Applied Mathematics and Computation 218 (2012) 5580–5587

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Hierarchical least squares based iterative estimation algorithm for multivariable Box–Jenkins-like systems using the auxiliary model

Zhening Zhang [a], Jie Jia [b], Ruifeng Ding [b,c,*]

[a] Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi 214122, PR China
[b] Institute of Aerospace Information and Security Technology, Nanchang Hangkong University, Nanchang 330063, PR China
[c] School of Internet of Things Engineering, Jiangnan University, Wuxi 214122, PR China

ARTICLE INFO

Keywords:
System modelling
Least squares
Parameter estimation
Auxiliary model identification
Hierarchical identification
Multivariable Box–Jenkins-like model

ABSTRACT

This paper presents a hierarchical least squares iterative algorithm to estimate the parameters of multivariable Box–Jenkins-like systems by combining the hierarchical identification principle and the auxiliary model identification idea. The key is to decompose a multivariable systems into two subsystems by using the hierarchical identification principle. As there exist the unmeasurable noise-free outputs and noise terms in the information vector, the solution is using the auxiliary model identification idea to replace the unmeasurable variables with the outputs of the auxiliary model and the estimated residuals. A numerical example is given to show the performance of the proposed algorithm.

1. Introduction

Parameter estimation is a basic method for system modelling and signal filtering [1–4]. The recursive or iterative algorithms can estimate the system parameters or find the solutions of matrix equations [5–13]. Recently, parameter estimation of multivariable systems has received much attention in system identification [14–18]. For example, Ding et al. presented a gradient based and a least squares based iterative estimation algorithms for multi-input multi-output systems [19]; Xiang et al. proposed a hierarchical least squares algorithm for single-input multiple-output systems [20]; Han and Ding studied the convergence of the multi-innovation stochastic gradient algorithm for multi-input multi-output systems [18]; Bao et al. presented a least squares based iterative parameter estimation algorithm for multivariable controlled autoregressive moving average systems [21]. Liu et al. studied the convergence of stochastic gradient estimation algorithm for multivariable ARX-like systems [22].

Furthermore, Wang presented a least squares-based recursive and iterative estimation for output error moving average (OEMA) systems using data filtering [23]; Wang and Ding derived an input–output data filtering based recursive least squares parameter estimation for CARARMA systems [24]; Ding et al. proposed a gradient based and a least-squares based iterative identification methods for OE and OEMA systems [25] and several recursive and iterative identification methods for Hammerstein systems [26]. Other identification methods can be found for Hammerstein OEMA systems, Hammerstein OEAR systems and Wiener systems in [27–34] and for linear regressive models [35–40].

Some novel identification methods are born often, e.g., the multi-innovation identification methods for linear and pseudo-linear regression models [18,41–51], the auxiliary model based identification methods and the hierarchical identification methods for dual-rate and non-uniformly sampled-data systems or missing-data systems [49,52–64].

* Corresponding author at: School of Internet of Things Engineering, Jiangnan University, Wuxi 214122, PR China.
E-mail addresses: zhening_zhang@163.com (Z. Zhang), jiajie757@126.com (J. Jia), rfding@yahoo.cn (R. Ding).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2011.11.051

Hierarchical identification is based on the decomposition and can deal with parameter estimation for multivariable systems. Ding et al. proposed a hierarchical gradient iterative algorithm and a hierarchical least squares iterative algorithm for multivariable discrete-time systems [65,66]; Han et al. presented a hierarchical least-squares based iterative identification algorithm for multivariable CARMA-like model [67]; Zhang et al. studied the hierarchical gradient based iterative estimation algorithm for multivariable output error moving average systems (i.e., multivariable OEMA-like systems) [67,68]. On the basis of the work in [68], this paper considers the identification problem of general stochastic multivariable Box–Jenkins-like systems with an ARMA noise disturbance.

The paper is organized as follows. Section 2 describes the problem formulation related to the multivariable Box–Jenkins-like systems. Section 3 derives the hierarchical least squares iterative algorithm for the multivariable Box–Jenkins-like systems. Section 4 provides a numerical example to illustrate the proposed method. Finally, we offer some concluding remarks in Section 5.

2. System description

Recently, Han et al. presented a hierarchical least squares based iterative identification algorithm for multivariable CARMA-like systems with moving average noises [67]:

$$\alpha(z)\boldsymbol{y}(t) = \boldsymbol{Q}(z)\boldsymbol{u}(t) + D(z)\mathsf{v}(t).$$

Zhang et al. derived a hierarchical gradient based iterative parameter estimation algorithm for multivariable output error moving average systems (i.e., multivariable OEMA systems [68]):

$$\boldsymbol{y}(t) = \frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t) + D(z)\mathsf{v}(t).$$

On the basis of the work in [67,68], this paper considers the following multivariable Box–Jenkins-like system (i.e., multivariable BJ-like model) shown in Fig. 1,

$$\boldsymbol{y}(t) = \frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t) + \frac{D(z)}{C(z)}\mathsf{v}(t), \tag{1}$$

where $\boldsymbol{y}(t) \in \mathbb{R}^m$ is the system output vector, $\boldsymbol{u}(t) \in \mathbb{R}^r$ is the system input vector, $\mathsf{v}(t) \in \mathbb{R}^m$ is a stochastic white noise vector with zero mean and variance σ^2, $\alpha(z)$ is a monic polynomial in the unit backward shift operator z^{-1} $[z^{-1}y(t) = y(t-1)]$, $\boldsymbol{Q}(z)$ is a matrix polynomial in z^{-1}, $C(z)$ and $D(z)$ is a polynomial in z^{-1}, and are defined by

$$\begin{aligned}
\alpha(z) &:= 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \cdots + \alpha_n z^{-n}, \quad \alpha_i \in \mathbb{R}^1,\\
\boldsymbol{Q}(z) &:= \boldsymbol{Q}_1 z^{-1} + \boldsymbol{Q}_2 z^{-2} + \cdots + \boldsymbol{Q}_n z^{-n}, \quad \boldsymbol{Q}_i \in \mathbb{R}^{m\times r},\\
C(z) &:= 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{n_c} z^{-n_c}, \quad c_i \in \mathbb{R}^1,\\
D(z) &:= 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{n_d} z^{-n_d}, \quad d_i \in \mathbb{R}^1.
\end{aligned}$$

The objective of this paper is to derive a hierarchical least squares based iterative estimation algorithm to identify the parameters or parameter matrix (α_i, $\boldsymbol{Q}_i$, c_i, d_i) from given input–output data $\{\boldsymbol{u}(t), \boldsymbol{y}(t)\colon t = 1, 2, \ldots\}$.

