文章编号:1002-0640(2006)12-0001-04

解耦的三轴气浮台非线性模糊变结构鲁棒控制问题*

周凤岐,贾 杰,周 军
(西北工业大学航天学院,陕西 西安 710072)

摘 要:针对飞行器全物理仿真三轴气浮台这一具有不确定性的、耦合的非线性系统,对其动力学耦合和可解耦性问题进行了分析、计算和证明。通过综合变结构控制和模糊控制,给出了一种新的非线性控制系统设计方法,此方法既可以避免变结构控制所固有的颤动现象,同时由于该模糊控制律的解析性,所以也具有实现简单,易于工程化的优点。仿真结果表明,给出的模糊变结构控制,对飞行器模型不确定性和外来干扰具有较强的和良好的跟踪性能。

关键词:三轴气浮台,耦合解耦,模糊逻辑,变结构控制,鲁棒性
中图分类号:O231.2 **文献标识码**:A

The Study of Nonlinear Fuzzy Variable Structure Robust Controlling Problem of the TACT based on Decoupling

ZHOU Feng-qi,JIA Jie,ZHOU Jun
(*College of Astronautics,Northwestern Polytechnical University,Xi'an 710072,China*)

Abstract:In this paper, the nonlinear dynamics system of the spacecraft triaxial attitude control air-bearings testbed(TACT) with uncertainty and coupling is analysised, calculated and proved, its general dynamical equation is given. This paper proves the TACT can be decoupled by using converse-system theory. Aiming at this nonlinear dynamics system, a new nonlinear decoupling controlling method is put forward through synthesizing variable structure and fuzzy theory. The simulation results show, aiming at uncertainty and outer disturbance, this method has efficient following performance.

Keywords:TACT,nonlinear dynamics,coupling and decoupling,variable structure,fuzzy theory

Introduction

The TACT is a class of system with uncertainty and strong coupling. If not dealing with uncertainty and coupling between the three axis, the dynamic and static indices of the system are influenced directly, sequentially stability of the system. So it is necessary to study the decoupling and uncertainty problems.

Based on analyzing, calculating the nonlinear dynamics system of the (TACT), this paper has studied coupling problem by using the nonlinear decoupling theory. Consequently a linear model is obtained, thereby the system can be analyzed and calculated by using linear system theory[1].

Aiming at the TACT system with uncertainty and outer disturbance, and because the customary nonlinear decoupling isn't sensitive to them, at the same time, variable structure controlling has strong robust toward the parameter uncertainty and outer disturbance and is fit for nonlinear object. But variable structure controlling has the inherent defect of high frequency quiver, so designing the variable structure controller without quiver has great meaningful for the aerocraft

收稿日期:2005-01-08 **修回日期**:2005-03-10
* **基金项目**:国家863航天基金资助(2002AA742043);武器装备科研基金资助(51420030401HK0335)。
作者简介:周凤岐(1936-),男,江苏常州市人,教授、博士生导师,主要从事飞行器导航、制导与控制方面的研究。

(C)1994-2024 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

controlling[2].

Accordingly, This paper combines nonlinear decoupling and variable structure theory with fuzzy controlling theory, the fuzzy variable structure controlling method of the TACT dynamics based on the nonlinear decoupling is investigated and then the numerical simulation is carried through. The simulation results show this method is effective. This has great meaningful for the subsequent designing of the TACT controlling system.

1 Nonlinear Dynamics Model of the TACT

Known by the angular momentum theorem, the dynamics model of the TACT with the reaction wheel(RW) is obtained as follows:

$$I\omega + \omega\times(I\omega + h_w) = -h_w + T_d \qquad (1)$$

where I represents the inertia matrix, ω represents the angle velocity matrix, h_ω represents the momentum matrix of the RW, T_d represents the outer disturbance.

The kinematics model is:

$$\begin{aligned}\omega_x &= \varphi\cos\theta - \psi\cos\varphi\sin\theta\\ \omega_y &= \theta - \psi\sin\varphi\\ \omega_z &= \varphi\sin\theta - \psi\cos\varphi\cos\theta\end{aligned} \qquad (2)$$

Supposing the input voltage of the RW is u and the output is rotate speed Ω, then the model of the RW is:

$$u = T_m K_e\Omega + K_e\Omega + \square f \qquad (3)$$

Where T_m represents the electromagnetism constant, K_e represents the steady gain.

