



# Two-Stage Least Squares based Iterative Identification Algorithm for Box-Jenkins Model

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Received: 23 Jun. 2013, Revised: 28 Oct. 2013, Accepted: 29 Oct. 2013

Published online: 1 May. 2014

**Abstract:** In order to improve the parameters estimation precision, a two-stage least squares iterative algorithm for Box-Jenkins models is presented, which is based on the interactive estimation theory of the hierarchical identification and the auxiliary model. The main idea of the algorithm is to decompose a Box-Jenkins system into two subsystems so as to identify each subsystem, respectively. In our algorithm, the dimensions of the involved covariance matrices in each subsystem turn to be small. The simulation results indicate that the proposed algorithm is effective and has a high computational efficiency.

**Keywords:** Box-Jenkins model, parameter estimation, hierarchical identification, covariance

## 1 Introduction

Actual system often has interference effect, and the interference is often with colored noise. Under the colored noise interference, many identification methods are used to estimate parameters for Box-Jenkins model [1, 2, 3, 4, 5]. In order to get unbiased estimation parameters of the model, the recursive extended least squares algorithm (RELS) for CARMA model was extended to Box-Jenkins model, and then the recursive generalized extended least squares algorithm (RGELS) was presented [6]. The structure and parameter estimation of Box-Jenkins model were obtained by using UD decomposition [7]. The deviation compensation least-squares identification method and its recursive form were pushed by using deviation compensation principle [8, 9]. The drawback of deviation compensation least squares identification method is that the parameters of noise model are not identified. But the recursive extended least-squares algorithm is simple and easily to realized, not only can identify the parameters of the system model, but also parameter estimation of the noise model can be obtained [10, 11].

Least-squares iteration identification method use all the data of the system in the process of every step iterative calculation, and extract information as much

as possible from the data to improve the parameter estimation accuracy [12, 13, 14]. For example, Ding et al. presented least squares based iterative algorithms for Hammerstein nonlinear ARMAX systems [14], and for OE and OEMA systems [15]; Han et al. gave a hierarchical least squares based iterative identification algorithm for a class of multivariable CARMA-like systems [16]; Zhang et al. Proposed a hierarchical gradient based iterative parameter estimation algorithm for multi-variable output error Moving average systems [17]; Bao et al. developed a least squares based iterative identification method for multi-variable controlled ARMA systems [18]; Ding et al. Presented a least squares based iterative algorithm for controlled auto-regressive auto-regressive moving average (CARARMA) systems [19]. Also, a least squares based iterative algorithm is developed for Box-Jenkins systems, and the precision of parameter estimation is improved compared with recursive generalized extended least squares algorithm [20].

However, because the dimensions of the involved covariance matrices are big, lead to large amount of calculation [20]. The idea of decomposition technique and the auxiliary model identification were adopted in this paper, and then the two-stage [21, 22, 23, 24] least-squares iteration identification method was proposed for Box-

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Jenkins model. The basic idea is to decompose a Box-Jenkins system into two subsystems and to identify each subsystem, respectively. Because the dimensions of the involved covariance matrices in each subsystem become small, the proposed algorithm has a high computational efficiency[25].

## 2 Model description

Let us introduce some notations first. “A:=X” or “X:=A” stands for “A is defined as X”; the symbol  $I$  stands for an identity matrix of appropriate size ( $n \times n$ ); the superscript  $T$  denotes the matrix transpose;  $I_n$  represents an  $n$ -dimensional column vector whose elements are all 1.

Consider the Box-Jenkins system, depicted in Fig.1,

$$y(t) = \frac{B(z)}{A(z)}u(t) + \frac{D(z)}{C(z)}v(t) \quad (1)$$

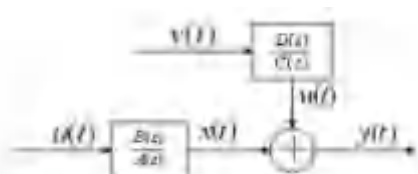


Fig. 1 A system described by the Box-Jenkins model

Where,  $y(t)$  and  $u(t)$  are the input and output sequences of the system, respectively,  $v(t)$  is a white noise sequence with zero mean and variance  $\sigma^2$ , and  $A(z)$ ,  $B(z)$ ,  $C(z)$  and  $D(z)$  are the polynomials in the unit backward shift operator  $Z^{-1}$  ( $[Z^{-1}y(t) = y(t-1)]$ ), and defined by,

