1)

pick w = 10^p1[…] where |w| = 2^p, so log2(|w|) = p

Case 1:

x = 10^α, y = 0^β, z = 0^(p-α-β)1[…]
i = 0
xy^0z = 10^(p-β)1[…]
p-β < p for all β > 0, so LP is not regular

Case 2:

x = ε, y = 10^β, z = 0^(p-β)1[…]
i = 2
xy^2z = 10^β10^p1[…]
β <= p-1
p-1 < p, so LP is not regular

2)

a)

pick w = 0^p1^2p

x = 0^α, y = 0^β, z = 0^(p-α-β)1^2p
i = 3
xy^3z = 0^(p+2β)1^2p
p+2β > 2p/2 for all β > 0, so LP is not regular

b)

(V, Σ, R, S)
V = {S, T}
Σ = {0,1}
R = { S => 11T,
T => 11T0 | ε}
S = S

3)

a)

pick w = a^nb^ma^mb^n s.t. n+m=p

Case 1:

x = a^α, y = a^(n-α)b^β, z = b^(m-β)a^nb^m
i = 0
xy^0z = a^αb^(m-β)a^mb^n
m-β != m for all β > 0, so LP is not regular

Case 2:

x = a^mb^α, y = b^β, z = b^(n-α-β)a^nb^m
i = 0
xy^0z = a^mb^(m-β)a^mb^n
m-β > m for all β > 0, so LP is not regular

b)

(V, Σ, R, S)

$V = \{ S, A,B,C \}$
$\Sigma = \{a,b\}$
$R = \{ S \Rightarrow aSb \mid bAa \mid aBb \mid \varepsilon$
$\quad A \Rightarrow bAa \mid aBb \mid \varepsilon$
$\quad B \Rightarrow aBb \mid \varepsilon \}$
$S = S$

4) Because the intersection of a context-free language and a regular language is context free, to create L' you can create L1 as a regular language $(\{S, A, B, C\}, \Sigma, \delta, S, \{C\})$

$\delta = \{ \delta(S, n, A), \delta(A, n, B), \delta(B, n, C), \delta(C, n, A)\}$ for n in $\Sigma$

then take the intersection of L and L1. This is equal to L'.