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Modeling and control for ultra-low altitude cargo airdrop

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Abstract

Purpose – The purpose of this paper is to model the aircraft-cargo's coupling dynamics during ultra-low altitude heavy cargo airdrop and to design the aircraft's robust flight control law counteracting its aerodynamic coefficients perturbation induced by ground effect and the disturbance from the sliding cargo inside.

Design/methodology/approach – Aircraft-cargo system coupling dynamics model in vertical plane is derived using the Kane method. Trimmed point is calculated when the cargo fixed in the cabin and then the approximate linearized motion equation of the aircraft upon it is derived. The robust stability and robust H_∞ optimal disturbance restraint flight control law are designed countering the aircraft's aerodynamic coefficients perturbation and the disturbance moment, respectively.

Findings – Numerical simulation shows the effectiveness of the proposed control law with elevator deflection as a unique control input.

Practical implications – The model derived and control law designed in the paper can be applied to heavy cargo airdrop integrated design and relevant parameters choice.

Originality/value – The dynamics model derived is closed, namely, the model can be called in numerical simulation free of assuming the values of parachute's extraction force or cargo's relative sliding acceleration or velocity as seen in many literatures. The modeling is simplified using Kane method rather than Newton's laws. The robust control law proposed is effective in guaranteeing the aircraft's flight stability and disturbance restraint performance in the presence of aerodynamic coefficients perturbation.

Keywords Flight control, H_∞ robust control, Kane method, Multi body system dynamics, Ultra-low altitude heavy cargo airdrop

Paper type Research paper

Introduction

Ultra-low altitude heavy cargo airdrop is of great significance in both military and civilian fields. Some critical problems need to be solved before it being applied widely. First of all, the interaction between aircraft and the cargo inside needs to be researched. Second, the aircraft's flight control law during heavy cargo airdrop is important. The features of ultra-low altitude heavy cargo airdrop are as follows:

- The aircraft's aerodynamic coefficient varies considerably due to ground effect (Liu et al., 2014).
- Huge disturbance moment is exerted on the aircraft by the sliding cargo inside the cabin.
- The aircraft flies at ultra-low altitude and low velocity during airdrop to avoid the heavy cargo from damaging when it hits ground, so the aircraft's angle of attack keeps high enough for sufficient lift, and it means that fairly robust flight control law should be designed elaborately to avoid the aircraft's angle of attack varying dramatically up to stall value in the presence of aerodynamic coefficients perturbation and huge disturbance moment.

In the last decade, much research has been documented on the dynamic, control and simulation problem of heavy cargo airdrop. Yang and Ke (2007) derived the cargo-parachute system multi-body dynamics model. Ke et al. (2006) studied the sliding motion of cargo in the cabin considering the action on the cargo by aircraft while neglecting that on the aircraft by the cargo. Conversely, Wu et al. (2012) focused on the action on the aircraft by the cargo. In his research, the rigid–flexible coupling dynamics model of the aircraft is derived using Lagrange analytical mechanics regarding the cargo as a particle and its relative sliding acceleration as constant. Literature studies show most aircraft-cargo coupling dynamics models are derived using Newton's laws and the modeling methods fall into two categories that are integral method (Chen et al., 2014) and isolation method (Chen and Shi, 2009; Feng et al., 2011; Zhang et al., 2014; Liu et al., 2015). In the former method, the interaction between the aircraft and the cargo needs not be considered as it is the inner action; however, the variation of the inertia moment of the whole system depending, on the cargo's position inside should be taken into account. Chen et al. (2014) calculate the whole system's moment of inertia adopting the assumption of constant sliding acceleration of the cargo inside the cabin. In the latter method, the interaction between the aircraft and the cargo is calculated, and the force and the

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moment on the aircraft by the cargo are considered as disturbance. In the above research, the system coupling dynamics model is not closed, as either the value of tension force of the extraction parachute or the cargo's relative sliding acceleration or velocity need to be assumed. In the field of flight control, Feng et al. (2011) derive the approximate linear motion equation of the aircraft with the cargo inside fixed at the center of mass of aircraft first; then the disturbance moment acted on it by the sliding cargo is transformed equivalently into the linear model's parameters perturbation, and the interval system's H_∞ robust control theory is applied to design its flight control law; Zhang et al. (2014) and Chen and Shi (2011) design the aircraft's flight control law based on the back-stepping method. Input–output linearization decoupling is conducted (Liu et al., 2015; Zhang and Shi, 2009; Xu and Sun, 2014) based on the differential geometric control theory, and then variable structure controller is designed for the pseudo-linear system. In the field of system simulation, Raissi et al. (2008) construct computer simulation system, while the mathematical models are not strict enough, for instance, the moment on the aircraft by the cargo is calculated roughly via multiplying the weight of the cargo by the distance between it and the center of mass of the aircraft. Thomas (2011) constructs the heavy cargo airdrop's desktop computer simulation system and cockpit real-time simulation system.

The common points on the modeling of aircraft-cargo multi-body system among the present literatures are as follows:
- the cargo is assumed as a particle;
- the modeling is based on Newton's laws mostly; and
- the model derived is not closed.

As to the flight control, the common point is that the deflection of elevator and the throttle opening degree are both considered as control input. While we know the response time of aircraft's thrust of engine to the throttle input is considerably long compared to the time the cargo sliding inside the cabin, so the existing simulation results are impractically ideal as they have not taken the dynamic characteristics of the "throttle input–thrust output" into account.

The paper derives the aircraft-cargo multi-body dynamics model during the cargo sliding inside the cabin using Kane method. The model derived is closed. The H_∞ control theory based aircraft robust stability flight control law when the cargo fixed and robust performance optimal flight control law when the cargo sliding inside the cabin are designed, respectively, counteracting the aerodynamic coefficients perturbation due to the ground effect and the disturbance moment acted by the cargo.

Kane method-based modeling of the aircraft-cargo multi-body dynamics system

This section begins with the system modeling along with the definition of some relevant coordinate systems.

The coordinate systems definition and transformation

The three coordinate systems are introduced as follows.

Ground inertia coordinate system ox_Ay_A

Ignoring the curvature of the Earth's surface and the Earth' rotation, the ground inertia coordinate system is defined as follows. The origin is fixed on some point of the ground. The positive direction of axis ox_A is along with the projection of the aircraft's velocity vector on the ground. Axis oy_A, perpendicular to axis ox_A, lies in the vertical plane, and its positive direction is upward. $\vec{a}_1$ and $\vec{a}_2$ are unit vectors lying on axes ox_A and oy_A, respectively, and their directions are the same as the positive directions of the corresponding axes, respectively.

Flight path coordinate system ox_By_B

The origin is the center of mass of the aircraft. The positive direction of axis ox_B is along with the velocity vector of the aircraft. Axis oy_B, perpendicular to axis ox_B, lies in the vertical plane, and its positive direction is upward. $\vec{b}_1$ and $\vec{b}_2$ are unit vectors lying on axes ox_B and oy_B, respectively, and their directions are the same as the positive directions of the corresponding axes, respectively.

