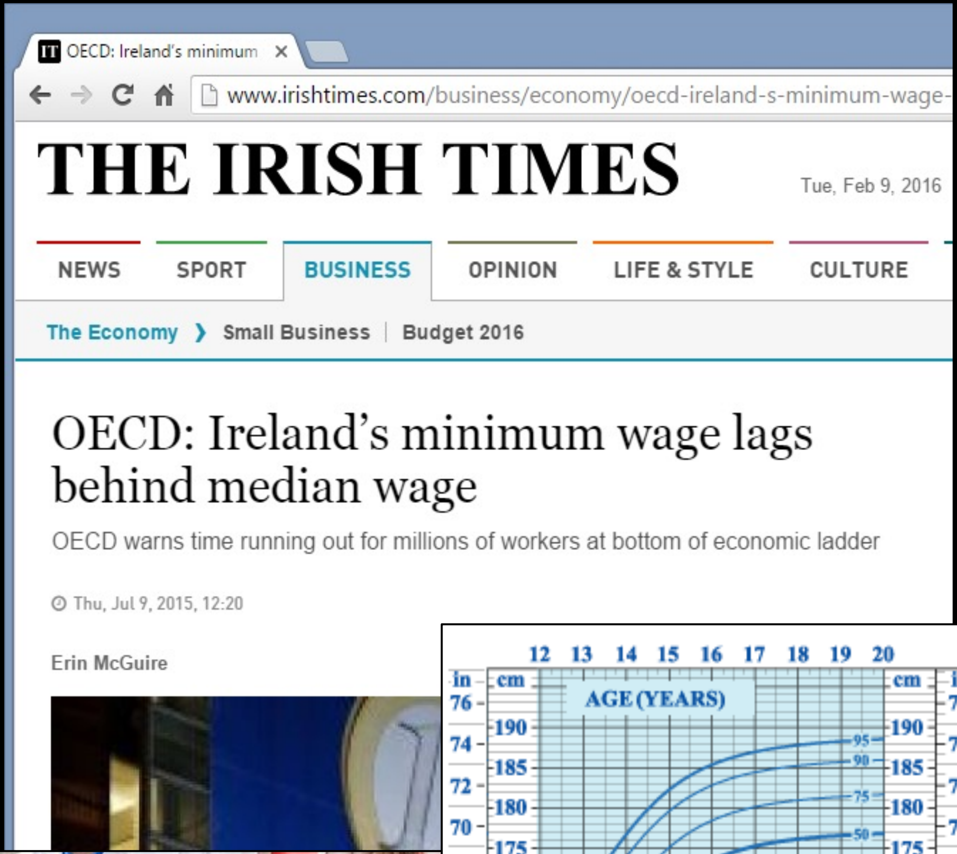
Selecting the kth ordered item from an unordered list



OECD: Ireland's minimum

www.irishtimes.com/business/economy/oecd-ireland-s-minimum-wage-

THE IRISH TIMES

Tue, Feb 9, 2016

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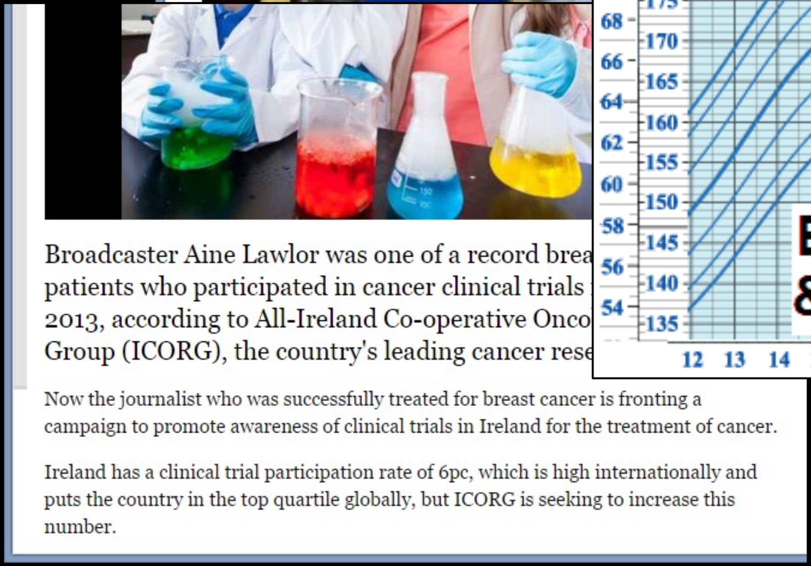
The Economy ❯ Small Business | Budget 2016

