14.31/14.310 Lecture 8

Probability---example

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Suppose $X \sim U[0,1]$ and $Y = -\log(X)/\lambda$, $\lambda > 0$. What is $f_Y(y)$?

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$$
\begin{aligned}
F_Y(y) &= P(Y <= y) \\
&= P(-\log(X)/\lambda <= y) \\
&= P(X >= \exp\{-\lambda y\}) \\
&= 1 - \exp\{-\lambda y\} \qquad \text{by definition of uniform}
\end{aligned}
$$

Probability---example

We have

$$F_Y(y) = 1 - \exp\{-\lambda y\} \qquad \text{for } y > 0$$

Probability---example

We have

$F_Y(y) = 1 - \exp\{-\lambda y\}$ for $y > 0$

So,

$f_Y(y) = dF_Y(y)/dy = \lambda\exp\{-\lambda y\}$ for $y > 0$

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Look familiar?

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Look familiar? It's the exponential distribution.

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Look familiar? It's the exponential distribution.

Let's take the inverse of the CDF that we found above, and see if it's the same function that we used to transform the uniform originally.

Probability---example

We have

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Probability---example

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We find the inverse function:

$$x = 1-\exp\{-\lambda y\}$$

$$1-x = \exp\{-\lambda y\}$$

$$\log(1-x) = -\lambda y$$

So, $Y = -\log(1-X)/\lambda$

Probability---example

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But we used $Y = -\log(X)/\lambda$

What's going on?

Probability---example

We have

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We find the inverse function:

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$$\log(1-x) = -\lambda y$$

So, $Y = -\log(1-X)/\lambda$

But we used $Y = -\log(X)/\lambda$

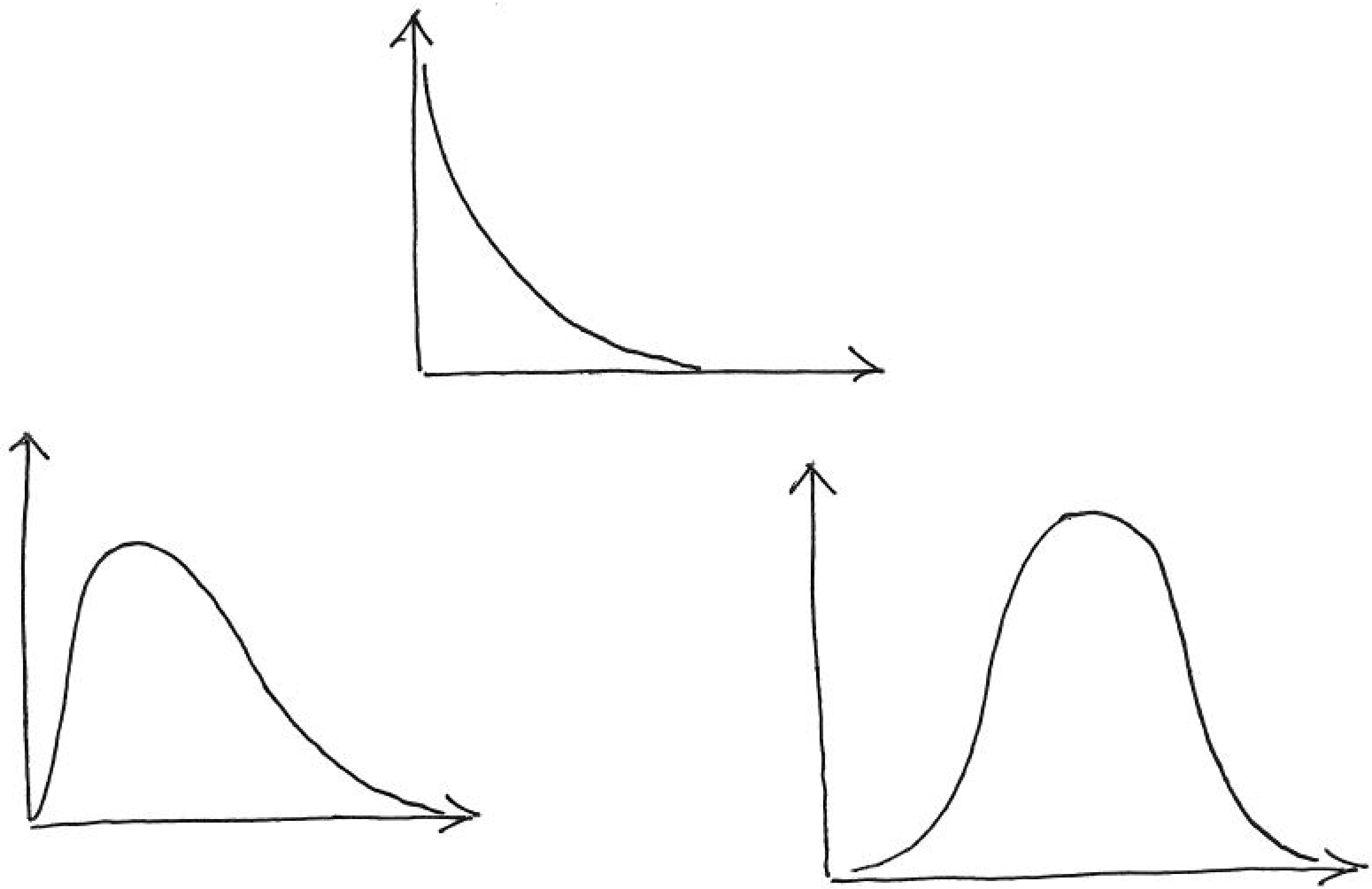
What's going on? Both work. If X is $U[0,1]$, $1-X$ is, too.

Probability---moments of a distribution

There is a lot of information in a PDF. Sometimes, too much information. Perhaps we don't care precisely what the shape of the distribution is but just want to summarize some of its most salient features---where it is centered, where it reaches its peak, how spread out it is, whether it is symmetric, how thick its tails are, etc.

