

Pandemic Flu Spread

Davud Davudov
ddavudov@gatech.edu

Reem Soliman
rsoliman3@gatech.edu

Simon Yee
syee30@gatech.edu

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Abstract

Influenza (flu) is a contagious viral infection that can cause mild to severe symptoms and life-threatening complications, including death, even in healthy children and adults. Influenza viruses spread mainly from one individual to another through coughing or sneezing. Less often, they can also spread through touching a contaminated surface and then touching the mouth, eyes, or nose. Individuals can pass flu on to others even before their own symptoms start and for a week or more after symptoms begin.

The problem definition

Consider a classroom of 21 elementary school kids. 20 of the kids are healthy (and susceptible to flu) on Day 1. Tommy (the 21st kid) walks in with the flu and starts interacting with his potential victims. To keep things simple, let's suppose that Tommy comes to school every day (whether or not he's sick) and will be infectious for 3 days. Thus, there are 3 chances for Tommy to infect the other kids — Days 1, 2, and 3. Suppose that the probability that he infects any individual susceptible kid on any of the three days is $p = 0.02$; and suppose that all kids and days are independent (so that you have i.i.d. $\text{Bern}(p)$ trials). If a kid gets infected by Tommy, he will then become infectious for 3 days as well, starting on the next day.

Assumptions

Simulating and modeling real life scenarios can be challenging, as there are many factors to consider. Our simulation currently operates under the current restraints and assumptions:

- Probability (p) with $p = 0.02$ infection rate may be interpreted as either the probability that infection is certain, with 0.02 being the likelihood of contact, or that the combination of being in contact and becoming infected is 0.02. Based on our design, we have a slight variation of the latter, where we assume everyday an active infected individual will make contact with every child that is not infected, and the probability that they will become infected is 0.02.
- In our base model, we are currently assuming re-infection is not possible. Once an individual has left their three day contagious period, they will not catch this flu again.
- Our base model treats days in a continuous manner, where hypothetically if we started the scenario on a Monday and an individual gets infected 5 days later (Friday), they will continue to make contact and infect into Saturday and Sunday.

- Our base model does not run for a set amount of days, it will only continue until someone can no longer infect. This will affect the averages and its distribution slightly. We standardize this later when aggregating the data.

Solving Parts A, B, C and D

- (a) What is the distribution of the number of kids that Tommy infects on Day 1?

Using the definition of Binomial Distribution: the binomial distribution is the probability distribution of a sequence of experiments where each experiment produces a binary outcome and where each of the outcomes is independent of all the others: $\text{Binary}(20, 0.02)$ [3]

As it can be seen from Figure 1, distribution of the kids infected on day 1 is binomial which is as expected.

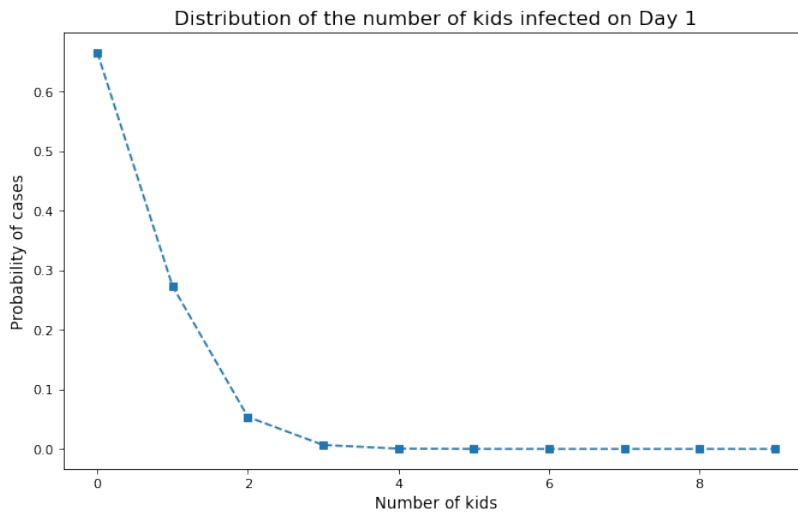


Figure 1: Distribution of the number of the kids infected on Day 1

- (b) What is the expected number of kids that Tommy infects on Day 1?