In order to simplify calculating, suppose $I_x = I_y = I_z \square J$. Input (2)、(3) into (1), the reduced nonlinear controlling dynamics model is obtained through deducibility as follows:

$$\begin{aligned}u_x &= -3I\cos\theta\varphi + 3I\varphi\theta\sin\theta + 3I\psi\cos\varphi\sin\theta -\\ &\quad 3I\psi\varphi\sin\varphi\sin\theta + 3I\psi\theta\cos\varphi\cos\theta + 3T_{dx} +\\ &\quad \square f_x\\ u_y &= -3I\theta - 3I\psi\psi\sin\varphi - 3I\psi\varphi\cos\varphi + 3T_{dy} + \square f_y\\ u_z &= -3I\varphi\sin\theta - 3I\varphi\theta\cos\theta - 3I\psi\cos\varphi\cos\theta +\\ &\quad 3I\psi\varphi\sin\varphi\cos\theta + 3I\psi\theta\cos\varphi\sin\theta + 3T_{dz} +\\ &\quad \square f_z\end{aligned} \qquad (4)$$

Supposing $x_1 = \varphi$　$x_2 = \varphi$　$x_3 = \theta$　$x_4 = \theta$　$x_5 = \psi$　$x_6 = \psi$, $u = [u_x \quad u_y \quad u_z]^T$　$y = [\varphi \quad \theta \quad \psi]^T = [x_1 \quad x_3 \quad x_5]^T$

the state and output equation of the system with parameter uncertainty and outer disturbance is obtained through (4).

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix} = \begin{bmatrix}x_2\\ x_6x_4\cos x_1\\ x_4\\ x_2x_4\tan x_1 - x_6x_2\sec x_1\sin^2 x_1 - \\ \quad x_6x_2\cos x_1x_6\\ -x_2x_4\sec x_1 + x_6x_2\mathrm{tg}x_1\end{bmatrix} +$$

$$\begin{bmatrix}0\\ -0.01\cos x_3u_x - 0.01\sin x_3u_z + E_2\\ 0\\ -0.01\tan x_1\sin x_3u_x - 0.01u_y + \\ \quad 0.01\tan x_1\cos x_3u_z + E_4\\ 0\\ -0.01\sec x_1\sin x_3u_x - 0.01\sec x_1\cos x_3u_z + E_6\end{bmatrix} =$$

$$f(x) + g(x)u \qquad (5)$$

$$y = \begin{bmatrix}\varphi\\ \theta\\ \psi\end{bmatrix} = \begin{bmatrix}x_1\\x_3\\x_5\end{bmatrix} = h(x) \qquad (6)$$

Seen from (5) and (6), this system is a nonlinear system with three inputs and three outputs, and has complex nonlinear coupling relations. For this system, designing the controlling system is very difficult. In order to design a controlling system with high performance, it must be decoupled.

2 Decoupling Proving

For the same nonlinear dynamics system as (5) and (6), the following theorem comes into existence: If and only if, for $i = 1, 2, \cdots, r$ (r represents the length of the input vector set), supposing ρ_i is the least positive integer that satisfies the two formulas as follows:

$$\frac{\partial}{\partial u}[f^k h_i] = 0 \quad k = 0, 1, \cdots, \rho_i - 1 \qquad (7)$$

$$\frac{\partial}{\partial u}[f^k h_i] \neq 0 \quad k = \rho_i \qquad (8)$$

where $f h_i = \left[\frac{\partial h_i}{\partial x}\right]^T f$; $f^k h_i = f(f^{k-1}h_i)$, then

$$\rho_i \leqslant n = 6 \qquad (9)$$

$$\det\left[\frac{\partial}{\partial u}(f^{\rho}h)\right] \neq 0 \qquad (10)$$

(C)1994-2024 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net

where，$f^{\rho}h = [f^{\rho_1}h_1 \quad f^{\rho_2}h_2 \quad \cdots \quad f^{\rho_r}h_r]^T$，the proving of the theorem references the literature[1]. For the TACT (5) and (6)，the following equations come into existence:

$$fh_1 = \left[\frac{\partial x_1}{\partial x}\right]^T f = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] f = x_2$$

$$\frac{\partial [fh_1]}{\partial u} = 0 \qquad (11)$$

$$f^2 h_1 = f(fh_1) = \left[\frac{\partial x_2}{\partial x}\right]^T f = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] f = f_2 \qquad (12)$$

$$\frac{\partial f^2 h_1}{\partial u} = [-0.01\cos x_3 \quad 0 \quad -0.01\sin x_3]^T$$

Clearly，$\frac{\partial f^2 h_1}{\partial u} \neq 0$，$\rho_1 = 2 < n$. The following equations can be obtained by the same method:

$$\frac{\partial [fh_2]}{\partial u} = 0，\frac{\partial f^2 h_2}{\partial u} = [-0.01\tan x_1 \quad -0.01 \quad 0.01\tan x_1 \cos x_3]^T \qquad (13)$$

$$\frac{\partial [fh_3]}{\partial u} = 0，\frac{\partial f^2 h_3}{\partial u} = [0.01\sec x_1 \sin x_3 \quad 0 \quad -0.01\sec x_1 \cos x_3]^T \qquad (14)$$

Clearly，$\frac{\partial f^2 h_2}{\partial u} \neq 0$，$\rho_2 = 2 < n$，$\frac{\partial f^2 h_3}{\partial u} \neq 0$，$\rho_3 = 2 < n$。

So the nonlinear dynamics system of the TACT is control－able and can be coupled through state feedback and dynamic compensation.

