Trigonometric Regularization and Continuation Method Based Time-Optimal Control of Hypersonic Vehicles

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Abstract: Aiming at the time-optimal control problem of hypersonic vehicles (HSV) in ascending stage, a trigonometric regularization method (TRM) is introduced based on the indirect method of optimal control. This method avoids analyzing the switching function and distinguishing between singular control and bang-bang control, where the singular control problem is more complicated. While in bang-bang control, the costate variables are unsmooth due to the control jumping, resulting in difficulty in solving the two-point boundary value problem (TPBVP) induced by the indirect method. Aiming at the easy divergence when solving the TPBVP, the continuation method is introduced. This method uses the solution of the simplified problem as the initial value of the iteration. Then through solving a series of TPBVP, it approximates to the solution of the original complex problem. The calculation results show that through the above two methods, the time-optimal control problem of HSV in ascending stage under the complex model can be solved conveniently.

Key words: hypersonic vehicle (HSV); optimal control; trigonometric regularization method (TRM); continuation method

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0 Introduction

The optimal control problem (OCP) of hypersonic vehicles (HSV) has always been a hot topic of research. The time-optimal control problem has great military significance and has attracted widespread attention[1-3]. In some military launches, the mission places an important requirement on the rapidity of the spacecraft reaching the designed state, which is necessary to develop the algorithm for time-optimal control.

The methods for solving OCP include direct[4] and indirect[5] methods.

The direct method transforms the OCP into a finite-dimensional nonlinear parameter optimization problem through parametric methods[6]. Zhang[7] proposed and verified the framework of solving multi-objective trajectory optimization problem of HSV based on pseudo-spectral methods. Yang et al.[8] combined extrapolation and bisection methods to solve the time-optimal control problem from a solution gained by convex optimization. The direct method has the merits of strong applicability, but it is computationally intensive and optimality unguaranteed.

The indirect method has high solution accuracy and the qualities of first-order optimality, but its calculation of the costate variables is a bit complicated. Based on the riccati equation, Qiao et al.[9] designed the control law of HSV with the quadratic energy-consuming index, and designed a spiral dive maneuvering trajectory. Considering time-consuming index, Zhang et al.[10] used a genetic algorithm to gain the costate variable initial value, and realized the trajectory optimization of HSV in ascending stage. In the indirect method, enormous effort may be taken to analyze the switching function, derive the optimal control, and calculate the costate variables that

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do not have physical meaning. It makes the calculation process too cumbersome. And the costate variables is difficult to solve.

The OCP of atmospheric vehicles is prone to singular optimal control, which is hard to solve. In order to avoid the problem of analyzing the switching function which may result in singular control, and solve the problem of easy divergence of two-point boundary value problem (TPBVP), this paper introduces the trigonometric regularization method (TRM). This method avoids the need to analyze the switching function by applying the necessary conditions of first-order optimality, and gains the optimal control easily. The continuation method is also introduced to make the TPBVP easier to converge. Finally, the solution of the OCP under the simplified model is used as the initial value in the iteration, and the solution of the HSV optimal time-consuming problem under the complex model is obtained by the continuation method. The simulation results show that the TRM and the continuation method effectively realize the optimal time-consuming control of the vehicle.

1 Trigonometric Regularization Method

Under the condition of unconstrained control, the optimal control can be easily obtained by utilizing the necessary conditions of first-order optimality

$$\frac{\partial H}{\partial u}=0 \tag{1}$$

where H is the Hamiltonian function and u the control.

In the HSV control problem, the control is usually magnitude constrained and Pontryagin's minimum principle (PMP) should be used

$$H(u^{*})=\min_{u} H \tag{2}$$

where "*" in the upper corner indicates the optimal value. In the optimal control problem of the vehicle, the Hamiltonian function can be reduced to the following form.

$$H=H_0+S\cdot u \tag{3}$$

where H_0 is the control-independent part of the function and S the switching function. In common, the Hamiltonian function and switching function are used to analyze and determine whether it is bang-bang control or singular control. When the switching function is constant equal to zero over a certain period, it is singular optimal control. And vice versa, it is bang-bang control.

The trigonometric regularization method[11-12] proposes to use the mathematical properties of the trigonometric function to express the magnitude constrained control as the sine function, which will make the necessary conditions of first-order optimality available and bring great convenience to solve the OCP.

