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Einstein held that the formalism of quantum mechanics involves "spooky actions at a distance." In the 1960s, Bell amplified this by showing that the predictions of quantum mechanics disagree with the results of any locally causal description. It should be appreciated that accepting nonlocal descriptions while retaining causality leads to a clash with relativity. Furthermore, the causal arrow of time by definition contradicts time-reversal symmetry. For these reasons, Wheeler and Feynman, Costa de Beauregard, Cramer, Price, and others have advocated abandoning microscopic causality. In this paper, a simplistic but concrete example of this line of thought is presented, in the form of a retro-causal toy model that is stochastic and provides an appealing description of the quantum correlations discussed by Bell. It is concluded that Einstein's "spooky actions" may occur "in the past" rather than "at a distance," resolving the tension between quantum mechanics and relativity and opening unexplored possibilities for future reformulations of quantum mechanics.

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I. INTRODUCTION

"…nobody understands quantum mechanics."
R. P. Feynman[1]

Quantum mechanics has a remarkable number of mathematical formulations, including Heisenberg's matrix mechanics,[2] Schrödinger's wave mechanics,[3] Feynman path integrals,[4] de Broglie–Bohm guiding waves,[5] Nelson's stochastic mechanics,[6] and more,[7] and many interpretations, including the Copenhagen interpretation,[8] Everett's many-worlds interpretation,[9] Cramer's transactional interpretation[10] and others.[11] In this maze, the voice of John Bell speaks with exceptional clarity.[12,13] In 1964, he provided a simple proof for the fact that the predictions of quantum mechanics cannot be reproduced by any locally causal mathematical description.[14] This fact is known as Bell's theorem.

Bell's theorem is a major obstacle to understanding quantum mechanics, in addition to other well-known difficulties such as the measurement problem[15] and the exponential complexity of the quantum mechanics description of many particles. Naturally, much discussion of the possibility of hidden assumptions in Bell's analysis has ensued.[16]

Bell's work followed a review of the hidden-variables problem in quantum mechanics[17] and is based on two-particle correlations,[18] generalizing those considered by Einstein, Podolsky, and Rosen[19] (EPR) in 1935. As a result, many subsequent publications have invoked assumptions that entered these earlier discussions, such as determinism[20] and realism.[21] Such assumptions are not necessary for a proof of Bell's theorem. However, there is one necessary assumption, which is only implicitly stated in the bulk of the literature regarding Bell's theorem, including the original, and this is the assumption of causality,[22] that is, the causal arrow of time. In terms of a critique of the logic used, this point is minor, because the assumption of locality, which is made explicit, clearly presupposes causality. Thus, the flaw in the analysis[23] may be adequately corrected by simply taking "locality" to be an abbreviation of "local causality." Indeed, the latter term is the one preferred by Bell in later publications.[24] However, the prevalent omission of an explicit mention of causality has consequences.

The problem becomes acute when special relativity and Lorentz invariance are brought into the discussion. Bell concluded that it would be necessary to choose a preferred reference frame in which the nonlocal effects of quantum mechanics would occur, defying the principle of relativity. For a while, Bell even advocated a return to the concept of the aether.[25] The conclusion that a tension exists between quantum mechanics and the principle of relativity is widely echoed to this day,[26] but the tension is based on three ingredients, quantum mechanics, relativity, and the causal arrow of time. Of these three, our confidence in the arrow of time should clearly be the weakest. After all, one is considering the microscopic realm, which is supposedly time-reversal symmetric.

The purpose of this paper is to review Bell's original 1964 contribution,[14] emphasizing rather than neglecting the assumption of causality. As an illustration of the character of the correlations between distant particles predicted by quantum mechanics, Bell included in Ref. 14 a simplistic toy model, which was necessarily nonlocal. This illustration is completed in Sec. III by introducing a similarly simplistic (but not as artificial) retro-causal toy model, which reproduces the same predictions. Surprisingly, such simple and explicit retro-causal models are not found in the literature,[27–29] despite the following two developments. (a) Restoring time-reversal symmetry at a fundamental level was achieved for classical charged particles by Wheeler and Feynman[30] and for quantum mechanics by Aharonov and co-workers.[31] (b) The relevance of retro-causation to the EPR discussion was noted repeatedly since 1953 by Costa de Beauregard,[32] Feynman,[33] and others,[34,35] was made the basis of a reinterpretation of quantum mechanics by Cramer[10] and was thoroughly discussed by Price.[36] These developments have been criticized by invoking interesting possibilities for causal loops.[37] Assuming that such loops can be avoided,[38] it may be hoped that the idea of retro-causation may one day lead to yet another reformulation of quantum mechanics, which could lead to significant progress in our understanding, perhaps analogous to the progress brought about by the introduction of path integrals.[4]

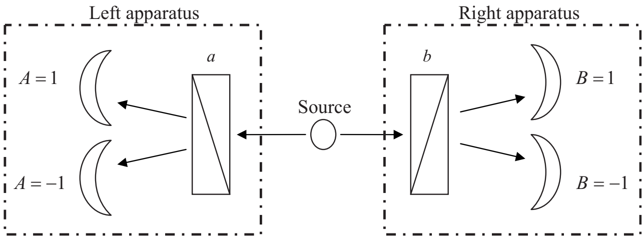

Fig. 1. Schematic of the setup considered by Bell (Ref. 14), adapted to photons. The circle represents the source of entangled pairs of photons, the dot-dashed boxes represent the measuring apparatuses, and the arrows mark the paths of the photons. Each apparatus contains a polarizing beam splitter (crossed rectangle), and a pair of detectors (crescents). The angles a and b extend out of the plane of the figure and measure the preferred polarization direction of each polarizing beam splitter (say, relative to the vertical). If the photon on the left (or the right) has the preferred polarization, it is deflected toward the detector labeled $A=1$ (or $B=1$), whereas if it has the perpendicular polarization, it reaches the other detector, resulting in $A=-1$ (or $B=-1$). The elements of the setup are considered perfect so that shortly after each time the source is activated, two detectors click, one on the left and one on the right, and the associated values of A and B may be recorded. For large distances between the source and the apparatuses, the apparatus settings (a and b) can be changed while the photons are in flight. The statistics of the results A and B are described by Eqs. (1) and (2) and can be measured by repeating the experiment many times.

