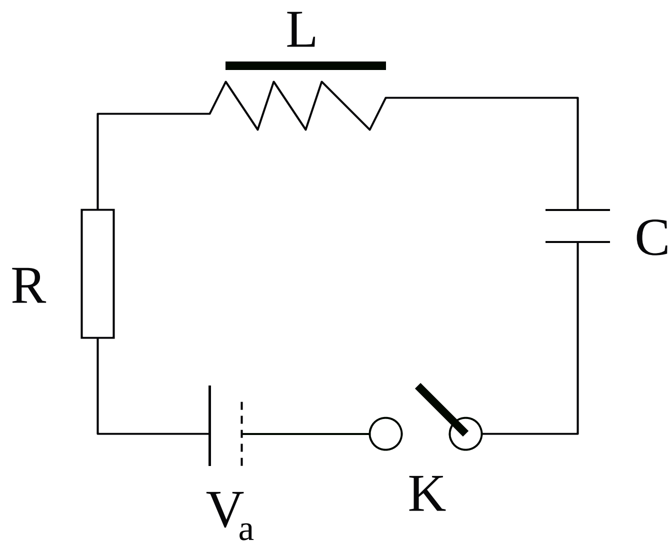
Runge-Kutta法
(4阶)

$$\begin{cases} k_1 = f(y_n, t_n) \\ k_2 = f(y_n + k_1 \Delta t / 2., t_{n+1/2}) \\ k_3 = f(y_n + k_2 \Delta t / 2., t_{n+1/2}) \\ k_4 = f(y_n + k_3 \Delta t, t_{n+1}) \end{cases}$$

局部截断误差： $\propto \Delta t^5$

RLC回路问题(二阶常微分方程):

$$L\frac{d^2Q}{dt^2}+\frac{dQ}{dt}R+\frac{Q}{C}=V_0\sin(Wt)$$



思路:

二阶微分方程变为一阶微分方程组

$$\begin{cases}\dfrac{dI}{dt}=\dfrac{1}{L}(V_0\sin(Wt)-IR-\dfrac{Q}{C})\\ \dfrac{dQ}{dt}=I\\ Q(t_0)=Q_0\\ I(t_0)=I_0\end{cases}$$

一般情况：

$$
\begin{cases}
\dfrac{dy}{dt} = f(y,x,t) \\
\dfrac{dx}{dt} = g(y,x,t) \\
y(0) = y_0 \\
x(0) = x_0
\end{cases}
$$

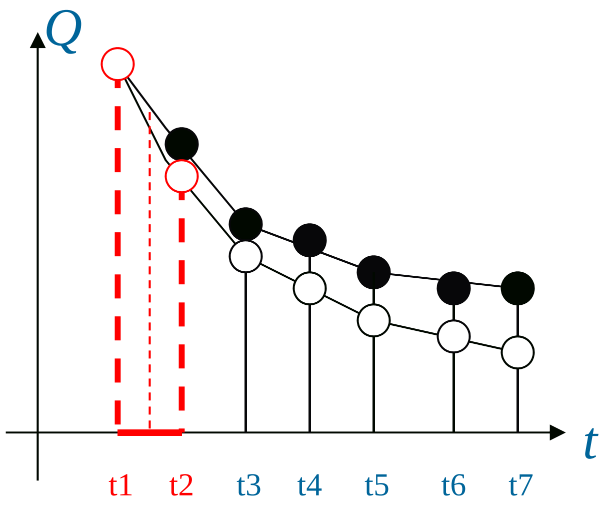
RLC回路问题：

$$\begin{cases} \dfrac{dI}{dt} = \dfrac{1}{L}(V_0 \sin(Wt) - IR - \dfrac{Q}{C}) = f(I,Q) \\ \dfrac{dQ}{dt} = I = g(I,Q) \\ Q(0) = Q_0 \\ I(0) = I_0 \end{cases}$$

用改进的欧拉法:

1. 用欧拉公式预测 I_{n+1}和Q_{n+1}:

$$\begin{cases} I_{n+1} = I_n + f(I_n, Q_n)\Delta t \\ Q_{n+1} = Q_n + f(I_n, Q_n)\Delta t = Q_n + I_n \Delta t \end{cases}$$



2. 计算 $f\,(I_{n+1},Q_{n+1})$ 和 $g\,(I_{n+1},Q_{n+1})$:

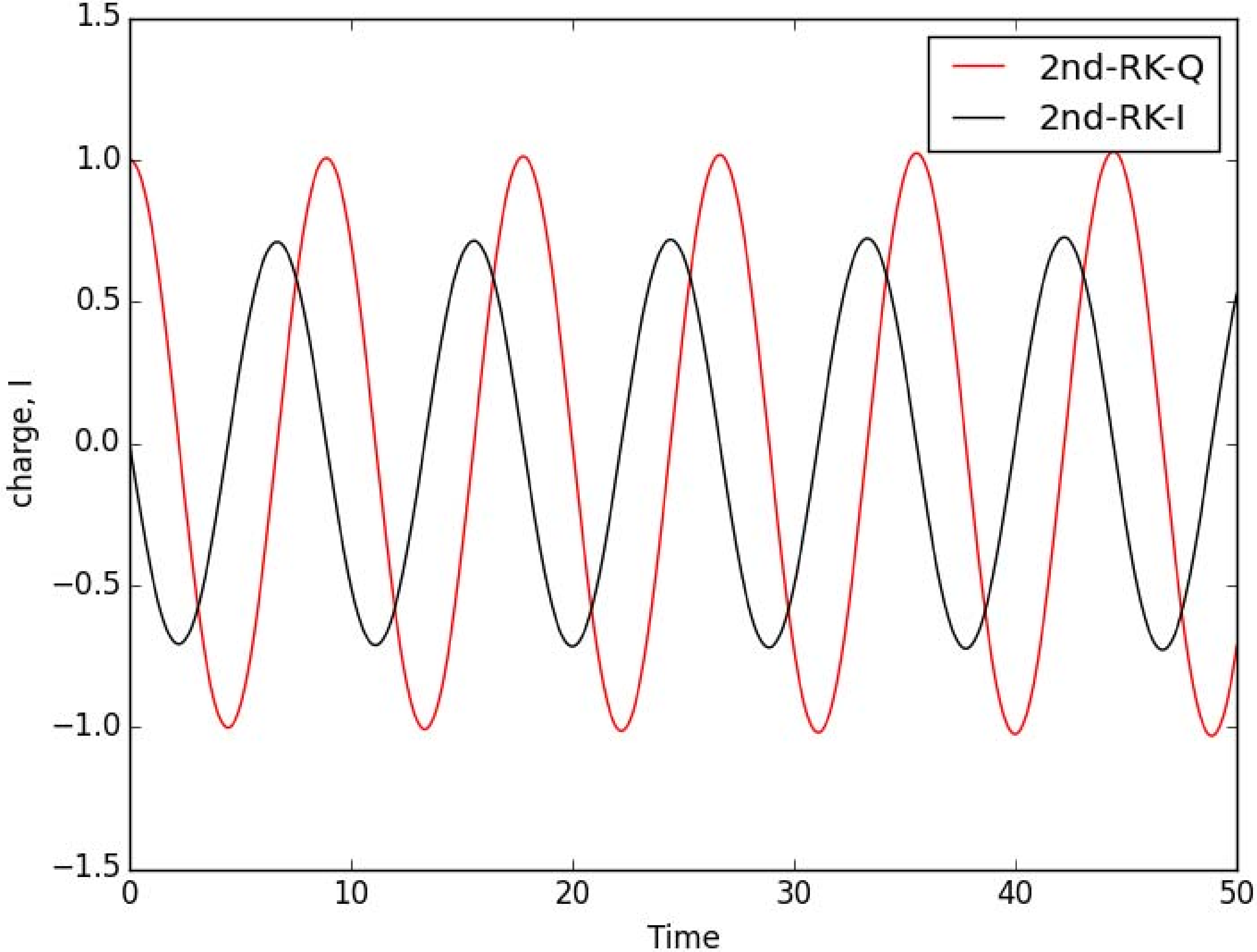
$$\begin{cases} f(I_{n+1}, Q_{n+1}) \\ g(I_{n+1}, Q_{n+1}) \end{cases}$$

3. 用改进的欧拉公式校正I_{n+1},Q_{n+1}:

