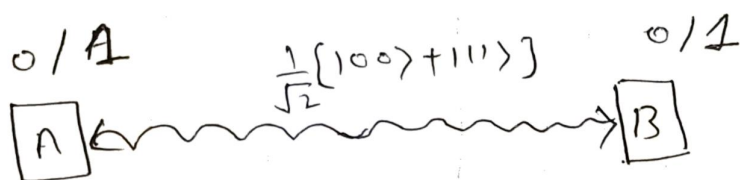


±



Here we can choose measurement direction

let  $A, A'$  is along  $z$  and  $x$  axis and corresponding operator

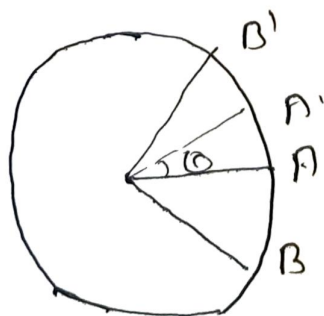
$Z, X$

Similarly

$\Rightarrow B, B'$  in  $w$  and  $v$  axis and corresponding operator are

$$|w\rangle = \frac{1}{\sqrt{2}}[Z+X], \quad |v\rangle = \frac{1}{\sqrt{2}}[Z-X]$$

$\Rightarrow$



⇒

$$A \xrightarrow{Z} \text{ (crossed out) }$$

$$A' \xrightarrow{X} \boxed{H} \rightarrow \text{ (crossed out) }$$

$$B \xrightarrow{W} \boxed{S} - \boxed{H} - \boxed{T} - \boxed{H} \rightarrow \text{ (crossed out) }$$

$$B' \xrightarrow{V} \boxed{S} - \boxed{H} - \boxed{T^+} - \boxed{H} \rightarrow \text{ (crossed out) }$$

$$Z = (|0\rangle, |1\rangle)$$

$$X = (|+\rangle, |-\rangle)$$

$$W = \frac{1}{\sqrt{2}} [Z + X]$$

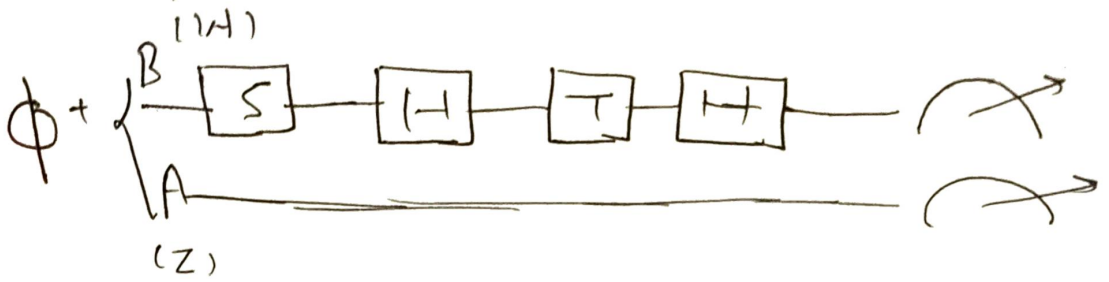
$$V = \frac{1}{\sqrt{2}} [Z - X]$$

⇒

$$A, B, AB, A', B', A'B, A'B'$$

⇒

$$S = E(A, B) + E(A, B') + E(A', B) - E(A', B')$$



A  $\longrightarrow$

For B

$$(I \otimes H) (I \otimes T) (H \otimes I) (I \otimes S) \left[ \frac{1}{\sqrt{2}} |100\rangle + |111\rangle \right]$$

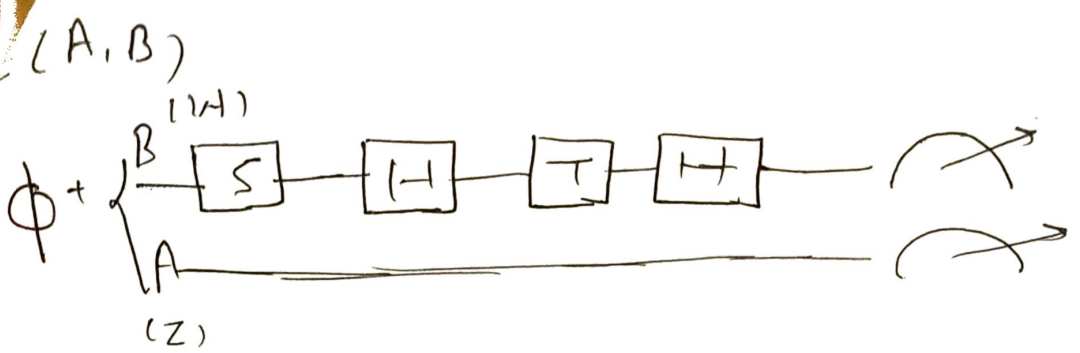
$$\frac{1}{\sqrt{2}} [ |10\rangle \otimes |10\rangle + |11\rangle \otimes i |11\rangle ]$$