$$A(z) := 1 + a_1z^{-1} + a_2z^{-2} + \dots + a_{n_a}z^{-n_a}$$

$$B(z) := b_1z^{-1} + b_2z^{-2} + \dots + b_{n_b}z^{-n_b}$$

$$C(z) := 1 + c_1z^{-1} + c_2z^{-2} + \dots + c_{n_c}z^{-n_c}$$

$$D(z) := 1 + d_1z^{-1} + d_2z^{-2} + \dots + d_{n_d}z^{-n_d}$$

Without loss of generality, assume that  $u(t) = 0$ ,  $y(t) = 0$ ,  $v(t) = 0$  for  $t \leq 0$ , and known orders  $(n_a, n_b, n_c, n_d)$ . The objective of this paper is to apply the decomposition technique and derive a two-stage least squares based iterative identification algorithm for estimating the system parameters, and decompose the original identification system into two subsystems with smaller order problem.

Define the system parameter vectors and the information vectors,

$$\theta := \begin{bmatrix} \theta_s \\ \theta_n \end{bmatrix} \in R^n$$

$$\theta_s := [a_1, a_2, \dots, a_{n_a}, b_1, b_2, \dots, b_{n_b}] \in R^{n_a+n_b}$$

$$\theta_n := [c_1, c_2, \dots, c_{n_c}, d_1, d_2, \dots, d_{n_d}] \in R^{n_c+n_d}$$

$$\varphi(t) := \begin{bmatrix} \varphi_s(t) \\ \varphi_n(t) \end{bmatrix} \in R^n$$

$$\varphi_s(t) := [-x(t-1), -x(t-2), \dots, -x(t-n_a),$$

$$u(t-1), u(t-2), \dots, u(t-n_b)]^T \in R^{(n_a+n_b)}$$

$$\varphi_n(t) := [-w(t-1), -w(t-2), \dots, -w(t-n_c),$$

$$v(t-1), v(t-2), \dots, v(t-n_d)]^T \in R^{(n_c+n_d)}$$

Define the intermediate variable,

$$x(t) := \frac{A(z)}{B(z)}u(t) \quad (2)$$

Or

$$\begin{aligned} x(t) &= [1 - A(z)]x(t) + B(z)u(t) \\ &= (-a_1z^{-1} - a_2z^{-2} - \dots - a_{n_a}z^{-n_a})x(t) \\ &\quad + (b_1z^{-1} + b_2z^{-2} + \dots + b_{n_b}z^{-n_b})u(t) \\ &= -a_1x(t-1) - a_2x(t-2) - \dots \\ &\quad - a_{n_a}x(t-n_a) + b_1u(t-1) + b_2u(t-2) \\ &\quad + \dots + b_{n_b}u(t-n_b) \\ &= \varphi_s^T(t)\theta_s \end{aligned}$$

$$w(t) := \frac{D(z)}{C(z)}v(t) \quad (3)$$

Or

$$\begin{aligned} w(t) &= [1 - C(z)]w(t) + D(z)v(t) \\ &= (-c_1z^{-1} - c_2z^{-2} - \dots - c_{n_c}z^{-n_c})w(t) \\ &\quad + (1 + d_1z^{-1} + d_2z^{-2} + \dots + d_{n_d}z^{-n_d})v(t) \\ &= -c_1w(t-1) - c_2w(t-2) - \dots \\ &\quad - c_{n_c}w(t-n_c) + v(t) + d_1v(t-1) \\ &\quad + d_2v(t-2) + \dots + d_{n_d}v(t-n_d) \\ &= \varphi_n^T(t)\theta_n + v(t) \end{aligned}$$

Replacing (1) with (2), (3), we have

$$\begin{aligned} y(t) &= x(t) + w(t) \\ &= \varphi_s^T(t)\theta_s + \varphi_n^T(t)\theta_n + v(t) \\ &= \varphi^T(t)\theta + v(t) \end{aligned} \quad (4)$$

