Merge Sort

More sorting?

Heapsort had complexity O(n log n), and sorted in-place.
Is there anything else worth looking at?

- Are there algorithms with better worst case complexity?
  - even if they have the same complexity, maybe the lower order terms are better?
- Are there algorithms with better average complexity?
- Do we really need to worry about in-place sorting?
- Are there other problem-solving strategies we could try?

Divide and Conquer

If a problem is very simple, solve it in a single step.
If a problem is too complex to solve in a single step,
    divide it into multiple pieces
    solve the individual pieces
    combine the pieces together to get a solution

Typically implemented using multiple recursion

A general problem solving strategy used throughout computing

Sorting by divide and conquer

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If a problem is too complex to solve in a single step,
    divide it into multiple pieces
    solve the individual pieces
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Typically implemented using multiple recursion

Sorting a list:
- If an input list is of size 1 (or 0), do nothing.
- If an input list is of size 2 or more
  - split it into two roughly equal sublists *#divide*
  - sort the first sublist *#conquer*
  - sort the second sublist
  - merge the two sublists into a combined sorted list *#combine*

41	37	35	62	29	39	54	27	60	25	40	56	51	48	43	51	43

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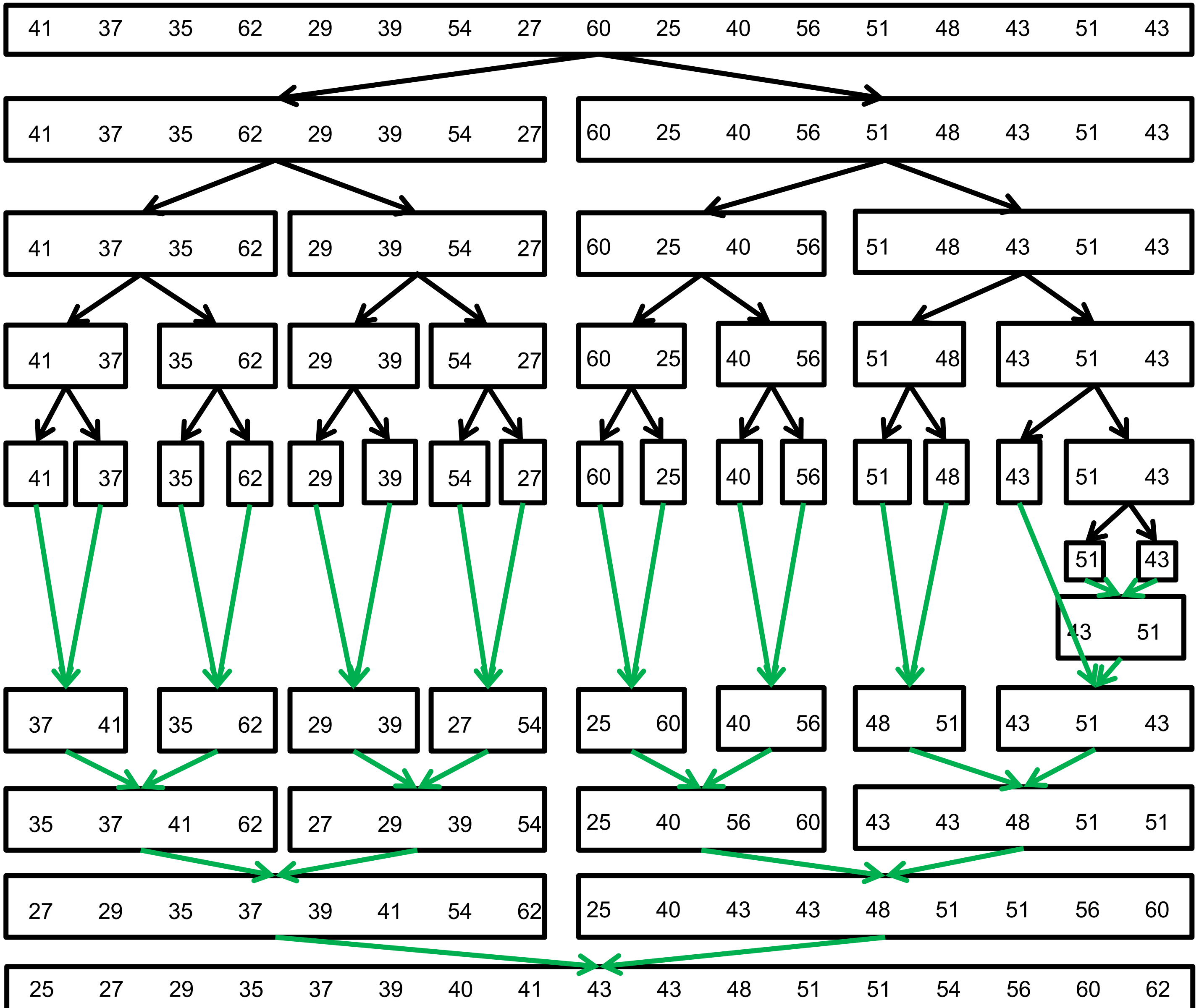


```
def mergesort(mylist):
    n = len(mylist)
    if n > 1:
        list1 = mylist[:n//2]
        list2 = mylist[n//2:]
        mergesort(list1)
        mergesort(list2)
        merge(list1, list2, mylist)
```