Assume that the magnitude of the control is

$$u_{\min}\leqslant u\leqslant u_{\max} \tag{4}$$

where $u_{\min}$, $u_{\max}$ are the lower and upper bound of control, respectively. Then let

$$u=\frac{u_{\max}-u_{\min}}{2}\sin u_{\mathrm{TRIG}}+\frac{u_{\max}+u_{\min}}{2} \tag{5}$$

where u_{TRIG} is trigonometric control, which is unconstrained by magnitude. By introducing the trigonometry, the problem of solving optimal control u^{*} is transformed into the problem of finding optimal trigonometric control u^{*}_{TRIG}. The optimal trigonometric control can be gained from Eq.(1). It also ensures that the actual control is limited as Eq.(4). After such transformation, the problem of analyzing the switching function is avoided. The actual control and the trigonometric control can be converted to each other as Eq.(5).

The control may jump during the flight and lead the costate variables unsmooth, which make TPBVP converted by indirect method hard to solve. Therefore, the error control is introduced to smooth the control. The smoothing effect is shown as Fig.1.

In order to achieve smoothing, while replacing the actual control with a sine control earlier, the er-

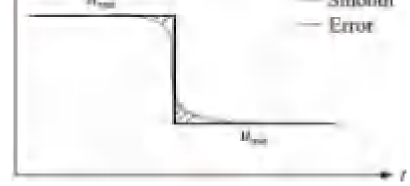

Fig.1 Control smoothing effect

ror control can take the form of a cosine function, thereby forming a control triangle, as shown in Fig.2 (assuming the control bound is 1), where the dashed line is the cosine error control, the green line is the sine smooth control, and the red line is the bound of the control. To achieve different smoothing effects, an error parameter ε is introduced. The smaller the ε, the weaker the error control effect, and the smooth control is closer to ideal control. For OCPs, this error control can be added to the integral index, so the performance index J in the OCP is rewritten as

$$J=\phi[x(t_f),t_f]+\int_0^{t_f}[L(x,u,t)+\varepsilon\frac{u_{\max}-u_{\min}}{2}\cos u_{\mathrm{TRIG}}]\mathrm{d}t \tag{6}$$

where ϕ is the terminal index, and $\int_0^{t_f}L\mathrm{d}t$ the original integral index.

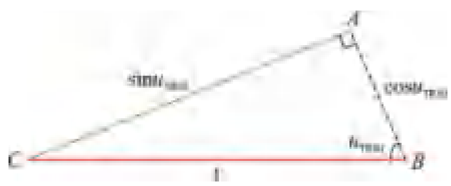

Fig.2 Control triangle

By utilizing the TRM, the optimal trigonometric control can be calculated conveniently, and the actual control can be reverted from Eq.(5). Then the OCP is converted to TPBVP of differential equations of states and costate variables.

2 Continuation Method

For solving the TPBVP, the current classical methods include shooting method, finite difference method and finite element method[13]. These methods solve the TPBVP through iteration. But if the initial value of iteration is not set properly, the calculation is likely to diverge[14]. For the TPBVP converted by the OCP, the costate variables have no physical meanings, so their initial values are difficult to give reasonably.

The continuation method is a method of transitioning from solving a simplified problem to solve a complex one. If the OCP can be simplified by ignoring some nonlinear terms, and its solution can be solved analytically, the continuation method is suitable to be used. Through taking the solution of the simplified problem as the initial value, and solving a series of TPBVP, the original complex OCP can be solved by the method[15-16].

Assuming n nonlinear terms need to be ignored in order to simplify the problem, n continuation parameters are introduced. Add these continuation parameters to each nonlinear term as multipliers, and make all parameters equal to zero. Then the complex problem reduces to a simplified one that can be solved. The solution of the simplified problem is set to be the initial value. Let $k_1=k_1+dk_1$ ($k_2, k_3, \cdots, k_n=0$), and solve the TPBVP induced by the indirect method. Continue with $k_1=k_1+dk_1$, let the previous solution as initial value and solve the TPBVP at current k_1. Repeat the process until $k_1=1$. Leave k_1 unchanged and follow this step to conduct $k_2, k_3, \cdots, k_n$. Fig.3 shows the schematic diagram of the continuation method. When the process completes, the solution of the original complex problem is obtained.