Aircraft body-*fixed* coordinate system ox_Cy_C

The origin is the center of mass of the aircraft. Axis ox_C is coincident with the longitudinal axis of the aircraft body, and its positive direction is forward, pointing the nose of the aircraft. Axis oy_C, perpendicular to axis ox_C, is in the plane of symmetry of the aircraft, and the positive direction of it is upward. $\vec{c}_1$ and $\vec{c}_2$ are unit vectors lying on axes ox_C and oy_C, respectively, and their directions are the same as the positive directions of the corresponding axes, respectively (Figure 1).

In accordance with the above definition, the vector groups $A = (\vec{a}_1; \vec{a}_2)$, $B = (\vec{b}_1; \vec{b}_2)$, $C = (\vec{c}_1; \vec{c}_2)$ are all unit orthogonal basis vector groups with the transformation relations among them as the following:

$$B = AT_{AB} \tag{1}$$

$$C = AT_{AC} \tag{2}$$

where

$$T_{AB} = \begin{pmatrix} \cos u & -\sin u \\ \sin u & \cos u \end{pmatrix} \tag{3}$$

$$T_{AC} = \begin{pmatrix} \cos h & -\sin h \\ \sin h & \cos h \end{pmatrix} \tag{4}$$

u and h are flight path angle and pitch angle of aircraft, respectively.

Analysis of kinematics and mechanics

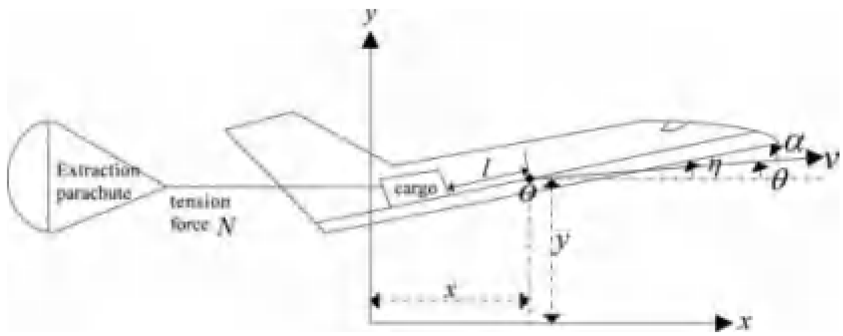
The position vector of the center of mass of aircraft relative to the origin of ground inertia coordinate system can be expressed as:

$$\vec{r}_a = A\begin{pmatrix} x \\ y \end{pmatrix} \tag{5}$$

where x and y are, respectively, the longitudinal flight range and the flight height of the aircraft.

The time derivative of equation (5) is:

Figure 1 The sketch of heavy cargo airdrop



$$\vec{v}_a = A\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} \quad (6)$$

In the same way, the time derivative of equation (6) is:

$$\vec{a}_a = A\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} \quad (7)$$

where, $\vec{v}_a$ and $\vec{a}_a$ are, respectively, the velocity and acceleration vector of the aircraft.

The position vector of the center of mass of cargo relative to the origin of ground inertia coordinate system is:

$$\vec{r}_c = \vec{r}_a + \vec{l} \quad (8)$$

where

$$\vec{l} = C\begin{pmatrix}l\\ 0\end{pmatrix} \quad (9)$$

and l is the distance of the cargo relative to the center of mass of the aircraft.

The velocity vector of the cargo relative to the origin of ground inertia coordinate system is:

$$\vec{v}_c = \frac{d\vec{r}_c}{dt} \quad (10)$$

Substituting equations (2), (4), (5), (8) and (9) into equation (10) yields the following:

$$\vec{v}_c = A\begin{pmatrix}\dot{x} + \dot{l}\cos\theta - l\dot{\theta}\sin\theta\\ \dot{y} + \dot{l}\sin\theta + l\dot{\theta}\cos\theta\end{pmatrix} \quad (11)$$

The acceleration of the cargo can be obtained by conducting time derivative ofequation (11), as the following:

$$\vec{a}_c = A\frac{d}{dt}\begin{pmatrix}\dot{x} + \dot{l}\cos\theta - l\dot{\theta}\sin\theta\\ \dot{y} + \dot{l}\sin\theta + l\dot{\theta}\cos\theta\end{pmatrix} \quad (12)$$

The attitude angular velocity vectors of the aircraft and the cargo inside are, respectively:

$$\vec{\omega}_a = \vec{c}_3\dot{\theta} = \vec{a}_3\dot{\theta} \quad (13)$$

$$\vec{\omega}_c = \vec{c}_3\dot{\theta} = \vec{a}_3\dot{\theta} \quad (14)$$

where $\vec{a}_3$ is a unit vector perpendicular to both $\vec{a}_1$ and $\vec{a}_2$, that is $\vec{a}_3 = \vec{a}_1 \times \vec{a}_2$ and $\vec{c}_3 = \vec{c}_1 \times \vec{c}_2$. Obviously, $\vec{c}_3 = \vec{a}_3$.

Time derivative of equations (13) and (14) yields the attitude angular acceleration vectors of the aircraft and the cargo, respectively, as the following:

$$\vec{\varepsilon}_a = \vec{a}_3\ddot{\theta} \quad (15)$$

$$\vec{\varepsilon}_c = \vec{a}_3\ddot{\theta} \quad (16)$$

The active and inertia forces upon the aircraft will be analyzed below.

The gravity of aircraft is:

$$\vec{G}_a = A\begin{pmatrix}0\\ -m_a g\end{pmatrix} \quad (17)$$

where m_a is the mass of the aircraft and g is the gravitational constant.

The aerodynamic force upon the aircraft is:

$$\vec{R} = B\begin{pmatrix}-Q\\ Y\end{pmatrix} \quad (18)$$

where Q and Y are, respectively, the aerodynamic drag and lift upon the aircraft.

The thrust vector of the engine of the aircraft is:

$$\vec{P} = C\begin{pmatrix}P\\ 0\end{pmatrix} \quad (19)$$

where P is the amplitude of the vector $\vec{P}$.

The action between the aircraft and cargo is ideal constraint as the friction between the rail in the cabin and the cargo is very small in engineering practice.

The inertia force upon the aircraft induced by its linear acceleration $\vec{a}_a$ is:

$$\vec{F}_a^{(I)} = -m_a \vec{a}_a \tag{20}$$

The moments of active and inertia force upon the aircraft will be analyzed below.

The active moment of force acted on the aircraft is only the aerodynamic moment assuming that the direction of the engine thrust coincides with the longitudinal axis of the aircraft:

$$\vec{M}_a^{(R)} = \vec{c}_3 M_a^{(R)} = \vec{a}_3 M_a^{(R)} \tag{21}$$

The detailed expression of $Q; Y; M_a^{(R)}$ will be given below.