II. BELL'S THEOREM

To briefly review Bell's theorem, we will describe a specific physical system, spell out the predictions that quantum mechanics makes for the correlations it exhibits, give the most general expression for these correlations, which could be generated by a locally causal mathematical model, and show that this expression is incompatible with the predictions of quantum mechanics. Bell's theorem has been reviewed many times, but the assumption of causality is generally not stated adequately. We will deviate from Ref. 14 by considering correlated photons rather than spin one-half particles because the expressions are slightly simpler (they contain no minus signs), many of the experimental tests involve photons,[39] and photon polarization is more familiar than spin.

Consider an idealized experimental setup involving a source of pairs of photons and two spatially separated measurement apparatuses, as sketched in Fig. 1. Every time the source is activated, two photons are emitted, with one reaching the left apparatus and the other reaching the right apparatus. The photon polarizations are described by a quantum mechanics state ψ of vanishing total angular momentum and even parity,[40] corresponding, for example, to emission from Ca atoms, as used in many experiments.[39] These atoms are activated by excitation into a state of zero angular momentum, $J=0$, and emit the two photons through a radiative cascade, passing through an intermediate state with $J=1$ before reaching their $J=0$ ground state. Each measurement apparatus contains a polarizing beam splitter, with a preferred orientation denoted by a for the one on the left and by b for the one on the right. These orientations are angles, defined modulo π. The measurement result on the left is denoted by $A=1$ if the photon is detected to be polarized along a and by $A=-1$ if its polarization is found to be perpendicular to a. The result of the measurement on the right is similarly denoted by B.

For a given choice of the orientations a and b (the free variables), quantum mechanics provides the probabilities, $P(A,B|a,b,\psi)$, for the four outcomes, $A=\pm 1, B=\pm 1$. It is convenient to specify these probabilities by stating the expectation values of the individual outcomes,

$$\langle A\rangle = \langle B\rangle = 0, \tag{1}$$

and the correlator,

$$P_{\mathrm{QM}}(a,b) \equiv \langle AB\rangle = \cos(2a-2b). \tag{2}$$

The quantum mechanical description is slightly different if the measurement on one side, say, on the left, is performed before the other, although Eqs. (1) and (2) are retained. In such cases, the probabilities on the left are evaluated through an expression of the form $P(A|a,\psi)$, and the wavefunction "collapses" into a simpler one, $\varphi(A,a,\psi)$, which is then used to evaluate the probabilities on the right in the form $P(B|b,\varphi)$. Note that even if the nonlocal collapse is avoided, as in the many-worlds interpretation, and ψ is taken to evolve smoothly into a combination of $\varphi(+,a,\psi)$ and $\varphi(-,a,\psi)$, the description must still use an expression of the form $P(B|b,\varphi)$ to reproduce the predictions for the correlator, Eq. (2). It is thus nonlocal in the sense relevant to the present discussion—the evolution of ψ may be "local" in configuration space, but it is not local in the three-dimensional physical space.[41] The nonlocality of quantum mechanics cannot be used to send signals immediately from one location to another. This no-signaling condition is manifested here in Eq. (1), which specifies, for example, that the probabilities for the different outcomes on the right do not depend on the free parameter on the left, $\langle B\rangle=0$ regardless of a.

To discuss general mathematical descriptions conforming with local causality, we have to fully specify input and output variables, and possibly internal variables, and to associate space-time locations with each of them. Inputs are taken as causes and outputs as effects. To be able to identify the meaning of the causal arrow of time, we must avoid using a definition that automatically takes causes to precede their effects.[23] For example, Newton's equations with the initial positions and momenta treated as inputs form a causal description, as an external force applied at an intermediate time (treated as an additional input) will have consequences only for later times. This classical description becomes retrocausal if the final position and momenta are treated as free input variables instead.

For the setup we have considered, a and b are inputs, A and B (or rather, their probability distribution) are outputs, and they are all associated with the time of measurement. Bell introduced the notation λ for the set of all the properties of each pair of photons just before the measurement is made on them. This set may be empty, may contain discrete or continuous variables, or may also contain functions or more complicated constructs. In standard quantum mechanics, there is a single possibility, $\lambda=\psi$, but a general description may involve stochastic elements, requiring a probability distribution $\rho(\lambda)$. More generally, we could consider a distribution of the type $\rho(\lambda|a,b)$, but this possibility will be taken up only in Sec. III because the assumption of *causality* would be violated if λ (or its distribution) were to depend on the free variables associated with the time of measurement.[42] The assumption of *locality* implies that A does not depend on the orientation b of the distant beam splitter or the measurement result B there, and similarly, B does not depend on a and on

A. Thus, a general locally causal description would specify, in addition to the character of λ, the distributions $\rho(\lambda)$, $P(A|a,\lambda)$, and $P(B|b,\lambda)$, which must be nonnegative and normalized. The correlator of interest may then be expressed as

$$\langle AB\rangle = \int \mathrm{d}\lambda \rho(\lambda) \sum_{AB} ABP(A|a,\lambda)P(B|b,\lambda), \tag{3}$$

where the integration over λ is understood as a summation if λ varies over a discrete set (and is absent if λ has only one possible value).

