4. Solving System of Linear Equations

4.1 Creating Polynomial Equations and solving for roots

In [3]:
```
import sympy as sp
sp.init_printing()
```

In [4]:
```
#Defining symbols
x,y,z = sp.symbols("x,y,z")
```

In [9]:
```
#Creating equation
eq1 = sp.Eq(x**2-2*x+3,0)
eq1
```

Out[9]: $x^2 - 2x + 3 = 0$

In [4]:
```
#Substituting value for x in the equarion
eq1.subs(x,2)
```

Out[4]: False

In [11]:
```
#Solving equation/Finding root
sp.solveset(eq1,x)
```

Out[11]: $\{1 - \sqrt{2}i, 1 + \sqrt{2}i\}$

In [12]:
```
eq2 = sp.Eq(x**2-4,0)
eq2
```

Out[12]: $x^2 - 4 = 0$

In [21]:
```
eq3=sp.factor(eq2)
eq3
```

Out[21]: $(x - 2)(x + 2) = 0$

In [22]:
```
sp.expand(eq3)
```

Out[22]: $x^2 - 4 = 0$

In [24]:
```
sp.simplify(eq3)
```
Out[24]: $x^2 = 4$

In [25]:
```
sp.solveset(eq2)
```
Out[25]: $\{-2, 2\}$

In [14]:
```
eq3=sp.Eq((x-1)/(2*x**2-7*x+5),0)
eq3
```
Out[14]: $$\frac{x-1}{2x^2-7x+5} = 0$$

In [15]:
```
sp.simplify(eq3)
```
Out[15]: $$\frac{1}{2x-5} = 0$$

In [16]:
```
eq4=sp.factor(eq3)
eq4
```
Out[16]: $$\frac{1}{2x-5} = 0$$

In [17]:
```
sp.expand(eq4)
```
Out[17]: $$\frac{1}{2x-5} = 0$$

In [18]:
```
soln=sp.solveset(eq3,x)
soln
```
Out[18]: $\emptyset$

4.2 Creating system of linear Equations and solving

Solve the system of linear equations

$$\begin{aligned} x - y + z &= 13 \\ 2x + 5y - 3z &= -13 \\ 4x - y - 6z &= -4 \end{aligned}$$

In [19]:
```
#Defining symbols

x,y,z=sp.symbols("x,y,z")
```

In [20]:
```
#Creating system of equations (Method 1)
system_eqn = [x-y+z-13,2*x+5*y-3*z+13,4*x-y-6*z+4]
display(system_eqn)
```

$$[x - y + z - 13,\ 2x + 5y - 3z + 13,\ 4x - y - 6z + 4]$$

In [21]:
```
#Creating system of equations (Method 2)
eqn1=sp.Eq(x-y+z,13)
eqn2=sp.Eq(2*x+5*y-3*z,-13)
eqn3=sp.Eq(4*x-y-6*z,-4)
display(eqn1)
display(eqn2)
display(eqn3)
system=[eqn1,eqn2,eqn3]
system
```

$$x - y + z = 13$$

$$2x + 5y - 3z = -13$$

$$4x - y - 6z = -4$$

Out[21]: $[x - y + z = 13,\ 2x + 5y - 3z = -13,\ 4x - y - 6z = -4]$

In [22]:
```
#Solving system_eqn
soln1 = sp.solve(system_eqn,x,y,z)
display(soln1)
```

$$\{x : 6,\ y : -2,\ z : 5\}$$

In [23]:
```
soln2=sp.solve([eqn1,eqn2,eqn3],x,y,z)
soln2
```

Out[23]: $\{x : 6,\ y : -2,\ z : 5\}$

In [24]:
```
#Substituting the solution in first equation
eqn1.subs([(x,6),(y,-2),(z,5)])
```

Out[24]: True

In [25]:
```
#Substituting the solution in second equation
eqn2.subs([(x,6),(y,-2),(z,5)])
```

Out[25]: True

In [26]:
```
#Substituting the solution in third equation
eqn3.subs([(x,6),(y,-2),(z,5)])
```

Out[26]: True

Solve the system of linear equations

$$x + y + z = 7$$
$$3x - 2y - z = 4$$
$$x + 6y + 5z = 24$$

In [51]:
```
eqnn1=sp.Eq(x+y+z,7)
eqnn2=sp.Eq(3*x-2*y-1*z,4)
eqnn3=sp.Eq(x+6*y+5*z,24)
syst_eqn = [eqnn1,eqnn2,eqnn3]
syst_eqn
```