The inertia moment induced by aircraft's attitude angular acceleration $\vec{\varepsilon}_a$ is:

$$\vec{M}_a^{(I)} = -J_a \vec{\varepsilon}_a \tag{22}$$

where J_a is the moment of inertia of the aircraft with respect to its center of mass.

The resultant force and moment of force acted on the aircraft are, respectively:

$$\vec{F}_a^{(total)} = \vec{G}_a + \vec{R} + \vec{P} + \vec{F}_a^{(I)} \tag{23}$$

$$\vec{M}_a^{(total)} = \vec{M}_a^{(R)} + \vec{M}_a^{(I)} \tag{24}$$

The active and inertia forces upon the cargo will be analyzed below.

The gravity of the cargo is:

$$\vec{G}_c = A\begin{pmatrix} 0 \\ -m_c g \end{pmatrix} \tag{25}$$

where m_c is the mass of the cargo.

The tension force vector of the extraction parachute upon the cargo is:

$$\vec{T} = B\begin{pmatrix} -T \\ 0 \end{pmatrix} \tag{26}$$

where T is the amplitude of the vector $\vec{T}$.

The inertia force upon the cargo induced by its linear acceleration $\vec{a}_c$ is:

$$\vec{F}_c^{(I)} = -m_c \vec{a}_c \tag{27}$$

The moments of active and inertia force upon the cargo will be analyzed below.

The active moment of force acted on the cargo is zero assuming that the tension force of the extraction parachute passes through the center of mass of it. So

$$\vec{M}_c = 0 \tag{28}$$

The inertia moment of force induced by the cargo's attitude angular acceleration $\vec{\varepsilon}_c$ is:

$$\vec{M}_c^{(I)} = -J_c \vec{\varepsilon}_c \tag{29}$$

where J_c is the moment of inertia of the cargo with respect to its center of mass.

The resultant force and moment of force acted on the cargo are, respectively:

$$\vec{F}_c^{(total)} = \vec{G}_c + \vec{T} + \vec{F}_c^{(I)} \tag{30}$$

$$\vec{M}_c^{(total)} = \vec{M}_c + \vec{M}_c^{(I)} \tag{31}$$

Equilibrium equation of generalized force

The essence of Kane method is to project all the active and inertia forces on the directions of the generalized coordinate curves, converting them into generalized forces, and then to write the equilibrium equations of the generalized forces. So it is also called as generalized D' Alembert's principle. For the multi-body system of aircraft-cargo researched in the paper, x, y, l, and θ are defined as generalized coordinates, so the matrices of projection (called as partial velocity and/or partial angular velocity in Kane method) can be calculated out according to equations (6), (11), (13) and (14) as follows:

$$\begin{cases} \dfrac{\partial \vec{v}_a}{\partial \dot{x}} = A\begin{pmatrix} 1 \\ 0 \end{pmatrix}; \ \dfrac{\partial \vec{v}_a}{\partial \dot{y}} = A\begin{pmatrix} 0 \\ 1 \end{pmatrix}; \ \dfrac{\partial \vec{v}_a}{\partial \dot{l}} = 0; \ \dfrac{\partial \vec{v}_a}{\partial \dot{\theta}} = 0 \\ \dfrac{\partial \vec{v}_c}{\partial \dot{x}} = A\begin{pmatrix} 1 \\ 0 \end{pmatrix}; \ \dfrac{\partial \vec{v}_c}{\partial \dot{y}} = A\begin{pmatrix} 0 \\ 1 \end{pmatrix}; \ \dfrac{\partial \vec{v}_c}{\partial \dot{l}} = A\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}; \ \dfrac{\partial \vec{v}_c}{\partial \dot{\theta}} = A\begin{pmatrix} -l\sin\theta \\ l\cos\theta \end{pmatrix} \\ \dfrac{\partial \vec{V}_a}{\partial \dot{x}} = 0; \ \dfrac{\partial \vec{V}_a}{\partial \dot{y}} = 0; \ \dfrac{\partial \vec{V}_a}{\partial \dot{l}} = 0; \ \dfrac{\partial \vec{V}_a}{\partial \dot{\theta}} = \vec{a}_3 \\ \dfrac{\partial \vec{V}_c}{\partial \dot{x}} = 0; \ \dfrac{\partial \vec{V}_c}{\partial \dot{y}} = 0; \ \dfrac{\partial \vec{V}_c}{\partial \dot{l}} = 0; \ \dfrac{\partial \vec{V}_c}{\partial \dot{\theta}} = \vec{a}_3 \end{cases} \tag{32}$$

The equilibrium equations of the generalized forces are:

$$\begin{aligned} &\vec{F}_a^{(total)} \cdot \frac{\partial \vec{v}_a}{\partial \dot{q}_i} + \vec{F}_c^{(total)} \cdot \frac{\partial \vec{v}_c}{\partial \dot{q}_i} + \vec{M}_a^{(total)} \cdot \frac{\partial \vec{V}_a}{\partial \dot{q}_i} \\ &\quad + \vec{M}_c^{(total)} \cdot \frac{\partial \vec{V}_c}{\partial \dot{q}_i} \\ &= 0 \ (i = 1 \sim 4) \end{aligned} \tag{33}$$

where $q_1 = x$, $q_2 = y$, $q_3 = l$, $q_4 = \theta$.

Substituting equations (1) ~ (4), (7), (12), (15) ~ (32) into equation (33) yields the aircraft-cargo system coupling dynamics equations, as follows:

$$\begin{cases} (m_a + m_c)\ddot{x} + m_c\ddot{l}\cos\theta - m_c l\ddot{\theta}\sin\theta - m_c l\dot{\theta}^2\cos\theta - 2m_c\dot{l}\dot{\theta}\sin\theta \\ \quad -P\cos\theta + Q\cos\alpha + Y\sin\alpha + T\cos\alpha = 0 \\ (m_a + m_c)\ddot{y} + m_c\ddot{l}\sin\theta + m_c l\ddot{\theta}\cos\theta - m_c l\dot{\theta}^2\sin\theta + 2m_c\dot{l}\dot{\theta}\cos\theta \\ \quad -P\sin\theta + Q\sin\alpha - Y\cos\alpha + T\sin\alpha + (m_a + m_c)g = 0 \\ m_c\ddot{x}\cos\theta + m_c\ddot{y}\sin\theta + m_c\ddot{l} - m_c l\dot{\theta}^2 + T\cos(\theta - \alpha) + m_c g\sin\theta = 0 \\ m_c l\ddot{x}\sin\theta - m_c l\ddot{y}\cos\theta - \left(J_a + J_c + m_c l^2\right)\ddot{\theta} - 2m_c l\dot{l}\dot{\theta} \\ \quad + Tl\sin(\theta - \alpha) + M_a^{(R)} - m_c g l\cos\theta = 0 \end{cases} \tag{34}$$

We know:

$$\begin{cases} \dot{x} = v\cos u \\ \dot{y} = v\sin u \end{cases} \tag{35}$$

where v is the flight velocity of the aircraft.