Bell went through three steps to prove that the predictions of quantum mechanics for the two-photon correlator, Eq. (2), cannot be reproduced in this manner. The first step follows EPR in deducing that A (and similarly, B) cannot be stochastic and must instead be completely determined by a and λ. Indeed, when $a=b$, quantum mechanics predicts perfect correlations between the measurements of the two photons—the value of A must in this case be equal to B with a probability of 100%. Thus, when $b=a$, the probabilities $P(A|a,\lambda)$ in Eq. (3) must be equal to either 0% or 100%, and we may use a function $A(a,\lambda)$ instead. Because A is assumed independent of b, the same must apply when $b\neq a$ as well. For the perfect correlations to hold, the similarly defined $B(b,\lambda)$ must share this dependence, $B(b,\lambda)=A(b,\lambda)$. Physically, this dependence would imply that the information within λ that determines the polarizations A and B is "duplicated" at the source and "carried" by each photon individually.

The second step of Bell's proof consists of writing the corresponding expression for the correlator,

$$P_{\mathrm{Bell}}(a,b) = \int \mathrm{d}\lambda \rho(\lambda) A(a,\lambda)A(b,\lambda), \tag{4}$$

and deriving an inequality for correlators of this type. The derivation uses the fact that $\rho(\lambda)$ is never negative and is normalized and that $A^2=1$. It involves introducing a third orientation c and noting that from

$$P_{\mathrm{Bell}}(a,b) - P_{\mathrm{Bell}}(a,c) = \int \mathrm{d}\lambda \rho(\lambda) A(a,\lambda)A(b,\lambda) \times[1-A(b,\lambda)A(c,\lambda)], \tag{5}$$

we can obtain, by taking the absolute value of the integrand,

$$|P_{\mathrm{Bell}}(a,b) - P_{\mathrm{Bell}}(a,c)| \leq \int \mathrm{d}\lambda \rho(\lambda)[1-A(b,\lambda)A(c,\lambda)] = 1 - P_{\mathrm{Bell}}(b,c). \tag{6}$$

Equation (6) is the original Bell's inequality, adapted to photon polarizations.

The third step consists of substituting Eq. (2) for the correlators within the inequality and noting that it is violated. The violation arises for nearby values of b and c, for which the left-hand side is linear in $|b-c|$, whereas the right-hand side is quadratic.

Thus, the quantum phenomena described by Eq. (2) are incompatible with any locally causal model, defying Einstein's expectations.[19,43] Notice that the difficulty is not with the use of a nonlocal description—such descriptions are common, for example, the Liouville equation—but with the incompatibility with a more detailed locally causal description. Many phenomena that are typically considered peculiarly quantum can be imitated by an underlying locally causal dynamics,[44] including the EPR phenomenon and quantum teleportation.[45] In contrast, the phenomena identified by Bell necessarily violate local causality.

III. RETRO-CAUSAL MODEL

Research activity on Bell's theorem was slow at first but has intensified considerably in recent decades.[46] A "Bell inequality" involving four rather than three different orientations was derived from Eq. (4), that is, for local deterministic hidden variables,[47] and then rederived from Eq. (3), that is, directly from local causality.[48] Violations of this inequality have been observed experimentally.[39] Imperfections in the experimental setup (called "loopholes") have been identified and addressed.[49] The quantum mechanics formalism and its predictions have thus been eliminated from the discussion, yielding a direct demonstration that local causality (a.k.a. "local realism") is inconsistent with observations.[18] Systems of three or four particles have been considered, and it was predicted[50] and experimentally demonstrated[51] that violations of local causality in some systems are even more explicit, that is, they can be identified without recourse to inequalities or to situations in which the quantum mechanical predictions are probabilistic and cannot be made with 100% certainty. Related research has been performed in quantum computation and quantum cryptography.[52] Many articles are more philosophical, addressing, for example, the nature of physical reality. Although Bell's mathematical approach can be used to illuminate such issues,[53] they can be very difficult to settle, absent significant technical advances, as exemplified by the controversy regarding the reality of atoms in the late 19th century.

In this paper, we shall avoid all of these issues and consider instead the following question: If locally causal descriptions are ruled out, what type of mathematical descriptions can reproduce quantum mechanics? The theoretical description, which served to motivate Bell and was later forcefully advocated by him,[54] is Bohmian mechanics,[5] a redevelopment of an early idea of de Broglie,[55] which treats the wavefunction as a guiding wave for particle configurations. For N particles, this description involves supplementing the wavefunction $\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_N,t)$ by N coordinates $\mathbf{R}_j(t)$. These configuration coordinates are chosen at random according to the standard $\Psi^+\Psi$ weights at the initial time[56] and then evolved according to the standard velocities, $(\hbar/M_j)\mathrm{Im}(\Psi^+\nabla_{\mathbf{r}_j}\Psi)/\Psi^+\Psi$, evaluated at the current configuration $\{\mathbf{r}_k=\mathbf{R}_k(t)\}_{k=1,\ldots,N}$ (M_j is the mass of the jth particle). The continuity equation associated with the Schrodinger evolution of Ψ guarantees that the coordinates $\mathbf{R}_j(t)$ will continue to be distributed according to $\Psi^+\Psi$ at later times. Taking the $\mathbf{R}_j(t)$ as the predictions for positions at the time of measurement thus reproduces standard quantum mechanics, as all quantum measurements can be reduced to measurements of positions.