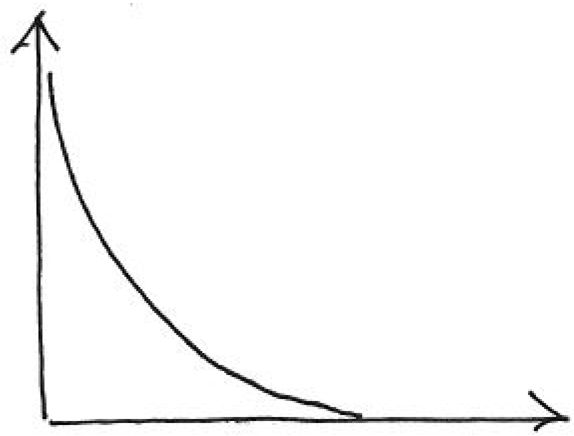
We can define the <u>moments</u> of a distribution to help us summarize some of these most salient features.

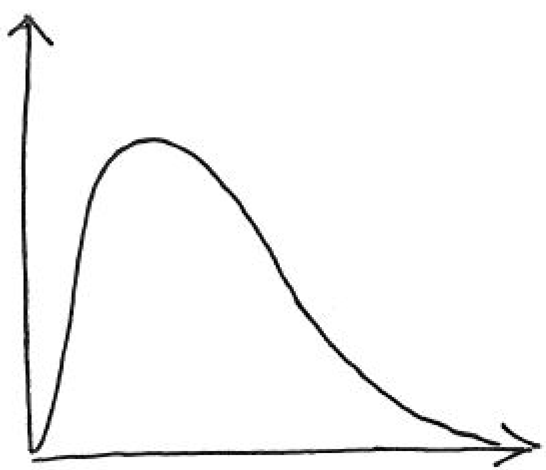
Probability---moments of a distribution

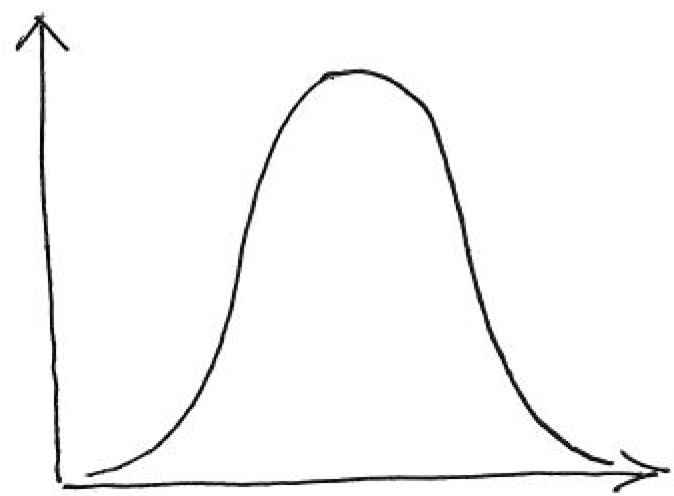


Probability--moments of a distribution

For instance, mean, median, and mode all describe where the distribution is located, or centered.

