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Variable structure guidance law for attacking surface maneuver targets

Han Yanhua & *Xu Bo*
Coll. of Astronautics, Nanjing Univ. of Aeronautics and Astronautics, Nanjing 210016, P. R. China
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Abstract: The characteristics of surface maneuver targets are analyzed and a 3-D relative motion model for missiles and targets is established. A variable structure guidance law is designed considering the characteristics of targets. In the guidance law, the distance between missiles and targets as well as the missile-target relative velocity are all substituted by estimation values. The estimation errors, the target's velocity, and the maneuver acceleration are all treated as bounded disturbance. The guidance law proposed can be implemented conveniently in engineering with little target information. The performance of the guidance system is analyzed theoretically and the numerical simulation result shows the effectiveness of the guidance law.

Keywords: air-to-surface missile, guidance law, variable structure control, surface maneuver target.

1. Introduction

In modern airland battle, maneuver targets are comprised of air targets and surface targets. There have been large numbers of homing guidance laws attacking air maneuver targets[1−3]. As a sort of important tactics targets, tanks and panzers are surface maneuver targets. There are two distinct characteristics in surface maneuver target compared with air maneuver target: firstly, it is difficult to measure the distance between missiles and targets directly owing to the complexity of background; secondly, the velocity and maneuver acceleration of targets are small. A variable structure guidance law is proposed to attack surface maneuver targets considering the characteristics mentioned above. There are three advantages in the proposed guidance law here: firstly, only the missile-target line-of-sight angle and the angular velocity are needed to measure online; secondly, the distance between missiles and targets as well as the relative velocity between them adopted in the guidance law are substituted by estimated values, and the estimation errors as well as the target's velocity and maneuver acceleration are treated as bounded disturbance; lastly, the above disturbance can be resisted successfully if the parameters of the guidance law are appropriately chosen.

2. Model of missile-target relative motion

Figure 1 shows the motion of missiles and surface targets.

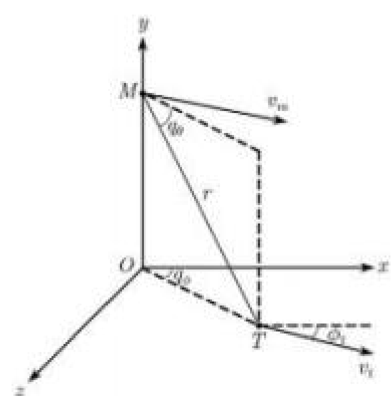

Fig. 1 The figure of missile's and targets' motion

In Fig.1, M, T, and MT denote missiles, targets, and the missile-target line of sight respectively. r is the distance between missiles and targets, q_θ is the obliquity of MT, and q_ϕ is the azimuth of MT. v_t denotes the velocity of targets and ϕ_t is the azimuth of v_t. The obliquity of v_t is zero since it is supposed that the target is moving on the ground level all the time.

v_m denotes the velocity of missiles, and the obliquity and azimuth of v_m are denoted by θ_m and ϕ_m, respectively. The definition of θ_m and ϕ_m is similar to the definitions of q_θ and q_ϕ.

Th rectangular coordinate of missiles is denoted by $(x_m, y_m, z_m)^{\mathrm{T}}$ and the rectangular coordinate of targets is denoted by $(x_t, 0, z_t)^{\mathrm{T}}$; then the motion equations of missiles and targets are as follows

$$\begin{cases} \dot{x}_m = v_m \cos\theta_m \cos\phi_m \\ \dot{y}_m = v_m \sin\theta_m \\ \dot{z}_m = -v_m \cos\theta_m \sin\phi_m \end{cases} \tag{1}$$

$$\begin{cases} \dot{x}_t = v_t \cos\phi_t \\ \dot{y}_t = 0 \\ \dot{z}_t = -v_t \sin\phi_t \end{cases} \tag{2}$$