OECD: Ireland's minimum wage lags behind median wage

OECD warns time running out for millions of workers at bottom of economic ladder

Thu, Jul 9, 2015, 12:20

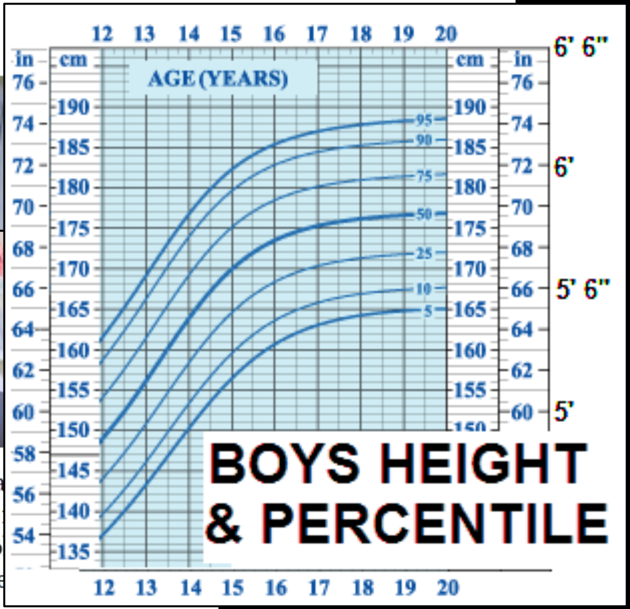
Erin McGuire



Broadcaster Aine Lawlor was one of a record brea
patients who participated in cancer clinical trials
2013, according to All-Ireland Co-operative Onco
Group (ICORG), the country's leading cancer rese

Now the journalist who was successfully treated for breast cancer is fronting a campaign to promote awareness of clinical trials in Ireland for the treatment of cancer.

Ireland has a clinical trial participation rate of 6pc, which is high internationally and puts the country in the top quartile globally, but ICORG is seeking to increase this number.



Revision: find the position of an item *target* in an array-based list of length *n*

Unsorted list:

```
i = 0
while i < length of list
   if list[i] == target
      return i
   i +=1
return -1
```

Linear search

O(n)

Sorted list:

```
s=0, e=n-1
while s ≤ e
    m= s + (e-s)//2
    if list[m] > target
        e = m-1
    elif list[m] < target
        s = m+1
    else
        return m
return -1
```