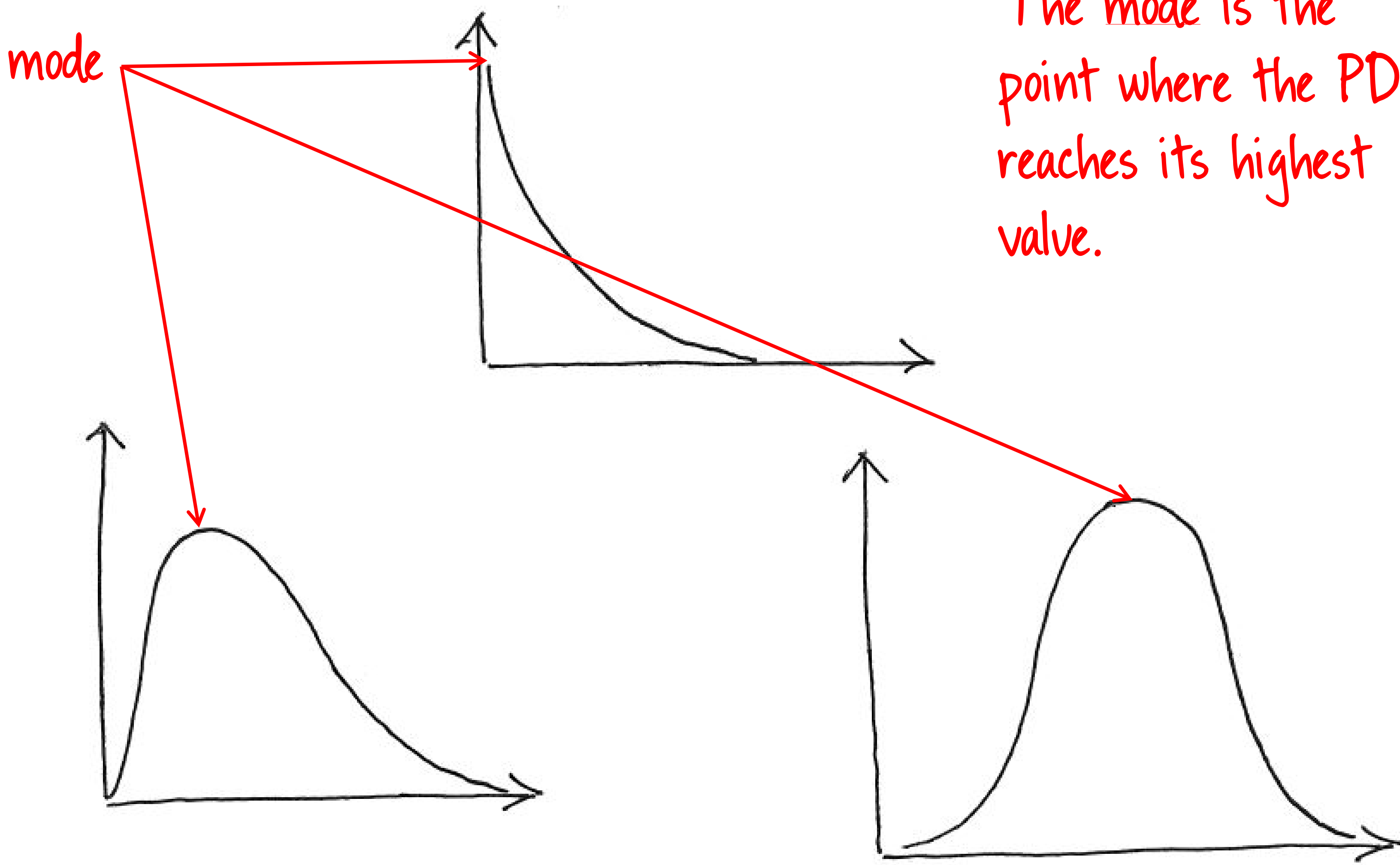






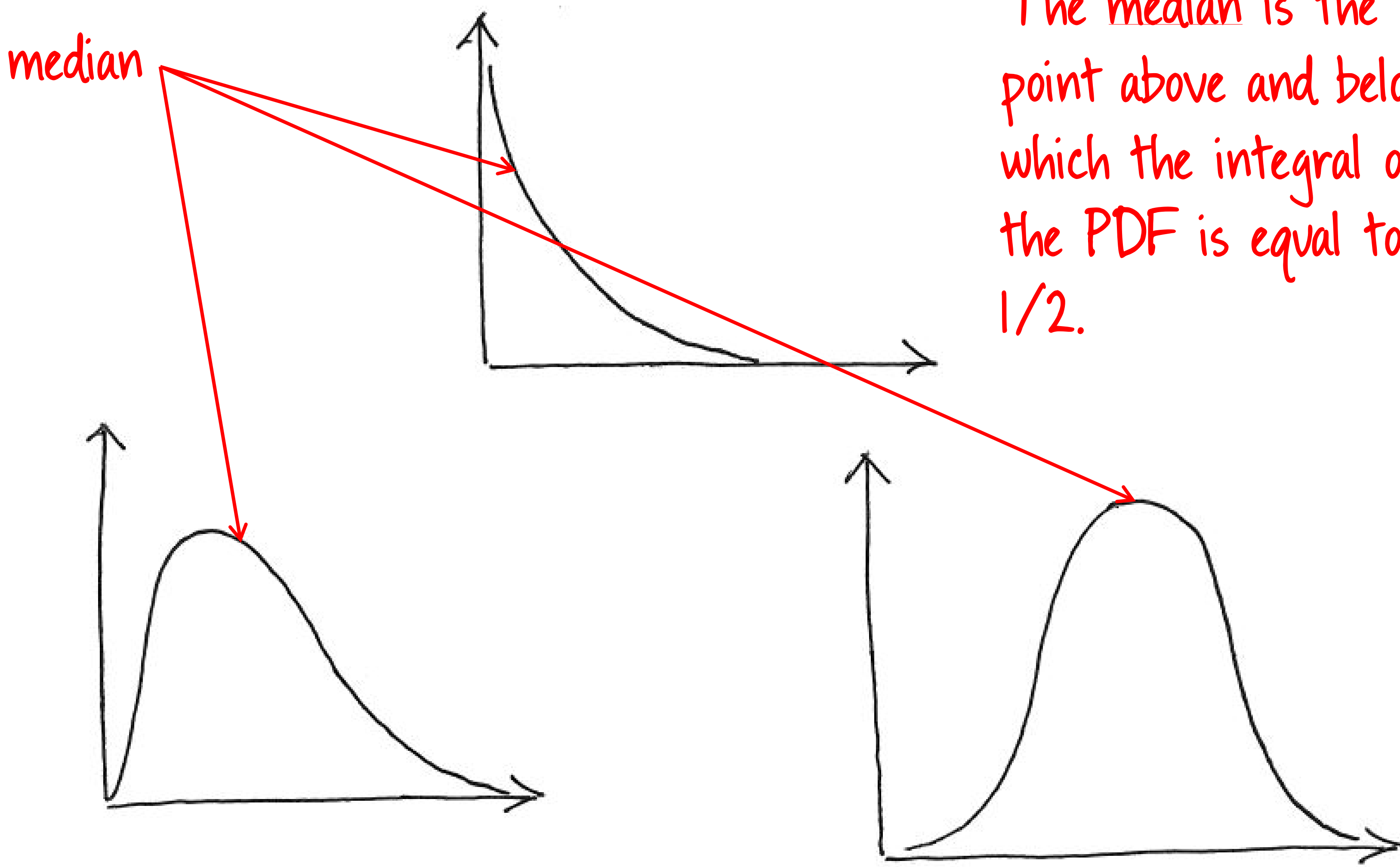
Probability--moments of a distribution

The mode is the point where the PDF reaches its highest value.