3 Design of The Nonlinear Fuzzy Variable Vtructure Controller based on Decoupling

For the forementioned nonlinear dynamics system of the TACT，the section two educes: $\rho_1 = \rho_2 = \rho_3 = 2$，and the Matrices $F(x)$ $G(x)$ can be calculated through the nonlinear differential geometry theory[3]:

$$F(x) = \begin{bmatrix} x_6 x_4 \cos x_1 \\ x_2 x_4 \tan x_1 - x_6 x_2 \sec x_1 \sin^2 x_1 - \\ x_6 x_2 \cos x_1 \\ - x_2 x_4 \sec x_1 + x_6 x_2 \tan x_1 \end{bmatrix} \qquad (15)$$

$$G(x) = \begin{bmatrix} -\cos x_3 & 0 & -\sin x_3 \\ -\tan x_3 \sin x_3 & -1 & \tan x_1 \cos x_3 \\ \sec x_1 \sin x_3 & 0 & -\sec x_1 \cos x_3 \end{bmatrix} \qquad (16)$$

Then the system controller can be designed as follows:

$$u = G^{-1}(x)(v_1 + v_2 - F(x)) \qquad (17)$$

Let:
$$e = y - y_r \qquad (18)$$

Not consider disturbance and due to $s_i(t) = 0$ $\dot{s}_i(t) = 0$，v_1 is obtained:

$$v_1 = \begin{bmatrix} -c_{11}e_1 - c_{12}\dot{e}_1 \\ -c_{21}e_2 - c_{22}\dot{e}_2 \\ -c_{31}e_3 - c_{32}\dot{e}_3 \end{bmatrix} \qquad (19)$$

For the fuzzy variable structure v_2 with boundary，supposing the thickness of the boundary is $\delta > 0$，and design $k = [k_1 \quad k_2 \quad k_3]^T$ $\| \Delta f \| \leqslant k_i$:

$$v_2 = \begin{cases} -k_i w s_i(t) / \delta & |s_i(t)| \leqslant \delta \\ -k_i \operatorname{sgn}(s_i(t)) & \text{others} \end{cases} \qquad (20)$$

Establishing the fuzzy relation of s and v_2，s is standardized as $s_g = s/\delta$，designing the language values of s_g and v_2 are: {PB，PM，PS，ZE，NS，NM，NB}，supposing the fuzzy sets of s_g and v_2 are as follows respectively:

$\{A_{-3}, A_{-2}, A_{-1}, A_0, A_1, A_2, A_3\}$ $\{B_{-3}, B_{-2}, B_{-1}, B_0, B_1, B_2, B_3\}$

where the corresponding language values of A_i and B_i，$i = -3, \cdots, 3$ are: PB，PM，PS，ZE，NS，NM，NB.

The forms of the corresponding subjection functions are showed in figure 1，2:

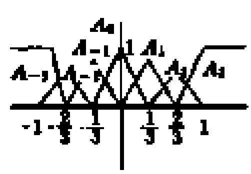

Fig 1 the form of the subjection function of A

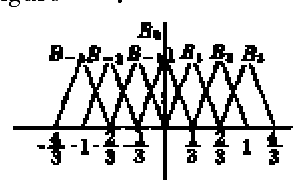

Fig 2 the form of the subjection function of B

Designing seven fuzzy controlling rules as follows: if s_g obeys A_i，then v_2 obeys B_{-i}，$i = -3, \cdots, 3$. Supposing the apex values of the subjection functions of the fuzzy set B_i is $PV(B_i)$，then

$$PV(B_i) = i/3 \quad i = -3, \cdots, 3 \qquad (21)$$

And then adopting the non-fuzzy decision method of literature[4]，the accurate output value of the fuzzy controller after quantitateing can be calculated as follows:

$$v_2 = k \sum_{i=-3}^{3} A_i(s_g) PV(B_{-i}) / \sum_{i=-3}^{3} A_i(s_g) \qquad (22)$$

From the formula above，the accurate output

(C)1994-2024 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

value v_2 of the fuzzy controller after quantitating can be calculated as formula(23):

The aim of removing quiver can be achieved by regulateing a, b.