Slicing creates a new list each time, so not in-place.
But it is difficult to write an in-place mergesort
without increasing the time complexity

Merge:

35 37 41 62 27 29 39 54

```
def merge(list1, list2, mylist):
    f1 = 0
    f2 = 0
    while f1 + f2 < len(mylist):
        if f1 == len(list1):
            mylist[f1+f2] = list2[f2]
            f2 += 1
        elif f2 == len(list2):
            mylist[f1+f2] = list1[f1]
            f1 += 1
        elif list2[f2] < list1[f1]:
            mylist[f1+f2] = list2[f2]
            f2 += 1
        else:
            mylist[f1+f2] = list1[f1]
            f1 += 1
```

Note: written for clarity. Repeated code is not a good idea, so should rewrite to require only 1 test in loop body

```python
def merge(list1, list2, mylist):
    f1 = 0
    f2 = 0
    while f1 + f2 < len(mylist):
        if f1 == len(list1):
            mylist[f1+f2] = list2[f2]
            f2 += 1
        elif f2 == len(list2):
            mylist[f1+f2] = list1[f1]
            f1 += 1
        elif list2[f2] < list1[f1]:
            mylist[f1+f2] = list2[f2]
            f2 += 1
        else:
            mylist[f1+f2] = list1[f1]
            f1 += 1
```

Analysis: (|mylist| = n)

round the loop n times

inside the loop, at most 3 tests, 2 calls to len(.), and 2 assignments

So O(1) inside the loop.

So function has worst case time O(n)

Writing the result into the 3rd input list, so we do not occupy any extra space.

If we just count how many times we compare two list items, it will be **less than n** (but still O(n))

```
def mergesort(mylist):
    n = len(mylist)
    if n > 1:
        list1 = mylist[:n//2]
        list2 = mylist[n//2:]
        mergesort(list1)
        mergesort(list2)
        merge(list1, list2, mylist)
```

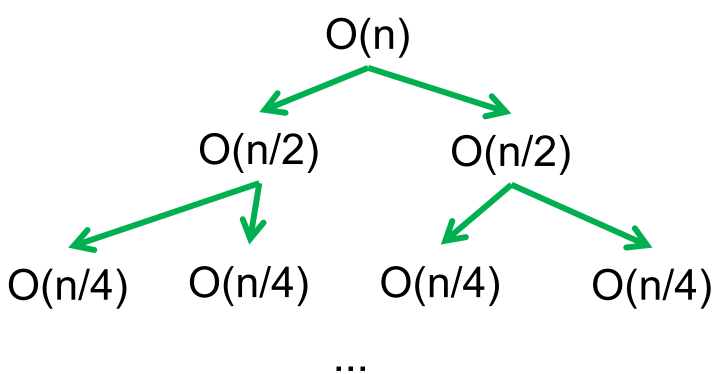
<u>Analysis:</u>

Each call (without recursion) takes:

- n assignments to create the slices
- O(n) for the merge function

So O(n)

Each recursive call is for a list of size n/2, and so takes O(n/2), etc.



So we have O(n) at each level in the call tree.

The depth of the tree is either $\log_2(n)$ or $\log_2(n) + 1$

So O(n log n) in total

Each call creates new smaller lists, so space complexity (of this implementation) is same as time – O(n log n)

Alternative Analysis: recurrence equations

The base case is $O(1)$.
Time to sort a list of length 1, $t(1)$ is just d, for some constant d.

Merge is $O(n)$, so we will write as $c*n$
Time to sort a list of length n, $t(n)$, for $n > 1$, is then:

$t(n) = 2*t(n/2) + c*n$. But $t(n/2)$ must then be $2*t(n/4) + c*n/2$. So

$t(n) = 2*(2*t(n/4) + c*n/2) + c*n = 4*t(n/4) + 2c*n = \ldots 8*t(n/8) + 3c*n$

So $t(n) = 2^k*t(n/2^k) + kc*n$ This eventually stops when the list is of size 1, which happens when $k = \log_2 n$.

