

1) L_P is not regular.

let $w = a^p b^p c^{p+1} d^p$.

so then $x = a^\alpha$, $y = a^\beta$, $z = a^{p-\alpha-\beta} b^p c^{p+1} d^p$

let $i = 3$

then $xyz = a^{p+2\beta} b^p c^{p+1} d^p$

since $\beta \geq 1$, $p+2\beta > p+1$, so xy^3z is not in L_P

L_P is not context-free (Assuming that winning by majority is not in the language)

let $w = a^0 b^3 c^p d^p$

then there are the following cases:

1) vx Contains only b 's

2) vx contains b 's and c 's

3) vx contains only c 's

4) vx contains c 's and d 's

5) vx contains only d 's

In cases 1, 2, 4, and 5, setting $i=0$ would either remove the plurality or make it a majority.

In case 3, setting $i=0$ would remove the plurality.

so uv^0wx^0y is not in L_P

2) a) L_M is not regular

let $w = a^0 b^p c^{p+1} d^p$

then either

$x = c^\alpha$, $y = c^\beta$, $z = c^{p-\alpha-\beta+1} d^p$

let $i = 0$

then $xyz = a^0 b^0 c^{p-\beta+1} d^p$

since $\beta \geq 1$, $p-\beta+1 \leq p$, so xy^0z is not in L_M

b) $M = (Q, \Sigma, \Gamma, \delta, q_0, \{q_3\})$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b, c, d\}$

$\Gamma = \Sigma \cup \{\$ \}$

$\delta =$

$\delta(q_0, a, \epsilon) \rightarrow q_0, a$

$\delta(q_0, b, \epsilon) \rightarrow q_1, b$

$\delta(q_1, b, \epsilon) \rightarrow q_1, b$

$\delta(q_1, c, \epsilon) \rightarrow q_2, \epsilon$

$\delta(q_2, c, \epsilon) \rightarrow q_2, \epsilon$

$\delta(q_2, c, a) \rightarrow q_2, \epsilon$

$\delta(q_2, c, \$) \rightarrow q_2, c$

$\delta(q_2, c, c) \rightarrow q_2, cc$

$\delta(q_2, d, \epsilon) \rightarrow q_3, \epsilon$

$\delta(q_3, d, \epsilon) \rightarrow q_3, \epsilon$

$\delta(q_3, d, \$) \rightarrow q_4, \epsilon$

$\delta(q_4, *, \epsilon) \rightarrow q_4, \epsilon$

3) a)

G:

S \rightarrow ABCD
A \rightarrow aAc
A \rightarrow B
B \rightarrow bBc
B \rightarrow C
C \rightarrow cC
C \rightarrow cD
D \rightarrow cDd
D \rightarrow ϵ

b)

G:

S \rightarrow AS1
S1 \rightarrow BS2
S2 \rightarrow CD
A \rightarrow A1A2
A \rightarrow C1C
A \rightarrow C1D | C1
A2 \rightarrow AC1
A1 \rightarrow a
B \rightarrow B1b2
B \rightarrow C1C
B \rightarrow C1D | C1
B2 \rightarrow BC1
B1 \rightarrow b
C \rightarrow C1C
C \rightarrow C1D | C1
C1 \rightarrow c
D \rightarrow C1D2
D2 \rightarrow DD1 | D1
D1 \rightarrow d

4) L_W is not regular

let $w = a^p b^{p+1} c^{(p+1)*3} d^p$

then $x = a^\alpha$, $y = a^\beta$, $z = a^{p-\alpha-\beta} b^{p+1} c^{(p+1)*3} d^p$

let $i = 3$

then $xyz = a^{p+2\beta} b^{p+1} c^{(p+1)*3} d^p$

since $\beta \geq 1$, $p+2\beta > p+1$, so xy^3z is not in L_M