Probability--moments of a distribution

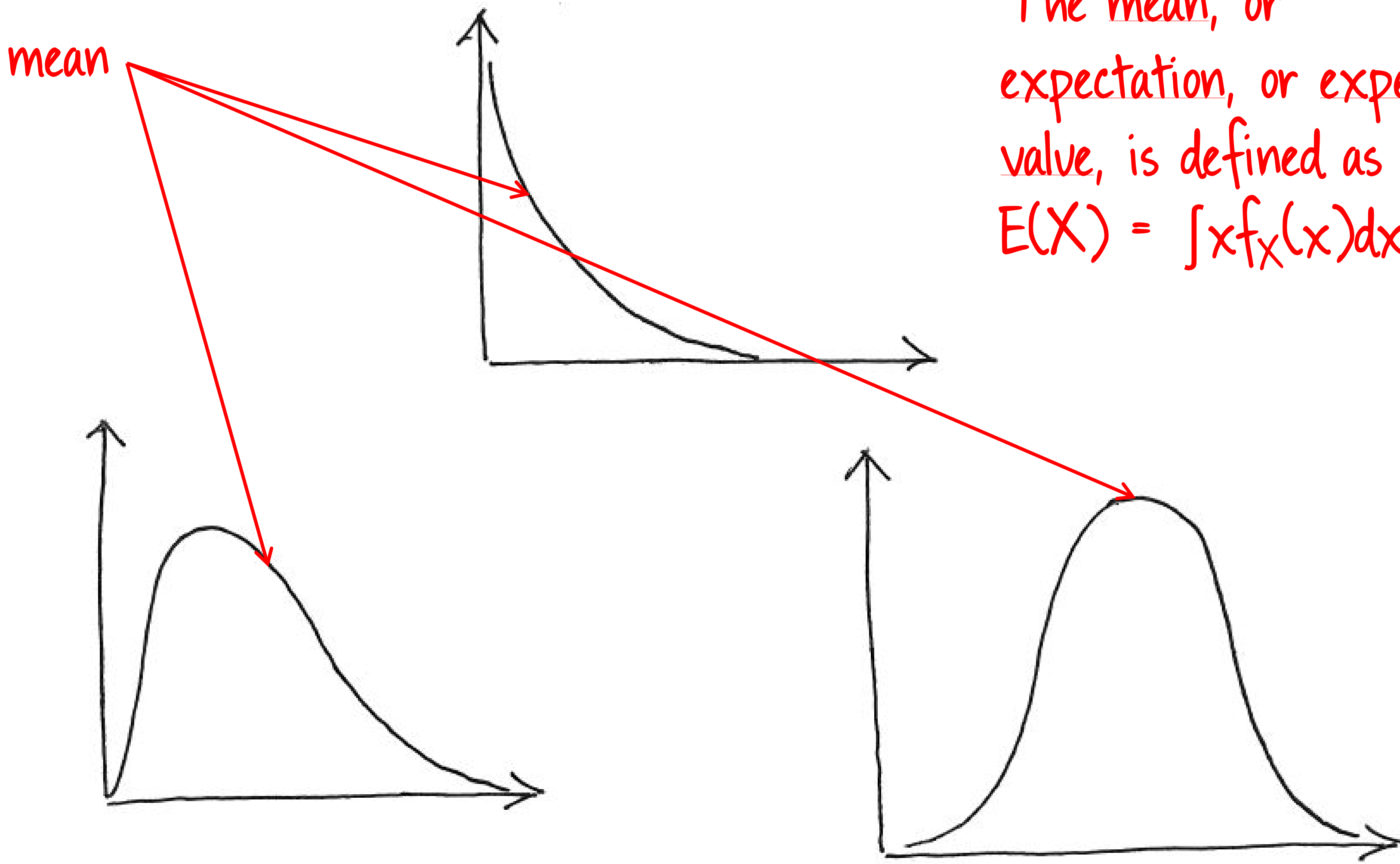
The median is the point above and below which the integral of the PDF is equal to 1/2.



Probability--moments of a distribution

The mean, or expectation, or expected value, is defined as

$$E(X) = \int x f_X(x)\,dx$$



Probability---moments of a distribution

So if we need to find the expectation of a continuous random variable, we just integrate the PDF times the value over the support:

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Discrete analog:

$$E(X) = \sum x f_X(x)$$

Probability--moments of a distribution

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If we think of the PDF literally as a density, the expectation is the balancing point of the density.

I will use "mean," "expectation," and "expected value" interchangeably.

Probability---example

Suppose $f_X(x) = \lambda \exp\{-\lambda x\}$ $x >= 0$

Probability---example

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Exponential distribution

Probability---example

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Probability---example

Suppose $f_X(x) = \lambda \exp\{-\lambda x\}$ $x >= 0$

Then,

$$E(X) = \int x f_X(x)dx$$

$$= \int x \lambda \exp\{-\lambda x\}dx$$

Probability---example

Suppose $f_X(x) = \lambda\exp\{-\lambda x\}$ $x >= 0$

Then,

$$E(X) = \int x f_X(x)dx$$

$$= \int x\lambda\exp\{-\lambda x\}dx$$

$$= \ldots$$

$$= 1/\lambda$$

Probability---example

Suppose $f_X(x) = \lambda \exp\{-\lambda x\}$ $x >= 0$

Then,

$$E(X) = \int x f_X(x)dx$$

$$= \int x \lambda \exp\{-\lambda x\}dx$$

$$= \ . \ . \ .$$

$$= 1/\lambda$$

Well, it turns out that this integral is a bit of a pain (integration by parts).

Probability---auctions

We're going to take a little side trip into auction theory. What do auctions have to do with probability? Well, typically, the winner of an auction is the highest bidder. So, if we want to model and analyze how auctions work, an obvious thing to do is to model bids in an auction as a i.i.d. random sample and the winning bid as the n^{th} order statistic from that random sample. That's what we'll do when we try to analytically answer the question of whether a seller should sell a product with a posted price or auction it off.

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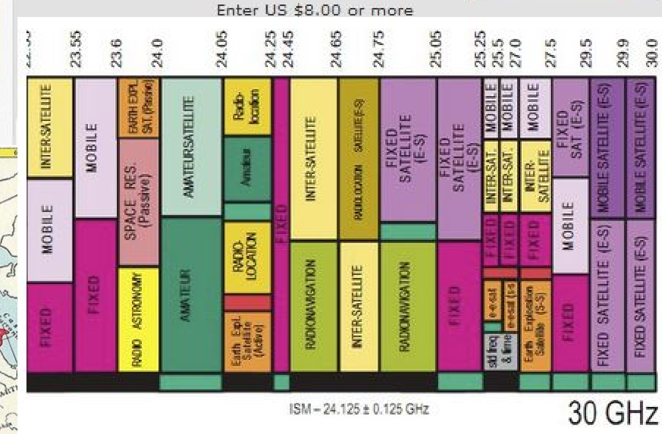
And we'll also compute some expectations.

Probability---auctions

Here are some things that are sold at auction:



Probability---auction of the Roman Empire

The Praetorian Guard auctioned off the Roman Empire in 193 CE to the highest bidder. Marcus Didius Severus Julianus won, paying 25,000 sesterces/soldier. He served as emperor for 66 days before being executed by the Praetorian Guard. (I imagine bids for the follow-on auction were lower, if one was actually held.)

"Auction" comes from the Latin "auctio", or increase.

