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Experimental test of CHSH inequality for non-maximally entangled states

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Abstract

A method of testing the maximal violation of the CHSH inequality for non-maximally entangled pure states is presented and experimentally realized. © 2001 Elsevier Science B.V. All rights reserved.

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The Einstein–Podolsky–Rosen (EPR) paradox has been discussed for more than sixty years [1] to verify the completeness of the quantum mechanics description of nature. Further work about the EPR state was carried out by Bell who celebrated Bell inequality [2] to point out the contradiction of quantum mechanics with local realism. By using of Bell-type inequalities people can experimentally test whether the local hidden variables (LHV) theory or quantum theory is right.

Up to now, Bell-type inequalities have been tested by many experiments [3–5]. Most of the experiments used maximally entangled states of two qubits to test Bell inequalities. But, in fact, it has been proved [6] that any entangled pure state of two qubits violates some Bell inequality. That is, for bipartite pure states, "entangled" means "Bell inequality violating". So, it will be something important to experimentally test that to what extent a non-maximally entangled bipartite pure state can violate the Bell inequalities. The value of violation can be viewed as the measurement of "entanglement".

In this Letter we perform an experiment to test CHSH [7] inequality (one of the Bell-type inequalities) for a broad range of non-maximally entangled bipartite pure states. The polarization entanglement of two photons produced in the spontaneous parametric down-conversion (SPDC) process is used. The polarization angle settings of the measurement are selected to maximize the violation of CHSH inequality.

In our experiment, the non-maximally polarization entangled photon pairs are conveniently produced in the SPDC process using a two-crystal geometry [8]. We use two adjacent, identically cut BBO crystals ($5.0 \times 5.0 \times 0.59$ mm, optic axis cut at $\theta_{pm} = 33.9°$), and operate them with type-I phase matching. These two crystals are oriented with their optic axes aligned in perpendicular planes; i.e., the first (second) crystals optic axis and the pump laser define the vertical (horizontal) plane. As has been shown in [8], when the pump beam is polarized at $e^{i\Phi}\sin\theta|H\rangle + \cos\theta|V\rangle$ (H and V denote horizontal and vertical polarization),

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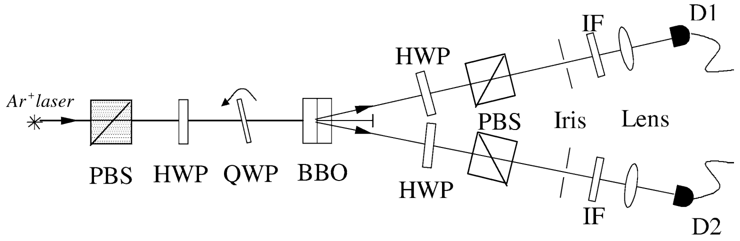

Fig. 1. The experiment setup of producing non-maximally entangled states and measuring the value of S.

the emitted photon pairs will be in the non-maximally entangled state $\cos\theta|HH\rangle + e^{i\Phi}\sin\theta|VV\rangle$. When $\theta = 45°$ and $\Phi = 0$ we get the maximally entangled state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$.

Fig. 1 shows the experimental setup used to produce the non-maximally entangled states and measure the value proposed in CHSH inequality. The 1.7-mm-diam. pump beam at 351.1 nm is provided by an Ar^+ laser. After passing through a polarizing beam splitter (PBS) and a suitably oriented rotatable half wave plate (HWP), the pump beam is turned into the polarization state of $\sin\theta|H\rangle + \cos\theta|V\rangle$ and then directed to the two BBO crystals. The PBS is used to give a pure polarization state $|H\rangle$ and the HWP is used to rotate the state $|H\rangle$ to any linear polarization state. The tiltable quarter wave plate (QWP) behind the HWP is used to compensate for the phase difference between the $|H\rangle$ and $|V\rangle$ components which caused by the BBO crystals. After the SPDC process in BBO crystals, the degenerate frequency photons at 702.2 nm are emitted into a cone of half-opening angle 3.0°. The HWP and PBS in the two arms are used as polarization analyzers, i.e., with a PBS preceded by an adjustable HWP (for 702 nm), we can measure an arbitrary given linear polarization state of a photon. After passing through a 2-mm-diam. iris and an interference filter (IF) centered at 702 nm (FWHM $\approx$ 4 nm), the light is detected by single photon detectors — silicon avalanche photodiodes (EG&G, SPCM-AQR), with efficiencies of $\sim$70% and dark count rates of order 25 s^{-1}. The outputs of the detectors are recorded in coincidence using a time-to-amplitude converter and single-channel analyzer (EG&G, TAC/SCA). A 5 ns time window is used in the TAC/SCA to capture true coincidences. For the polarization analyzer angle settings θ_1, θ_2 in each arm, we record the coincidence rate as $C(\theta_1, \theta_2)$.

In the following, we will discuss how to obtain the maximal violation of CHSH inequality with recorded coincidence rates for non-maximally entangled states.

In CHSH inequality [7], the proposed value S should not be more than 2 for LHV theories (of any bipartite systems), S is calculated as

$$S = E(\theta_1, \theta_2) + E(\theta_1, \theta_2') + E(\theta_1', \theta_2') - E(\theta_1', \theta_2). \tag{1}$$

where $E(\theta_1, \theta_2)$ is given by

$$E(\theta_1, \theta_2) = \frac{C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) - C(\theta_1, \theta_2^\perp) - C(\theta_1^\perp, \theta_2)}{C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) + C(\theta_1, \theta_2^\perp) + C(\theta_1^\perp, \theta_2)}. \tag{2}$$

Here, $\theta_i^\perp = \theta_i + 90°$, $i = 1, 2$.