If $X \sim B(n, p)$, that is, X is a binomially distributed random variable, n being the total number of experiments and p the probability of each experiment yielding a successful result, then the expected value of X is $E[X] = n * p$. The expected number of kids that Tommy infects in day one is: $E[X] = n * p = 20 * 0.02 = 0.4$. See Figure 2

- (c) What is the expected number of kids that are infected by Day 2 (you can count Tommy if you want)?

Excluding Tommy, from Figure 2, the expected number of kids that are infected by Day 2 is around 0.94

- (d) Simulate the number of kids that are infected on Days 1,2,. . . . Do this many times. What are the (estimated) expected numbers of kids that are infected by Day i , $i = 1, 2, \dots$? Produce a histogram detailing how long the "epidemic" will last.

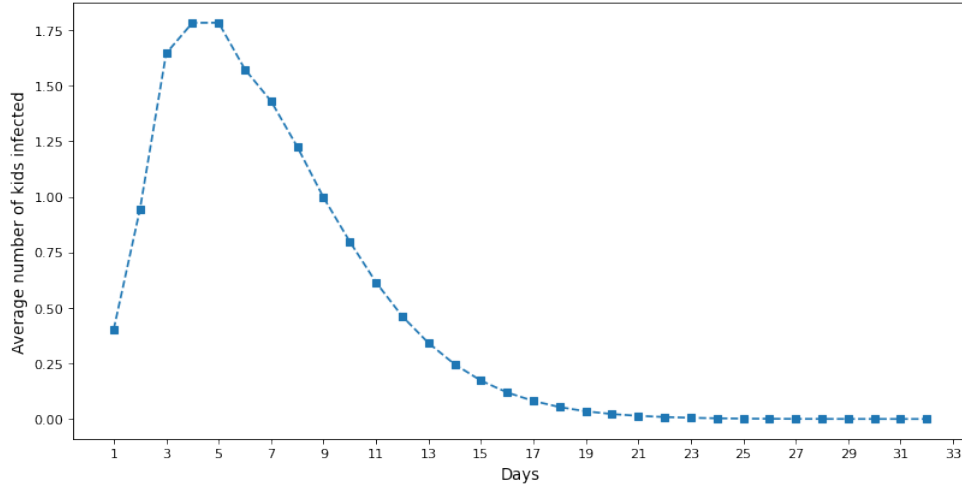


Figure 2: Average number of the kids infected on days

It should be noted that this has been simulated with assumption that kids can meet with each other all 7 days in week. Alternatively, considering only weekdays for the kids to be able to meet and get infected, then average number of cases is shown in Figure 3. In this case also, another major assumption is that first day is Monday. Thus we can see that first 5 days from two cases are identical. However, we observe significant reduction during weekends (day 6 and 7) and some increase in the following weekdays (day 8, 9 and 10).