$$= \frac{1}{\sqrt{2}} \left[ |10\rangle \otimes \left[ \frac{|10\rangle + |11\rangle}{\sqrt{2}} \right] + |11\rangle \otimes i \left[ \frac{|10\rangle - |11\rangle}{\sqrt{2}} \right] \right]$$

$$= \frac{1}{2} \left[ |10\rangle \otimes [ |10\rangle + e^{i\pi/4} |11\rangle ] + |11\rangle \otimes i [ |10\rangle - e^{i\pi/4} |11\rangle ] \right]$$

$$\frac{1}{2} \left[ |10\rangle \otimes \left[ \frac{(|10\rangle + |11\rangle)}{\sqrt{2}} + e^{i\pi/4} \frac{(|10\rangle - |11\rangle)}{\sqrt{2}} \right] + |11\rangle \otimes i \left[ \frac{|10\rangle + |11\rangle}{\sqrt{2}} - e^{i\pi/4} \frac{(|10\rangle - |11\rangle)}{\sqrt{2}} \right] \right]$$

$$\frac{1}{2\sqrt{2}} \left[ |100\rangle + |101\rangle + |100\rangle e^{i\pi/4} - e^{i\pi/4} |101\rangle + i |110\rangle + i |111\rangle - i e^{i\pi/4} |110\rangle - i e^{i\pi/4} |111\rangle \right]$$



A  $\rightarrow$

For B

$$\begin{aligned}
 & (I \otimes H) (I \otimes T) (H \otimes I) (I \otimes S) \left[ \frac{1}{\sqrt{2}} |00\rangle + |11\rangle \right] \\
 &= \frac{1}{\sqrt{2}} [ |0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle ] \\
 &= \frac{1}{\sqrt{2}} [ |0\rangle \otimes \left[ \frac{|0\rangle + |1\rangle}{\sqrt{2}} \right] + |1\rangle \otimes \left[ \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] ] \\
 &= \frac{1}{2} [ |0\rangle \otimes [ |0\rangle + e^{i\pi/4} |1\rangle ] + |1\rangle \otimes i [ |0\rangle - e^{i\pi/4} |1\rangle ] ] \\
 &= \frac{1}{2} [ |0\rangle \otimes \left[ \frac{|0\rangle + |1\rangle}{\sqrt{2}} \right] + e^{i\pi/4} [ |0\rangle - |1\rangle ] \\
 &\quad + |1\rangle \otimes i \left[ \frac{|0\rangle + |1\rangle}{\sqrt{2}} - e^{i\pi/4} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] ] \\
 &= \frac{1}{2\sqrt{2}} [ |00\rangle + |01\rangle + |00\rangle e^{i\pi/4} - e^{i\pi/4} |01\rangle \\
 &\quad + i |110\rangle + i |111\rangle - i e^{i\pi/4} |110\rangle - i e^{i\pi/4} |101\rangle ]
 \end{aligned}$$

$$\text{output} = \left[ \frac{1 + e^{i\pi/4}}{2\sqrt{2}} |00\rangle + \frac{1 - e^{i\pi/4}}{2\sqrt{2}} |01\rangle + i \frac{1 - e^{i\pi/4}}{2\sqrt{2}} |10\rangle + i \frac{1 + e^{i\pi/4}}{2\sqrt{2}} |11\rangle \right]$$

$$\Rightarrow p_{00} = \left| \frac{1 + e^{i\pi/4}}{2\sqrt{2}} \right|^2 = \frac{2 + \sqrt{2}}{8}$$

$$p_{01} = \left| \frac{1 - e^{i\pi/4}}{2\sqrt{2}} \right|^2 = \frac{2 - \sqrt{2}}{8}$$

$$p_{10} = \left| i \frac{1 - e^{i\pi/4}}{2\sqrt{2}} \right|^2 = \frac{2 - \sqrt{2}}{8}$$