## 3 Two-stage least squares based iterative identification algorithm

The basic idea of two-stage least squares iterative identification algorithm is decomposing the system into two subsystems, and decomposing the parameters and information vector respectively into two parameter sub vectors and information sub vectors, and then using hierarchical identification of the interactive estimation theory and the auxiliary model to identify parameters of each subsystem. Define two intermediate variables:

$$y_1(t) := y(t) - \varphi_n^T(t)\theta_n \quad (5)$$

$$y_2(t) := y(t) - \varphi_s^T(t)\theta_s \quad (6)$$



From (5) and (6), the system in (4) can be decomposed into two “fictitious” subsystems:

$$y_1(t) = \varphi_s^T(t) \theta_s + v(t) \quad (7)$$

$$y_2(t) = \varphi_n^T(t) \theta_n + v(t) \quad (8)$$

Consider the data from  $t = L, L \gg n$ , and define the stacked output vectors  $Y(L)$ ,  $Y_1(L)$  and  $Y_2(L)$ , the stacked information matrices  $\Phi_s(L)$  and  $\Phi_n(L)$ , and the stacked white noise vector  $V(L)$  as:

$$Y(L) := \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(L) \end{bmatrix} \in R^L \quad V(L) := \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(L) \end{bmatrix} \in R^L$$

$$Y_1(L) := \begin{bmatrix} y_1(1) \\ y_1(2) \\ \vdots \\ y_1(L) \end{bmatrix} \in R^L \quad Y_2(L) := \begin{bmatrix} y_2(1) \\ y_2(2) \\ \vdots \\ y_2(L) \end{bmatrix} \in R^L$$

$$\Phi_s(L) := \begin{bmatrix} \varphi_s^T(1) \\ \varphi_s^T(2) \\ \vdots \\ \varphi_s^T(L) \end{bmatrix} \in R^{L \times (n_a + n_b)} \quad \Phi_n(L) := \begin{bmatrix} \varphi_n^T(1) \\ \varphi_n^T(2) \\ \vdots \\ \varphi_n^T(L) \end{bmatrix} \in R^{L \times (n_c + n_d)}$$

Note that  $Y(L)$ ,  $\Phi_s(L)$  and  $\Phi_n(L)$  contain all the measured data  $\{u(t), y(t) : t = 1, 2, \dots, L\}$ , from (5) and (6), we have

$$Y_1(L) = Y(L) - \Phi_n(L) \theta_n \quad (9)$$

$$Y_2(L) = Y(L) - \Phi_s(L) \theta_s \quad (10)$$

From (7) and (8), we have

$$Y_1(L) = \Phi_s(L) \theta_s + V(L)$$

$$Y_2(L) = \Phi_n(L) \theta_n + V(L)$$

Define two quadratic criterion functions:

$$J_1(\theta_s) := \|Y_1(L) - \Phi_s \theta_s\|^2$$

$$J_2(\theta_n) := \|Y_2(L) - \Phi_n \theta_n\|^2$$

For these two optimization problems, letting the partial derivatives of  $J_1(\theta_s)$  and  $J_2(\theta_n)$  with respect to  $\theta_s$  and  $\theta_n$  be zero gives:

$$\frac{\partial J_1(\theta_s)}{\partial \theta_s} = -2\Phi_s^T(L)[Y_1(L) - \Phi_s(L)\theta_s] = 0$$

$$\frac{\partial J_2(\theta_n)}{\partial \theta_n} = -2\Phi_n^T(L)[Y_2(L) - \Phi_n(L)\theta_n] = 0$$

Assume that the information vectors  $\varphi_s(t)$  and  $\varphi_n(t)$  are persistently exciting, that is,  $[\Phi_s^T(L)\Phi_s(L)]$  and  $[\Phi_n^T(L)\Phi_n(L)]$  are non-singular. From the above two equations, we can obtain the following least squares estimates of the parameter vectors  $\theta_s$  and  $\theta_n$ :

$$\hat{\theta}_s = [\Phi_s^T(L)\Phi_s(L)]^{-1}\Phi_s^T(L)Y_1(L) \quad (11)$$

$$\hat{\theta}_n = [\Phi_n^T(L)\Phi_n(L)]^{-1}\Phi_n^T(L)Y_2(L) \quad (12)$$

Substituting (9) into (11) and (10) into (12) gives

$$\hat{\theta}_s = [\Phi_s^T(L)\Phi_s(L)]^{-1}\Phi_s^T(L)[Y(L) - \Phi_n(L)\theta_n] \quad (13)$$

$$\hat{\theta}_n = [\Phi_n^T(L)\Phi_n(L)]^{-1}\Phi_n^T(L)[Y(L) - \Phi_s(L)\theta_s] \quad (14)$$