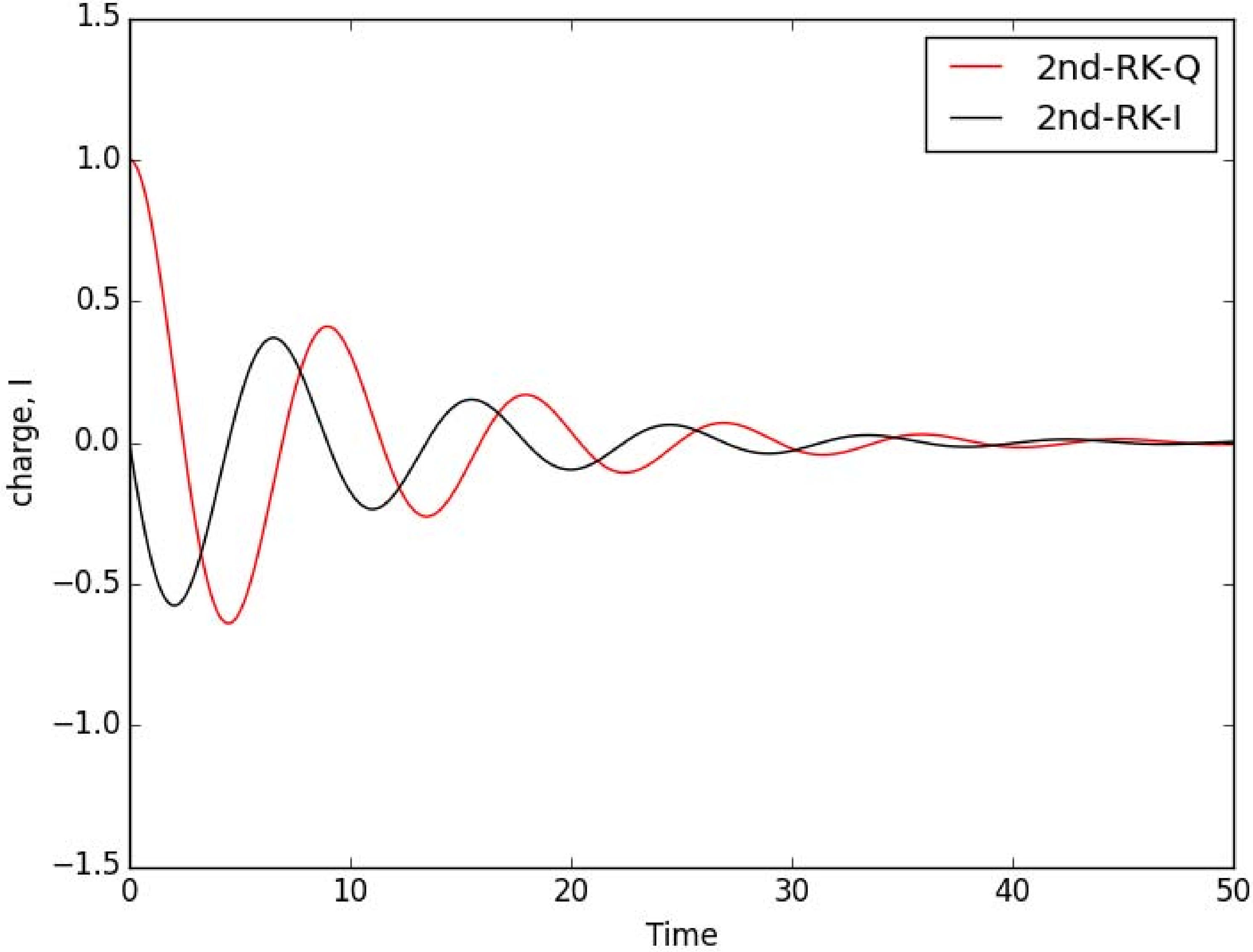
$$\begin{cases} I_{n+1} = I_n + \left(f(I_n, Q_n) + f(I_{n+1}, Q_{n+1})\right)\Delta t / 2 \\ Q_{n+1} = Q_n + \left(g(I_n, Q_n) + g(I_{n+1}, Q_{n+1})\right)\Delta t / 2 \end{cases}$$

计算结果:

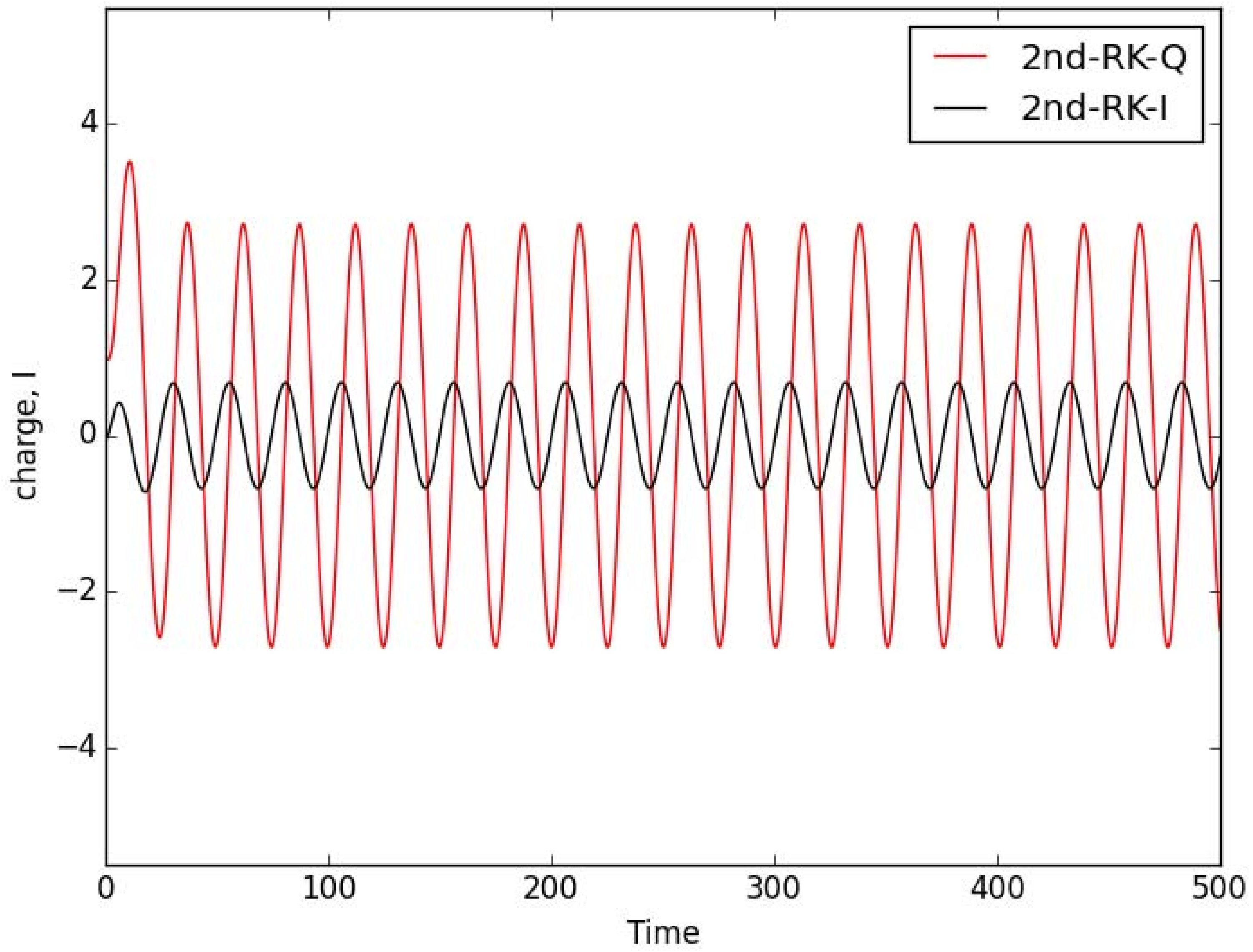
R=0; L=2, C=2, W=0, V0=0



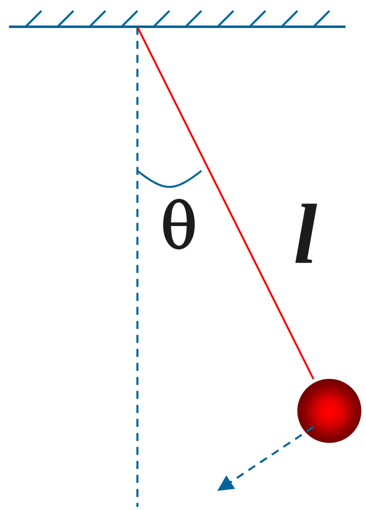
R=0.2; L=2, C=2, W=0, V0=0



R=1.0; L=9.0, C=8.0, W=0.25, V0=0.7



例：单摆



$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$ (小角度近似时)

解析解： $\theta = \theta_0 \sin(\Omega t + \phi_0)$

简谐振动

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

$$\begin{cases} \dfrac{d\omega}{dt} = f(\theta, \omega, t) = -\dfrac{g}{l}\theta \\ \dfrac{d\theta}{dt} = g(\theta, \omega, t) = \omega \end{cases}$$

4th order Runge-Kutta:

$$\omega_{n+1} = \omega_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\cdot\Delta t$$

$$\theta_{n+1} = \theta_n + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)\cdot\Delta t$$

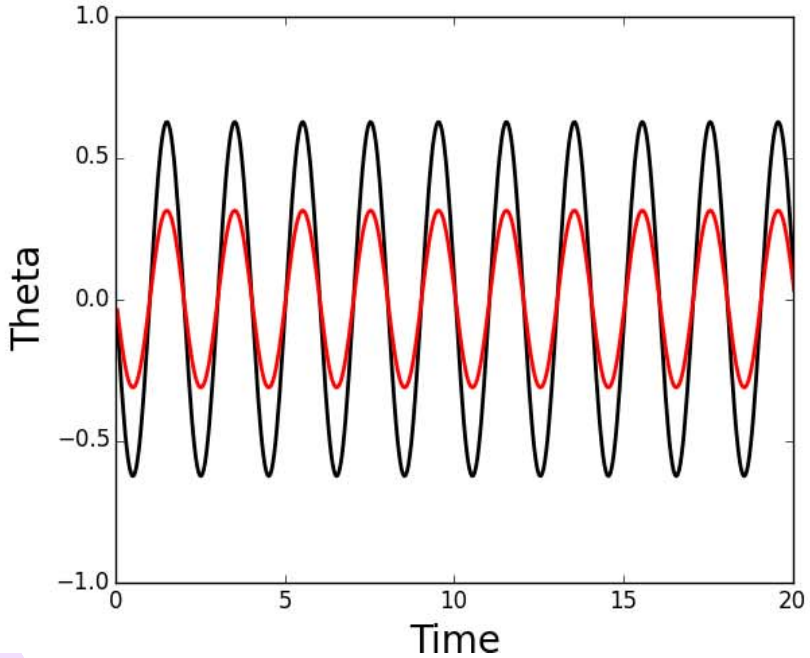
$$\begin{cases} \dfrac{d\omega}{dt} = f(\theta, \omega, t) = -\dfrac{g}{l}\theta \\ \dfrac{d\theta}{dt} = g(\theta, \omega, t) = \omega \end{cases}$$

$$\omega_{n+1} = \omega_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \cdot \Delta t$$