Bohmian mechanics has advantages. Significantly, it does not suffer from the measurement problem. It also follows standard quantum mechanics quite closely—the added coordinates are local quantities obeying local rules, and the nonlocal effects present are clearly due to the use of the wavefunction. To illustrate such nonlocal effects, Bell included in Ref. 14 a simplistic toy model, which when adapted to photons, takes the initial condition λ to be a uniformly distributed random angle, $\rho(\lambda)=\text{constant}$ and takes $B(b,\lambda)$ to be 1 if $|\lambda-b|<\pi/4$ and -1 otherwise (recall that the angles are defined modulo π). Nonlocality enters in the b-dependence of $A(a,b,\lambda)$, which is taken as 1 if $|\lambda-a'|<\pi/4$ and -1 otherwise, where the angle a' is (somewhat artificially) chosen so that $1-(4/\pi)(b-a')=\cos(2a-2b)$. Because the distribution of λ is uniform, the correlator depends linearly on $|b-a'|$ in the range $[0,\pi/2]$. This model succeeds in reproducing the quantum mechanics predictions of Eqs. (1) and (2) by directly breaking locality and without violating causality.

The realization that Bell's theorem assumes local causality, rather than only locality, leads naturally to a consideration of mathematical descriptions that break causality altogether. A similarly simplistic noncausal toy model may be obtained[57] by taking λ to be an angle that accepts one of the values a, $a+\pi/2$, b, $b+\pi/2$, with equal probabilities, that is, by using

$$\rho(\lambda|a,b)=\frac{1}{4}\left[\delta(\lambda-a)+\delta\left(\lambda-a-\frac{\pi}{2}\right)+\delta(\lambda-b)+\delta\left(\lambda-b-\frac{\pi}{2}\right)\right]. \tag{7}$$

It is appropriate to take λ to represent the linear polarization of the photons belonging to each pair. The model thus assumes that the photons are emitted by the source with polarizations that "anticipate" the directions of the apparatuses to be encountered in the future—a blatant and explicit violation of causality. The subsequent interaction with each apparatus follows the standard probability rules for single-photon polarization measurements (Malus' law[58]),

$$p(A|a,\lambda)=\begin{cases}\cos^2(a-\lambda) & A=1\\ \sin^2(a-\lambda) & A=-1,\end{cases} \tag{8}$$

together with the corresponding expression for $p(B|b,\lambda)$.

It is straightforward to derive Eqs. (1) and (2) from Eqs. (7) and (8). Each of the possible values of λ separately leads to the quantum mechanical correlations of Eq. (2). For example, if $\lambda=a+\pi/2$ [the second possibility in Eq. (7)], then $A=-1$ with certainty, and $B=1$ or -1 with probabilities $\sin^2(a-b)$ and $\cos^2(a-b)$, respectively. (When $b=a$ or $b=a+\pi/2$, the values of both A and B are selected with certainty, resulting in perfect correlations.) The resulting contribution to the correlator,

$$\langle AB\rangle_{\lambda=a+\pi/2}=-\sin^2(a-b)+\cos^2(a-b), \tag{9}$$

is equal to $P_{\mathrm{QM}}(a,b)$. Because this result is obtained for each one of the four possible values of λ, any model that makes these choices with other weights, including models that would choose one of the possible values of λ with probability 100%, will reproduce Eq. (2). It is necessary to introduce λ as a *stochastic* variable with equal probabilities for $\lambda=a$ and for $\lambda=a+\pi/2$, and similarly for $\lambda=b,b+\pi/2$, to also reproduce Eq. (1), for example, through

$$\begin{aligned}\tfrac{1}{2}\langle B\rangle_{\lambda=a}+\tfrac{1}{2}\langle B\rangle_{\lambda=a+\pi/2}&=\tfrac{1}{2}(\cos^2(a-b)-\sin^2(a-b))\\&\quad+\tfrac{1}{2}(\sin^2(a-b)-\cos^2(a-b))\\&=0.\end{aligned} \tag{10}$$

In Eq. (7), equal weights have been chosen for all values of λ for symmetry reasons (aesthetics). If one of the measurements, say, the one on the left, is performed before the other, then avoiding causal loops requires[59] using an asymmetric version of the toy model, with $\rho(\lambda|a)=(1/2)[\delta(\lambda-a)+\delta(\lambda-a-\pi/2)]$ instead of Eq. (7) (that is, λ is independent of b). No change in the prescription for the probabilities of A and B, Eq. (8), is required.

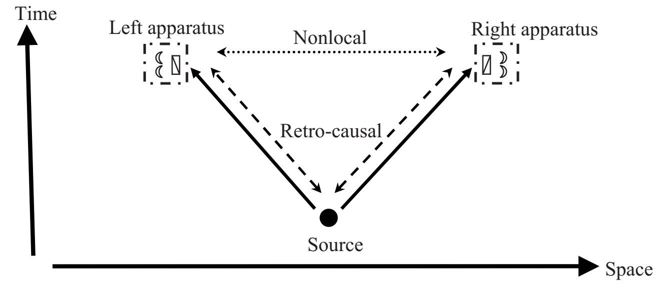

Fig. 2. Space-time sketch comparing alternatives for models of quantum correlations. The full arrows describe the paths of the photons from the source to the measuring apparatuses. In directly nonlocal models (dotted double arrow), the mathematical description of the state of the system propagates linearly from the past into the future, but output variables corresponding (for example) to the left apparatus may depend on variables describing the right apparatus. In retro-causal models (dashed arrows), the propagation of information from the apparatuses backward in time to the source is allowed, and thus no instantaneous "action at a distance" is needed. In these models, the variables that carry information, which has propagated into the past, represent microscopic physical quantities (for example, attributes of "quantum fluctuations"), which must be inaccessible to the macroscopic level; otherwise, causal paradoxes would arise.