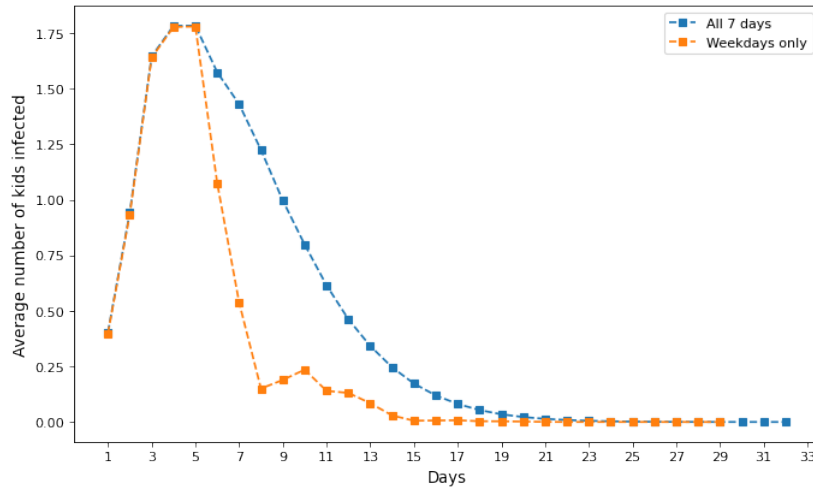


Figure 3: Average number of the kids infected on days

Alternatively, it can also be simulated with different starting days for the school. In this scenario, as the first the of the school all 5 days are simulated individually and results are shown in Figure 4.

It can be seen that, comparing with previous scenarios, only first day is same and in all following days there are less average number of kids getting infected which is an expected result.

In light of COVID-19, we simulated a scenario with a "stay at home" policy. The simulation is under the assumption that the infected only have one day to infect,

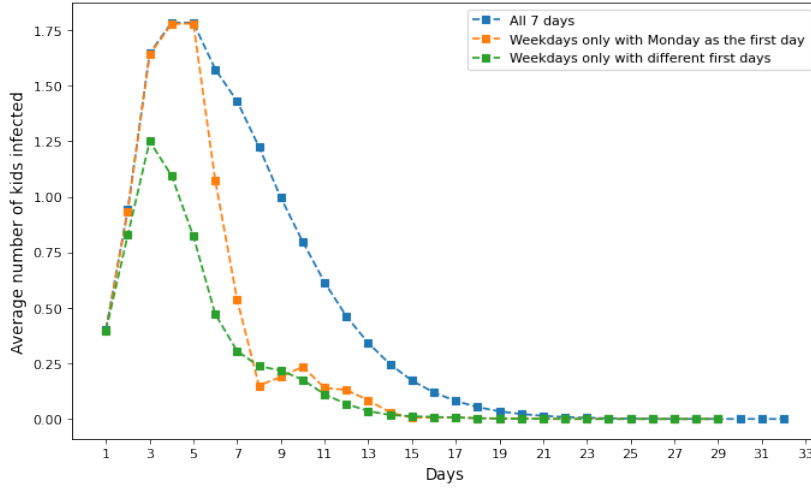


Figure 4: Average number of the kids infected on days

because parents or teachers will force students to stay at home given that symptoms will show by the end of the first day. When only given one day to infect, the average infected days dramatically reduces from an average of 32 to just about 10 days.

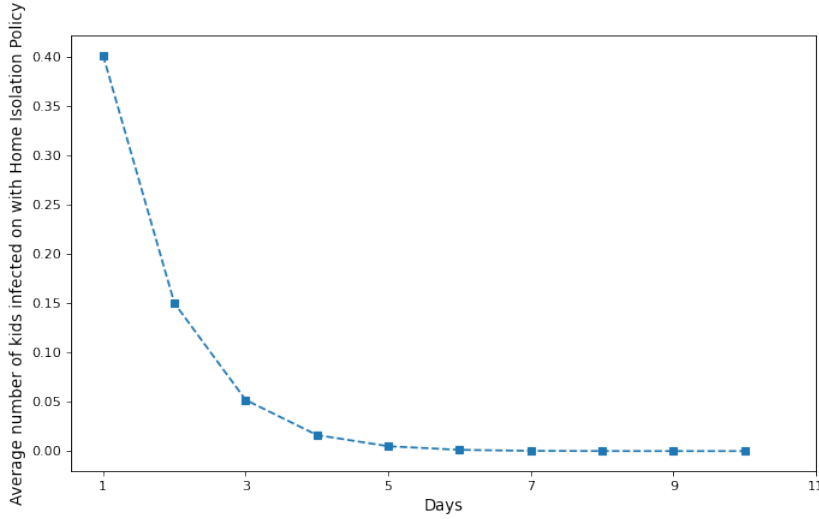


Figure 5: Average number of the kids infected on with Home Isolation Policy

Finally, the histogram of how long epidemic lasted (excluding Tommy) is illustrated in Figure 6. The maximum days epidemic lasted is 30 days whereas average is around 6.6. It can also be seen around 30% of the cases epidemic never spread (Tommy was not able to infect anyone). However, in case Tommy was able to infect someone on the first day, then minimum number of days epidemic lasted is 3, which is number of days infection takes. Thus minimum number of days epidemic lasted is either 0 or 3.