However, the right-hand sides of (13) and (14) contain the unknown parameter  $\theta_s$  and  $\theta_n$ , and  $\Phi_s(L)$  and  $\Phi_n(L)$  contain the unknown variable  $x(t-i)$ ,  $w(t-i)$  and  $v(t-i)$ , respectively, it is impossible to compute the estimates  $\hat{\theta}_s$  and  $\hat{\theta}_n$ . In order to solve the difficult, the approach is based on the hierarchical identification principle. As follows: let  $k = 1, 2, 3, \dots$  be an iteration variable,  $\hat{\theta}_k = \begin{bmatrix} \hat{\theta}_{s,k} \\ \hat{\theta}_{n,k} \end{bmatrix}$  be the

iterative estimate of  $\theta_k = \begin{bmatrix} \theta_{s,k} \\ \theta_{n,k} \end{bmatrix}$  at iteration  $k$ , and  $\hat{x}_{k-1}(t-i)$ ,  $\hat{w}_{k-1}(t-i)$  and  $\hat{v}_{k-1}(t-i)$  be the iterative estimate of  $x(t-i)$ ,  $w(t-i)$  and  $v(t-i)$  at iteration  $k-1$ , and define

$$\hat{\phi}_k(t) := \begin{bmatrix} \hat{\phi}_{s,k}(t) \\ \hat{\phi}_{n,k}(t) \end{bmatrix} \in R^{n_a + n_b + n_c + n_d}$$

$$\hat{\phi}_{s,k}(t) := [-\hat{x}_{k-1}(t-1), -\hat{x}_{k-1}(t-2), \dots, -\hat{x}_{k-1}(t-n_a), u(t-1), u(t-2), \dots, u(t-n_b)]^T \in R^{n_a + n_b}$$

$$\hat{\phi}_{n,k}(t) := [-\hat{w}_{k-1}(t-1), -\hat{w}_{k-1}(t-2), \dots, -\hat{w}_{k-1}(t-n_c), \hat{v}_{k-1}(t-1), \hat{v}_{k-1}(t-2), \dots, \hat{v}_{k-1}(t-n_d)]^T \in R^{n_c + n_d}$$

Replacing  $t$  with  $t-i$  in (2), we have:

$$x(t-i) = \varphi_s^T(t-i) \theta_s$$

Replacing  $\varphi_s(t-i)$  and  $\theta_s$  with  $\hat{\phi}_{s,k}(t-i)$  and  $\hat{\theta}_{s,k}(t)$ , the estimate  $\hat{x}_k(t-i)$  of  $x(t-i)$  can be computed by

$$\hat{x}_k(t-i) = \hat{\phi}_{s,k}^T(t-i) \hat{\theta}_{s,k}(t) \quad (15)$$

From (3), we have

$$w(t-i) = y(t-i) - x(t-i)$$

Replacing  $\hat{x}_k(t-i)$  with  $x(t-i)$ , the estimate  $\hat{w}_k(t-i)$  of  $w(t-i)$  can be computed by

$$\begin{aligned} \hat{w}_k(t-i) &= y(t-i) - \hat{x}_k(t-i) \\ &= y(t-i) - \hat{\phi}_{s,k}^T(t-i) \hat{\theta}_{s,k}(t) \end{aligned} \quad (16)$$

From (3), we have

$$v(t-i) = w(t-i) - \varphi_n^T(t-i) \theta_n$$

Replacing  $w(t-i)$ ,  $\varphi_n(t-i)$  and  $\theta_n$  with  $\hat{w}_k(t-i)$ ,  $\hat{\phi}_{n,k}(t-i)$  and  $\hat{\theta}_{n,k}(t)$ , the estimate  $\hat{v}_k(t-i)$  of  $v(t-i)$  can be computed by

$$\hat{v}_k(t-i) = \hat{w}_k(t-i) - \hat{\phi}_{n,k}^T(t-i) \hat{\theta}_{n,k}(t) \quad (17)$$