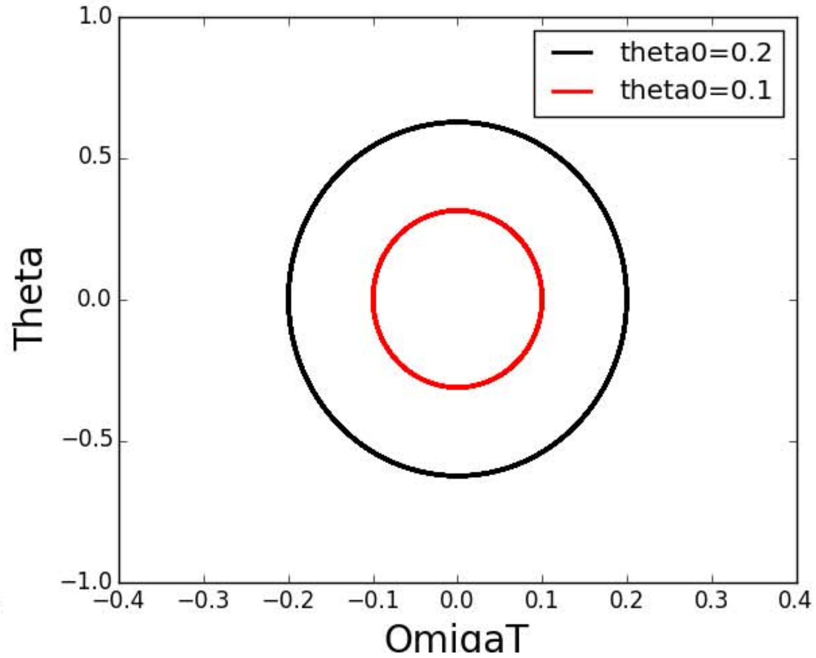
$$\theta_{n+1} = \theta_n + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \cdot \Delta t$$

$$\begin{cases} k_1 = -\dfrac{g}{l}\theta_n \\ l_1 = \omega_n \\ k_2 = -\dfrac{g}{l}\theta_{n+1/2} = -\dfrac{g}{l}(\theta_n + l_1 \cdot \Delta t / 2.) \\ l_2 = \omega_{n+1/2} = \omega_n + k_1 \cdot \Delta t / 2. \\ k_3 = -\dfrac{g}{l}\theta_{n+1/2} = -\dfrac{g}{l}(\theta_n + l_2 \cdot \Delta t / 2.) \\ l_3 = \omega_{n+1/2} = \omega_n + k_2 \cdot \Delta t / 2. \\ k_4 = -\dfrac{g}{l}\theta_{n+1} = -\dfrac{g}{l}(\theta_n + l_3 \cdot \Delta t) \\ l_4 = \omega_{n+1} = \omega_n + k_3 \cdot \Delta t \end{cases}$$

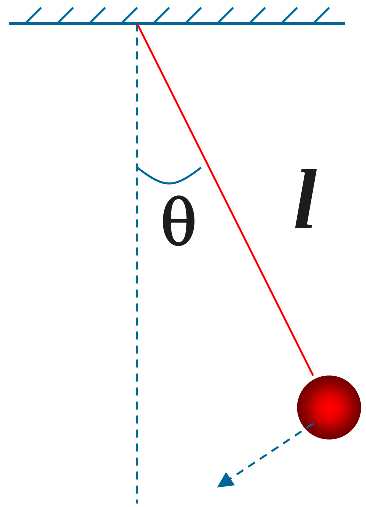
轨迹图：


1.0
0.5
0.0
−0.5
−1.0
Theta
0
5
10
15
20
Time

相空间轨迹图：


theta0=0.2
theta0=0.1
1.0
0.5
0.0
−0.5
−1.0
Theta
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4
OmigaT

例：单摆(有阻尼)



$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt}$$

阻尼项

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt}$$

$$\downarrow$$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\theta - q\omega \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\theta - q\omega \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

$$\omega_{n+1} = \omega_n + \frac{1}{6}(k1 + 2k2 + 2k3 + k4) \cdot \Delta t$$

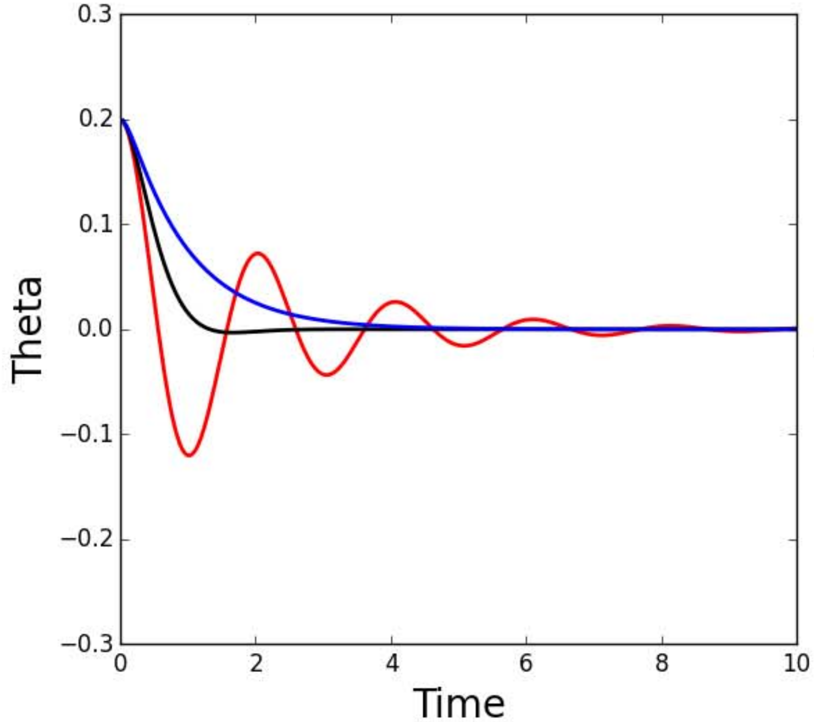
$$\theta_{n+1} = \theta_n + \frac{1}{6}(l1 + 2l2 + 2l3 + l4) \cdot \Delta t$$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\theta - q\omega \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

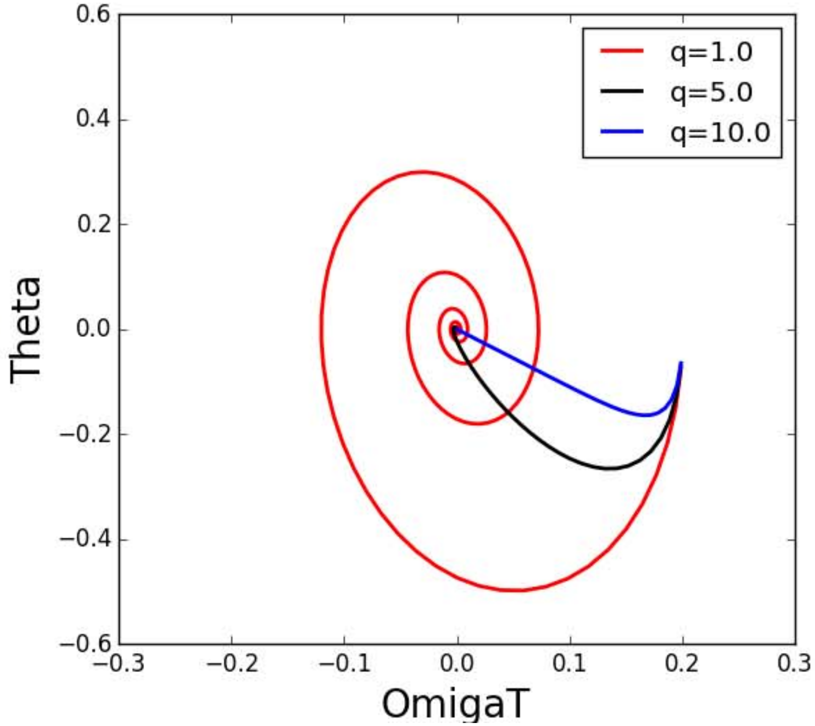
$$\omega_{n+1} = \omega_n + \frac{1}{6}(k1 + 2k2 + 2k3 + k4)\cdot\Delta t$$
$$\theta_{n+1} = \theta_n + \frac{1}{6}(l1 + 2l2 + 2l3 + l4)\cdot\Delta t$$

$$\begin{cases} k1 = -\dfrac{g}{l}\theta_n - q\omega_n \\ l1 = \omega_n \\ k2 = -\dfrac{g}{l}\theta_{n+1/2} - q\omega_{n+1/2} = -\dfrac{g}{l}(\theta_n + l1\cdot\Delta t/2.) - q(\omega_n + k1\cdot\Delta t/2) \\ l2 = \omega_{n+1/2} = \omega_n + k1\cdot\Delta t/2. \\ k3 = -\dfrac{g}{l}\theta_{n+1/2} - q\omega_{n+1/2} = -\dfrac{g}{l}(\theta_n + l2\cdot\Delta t/2.) - q(\omega_n + k2\cdot\Delta t/2) \\ l3 = \omega_{n+1/2} = \omega_n + k2\cdot\Delta t/2. \\ k4 = -\dfrac{g}{l}\theta_{n+1} - q\omega_{n+1} = -\dfrac{g}{l}(\theta_n + l3\cdot\Delta t) - q(\omega_n + k3\cdot\Delta t) \\ l4 = \omega_{n+1} = \omega_n + k3\cdot\Delta t \end{cases}$$