The equations of missile-target relative motion can be deduced by considering Eqs. (1) and (2) according to Fig. 1

$$\begin{aligned} \dot{r} = &-v_m[\cos\theta_m \cos q_\theta \cos(q_\phi - \phi_m) + \\ &\sin\theta_m \sin q_\theta] + v_t \cos q_\theta \cos(q_\phi - \phi_t) \end{aligned} \tag{3}$$

$$\begin{aligned} r\dot{q}_\theta = &v_m[\cos\theta_m \sin q_\theta \cos(q_\phi - \phi_m) - \\ &\sin\theta_m \cos q_\theta] - v_t \sin q_\theta \cos(q_\phi - \phi_t) \end{aligned} \tag{4}$$

$$\begin{aligned} r\dot{q}_\phi \cos q_\theta = &v_m \cos\theta_m \sin(q_\phi - \phi_m) - \\ &v_t \sin(q_\phi - \phi_t) \end{aligned} \tag{5}$$

The dynamical equations of the center of the missile's mass are [4]

$$\begin{cases} v_m\dot{\theta}_m = -g\cos\theta_m + u_1 \\ v_m\dot{\phi}_m \cos\theta_m = -u_2 \end{cases} \tag{6}$$

where, u_1 and u_2 denote the force imposed on the missile along the Y axis and the Z axis of the flight path coordinate system.

The motion of missiles and targets has been described adequately by equations[1−6]. It is evident that the missile-target relative motions in the vertical surface and in the horizontal surface are coupling with each other. The relative motion model can be simplified adequately for the design of a guidance law. The model error from the simplification can be compensated by the guidance law since it is a closed loop.

The relative motion between missiles and targets in the horizontal surface is small; then, $\cos(q_\phi - \phi_m) \cong 1$. Equation (4) can be rewriten accordingly as follows

$$r\dot{q}_\theta = v_m \sin(q_\theta - \theta_m) - v_t \sin q_\theta \cos(q_\phi - \phi_t) \tag{7}$$

The following equations can be obtained via differentiation operation about time to Eqs. (7) and (5) considering (6)

$$\begin{aligned} r\ddot{q}_\theta = &-2\dot{r}\dot{q}_\theta + g\cos\theta_m \cos(q_\theta - \theta_m) - \\ &u_1 \cos(q_\theta - \theta_m) + v_t\dot{q}_\phi \sin q_\theta \sin(q_\phi - \phi_t) - \\ &[\dot{v}_t \cos(q_\phi - \phi_t) + v_t\dot{\phi}_t \sin(q_\phi - \phi_t)] \sin q_\theta \end{aligned} \tag{8}$$

$$\begin{aligned} r\ddot{q}_\phi \cos q_\theta = &\dot{q}_\phi[r\dot{q}_\theta \sin q_\theta - \dot{r}\cos q_\theta + \\ &v_m \cos\theta_m \cos(q_\phi - \phi_m) - v_t \cos(q_\phi - \phi_t)] + \\ &(g\cos\theta_m - u_1)\sin\theta_m \sin(q_\phi - \phi_m) + \\ &u_2 \cos(q_\phi - \phi_m) - \dot{v}_t \sin(q_\phi - \phi_t) + \\ &v_t\dot{\phi}_t \cos(q_\phi - \phi_t) \end{aligned} \tag{9}$$

where, $\dot{v}_t$ and $v_t\dot{\phi}_t$ are the target's tangent acceleration and the normal acceleration, respectively, denoted by a_{tt} and a_{tn}; therefore,

$$\begin{aligned} &\dot{v}_t \cos(q_\phi - \phi_t) + v_t\dot{\phi}_t \sin(q_\phi - \phi_t) = \\ &\sqrt{a_{tt}^2 + a_{tn}^2} \cos(q_\phi - \phi_t - \lambda) \triangleq w_1 \\ &\dot{v}_t \sin(q_\phi - \phi_t) - v_t\dot{\phi}_t \cos(q_\phi - \phi_t) = \\ &\sqrt{a_{tt}^2 + a_{tn}^2} \sin(q_\phi - \phi_t - \lambda) \triangleq w_2 \end{aligned}$$

where, $\cos\lambda = \dfrac{a_{tt}}{\sqrt{a_{tt}^2 + a_{tn}^2}}$. Then, Eqs. (8) and (9) can be rewritten respectively as follows

$$\begin{aligned} r\ddot{q}_\theta = &-2\dot{r}\dot{q}_\theta + g\cos\theta_m \cos(q_\theta - \theta_m) - \\ &u_1 \cos(q_\theta - \theta_m) + v_t\dot{q}_\phi \sin q_\theta \sin(q_\phi - \phi_t) - \\ &w_1 \sin q_\theta \end{aligned} \tag{10}$$

