

$$1. \sum_{i=1}^n \frac{1}{2^i} = 1 - \frac{1}{2^n}$$

$$\text{Base Case: } n = 1 \quad \sum_{i=1}^n \frac{1}{2^i} = \frac{1}{2^1} = 1 - \frac{1}{2^1} = \frac{1}{2}$$

$$\text{Recursive Case: } n = k \quad \sum_{i=1}^k \frac{1}{2^i} = \sum_{i=1}^{k-1} \frac{1}{2^i} + \frac{1}{2^k} = 1 - \frac{1}{2^{k-1}} + \frac{1}{2^k} = 1 - \frac{1}{2^{k-1}} + \frac{1}{2^k} = 1 - \frac{2}{2^k} + \frac{1}{2^k} = 1 - \frac{1}{2^k}$$

$$2. \text{ a) } \lim_{n \rightarrow \infty} \frac{\frac{n^2-n}{2}}{6n} = \frac{1}{2} \lim_{n \rightarrow \infty} \frac{n(n-1)}{6n} = \frac{1}{2} \lim_{n \rightarrow \infty} \frac{n-1}{6} = \infty \text{ SO } g(n) \in O(f(n))$$

$$\text{b) } \lim_{n \rightarrow \infty} \frac{n+\log(n)}{n\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{n}{n\sqrt{n}} + \frac{\log(n)}{n\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{n}{n\sqrt{n}} + \lim_{n \rightarrow \infty} \frac{\log(n)}{n\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{\log(n)'}{(n\sqrt{n})'} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{\frac{3\sqrt{n}}{2}} = 2 \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{2\sqrt{n}} = 0 \text{ SO } f(n) \in O(g(n))$$

$$\text{c) } \lim_{n \rightarrow \infty} \frac{2(\log(n))^2}{\log(n)+1} = 2 \lim_{n \rightarrow \infty} \frac{\log(n)^2}{\log(n)+1} = 2 \lim_{n \rightarrow \infty} \frac{\frac{2\log(n)}{n}}{\frac{1}{n}} = 4 \lim_{n \rightarrow \infty} \log(n) = \infty \text{ SO } g(n) \in O(f(n))$$

$$3. \text{ a) } \lim_{n \rightarrow \infty} \frac{n^2+3n+4}{6n+7} = \infty \text{ SO } f(n) \in \Omega(g(n))$$

$$\text{b) } \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\log(n+3)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\log(n)\log(3)} = \log(3) \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\log(n)} = \log(3) \lim_{n \rightarrow \infty} \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n}} = \log(3) \lim_{n \rightarrow \infty} \frac{n}{2\sqrt{n}} = \log(3) \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{2} = \infty \text{ SO } f(n) \in \Omega(g(n))$$

$$\text{c) } \lim_{n \rightarrow \infty} \frac{2^n - n^2}{n^4 + n^2} = \lim_{n \rightarrow \infty} \frac{2^n}{n^4 + n^2} - \lim_{n \rightarrow \infty} \frac{n^2}{n^4 + n^2} = \lim_{n \rightarrow \infty} \frac{2^n}{n^4 + n^2} - \lim_{n \rightarrow \infty} \frac{n^2}{n^2(n^2+1)} = \lim_{n \rightarrow \infty} \frac{2^n}{n^4 + n^2} - \lim_{n \rightarrow \infty} \frac{1}{n^2+1} = \lim_{n \rightarrow \infty} \frac{2^n}{n^4 + n^2} = \infty \text{ SO } f(n) \in \Omega(g(n))$$

$$4. \lim_{n \rightarrow \infty} \frac{(n+1)^2}{n^2} = \lim_{n \rightarrow \infty} \frac{n^2+2n+1}{n^2} = 1 \text{ SO } (n+1)^2 \in \theta(n^2) \text{ SO } (n+1)^2 \in O(n^2)$$

$$5. \lim_{n \rightarrow \infty} \frac{\log(n)}{n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{1} = \lim_{n \rightarrow \infty} \frac{1}{n} = 0 \text{ SO } \log(n) \in O(n)$$

$$6. \text{ a) } f(n) = \frac{1}{3}(n^3 - n) \\ \text{b) } O(n^3)$$

$$7. \text{ a) The threads are distributed evenly.}$$

b) The variables outside the parallel region are totally independent of the variables inside it. Inside the region, a always equals $i+1$. Outside, they remain whatever the arbitrary values chosen are. If they are not initialized, it will not compile. If they are not assigned, the values are 0.

c) The summation with reduction takes a good deal less time than without.

d) Static assigns iterations 0-39 to thread 0, 40-79 to thread 1, 80-119 to thread 2, 120-159 to thread 3, and 160-199 to thread 4. Dynamic assigns iterations as a new thread opens up, which is almost always (except for very low values of i) ends up evenly distributing the iterations evenly between the 5 threads. Runtime appears to do exactly the same thing as dynamic.