轨迹图：

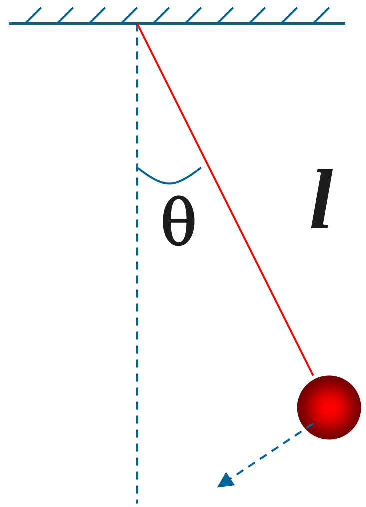


相空间轨迹图：



$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt}$$

例：单摆(阻尼+策动力)



$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt} + F\sin(Wt)$$

阻尼项　　　　策动力

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt} + F\sin(Wt)$$

$$\downarrow$$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\theta - q\omega + F\cdot\sin(Wt) \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\theta - q\omega + F\cdot\sin(Wt) \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

$$t = t + \Delta t$$

$$\omega_{n+1} = \omega_n + \frac{1}{6}(k1 + 2k2 + 2k3 + k4)\cdot\Delta t$$

$$\theta_{n+1} = \theta_n + \frac{1}{6}(l1 + 2l2 + 2l3 + l4)\cdot\Delta t$$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\theta - q\omega + F\cdot\sin(Wt) \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

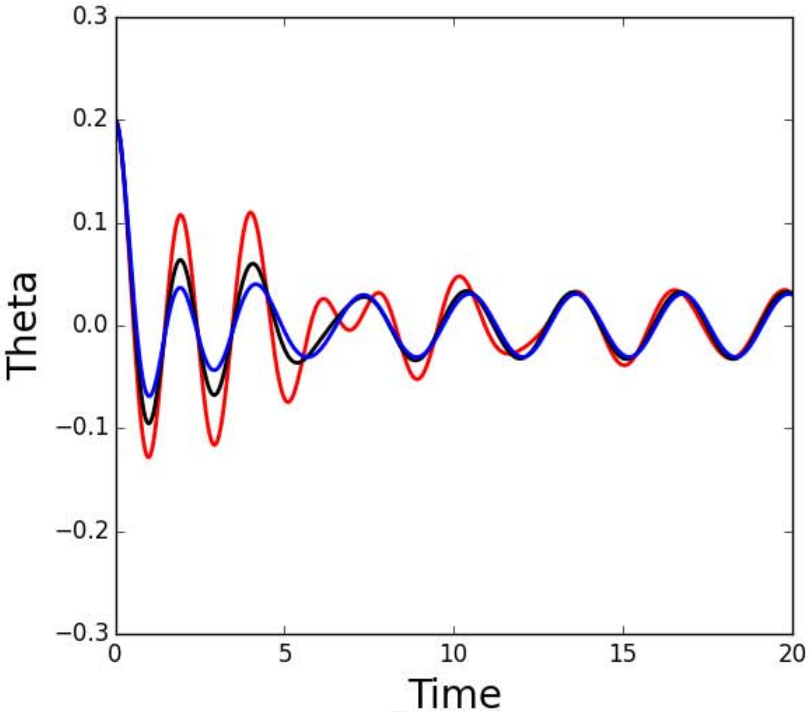
$$t = t + \Delta t$$

$$\omega_{n+1} = \omega_n + \frac{1}{6}(k1 + 2k2 + 2k3 + k4)\cdot\Delta t$$

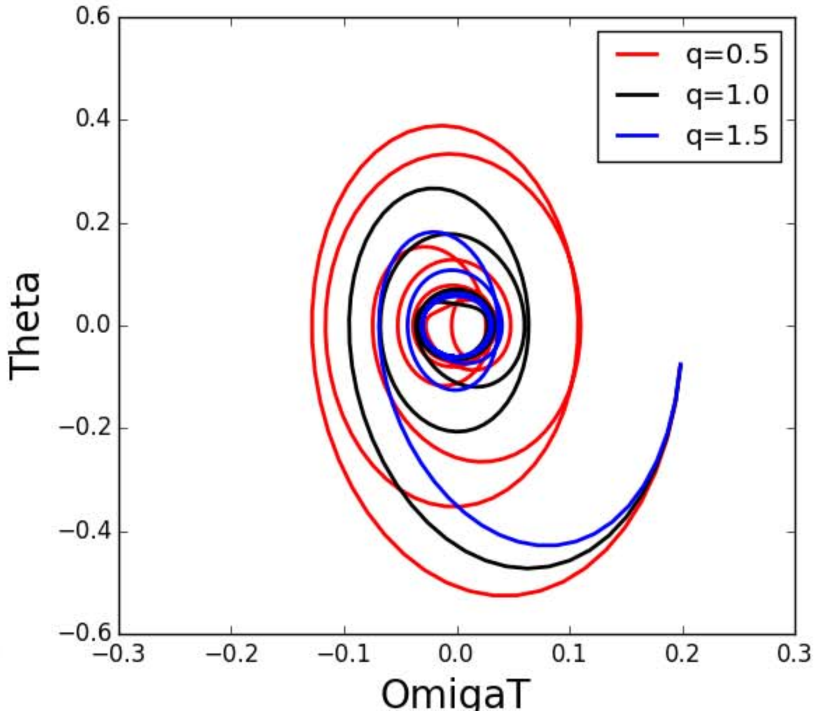
$$\theta_{n+1} = \theta_n + \frac{1}{6}(l1 + 2l2 + 2l3 + l4)\cdot\Delta t$$

$$\begin{cases} k1 = \dfrac{g}{l}\theta_n - q\omega_n + F\sin(Wt_n) \\ l1 = \omega_n \\ k2 = \dfrac{g}{l}(\theta_n + l1\cdot\Delta t/2.) - q(\omega_n + k1\cdot\Delta t/2) + F\sin(W(t_n + \Delta t/2)) \\ l2 = \omega_n + k1\cdot\Delta t/2. \\ k3 = \dfrac{g}{l}(\theta_n + l2\cdot\Delta t/2.) - q(\omega_n + k2\cdot\Delta t/2) + F\sin(W(t_n + \Delta t/2)) \\ l3 = \omega_n + k2\cdot\Delta t/2. \\ k4 = \dfrac{g}{l}(\theta_n + l3\cdot\Delta t) + q(\omega_n + k3\cdot\Delta t) + F\sin(W(t_n + \Delta t)) \\ l4 = \omega_n + k3\cdot\Delta t \end{cases}$$

轨迹图：

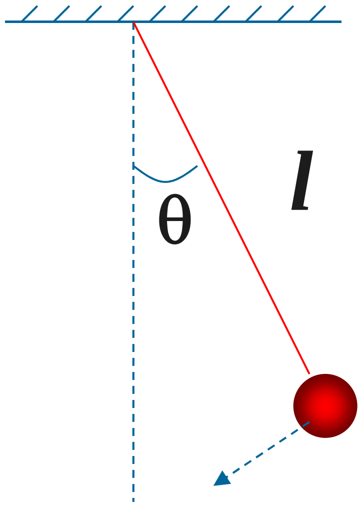


相空间轨迹图：



$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt} + F\sin(Wt)$$

例：单摆(阻尼+策动力+大角度)



$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F\sin(Wt)$$

阻尼项　　　　策动力

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F\sin(Wt)$$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\sin\theta - q\omega + F\cdot\sin(Wt) \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\sin\theta - q\omega + F\cdot\sin(Wt) \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

$$t = t + \Delta t$$

$$\omega_{n+1} = \omega_n + \frac{1}{6}(k1 + 2k2 + 2k3 + k4)\cdot\Delta t$$

$$\theta_{n+1} = \theta_n + \frac{1}{6}(l1 + 2l2 + 2l3 + l4)\cdot\Delta t$$

if： $\theta > \pi$

then $\theta = \theta - 2\pi$

if： $\theta < -\pi$

then $\theta = \theta + 2\pi$

$$\begin{cases} \dfrac{d\omega}{dt} = -\dfrac{g}{l}\sin\theta - q\omega + F\cdot\sin(Wt) \\ \dfrac{d\theta}{dt} = \omega \end{cases}$$