3. The hierarchical least squares based iterative algorithm

Let

$$\boldsymbol{x}(t) := \frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t) \in \mathbb{R}^m, \tag{2}$$

$$\boldsymbol{w}(t) := \frac{D(z)}{C(z)}\mathsf{v}(t) \in \mathbb{R}^m. \tag{3}$$

Substituting (2) and (3) into (1) gives

$$\boldsymbol{y}(t) = \boldsymbol{x}(t) + \boldsymbol{w}(t). \tag{4}$$

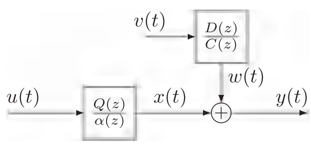

Fig. 1. The multivariable systems based on Box–Jenkins-like model.

Define the parameter vectors a, $\boldsymbol{c}$, $\boldsymbol{d}$ and $\boldsymbol{\vartheta}$, the parameter matrix h, the input information vector $\mathrm{u}(t)$ and the information matrices $\mathrm{U}(L)$ and $\mathrm{W}(L)$ as follows

$$\mathrm{a} := \begin{bmatrix} \mathrm{a}_1 \\ \mathrm{a}_2 \\ \vdots \\ \mathrm{a}_n \end{bmatrix} \in \mathbb{R}^{n}, \quad \boldsymbol{c} := \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n_c} \end{bmatrix} \in \mathbb{R}^{n_c}, \quad \boldsymbol{d} := \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{n_d} \end{bmatrix} \in \mathbb{R}^{n_d},$$

$$\boldsymbol{\vartheta} := \begin{bmatrix} \mathrm{a} \\ \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix} \in \mathbb{R}^{n+n_c+n_d},$$

$$\begin{aligned}
&\mathrm{h}^{\mathrm{T}} := [\boldsymbol{Q}_1, \boldsymbol{Q}_2, \cdots, \boldsymbol{Q}_n] \in \mathbb{R}^{m\times(nr)},\\
&\mathrm{u}(t) := \left[\boldsymbol{u}^{\mathrm{T}}(t-1), \boldsymbol{u}^{\mathrm{T}}(t-2), \cdots, \boldsymbol{u}^{\mathrm{T}}(t-n)\right]^{\mathrm{T}} \in \mathbb{R}^{(nr)},\\
&/(t) := [\boldsymbol{x}(t-1), \boldsymbol{x}(t-2), \cdots, \boldsymbol{x}(t-n)] \in \mathbb{R}^{m\times n},\\
&\mathrm{w}(t) := [/(t), \boldsymbol{w}(t-1), \boldsymbol{w}(t-2), \cdots, \boldsymbol{w}(t-n_c), -\mathrm{V}(t-1), -\mathrm{V}(t-2), \cdots, -\mathrm{V}(t-n_d)] \in \mathbb{R}^{m\times(n+n_c+n_d)}.
\end{aligned}$$

From (1)–(4), we have

$$\boldsymbol{x}(t) = -/(t)\mathrm{a} + \mathrm{h}^{\mathrm{T}}\mathrm{u}(t), \tag{5}$$

and

$$\boldsymbol{y}(t) + \mathrm{w}(t)\boldsymbol{\vartheta} = \mathrm{h}^{\mathrm{T}}\mathrm{u}(t) + \mathrm{V}(t). \tag{6}$$

The identification model in (6) contain both a parameter vector ϑ and a parameter matrix h. Define the matrix norm $\|\boldsymbol{X}\|^2 := \mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]$. Next, we use the hierarchical identification principle in [65,66] and define two criterion function,

$$J(\boldsymbol{\vartheta}, \mathrm{h}) := \sum_{j=1}^{L} \|\boldsymbol{y}(j) + \mathrm{w}(j)\boldsymbol{\vartheta} - \mathrm{h}^{\mathrm{T}}\mathrm{u}(j)\|^2$$

Let k = 1,2, ..., be an iteration variable, 1_k be the time-varying convergence factor, and $\hat{\boldsymbol{\vartheta}}_k := \begin{bmatrix} \hat{\mathrm{a}}_k \\ \hat{\boldsymbol{c}}_k \\ \hat{\boldsymbol{d}}_k \end{bmatrix}$ and $\hat{\mathrm{h}}_k$ be the parameter estimates of $\vartheta = \begin{bmatrix} \mathrm{a} \\ \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix}$ and h at iteration k, respectively. Using the Newton method and minimizing $J(\vartheta,\mathrm{h})$ leads to the iterative algorithm of estimating ϑ and h:

$$\begin{aligned}
\hat{\boldsymbol{\vartheta}}_k &= \hat{\boldsymbol{\vartheta}}_{k-1} - 1_k \left[\frac{\partial^2 J(\hat{\boldsymbol{\vartheta}}_{k-1}, \hat{\mathrm{h}}_{k-1})}{\partial\boldsymbol{\vartheta}\partial\boldsymbol{\vartheta}^{\mathrm{T}}}\right]^{-1} \frac{\partial J(\hat{\boldsymbol{\vartheta}}_{k-1}, \hat{\mathrm{h}}_{k-1})}{\partial\boldsymbol{\vartheta}}\\
&= \hat{\boldsymbol{\vartheta}}_{k-1} - 1_k \left[\sum_{t=1}^{L} \mathrm{w}^{\mathrm{T}}(t)\mathrm{w}(t)\right]^{-1} \sum_{t=1}^{L} \mathrm{w}^{\mathrm{T}}(t)\left[\boldsymbol{y}(t) + \mathrm{w}(t)\hat{\boldsymbol{\vartheta}}_{k-1} - \hat{\mathrm{h}}_{k-1}^{\mathrm{T}}\mathrm{u}(t)\right],
\end{aligned} \tag{7}$$