This retro-causal model is appealing in its simplicity. As should be expected for such models,[36] λ represents an inaccessible quantity, which cannot be simply measured at the source. If it were measurable, we could send signals to a and b instructing them to avoid the values $\lambda,\lambda+\pi/2$, resulting in a paradoxical situation. For a source consisting of a Ca atom, we may easily imagine an attempt to measure the polarization λ by interrogating the intermediate $J=1$ state of the radiative cascade and measuring the relevant component of its spin. This measurement would destroy the EPR correlations of the photon pair, just as a "which path" measurement destroys the interference pattern in a two-slit experiment. The inaccessibility of λ thus reflects the inaccessibility of which path variables in standard quantum mechanics and should not be surprising.

Note that the assumptions of locality and causality are inseparable in two distinct senses (see Fig. 2). First, they imply each other in the sense that violations of causality imply indirect violations of locality (if λ depends on a and B depends on λ, then B depends indirectly on a), and in a relativistic setting, violations of locality, which are instantaneous in one reference frame, violate causality in other reference frames. Secondly, within the framework considered, any noncausal model can be "translated" into a nonlocal model, and vice versa. For example, λ of Eq. (7) can be replaced by a random integer n from 1 to 4, thus becoming independent of a and b, and then an angle $\lambda'(n,a,b)$ can be defined as associated with a later time, but in a manner such that λ' plays precisely the role of the previous λ. Conversely, any nonlocal model can be translated into a retro-causal one, by evaluating A and B "at the source," and including them as elements of the set λ.[60] Although the class of nonlocal models and the class of noncausal models are thus "mathemati-

Table I. Different ways of reproducing the Bell's-inequality-violating statistics of Eqs. (1) and (2). Bell's nonlocal model is naturally generalized by Bohmian mechanics. Finding general noncausal models remains a challenge.

	Initial conditions λ	Dynamics determining results A and B
Quantum mechanics	λ is the wavefunction ψ.	*Stochastic* and *nonlocal*, $P(A,B\|a,b,\lambda)$ provided.
Bohmian mechanics	λ consists of the wavefunction ψ and initial particle positions $\mathbf{R}_i$; the coordinates $\mathbf{R}_i$ are *stochastic*, distributed according to ψ.	*Deterministic* and *nonlocal*, $A(a,b,\lambda)$ and $B(a,b,\lambda)$ specified by the dynamics.
Bell's nonlocal toy model	$\lambda \in [0,\pi)$ is an angle, and is *stochastic*, uniformly distributed.	*Deterministic* and *nonlocal*, $A(a,b,\lambda)$ and $B(b,\lambda)$ provided.
Retro-causal toy model	$\lambda \in [0,\pi)$ is an angle. It depends on a and b in a *stochastic*, *retro-causal* manner. $P(\lambda\|a,b)$ is discrete, Eq. (7).	*Stochastic* (except for $a=\lambda,\lambda+\pi/2$), $P(A\|a,\lambda)$ and $P(B\|b,\lambda)$ provided by Eq. (8).

cally equivalent," physically, locality and causality are quite distinct. In particular, direct nonlocality clashes with the principles of relativity.

IV. CONCLUDING REMARKS

We have seen that mathematical models, which reproduce the quantum correlations of pairs of photons [Eqs. (1) and (2)], can be either directly nonlocal or retro-causal. A more detailed comparison is given in Table I, which compares standard quantum mechanics, Bohmian mechanics,[61] Bell's nonlocal toy model, and the retro-causal toy model. The first three are causal and directly nonlocal descriptions. They differ in the manner in which probabilities enter: In quantum mechanics the results are probabilistic, whereas in Bohmian mechanics and in the nonlocal toy model, the initial conditions are random and the dynamic rules are deterministic. The last row in Table I corresponds to the noncausal toy model [Eqs. (7) and (8)]. Additional models could be included in the table. For example, in the 1950s, Bohm also considered a description in which the Schrodinger wavefunction serves to guide particles with diffusive, rather than deterministic, dynamics.[62] It would be especially relevant to include additional retro-causal descriptions, but those available in the literature[31,63,64] are not valid candidates for the present comparison, as they do not provide explicit expressions for the measurement results A and B or their probabilities, treating them instead as given "final conditions."

The comparison of Table I leads naturally to the question: Can the retro-causal stochastic toy model be generalized to all quantum phenomena, in analogy with the fact that Bohmian mechanics provides a general description belonging to the same "deterministic-nonlocal" class as the simple model of Bell? Although a direct generalization would be desirable, it would not resolve the measurement problem because the variables associated with measurement apparatuses are treated here very differently from other physical variables. A more ambitious goal would be to seek a reformulation that, at a fundamental level, describes quantum fluctuations that are not subject to the causal arrow of time. Achieving agreement with the predictions of quantum mechanics, with the macroscopic causal arrow of time and with the rule of no nonlocal signaling,[65] would require breaking the time-reversal symmetry, perhaps by postulating a low entropy in the past and including dissipation in the description of measurement devices, in analogy to current descriptions of heat baths.[66]

When compared to standard quantum mechanics, these ideas lead to the view that quantum mechanics is a description following only the irreversibly registered part of the information regarding a system. Representing all of this information up to a given instant in time requires the exponentially complex structure of the wavefunction.[67] Such a view makes it appear natural to have conservation of information—unitary evolution—punctuated by "jumps" whenever additional information is registered. In contrast, the presently available retro-causal descriptions of general quantum phenomena[10,31] employ the wavefunctions in a manner that allows them to propagate backward in time. When applied to systems with several particles, such descriptions would involve a preferred reference frame and thus would not avoid the clash with relativity. Moreover, use of wavefunctions involves direct nonlocality instead of taking advantage of the indirect route offered by retro-causality. It is clearly desirable to explore the indirect route as well.