For a given non-maximally entangled pure state $|\Psi(\theta, \Phi)\rangle = \cos\theta|HH\rangle + e^{i\Phi}\sin\theta|VV\rangle$ (without loss of generality, we may assume $\theta \in [0, \pi/4]$ and $\Phi = 0$ [9]), we have to choose proper polarization analyzer angle settings θ_1, θ_2, θ_1', θ_2' in Eq. (1) to obtain maximal violation of CHSH inequality.

It is easy to get the expression of S for the state $|\Psi(\theta)\rangle = \cos\theta|HH\rangle + \sin\theta|VV\rangle$

$$\begin{aligned} S = {} & \cos 2\theta_1(\cos 2\theta_2 + \cos 2\theta_2') \\ & + \sin 2\theta \sin 2\theta_1(\sin 2\theta_2 + \sin 2\theta_2') \\ & + \cos 2\theta_1'(\cos 2\theta_2' - \cos 2\theta_2) \\ & + \sin 2\theta \sin 2\theta_1'(\sin 2\theta_2' - \sin 2\theta_2). \end{aligned} \tag{3}$$

After analyzing the following four equations related to S

$$\frac{\partial S}{\partial \theta_1} = 0, \qquad \frac{\partial S}{\partial \theta_2} = 0, \qquad \frac{\partial S}{\partial \theta_1'} = 0, \qquad \frac{\partial S}{\partial \theta_2'} = 0 \tag{4}$$

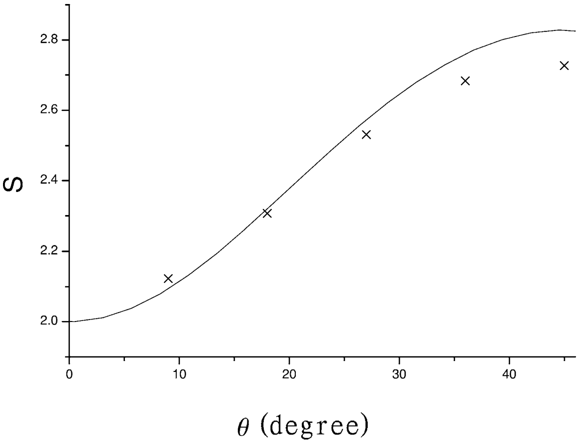

Fig. 2. The experiment results (denoted by $\times$) of S for five states, corresponding to $\theta = 9°, 18°, 27°, 36°, 45°$. The solid curve is the theoretical calculation results of S.

we found that for maximal S, the settings θ_1, θ_2, θ_1', θ_2' is not unique but in a region for a given θ. For convenience, we choose a set of special settings from the region. The special settings satisfy an extra symmetry requirement as

$$\theta_1 = -\theta_2', \qquad \theta_2 = -\theta_1'. \tag{5}$$

Combined with Eq. (5), we can easily solve Eq. (4) as

$$\begin{aligned}
\theta_1 &= -\frac{1}{4}\left[\cos^{-1}\left(\frac{\sqrt{1+4x^2}}{1+2x}\right) - \sin^{-1}\left(2\sqrt{\frac{x}{1+4x^2}}\right)\right],\\
\theta_2 &= \frac{1}{4}\left[\cos^{-1}\left(\frac{\sqrt{1+4x^2}}{1+2x}\right) + \sin^{-1}\left(\sqrt{\frac{x}{1+4x^2}}\right)\right],\\
\theta_1' &= -\theta_2,\\
\theta_2' &= -\theta_1,
\end{aligned} \tag{6}$$

where $x = \sin\theta\cos\theta$.

Now we can easily calculate out the maximal violation of CHSH inequality for a given non-maximally entangled state $\cos\theta|HH\rangle + \sin\theta|VV\rangle$ with Eqs. (3), (5) and (6). The theoretical calculation results of S for $\theta \in [0, \pi/4]$ is shown in Fig. 2. When $\theta = \pi/4$, S equals to $2\sqrt{2}$, and when $\theta = 0$, S equals to 2. With the settings discussed above, we obtained the experiment results as shown in Fig. 2. For the maximally entangled state, we obtained the violation of about 10σ in 100 s and the data is 2.7277 ± 0.0719. And we can see that the experiment results basically agree with the theoretical calculation for other non-maximally entangled state. The differences between them comes mainly from the imperfection of the state preparation. There are two reasons. One is the impurity of states, that is, the state we measured is in fact not a pure state (in this experiment, the maximally entangled state produced has a visibility of about 98.0%). The other reason is the deviation of θ, which comes from the uncontrollable inequal loss of $|HH\rangle$ and $|VV\rangle$ components during the state preparation stage. From Fig. 2, it is obviously that as "entanglement" of the bipartite pure state decreases, the "violation" of CHSH inequality decreases too. However, when θ is close to 0, the "violation" has begun.

In this experiment, the total detection efficiency is still far from the value 81%, which is required for solving the so called "detection loophole" of Bell states. But we can still say that we have verified the equivalence between "entanglement" and "violation" of Bell-type inequality for bipartite pure states. However, further works are still needed in study of the relationship between "entanglement" and "violation" of Bell-type inequalities for mixed bipartite states or multipartite states. In these cases, the situation may be more subtle and thus more complicated.

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