Out[51]: $[x + y + z = 7, \ 3x - 2y - z = 4, \ x + 6y + 5z = 24]$

In [52]:
```
soln = sp.solve(syst_eqn,x,y,z)
soln
```

Out[52]: $\left\{x : \frac{18}{5} - \frac{z}{5}, \ y : \frac{17}{5} - \frac{4z}{5}\right\}$

In [53]:
```
eqnn1.subs([(x,18/5-z/5),(y,17/5-4*z/5),(z,z)])
```

Out[53]: True

In [55]:
```
eqnn2.subs([(x,18/5-z/5),(y,17/5-4*z/5),(z,z)])
```

Out[55]: False

In [54]:
```
eqnn3.subs([(x,18/5-z/5),(y,17/5-4*z/5),(z,z)])
```

Out[54]: True

Solve the system of linear equations

$$x - 3y + z = 4$$
$$-x + 2y - 5z = 3$$
$$5x - 13y + 13z = 8$$

In [56]:
```
eqs1=sp.Eq(x-3*y+z,4)
eqs2=sp.Eq(-x+2*y-5*z,3)
eqs3=sp.Eq(5*x-13*y+13*z,8)
system=[eqs1,eqs2,eqs3]
system
```

Out[56]: $[x - 3y + z = 4, \ -x + 2y - 5z = 3, \ 5x - 13y + 13z = 8]$

In [57]:
```
soln = sp.solve(system,x,y,z)
soln
```

Out[57]: []

4.3 Getting row echelon form and reduced row echelon from of an augmented matrix

Get the augmented matrix of the following system and get its row echelon form and reduced row echelon form.

$$\begin{aligned} x - y + z &= 13 \\ 2x + 5y - 3z &= -13 \\ 4x - y - 6z &= -4 \end{aligned}$$

In [62]:
```
#Augmented Matrix
A = sp.Matrix(3,4,[1,-1,1,13,2,5,-3,-13,4,-1,-6,-4])
A
```

Out[62]:

$$\begin{bmatrix} 1 & -1 & 1 & 13 \\ 2 & 5 & -3 & -13 \\ 4 & -1 & -6 & -4 \end{bmatrix}$$

In [72]:
```
# Row echelon form
A.echelon_form()
```

Out[72]:

$$\begin{bmatrix} 1 & -1 & 1 & 13 \\ 0 & 7 & -5 & -39 \\ 0 & 0 & -55 & -275 \end{bmatrix}$$

In [75]:
```
#Reduced row echelon form
A.rref(pivots=False)
```

Out[75]:

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

4.4 Performing elementary row operations

Three elementary row operations are

1. Row swap: $n \leftrightarrow m$
2. Replace a row by constant k times the row:
$$n \rightarrow kn$$
3. Replace row n by row n plus constant k times row m:
$$n \rightarrow n + km$$

Syntax to perform elementary row operations on augmented matrix A are:

```
1. A.elementary_row_op(op='n<->m',row1=n,row2=m)
2. A.elementary_row_op(op='n->kn',row=n,k=c)
3. A.elementary_row_op(op='n->n+km',row1=n,row2=m,k=a)
```

Solve this system by performing elementary row operations

$$\begin{aligned} x - y + z &= 13 \\ 2x + 5y - 3z &= -13 \\ 4x - y - 6z &= -4 \end{aligned}$$

In [63]:
```
#Augmented Matrix
A = sp.Matrix(3,4,[1,-1,1,13,2,5,-3,-13,4,-1,-6,-4])
A
```

Out[63]:
$$\begin{bmatrix} 1 & -1 & 1 & 13 \\ 2 & 5 & -3 & -13 \\ 4 & -1 & -6 & -4 \end{bmatrix}$$

In [64]:
```
A_1=A.elementary_row_op(op='n->n+km', row1=1,row2=0, k=-2)
A_1
```

Out[64]:
$$\begin{bmatrix} 1 & -1 & 1 & 13 \\ 0 & 7 & -5 & -39 \\ 4 & -1 & -6 & -4 \end{bmatrix}$$

In [65]:
```
A_2=A_1.elementary_row_op(op='n->n+km', row1=2,row2=0, k=-4)
A_2
```

Out[65]:
$$\begin{bmatrix} 1 & -1 & 1 & 13 \\ 0 & 7 & -5 & -39 \\ 0 & 3 & -10 & -56 \end{bmatrix}$$