Certainly,  $v(t-i)$  can be computed by

$$v(t-i) = y(t-i) - \varphi^T(t-i) \theta$$

Replacing  $\varphi(t)$ ,  $\theta$  with  $\hat{\phi}_k(t)$ ,  $\hat{\theta}_k(t)$  the estimate  $\hat{v}_k(t-i)$  of  $v(t-i)$  can be computed by

$$\hat{v}_k(t-i) = y(t-i) - \hat{\phi}_k^T(t-i) \hat{\theta}_k(t) \quad (18)$$

To sum up, two-stage least squares based iterative identification algorithm as follow:

$$\hat{\theta}_{s,k} = [\hat{\Phi}_{s,k}^T(L) \hat{\Phi}_{s,k}(L)]^{-1} \hat{\Phi}_{s,k}^T(L) [Y(L) - \hat{\Phi}_{n,k}(L) \hat{\theta}_{n,k-1}] \quad (19)$$

$$\hat{\theta}_{n,k} = [\hat{\Phi}_{n,k}^T(L) \hat{\Phi}_{n,k}(L)]^{-1} \hat{\Phi}_{n,k}^T(L) [Y(L) - \hat{\Phi}_{s,k}(L) \hat{\theta}_{s,k}] \quad (20)$$

$$Y(L) = [y(1), y(2), \dots, y(L)]^T \quad (21)$$

$$\hat{\Phi}_{s,k}(L) = [\hat{\phi}_{s,k}(1), \hat{\phi}_{s,k}(2), \dots, \hat{\phi}_{s,k}(L)]^T \quad (22)$$

$$\hat{\Phi}_{n,k}(L) = [\hat{\phi}_{n,k}(1), \hat{\phi}_{n,k}(2), \dots, \hat{\phi}_{n,k}(L)]^T \quad (23)$$

$$\hat{\phi}_{s,k}(t) = [-\hat{x}_{k-1}(t-1), -\hat{x}_{k-1}(t-2), \dots, -\hat{x}_{k-1}(t-n_a), u(t-1), u(t-2), \dots, u(t-n_b)]^T \quad (24)$$

$$\hat{\phi}_{n,k}(t) = [-\hat{w}_{k-1}(t-1), -\hat{w}_{k-1}(t-2), \dots, -\hat{w}_{k-1}(t-n_c), \hat{v}_{k-1}(t-1), \hat{v}_{k-1}(t-2), \dots, \hat{v}_{k-1}(t-n_d)]^T \quad (25)$$

$$\hat{\phi}_k(t) = [\varphi_{s,k}^T(t), \varphi_{n,k}^T(t)]^T \quad (26)$$

$$\hat{\theta}_k = [\hat{\theta}_{s,k}^T, \hat{\theta}_{n,k}^T]^T \quad (27)$$

$$\hat{x}_k(t-i) = \hat{\phi}_{s,k}^T(t-i) \hat{\theta}_{s,k} \quad (28)$$

$$\hat{w}_k(t-i) = y(t-i) - \hat{\phi}_{s,k}^T(t-i) \hat{\theta}_{s,k} \quad (29)$$

$$\hat{v}_k(t-i) = y(t-i) - \hat{\phi}_k^T(t-i) \hat{\theta}_k \quad (30)$$

The flow chart of computing the parameter estimates  $\hat{\theta}_{s,k}$  and  $\hat{\theta}_{n,k}$  is shown in Fig.2.

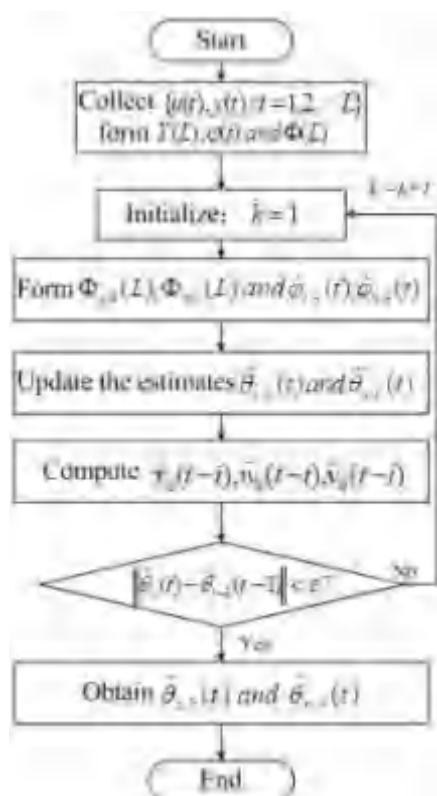


Fig. 2 The flow chart for computing the two-stage LSI parameter estimates

## 4 The least squares based iterative identification algorithm

To compare the algorithm, the least-squares iterative algorithm for Box-Jenkins model was given in this section, as follows:

