3. Cofactor Matrix and Adjugate Matrix

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0.1 3. Minor, Cofactor, Cofactor Matrix and Adjugate matrix

[2]:
```
import sympy as sp
sp.init_printing()
```

[3]:
```
M1=sp.Matrix(3,3,[1,2,3,-1,2,-2,2,0,1])
M1
```

[3]:
$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & -2 \\ 2 & 0 & 1 \end{bmatrix}$$

[9]:
```
#To get minor of each element
M1.minor(1,0)
```

[9]:
$$2$$

[10]:
```
#To get cofactor of each element
M1.cofactor(1,0)
```

[10]:
$$-2$$

[11]:
```
#To get minor submatrix of each element
M1.minor_submatrix(0,0)
```

[11]:
$$\begin{bmatrix} 2 & -2 \\ 0 & 1 \end{bmatrix}$$

[12]:
```
M1.minor_submatrix(1,1)
```

[12]:
$$\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$

[5]:
```
cofactor=M1.cofactor_matrix()
cofactor
```

[5]:
$$\begin{bmatrix} 2 & -3 & -4 \\ -2 & -5 & 4 \\ -10 & -1 & 4 \end{bmatrix}$$

[13]:
```
#Get the adjoint of the matrix M3
M1.adjugate()
```

[13]:
$$\begin{bmatrix} 2 & -2 & -10 \\ -3 & -5 & -1 \\ -4 & 4 & 4 \end{bmatrix}$$

[14]:
```
#Adjugate is the transpose of cofactor matrix
cofactor.T
```

[14]:
$$\begin{bmatrix} 2 & -2 & -10 \\ -3 & -5 & -1 \\ -4 & 4 & 4 \end{bmatrix}$$

3.1 Inverse of a matrix using adjugate matrix

[15]:
```
A = sp.Matrix(3,3,[1,2,3,2,5,3,1,0,8])
A
```

[15]:
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

[18]:
```
#Check if the inverse exist by checking determinant is zero or not
det_A = sp.det(A)
det_A
```

[18]:
$$-1$$

[20]:
```
#Getting adjoint matrix
Adj_A = A.adjugate()
Adj_A
```

[20]:
$$\begin{bmatrix} 40 & -16 & -9 \\ -13 & 5 & 3 \\ -5 & 2 & 1 \end{bmatrix}$$

The formula for finding inverse of non singular matrix A is

$$A^{-1} = \frac{1}{det(A)} Adj(A)$$

[21]:
```
Inv_A = (1/det_A)*Adj_A
Inv_A
```

[21]:
$$\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

[24]:
```
#Verify if the inverse is right by checking if the product of A
#and its inverse is identity
A*Inv_A
```

[24]:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

0.1.1 Exercises

Check if the following matrices are invertible. If so, find the inverse using adjugate matrix.

1. $\begin{pmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{pmatrix}$

2. $\begin{pmatrix} -1 & 0 & 1 & 0 \\ 2 & 3 & -2 & 6 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & 5 \end{pmatrix}$