$$\begin{aligned}
\hat{\mathrm{h}}_k &= \hat{\mathrm{h}}_{k-1} - 1_k \left[\frac{\partial^2 J(\hat{\boldsymbol{\vartheta}}_{k-1}, \hat{\mathrm{h}}_{k-1})}{\partial\mathrm{h}\partial\mathrm{h}^{\mathrm{T}}}\right]^{-1} \left[\frac{\partial J(\hat{\boldsymbol{\vartheta}}_{k-1}, \hat{\mathrm{h}}_{k-1})}{\partial\mathrm{h}}\right]^{\mathrm{T}}\\
&= \hat{\mathrm{h}}_{k-1} + 1_k \left[\sum_{t=1}^{L} \mathrm{u}(t)\mathrm{u}^{\mathrm{T}}(t)\right]^{-1} \sum_{t=1}^{L} \mathrm{u}(t)\left[\boldsymbol{y}(t) + \mathrm{w}(t)\hat{\boldsymbol{\vartheta}}_{k-1} - \hat{\mathrm{h}}_{k-1}^{\mathrm{T}}\mathrm{u}(t)\right]^{\mathrm{T}}.
\end{aligned} \tag{8}$$

However, the information vector $\mathrm{w}(t)$ contains the unknown inner vectors $\boldsymbol{x}(t-i)$ and the unknown noise vectors $\boldsymbol{w}(t-i)$ and $\boldsymbol{v}(t-i)$, the algorithm in (7) and (8) cannot be implemented. The solution here is based on the auxiliary model identification idea to construct an auxiliary model [52,55,63], and the $\boldsymbol{x}(t-i)$ is replaced with the output $\hat{\boldsymbol{x}}_k(t-i)$ of the auxiliary model (refer to $\hat{\boldsymbol{x}}_k(t-i)$ to the estimates of $\boldsymbol{x}(t-i)$ at iteration k), and the unknown noise terms $\boldsymbol{w}(t-i)$ and $\boldsymbol{v}(t-i)$ are replaced with their estimates $\hat{\boldsymbol{w}}_k(t-i)$ and $\hat{\mathrm{V}}_k(t-i)$, respectively. Let

$$\begin{aligned}
\hat{\mathrm{w}}_k(t) := [&\hat{/}_k(t), \hat{\boldsymbol{w}}_{k-1}(t-1), \hat{\boldsymbol{w}}_{k-1}(t-2), \cdots, \hat{\boldsymbol{w}}_{k-1}(t-n_c),\\
&-\hat{\mathrm{V}}_{k-1}(t-1), -\hat{\mathrm{V}}_{k-1}(t-2), \cdots, -\hat{\mathrm{V}}_{k-1}(t-n_d)] \in \mathbb{R}^{m\times(n+n_c+n_d)},\\
\hat{/}_k(t) := [&\hat{\boldsymbol{x}}_{k-1}(t-1), \hat{\boldsymbol{x}}_{k-1}(t-2), \cdots, \hat{\boldsymbol{x}}_{k-1}(t-n)] \in \mathbb{R}^{m\times n}.
\end{aligned}$$

Replacing $/(t)$, h and a in (5) with $\hat{/}_k(t)$, $\hat{\mathrm{h}}_k(t)$ and $\hat{\mathrm{a}}_k$ gives the estimate $\hat{\boldsymbol{x}}_k(t)$ of $x(t)$ can be computed by

$$\hat{\boldsymbol{x}}_k(t) = -\hat{/}_k(t)\hat{\mathrm{a}}_k + \hat{\mathrm{h}}_k^{\mathrm{T}}\mathrm{u}(t),$$

From (4) and (6), we have

$$\boldsymbol{w}(t) = \boldsymbol{y}(t) - \boldsymbol{x}(t), \tag{9}$$

$$\mathrm{V}(t) = \boldsymbol{y}(t) + \mathrm{w}(t)\boldsymbol{\vartheta} - \mathrm{h}^{\mathrm{T}}\mathrm{u}(t). \tag{10}$$

Replacing $\boldsymbol{x}(t)$ in (9) with $\hat{\boldsymbol{x}}_k(t)$, and replacing $\mathrm{w}(t)$, $\boldsymbol{\vartheta}$, and h in (10) with $\hat{\mathrm{w}}_k(t)$, $\hat{\boldsymbol{\vartheta}}_k$, and $\hat{\mathrm{h}}_k$, respectively, the estimates $\hat{\boldsymbol{w}}_k(t)$ of $\hat{\boldsymbol{w}}(t)$ and the estimates $\hat{\mathrm{V}}_k(t)$ of $\hat{\mathrm{V}}(t)$ can be computed by

$$\hat{\boldsymbol{w}}_k(t) = \boldsymbol{y}(t) - \hat{\boldsymbol{x}}_k(t),$$

$$\hat{\mathrm{V}}_k(t) = \boldsymbol{y}(t) + \hat{\mathrm{w}}_k(t)\hat{\boldsymbol{\vartheta}}_k - \hat{\mathrm{h}}_k^{\mathrm{T}}\mathrm{u}(t).$$

The convergency factor l_k is taken as $\mathrm{l}_k = 1$, referring to as Lemma 2 in [7].