Considering only weekdays, the histogram of how long epidemic lasted is illustrated in Figure 7. For this scenario the maximum days epidemic lasted is 27 days whereas average is around 4.8 which both are less than previous scenario. Analyzing results from Figure 7, another interesting observation is that, the number of cases where

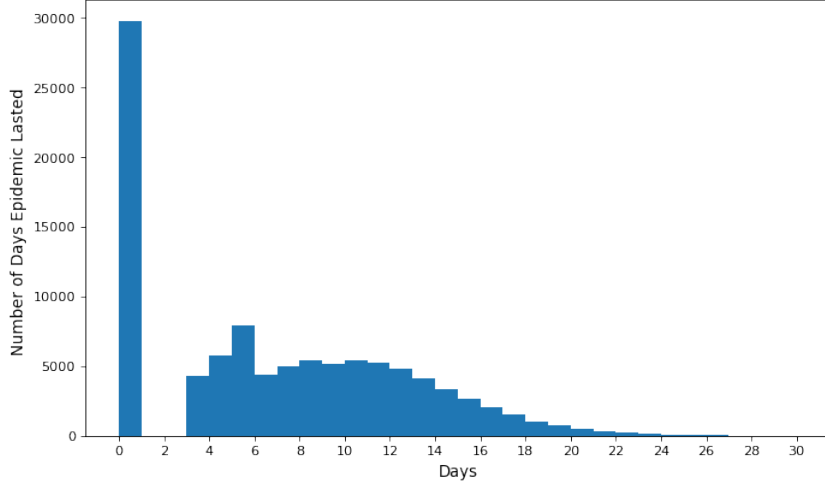


Figure 6: Histogram of days epidemic lasted

epidemic ended on the first weekend increases significantly.

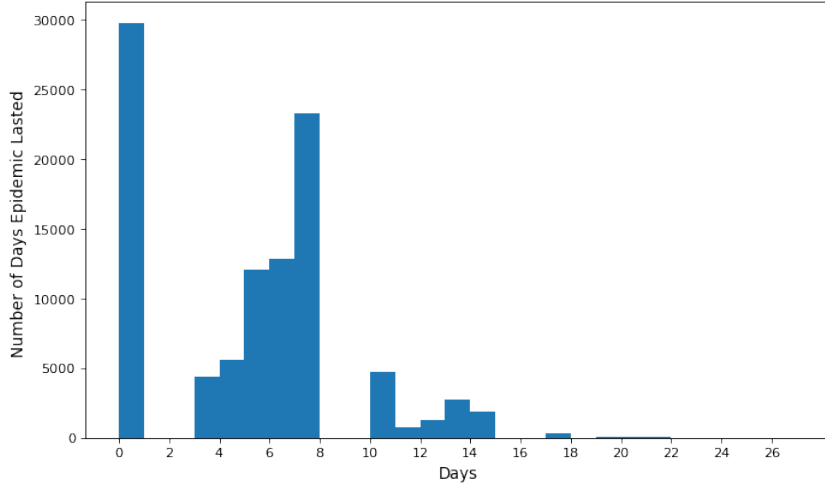


Figure 7: Histogram of days epidemic lasted

Sensitivity Analysis

To understand impact of infection rate (infection probability), infection days and population size, we simulated multiple cases. Two population sizes of 20 and 200 are considered. For each case, 5 infection rates with 2,4,6,8, and 10% and 5 infection days with 1,3,5,7 and 9 are considered. In all cases base simulation case is considered (all 7 days contact). We also introduce new parameter as multiplication of infection probability with infection days (**evaluative parameter** hereafter). The **evaluative parameter** can be also be expressed as ratio of infection rate to recovery rate ($\frac{1}{infection\ days}$) and it can be further linked to the basic reproduction ratio from SIR model. All results are summarized in **Table 1** and **Table 2**. [1]