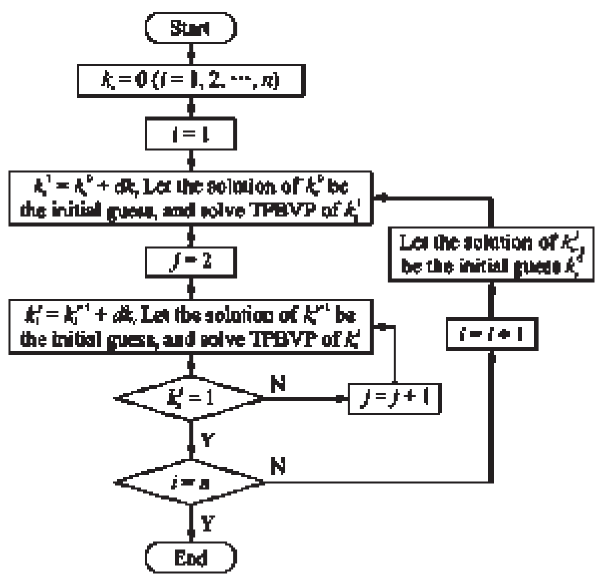

Fig.3 Diagram of the continuation method

3 The Problem of Optimal Control of HSV

The equations of motion of HSV are listed as follows

$$\begin{cases}\dot{y}=v_y\\ \dot{v}_x=-\dfrac{v_xv_y}{R_e+y}+\dfrac{1}{m}u\cos\eta-\dfrac{D\cos\theta}{m}-\dfrac{L\sin\theta}{m}\\ \dot{v}_y=\dfrac{v_x{}^2}{R_e+y}-\dfrac{\mu}{(R_e+y)^2}+\dfrac{1}{m}u\sin\eta-\\ \quad\dfrac{D\sin\theta}{m}+\dfrac{L\cos\theta}{m}\\ \dot{m}=-\dfrac{u}{I_{sp}}\end{cases}\tag{7}$$

where y is the flight height, m the vehicle mass, μ the earth's gravitational constant, R_e the radius of the earth, I_{sp} the fuel specific impulse, u the thrust of the engine, η the angle of thrust relative to the local horizontal direction; v_x, v_y are the projections of the velocity vector in the local horizontal and vertical direction, respectively, and L, D the aerodynamic lift and drag, respectively. And $\theta=\arctan\left(\dfrac{v_y}{v_x}\right)$ is the flight path angle. The downrange of the HSV is omitted here. The term $\dfrac{v_xv_y}{R_e+y}$ is the coriolis acceleration and $\dfrac{v_x{}^2}{R_e+y}$ is the centrifugal acceleration.

L and D are calculated by

$$\begin{cases}L=\dfrac{1}{2}C_L\rho v^2S\\ D=\dfrac{1}{2}C_D\rho v^2S\end{cases}\tag{8}$$

where C_L, C_D are the lift and drag coefficients, respectively; $v=\sqrt{v_x^2+v_y^2}$; S is the reference area and ρ the air density

$$\rho=\rho_0\mathrm{e}^{-\frac{y}{h}}\tag{9}$$

where ρ_0 is the density at zero altitude and h the scalar height factor[17].

The vehicle initial states are set as

$$\begin{cases}y(0)=0,\ v_x(0)=0,\\ v_y(0)=0,\ m(0)-m_0=0\end{cases}\tag{10}$$

The desired final states in ascending stage are

$$y(t_f)-y_f^{\#}=0,\ v_x(t_f)-v_{xf}^{\#}=0,\ v_y(t_f)=0\tag{11}$$

where "#" in the upper corner indicates the desired value.