Replacing $\mathrm{w}(t)$ in (7) with $\hat{\mathrm{w}}_k(t)$, we can obtain the hierarchical least squares based iterative algorithm for multivariable Box–Jenkins-like systems (the BJ-like-HLSI algorithm for short)

$$\hat{\boldsymbol{\vartheta}}_k = \hat{\boldsymbol{\vartheta}}_{k-1} - \left[\sum_{t=1}^{L}\hat{\mathrm{w}}_k^{\mathrm{T}}(t)\hat{\mathrm{w}}_k(t)\right]^{-1}\sum_{t=1}^{L}\hat{\mathrm{w}}_k^{\mathrm{T}}(i)\left[\boldsymbol{y}(t) + \hat{\mathrm{w}}_k(t)\hat{\boldsymbol{\vartheta}}_{k-1} - \hat{\mathrm{h}}_{k-1}^{\mathrm{T}}\mathrm{u}(t)\right], \tag{11}$$

$$\hat{\mathrm{h}}_k = \hat{\mathrm{h}}_{k-1} + \left[\sum_{t=1}^{L}\mathrm{u}(i)\mathrm{u}^{\mathrm{T}}(t)\right]^{-1}\sum_{t=1}^{L}\mathrm{u}(t)\left[\boldsymbol{y}(t) + \hat{\mathrm{w}}_k(t)\hat{\boldsymbol{\vartheta}}_{k-1} - \hat{\mathrm{h}}_{k-1}^{\mathrm{T}}\mathrm{u}(t)\right]^{\mathrm{T}}, \tag{12}$$

$$\mathrm{u}(t) = \left[\boldsymbol{u}^{\mathrm{T}}(t-1), \boldsymbol{u}^{\mathrm{T}}(t-2), \cdots, \boldsymbol{u}^{\mathrm{T}}(t-n)\right]^{\mathrm{T}}, \tag{13}$$

$$\hat{\mathrm{w}}_k(t) = \left[\hat{/}_k(t), \hat{\boldsymbol{w}}_{k-1}(t-1), \hat{\boldsymbol{w}}_{k-1}(t-2), \cdots, \hat{\boldsymbol{w}}_{k-1}(t-n_c), -\hat{\mathrm{V}}_{k-1}(t-1), -\hat{\mathrm{V}}_{k-1}(t-2), \cdots, -\hat{\mathrm{V}}_{k-1}(t-n_d)\right], \tag{14}$$

$$\hat{/}_k(t) = [\hat{\boldsymbol{x}}_{k-1}(t-1), \hat{\boldsymbol{x}}_{k-1}(t-2), \cdots, \hat{\boldsymbol{x}}_{k-1}(t-n)], \tag{15}$$

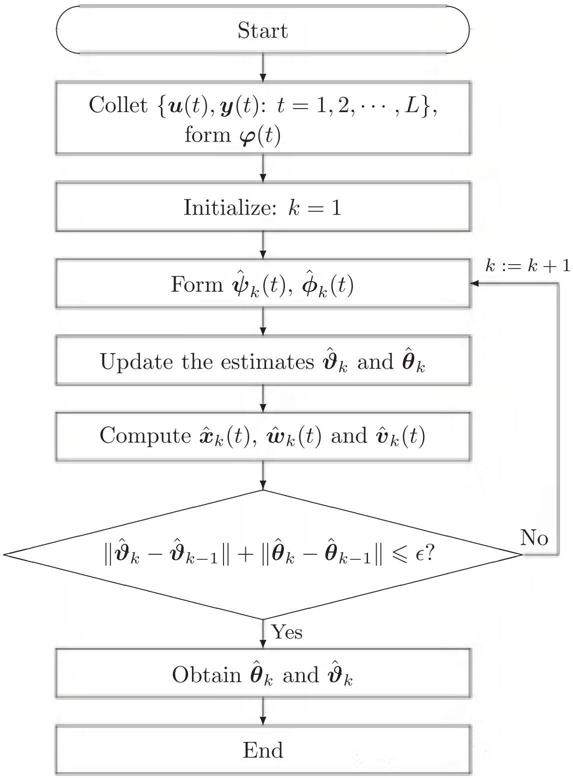

Fig. 2. The flowchart of computing the BJ-like-HLSI estimates $\hat{\vartheta}_k$ and $\hat{\mathrm{h}}_k$.

$$\hat{\boldsymbol{\vartheta}}_k = \begin{bmatrix} \hat{\mathrm{a}}_k \\ \hat{\boldsymbol{c}}_k \\ \hat{\boldsymbol{d}}_k \end{bmatrix}, \tag{16}$$

$$\hat{\boldsymbol{x}}_k(t) = -\hat{\ell}_k(t)\hat{\mathrm{a}}_k + \hat{\mathrm{h}}_k^{\mathrm{T}}\mathrm{u}(t), \tag{17}$$

$$\hat{\boldsymbol{w}}_k(t) = \boldsymbol{y}(t) - \hat{\boldsymbol{x}}_k(t), \tag{18}$$

$$\hat{\mathrm{V}}_k(t) = \boldsymbol{y}(t) + \hat{\mathrm{w}}_k(t)\hat{\boldsymbol{\vartheta}}_k - \hat{\mathrm{h}}_k^{\mathrm{T}}\mathrm{u}(t). \tag{19}$$

To summarize, we list the steps of computing the parameter estimation $\hat{\boldsymbol{\vartheta}}_k$ and $\hat{\mathrm{h}}_k$ in the HLSI algorithm as follows.