$$t = t + \Delta t$$

$$\omega_{n+1} = \omega_n + \frac{1}{6}(k1 + 2k2 + 2k3 + k4)\cdot\Delta t$$

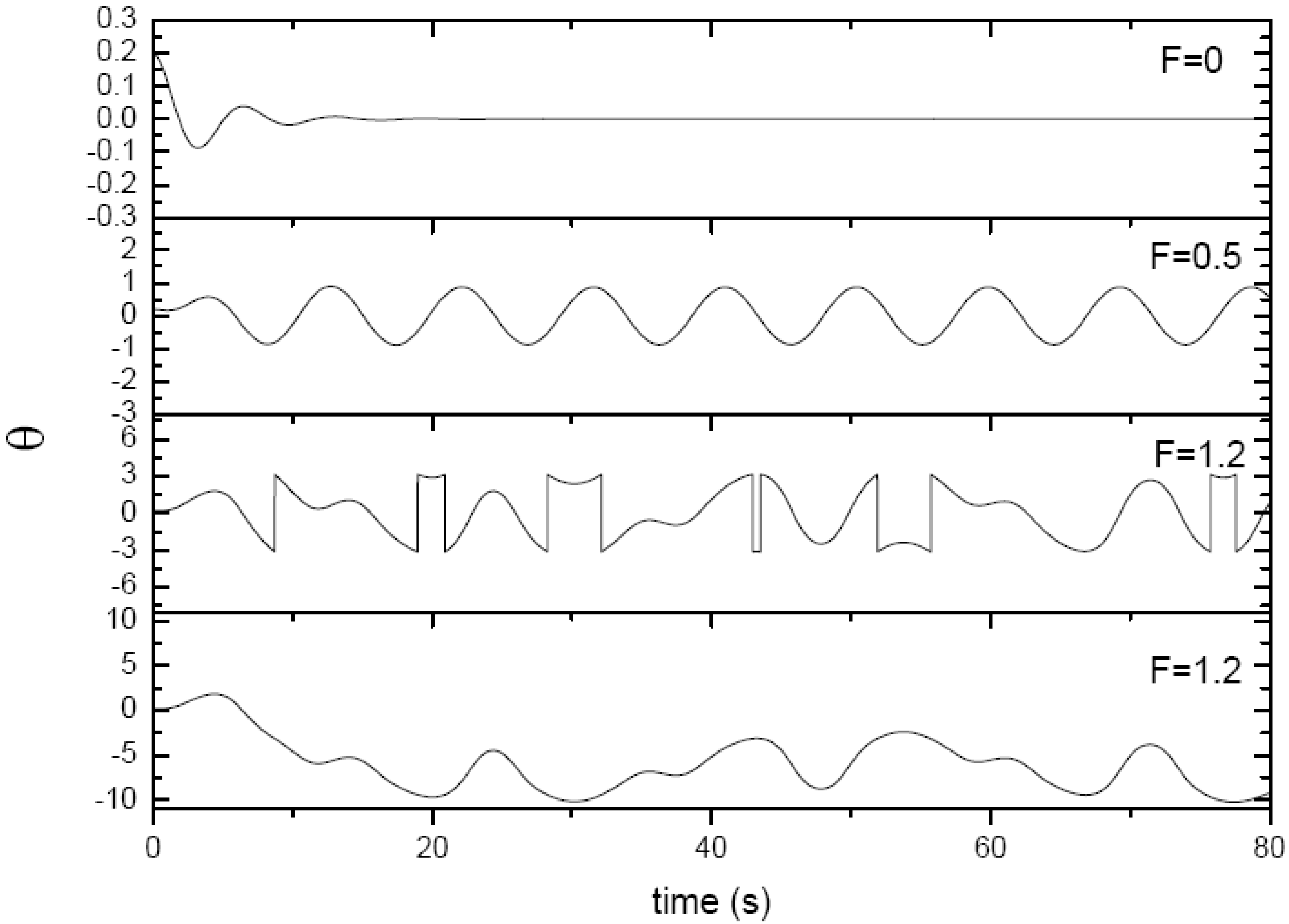
$$\theta_{n+1} = \theta_n + \frac{1}{6}(l1 + 2l2 + 2l3 + l4)\cdot\Delta t$$

$$\begin{cases} k1 = -\dfrac{g}{l}\sin\theta_n - q\omega_n + F\sin(Wt) \\ l1 = \omega_n \\ k2 = -\dfrac{g}{l}\sin(\theta_n + l1\cdot\Delta t/2.) - q(\omega_n + k1\cdot\Delta t/2) + F\sin(Wt) \\ l2 = \omega_n + k1\cdot\Delta t/2. \\ k3 = -\dfrac{g}{l}\sin(\theta_n + l2\cdot\Delta t/2.) - q(\omega_n + k2\cdot\Delta t/2) + F\sin(Wt) \\ l3 = \omega_n + k2\cdot\Delta t/2. \\ k4 = -\dfrac{g}{l}\sin(\theta_n + l3\cdot\Delta t) + q(\omega_n + k3\cdot\Delta t) + F\sin(Wt) \\ l4 = \omega_n + k3\cdot\Delta t \end{cases}$$

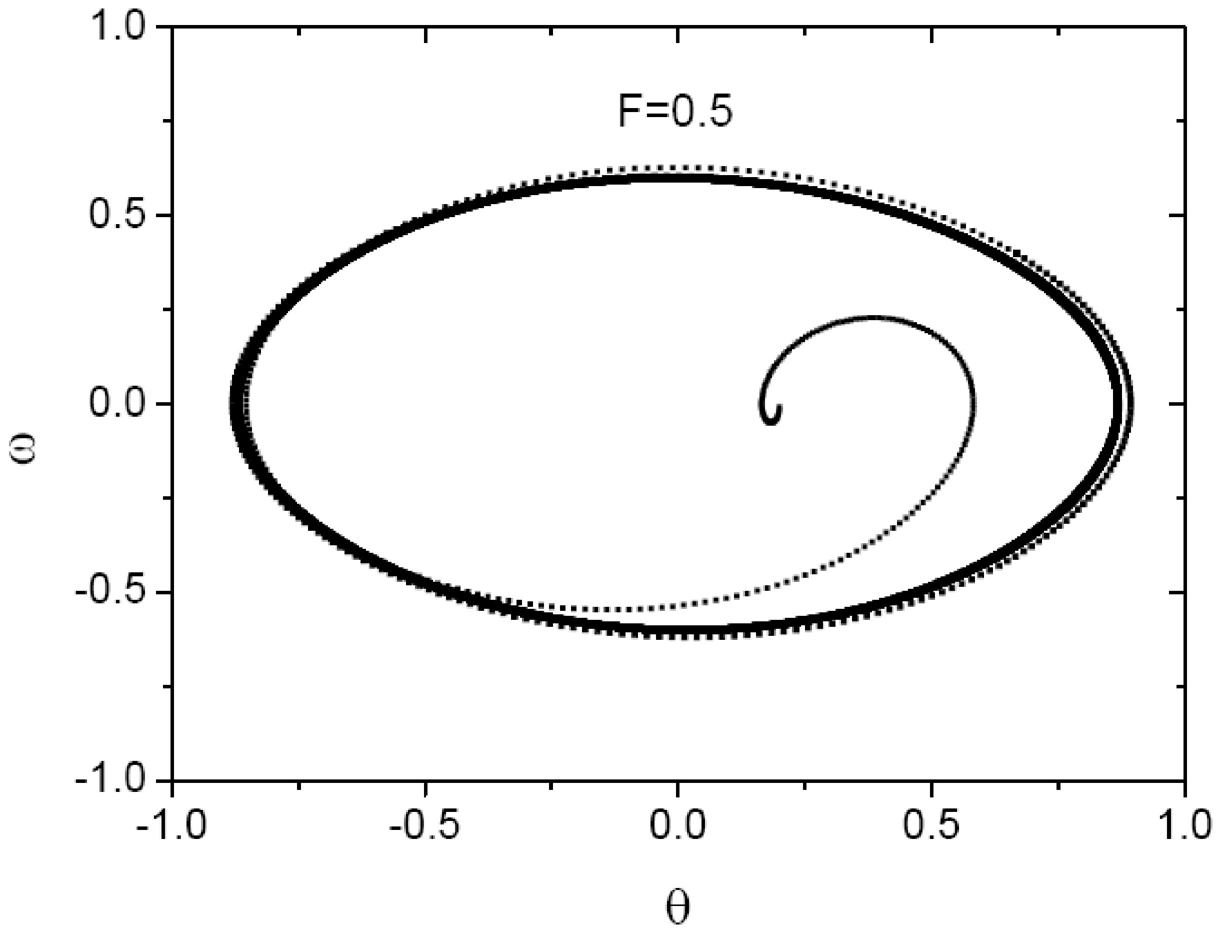
轨迹图：

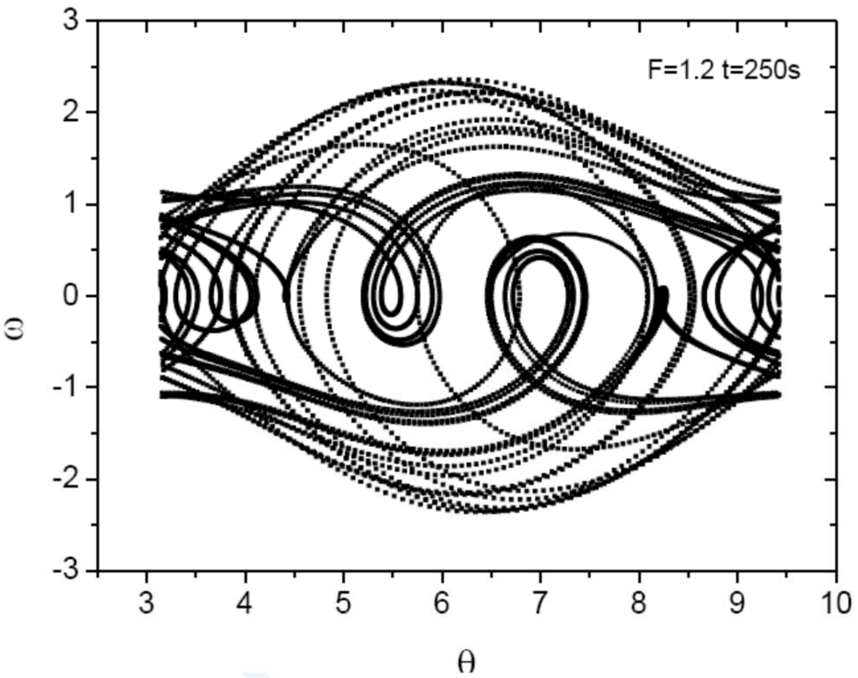
$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F\sin(Wt)$$