The engine thrust is limited as

$$0\leqslant u\leqslant u_{\max}\tag{12}$$

The performance index is

$$J=t_f\tag{13}$$

According to the TRM described previously, the thrust should be converted as

$$u=\frac{u_{\max}}{2}\sin u_{\mathrm{TRIG}}+\frac{u_{\max}}{2}\tag{14}$$

And the Hamiltonian function is

$$\begin{aligned}H=&\frac{u_{\max}}{2}\varepsilon\cos u_{\mathrm{TRIG}}+\lambda_yv_y+\\&\lambda_{vx}\left(\frac{-v_xv_y}{R_e+y}+\frac{1}{m}u\cos\eta-\frac{D\cos\theta}{m}\right)+\\&\lambda_{vy}\left[\frac{v_x{}^2}{R_e+y}-\frac{\mu}{(R_e+y)^2}+\right.\\&\left.\frac{1}{m}u\sin\eta-\frac{D\sin\theta}{m}\right]-\lambda_m\frac{u}{I_{sp}}\end{aligned}\tag{15}$$

where the term $\dfrac{u_{\max}}{2}\varepsilon\cos u_{\mathrm{TRIG}}$ is the error control. According to the PMP, the optimal thrust angle should satisfy

$$\begin{cases}\cos\eta=\dfrac{-\lambda_{vx}}{\sqrt{\lambda_{vx}^2+\lambda_{vy}^2}}\\ \sin\eta=\dfrac{-\lambda_{vy}}{\sqrt{\lambda_{vx}^2+\lambda_{vy}^2}}\end{cases}\tag{16}$$

The optimal thrust can be easily derived out as follows according to the necessary conditions of first-order optimality

$$u_{\mathrm{TRIG}}=\begin{cases}\arctan\left(\dfrac{\dfrac{\lambda_{vx}\cos\eta+\lambda_{vy}\sin\eta}{m}-\dfrac{\lambda_m}{I_{sp}}}{\varepsilon}\right)\\ \arctan\left(\dfrac{\dfrac{\lambda_{vx}\cos\eta+\lambda_{vy}\sin\eta}{m}-\dfrac{\lambda_m}{I_{sp}}}{\varepsilon}\right)+\pi\end{cases}\tag{17}$$

The two triangular control should be reverted to the actual control by Eq.(14), and substituted into Eq.(15). The optimal control should choose the one that minimize Eq.(15).

According to the optimal control theory, two bound conditions are supplemented

$$\{H(t_f)+1=0,\lambda_m(t_f)=0\}\tag{18}$$

The differential equations for costate variables are

$$
\begin{cases}
\dot{\lambda}_y = -\dfrac{\partial H}{\partial y} = -\dfrac{\lambda_{vx} v_x v_y}{(R_e + y)^2} + \dfrac{\lambda_{vy} v_x^2}{(R_e + y)^2} - \\
\quad \dfrac{2\mu\lambda_{vy}}{(R_e + y)^3} - (\lambda_{vx}\cos\theta + \lambda_{vy}\sin\theta)\dfrac{D}{mh} + \\
\quad (\lambda_{vy}\cos\theta - \lambda_{vx}\sin\theta)\dfrac{L}{mh} \\
\dot{\lambda}_{vx} = -\dfrac{\partial H}{\partial v_x} = \dfrac{\lambda_{vx} v_y}{R_e + y} - \dfrac{2\lambda_{vy} v_x}{R_e + y} + \\
\quad \lambda_{vx}\dfrac{C_D \rho_0 e^{-\frac{y}{h}} S(2v_x^2 + v_y^2)}{2mv_x\sqrt{1 + \dfrac{v_y^2}{v_x^2}}} + \lambda_{vy}\dfrac{C_D \rho_0 e^{-\frac{y}{h}} S v_y}{2m\sqrt{1 + \dfrac{v_y^2}{v_x^2}}} + \\
\quad \lambda_{vx}\dfrac{C_L \rho_0 e^{-\frac{y}{h}} S v_y}{2m\sqrt{1 + \dfrac{v_y^2}{v_x^2}}} - \lambda_{vy}\dfrac{C_L \rho_0 e^{-\frac{y}{h}} S(2v_x^2 + v_y^2)}{2mv_x\sqrt{1 + \dfrac{v_y^2}{v_x^2}}} \\
\dot{\lambda}_{vy} = -\dfrac{\partial H}{\partial v_y} = -\lambda_y + \dfrac{\lambda_{vx} v_x}{R_e + y} + \\
\quad \lambda_{vx}\dfrac{C_D \rho_0 e^{-\frac{y}{h}} S v_y}{2m\sqrt{1 + \dfrac{v_y^2}{v_x^2}}} + \lambda_{vy}\dfrac{C_D \rho_0 e^{-\frac{y}{h}} S(v_x^2 + 2v_y^2)}{2mv_x\sqrt{1 + \dfrac{v_y^2}{v_x^2}}} + \\
\quad \lambda_{vx}\dfrac{C_L \rho_0 e^{-\frac{y}{h}} S(2v_x^2 + v_y^2)}{2mv_x\sqrt{1 + \dfrac{v_y^2}{v_x^2}}} - \lambda_{vy}\dfrac{C_L \rho_0 e^{-\frac{y}{h}} S v_y}{2m\sqrt{1 + \dfrac{v_y^2}{v_x^2}}} \\
\dot{\lambda}_m = -\dfrac{\partial H}{\partial m} = \dfrac{u}{m^2}(\lambda_{vx}\cos\eta + \lambda_{vy}\sin\eta) - \\
\quad \dfrac{D}{m^2}\left(\lambda_{vx}\cos\theta + \lambda_{vy}\sin\theta\right) + \\
\quad \dfrac{L}{m^2}\left(\lambda_{vy}\cos\theta - \lambda_{vx}\sin\theta\right)
\end{cases}
\tag{19}
$$