1. Collect the input/output data $\{u(t), y(t): t = 1, 2, \ldots, L\}$ (L is the data length), give a small positive number ϵ and form $\mathrm{u}(t)$ by (13).
2. Let $k = 1$, $\hat{\boldsymbol{x}}_0(t) = \mathbf{1}_{m\times 1}/p_0$, $\hat{\boldsymbol{w}}_0(t) = \mathbf{1}_{m\times 1}/p_0$, $\hat{\mathrm{V}}_0(t) = \mathbf{1}_{m\times 1}/p_0$, $\hat{\boldsymbol{\vartheta}}_k = \mathbf{1}_{n+n_c+n_d}/p_0$, $\hat{\mathrm{h}}_k^{\mathrm{T}} = \mathbf{1}_{m\times(nr)}/p_0$, with $\mathbf{1}_{m\times nr}$ being an $m \times nr$ matrix whose elements are 1.
3. Form $\hat{\ell}_k(t)$ by (15) and $\hat{\mathrm{w}}_k(t)$ by (14).
4. Update the estimates $\hat{\boldsymbol{\vartheta}}_k$ by (11) and $\hat{\mathrm{h}}_k$ by (12).
5. Read $\hat{\mathrm{a}}_k$ from $\hat{\boldsymbol{\vartheta}}_k$ by (16), and compute $\hat{\boldsymbol{x}}_k(t), \hat{\boldsymbol{w}}_k(t)$ and $\hat{\mathrm{V}}_k(t)$ by (17) to (19), respectively.
6. Compare $\hat{\boldsymbol{\vartheta}}_k$ with $\hat{\boldsymbol{\vartheta}}_{k-1}$, and $\hat{\mathrm{h}}_k$ with $\hat{\mathrm{h}}_{k-1}$, respectively, if

$$\|\hat{\boldsymbol{\vartheta}}_k - \hat{\boldsymbol{\vartheta}}_{k-1}\| + \|\hat{\mathrm{h}}_k - \hat{\mathrm{h}}_{k-1}\| \leqslant \epsilon,$$

we terminate this process and obtain the estimates $\hat{\boldsymbol{\vartheta}}_k$ and $\hat{\mathrm{h}}_k$; otherwise, increase k by 1 and go to step 3.

The flowchart of computing the parameter estimates $\hat{\boldsymbol{\vartheta}}_k$ and $\hat{\mathrm{h}}_k$ in the HLSI algorithm in (11)–(19) is shown in Fig. 2.

4. Example

This section provides an example to illustrate the performance of the BJ-like-HLSI algorithm. Consider the following two-input and two-output system

$$\boldsymbol{y}(t) = \frac{\boldsymbol{Q}(z)}{\mathrm{a}(z)}\boldsymbol{u}(t) + \frac{D(z)}{C(z)}\mathrm{V}(t),$$

$$\boldsymbol{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \quad \boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \quad \mathrm{V}(t) = \begin{bmatrix} \mathrm{V}_1(t) \\ \mathrm{V}_2(t) \end{bmatrix},$$

$$\mathrm{a}(z) = 1 + 0.60z^{-1}, \quad C(z) = 1 + 0.2z^{-1}, \quad D(z) = 1 - 0.9z^{-1},$$

$$\boldsymbol{Q}(z) = \begin{bmatrix} 2.00 & 1.00 \\ 1.00 & 2.00 \end{bmatrix} z^{-1}.$$

Table 1
The parameter estimates and errors ($L = 1000$).

k	a_1	$\boldsymbol{Q}_1(1,1)$	$\boldsymbol{Q}_1(1,2)$	$\boldsymbol{Q}_1(2,1)$	$\boldsymbol{Q}_1(2,2)$	c_1	d_1	d (%)
1	0.60235	1.94132	1.03767	1.26255	1.96989	0.21428	−0.86360	8.24653
2	0.59693	1.98971	1.04391	1.12488	1.99465	0.20691	−0.84962	4.25064
3	0.59633	1.98696	1.04291	1.13051	1.99795	0.17751	−0.90067	4.17800
4	0.59652	1.98987	1.04616	1.12979	1.99195	0.18593	−0.89469	4.15793
5	0.59651	1.98946	1.04615	1.12946	1.99266	0.18826	−0.89306	4.14426
True values	0.60000	2.00000	1.00000	1.00000	2.00000	0.20000	−0.90000	

Table 2
The parameter estimates and errors ($L = 2000$).

k	a_1	$\boldsymbol{Q}_1(1,1)$	$\boldsymbol{Q}_1(1,2)$	$\boldsymbol{Q}_1(2,1)$	$\boldsymbol{Q}_1(2,2)$	c_1	d_1	d (%)
1	0.61989	1.94420	0.96592	0.97183	2.16081	0.22393	−0.85987	5.46714
2	0.61039	1.96661	0.92295	0.98233	2.03346	0.20750	−0.84217	3.27086
3	0.60866	1.96600	0.92226	0.97964	2.03347	0.18133	−0.87310	2.96893
4	0.60837	1.96594	0.92239	0.98093	2.03460	0.17523	−0.88627	2.92891
5	0.60832	1.96589	0.92246	0.98116	2.03473	0.17769	−0.88585	2.91163
True values	0.60000	2.00000	1.00000	1.00000	2.00000	0.20000	−0.90000	

Table 3
The parameter estimates and errors ($L = 3000$).

k	a_1	$\mathbf{Q}_1(1,1)$	$\mathbf{Q}_1(1,2)$	$\mathbf{Q}_1(2,1)$	$\mathbf{Q}_1(2,2)$	c_1	d_1	δ (%)
1	0.58546	1.94664	0.96540	0.90672	1.97771	0.18887	−0.89512	3.48343
2	0.58953	1.96579	0.96459	0.99421	1.96899	0.19017	−0.89527	1.80384
3	0.59137	1.96593	0.96496	0.99460	1.97106	0.19024	−0.89771	1.74927
4	0.59167	1.96583	0.96476	0.99490	1.97108	0.19182	−0.89627	1.74719
5	0.59171	1.96591	0.96482	0.99487	1.97104	0.19223	−0.89576	1.74460
True values	0.60000	2.00000	1.00000	1.00000	2.00000	0.20000	−0.90000	

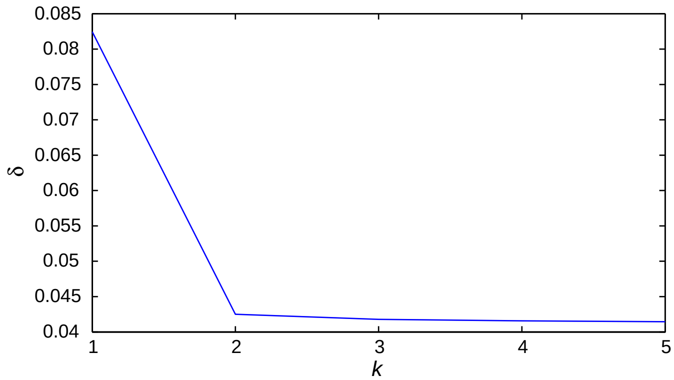

Fig. 3. The parameter estimation errors δ versus k ($L = 1000$).