$t(n) = 2^{\log_2 n}*t(n/2^{\log_2 n}) + (\log_2 n * c*n)$ But $2^{\log_2 n} = n$, so

$t(n) = n*t(n/n) + (\log_2 n * c*n)$ and $t(n/n) = t(1)$, which is just d

$t(n) = d*n + (\log_2 n)*c*n$ which is $O(n \log n)$

Alternative Mergesort implementations

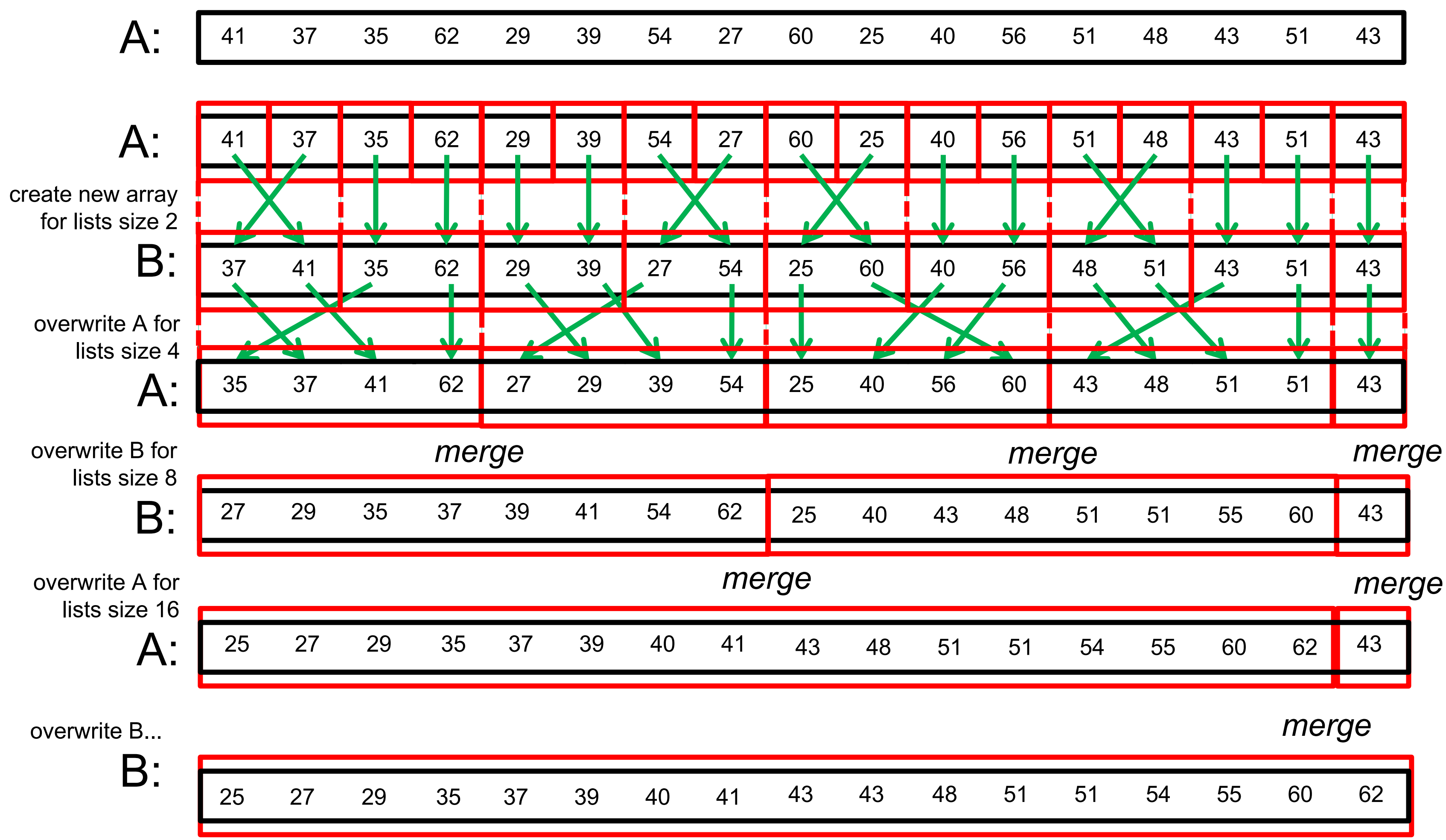
1. Implementing mergesort on linked lists is easier

2. Mergesort on arrays can be implemented bottom-up rather than top down, using just O(n) extra space:

Create a new empty list of size n, called list B
View list A as being n separate lists each of size 1 ➔ next slide
For each pair of cells in original list (list A)
    merge into sorted pair in corresponding cells in list B
For each successive group of two pairs (4 cells) in list B
    merge into sorted group of 4 in corresponding cells in list A
For each successive group of two 4-tuples (8 cells) in list A
    merge into sorted group of 8 in corresponding cells in list B
...

    continuing until entire list is sorted.

Treat each cell in the array as though it were a list of size 1



Note: implementing this successfully is tricky …

Apply bottom-up mergesort

41 37 62 35 54 27 39 29

Mergesort summary

Mergesort is an example of “Divide and Conquer”

- keep breaking the problem down into smaller chunks until they are trivial to solve, and combine the results back together to create a solution to the original problem.

Mergesort has complexity O(n log n) for time and space.

In practice, Mergesort tends to be faster than Heapsort

Next Lecture

Quicksort