The time derivative of equation (35) yields:

$$\begin{cases} \ddot{x} = \dot{v}\cos u - v\dot{u}\sin u \\ \ddot{y} = \dot{v}\sin u \; 1 \; v\dot{u}\cos u \end{cases} \tag{36}$$

We define:

$$v = \dot{h} \tag{37}$$

$$V = \dot{l} \tag{38}$$

$$a = h - u \tag{39}$$

then substitute equations (36) ∼ (39) into equation (34), the following can be obtained:

$$\begin{cases} (m_a \, 1 \, m_c)\dot{v}\cos u - (m_a \, 1 \, m_c)v\dot{u}\sin u \, 1 \, m_c\dot{V}\cos h - m_c l\dot{v}\sin h \\ -m_c l v^2\cos h - 2m_c V v\sin h - P\cos h \, 1 \, Q\cos u \, 1 \, Y\sin u \, 1 \, T\cos u = 0 \\ \\ (m_a \, 1 \, m_c)\dot{v}\sin u \, 1 \, (m_a \, 1 \, m_c)v\dot{u}\cos u \, 1 \, m_c\dot{V}\sin h \, 1 \, m_c l\dot{v}\cos h \\ -m_c l v^2\sin h \, 1 \, 2m_c V v\cos h - P\sin h \, 1 \, Q\sin u - Y\cos u \\ \; 1 \, T\sin u \, 1 \, (m_a \, 1 \, m_c)g = 0 \\ \\ m_c\dot{v}\cos a \, 1 \, m_c v\dot{u}\sin a \, 1 \, m_c\dot{V} - m_c l v^2 \, 1 \, T\cos a \, 1 \, m_c g\sin h = 0 \\ \\ m_c l\dot{v}\sin a - m_c l v\dot{u}\cos a - \left(J_a \, 1 \, J_c \, 1 \, m_c l^2\right)\dot{v} \\ -2m_c lV v \, 1 \, Tl\sin a \, 1 \, M_a^{(R)} - m_c gl\cos h = 0 \end{cases} \tag{40}$$

where:

$$\begin{cases} Q = qS\left[C_{Q0} \, 1 \, C_Q^a a^2 \, 1 \, C_Q^{d_{stb}}(a \, 1 \, d_{stb})^2\right] \\ Y = qS\left(C_{Y0} \, 1 \, C_Y^a a \, 1 \, C_Y^{d_{stb}} d_{stb} \, 1 \, C_Y^{d_e} d_e\right) \\ M_a^{(R)} = qSb\left(m^a a \, 1 \, m^{d_{stb}} d_{stb} \, 1 \, m^v v \, 1 \, m^{d_e} d_e\right) \end{cases} \tag{41}$$

$$q = \frac{1}{2} r v^2 \tag{42}$$

In the above equations, q is the dynamic pressure upon the aircraft. r is the air density. S and b are the aerodynamic characteristic area and length of the aircraft, respectively. d_{stb} is the deflection of the horizontal stabilizer of the aircraft which remains at trimmed value throughout the ultra-low altitude heavy cargo airdrop. $C_{Q0}; C_Q^a; C_Q^{d_{stb}}; C_{Y0}; C_Y^a; C_Y^{d_{stb}}; C_Y^{d_e}; m^a; m^{d_{stb}}; m^v; m^{d_e}$ are the aerodynamic coefficients of the aircraft.

Now we calculate the tension force of the extraction parachute neglecting the parachute's random flutter in the turbulence, the mass of it, and the elasticity of its suspension line.

Substituting equations (1), (3), (35), (37) ∼ (39) into equation (11) yields the expression of the velocity vector of the cargo in the flight path coordinate system, as follows:

$$\vec{v}_c = B\begin{pmatrix} v \, 1 \, V\cos a - l v\sin a \\ l v\cos a \, 1 \, V\sin a \end{pmatrix} \tag{43}$$

The velocity of the parachute $\vec{v}_p$ is the same as that of the cargo under the assumptions aforementioned, so $\vec{v}_p = \vec{v}_c$, and the aerodynamic drag of the parachute and the extraction force of it on the cargo can be estimated as the following:

$$\begin{aligned} T &= \frac{1}{2} r\|\vec{v}_p\|^2 S_p = \frac{1}{2} r\|\vec{v}_c\|^2 S_p \\ &= \frac{1}{2} r\left(l^2 v^2 - 2lv v\sin a \, 1 \, v^2 \, 1 \, 2vV\cos a \, 1 \, V^2\right)S_p \end{aligned} \tag{44}$$

where S_p is the characteristic area of the extraction parachute.

Equations (35), (37)-(42) and (44) are the dynamics model of aircraft-cargo multi-body coupling system with the deflection of elevator d_e as control input.

The aircraft's robust flight control law design

There are two critical stages in ultra-low altitude heavy cargo airdrop. In the first stage, the aircraft flies at predetermined altitude and velocity with the cargo fixed in the cabin, while their centers of mass coinciding with each other. The cargo can be regarded as a part of the "aircraft" with the total mass $m_a \, 1 \, m_c$ and inertia moment $J_a \, 1 \, J_c$. The goal of the control law is to realize aircraft's robust stability in the presence of the aerodynamic coefficients perturbation induced by ground effect. In the second stage, the cargo slides outward under the extraction force of the parachute until separating from the aircraft. The goal of the control law is to realize robust disturbance restraint. That is to say, the aircraft's state error response to the disturbance moment from the cargo can be attenuated within a low level in the presence of the aircraft's aerodynamic coefficients perturbation.

Robust stability control law design when the cargo fixed

Just as mentioned above, the aircraft combined with the cargo inside can be regarded as an "aircraft" with total mass $m_a \, 1 \, m_c$ and inertia moment $J_a \, 1 \, J_c$ when the cargo fixed in the aircraft and their centers of mass coinciding with each other. The "aircraft" state equation can be written as:

$$\begin{cases} \dot{y} = v\sin(h - a) \\ \dot{v} = \dfrac{P\cos a - Q}{m_a \, 1 \, m_c} - g\sin(h - a) \\ \dot{a} = v \, 1 \, \dfrac{-P\sin a - Y}{(m_a \, 1 \, m_c)v} \, 1 \, \dfrac{g}{v}\cos(h - a) \\ \dot{v} = \dfrac{M_R}{J_a \, 1 \, J_c} \\ \dot{h} = v \end{cases} \tag{45}$$

where Q, Y, $M_a^{(R)}$ can be calculated according toequation (41).

We define the trimmed state, thrust of engine, deflection of horizontal stabilizer and elevator as $X^* = (y^*, v^*, a^*, v^*, h^*)^T$, P^*, d_{stb}^* and d_e^*, respectively. The values of y^*, v^* are predetermined, while $v^* = 0$ and $h^* = a^*$ are self-evident. The function of the horizontal stabilizer is to make d_e^* be 0. The values of the trimmed variables $P^*; a^*; d_{stb}^*$ can be obtained by solving equilibrium equations.