$$\begin{aligned} r\ddot{q}_\phi \cos q_\theta = &\dot{q}_\phi[r\dot{q}_\theta \sin q_\theta - \dot{r}\cos q_\theta + \\ &v_m \cos\theta_m \cos(q_\phi - \phi_m) - v_t \cos(q_\phi - \phi_t)] + \\ &(g\cos\theta_m - u_1)\sin\theta_m \sin(q_\phi - \phi_m) + \\ &u_2 \cos(q_\phi - \phi_m) - w_2 \end{aligned} \tag{11}$$

w_1 and w_2 are the functions of the target's maneuver acceleration a_{tt} and a_{tm}, and these can be treated as disturbance here; thus, Eqs. (10) and (11) are the basis of designing the guidance law. Supremum of a_{tt}, a_{tn}, and v_t are supposed as follows since the target's maneuver acceleration and velocity are bounded: $\sup a_{tt} = a_{tt\sup}$, $\sup a_{tn} = a_{tn\sup}$, $\sup v_t = v_{t\sup}$. Then, it can be obtained that $\sup\sqrt{a_{tt}^2 + a_{tn}^2} \leqslant \sqrt{a_{tt\sup}^2 + a_{tn\sup}^2} \triangleq a_{t\sup}$.

3. Variable structure guidance law

The switching surface equation can be designed as

$$s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} \dot{q}_\theta \\ \dot{q}_\phi \end{pmatrix} = 0$$

The reaching law is designed as follows, which guarantees the switching surface as sliding surface[5−6]

$$\dot{s}_1 = -\frac{1}{r}k_1 s_1 - \frac{1}{r}\varepsilon_1 \text{sign } s_1 \tag{12}$$

$$\dot{s}_2 = -\frac{1}{r\cos q_\theta}k_2 s_2 - \frac{1}{r\cos q_\theta}\varepsilon_2 \text{ sign } s_2 \tag{13}$$

The following equation can be obtained according to Eqs. (10) and (12)

$$\begin{aligned} u_1 = &(k_1 - 2\dot{r})\,\dot{q}_\theta \sec(q_\theta - \theta_m) + \\ &\varepsilon_1 \text{ sign } \dot{q}_\theta \sec(q_\theta - \theta_m) + g\cos\theta_m + \\ &v_t \dot{q}_\phi \sin q_\theta \tan(q_\phi - \phi_t) - w_1 \sin q_\theta \sec(q_\theta - \theta_m) \end{aligned}$$

Considering that it is difficult to measure the target's velocity v_t and the maneuver acceleration w_1, as well as the accurate relative velocity $\dot{r}$ between missiles and targets, which can be estimated approximately as $\dot{R}$ according to the missile's velocity, the guidance law can be designed as

$$\begin{aligned} u_1 = &\left(k_1 - 2\dot{R}\right)\dot{q}_\theta \sec(q_\theta - \theta_m) + \\ &\varepsilon_1 \text{ sign } \dot{q}_\theta \sec(q_\theta - \theta_m) + g\cos\theta_m \end{aligned} \tag{14}$$

and according to Eq. (3)

$$\begin{aligned} \dot{R} = &-v_m[\cos\theta_m \cos q_\theta \cos(q_\phi - \phi_m) + \\ &\sin\theta_m \sin q_\theta] \end{aligned} \tag{15}$$

The surface target's velocity and maneuver are small when compared with that of missiles, and therefore, the approximation above is reasonable, and the error caused by the approximation can be compensated by the variable structure guidance law. A detailed analysis on this can be seen later in the article.

Similarly, the following equation can be derived from Eqs. (11) and (13)

$$\begin{aligned} u_2 = &\dot{q}_\phi[\dot{R}\cos q_\theta - r\dot{q}_\theta \sin q_\theta - \\ &v_m \cos\theta_m \cos(q_\phi - \phi_m) - k_2]\sec(q_\phi - \phi_m) - \\ &\varepsilon_2 \text{ sign } \dot{q}_\phi \sec(q_\phi - \phi_m) + \\ &(u_1 - g\cos\theta_m)\sin\theta_m \tan(q_\phi - \phi_m) \end{aligned} \tag{16}$$

In Eq. (16), the distance r between missiles and targets can be expressed as $r = -h\cdot\csc q_\theta$, where, h is the altitude of missiles to the ground surface, which can be measured by the missile-borne radar.