In [72]:
```
A_3=(A_2.elementary_row_op(op='n->n+km', row1=2,row2=1, k=-3/7))
A_3
```

Out[72]:
$$\begin{bmatrix} 1 & -1 & 1 & 13 \\ 0 & 7 & -5 & -39 \\ 0 & 0 & -7.85714285714286 & -39.2857142857143 \end{bmatrix}$$

In [73]:
```
sp.nsimplify(A_3.row(2)[2])
```

Out[73]:
$$-\frac{55}{7}$$

In [74]:
```
A_4=A_3.elementary_row_op(op='n->kn',row=2,k=-7/55)
A_4
```

Out[74]:
$$\begin{bmatrix} 1 & -1 & 1 & 13 \\ 0 & 7 & -5 & -39 \\ 0 & 0 & 1.0 & 5.0 \end{bmatrix}$$

In [75]:
```
A_5=A_4.elementary_row_op(op='n->n+km',row1=1,row2=2,k=5)
A_5
```

Out[75]:
$$\begin{bmatrix} 1 & -1 & 1 & 13 \\ 0 & 7 & 0 & -14.0 \\ 0 & 0 & 1.0 & 5.0 \end{bmatrix}$$

In [76]:
```
A_6=A_5.elementary_row_op(op='n->n+km',row1=0,row2=2,k=-1)
A_6
```

Out[76]:
$$\begin{bmatrix} 1 & -1 & 0 & 8.0 \\ 0 & 7 & 0 & -14.0 \\ 0 & 0 & 1.0 & 5.0 \end{bmatrix}$$

In [77]:
```
A_7=A_6.elementary_row_op(op='n->kn',row=1,k=1/7)
A_7
```

Out[77]:
$$\begin{bmatrix} 1 & -1 & 0 & 8.0 \\ 0 & 1.0 & 0 & -2.0 \\ 0 & 0 & 1.0 & 5.0 \end{bmatrix}$$

In [78]:
```
A_8=A_7.elementary_row_op(op='n->n+km',row1=0,row2=1,k=1)
A_8
```

Out[78]:
$$\begin{bmatrix} 1 & 0 & 0 & 6.0 \\ 0 & 1.0 & 0 & -2.0 \\ 0 & 0 & 1.0 & 5.0 \end{bmatrix}$$

4.5 Solving using gauss_jordan_solve

Solve this system using gauss_jordan_solve

$$\begin{aligned} x - y + z &= 13 \\ 2x + 5y - 3z &= -13 \\ 4x - y - 6z &= -4 \end{aligned}$$

In [126]:
```
A=sp.Matrix(3,3,[1,-1,1,2,5,-3,4,-1,-6])
A
```

Out[126]:
$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 5 & -3 \\ 4 & -1 & -6 \end{bmatrix}$$

In [127]:
```
B = sp.Matrix([13,-13,-4])
B
```

Out[127]:
$$\begin{bmatrix} 13 \\ -13 \\ -4 \end{bmatrix}$$

In [130]:
```
A.gauss_jordan_solve(B)
```

Out[130]:
$$\left(\begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix}, \ [] \right)$$

4.6 Solving system using inverse of coefficient matrix

If $Ax = b$ is a system of linear equations then $x = A^{-1}b$

Solve this system using inverse of coefficient matrix

$$x - y + z = 13$$
$$2x + 5y - 3z = -13$$
$$4x - y - 6z = -4$$

In [4]:
```
A=sp.Matrix(3,3,[1,-1,1,2,5,-3,4,-1,-6])
A
```

Out[4]:
$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 5 & -3 \\ 4 & -1 & -6 \end{bmatrix}$$

In [5]:
```
b=sp.Matrix([13,-13,-4])
b
```

Out[5]:
$$\begin{bmatrix} 13 \\ -13 \\ -4 \end{bmatrix}$$

In [7]:
```
A_inv=A.inv()
A_inv
```

Out[7]:
$$\begin{bmatrix} \frac{3}{5} & \frac{7}{55} & \frac{2}{55} \\ 0 & \frac{2}{11} & -\frac{1}{11} \\ \frac{2}{5} & \frac{3}{55} & -\frac{7}{55} \end{bmatrix}$$

In [21]:
```
solution = A_inv*b
display(solution)
```

$$\begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix}$$

In [ ]:
```
```