The approximate linear equation upon the trimmed point can be written as follows:

$$D\dot{X} = A \cdot DX 1 B_2 \cdot U \tag{46}$$

where $DX = X - X^*$ is the state error. $X = (y, v, a, v, h)^T$ is the flight state. $U = d_e$ is the control input.

The values of the trimmed thrust of engine, angle of attack, deflection of horizontal stabilizer and the corresponding system matrix and control matrix are calculated out, as follows, using the parameters listed in Table I:

$$\begin{cases} P^* = 1{:}4753 \times 10^5(\mathrm{N}) \\ a^* = 0{:}0401(\mathrm{rad}) = 2{:}30^o \\ d_{stb}^* = -0{:}1044(\mathrm{rad}) = -5{:}98^o \end{cases} \tag{47}$$

$$A = \begin{pmatrix} 0 & 0 & -75 & 0 & 75 \\ 0 & -0{:}0262 & 9{:}2327 & 0 & -9{:}8 \\ 0 & -0{:}0035 & -0{:}6080 & 1 & 0 \\ 0 & 0 & -1{:}8293 & -8{:}9567 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{48}$$

$$B_2 = \begin{pmatrix} 0 \\ 0 \\ -0{:}0291 \\ -0{:}6912 \\ 0 \end{pmatrix} \tag{49}$$

The above trimmed point and the corresponding approximate linear model upon it is obtained according to the nominal values of the aircraft's aerodynamic lift coefficients. While in practice, the coefficients will be perturbed by ground effect. The real values of them are $C_{Y0} = C_{Y0}^* 1 DC_{Y0}$ and $C_Y^a = C_Y^{a*} 1 DC_Y^a$, respectively, where C_{Y0}^*, C_Y^{a*} are the nominal values, and DC_{Y0}, DC_Y^a are the perturbation values. The nominal trimmed point X^* calculated above will be deviated from the real one X^{**} by $dX^* = X^* - X^{**}$. Meanwhile, the perturbation of the approximate linear model's system matrix $DA = A - A^*$ comes into existence, where A and A^* are the real and nominal values, respectively. The approximate linear model upon the nominal trimmed point X^* can be easily derived out as follows:

$$D\dot{X} = (A^* 1 DA) \cdot DX 1 B_2U 1 A^* \cdot dX^* \tag{50}$$

where:

$$DA = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -0{:}0013\ DC_{Y0} & -0{:}0980\ DC_Y^a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{51}$$

The matrix A^* takes the same value as in equation (48).

Equation (50) represents a linear system with external disturbance $A^* \cdot dX^*$. The aerodynamic lift coefficients vary exponentially regarding to the aircraft's flight height (Yang and Deng, 1997), that is, within the height range of the ground effect, aerodynamic coefficients increase along with the decrease of the aircraft's flight height; however, its rate of change drastically decreases up to 0. The lift coefficients' rate of change approaches to 0 on the ultra-low altitude heavy cargo airdrop condition, so it is reasonable that the lift coefficients perturbation DC_{Y0}, DC_Y^a as well as dX^* and $A^* \cdot dX^*$ are considered as constant, when designing the control law. We know, the state error DX cannot converge to 0 only by the state feedback regulator $U = K \cdot DX$ in the presence of constant disturbance. The flight altitude is essential to the safety of both the aircraft and cargo, so it is necessary to introduce the time integral of the aircraft's flight height:

$$s = \int_0^t Dy(t)dt \tag{52}$$

as a new state in designing the control law. We define an augmented state vector:

$$D\overline{X} = \begin{pmatrix} DX \\ s \end{pmatrix} \tag{53}$$

The time derivative of equation (53), combining with equations (50) and (52), yields the augmented state equation as follow:

$$D\dot{\overline{X}} = (\overline{A}^* 1 D\overline{A}) \cdot D\overline{X} 1 \overline{B}_2 \cdot U 1 \overline{v} \tag{54}$$

where:

$$\overline{A}^* = \begin{pmatrix} A^* & 0_{5\times1} \\ (1; 0_{1\times4}) & 0 \end{pmatrix}; \quad D\overline{A} = \begin{pmatrix} DA & 0_{5\times1} \\ 0_{1\times5} & 0 \end{pmatrix};$$

$$\overline{B}_2 = \begin{pmatrix} B_2 \\ 0 \end{pmatrix}; \quad \overline{v} = \begin{pmatrix} A^* \cdot dX^* \\ 0 \end{pmatrix} \tag{55}$$

Substituting equation (51) into equation (55) yields:

Table I. Some parameter's nominal values

Parameters	Values
m_a/kg	110×10^3
S/m^2	320
q/(kg/m^3)	1.225
v^*/(m/s)	75
C_Y^a	6.0707
m_c/kg	40 × 103
b/m	6
g/(m/s^2)	9.8
C_{Y0}^*	1.1475
$m_z^{d_e}$	–1.0585

$$\mathrm{D}\overline{A} = \begin{pmatrix} 0 & 0 & 0 & 0_{1\times 3} \\ 0 & 0 & 0 & 0_{1\times 3} \\ 0 & -0{:}0013\ \mathrm{DC}_{Y0} & -0{:}0980\ \mathrm{DC}_Y^a & 0_{1\times 3} \\ 0_{3\times 1} & 0_{3\times 1} & 0_{3\times 1} & 0_{3\times 3} \end{pmatrix} \tag{56}$$

Factorization of it yields:

$$\mathrm{D}\overline{A} = \mathrm{ERF} \tag{57}$$

where:

$$E = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -0{:}0013 & -0{:}0980 \\ 0_{3\times 1} & 0_{3\times 1} \end{pmatrix}; R = \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix}; R_1 = \frac{\mathrm{DC}_{Y0}}{|\mathrm{DC}_{Y0}|_{sup}};$$
$$R_2 = \frac{\mathrm{DC}_Y^a}{|\mathrm{DC}_Y^a|_{sup}}; F = \begin{pmatrix} 0 & |\mathrm{DC}_{Y0}|_{sup} & 0 & 0_{1\times 3} \\ 0 & 0 & |\mathrm{DC}_Y^a|_{sup} & 0_{1\times 3} \end{pmatrix} \tag{58}$$

$|\mathrm{DC}_{Y0}|_{sup}$ and $|\mathrm{DC}_Y^a|_{sup}$ are the supremums of $|\mathrm{DC}_{Y0}|$ and $|\mathrm{DC}_Y^a|$, respectively, so $|\sum^1| \le 1$, $|\sum^2| \le 1$, and:

$$R^T R = \begin{pmatrix} R_1{}^2 & 0 \\ 0 & R_2{}^2 \end{pmatrix} \le I \tag{59}$$

For the linear system with bounded parameters perturbation satisfying equations (57) and (59), the necessary and sufficient condition that there exists a state feedback controller $U = K \cdot \mathrm{D}\overline{X}$ making the corresponding closed-loop system:

$$\mathrm{D}\dot{\overline{X}} = \left(\overline{A}^* 1\ \mathrm{D}\overline{A} 1\ \overline{B}_2 \cdot K\right) \cdot \mathrm{D}\overline{X} 1\ \overline{v} \tag{60}$$

quadratically stable is that there exists a proper parameter $« > 0$ making the following Riccati inequality:

$$P\overline{A}^* 1\ \overline{A}^{*T} P 1\ P\left(EE^T - «^{-2}\overline{B}_2\overline{B}_2{}^T\right)P 1\ F^T F < 0 \tag{61}$$

has a positive definite solution. Furthermore, if the parameter « exists, then the corresponding state feedback gain matrix is (Shen, 1996):

$$K = -\frac{1}{2«^2}\overline{B}_2{}^T P \tag{62}$$

The inequality (61) is equivalent to the following Riccati equation:

$$P\overline{A}^* 1\ \overline{A}^{*T} P 1\ P\left(EE^T - «^{-2}\overline{B}_2\overline{B}_2{}^T\right)P 1\ F^T F 1\ sI = 0 \tag{63}$$

where $s > 0$ can be selected freely within proper range.

