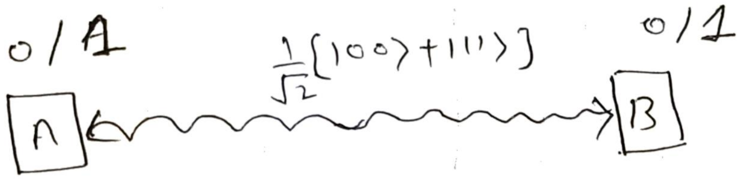
Rajkumar

Roll NO :: mn23iqt002

I



Here we can choose measurment direction

let A, A' is along Z and X axis and corresponding oprator

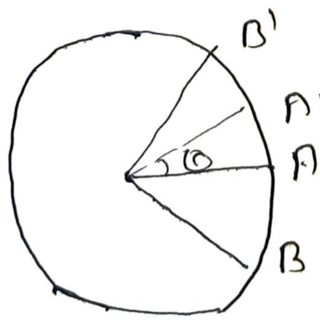
Z, X

similarly

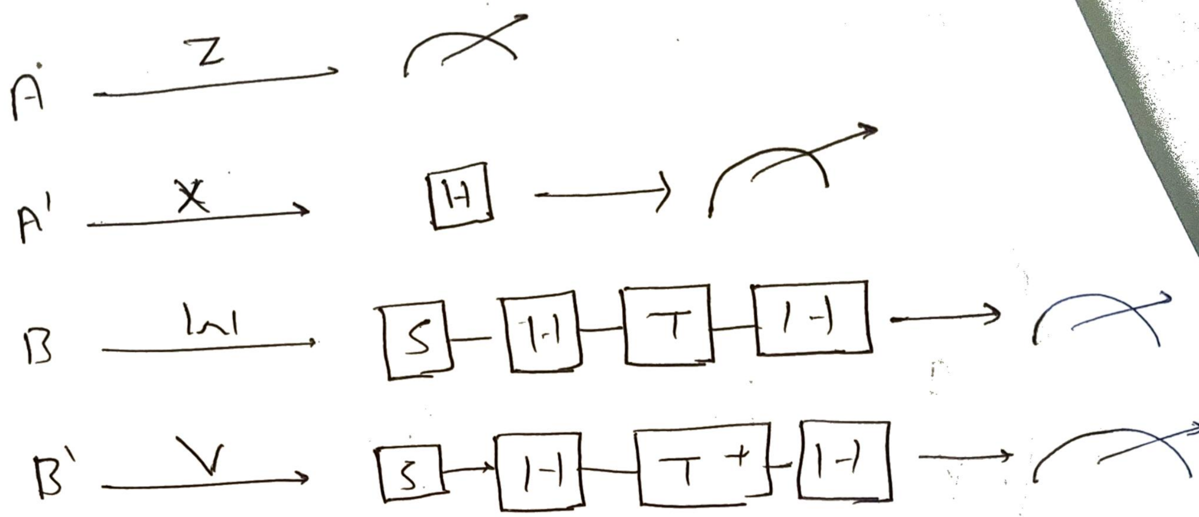
$\Rightarrow$ B, B' in W and V axis and corresponding oprator are

$|w| = \frac{1}{\sqrt{2}}[Z+X]$, $V = \frac{1}{\sqrt{2}}[Z+X]$

$\Rightarrow$



$\Rightarrow$



$Z = (|0\rangle, |1\rangle)$

$X = (|+\rangle, |-\rangle)$

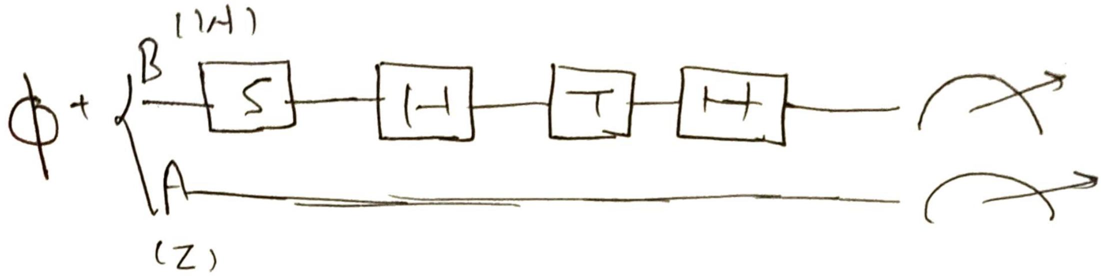
$W = \frac{1}{\sqrt{2}}[Z + X]$

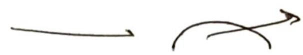
$V = \frac{1}{\sqrt{2}}[Z - X]$

$\Rightarrow$ A, B, $A B'$, A', B, A', B'