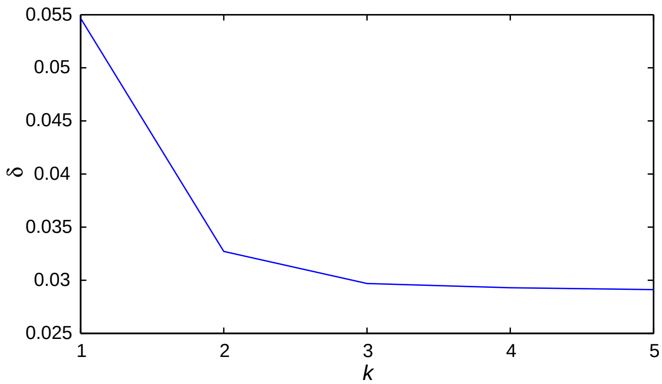

Fig. 4. The parameter estimation errors δ versus k ($L = 2000$).

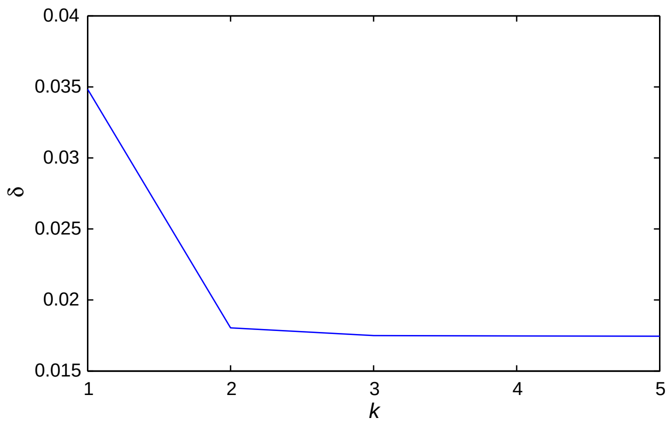

Fig. 5. The parameter estimation errors δ versus k ($L = 3000$).

In simulation, the inputs $\{u_1(t)\}$ and $\{u_2(t)\}$ are taken as two independent persistent excitation signal sequences with zero mean and unit variance, and $\{v_1(t)\}$ and $\{v_2(t)\}$ as two white noise sequences with zero mean and variances $\sigma_1^2 = \sigma_2^2 = 2.00^2$. We apply the proposed BJ-like-HLSI algorithm in (11)–(19) to estimate the parameters of this example system. The parameter estimates and their errors with different data lengths $t = L = 1000$, 2000 and 3000 are shown in Tables 1–3, and the parameter estimation errors

$$\delta := \sqrt{\frac{\|\hat{\vartheta}_k - \vartheta\|^2 + \|\hat{h}_k - h\|^2}{\|\vartheta\|^2 + \|h\|^2}}$$

versus k are shown in Figs. 3–5.

- The parameter estimation errors δ given by the BJ-like-HLSI algorithm gradually become small as the iteration k increases.
- When the noise variances are certain, the parameter estimation errors δ become small as the data length L increases.

5. Conclusions

This paper derives a hierarchical least squares algorithm for multivariable Box–Jenkins-like systems. The simulation results show that the proposed algorithm can estimate the parameters of multivariable systems. The proposed method can combine other multi-innovation identification methods [69–72] to study identification problems of linear or nonlinear systems with colored noises.

References

[1] Y. Zhang, G.M. Cui, Bias compensation methods for stochastic systems with colored noise, Applied Mathematical Modelling 35 (4) (2011) 1709–1716.

[2] Y.S. Xiao, F. Ding, Y. Zhou, M. Li, J.Y. Dai, On consistency of recursive least squares identification algorithms for controlled auto-regression models, Applied Mathematical Modelling 32 (11) (2008) 2207–2215.

[3] H. Habbi, M. Kidouchea, M. Zelmat, Data-driven fuzzy models for nonlinear identification of a complex heat exchanger, Applied Mathematical Modelling 35 (3) (2011) 1470–1482.

[4] A. Zymnis, S. Boyd, D. Gorinevsky, Mixed linear system estimation and identification, Signal Processing 90 (3) (2010) 966–971.

[5] M. Dehghan, M. Hajarian, An efficient algorithm for solving general coupled matrix equations and its application, Mathematical and Computer Modelling 51 (9-10) (2010) 1118–1134.

[6] F. Ding, T. Chen, Gradient based iterative algorithms for solving a class of matrix equations, IEEE Transactions on Automatic Control 50 (8) (2005) 1216–1221.

[7] F. Ding, T. Chen, Iterative least squares solutions of coupled Sylvester matrix equations, Systems & Control Letters 54 (2) (2005) 95–107.

[8] F. Ding, T. Chen, On iterative solutions of general coupled matrix equations, SIAM Journal on Control and Optimization 44 (6) (2006) 2269–2284.

[9] F. Ding, P.X. Liu, J. Ding, Iterative solutions of the generalized Sylvester matrix equations by using the hierarchical identification principle, Applied Mathematics and Computation 197 (1) (2008) 41–50.

[10] L. Xie, J. Ding, F. Ding, Gradient based iterative solutions for general linear matrix equations, Computers & Mathematics with Applications 58 (7) (2009) 1441–1448.

[11] F. Ding, Transformations between some special matrices, Computers & Mathematics with Applications 59 (8) (2010) 2676–2695.

[12] J. Ding, Y.J. Liu, F. Ding, Iterative solutions to matrix equations of form AiXBi=Fi, Computers & Mathematics with Applications 59 (11) (2010) 3500–3507.