Highest bidder is the "emptor."

Auctions also used to sell plunder, household effects, slaves, wives, commodities.

Probability---auction tulip bulbs

In 1600's, traders from the Ottoman Empire brought tulip bulbs back to Holland. The product was novel, so demand was unknown. Furthermore, it takes 7-12 years to go from a seed to a tradable bulb, so supply is fixed in the short run.

Demand was high, and traders invented a mechanism called the "Dutch Auction," when you start with a high price and decrease until someone buys.

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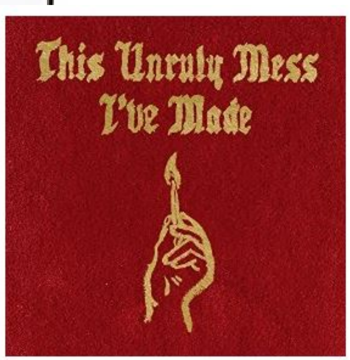
Options and futures contracts also pioneered during the Tulip Panic

Probability---auctions

Here are some things sold at a posted price:



This Unruly Mess I've Made [Explicit]

Macklemore & Ryan Lewis

Expected February 26, 2016

Pre-order Price Guarantee. Learn more.

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See all 4 formats and editions

MP3	Audio CD
$11.49	$9.99 Prime
	1 New from $9.99

Probability---auctions

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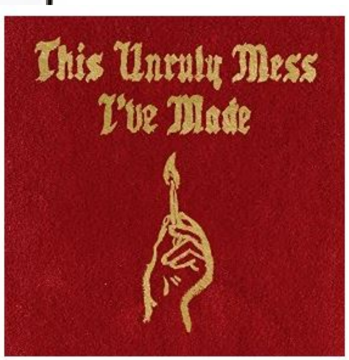
Actually most things on eBay today



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1. Transaction costs

   Both buyer and seller in an auction have to exert effort to monitor and then often wait for the outcome.

2. Information

   Seller receives free information on the value of the good (in fact, possibly information on the whole distribution of buyers' values).

Probability---auctions

Goods likely to be auctioned:

1. unique goods
2. expensive goods (transactions costs might not scale with price, but foregone surplus from uncertainty might)
3. goods where characteristics costly to assess
4. goods where buyers know more than sellers
5. goods with heterogeneity in buyer valuation

Probability---auctions

Let's consider a simple model to illustrate the point about information.

There are N potential buyers of some good. Their valuations are independent and distributed uniformly on the unit interval, [0,1].

The seller can offer the good, at no cost, at a posted price or auction it off. The seller knows the distribution of valuations, but does not know the individual realizations.

Probability---auctions

Posted price:

Set the price at p, sell the good if there are any $V_i >= p$

The expected profit:

$$E(\pi(p)) = pP(V_i >= p \text{ for at least one } i)$$

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What you get if you sell

1 - CDF of the n^{th} order statistic from V[0,1] evaluated at p

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How does this square with the formula we saw for expectation? Just apply the discrete formula and note that the first term is zero.

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Now for a little economics: we assume that the seller will choose p to maximize his profit. We figure out this optimal p by taking the derivative of expected profit with respect to price, setting equal to zero, and solving for price.

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$$= p(1-p^N)$$

So $d\pi/dp = 1 - (N+1)p^N$ and the optimal price is $\sqrt[N]{\frac{1}{N+1}}$

Futhermore, the expected profit under that optimal price is

$$\frac{N}{N+1}\sqrt[N]{\frac{1}{N+1}}$$

Probability---auctions

Which gives rise to the following table:

N	p	$E\Pi$
1	$\frac{1}{2}$	$\frac{1}{4}$
2	$\sqrt{\frac{1}{3}} \simeq .58$	$\frac{2}{3}\sqrt{\frac{1}{3}} \simeq .38$
3	$\sqrt[3]{\frac{1}{4}} \simeq .63$	$\frac{3}{4}\sqrt[3]{\frac{1}{4}} \simeq .47$
$\vdots$	$\vdots$	$\vdots$
9	$\sqrt[9]{\frac{1}{10}} \simeq .77$	$\frac{9}{10}\sqrt[9]{\frac{1}{10}} \simeq .70$

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Posted price is rising as the number of potential buyers goes up. Expected profits also go up. This is a consequence of the distribution of the n^{th} order statistic and how it changes at n increases.

Probability---auctions

Auction:

We will assume an "English Auction," where the price of the good will gradually increase and potential buyers stay in the bidding until $p > V_i$, where V_i is buyer i's valuation for the product. When only one buyer is left, he gets the good at $p=V_{(N-1)}$, the second-highest valuation.

Probability---auctions

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Why?

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This is the classic "open outcry" auction that we always see on TV shows, etc.

Probability---auctions

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To compute expected profits here, we will need the distribution of the N-1st order statistic from U[0,1].

$$f_{(N-1)}(x) = N(N-1)(1-x)x^{N-2} \qquad \text{for } 0 <= x <= 1$$

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Probability---auctions

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Here we use the continuous formula to calculate expectation

Probability---auctions

Auction:

$$\text{So } E(\pi(N)) = \int_{[0,1]} N(N-1)(1-x)x^{N-2}x\,dx$$

$$= N(N-1)\int_{[0,1]}(x^{N-1}-x^N)\,dx$$

Probability---auctions

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$= N(N-1)\int_{[0,1]} (x^{N-1}-x^{N})dx$

$= (N-1)/(N+1)$

Probability---auctions

Which gives rise to the following table:

N	$E\Pi$
1	0
2	$1/3$
3	$1/2$
$\vdots$	$\vdots$
9	.80

Probability---auctions

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⋮	⋮	⋮
9	.80	$\frac{9}{10}\sqrt[9]{\frac{1}{10}} \simeq .70$
	auction	posted price

auction does better for $N > 2$ (this is general)

Probability---auctions

So what did this model tell us (conditional on assumptions)?

1. The seller will do better with an auction when N is large enough.
2. This is true even though the seller needed to know the distribution of valuations to set an optimal posted price and did not need to know that distribution for the auction. (If the seller is wrong about the distribution of valuations, the fixed price does really badly.)
3. This model does not have transactions costs in it.