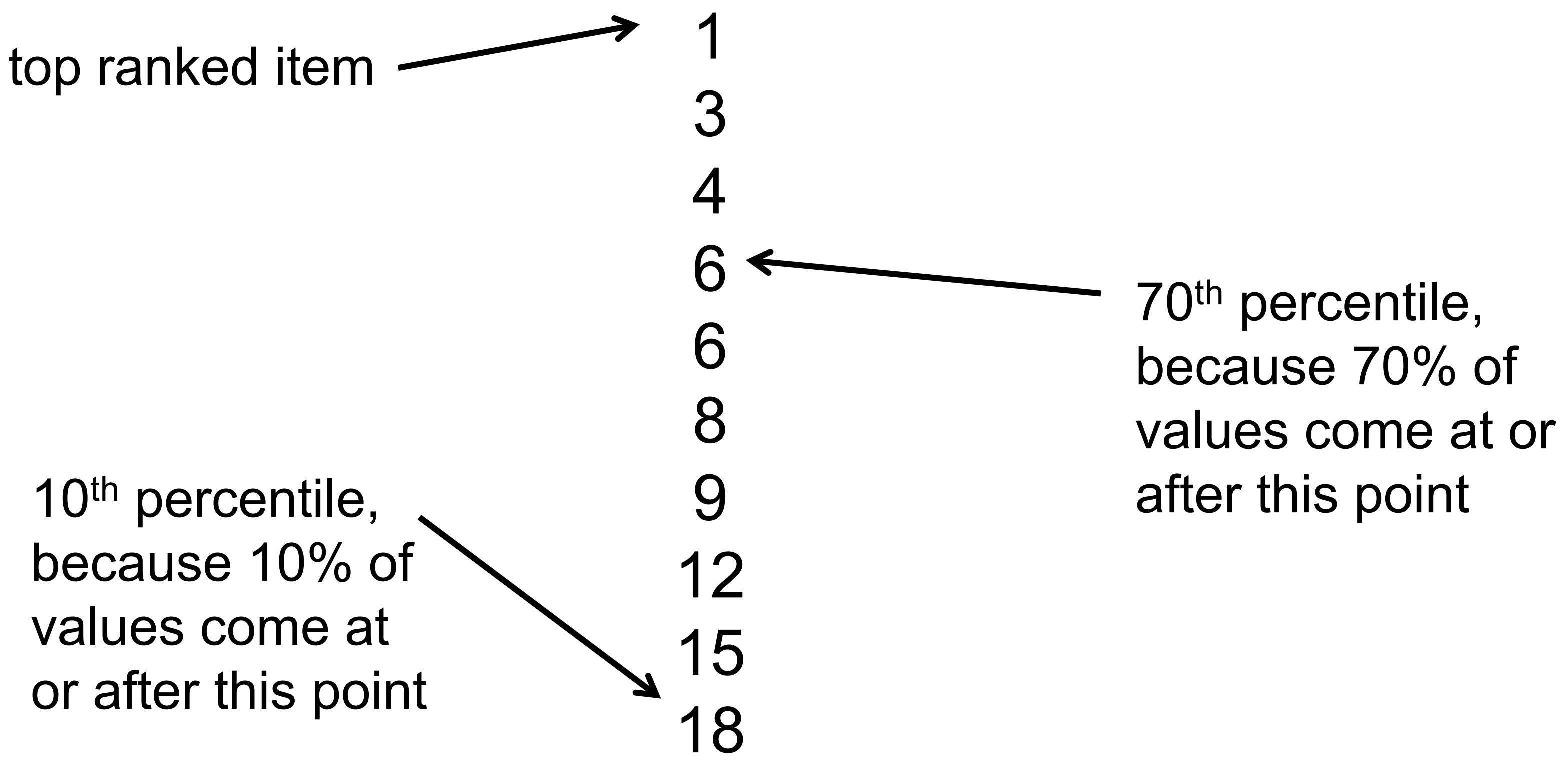
Binary search

O(log n)

The loop divides the list in two each iteration – you can only do that log n times until you get a list of size 1

A common task in data analytics, scientific computing, social sciences, statistics, economics, etc is identifying *percentiles* in a collection of items.

Given a set of items, the kth percentile is the value below which k% of the items are ranked.


top ranked item
1
3
4
6
6
8
9
12
15
18
70th percentile, because 70% of values come at or after this point
10th percentile, because 10% of values come at or after this point

The *median* is a measure of the average of a collection.

- less influenced by extreme values than the *mean*

The median is the 50th percentile.

The upper *quartile* is the subset of values that are 'better' than the 75th percentile.

The lower *quartile* is the subset of values that are 'poorer' than the 25th percentile.

Given an unsorted list of items, how should we compute the ith percentile? how should we compute the median?