q=1/2, l=g=9.8, W=2/3, dt=0.04

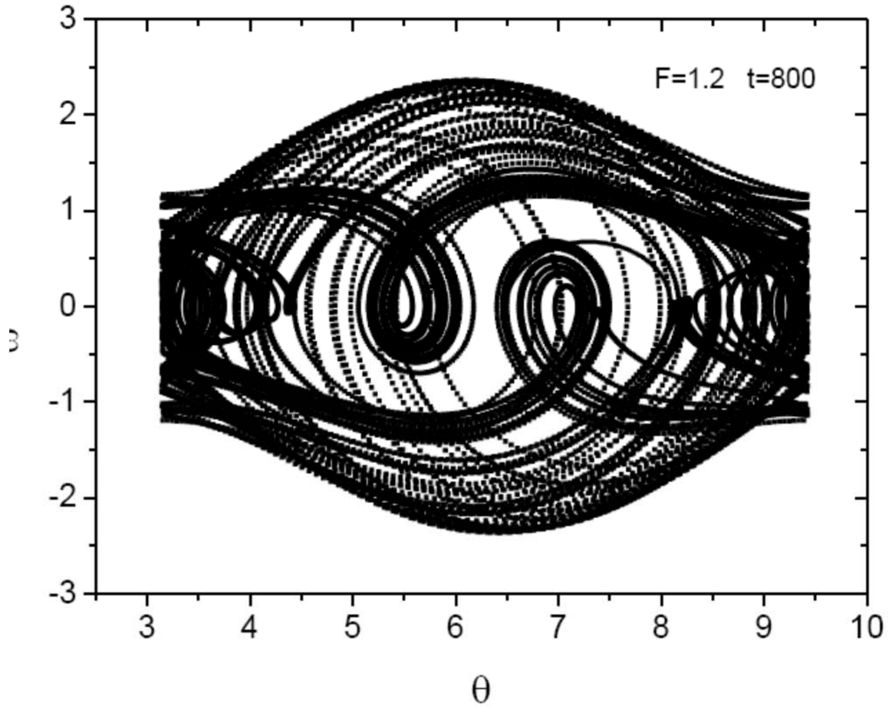


相空间轨迹图：



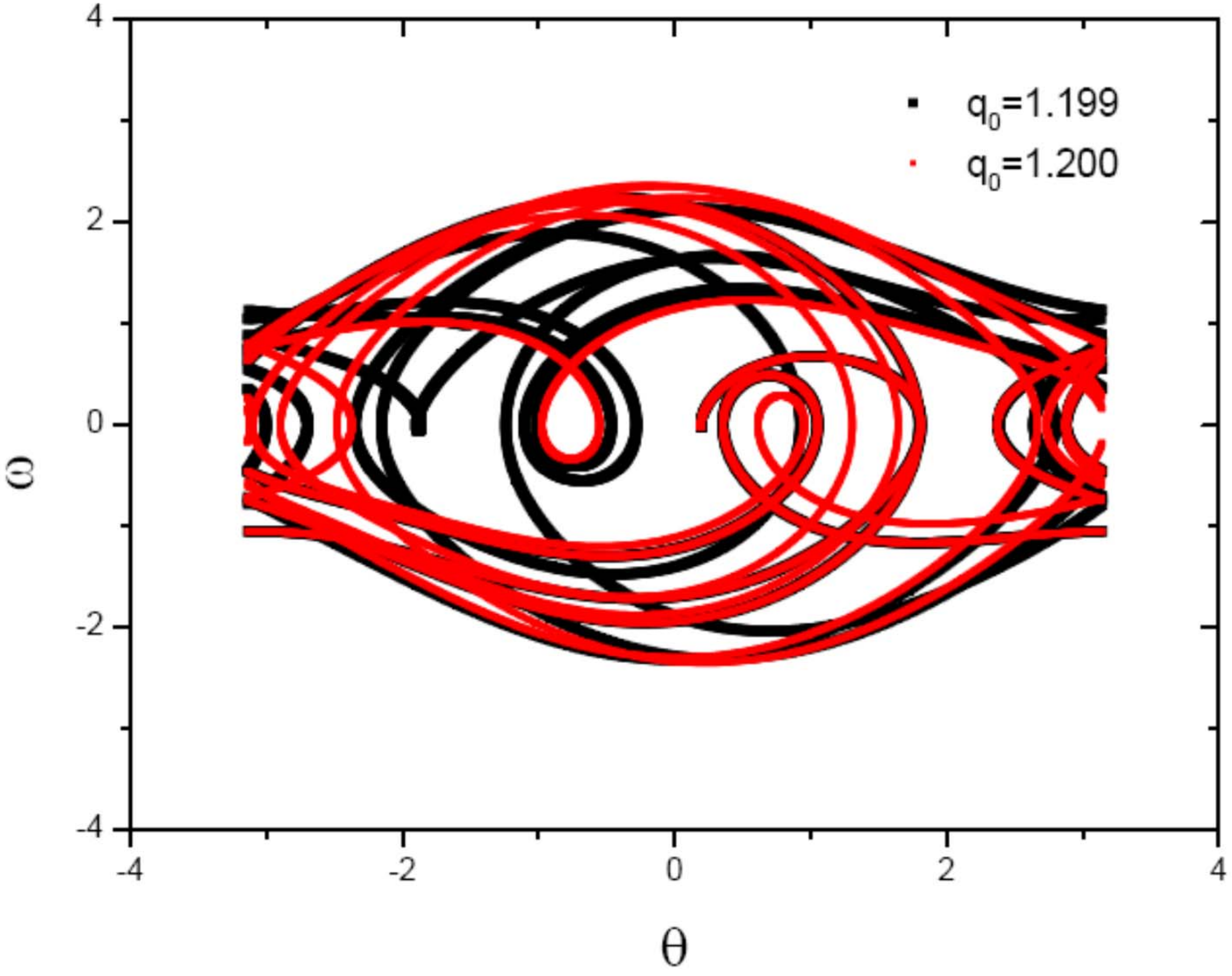


F=1.2 t=250s
ω
θ

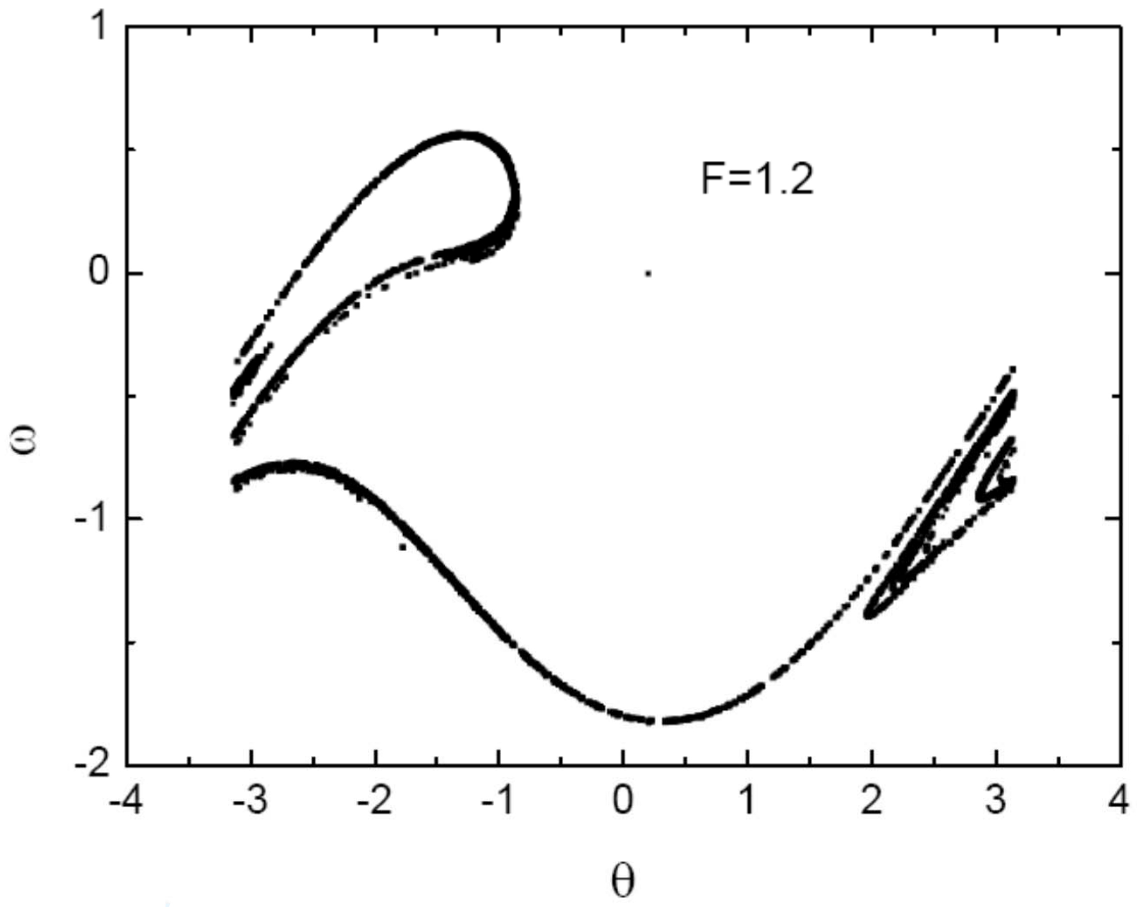


F=1.2 t=800
θ

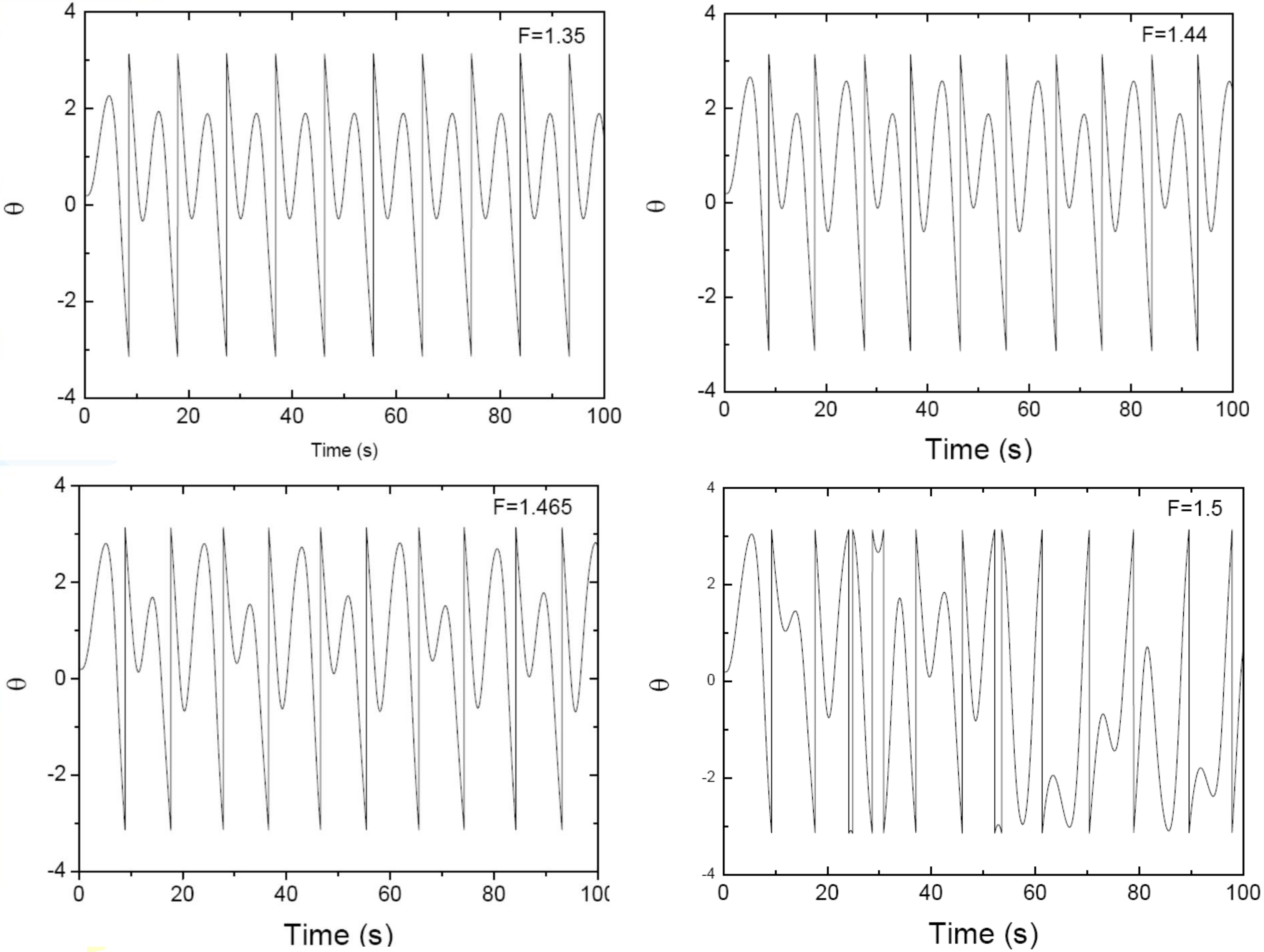
对条件的敏感性：$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta - q\frac{d\theta}{dt} + F\sin(Wt)$$