Results indicate that for the case of low infection rate and low infection days

Table 1: Sensitivity analysis for 20 kids

Inf Prob	Inf Days	Prob * Days	Ep. Days Max	Ep. Days Ave	Num Inf Max	Num Inf Ave
0.02	1	0.02	9	0.51	0.40	0.07
0.04	1	0.04	13	1.38	0.80	0.16
0.06	1	0.06	14	2.63	1.25	0.37
0.08	1	0.08	14	3.82	2.12	0.66
0.10	1	0.10	15	4.45	3.30	0.87
0.02	3	0.06	32	6.60	1.78	0.46
0.04	3	0.12	37	10.80	6.18	1.24
0.06	3	0.18	25	9.39	10.59	2.27
0.08	3	0.24	19	8.03	13.27	3.11
0.10	3	0.30	16	7.11	15.49	3.74
0.02	5	0.10	56	16.58	5.01	1.11
0.04	5	0.20	38	14.14	12.43	2.53
0.06	5	0.30	27	11.39	16.35	3.69
0.08	5	0.40	21	9.91	18.24	4.76
0.10	5	0.50	15	9.01	19.48	6.67
0.02	7	0.14	66	22.68	8.73	1.80
0.04	7	0.28	36	16.25	16.04	3.86
0.06	7	0.42	24	13.32	19.02	5.83
0.08	7	0.56	22	11.87	19.86	6.36
0.10	7	0.70	19	11.01	19.98	7.37
0.02	9	0.18	67	26.04	11.80	2.53
0.04	9	0.36	37	18.21	18.22	4.86
0.06	9	0.54	26	15.31	19.86	6.92
0.08	9	0.72	23	13.87	19.99	7.83
0.10	9	0.90	19	13.01	20.00	9.47

Table 2: Sensitivity analysis for 200 kids

Inf Prob	Inf Days	Prob * Days	Ep. Days Max	Ep. Days Ave	Num Inf Max	Num Inf Ave
0.02	1	0.02	12	6.38	71.5	16.0
0.04	1	0.04	6	4.11	116.9	33.1
0.06	1	0.06	5	3.29	95.8	39.8
0.08	1	0.08	4	3.00	132.0	49.8
0.10	1	0.10	4	3.00	154.8	49.8
0.02	3	0.06	12	7.52	174.8	49.9
0.04	3	0.12	7	5.99	191.0	85.6
0.06	3	0.18	6	5.14	198.8	99.8
0.08	3	0.24	6	5.00	199.0	99.8
0.10	3	0.30	6	5.00	199.0	99.8
0.02	5	0.10	13	9.48	197.4	76.8
0.04	5	0.20	10	7.99	200.0	99.9
0.06	5	0.30	9	7.14	200.0	111.0
0.08	5	0.40	8	7.00	200.0	124.9
0.10	5	0.50	7	7.00	200.0	142.7
0.02	7	0.14	14	11.48	200.0	99.9
0.04	7	0.28	12	9.99	200.0	116.6
0.06	7	0.42	10	9.14	200.0	139.9
0.08	7	0.56	10	9.00	200.0	139.9
0.10	7	0.70	9	9.00	200.0	155.4
0.02	9	0.18	17	13.47	200.0	105.8
0.04	9	0.36	14	11.99	200.0	128.5
0.06	9	0.54	12	11.14	200.0	149.9
0.08	9	0.72	12	11.00	200.0	149.9
0.10	9	0.90	11	11.00	200.0	163.5

(low evaluative parameter), average days epidemic take few to last is low. This is because epidemic doesn't get chance to spread. Similarly, for the case of high infection rate and high infection days (high evaluative parameter), average days epidemic take few to last is also low. This is however is because it takes very short time to infect everyone and diseases is spread to everyone in very short time. On the other hand, for the case of low infection rate but high infection days, it takes much longer for the disease to end. For both 20 and 200 kids scenarios, highest values for average epidemic days is observed for the cases of 2%, 4% infection rates with 5,7,and 9 days infection days. This indicates that unless infected people are recovered and/or isolated fast from the others, it will take much longer for the disease to end. Please note that based on assumptions epidemic ends either there is no one infected or everyone is infected.