Since we know the length of the list, we can calculate the position *k* in the rank order that corresponds to the ith percentile. Task is then to find the k^{th} ordered element.

```
6  4  12  9  6  15  3  18  1  8
```

What is the algorithm?

Method 1:

```
repeat k times:
    find the biggest item in the list
    replace that item with None
report the most recent item found
```

Analysis:
Each iteration is $O(n)$, so $O(k*n)$

For small $k << \log n$, this gives $k*O(n) = O(n)$
For $k \sim= \log n$, this is $O(n \log n)$
For large $k >> \log n$, this is $O(n^2)$

Method 2:

```
for each element in list:
    add element to a sorted linked list
step through linked list to position k
```

Analysis:
Worst case is for a sorted list, additions require
$O(1 + 2 + 3 + ... + n) = O(n^2)$
followed by k steps, so $O(n^2)$

or, if we have access to both ends of the list, worst case is when each element should go in the middle of the elements that came before it, so we get $O(1+2+3+... + n/2)$, which is $0.5*(n/2)*(n/2+1) = 0.5*(n*n/4 + n/2)$ which is still $O(n^2)$

Similar analysis for adding to a sorted array-based list, but instead of $O(n^2)$ comparisons, need $O(n^2)$ assignments for a reverse sorted list to shift up each item

Method 3:

sort the list by preferred sorting algorithm
read item in position k

Analysis:
For merge or heap sort, $O(n \log n)$
For quicksort, $O(n^2)$, but $O(n \log n)$ on average

Method 4:

apply quicksort idea to identify element in position k

```
pick a pivot
sweep through list to partition around the pivot and move pivot into place
if pivot in position k
    return pivot
else if more items 'less than or equal to pivot' than k
    recurse on all items less than or equal to pivot
else
    recurse on all items greater than pivot
```

'decrease and conquer'

Note: don't sort every sublist – all we are doing is picking a pivot from some part of the list, dividing elements from that part, choosing *one* of the divisions, and recursing.

Analysis of method 4:
- worst case – always choose a pivot that decreases the list by just 1 item, so n + (n-1) + (n-2) + .. + (n-(n-(k+1))), which is $O(n^2)$ for small k

- average case

By a similar argument to the one for quicksort, it is possible to show that the average time for a list of length n is O(n).

Informally, assume that pivot selection will, on average, split the (sub-)list into two approximately equal parts. So, on average, we only need to do O(n + n/2 + n/4 + n/8 + … + 1), which is O(2n), which is O(n) Comparisons.

In-Class exercise

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
6	4	12	9	6	15	3	18	1	8

Method 4 (improved?):

```
pick a pivot
sweep through list to partition 3 ways: less than, equal to, greater than pivot
if more  items 'less than pivot' than k
   recurse on all items less than pivot
else if more items 'less than or equal to pivot' than k
   return pivot
else
   recurse on all items greater than pivot
```

Need to revise the pseudocode for partitioning. In Hoare partitioning, maintain a record of 1st pivot and last pivot in group of pivots. When forward search finds a value less than pivot, swap with 1st pivot, then increment 1st pivot position and last pivot position. When forward search finds a copy of the pivot, increment last pivot position. When swapping forward and backward search items, if the smaller item was less than pivot, also need to swap with 1st pivot position, etc; if smaller item was equal to pivot, need to increment last pivot position.

Exercise: write out this pseudocode in full, and check it works on lists with multiple copies of the pivot.

Practical considerations:

We can implement quickselect in-place, in the same style as in-place quicksort.

Efficient implementation in python is not so obvious ...

Exercise:
 - develop versions of quickselect which:
(i) creates lists less, equal and more using python lists and append
(ii) creates lists less, equal and more using 3 list comprehensions
(iii) implements in place
- and algorithms which
(iv) pre-sort with mergesort, heapsort or quicksort, then select
(v) pre-sort with python list sort, then select

Which is fastest on average?

Next lecture

Graphs and Graph Algorithms