The sign function can be substituted by a saturation function in the guidance law to avoid vibration; then,

$$\begin{aligned} u_1 = &\left(k_1 - 2\dot{R}\right)\dot{q}_\theta \sec(q_\theta - \theta_m) + \\ &\varepsilon_1 \dot{q}_\theta / (|\dot{q}_\theta| + \delta)\sec(q_\theta - \theta_m) + g\cos\theta_m \end{aligned}$$

$$\begin{aligned} u_2 = &\dot{q}_\phi[\dot{R}\cos q_\theta - r\dot{q}_\theta \sin q_\theta - \\ &v_m \cos\theta_m \cos(q_\phi - \phi_m) - k_2]\sec(q_\phi - \phi_m) - \\ &\varepsilon_2 \dot{q}_\phi / (|\dot{q}_\phi| + \delta)\sec(q_\phi - \phi_m) + \\ &(u_1 - g\cos\theta_m)\sin\theta_m \tan(q_\phi - \phi_m) \end{aligned}$$

4. System analysis

The following equations can be obtained based on equations (10), (14) and (11), (16) respectively, considering the definition of the variable structure switching function s

$$\begin{aligned} \dot{s}_1 = &-\frac{1}{r}(k_1 + 2\Delta\dot{r})\,s_1 - \\ &\frac{1}{r}[\varepsilon_1 \text{sign } s_1 + w_1 \sin q_\theta - \\ &v_t \dot{q}_\phi \sin q_\theta \sin(q_\phi - \phi_t)] \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{s}_2 = &-\frac{1}{r\cos q_\theta}[k_2 + \Delta\dot{r}\cos q_\theta + \\ &v_t \cos(q_\phi - \varphi_t)]s_2 - \\ &\frac{1}{r\cos q_\theta}(\varepsilon_2 \text{sign } s_2 + w_2) \end{aligned} \tag{18}$$

where, $\Delta\dot{r} = \dot{r} - \dot{R}$ is the error of the missile-target relative velocity's estimation value. $|\Delta\dot{r}|_{\sup} \triangleq \sup|\Delta\dot{r}| \leqslant v_{t\sup}$ can be obtained according to Eqs. (3) and (15).

It is concluded from Eqs. (17), (18) and the definitions of w_1 and w_2 that s_2 will reach zero in finite time only if $k_2 = k_2^* + |\Delta\dot{r}|_{\sup} + v_{t\sup}$ and $\varepsilon_2 > a_{t\sup}$, where, k_2^* is a positive constant, so that $\dot{q}_\phi = 0$ and $v_t\dot{q}_\phi = 0$. In the same way, s_1 will reach zero in finite time only if $k_1 = k_1^* + 2|\Delta\dot{r}|_{\sup}$ and $\varepsilon_1 > a_{t\sup}$, where, k_1^* is a positive constant, so that $\dot{q}_\theta = 0$. The conclusion that $s = \begin{pmatrix} \dot{q}_\theta \\ \dot{q}_\phi \end{pmatrix} = 0$ can be obtained in finite time under the variable structure guidance law.

5. Numerical simulation

The effectiveness of the guidance law is validated by numerical simulation in which a type of foreign tank is adopted as the imaginary enemy target. In the simulation, the target moves at the speed of $v_t = v_{tup} =$ 20 m/s (equal to 72 km/h, which is the full speed of this type of tank), and snaking maneuver is carried out by the target to avoid attack from missile. The azimuth of the target's velocity varies strongly from $-60°$ to $+60°$ ruled by sine function. The target's tangent acceleration is $a_{tt} = 0$ m/s^2 since it is supposed that the target moves at constant speed. Its normal acceleration a_{tn} varies from 0 m/s^2 to 33.4 m/s^2; therefore, $a_{t\sup} \triangleq \sqrt{a_{tt\sup}^2 + a_{tn\sup}^2} = a_{tn\sup} = 33.4$ m/s^2.

