Few sample problems

Question-1 Suppose $|a\rangle = \frac{1}{\sqrt{3}}|+\rangle + \sqrt{\frac{2}{3}}\,|-\rangle$ and $|b\rangle = \frac{2}{3}|+\rangle - \frac{\sqrt{5}}{9}\,|-\rangle$ with 75% of systems prepared in state $|a\rangle$ and 25% of the systems prepared in state $|b\rangle$ then

(a) Write down the density operators ρ_a and ρ_b.
(b) Compute the density operator for the ensemble.
(c) A measurement is made then what are the probabilities of finding $|+\rangle$ and $|-\rangle$?
(d) Instead of question (c), what are the probabilities of finding $|0\rangle$ and $|1\rangle$

Question-2 write the spectral decomposition if followings are given

$$|\psi\rangle = \frac{1}{2}|0\rangle - \frac{i}{2}\,|1\rangle + \frac{1}{\sqrt{2}}\,|2\rangle$$

$$A = \begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix}$$

Question-3 if $|\psi\rangle = \frac{1}{\sqrt{8}}|00\rangle + \sqrt{\frac{3}{8}}\,|01\rangle + \frac{1}{2}\,|10\rangle + \frac{1}{2}\,|11\rangle$ t Express in $\sigma_x \otimes \sigma_y$ spectral decompositions.

Question-4 Find the density matrix for given state $|\psi_+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|\psi_-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$