These concluding remarks are very much in line with Bell's later views, as evidenced by the concluding paragraph of his 1990 recapitulation:[24]

> "The more closely one looks at the fundamental laws of physics, the less one sees of the laws of thermodynamics. The increase of entropy emerges only for large complicated systems, in an approximation depending on 'largeness' and 'complexity.' Could it be that causal structure emerges only in something like a 'thermodynamic' approximation, where the notions 'measurement' and 'external field' become legitimate approximations? Maybe that is part of the story, but I do not think it can be all. Local commutativity does not for me have a thermodynamic air about it…"

It is argued here that because Bell took the causal arrow of time for granted, he overinterpreted local commutativity, as this time-reversal symmetric property is routinely taken to represent local causality, which is not symmetric. Adding retro-causation to the picture, possibly together with stochasticity, may perhaps provide "the other part of the story." To quote Bell again,[68]

> "Let us hope that these analyses may one day be illuminated, perhaps harshly, by some simple constructive model. …what is proved by impossibility proofs is lack of imagination."

Given the persistence of pre-20th-century ideas on causality and determinism, it is likely that many readers will not entertain high hopes for this line of enquiry. However, even the unconvinced should heed the call to prominently mention the symmetry-breaking causal arrow of time whenever the assumptions leading to Bell's theorem are discussed.

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[9]H. Everett III, "'Relative state' formulation of quantum mechanics," Rev. Mod. Phys. **29**, 454–462 (1957).

[10]J. G. Cramer, "The transactional interpretation of quantum mechanics," Rev. Mod. Phys. **58**, 647–687 (1986).

[11]For a review, see F. Laloë, "Do we really understand quantum mechanics? Strange correlations, paradoxes and theorems," Am. J. Phys. **69**, 655–701 (2001).

[12]Bell's relevant publications are collected in *John S. Bell on the Foundations of Quantum Mechanics*, edited by M. Bell, K. Gottfried, and M. Veltman (World Scientific, Singapore, 2001) and in Ref. 13.

[13]J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, revised edition (Cambridge U. P., Cambridge, 2004).

[14]J. S. Bell, "On the Einstein–Podolsky–Rosen paradox," Physics **1**, 195–200 (1964) (Ref. 13, Chap. 2).

[15]J. Bell, "Against 'measurement'," Phys. World **3** (4), 33–40 (1990) (Ref. 13, Chap. 23).

[16]For a recent example, see G. Blaylock, "The EPR paradox, Bell's inequality, and the question of locality," Am. J. Phys. **78**, 111–120 (2010); T. Maudlin, "What Bell proved: A reply to Blaylock," *ibid.* **78**, 121–125 (2010).

[17]J. S. Bell, "On the problem of hidden variables in quantum mechanics," Rev. Mod. Phys. **38**, 447–452 (1966) (Ref. 13, Chap. 1).

[18]D. Bohm and Y. Aharonov, "Discussion of experimental proof for the paradox of Einstein, Rosen and Podolsky," Phys. Rev. **108**, 1070–1076 (1957).

[19]A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?," Phys. Rev. **47**, 777–780 (1935).

[20]See J. S. Bell, "Bertlmann's socks and the nature of reality," J. Phys. (Paris) **42**, C2 41–61 (1981) (Ref. 13, Chap. 16), especially note 10.

[21]T. Norsen, "Against 'Realism'," Found. Phys. **37**, 311–340 (2007).

[22]The word "causality" appears in early publications, including Ref. 14, with the meaning of "determinism," often in phrases such as "complete causality" or "ideal causality." It was used by Bell in the sense of an arrow of time in later publications.

[23]For example, J. S. Bell, "Free variables and local causality," Epistemological Letters, **15** (1977) (Ref. 13, Chap. 12) says, "'It has been assumed that the settings of instruments are in some sense free variables….' For me this means that the values of such variables have implications only in their future light cones." The flaw is in adopting this interpretation of "free variables," an ostensibly time-reversal symmetric concept, without mentioning that the causal arrow of time is involved as a separate assumption.

[24]For example, J. S. Bell, "La nouvelle cuisine," in *Between Science and Technology*, edited by A. Sarlemijn and P. Kroes (Elsevier/North-Holland, New York/Amsterdam, 1990), pp. 97–115 (Ref. 13, Chap. 24).

[25]See the conclusions in Ref. 14, Ref. 20 or J. S. Bell, "Introductory remarks," Phys. Rep. **137**, 7–9 (1986) ("Speakable and unspeakable in quantum mechanics," Ref. 13, Chap. 18). See also J. S. Bell, "How to teach special relativity," Progress in Scientific Culture **1** (2), 1–13 (1976) (Ref. 13, Chap. 9). He retreated from this position in J. S. Bell, "Are there quantum jumps?," in *Schrödinger. Centenary of a Polymath* (Cambridge U. P., Cambridge, 1987) (Ref. 13, Chap. 22).