Eqs.(7, 10, 11, 14, 16—19) constitute the TPBVP of states and costate variables, where Eqs.(7, 19) are the ordinary differential equations and Eqs.(10, 11, 18) consist the boundary conditions.

Apparently, the differential equations of costate variables are quite complex due to the strong nonlinear terms of coriolis acceleration, centrifugal acceleration, aerodynamic lift and drag. Therefore, these make it impossible to solve the TPBVP directly.

However, when the model is simplified, the solution of the OCP can be solved analytically. The continuation method can be used here to solve the HSV time-optimal problem under the complex model.

Assuming the earth is an infinitely large plane and the aerodynamics is unconsidered. The simplified equations of motion can be described as

$$
\begin{cases}
\dot{y} = v_y, \dot{v}_x = \dfrac{u}{m}\cos\eta \\
\dot{v}_y = \dfrac{u}{m}\sin\eta - g, \dot{m} = -\dfrac{u}{I_{sp}}
\end{cases}
\tag{20}
$$

where the four nonlinear terms are ignored and g is the gravity acceleration which seen as a constant. So Eqs.(10—13, 20) form the simplified time-optimal problem of HSV.

The differential equations for costate variables of simplified model are

$$
\begin{cases}
\dot{\lambda}_y = 0 \\
\dot{\lambda}_{vx} = 0, \quad \dot{\lambda}_{vy} = -\lambda_y \\
\dot{\lambda}_m = \dfrac{u(\lambda_{vx}\cos\eta + \lambda_{vy}\sin\eta)}{m^2}
\end{cases}
\tag{21}
$$

The simplified problem can be solved analytically by analyzing the corresponding switching function and conducting definite integral calculations on Eq.(21). The analysis and calculation process is omitted here. Then the solution of the simplified model can be gained, among which the costate curves of the simplified problem are shown as Fig.4.

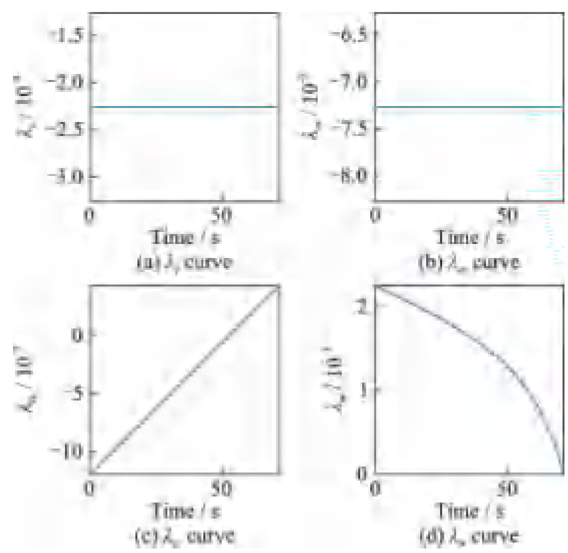

Fig.4 Costate curves of simplified problem

The continuation method is utilized through imbedding continuation parameters into the original model. Parameters k_1, k_2, k_3, k_4 are introduced as multipliers of the four nonlinear terms. The original model is modified as