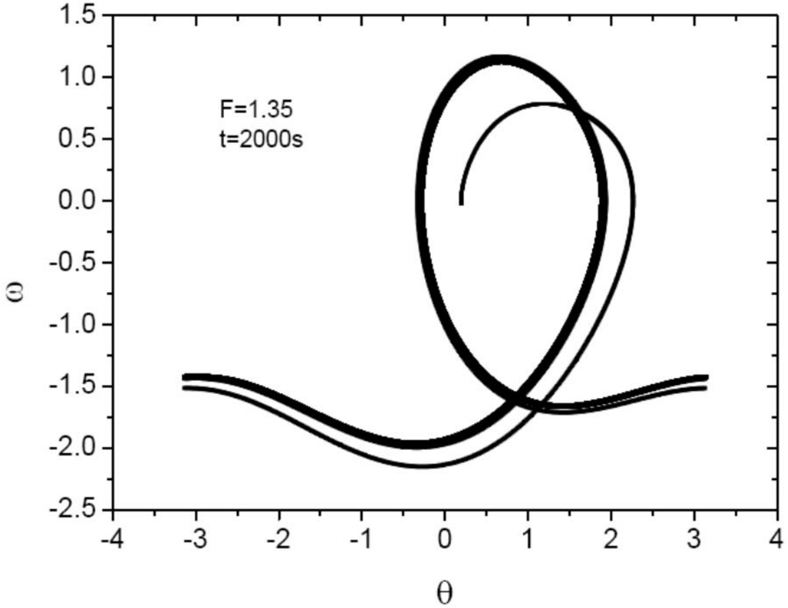
Poincare 截面：



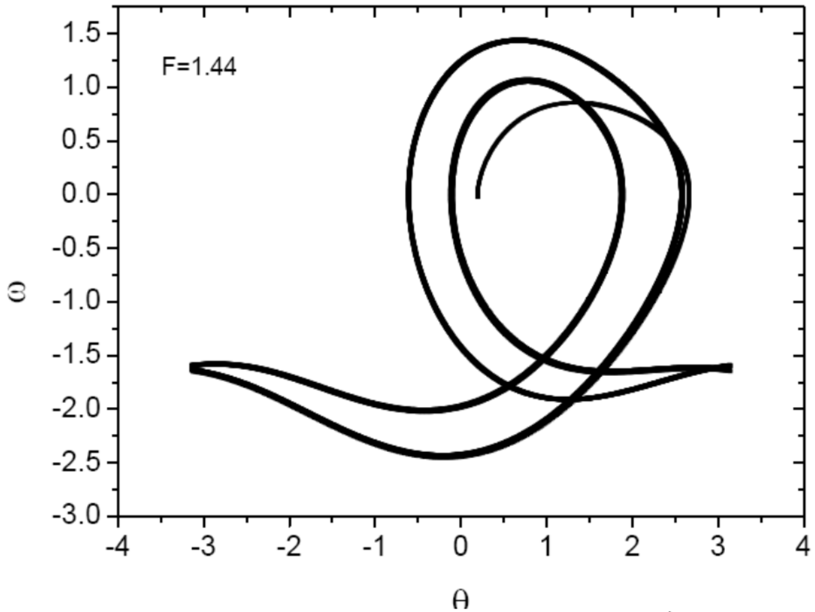
规则运动向混沌运动的过渡



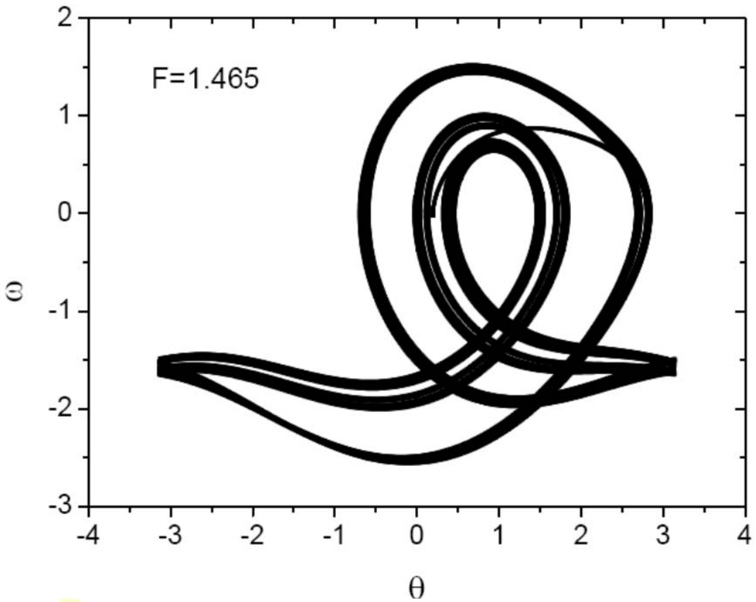
相空间轨迹图：



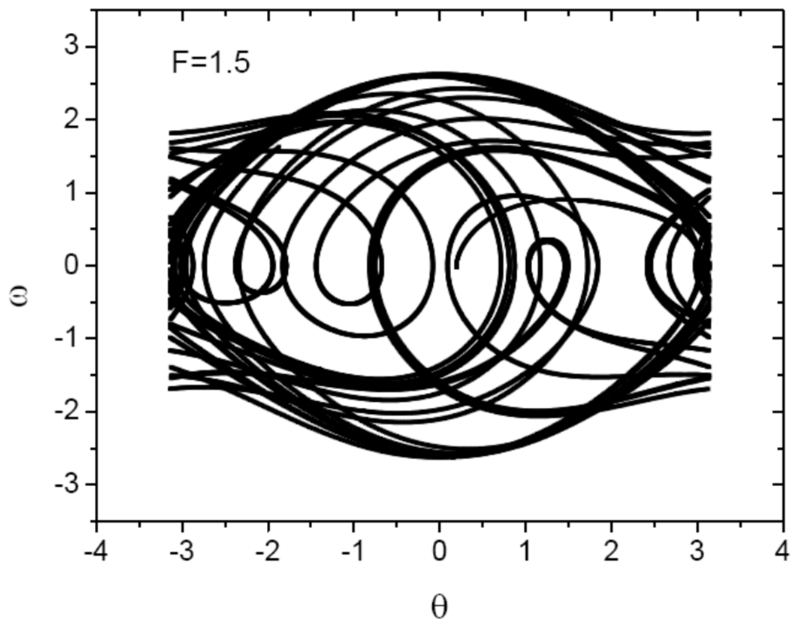
F=1.35
t=2000s
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-4
-3
-2
-1
0
1
2
3
4
θ
θ



F=1.44
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-4
-3
-2
-1
0
1
2
3
4
θ
θ

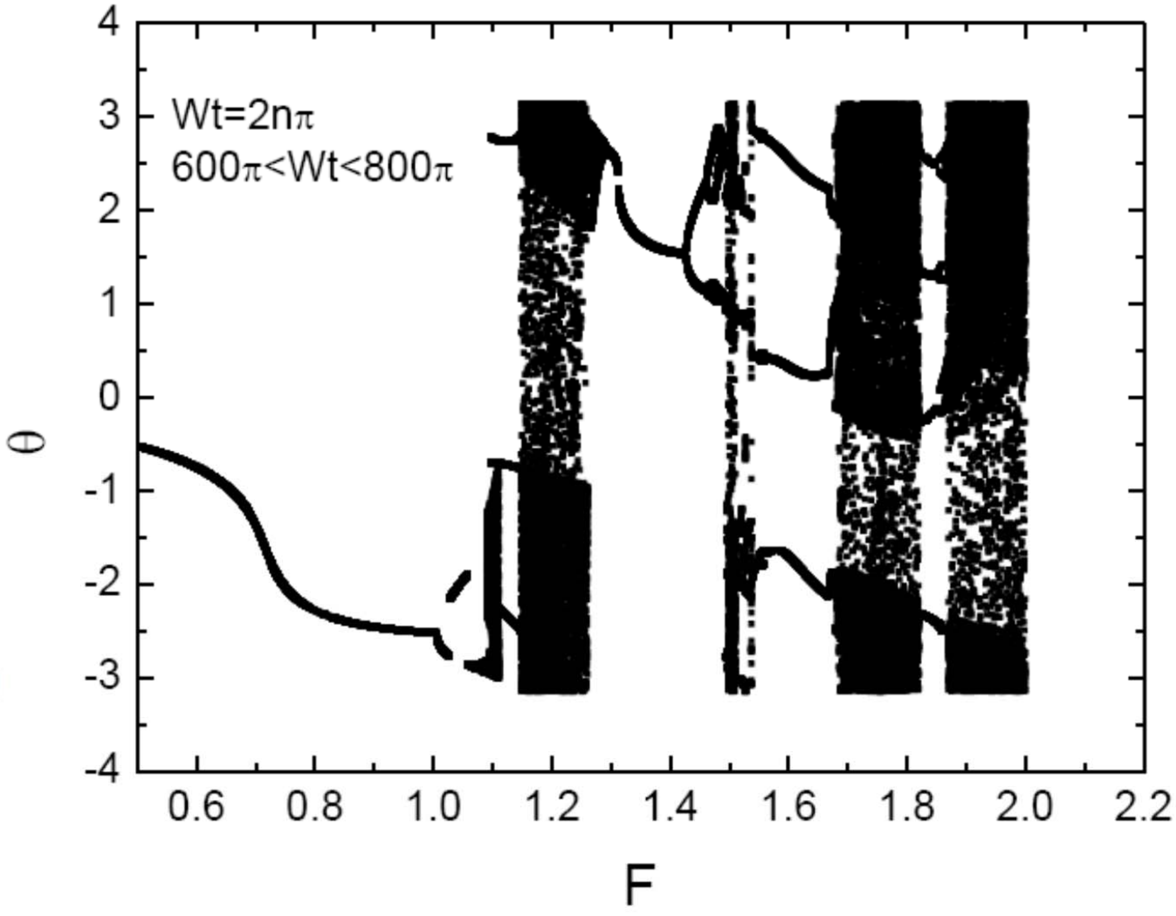


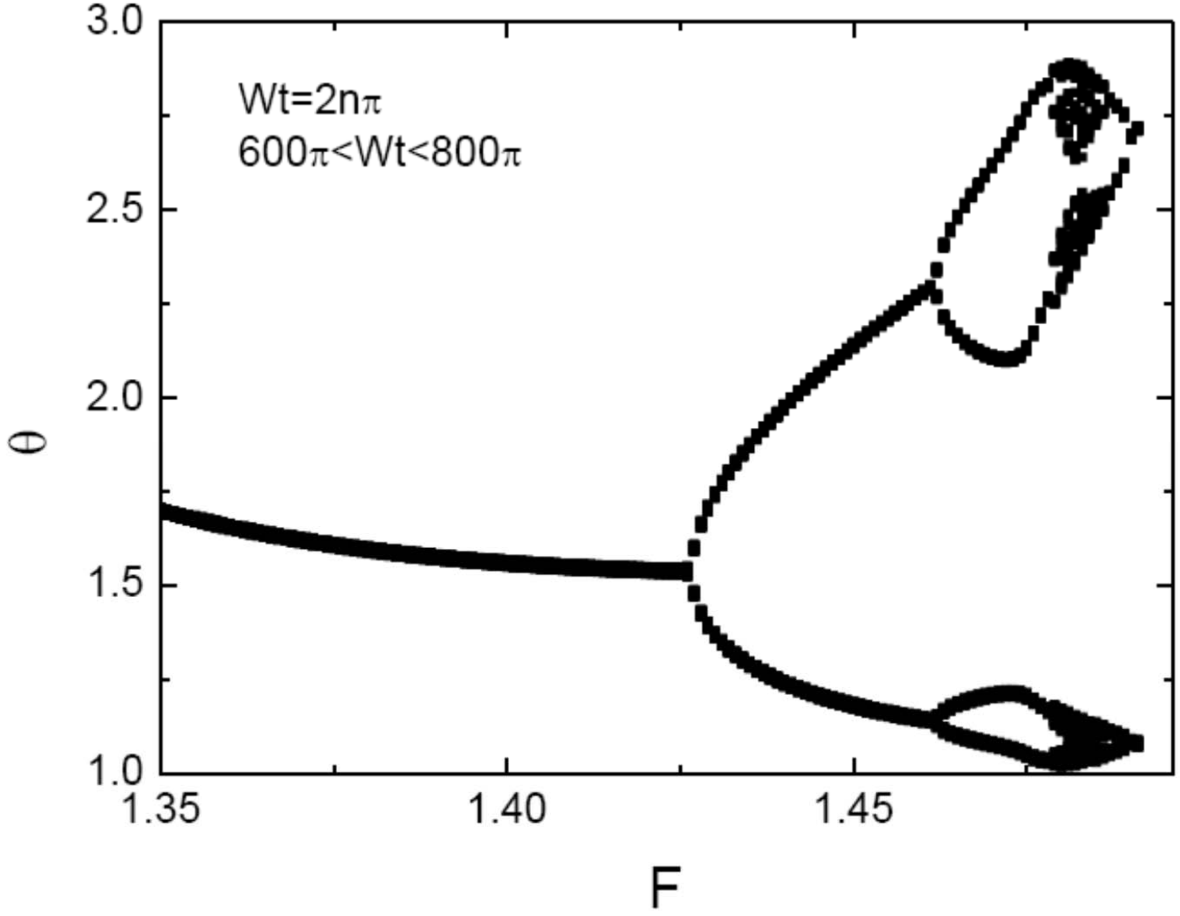
F=1.465
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4
θ
θ



F=1.5
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4
θ
θ

倍周期分岔





Wt=2nπ
600π<Wt<800π
3.0
2.5
2.0
1.5
1.0
θ
1.35
1.40
1.45
F

作业： Lorenz Model

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = -xz + rx - y$$

$$\frac{dz}{dt} = xy - bz$$

1. 当σ＝10；b=8/3；r=25时，画出z-x平面中的轨迹；
2. 当σ＝10；b=8/3时，改变r值，计算z-x平面中的轨迹;

要求：写出详细算法(流程图或计算公式)；

编写程序(Python)；

给出计算结果(图形)，并对结果加以分析讨论。