In the paper, the relative perturbation of the aerodynamic lift coefficients is assumed as 10 per cent, so $|\mathrm{DC}_{Y0}|_{sup} = 0{:}1$; $|\mathrm{DC}_Y^a|_{sup} = 0{:}6$. Substituting them into equation (58) calculates matrix F, then the aircraft's state feedback gain matrix, denoted by K_I, can be calculated out according to equation (62) and (63), as follows:

$$K_I = \left(0{:}1314\ 0{:}1449\ -13{:}4033\ 1{:}5016\ 27{:}5562\ 0{:}0131\right) \tag{64}$$

Robust disturbance restraint control law design when the cargo sliding inside the cabin

The cargo slides in the cabin outward under the extraction force of the parachute as soon as it is unlocked, and then exerts disturbance moment upon the aircraft and affect its pitching motion primarily. The approximate linear model can be modified from equation (54), as follows:

$$\mathrm{D}\dot{\overline{X}} = \left(\overline{A}^* 1\ \mathrm{D}\overline{A}\right) \cdot \mathrm{D}\overline{X} 1\ \overline{B}_1 \cdot w 1\ \overline{B}_2 \cdot U 1\ \overline{v} \tag{65}$$

where $w = \frac{M_c}{J_a}$ is the aircraft's pitching angular acceleration induced by M_c, while M_c is the disturbance moment from the cargo. $\overline{B}_1 = (0; 0; 0; 1; 0; 0)^T$.

Now our task is to design the aircraft's robust disturbance restraint control law which guarantees the system's internally stable and minimizes the gain of state error response $\mathrm{D}\overline{X}(t)$ to the external disturbance w(t) in the presence of the bounded parameters perturbation, meanwhile the energy consumption of control input should be taken into account. In the light of the above consideration, the following performance index is adopted:

$$J = \int_0^{t_f} \left(\mathrm{D}\overline{X}^T \cdot Q \cdot \mathrm{D}\overline{X} 1\ U^T \cdot R \cdot U\right) dt \tag{66}$$

We know, the performance of control system depends on rather the relative than the absolute value of weight coefficients in the performance index, meanwhile R in equation (66) is a scalar, so we would rather order that R = 1 without loss of generality.

Equation (66) can be rewritten in the form of energy of the evaluation signal z, as follows:

$$J = \int_0^{t_f} z^T z dt = \int_0^{t_f} \|z\|^2 dt \tag{67}$$

where:

$$z = C \cdot \mathrm{D}\overline{X} 1\ D \cdot U \tag{68}$$

$$\left\{ C = \begin{pmatrix} \sqrt{Q} \\ 0_{1\times 6} \end{pmatrix}; D = \begin{pmatrix} 0_{6\times 1} \\ \sqrt{R} \end{pmatrix} \right. \tag{69}$$

Now we would design the state feedback controller $U = K \cdot \mathrm{D}\overline{X}$, which guarantees the following closed-loop system:

$$\mathrm{D}\dot{\overline{X}} = \left(\overline{A}^* 1\ \mathrm{D}\overline{A} 1\ \overline{B}_2 \cdot K\right) \cdot \mathrm{D}\overline{X} 1\ \overline{B}_1 \cdot w 1\ \overline{v} \tag{70}$$

internally stable and minimizes the energy gain of the signal z to the external input signal w in the presence of the parameters perturbation $\mathrm{D}\overline{A} = \mathrm{ERF}$ satisfying equation (59).

Now we introduce a theorem as follows:

[Theorem] The necessary and sufficient condition of that the closed-loop system:

$$\begin{cases} \mathrm{D}\overline{\mathrm{X}} = \left(\overline{\mathrm{A}}^{*} \ 1 \ \mathrm{D}\overline{\mathrm{A}} \ 1 \ \overline{\mathrm{B}}_2 \cdot \mathrm{K}\right) \cdot \mathrm{D}\overline{\mathrm{X}} \ 1 \ \overline{\mathrm{B}}_1 \cdot \mathrm{w} \ 1 \ \overline{\mathrm{v}} \\ \mathrm{z} = (\mathrm{C} \ 1 \ \mathrm{D} \cdot \mathrm{K}) \cdot \mathrm{D}\overline{\mathrm{X}} \end{cases} \tag{71}$$

is internally stable meanwhile $\|\mathrm{T}_{\mathrm{zw}}(\mathrm{s})\|_{\infty} < 1$ is that there exists parameter $/ > 0$ making the following Riccati inequality has a positive definite solution $\mathrm{P} > 0$:

$$\mathrm{P}\overline{\mathrm{A}}^{*} \ 1 \ \overline{\mathrm{A}}^{*\mathrm{T}}\mathrm{P} \ 1 \ \mathrm{P}\left(\overline{\mathrm{B}}_1\overline{\mathrm{B}}_1{}^{\mathrm{T}} \ 1 \ /^{2}\mathrm{E}\mathrm{E}^{\mathrm{T}} - \overline{\mathrm{B}}_2\overline{\mathrm{B}}_2{}^{\mathrm{T}}\right)\mathrm{P}$$

$$1 \ \mathrm{C}^{\mathrm{T}}\mathrm{C} \ 1 \ \frac{1}{/^{2}}\mathrm{F}^{\mathrm{T}}\mathrm{F} < 0 \tag{72}$$

and, if the parameter $/$ exists, the corresponding matrix is $\mathrm{K} = -\overline{\mathrm{B}}_2{}^{\mathrm{T}}\mathrm{P}$.(Shen, 1996)

Obviously, the problem of gain minimization we would resolve here is not the one which can be resolved directly using the theorem mentioned above, so it is necessary to transform our problem into the one applicative for the theorem.