The velocity of missile is supposed as 250 m/s. The initial altitude, ballistic obliquity, and ballistic azimuth of missile are 3 000 m, 20°, and 30°, respectively. The parameters of guidance law are chosen as $k_1 = k_2 = 80$ (m/s) and $\varepsilon_1 = \varepsilon_2 = 68$ (m/s^2) according to the principle mentioned above in the system analysis. What's more, the sign function is substituted by a saturation function in the guidance law to avoid vibration and the parameter of the saturation function is $\delta = 0.01(\mathrm{s}^{-1})$.

In simulation, the overload command imposed on missile along the Y axis and the Z axis of the flight path coordinate system is restricted to the scope of 0–10 g and 0–8 g, respectively, considering that the missile's available overload is limited in practice. The Miss distance is only 0.2 m, which can satisfy the tactics index. Fig.2–Fig.4 show the results of numerical simulations.

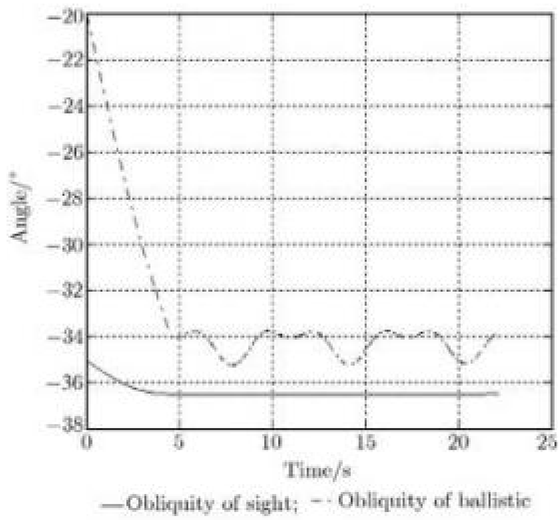

Fig. 2 Time history of sight and ballistic obliquity

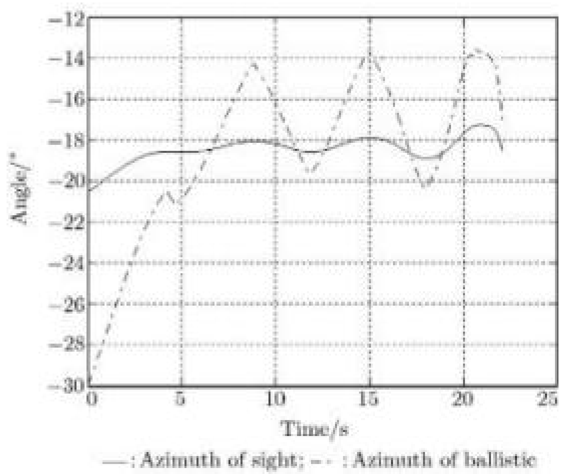

Fig. 3 Time history of sight and ballistic azimuth

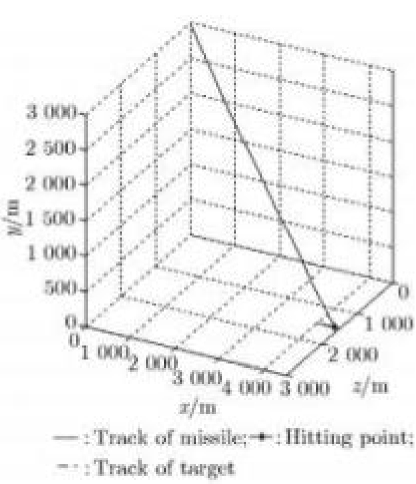

Fig. 4 3-D track of missile and target

6. Conclusion

Satisfactory anti-disturbance ability and high guidance precision are obtained by the variable structure guidance law proposed in this article. At the same time, the guidance law is simple and requires little target information. The guidance law can be used in the overall guidance process of airborne-antitank missiles, and the intermediate and terminal guidance of ground based antitank missiles with high ballisitic.

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Han Yanhua was born in 1976. He is a Ph. D. He is a lecturer of Nanjing University of Aeronautics and Astronautics. His research fields include application of modern control theory in missile and spacecraft's guidance and control system. E-mail: hanyanhua@nuaa.edu.cn