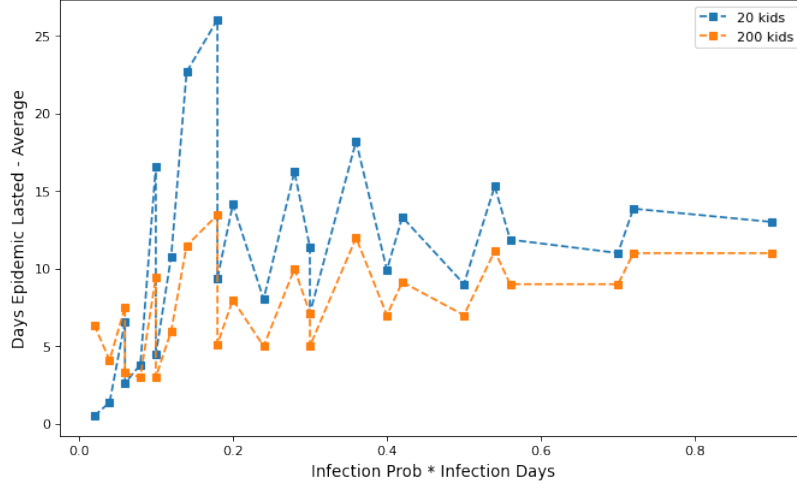


Figure 8: Average of days epidemic lasted

Figure 8 also shows average value of epidemic days versus **evaluative parameter**. As discussed before, for high and low values of **evaluative parameter** epidemic ends fast but it takes longer for in-between values of **evaluative parameter**. Moreover, comparing 20 kids with 200 kids scenario, it can also be observed that, except for the 1 day infection days case, for all other cases, it takes much shorter for the epidemic to end for the 200 kids case. This is because for high density population disease have much more chance to spread and in most cases everyone get infected easily.

To analyze number of kids infected, Figure 9 shows maximum number of the case in single day during epidemic versus **evaluative parameter**. Results indicate that as the maximum number of the maximum cases in single day increases as the **evaluative parameter** increases. It can also be observed that for the case of 20 kids, when **evaluative parameter** is more than 0.35, it gets to plateau and at least 90% of population (18 kids) are infected in single day. For the case of 200 kids however, this threshold is much lower, which shows that when **evaluative parameter** is more than 0.12, it gets to plateau and at least 96% of population (191 kids) are infected in single day. This indicates that, it is much critical to lower **evaluative parameter** in high populations. Moreover, comparing 10% infection rate and 1 day infection rate with 2% infection rate and 5 day infection days (both with 0.1 **evaluative parameter**), it is observed that in latter case there are much more infected kids. This also indicates that, infection days (recovery rate) is more critical compared infection rate and although infection rate is very high total infection can be lowered with fast isolation. On the other hand, even infection rate is low (2%), if infection days is long then it will spread to everyone.

