Exercise Sheet 1

(1)

$$\alpha^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 2 & 4 \end{bmatrix} \qquad \beta^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 1 & 2 & 3 \end{bmatrix}$$

$$\alpha\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 3 & 1 & 4 \end{bmatrix} \qquad \beta^{-1}\alpha\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 1 & 4 & 2 \end{bmatrix}$$

(2)

i) (143) ii) $\cancel{\varepsilon}(12)$ iii) $(153)(24)$

(3)

i) $(1)(12)(23)(13)(123)(132)$

ii) $(1)(12)(23)(34)(13)(14)(24)(123)(124)(142)$
$(134)(132)(143)(234)(243)$
$(1234)(1324)(1423)(1432)(1243)(1342)$

4.

i) (2453) ii) $(134862)(57)$

ii) (23)

5.

$\alpha^2 = (13)(24)$ $\alpha^3 = (1234)$

$\alpha^4 = \varepsilon$ $\alpha^5 = (1432)$

$$\alpha^{4k} = \varepsilon$$
$$\alpha^{4k+1} = \cancel{\varepsilon}\ \alpha$$
$$\alpha^{4k+2} = \alpha^2$$
$$\alpha^{4k+3} = \alpha^3$$

$$\alpha^{2023} = \alpha^{2023 \% 4} = \alpha^3$$
$$\downarrow$$
$$(1\,2\,3\,4)$$

(6) <u>Proof</u>

<u>For $k=1$</u>

$$(\alpha\beta\alpha^{-1})^1 = \alpha\beta^{1}\alpha^{-1}$$
$$\alpha\beta\alpha^{-1} = \alpha\beta\alpha^{-1}$$

<u>For $k = n$</u>

Assume that $(\alpha\beta\alpha^{-1})^{n} = \alpha\beta^{n}\alpha^{-1}$

<u>Proof for $k = n+1$</u>

$$(\alpha\beta\alpha^{-1})^{n+1} = \alpha\beta^{n+1}\alpha^{-1}$$
$$(\alpha\beta\alpha^{-1})^{n}(\alpha\beta\alpha^{-1}) = \alpha\beta^{n+1}\alpha^{-1}$$
$$(\alpha\beta^{n}\alpha^{-1})(\alpha\beta\alpha^{-1}) = \alpha\beta^{n+1}\alpha^{-1}$$
$$(\alpha\beta^{n})\cancel{(\alpha^{-1}\alpha)}(\beta\alpha^{-1}) = \alpha\beta^{n+1}\alpha^{-1}$$
$$\alpha\beta^{n}\beta\alpha^{-1} = \alpha\beta^{n+1}\alpha^{-1}$$
$$\alpha\beta^{n+1}\alpha^{-1} = \alpha\beta^{n+1}\alpha^{-1}$$

362 236

3642

2364

642 264

(7) i) $\alpha = (263)$ (45)

$\beta = (15)(246)$

$\alpha^{-1} = (236)(45)$

$\beta^{-1} = (15)(264)$

$\alpha\beta = (14325)$

$\beta\alpha = (15)(34)$

ii) (23)(26)(45)

(8) If $(\alpha\beta)^{-1}(\alpha\beta) = \varepsilon$

$(\beta^{-1}\alpha^{-1})(\alpha\beta) = \varepsilon$

$\beta^{-1}\cancel{\alpha^{-1}\alpha}\beta = \varepsilon$

$\beta^{-1}\beta = \varepsilon$

$\varepsilon = \varepsilon$

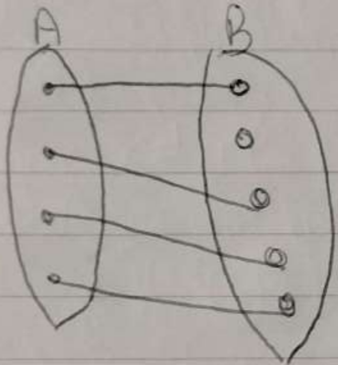
62 236 3642

9.

i) $(a_m, a_{m-1} \ldots\ldots a_1)$

ii) $(a_{m_k} \ldots\ldots a_1)(a_{m_1} \ldots\ldots a_1)$

10. Injective means that every value in the codomain has 0 or 1 connected values from the domain.



However if you map from A → A, both the codomain and the domain have the same amount of elements, meaning it's not possible for a value in the codomain to not be connected. If one element in the codomain does not have a value connected to it, then the function ~~is only surjective~~, is not injective.