Probability---eBay's switch from auctions

As I mentioned earlier, eBay, founded in 1995 as an online auction site exclusively, now only has about 15% of its listings in auctions.

Probability---eBay's switch from auctions

Here's a graph illustrating that point*:

*From NBER working Paper by Einav, Farronato, Levin, and Sundaresan

Figure 1: Auction Share on eBay over Time

For each month, the figure shows the average daily share of active eBay listings (black) and transaction revenues (gray) from pure auction listings out of all pure auction and posted price listings. Less common formats, such as hybrid auctions, are not included. The sharp drop in Fall 2008 coincides with a decision in September 2008 to allow "good till canceled" posted price listings (see Section 7).

Probability---eBay's switch from auctions

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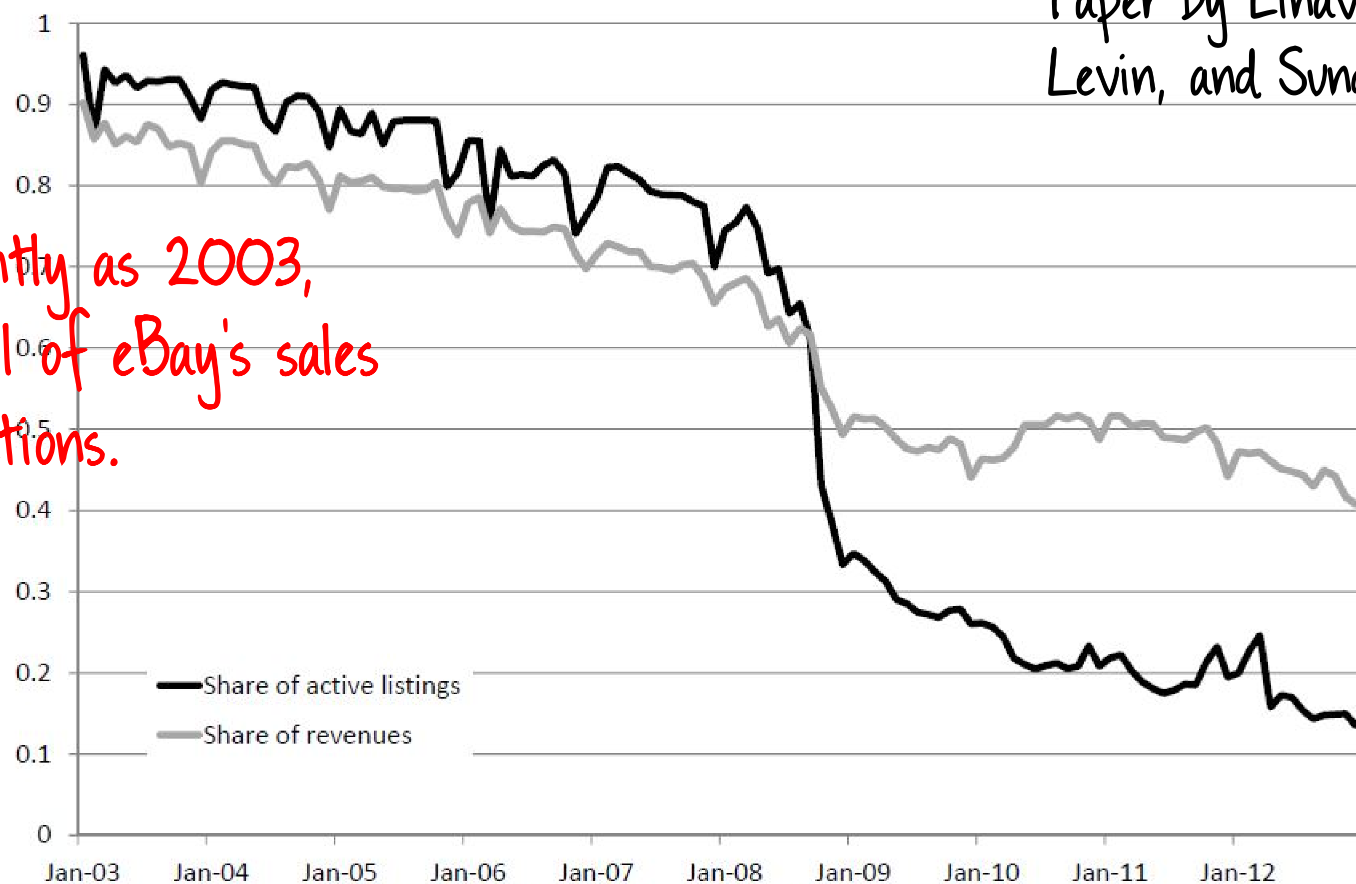
*From NBER working Paper by Einav, Farronato, Levin, and Sundaresan

As recently as 2003, almost all of eBay's sales were auctions.

Figure 1: Auction Share on eBay over Time



For each month, the figure shows the average daily share of active eBay listings (black) and transaction revenues (gray) from pure auction listings out of all pure auction and posted price listings. Less common formats, such as hybrid auctions, are not included. The sharp drop in Fall 2008 coincides with a decision in September 2008 to allow "good till canceled" posted price listings (see Section 7).