Finally, Figure 10 shows average number of kids infected per day as a function

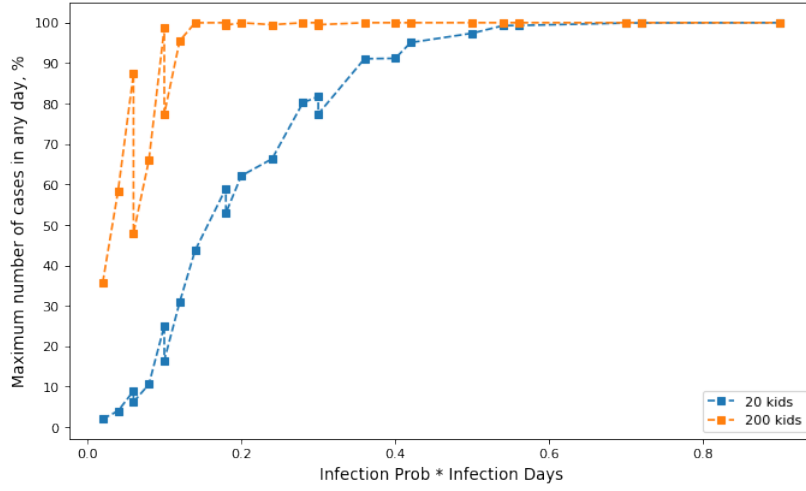


Figure 9: Maximum number of kids infected in single day

of **evaluative parameter**. Consistent with previous results, it is observed that average number of infected kids increases as the **evaluative parameter**. Moreover, this value is much higher for the case of 200 kids scenario.

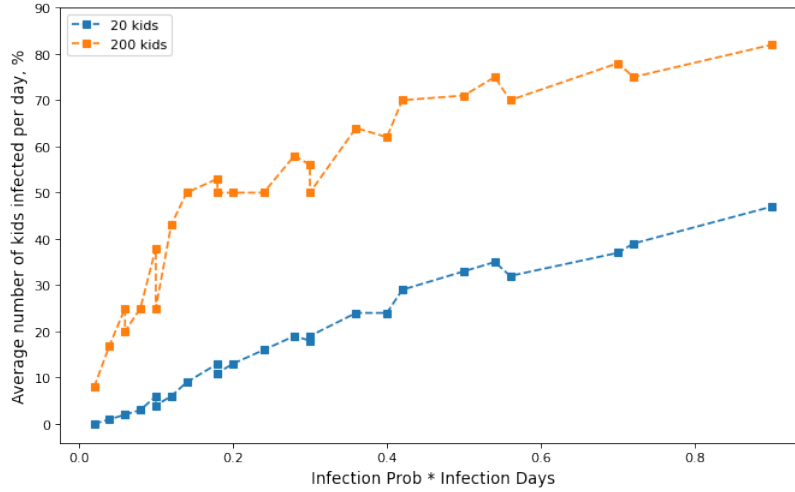


Figure 10: Average number of kids infected per day

Other Methods

Modelling Using Arena Software

We were able to model part a of the problem using Arena Software. In the Assign module, we defined an attribute named infected, its values is given for each student with the function $\text{DISC}(0.02, 1, 1, 0)$, where 0.02 is the probability of infection. Figure 11

We had 10,000 replication, and got expected number of infected students around 0.4 which matches the Python code and the Binomial Distribution Expected Value math.

Figure 12

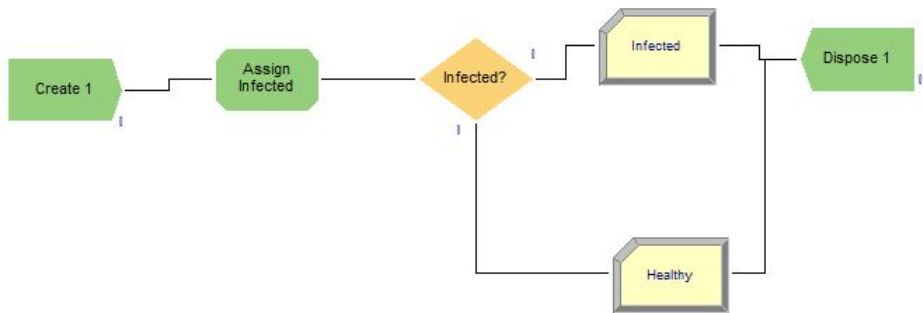


Figure 11: Arena Model for Day One



Figure 12: Arena Model for Day One Output Report

Using The SIR model

The SIR model is one of the simplest compartmental models, and many models are derivatives of this basic form. The model consists of three compartments:-