[13] L. Xie, Y.J. Liu, H.Z. Yang, Gradient based and least squares based iterative algorithms for matrix equations $AXB + CX^TD=F$, Applied Mathematics and Computation 217 (5) (2010) 2191–2199.

[14] Y. N Cao, Z.Q. Liu, Signal frequency and parameter estimation for power systems using the hierarchical identification principle, Mathematical and Computer Modelling 52 (5-6) (2010) 854–861.

[15] Y.Y. Fu, C.J. Wu, J.T. Jeng, C.N. Ko, Identification of MIMO systems using radial basis function networks with hybrid learning algorithm, Applied Mathematics and Computation 213 (1) (2009) 184–196.

[16] H. Peng, G. Kitagawa, J. Wu, K. Ohtsu, Multivariable RBF-ARX model-based robust MPC approach and application to thermal power plant, Applied Mathematical Modelling 35 (7) (2011) 3541–3551.

[17] M. Ahmadia, H. Mojallali, Identification of multiple-input single-output Hammerstein models using Bezier curves and Bernstein polynomials, Applied Mathematical Modelling 35 (4) (2011) 1969–1982.

[18] L.L. Han, F. Ding, Multi-innovation stochastic gradient algorithms for multi-input multi-output systems, Digital Signal Processing 19 (4) (2009) 545–554.

[19] F. Ding, Y.J. Liu, B. Bao, Gradient based and least squares based iterative estimation algorithms for multi-input multi-output systems. in: Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, 2011, in press. doi:10.1177/0959651811409491

[20] L.L. Xiang, L.B. Xie, Y.W. Liao, R.F. Ding, Hierarchical least squares algorithms for single-input multiple-output systems based on the auxiliary model, Mathematical and Computer Modelling 52 (5-6) (2010) 918–924.

[21] B. Bao, Y.Q. Xu, J. Sheng, R.F. Ding, Least squares based iterative parameter estimation algorithm for multivariable controlled ARMA system modelling with finite measurement data, Mathematical and Computer Modelling 53 (9-10) (2011) 1664–1669.

[22] Y.J. Liu, J. Sheng, R.F. Ding, Convergence of stochastic gradient estimation algorithm for multivariable ARX-like systems, Computers & Mathematics with Applications 59 (8) (2010) 2615–2627.

[23] D.Q. Wang, Least squares-based recursive and iterative estimation for output error moving average (OEMA) systems using data filtering, IET Control theory and Applications 5 (14) (2011) 1648–1657.

[24] D.Q. Wang, F. Ding, Input-output data filtering based recursive least squares parameter estimation for CARARMA systems, Digital Signal Processing 20 (4) (2010) 991–999.

[25] F. Ding, P.X. Liu, G. Liu, Gradient based and least-squares based iterative identification methods for OE and OEMA systems, Digital Signal Processing 20 (3) (2010) 664–677.

[26] F. Ding, P.X. Liu, G. Liu, Identification methods for Hammerstein nonlinear systems, Digital Signal Processing 21 (2) (2011) 215–238.

[27] D.Q. Wang, Y.Y. Chu, F. Ding, Auxiliary model-based RELS and MI-ELS algorithms for Hammerstein OEMA systems, Computers & Mathematics with Applications 59 (9) (2010) 3092–3098.

[28] D.Q. Wang, Y.Y. Chu, G.W. Yang, F. Ding, Auxiliary model-based recursive generalized least squares parameter estimation for Hammerstein OEAR systems, Mathematical and Computer Modelling 52 (1-2) (2010) 309–317.