$$\begin{cases} \dot{y}=v_y \\ \dot{v}_x=-k_1\dfrac{v_x v_y}{R_e+y}+\dfrac{1}{m}u\cos\eta- \\ \qquad k_3\dfrac{D\cos\theta}{m}-k_4\dfrac{L\sin\theta}{m} \\ \dot{v}_y=k_2\dfrac{{v_x}^2}{R_e+y}-\dfrac{\mu}{(R_e+y)^2}+\dfrac{1}{m}u\sin\eta- \\ \qquad k_3\dfrac{D\sin\theta}{m}+k_4\dfrac{L\cos\theta}{m} \\ \dot{m}=-\dfrac{u}{I_{sp}} \end{cases} \tag{22}$$

The costate equations should be modified correspondingly. The modified equations of costate variables and Eqs.(10, 11, 14, 16, 17, 22) form another TPBVP in the continuation model.

When $k_1, k_2, k_3, k_4=0$, Eq.(22) reduces to the simplified problem which has been solved analytically. Then the solution acts as an initial value of the first step of continuation. Firstly, the process starts from k_1. It should be noted that at each step of continuation, the previous solution must be set as initial value at the current step. When $k_1=1$, progress k_2, k_3, k_4 in the same way, and the solution of the time-optimal problem of HSV under the complex model is obtained.

The parameters used in the problem are shown in Table 1.

Table 1 Parameters used in the problem

Parameter	Value
u_{max}/N	9.2×10^5
I_{sp}/(m·s^{-1})	3 000
m_0/kg	3×10^4
$v_{xf}^{\#}$/(m·s^{-1})	3.4×10^3
$y_f^{\#}$/m	3×10^4
R_e/m	6.378×10^6
μ/(m^3·s^{-2})	3.983×10^{14}

The simulation results are shown as Figs.5—10. As can be seen from Figs.5—6, the HSV reaches the desired final states. Fig.8 shows that the HSV flies at the maximum thrust all the way, and the fuel is consumed at a constant speed as Fig.6 shown. The second plot of Fig.8 shows that the thrust angle is slowly becoming negative, so that the vertical velocity becomes zero in the final. The switching func-