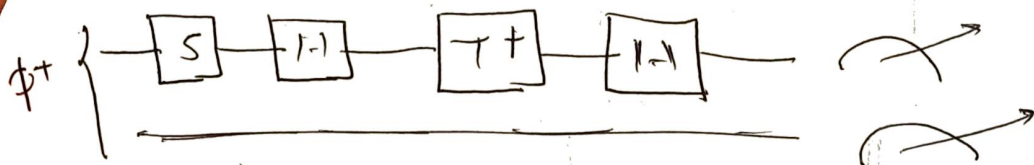
$$p_{11} = \left| i \frac{1 + e^{i\pi/4}}{2\sqrt{2}} \right|^2 = \frac{2 + \sqrt{2}}{8}$$

$$\Rightarrow \text{9.c} \quad E = (A, B) = p_{00} - p_{01} - p_{10} + p_{11}$$

$$= 2 \left( \frac{2 + \sqrt{2}}{8} \right) - 2 \left( \frac{2 - \sqrt{2}}{8} \right)$$

$$= \frac{4\sqrt{2}}{8} = \frac{1}{\sqrt{2}}$$

B')



$$\begin{aligned}
 & (I \otimes H) (I \otimes T) (I \otimes H) (I \otimes S) \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} |10\rangle \\ |11\rangle \end{pmatrix} \right\} \\
 &= \frac{1}{\sqrt{2}} \left\{ |10\rangle \otimes |10\rangle + |11\rangle \otimes i|11\rangle \right\} \\
 &= \frac{1}{\sqrt{2}} \left\{ |10\rangle \otimes \left( \frac{|10\rangle + |11\rangle}{\sqrt{2}} \right) + |11\rangle \otimes i \left( \frac{|10\rangle - |11\rangle}{\sqrt{2}} \right) \right\} \\
 &= \frac{1}{\sqrt{2}} \left\{ |10\rangle \otimes \left[ \frac{|10\rangle + e^{-i\pi/4}|11\rangle}{\sqrt{2}} \right] + |11\rangle \otimes i \left[ \frac{|10\rangle - e^{-i\pi/4}|11\rangle}{\sqrt{2}} \right] \right\} \\
 &= \frac{1}{2} \left\{ |10\rangle \otimes \left[ \frac{(|10\rangle + |11\rangle)}{\sqrt{2}} \right] + \frac{e^{-i\pi/4}}{\sqrt{2}} (|10\rangle - |11\rangle) \right. \\
 &\quad \left. + |11\rangle \otimes i \left[ \left( \frac{|10\rangle + |11\rangle}{\sqrt{2}} \right) - e^{-i\pi/4} \left( \frac{|10\rangle - |11\rangle}{\sqrt{2}} \right) \right] \right\} \\
 &= \frac{1}{2\sqrt{2}} \left\{ |100\rangle + |101\rangle + e^{-i\pi/4} |100\rangle - e^{-i\pi/4} |101\rangle \right. \\
 &\quad \left. + i [ |110\rangle + |111\rangle ] - i e^{-i\pi/4} |110\rangle \right. \\
 &\quad \left. + i e^{-i\pi/4} |111\rangle \right\} \\
 &= \frac{1}{2\sqrt{2}} \left\{ (1 + e^{-i\pi/4}) |100\rangle + (1 - e^{-i\pi/4}) |101\rangle \right. \\
 &\quad \left. + i [1 - e^{-i\pi/4}] |110\rangle \right. \\
 &\quad \left. + i [1 + e^{-i\pi/4}] |111\rangle \right\}
 \end{aligned}$$

$$\Rightarrow P_{00} = \left| \frac{1 + e^{-i\pi/4}}{2\sqrt{2}} \right|^2 = \frac{2 + \sqrt{2}}{8}$$

$$P_{01} = \left| \frac{1 - e^{-i\pi/4}}{2\sqrt{2}} \right|^2 = \frac{2 - \sqrt{2}}{8}$$

$$P_{10} = \left| i \frac{(1 + e^{-i\pi/4})}{2\sqrt{2}} \right|^2 = \frac{2 - \sqrt{2}}{8}$$

$$P_{11} = \left| i \frac{(1 - e^{-i\pi/4})}{2\sqrt{2}} \right|^2 = \frac{2 + \sqrt{2}}{8}$$

g.c  
 $\Rightarrow$

$$E(A, B') = P_{00} - P_{01} - P_{10} + P_{11}$$

$$= \frac{2 + \sqrt{2}}{8} - \frac{(2 - \sqrt{2})}{8} - \frac{(2 - \sqrt{2})}{8} + \frac{2 + \sqrt{2}}{8}$$

$$= \frac{4\sqrt{2}}{8} = \frac{1}{\sqrt{2}}$$

Similarly we find

$$E(A, B) = \frac{1}{\sqrt{2}}$$

$$E(A', B') = -\frac{1}{\sqrt{2}}$$



$$S = |E(A, B) + E(A, B') + E(A', B) - E(A', B')|$$

$$= \left| \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right|$$

$$= \frac{4}{\sqrt{2}}$$

$$S = 2\sqrt{2}$$

$$\Rightarrow S = 2\sqrt{2}$$

$\Rightarrow$  It is maximum amount that  $S$  can be using quantum mechanics

$$\Rightarrow \boxed{|S| \leq 2\sqrt{2}}$$

and This is the maximum amount that CHSH inequality can be violated by quantum mechanics