$\Rightarrow$

$$S = E(A, B) + E(A, B') + E(A', B') - E(A', B')$$



A 

For B

$$(I\otimes H)(I\otimes T)(I\otimes H)(I\otimes S)\left[\frac{1}{\sqrt{2}}|00\rangle+|11\rangle\right]$$

$$\frac{1}{\sqrt{2}}\left[|0\rangle\otimes|0\rangle + |1\rangle\otimes i|1\rangle\right]$$

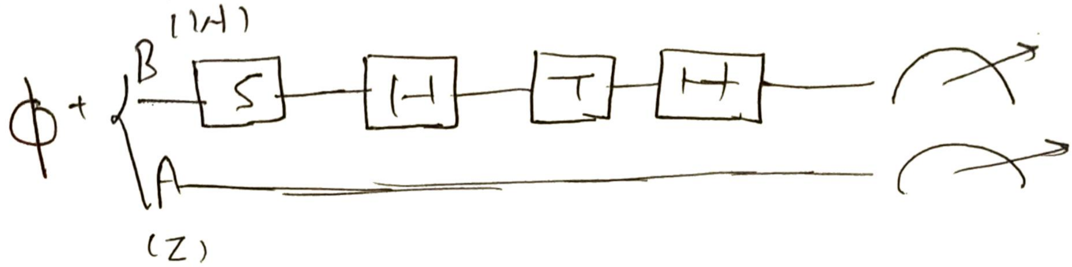
$$=\frac{1}{\sqrt{2}}\left\{|0\rangle\otimes\left[\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right] + |1\rangle\otimes i\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right\}$$

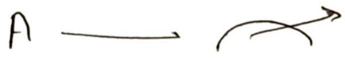
$$=\frac{1}{2}\left[|0\rangle\otimes\left(|0\rangle+e^{i\pi/4}|1\rangle\right] + |1\rangle\otimes i\left[|0\rangle-e^{i\pi/4}|1\rangle\right]\right]$$

$$\frac{1}{2}\left[|0\rangle\otimes\left[\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)+e^{i\pi/4}\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]\right] + |1\rangle\otimes i\left[\frac{|0\rangle+|1\rangle}{\sqrt{2}} - e^{i\pi/4}\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right]\right]$$

$$\frac{1}{2\sqrt{2}}\Big[|00\rangle+|01\rangle+|00\rangle e^{i\pi/4} - e^{i\pi/4}|01\rangle + i|10\rangle + i|11\rangle - ie^{i\pi/4}|10\rangle - ie^{i\pi/4}|011\rangle\Big]$$

(A, B)





For B

$$(I\otimes H)(I\otimes T)(I\otimes H)(I\otimes S)\left[\frac{1}{\sqrt{2}}|00\rangle + |11\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle\otimes|0\rangle + |1\rangle\otimes i|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle\otimes\left[\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right] + |1\rangle\otimes i\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{2}\left[|0\rangle\otimes\left(|0\rangle + e^{i\pi/4}|1\rangle\right] + |1\rangle\otimes i\left[|0\rangle - e^{i\pi/4}|1\rangle\right]\right]$$

$$\frac{1}{2}\left[|0\rangle\otimes\left[\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) + e^{i\pi/4}\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]\right]\right.$$
$$\left. + |1\rangle\otimes i\left[\frac{|0\rangle+|1\rangle}{\sqrt{2}} - e^{i\pi/4}\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right]\right]$$

$$= \frac{1}{2\sqrt{2}}\left[|00\rangle + |01\rangle + |00\rangle e^{i\pi/4} - e^{i\pi/4}|01\rangle\right.$$
$$+ i|10\rangle + i|11\rangle - ie^{i\pi/4}|10\rangle$$
$$\left. - ie^{i\pi/4}|011\rangle\right]$$

output $= \left[\frac{1+e^{i\pi/4}}{2\sqrt{2}} |00\rangle + \frac{1-e^{i\pi/4}}{2\sqrt{2}} |01\rangle \right.$

$\qquad + i \frac{(1-e^{i\pi/4})}{2\sqrt{2}} |10\rangle$

$\qquad \left. + i \frac{(1+e^{i\pi/4})}{2\sqrt{2}} |11\rangle \right]$

$$\Rightarrow \quad P_{00} = \left| \frac{(1+e^{i\pi/4})}{2\sqrt{2}} \right|^2 = \frac{2+\sqrt{2}}{8}$$

$$P_{01} = \left| \frac{1-e^{i\pi/4}}{2\sqrt{2}} \right|^2 = \frac{2-\sqrt{2}}{8}$$

$$P_{10} = \left| \frac{i(1-e^{i\pi/4})}{2\sqrt{2}} \right|^2 = \frac{2-\sqrt{2}}{8}$$

$$P_{11} = \left| \frac{i(1+e^{i\pi/4})}{2\sqrt{2}} \right|^2 = \frac{2+\sqrt{2}}{8}$$