Probability---eBay's switch from auctions

Here's a graph illustrating that point*:

*From NBER working Paper by Einav, Farronato, Levin, and Sundaresan

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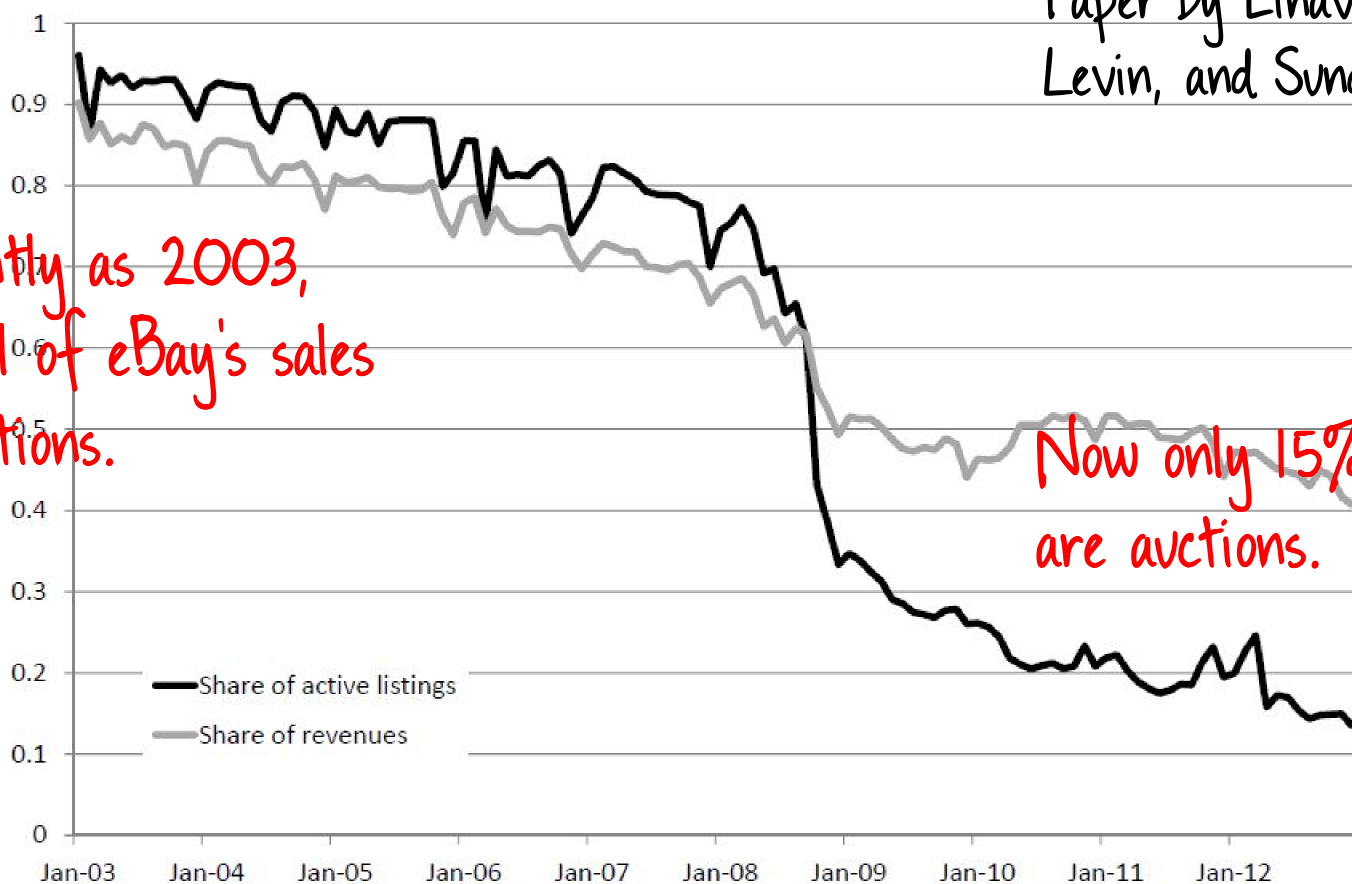

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Now only 15% of listings are auctions.

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Figure 2: Google Search Volume for Online Auctions and Online Prices