$$\hat{\theta}_k = [\hat{\Phi}_k^T(L) \hat{\Phi}_k(L)]^{-1} \hat{\Phi}_k^T(L) Y(L)$$

$$Y(L) = [y(1), y(2), \dots, y(L)]^T$$

$$\hat{\Phi}_k(L) = [\hat{\phi}_k(1), \hat{\phi}_k(2), \dots, \hat{\phi}_k(L)]^T$$

$$\hat{\phi}_k(t) = \begin{bmatrix} \hat{\phi}_{s,k}(t) \\ \hat{\phi}_{n,k}(t) \end{bmatrix}$$

$$\hat{\phi}_{s,k}(t) = [-\hat{x}_{k-1}(t-1), -\hat{x}_{k-1}(t-2), \dots, -\hat{x}_{k-1}(t-n_a), u(t-1), u(t-2), \dots, u(t-n_b)]^T$$

$$\hat{\phi}_{n,k}(t) = [-\hat{w}_{k-1}(t-1), -\hat{w}_{k-1}(t-2), \dots, -\hat{w}_{k-1}(t-n_c), \hat{v}_{k-1}(t-1), \hat{v}_{k-1}(t-2), \dots, \hat{v}_{k-1}(t-n_d)]^T$$

$$\hat{\theta}_k = [\hat{\theta}_{s,k}^T, \hat{\theta}_{n,k}^T]^T$$

$$\hat{x}_k(t-i) = \hat{\phi}_{s,k}^T(t-i) \hat{\theta}_{s,k}$$

$$\hat{w}_k(t-i) = y(t-i) - \hat{\phi}_{s,k}^T(t-i) \hat{\theta}_{s,k}$$

$$\hat{v}_k(t-i) = y(t-i) - \hat{\phi}_k^T(t-i) \hat{\theta}_k$$

## 5 Example

Consider the following simulation system of Box-Jenkins model:

$$y(t) = \frac{B(z)}{A(z)} u(t) + \frac{D(z)}{C(z)} v(t)$$

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} = 1 + 1.6z^{-1} + 0.8z^{-2}$$

$$B(z) = b_1 z^{-1} + b_2 z^{-2} = 0.412z^{-1} + 0.309z^{-2}$$

$$C(z) = 1 + c_1 z^{-1} = 1 + 0.8z^{-1}$$

$$D(z) = 1 + d_1 z^{-1} = 1 - 0.64z^{-1}$$

$$\theta_s = [a_1, a_2, b_1, b_2]^T = [1.6, 0.8, 0.412, 0.309]^T$$

$$\theta_n = [c_1, d_1]^T = [0.8, -0.64]^T$$

$$\theta = \begin{bmatrix} \theta_s \\ \theta_n \end{bmatrix}$$

In simulation,  $\{u(t)\}$  is taken as an uncorrelated persistent excitation signal sequence with zero mean and unit variance,  $\{v(t)\}$  as a white noise sequence with zero mean and variance  $\sigma^2$ .

Table 1 The two-stage LSI estimates and errors (L=3000)

k	$a_1$	$a_2$	$b_1$	$b_2$	$c_1$	$d_1$	$\delta(\%)$
1	0.91797	0.25697	0.41571	0.04388	0.73201	-0.55031	43.21152
5	1.49932	0.72201	0.41788	0.28496	0.80454	-0.63611	6.11205
10	1.59371	0.79125	0.41768	0.32386	0.80813	-0.63307	1.03478
15	1.56621	0.77580	0.41767	0.31252	0.80833	-0.63235	2.05145
20	1.57820	0.78179	0.41765	0.31749	0.80829	-0.63250	1.51456
25	1.57264	0.77918	0.41766	0.31518	0.80831	-0.63241	1.74772
26	1.57605	0.78076	0.41765	0.31660	0.80830	-0.63246	1.60218
27	1.57310	0.77939	0.41766	0.31537	0.80831	-0.63242	1.72760
True	1.60000	0.80000	0.41200	0.30900	0.80000	-0.64000	0.00000