The transfer function from w to z in closed-loop system depicted by equation (71) is:

$$\mathrm{T}_{\mathrm{zw}}(\mathrm{s}) = (\mathrm{C} \ 1 \ \mathrm{DK}) \cdot \left[\mathrm{sI} - \left(\overline{\mathrm{A}}^{*} \ 1 \ \mathrm{D}\overline{\mathrm{A}} \ 1 \ \overline{\mathrm{B}}_2 \cdot \mathrm{K}\right)\right]^{-1} \cdot \overline{\mathrm{B}}_1 \tag{73}$$

We notice that $\|\mathrm{T}_{\mathrm{zw}}(\mathrm{s})\|_{\infty} < g$ is equivalent to the following:

$$\left\|(\mathrm{C} \ 1 \ \mathrm{DK}) \cdot \left[\mathrm{sI} - \left(\overline{\mathrm{A}}^{*} \ 1 \ \mathrm{D}\overline{\mathrm{A}} \ 1 \ \overline{\mathrm{B}}_2 \cdot \mathrm{K}\right)\right]^{-1} \cdot g^{-1}\overline{\mathrm{B}}_1\right\|_{\infty} < 1 \tag{74}$$

The transfer function inequation (74) is just that of the following system:

$$\begin{cases} \mathrm{D}\overline{\mathrm{X}} = \left(\overline{\mathrm{A}}^{*} \ 1 \ \mathrm{D}\overline{\mathrm{A}} \ 1 \ \overline{\mathrm{B}}_2 \cdot \mathrm{K}\right) \cdot \mathrm{D}\overline{\mathrm{X}} \ 1 \ g^{-1}\overline{\mathrm{B}}_1 \cdot \mathrm{w} \ 1 \ \overline{\mathrm{v}} \\ \mathrm{z} = (\mathrm{C} \ 1 \ \mathrm{D} \cdot \mathrm{K}) \cdot \mathrm{D}\overline{\mathrm{X}} \end{cases} \tag{75}$$

Now the problem turns into seeking $g = g_{\min} > 0$ and the corresponding matrix K satisfying equation (74), where $g_{\min}$ is the optimal (minimum) index of disturbance attenuation. According to the theorem aforementioned, the necessary and sufficient condition of $\|\mathrm{T}_{\mathrm{zw}}(\mathrm{s})\|_{\infty} < g$ is that there exists parameter $/ > 0$ making the following Riccati inequality has a positive definite solution $\mathrm{P} > 0$:

$$\mathrm{P}\overline{\mathrm{A}}^{*} \ 1 \ \overline{\mathrm{A}}^{*\mathrm{T}}\mathrm{P} \ 1 \ \mathrm{P}\left(g^{-2}\overline{\mathrm{B}}_1\overline{\mathrm{B}}_1{}^{\mathrm{T}} \ 1 \ /^{2}\mathrm{E}\mathrm{E}^{\mathrm{T}} - \overline{\mathrm{B}}_2\overline{\mathrm{B}}_2{}^{\mathrm{T}}\right)\mathrm{P}$$

$$1 \ \mathrm{C}^{\mathrm{T}}\mathrm{C} \ 1 \ \frac{1}{/^{2}}\mathrm{F}^{\mathrm{T}}\mathrm{F} < 0 \tag{76}$$

and, if the parameter $/$ exists, the corresponding state feedback gain matrix is:

$$\mathrm{K} = -\overline{\mathrm{B}}_2{}^{\mathrm{T}}\mathrm{P} \tag{77}$$

The inequality equation (76) is equivalent to the following Riccati equation:

$$\mathrm{P}\overline{\mathrm{A}}^{*} \ 1 \ \overline{\mathrm{A}}^{*\mathrm{T}}\mathrm{P} \ 1 \ \mathrm{P}\left(g^{-2}\overline{\mathrm{B}}_1\overline{\mathrm{B}}_1{}^{\mathrm{T}} \ 1 \ /^{2}\mathrm{E}\mathrm{E}^{\mathrm{T}} - \overline{\mathrm{B}}_2\overline{\mathrm{B}}_2{}^{\mathrm{T}}\right)\mathrm{P}$$

$$1 \ \mathrm{C}^{\mathrm{T}}\mathrm{C} \ 1 \ \frac{1}{/^{2}}\mathrm{F}^{\mathrm{T}}\mathrm{F} \ 1 \ s\mathrm{I} = 0 \tag{78}$$

where $s > 0$ can be selected freely within proper range.

The supremums of the aerodynamic lift coefficients' perturbation are assumed as $|\mathrm{DC}_{\mathrm{Y}0}|_{\sup} = 0{:}1; |\mathrm{DC}_{\mathrm{Y}}^{a}|_{\sup} = 0{:}6$ like before. Substituting them into equation (58) calculates matrix F. Substituting Q = diag(0,0,0,0,20,0) into equation (69) yields the value of matrix C. With the parameters s and $/$ taking the values $s = 10^{-4}$ and $/ = 1$, the minimum $g_{\min} = 1.5$ making the Riccati equation (78) has a positive definite solution is obtained using bi-search procedure. Finally, the optimal state feedback gain matrix is calculated out as follows according to equation (77):

$$\mathrm{K}_{\mathrm{II}} = \left(0{:}0340 \ \ 0{:}2972 \ \ -8{:}8719 \ \ 3{:}1482 \ \ 37{:}0983 \ \ 0{:}0014\right) \tag{79}$$

Numerical simulation

The overall process of ultra-low altitude heavy cargo airdrop can be divided into the following sub-stages:

- aircraft downslides;
- the ultra-low altitude-level flight with cargo fixed in the cabin;
- the flight with the cargo sliding inside the cabin; and
- aircraft climbs after the cargo separation.

The paper researches the system dynamics modeling and the aircraft's control law concerning the second and third sub-stages above, so the numerical simulation focuses on the two sub-stages mainly. The simulation scenario is considered as the following: the aircraft flies from it's initial states containing ultra low altitude and velocity predetermined, as well as the other trimmed states, while the cargo is fixed on the center of mass of the aircraft. The robust stability flight control law works in the sub-stage. After the error of flight height has been eliminated, the aircraft continues to fly at the ultra-low altitude for seconds. Then with the extraction parachute deployment, the cargo slides outward in the cabin till separation from the aircraft. The robust disturbance restraint control law works in the sub-stage. The deflection amplitude of the elevator is limited so as to let the simulation close to the practical condition. The simulation continues for seconds after the cargo separation, while the throttle and horizontal stabilizer keep at the original trimmed values and the elevator deflection keeps at zero so as to observe the natural response of the aircraft (Table II).

In addition, the position of the aircraft-cargo system's common center of mass (denoted by x_{cm}) relative to that of the aircraft, and the aircraft-cargo system's common moment of inertia (denoted by $\mathrm{J}_{\mathrm{sys}}$) with respect to the system's common center of mass are both calculated out according to the equations (80) and (81), in spite that the two variables are both non-important in the modeling method adopted by the paper, as the aircraft rather than the whole system (namely the aircraft combined with the cargo inside) is treated as controlled plant, so the action upon the aircraft from the cargo is treated as

Table II. Some parameters called by numerical simulation

Parameters	Values
J_a/(kg·m^2)	9×10^6
L_2/m(the length of the rail for cargo sliding in the cabin)	10
y^*/m(the predetermined airdrop height)	5
J_c/(kg·m$_2$)	1.13×106
S_p/m$_2$	50.27
$\vert d_e\vert_{sup}$/deg	30

external disturbance when we design the aircraft's flight control law:

$$x_{cm} = \frac{m_c}{m_a \; 1 \; m_c} l \tag{80}$$

$$J_{sys} = J_a \; 1 \; J_c \; 1 \; m_a x_{cm}{}^2 \; 1 \; m_c(l - x_{cm})^2 \tag{81}$$

The following are the results of the numerical simulation.