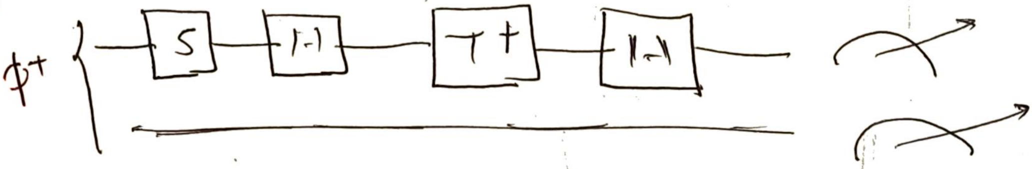
Q.C

$$\Rightarrow \quad E = (A, B) = P_{00} - P_{01} - P_{10} + P_{11}$$

$$= 2\left(\frac{2+\sqrt{2}}{8}\right) - 2\left(\frac{2-\sqrt{2}}{8}\right)$$

$$= \frac{4\sqrt{2}}{8} = \frac{1}{\sqrt{2}}$$

B')



$$(I\otimes H)(I\otimes T^{\dagger})(I\otimes H)(I\otimes S)\left[\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)\right]$$

$$=\frac{1}{\sqrt{2}}\left[|0\rangle\otimes|0\rangle+|1\rangle\otimes i|1\rangle\right]$$

$$=\frac{1}{\sqrt{2}}\left[|0\rangle\otimes\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)+|1\rangle\otimes i\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right]$$

$$=\frac{1}{\sqrt{2}}\left[|0\rangle\otimes\left[\frac{|0\rangle+e^{-i\pi/4}|1\rangle}{\sqrt{2}}\right]+|1\rangle\otimes i\left(\frac{|0\rangle-e^{-i\pi/4}|1\rangle}{\sqrt{2}}\right)\right.$$

$$=\frac{1}{2}\left[|0\rangle\otimes\left[\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)+\frac{e^{-i\pi/4}}{\sqrt{2}}(|0\rangle-|1\rangle)\right]\right.$$

$$+|1\rangle\otimes i\left[\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)-e^{-i\pi/4}\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right.$$

$$=\frac{1}{2\sqrt{2}}\left[|00\rangle+|01\rangle+e^{-i\pi/4}|00\rangle-e^{-i\pi/4}|01\rangle\right.$$

$$+i[|10\rangle+|11\rangle]-ie^{-i\pi/4}|10\rangle$$

$$\left.+ie^{-i\pi/4}|11\rangle\right]$$

$$=\frac{1}{2\sqrt{2}}\left[(1+e^{-i\pi/4})|00\rangle+(1-e^{-i\pi/4})|01\rangle\right.$$

$$+i[1-e^{-i\pi/4}]|10\rangle$$

$$+i(1+e^{-i\pi/4})|11\rangle$$

$\Rightarrow$ $P_{00} = \left|\dfrac{1+e^{-i\pi/4}}{2\sqrt{2}}\right|^2 = \dfrac{2+\sqrt{2}}{8}$

$P_{01} = \left|\dfrac{1-e^{-i\pi/4}}{2\sqrt{2}}\right|^2 = \dfrac{2-\sqrt{2}}{8}$

$P_{10} = \left|\dfrac{i(1+e^{-i\pi/4})}{2\sqrt{2}}\right| = \dfrac{2-\sqrt{2}}{8}$

$P_{11} = \left|\dfrac{i(1+e^{-i\pi/4})}{2\sqrt{2}}\right| = \dfrac{2+\sqrt{2}}{8}$

<u>9.c</u>

$\Rightarrow$

$E(A, B') = P_{00} - P_{01} - P_{10} + P_{11}$

$= \dfrac{2+\sqrt{2}}{8} - \dfrac{(2-\sqrt{2})}{8} - \dfrac{(2-\sqrt{2})}{8} + \dfrac{2+\sqrt{2}}{8}$

$= \dfrac{4\sqrt{2}}{8} = \dfrac{1}{\sqrt{2}}$

Similiarly we find

$E(A', B) = \dfrac{1}{\sqrt{2}}, \quad E(A', B') = -\dfrac{1}{\sqrt{2}}$

$$S = \left| E(A,B) + E(A,B') + E(A',B) - E(A',B') \right|$$

$$= \left| \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right|$$

$$= \frac{4}{\sqrt{2}}$$

$$S = 2\sqrt{2}$$

$\Longrightarrow$ $S = 2\sqrt{2}$

$\Rightarrow$ It is maximum amount that S can be using quantum mechanics

$\Longrightarrow$ $\boxed{|S| \leq 2\sqrt{2}}$

and This is the maximum amount that CHSH inequality can be violated by quantum mechanic