4 Theoretical Analysis and Example

The parameters of a TACT system are chosen as follows:

$I = 34$, $T_m = 180$, $K_e = 1/3000$; $a = 0.4$, $b = 0.4$; $c_{11} = c_{21} = c_{31} = 5.1$; $c_{12} = c_{22} = c_{32} = 25$; $\delta = 0.02$; $k_1 = k_2 = k_3 = 0.5$

Choosing: $[\varphi \quad \theta \quad \Psi] = [1.5\sin t \quad 0.5\sin t \quad 0.58]$, the original condition is $[\pi/3 \quad 0 \quad \pi/3 \quad 0 \quad \pi/3 \quad 0]$. The sampling time is 10s, the time interval is 0.01s.

The response curves of fuzzy variable structure controlling of the system based on decoupling are shown in Fig. 3～Fig. 8:

$$v_2 = \begin{cases} k & s_g < -1 \\ k\dfrac{(2b-3)s_g + 2(b-1)}{3(b-1)s_g + (3b-2)} & -1 \leqslant s_g < -\dfrac{2}{3} \\ k\dfrac{(a-2b)s_g + \dfrac{2}{3}(a-b)}{3(a-b)s_g + (2a-b)} & -\dfrac{2}{3} \leqslant s_g < -\dfrac{1}{3} \\ k\dfrac{-as_g}{3(1-a)s_g + 1} & -\dfrac{1}{3} \leqslant s_g < 0 \\ -k\dfrac{as_g}{3(a-1)s_g + 1} & 0 \leqslant s_g < \dfrac{1}{3} \\ -k\dfrac{(2b-3)s_g + \dfrac{2}{3}(a-b)}{3(b-a)s_g + (2a-b)} & \dfrac{1}{3} \leqslant s_g < \dfrac{2}{3} \\ -k\dfrac{(3-2b)s_g + 2(b-1)}{3(1-b)s_g + (3b-2)} & \dfrac{2}{3} \leqslant s_g < 1 \\ -k & s_g \geqslant 1 \end{cases} \qquad (23)$$

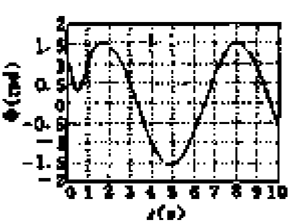

Fig 3 the response curve of φ

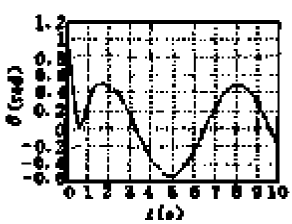

Fig 4 the response curve of θ

5 Conclusion

The dynamics model of the TACT is deduced through investigating it and is predigested. The

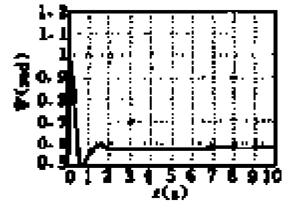

Fig 5 the response curve of Ψ

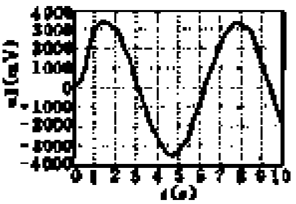

Fig 6 the response curve of $u1$

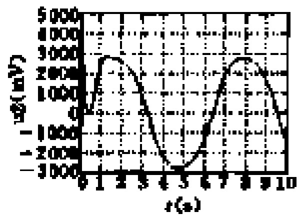

Fig 7 the response curve of $u2$

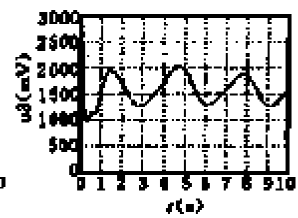

Fig 8 the response curve of $u3$

dynamic controlling equation of the TACT is given. That this system can be decoupled has be proved through the nonlinear differential geometry theory, the fuzzy variable structure controller based on decoupling is given. The quiver has been removed and the controlling precision has been improved.

It can been seen from the simulation results that the quiver is not only removed efficiently by adopting the fuzzy variable structure controlling given in this paper but has very strong robust for the model uncertainty and outer disturbance and the following error is correspondingly satisfying. In addition, the controller designed in this paper is continuous and the structure is simple too. This has established the base of designing the decoupling controlling system of the later TACT with high precision.

Reference Literatures:

[1] Xia X H. Gao W B. Controlling and Decoupling of Nonlinear Systems [M]. Beijing: Science Press, 1993.

[2] Rainer Palm. Robust Control by Fuzzy Sliding Mode[J]. Automatica, 1994, (30) 9: 12-15.

[3] Cheng D Z. The Geometry Theory of Nonlinear Systems[M]. Beijing: Science Press, 1988.

[4] Ying H. A Nonlinear Fuzzy Controller Rules is the Sum of a Global Two-dimension Multilevel Relay and a Local Nonlinear Proportional-intergral Controller[J]. Automatica, 1993, 29: 499-505.

(C)1994-2024 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net