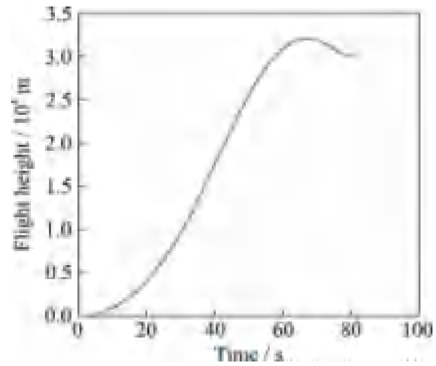

Fig.5 Height curve

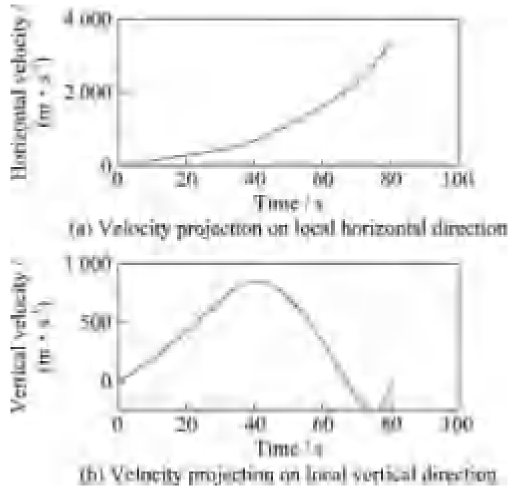

Fig.6 Velocity curves

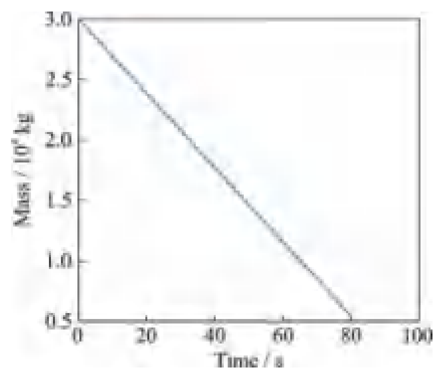

Fig.7 Mass curve

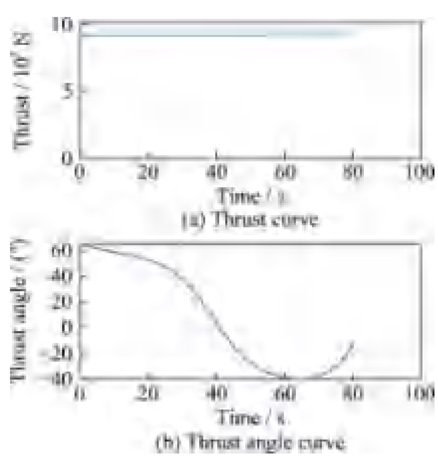

Fig.8 Control curves

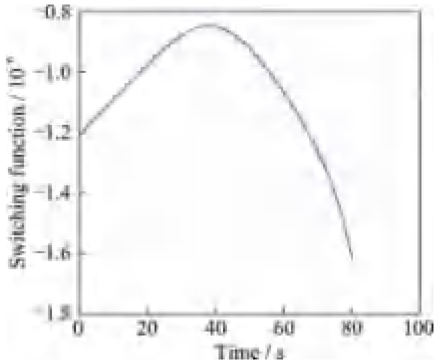

Fig.9 Switching function

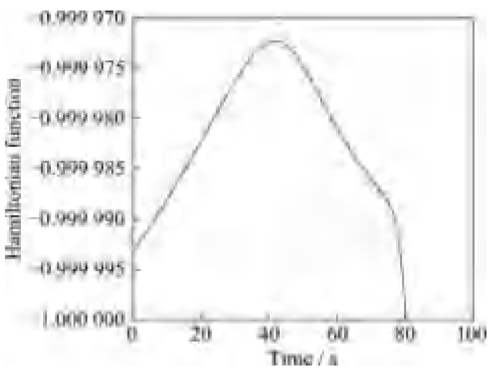

Fig.10 Hamiltonian function

tion in Fig.9 is always negative, supporting the correction of $u^* \equiv u_{\max}$. The Hamiltonian function in Fig.10 equals to -1 within the error of 10^{-4}, considering meets the necessary conditions of optimality. The slight fluctuations are due to the introduction of error control and the calculation error of the computer.

The paper tries to solve the problem by the traditional indirect method to verify the convenience and effectiveness of the above method. However, the traditional method to solve TPBVP of the optimal problem under such a complex coupled model always diverges, while the above method solves it successfully.

4 Conclusions

Based on the indirect method of optimal control, this paper introduces the TRM, which eliminates the need for switching function analysis. For the problem that the costate variables have no reasonable initial value guess in the TPBVP under the complex model, resulting in the divergence easily, the continuation method is introduced. The OCP under the complex model can be gradually approached and solved by taking the solution of the simpler problem as the initial value. Finally, through the analysis and simulation of the time-optimal problem of HSV in ascending stage, the effectiveness of the above method is successfully verified.

There are some constraints such as heat constraint and load constraint due to the structure of the vehicle during the flight. But those constraints are not considered in the paper. Further research may lay on the state constraints for the optimal control method.

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Author contributions Mr. **LIN Yujie** mainly designed the study and conducted the method analysis, performed the coding and simulation. Dr. **HAN Yanhua** mainly modeled the motion of the aircraft and participated in the editing and writing of the paper. The authors commented on the manuscript draft and approved the submission.

Competing interests The authors declare no competing interests.

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基于三角正则化和延拓法的高超声速飞行器时间最优控制

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摘要：针对高超声速飞行器(Hypersonic vehicles, HSV)上升段的最优控制问题，基于间接法引入了三角正则化方法(Trigonometric regularization method, TRM)。相较于传统间接法，该方法免于分析开关函数，从而区分bang-bang控制或难以求解的奇异最优控制。bang-bang控制情况会使协态变量在控制跃变处不平滑，从而引起由间接法转化的两点边值问题(Two-point boundary value problem, TPBVP)，导致求解困难。为了TPBVP求解更易收敛，引入了延拓法。该方法由简化问题的解作为延拓初值，通过求解一系列TPBVP，逼近至原复杂问题的解。计算结果表明，以上方法能够方便有效地求解高超声速飞行器上升段时间最优控制问题。

关键词：高超声速飞行器；最优控制；三角正则化法；延拓法