In the simulation curves above, the red small circle denotes the moment of deployment of the extraction parachute and the red small rectangle denotes the moment of the cargo separation. Figure 2 shows that the aircraft's flight height converges to the predetermined value quickly under the robust control law proposed. The velocity, angle of attack and pitch angle of the aircraft cannot converge to the trimmed values due to the nearly constant disturbance from the difference between the real and nominal lift coefficients values, while their steady errors are within the tolerance acceptable for the heavy cargo airdrop. At the moment of 15 s, the extraction parachute is deployed while the cargo being unlocked simultaneously, and then the cargo slides outward along the rail inside exerting increased disturbance moment on the aircraft. Figure 2 shows that the error response of the aircraft's flight height, velocity, angle of attack and pitch angle are all very small under the H_∞ robust control law proposed. Specifically speaking, the variation of the flight height, velocity, pitch angle and angle of attack are only 0.64 m, 0.18 m/s, 1.44°, and 0.42°, respectively. Figure 3 shows that the sliding time of the cargo in the cabin is

Figure 2 The aircraft's states and the deflection of its elevator

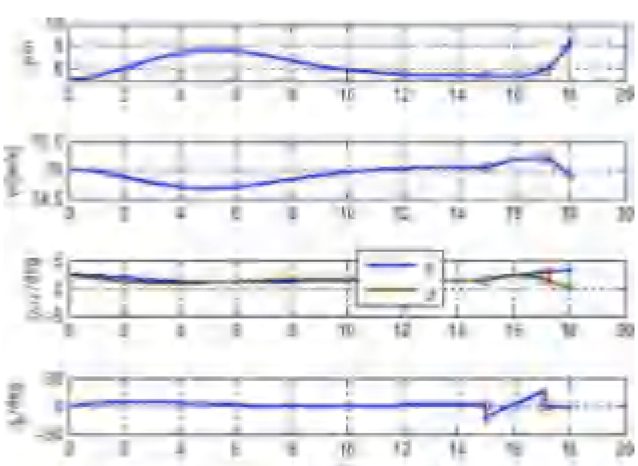
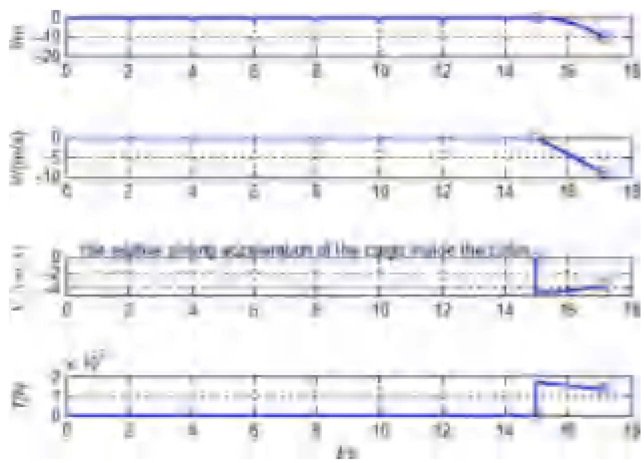


Figure 3 The Cargo's motion inside the cabin and the tension force of the extraction parachute



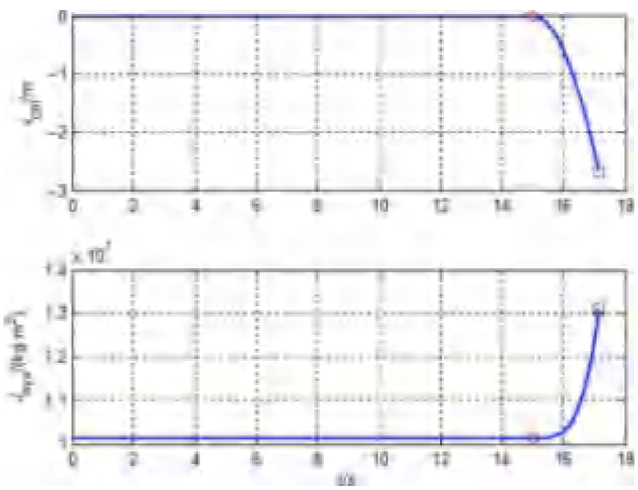
about 2.13 s. The relative sliding velocity of the cargo increases from the original value of 0 to the final of about 9.13 m/s. However, the tension force of the extraction parachute decreases from the original value of 1.73×10^5 N to the final of 1.35×10^5 N. The reason of it is that the cargo's and the parachute's relative backward velocity to the aircraft increases, so the parachute's absolute velocity in the air as well as the aerodynamic drag on it and then the extraction force to the cargo decreases. Thus we guess that the value of the cargo's relative sliding acceleration maybe decreases, which is verified by the 3rd subfigure of Figure 3. In fact, the cargo's relative sliding acceleration decreases from the original value of 4.6 m/s^2 to the final of 3.8 m/s^2. This differs from the assumption of the constant sliding acceleration value documented in many literatures. The aircraft flight simulation of 1-s duration time is conducted after the cargo separation so as to observe whether the dangerous state of the aircraft may appear. The simulation results show that the aircraft's flight height significantly increases due to the loss of the load on it. The angle of attack converges to a new equilibrium value (smaller than the original equilibrium one) under the stabilizing moment of the aircraft itself. The pitch angle's variation is inconspicuous due to the increase of flight path angle and the simultaneous decrease of the angle of attack. We see that no dangerous state of the aircraft appears.

Figure 4 shows that the position of the whole system's common center of mass relative to that of the aircraft varies to the final value of –2.67 m from the original one of 0, and the whole system's common moment of inertia increases to the final value of 1.308×10^7 kg·m^2 from the original one of 1.013×10^7 kg·m^2(that is the sum of J_a and J_c), while the cargo sliding outward inside the cabin of the aircraft.

Conclusion

The aircraft-cargo multi-body system dynamics model for the ultra-low altitude heavy cargo airdrop is derived using the Kane method. The model is closed, so neither the value of tension force of the extraction parachute nor the relative sliding acceleration and velocity of the cargo inside the cabin needs to be assumed in simulation. The H_∞ control theory based robust

Figure 4 The whole system's common center of mass and moment of inertia



stability and robust disturbance restraint control law for the aircraft are designed applied in the two critical sub-stages that are the flight with the cargo fixed and the cargo sliding in the cabin, respectively. Numerical simulation shows the effectiveness of the control laws proposed.

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