[29] D.Q. Wang, F. Ding, Least squares based and gradient based iterative identification for Wiener nonlinear systems, Signal Processing 91 (5) (2011) 1182–1189.
[30] F. Ding, T. Chen, Identification of Hammerstein nonlinear ARMAX systems, Automatica 41 (9) (2005). 1479-148.
[31] F. Ding, Y. Shi, T. Chen, Auxiliary model based least-squares identification methods for Hammerstein output-error systems, Systems & Control Letters 56 (5) (2007) 373–380.
[32] F. Ding, Y. Shi, T. Chen, Gradient-based identification methods for Hammerstein nonlinear ARMAX models, Nonlinear Dynamics 45 (1-2) (2006) 31–43.
[33] J. Chen, Y. Zhang, R.F. Ding, Auxiliary model based multi-innovation algorithms for multivariable nonlinear systems, Mathematical and Computer Modelling 52 (9-10) (2010) 1428–1434.
[34] D.Q. Wang, F. Ding, Extended stochastic gradient identification algorithms for Hammerstein-Wiener ARMAX systems, Computers & Mathematics with Applications 56 (12) (2008) 3157–3164.
[35] J. Chen, F. Ding, Modified stochastic gradient algorithms with fast convergence rates, Journal of Vibration and Control 17 (9) (2011) 1281–1286.
[36] J. Ding, Y. Shi, H.G. Wang, F. Ding, A modified stochastic gradient based parameter estimation algorithm for dual-rate sampled-data systems, Digital Signal Processing 20 (4) (2010) 1238–1249.
[37] F. Ding, P.X. Liu, H.Z. Yang, Parameter identification and intersample output estimation for dual-rate systems, IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans 38 (4) (2008) 966–975.
[38] F. Ding, H.Z. Yang, F. Liu, Performance analysis of stochastic gradient algorithms under weak conditions, Science in China Series F–Information Sciences 51 (9) (2008) 1269–1280.
[39] F. Ding, T. Chen, Performance bounds of the forgetting factor least squares algorithm for time-varying systems with finite measurement data, IEEE Transactions on Circuits and Systems–I: Regular Papers 52 (3) (2005) 555–566.
[40] W. Wang, F. Ding, J.Y. Dai, Maximum likelihood least squares identification for systems with autoregressive moving average noise, Applied Mathematical Modelling 36 (x) (2012), doi:10.1016/j.apm.2011.07.083.
[41] F. Ding, T. Chen, Performance analysis of multi-innovation gradient type identification methods, Automatica 43 (1) (2007) 1–14.
[42] Y.J. Liu, Y.S. Xiao, X.L. Zhao, Multi-innovation stochastic gradient algorithm for multiple-input single-output systems using the auxiliary model, Applied Mathematics and Computation 215 (4) (2009) 1477–1483.
[43] F. Ding, P.X. Liu, G. Liu, Auxiliary model based multi-innovation extended stochastic gradient parameter estimation with colored measurement noises, Signal Processing 89 (10) (2009) 1883–1890.
[44] J.B. Zhang, F. Ding, Y. Shi, Self-tuning control based on multi-innovation stochastic gradient parameter estimation, Systems & Control Letters 58 (1) (2009) 69–75.
[45] F. Ding, P. X Liu, G. Liu, Multi-innovation least squares identification for system modeling, IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics 40 (3) (2010) 767–778.
[46] F. Ding, Several multi-innovation identification methods, Digital Signal Processing 20 (4) (2010) 1027–1039.
[47] D.Q. Wang, F. Ding, Performance analysis of the auxiliary models based multi-innovation stochastic gradient estimation algorithm for output error systems, Digital Signal Processing 20 (3) (2010) 750–762.
[48] L. Xie, H.Z. Yang, F. Ding, Modeling and identification for non-uniformly periodically sampled-data systems, IET Control Theory & Applications 4 (5) (2010) 784–794.
[49] L.L. Han, F. Ding, Parameter estimation for multirate multi-input systems using auxiliary model and multi-innovation, Journal of Systems Engineering and Electronics 21 (6) (2010) 1079–1083.
[50] L.L. Han, F. Ding, Identification for multirate multi-input systems using the multi-innovation identification theory, Computers & Mathematics with Applications 57 (9) (2009) 1438–1449.
[51] Y.J. Liu, L. Yu, F. Ding, Multi-innovation extended stochastic gradient algorithm and its performance analysis, Circuits, Systems and Signal Processing 29 (4) (2010) 649–667.
[52] F. Ding, T. Chen, Combined parameter and output estimation of dual-rate systems using an auxiliary model, Automatica 40 (10) (2004) 1739–1748.
[53] F. Ding, T. Chen, Identification of dual-rate systems based on finite impulse response models, International Journal of Adaptive Control and Signal Processing 18 (7) (2004) 589–598.
[54] F. Ding, T. Chen, Least squares based self-tuning control of dual-rate systems, International Journal of Adaptive Control and Signal Processing 18 (8) (2004) 697–714.
[55] F. Ding, T. Chen, Parameter estimation of dual-rate stochastic systems by using an output error method, IEEE Transactions on Automatic Control 50 (9) (2005) 1436–1441.
[56] F. Ding, T. Chen, Hierarchical identification of lifted state-space models for general dual-rate systems, IEEE Transactions on Circuits and Systems–I: Regular Papers 52 (6) (2005) 1179–1187.
[57] F. Ding, T. Chen, A gradient based adaptive control algorithm for dual-rate systems, Asian Journal of Control 8 (4) (2006) 314–323.
[58] F. Ding, T. Chen, Z. Iwai, Adaptive digital control of Hammerstein nonlinear systems with limited output sampling, SIAM Journal on Control and Optimization 45 (6) (2007) 2257–2276.
[59] F. Ding, L. Qiu, T. Chen, Reconstruction of continuous-time systems from their non-uniformly sampled discrete-time systems, Automatica 45 (2) (2009) 324–332.
[60] Y.J. Liu, L. Xie, F. Ding, An auxiliary model based recursive least squares parameter estimation algorithm for non-uniformly sampled multirate systems, Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering 223 (4) (2009) 445–454.
[61] F. Ding, G. Liu, X.P. Liu, Partially coupled stochastic gradient identification methods for non-uniformly sampled systems, IEEE Transaction on Automatic Control 55 (8) (2010) 1976–1981.
[62] F. Ding, J. Ding, Least squares parameter estimation with irregularly missing data, International Journal of Adaptive Control and Signal Processing 24 (7) (2010) 540–553.
[63] F. Ding, G. Liu, X.P. Liu, Parameter estimation with scarce measurements, Automatica 47 (8) (2011) 1646–1655.
[64] L. Xie, H.Z. Yang, F. Ding, Recursive least squares parameter estimation for non-uniformly sampled systems based on the data filtering, Mathematical and Computer Modelling 54 (1–2) (2011) 315–324.
[65] F. Ding, T. Chen, Hierarchical gradient-based identification of multivariable discrete-time systems, Automatica 41 (2) (2005) 315–325.
[66] F. Ding, T. Chen, Hierarchical least squares identification methods for multivariable systems, IEEE Transactions on Automatic Control 50 (3) (2005) 397–402.
[67] H.Q. Han, L. Xie, F. Ding, X.G. Liu, Hierarchical least squares based iterative identification for multivariable systems with moving average noises, Mathematical and Computer Modelling 51 (9-10) (2010) 1213–1220.
[68] Z.N. Zhang, F. Ding, X.G. Liu, Hierarchical gradient based iterative parameter estimation algorithm for multivariable output error moving average systems, Computers & Mathematics with Applications 61 (3) (2011) 672–682.
[69] Y.J. Liu, D.Q. Wang, F. Ding, Least-squares based iterative algorithms for identifying Box-Jenkins models with finite measurement data, Digital Signal Processing 20 (5) (2010) 1458–1467.
[70] D.Q. Wang, G.W. Yang, R.F. Ding, Gradient-based iterative parameter estimation for Box-Jenkins systems, Computers & Mathematics with Applications 60 (5) (2010) 1200–1208.
[71] F. Ding, T. Chen, L. Qiu, Bias compensation based recursive least squares identification algorithm for MISO systems, IEEE Transactions on Circuits and Systems–II: Express Briefs 53 (5) (2006) 349–353.
[72] F. Ding, Y. Shi, T. Chen, Performance analysis of estimation algorithms of non-stationary ARMA processes, IEEE Transactions on Signal Processing 54 (3) (2006) 1041–1053.