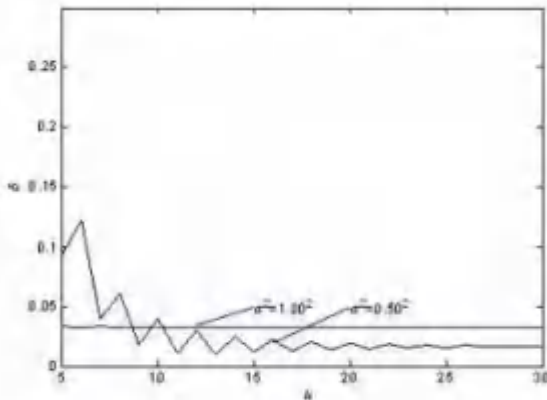


Fig. 3 the two-stage LSI parameter estimation errors  $\delta$  versus  $k$  ( $\sigma^2 = 0.50^2$  and  $\sigma^2 = 1.00^2$ )

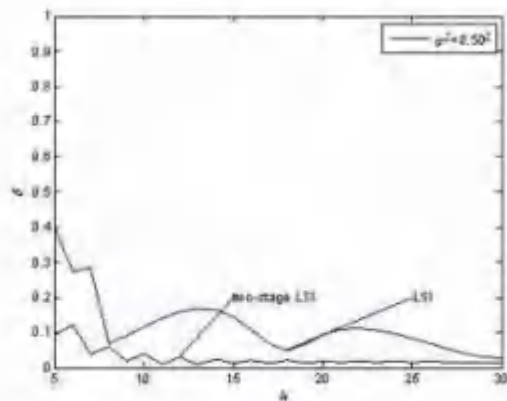


Fig. 4 the parameter estimation errors versus  $k$  for the algorithms ( $\sigma^2 = 0.50^2$ )

From the simulation results in Tables 1-2 and Figs.3-4, we can draw the following conclusions:

With the decrease of the noise mean variance  $\sigma^2$ , the accuracy of parameter estimation is gradually improved, compare to Fig.3;

Under the same data length and noise variance, with the increase of iteration numbers, the parameter estimation error is smaller and smaller, and the two-stage LSI is obviously superior to LSI from Fig.4

Table 2 The LSI estimates and errors ( $L=3000$ )

$k$	$a_1$	$a_2$	$b_1$	$b_2$	$c_1$	$d_1$	$\delta(\%)$
1	1.82146	0.93284	0.41597	0.38206	0.0000	0.0000	49.84331
5	1.53821	0.68932	0.41782	0.25655	0.78749	-0.58859	6.92539
10	1.37748	0.63592	0.41822	0.19424	0.99054	-0.67091	16.88002
15	1.58786	0.89152	0.41890	0.28227	0.82818	-0.60752	4.96624
20	1.72951	0.91571	0.41825	0.34128	0.65510	-0.66347	10.81336
25	1.63762	0.80134	0.41804	0.30280	0.73493	-0.67901	4.00652
26	1.61662	0.78350	0.41804	0.29403	0.75643	-0.67845	3.04469
27	1.59680	0.76839	0.41805	0.28577	0.77707	-0.67745	2.78985
True	1.60000	0.80000	0.41200	0.30900	0.80000	-0.64000	0.00000

## 6 Conclusion

This paper presents a two-stage LSI algorithm for Box-Jenkins models by using auxiliary model identification

idea and the decomposition technique. The proposed algorithm requires less computational load than the least squares based iterative algorithm. The simulation results show that the proposed algorithm has fast convergence rates and can generate highly accurate parameter estimation.

## Acknowledgement

This research was supported by the National Natural Science Foundation of China (No.61263012, 61263040, 61262034), by the Postdoctoral Foundation of China (No. 2012M510593), by the Aerospace Science Foundation (No. 20120156001), by the Key Project of Chinese Ministry of Education (No. 211087), by the Natural Science Foundation of Jiangxi Province (No.20114BAB 211020, 20132BAB201025), by the Young Scientist Foundation of Jiangxi Province (No. 20122BCB23017), and by the Science and Technology Research Project of the Education Department of Jiangxi Province (No.GJJ13302).

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