- S: The number of susceptible individuals. When a susceptible and an infectious individual come into "infectious contact", the susceptible individual contracts the disease and transitions to the infectious compartment.
- I: The number of infectious individuals. These are individuals who have been infected and are capable of infecting susceptible individuals.
- R for the number of removed (and immune) or deceased individuals. These are individuals who have been infected and have either recovered from the disease and entered the removed compartment, or died. It is assumed that the number of deaths is negligible with respect to the total population. This compartment may also be called "recovered". [4] [2]

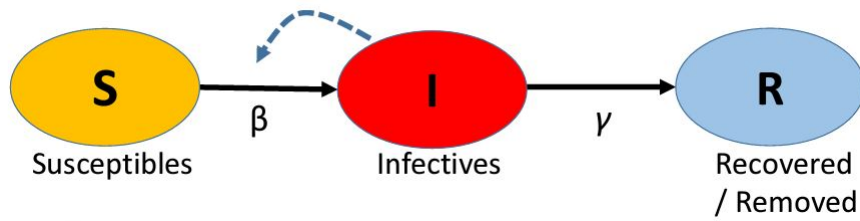


Figure 13: States in an SIR epidemic model and the rates at which individuals transition between them

In our problem, the initial state is that Tommy is the only one in the Infected group and the rest of the kids are in the Susceptible group. We assume that the kid who gets infected will not be infected again, so we apply SIR with immunity model.

Model Parameters:

1- Transmission rate (Beta): $\beta = \text{contact rate} * \text{probability of transmission}$.

2- Recovery rate (Gamma): $\gamma = 1 / \text{Infection Period}$.

Using the model parameters, we used Python to generate the graph below.

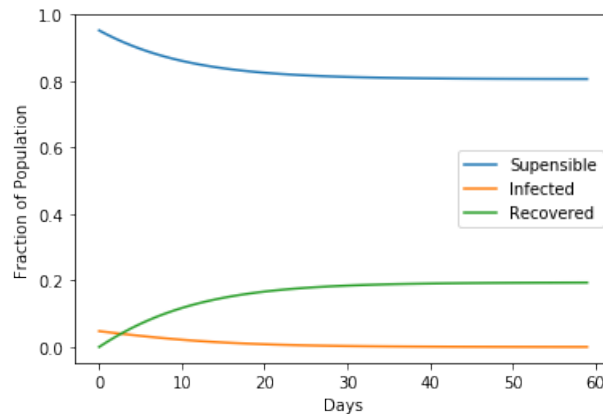


Figure 14: The SIR epidemic trajectory

With more digging and researches, we found that the SIR model is more intended for larger population like cities or countries.

Getting Day One Infection using the SIR model:

Let contact rate = 1 (7 days a week)

$\beta = \text{contact rate} * 0.02 = 0.02$.

Initial State: $S_0 = 20$, $I_0 = 1$, $R_0 = 0$

Number of Infected kids in day 1 (I_1) = $\beta * S_0 * I_0 = 0.02 * 20 * 1 = 0.4$.

Conclusion

In conclusion, our team found that simulating for a specific scenario is often unrealistic, complicated, and that there are many considerations that need to be taken

into account. We learned that a simulation may help us understand an approximate outcome, but real world data is needed to compare, and help adjust and understand what outcomes in our simulation are realistic and what is too idealistic. A small adjustment in variables may produce similar patterns, but completely different numerical results. It is important to consider carefully, even in a small, controlled classroom environment, what the objective is, what the outcome means, and what roles do variables play in the simulation. For any current and future projects, we learned it is important to understand how the methodology, results and variables all tie in with each other - because one issue in any of the three components will may produce an outcome that does not reflect the goal of the research.

References

- [1] Science Direct. *Sensitivity Analysis*. URL: <https://www.sciencedirect.com/topics/medicine-and-dentistry/sensitivity-analysis>. (accessed: 01.12.2021).
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