[26]See, for example, D. Z. Albert and R. Galchen, "A quantum threat to special relativity," Sci. Am. **300** (3), 32–39 (2009).

[27]H. Price, "Toy models of retrocausality," Stud. Hist. Philos. Mod. Phys. **39**, 752–761 (2008).

[28]For example, B. d'Espagnat, "Nonseperability and the tentative descriptions of reality," Phys. Rep. **110**, 201–264 (1984), refers to the work of Costa de Beauregard and states that "a 'backwards causality'…was never explicitly defined" (p. 246).

[29]Early versions of the present work are available at ⟨pirsa.org/06110017⟩ and ⟨arxiv.org/abs/0807.2041v1⟩.

[30]J. A. Wheeler and R. P. Feynman, "Interaction with the absorber as the mechanism of radiation," Rev. Mod. Phys. **17**, 157–181 (1945); "Classical electrodynamics in terms of direct interparticle action," *ibid.* **21**, 425–433 (1949).

[31]See Y. Aharonov and L. Vaidman, "The two-state vector formalism of quantum mechanics: An updated review," in *Time and Quantum Mechanics*, Lect. Notes Phys., Vol. 734, edited by J. G. Muga, R. Sala Mayato, and I. L. Egusquiza (Springer, Berlin, 2008), pp. 399–447.

[32]O. Costa de Beauregard, "Une réponse à l'argument dirigé par Einstein, Podolsky, et Rosen contre l'interprétation bohrienne des phénomènes quantiques," C. R. Hebd. Seances Acad. Sci. **236**, 1632–1634 (1953); "Time symmetry and the Einstein paradox," Nuovo Cimento B **42**, 41–64 (1977); "Time symmetry and the Einstein paradox–II," *ibid.* **51**, 267–279 (1979). The authors of Refs. 10 and 32 have, years later, taken their arguments much further, resulting in publications, which are, at best, speculative (including discussions of signaling backward in time and of paranormal phenomena, respectively). The transition made by these authors may constitute a remarkable history-of-science phenomenon but does not detract from the validity of the original arguments.

[33]R. P. Feynman, "Simulating physics with computers," Int. J. Theor. Phys. **21**, 467–488 (1982).

[34]R. I. Sutherland, "Bell's theorem and backwards-in-time causality," Int. J. Theor. Phys. **22**, 377–384 (1983), and references therein.

[35]It is interesting to note that N. Bohr, "Can quantum-mechanical description of physical reality be considered complete?," Phys. Rev. **48**, 696–702 (1935), already discusses a measurement apparatus set up *a la* EPR, which allows switching between position or momentum detection *after* the detected particle has continued its flight and is at a remote location. The fact that even after the particle has left, we are "still left with a *free choice* whether we wish to know the momentum of the particle or its initial position" (italics in original) did not deter Bohr, as he did not associate "physical reality" with the position and momentum coordinates, nor did he explicitly refer to the value one of these variables would have at a time before the choice was made. Instead, he pointed out that the choice would merely affect "*the possible types of predictions regarding the future behavior of the system*."

[36]H. Price, "A neglected route to realism about quantum mechanics," Mind **103**, 303–336 (1994); *Time's Arrow and Archimedes' Point* (Oxford U. P., Oxford, 1996).

[37]R. I. Sutherland, "A corollary to Bell's theorem," Nuovo Cimento B **88**, 114–118 (1985); T. Maudlin, *Quantum Non-Locality and Relativity* (Blackwell, Oxford, 1994).

[38]Wheeler and Feynman give an example of a retro-causal physical theory avoiding causal loops in Fig. 2 of the second article in Ref. 30.

[39] For example, A. Aspect, J. Dalibard, and G. Roger, "Experimental test of Bell's inequalities using time-varying analyzers," Phys. Rev. Lett. **49**, 1804–1807 (1982).

[40] C. A. Kocher and E. D. Commins, "Polarization correlation of photons emitted in an atomic cascade," Phys. Rev. Lett. **18**, 575–577 (1967); C. N. Yang, "Selection rules for the dematerialization of a particle into two photons," Phys. Rev. **77**, 242–245 (1950).

[41] That the many-worlds view is nonlocal in the sense relevant to Bell's theorem has been disputed and should be contrasted with, for example, the conclusions in Ref. 16.

[42] A. Shimony, M. A. Horne, and J. F. Clauser, "Comment on 'The theory of local beables'," Epistemological Lett., **13** (1976) [reproduced in Dialectica **39**, 97–102 (1985)] suggested denying the free-variable status of the apparatus settings a and b by introducing a conspiratorial setup in which the experimenters fail to exercise free will in studying Bell-type correlations. In this scenario, common causes *in the past* determine a and b (and λ). If this were the case, use of a distribution of the form $\rho(\lambda|a,b)$ could be justified without invoking retro-causation, that is, causes *in the future* (Ref. 36). This possibility has been called superdeterministic by Bell (Ref. 24). Rather than delving into a philosophical discussion of "free will," it suffices to point out that a and b do play the role of free variables in standard quantum mechanics, and that the purpose here is to characterize the class of models which are capable of reproducing quantum mechanics in this sense (see also Ref. 23).

[43] See, for example, N. D. Mermin, "Is the moon there when nobody looks? Reality and the quantum theory," Phys. Today **38** (4), 38–47 (1985).

[44] R. W. Spekkens, "Evidence for the epistemic view of quantum states: A toy theory," Phys. Rev. A **75**, 032110 (2007), and references therein.

[45] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, "Experimental quantum teleportation," Nature (London) **390**, 575–579 (1997).

