

1)

- a. Valid. If it visits a number lower than 363, it never visits a lower number. If it visits a number higher than 363, it never visits a higher number.
- b. Valid. Same reason.
- c. Invalid. It visits 911 then 912 2 steps later
- d. Valid. Same as a.
- e. Invalid. It visits 347 then 299.

2)

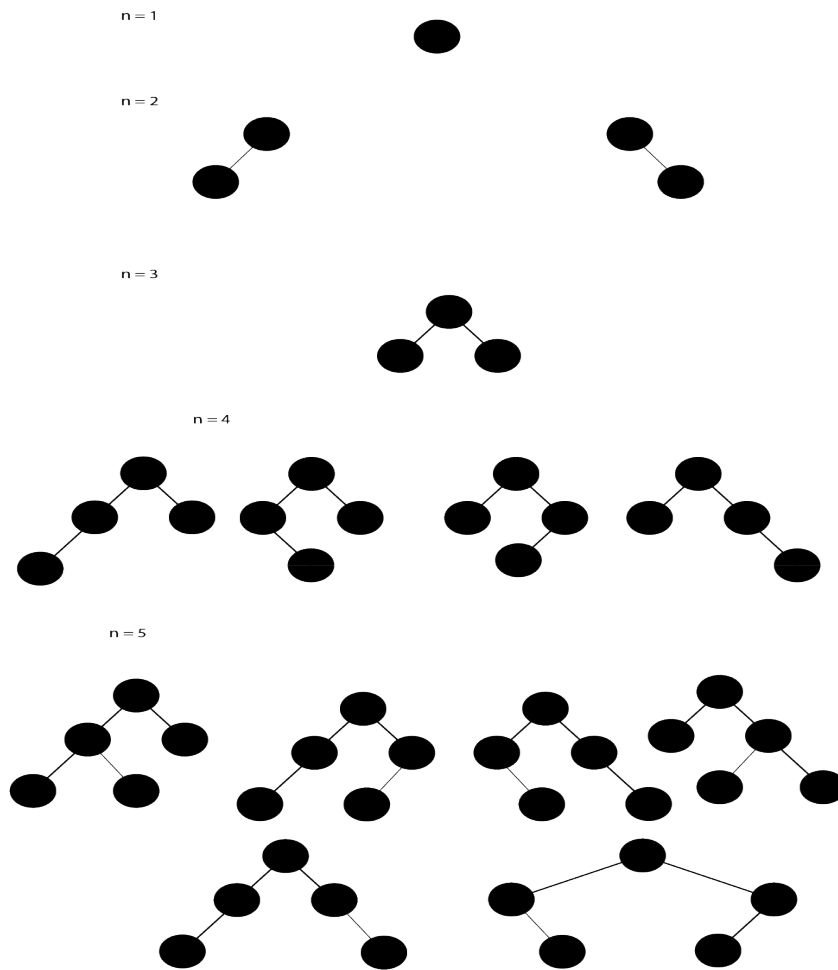
- a. `smallest(T)`
  - `if (empty(T)): return -1`
  - `if (leaf(T[0])): return T[0]`
  - `smallest(leftChild(T[0]))`
- b. `successor(T, k)`
  - `if (T[0] == k):`
    - `if (right(T[0])):`
      - `return smallest(right(T[0]))`
    - `if (!parent(T[0])): return -1 // no successor`
    - `if (parent(T[0]) > k): return parent(T[0])`
  - `if (T[0] > k):`
    - `if (!left(T[0])): return -1 //not in tree`
    - `return successor(left(T[0]), k)`
  - `if (T[0] < k):`
    - `if (!right(T[0])): return -1 //not in tree`
    - `return successor(right(T[0]), k)`

3)

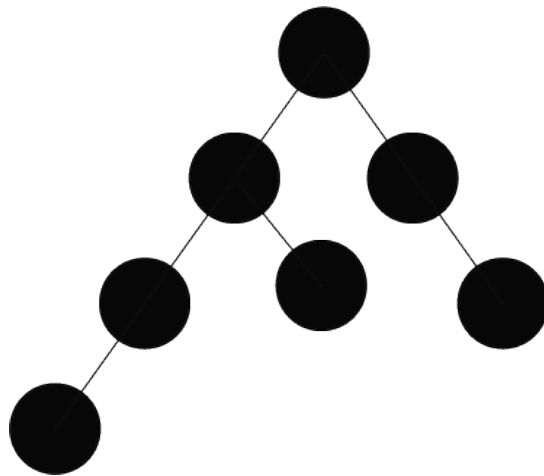
- a.  $A = \{\}, B = \{10, 9, 4\}, C = \{7\}, b=10 > c = 7$
- b. If a node has a right child, it's successor is the smallest descendent of that right child. If a node has a left child, it is not the smallest descendent. Therefore, the successor of a node with a right child must have no left child.  
If a node has a left child, it's predecessor is the largest descendent of that left child. If a node has a right child, it is not the largest descendent. Therefore, the predecessor of a node with a left child must have no right child.