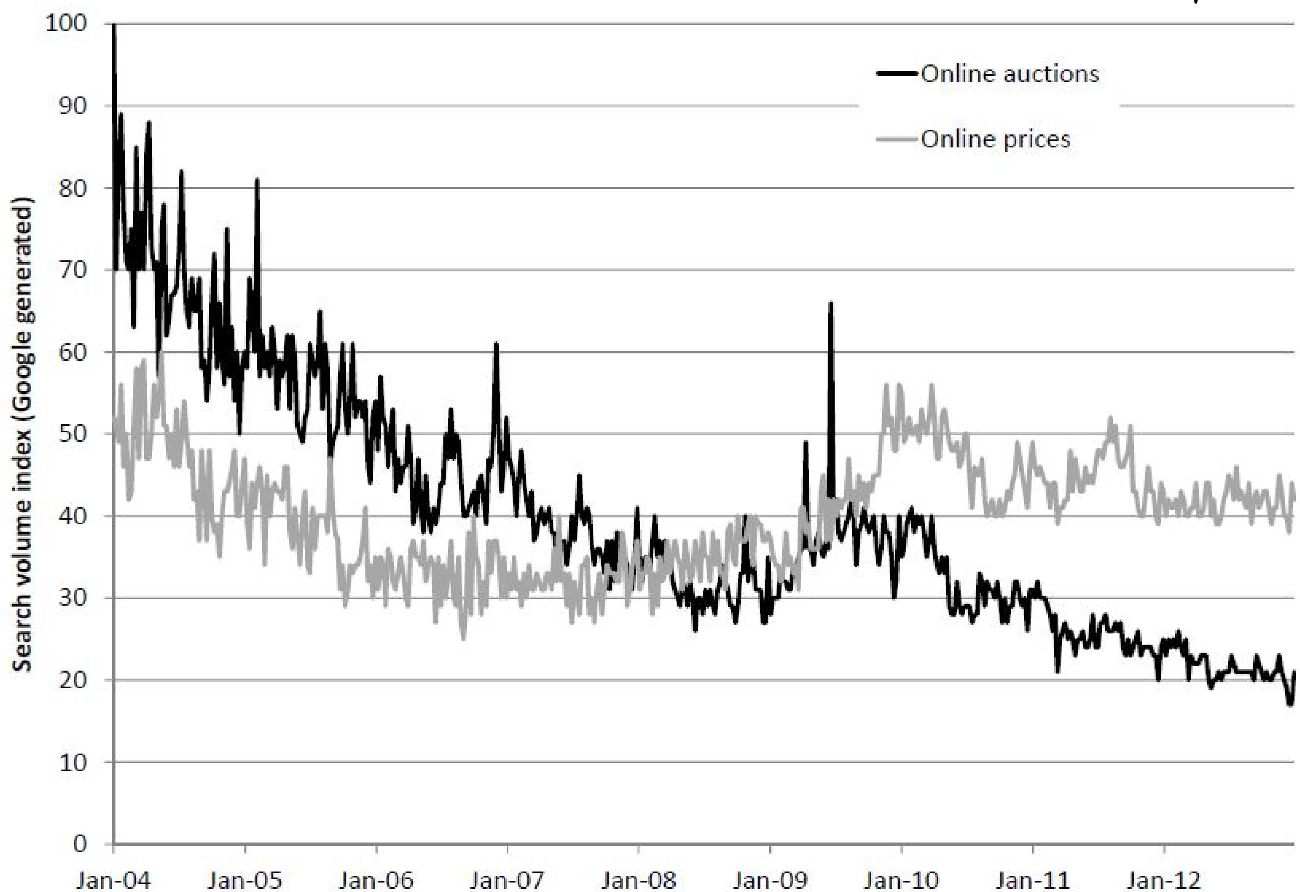

Figure presents results from "Google Trends" for search terms "online auctions" and "online prices." The y-axis is a Google generated index for the weekly volume of Google searches for each of the two search terms, which should make the weekly volume figures comparable over time and across the two search terms.

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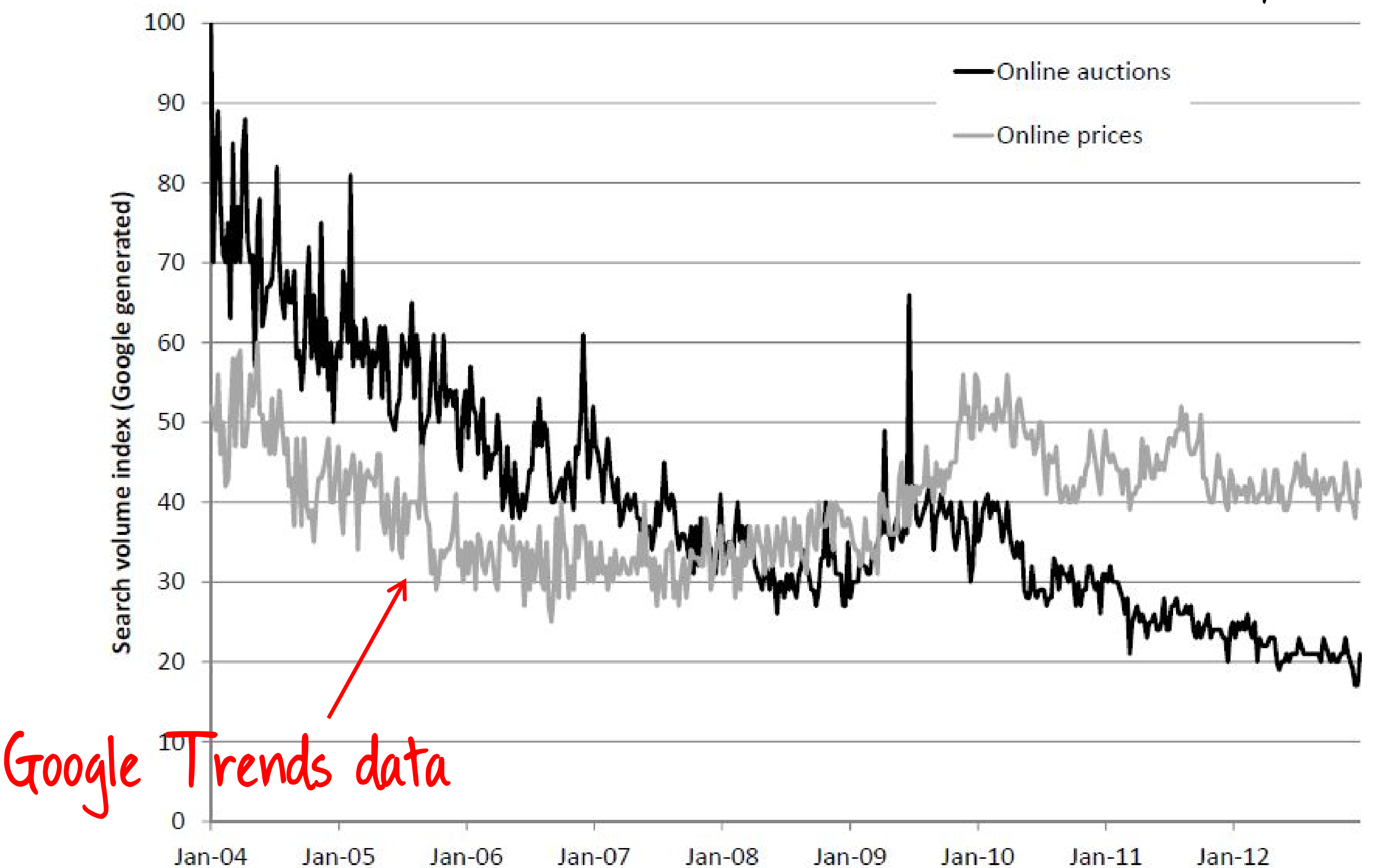

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What's going on?

We have a theory suggesting when a seller might prefer to sell an item with posted price versus auction.

That theory is useful for thinking about information asymmetries, but it is incomplete. (For instance, our model did not include anything about transactions costs.)

Furthermore, these eBay graphs suggest that something has been changing over time.

Probability---eBay's switch from auctions

Three broad hypotheses:

1. There's been a compositional shift of sellers and types of products on eBay.
2. Consumer tastes have changed---online auctions are not as fun and novel as they used to be.
3. The "price discovery" benefits of auctions have declined over time.

    Online search has made it easier to find prices for comparable items

    eBay itself has created thick national markets for lots of things that didn't exist before, which provide price information.

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Can decompose the shift over time from auction to posted price:

- How much have product categories shifted over time (towards more standardized products, away from unique products)?
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They find that these explanations only account for a fairly small fraction of the shift. Instead, the returns to sellers using auctions have diminished.