[46] The citation count for Ref. 14 has roughly doubled each decade and is now greater than 200/year.

[47] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, "Proposed experiment to test local hidden-variable theories," Phys. Rev. Lett. **23**, 880–884 (1969).

[48] J. F. Clauser and M. A. Horne, "Experimental consequences of objective local theories," Phys. Rev. D **10**, 526–535 (1974); see especially note 15 therein and note 10 in J. S. Bell, "Introduction to the hidden-variable question," in *Proceedings of the International School of Physics "Enrico Fermi," Course IL: Foundations of Quantum Mechanics*, edited by B. d'Espagnat (Academic, New York, 1971), pp. 171–181 (Ref. 13, Chap. 4).

[49] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, "Violation of Bell's inequality under strict Einstein locality conditions," Phys. Rev. Lett. **81**, 5039–5043 (1998); M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, "Experimental violation of a Bell's inequality with efficient detection," Nature (London) **409**, 791–794 (2001).

[50] D. M. Greenberger, M. A. Horne, and A. Zeilinger, "Multiparticle interferometry and the superposition principle," Phys. Today **46** (8), 22–29 (1993).

[51] D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, "Observation of three-photon Greenberger–Horne–Zeilinger entanglement," Phys. Rev. Lett. **82**, 1345–1349 (1999).

[52] See, for example, A. K. Ekert, "Quantum cryptography based on Bell's theorem," Phys. Rev. Lett. **67**, 661–663 (1991).

[53] It has become more-or-less standard to interpret the failure of Einstein's local realism as in the following example: H. A. Wiseman, "From Einstein's theorem to Bell's theorem: a history of quantum non-locality," Contemp. Phys. **47**, 79–88 (2006), uses quotes of Einstein to list the possible falsehoods identified by Bell's theorem as locality (the postulates of relativity); and the independent reality of distant events. However, focusing on causal mathematical descriptions (as the assumption of causality is obviously being made), it seems that one is merely distinguishing here between two types of nonlocality: Signals which propagate instantaneously, or nonlocal variables, which are not simply associated with particular positions in space. Both of these violate relativistic causality. See also Ref. 21.

[54] See, for example, J. S. Bell, "de Broglie–Bohm, delayed-choice, double-slit experiment, and density matrix," Int. J. Quantum Chem., Quantum Chem. Symp. **14**, 155–159 (1980) (Ref. 13, Chap. 14).

[55] G. Bacciagaluppi and A. Valentini, *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference* (Cambridge U. P., Cambridge, 2009).

[56] The notation allows for spin and spinors; else $|\Psi|^2$ would have been used.

[57] For example, Fig. 1 of Ref. 10 implicitly hints at this. See also J. G. Cramer, "Generalized absorber theory and the Einstein–Podolsky–Rosen paradox," Phys. Rev. D **22**, 362–376 (1980).

[58] This law was discovered well before single photons were considered and originally related to intensities of light. See, e.g., B. Kahr and K. Claborn, "The lives of Malus and his bicentennial law," ChemPhysChem **9**, 43–58 (2008).

[59] It is interesting to note that the causal loops that can be generated with the symmetric version of the toy model involve loss of predictive power, rather than the contradictions considered in Ref. 37. The agreement with quantum mechanics guarantees that contradictions cannot occur. See J. Berkovitz, "On predictions in retro-causal interpretations of quantum mechanics," Stud. Hist. Philos. Mod. Phys. **39**, 709–735 (2008).

[60] Translating the standard quantum mechanics of a single particle into a retro-causal model in this manner gives essentially the transactional interpretation of Ref. 10. Doing the same for the two-photon system gives the description of N. Rosen, "Bell's theorem and quantum mechanics," Am. J. Phys. **62**, 109–110 (1994) (This type of description could be the one the authors of Ref. 34 had in mind.) Note that Rosen associated λ with the time of measurement, yielding a nonlocal description (as with the abovementioned λ'), and did not mention causality and the arrow of time.

[61] The table refers to the standard, nonrelativistic version of Bohmian mechanics—it is irrelevant to the present discussion that adapting Bohmian mechanics to photons may require a relativistic extension.

[62] D. Bohm, *Causality and Chance in Modern Physics* (Routledge, London, 1957); see also Ref. 6.

[63] S. Goldstein and R. Tumulka, "Opposite arrows of time can reconcile relativity and nonlocality," Class. Quantum Grav. **20**, 557–564 (2003).

[64] R. Sutherland, "Causally symmetric Bohm model," Stud. Hist. Philos. Mod. Phys. **39**, 782–805 (2008).

[65] For a recent generalization of this rule, see M. Pawłowski, T. Paterek, D. Kaszlikowski, V. Scarani, A. Winter, and M. Zukowski, "Information causality as a physical principle," Nature (London) **461**, 1101–1104 (2009).

[66] Of course, such descriptions must avoid the causal loops in Ref. 37, which are generated by making the result of a measurement at one time determine which measurement will subsequently be performed (Ref. 38).

[67] Note that although classically simulating a quantum computer is widely conjectured to require exponentially many steps, it is known not to require a large memory—see, for example, E. Bernstein and U. Vazirani, "Quantum complexity theory," in *Proceedings of the 25th Annual ACM Symposium on Theory of Computing* (ACM, New York, 1993), pp. 11–20. The fact that the memory requirements are modest may indicate that the state of a quantum system, when it is not restricted to a single time, should not be so complicated.

[68] J. S. Bell, "On the impossible pilot wave," Found. Phys. **12**, 989–999 (1982) (Ref. 13, Chap. 17).