4)

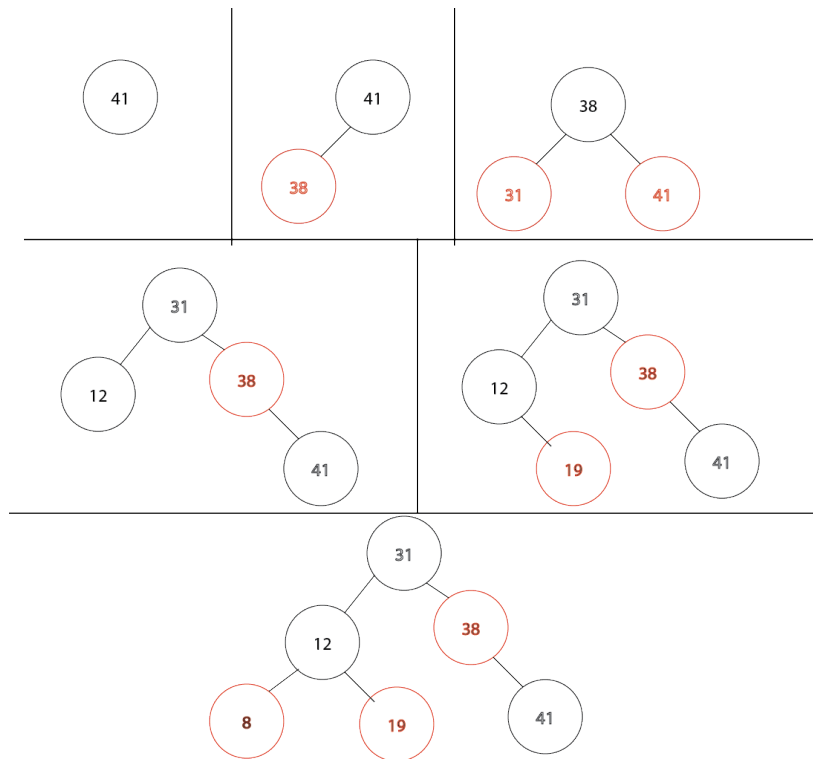
a.



b.



5)



6)

a.

- i. Nodes may have more than 1 value
- ii. Nodes may have more than 2 children

b.  $3 \leq \left\lfloor \log_{\frac{m}{2}} \left( \frac{100000001}{4} \right) \right\rfloor + 1 \rightarrow m = 585$

7)

a.

```
C(n,k):
  for (i=0; i<=min(n,k):
    table[i][i] = 1
  for(i=0; i<=n; i++):
    table[i][0] = 1
  for(i=1; i<=n; i++):
    for(j=1; j<i; j++):
      table[i][j] = table[i-1][j-1] + table[i-1][j]
  return table[n][k]
```

b. Time complexity:  $O(n^2)$

Space complexity:  $O(n \cdot k)$

c. A(n, k):

```
a = 0
for (i=2; i <=n; i++):
  for(j=1; j<i; j++):
    a++
return a
```

A(n, k):

```
a = 0
for(i=2; i<=n; i++):
  a+= i
return a
```

A(n,k):

```
a =  $n^2 - 3n$ 
return a
```

8)

a.

coinCollect(matrix[[]], x, y, coins, checked[[]], n, m): //this CANNOT BE parallelized.

```
checked[1][1] = 0
if (matrix[x][y] == 1):
  coins++
checked[x][y] = max(checked[x][y], coins)
if (!(x<n) && y<m): coinCollect(matrix, x, y+1, coins, checked, n, m)
if(!(y<m) && x<n): coinCollect(matrix, x+1, y, coins, checked, n, m)
if(x<n && y<m): coinCollect(matrix, x+1, y+1, coins, checked, n, m)
return checked[n][m]
```

b.

coinCollect(matrix[1][1], x, y, coins, checked[1][1], n, m): //this CANNOT BE parallelized.

checked[1][1] = 0

if (matrix[x][y] == 1):

coins++

checked[x][y] = max(checked[x][y], coins)

if (!(x < n) && y < m):

if (matrix[x][y+1] != 'x'): coinCollect(matrix, x, y+1, coins, checked, n, m)

if (!(y < m) && x < n):

if (matrix[x+1][y] != 'x'): coinCollect(matrix, x+1, y, coins, checked, n, m)

if (x < n && y < m):

if (matrix[x+1][y+1] != 'x'): coinCollect(matrix, x+1, y+1, coins, checked, n, m)

return checked[n][m]

c.

	1	2	3	4	5	6
1	<b>0</b>	×		●		
2	<b>1</b>	<b>1</b>	<b>1</b>	×	●	
3	<b>1</b>	<b>2</b>	<b>2</b>	×	●	
4	<b>1</b>	<b>2</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>4</b>
5	×	×	×	<b>3</b>	<b>4</b>	<b>4</b>

d. 6