Supplementary: Real-time Subsurface Control Variates

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We provide the detailed proof of our proposed exponential moving covariance matrix and how it is related to the largely adopted exponential moving weighted average (EMWA) covariance estimator. It serves as the basis for online control variates (CV) coefficient estimation for subsurface scattering with real-time adaptive sampling. A concrete example and the corresponding theory of why we select the CV coefficient and residual function is also provided.

Additional Key Words and Phrases: Real-time adaptive sampling, subsurface scattering, control variates, exponential moving covariance matrix

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1 EXPONENTIAL MOVING COVARIANCE

To assure that the exponential moving covariance matrix is correct, we first prove the basic building block that exponential moving covariance, is correct.

1.1 Variable-weight covariance

The derivation is based on [Finch 2009]. For exponential moving average and variance, please refer to the original paper. Denote n observations $x_1, ..., x_n$ and $y_1, ..., y_n$ from two random variable x, y. The weighted means for x and y are:

$$\mu_n = \frac{\sum_{i=1}^{n} w_{n,i} x_i}{\sum_{i=1}^{n} w_{n,i}}, \upsilon_n = \frac{\sum_{i=1}^{n} w_{n,i} y_i}{\sum_{i=1}^{n} w_{n,i}} \tag{1}$$

where the weights sum as

$$W_n = \sum_{i=1}^{n} w_{n,i}. \tag{2}$$

To keep

$$W_n \mu_n - w_{n,n} x_n = (W_n - w_{n,n}) \mu_{n-1}, \tag{3}$$

$$W_n \upsilon_n - w_{n,n} y_n = (W_n - w_{n,n}) \upsilon_{n-1} \tag{4}$$

we have the following constraint between W_n and W_{n-1}:

$$\frac{w_{n,j}}{\sum_{i=1}^{n-1} w_{n,i}} = \frac{w_{n-1,j}}{\sum_{i=1}^{n-1} w_{n-1,i}}, 1 \leq j \leq n-1. \tag{5}$$

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Then we can derive from the covariance between x and y

$$\begin{aligned} Cov_n(x,y) =& E_n((x-\mu_n)(y-\upsilon_n)) \\ =& \frac{1}{W_n}\sum_{i=1}^{n} w_{n,i}(x_i-\mu_n)(y_i-\upsilon_n) \end{aligned}$$

with $f_n(x,y) = (x-\mu_n)(y-\upsilon_n)$ and Eq. 5 that

$$W_n E_n(f_{n-1}(x,y)) = \sum_{i=1}^{n} w_{n,i} f_{n-1}(x_i,y_i) \tag{6}$$

$$= w_{n,n} f_{n-1}(x_n,y_n) + (W_n - w_{n,n})\frac{\sum_{i=1}^{n-1} w_{n-1,i} f_{n-1}(x_i,y_i)}{W_{n-1}} \tag{7}$$

$$= w_{n,n} f_{n-1}(x_n,y_n) + (W_n - w_{n,n})E_{n-1}(f_{n-1}(x,y)). \tag{8}$$

With $S_n = W_n Cov(x,y)$ we can have:

$$S_n = W_n E_n([x-\mu_n][y-\upsilon_n]) \tag{9}$$

$$= W_n E_n(([x-\mu_{n-1}]-[\mu_n-\mu_{n-1}])([y-\upsilon_{n-1}]-[\upsilon_n-\upsilon_{n-1}])) \tag{10}$$

$$\begin{aligned} =& W_n E_n([x-\mu_{n-1}][y-\upsilon_{n-1}]) + W_n E_n([\mu_n-\mu_{n-1}][\upsilon_n-\upsilon_{n-1}]) \\ & - W_n E_n([\mu_n-\mu_{n-1}][y-\upsilon_{n-1}]) - W_n E_n([x-\mu_{n-1}][\upsilon_n-\upsilon_{n-1}]) \end{aligned} \tag{11}$$

where the 3rd and 4th term can be simplified as

$$W_n E_n([\mu_n-\mu_{n-1}][y-\upsilon_{n-1}]) = W_n[\mu_n-\mu_{n-1}][\upsilon_n-\upsilon_{n-1}] \tag{12}$$

$$W_n E_n([x-\mu_{n-1}][\upsilon_n-\upsilon_{n-1}]) = W_n[\mu_n-\mu_{n-1}][\upsilon_n-\upsilon_{n-1}]. \tag{13}$$

The first term can be simplified with Eq. 8 as

$$W_n E_n([x-\mu_{n-1}][y-\upsilon_{n-1}]) = \tag{14}$$

$$= w_{n,n}[x_n-\mu_{n-1}][y_n-\upsilon_{n-1}] + \frac{W_n - w_{n,n}}{W_{n-1}} S_{n-1} \tag{15}$$

$$= \frac{W_n^2}{w_{n,n}}[\mu_n-\mu_{n-1}][\upsilon_n-\upsilon_{n-1}] + \frac{W_n - w_{n,n}}{W_{n-1}} S_{n-1} \tag{16}$$

After summing up the four terms, it leads to

$$S_n = \frac{W_n - w_{n,n}}{W_{n-1}} S_{n-1} + w_{n,n}(y_n-\upsilon_n)(x_n-\mu_{n-1}). \tag{17}$$

Then, the variable-weight covariance has$Cov_n(x,y) = S_n/W_n$

1.2 Exponential moving covariance

In exponential moving average, we have the general form

$$\mu_n = (1-\eta)^n x_0 + \sum_{i=1}^{n}(1-\eta)^{n-i}\eta x_i. \tag{18}$$

Therefore, we have $w_{n,i} = (1-\eta)^{n-i}\eta, 1 \le i \le n$, $w_{n,n} = \eta$ and $W_n = W_{n-1} = 1$. Then, we have

$$\begin{aligned} S_n =& \frac{W_n - w_{n,n}}{W_{n-1}}(S_{n-1} + \frac{W_{n-1} w_{n,n}}{W_n - w_{n,n}}(y_n-\upsilon_n)(x_n-\mu_{n-1})) \\ =& (W_n - w_{n,n})(S_{n-1} + w_{n,n}(y_n-\upsilon_{n-1})(x_n-\mu_{n-1})). \end{aligned} \tag{19}$$

Since we have

$$Cov_n(x,y) = \frac{S_n}{W_n} = S_n, \tag{20}$$

the final simplified exponential moving covariance is

$$Cov_n(x,y) = (1-\eta)Cov_{n-1}(x,y) + \eta(1-\eta)(x_n - \mu_{n-1})(y_n - v_{n-1}). \tag{21}$$

When $n = 0$, we have $Cov_n(x,y) = 0$.

1.3 EWMA covariance estimator

Note that Eq. 15 leads to the EWMA covariance estimator as

$$\widetilde{Cov}_n(x,y) = (1-\eta)\widetilde{Cov}_{n-1}(x,y) + \eta(x_n - \mu_{n-1})(y_n - v_{n-1}) \tag{22}$$

where the prediction is done with

$$\widetilde{Cov}_n(x,y) \approx W_n E_n([x - \mu_{n-1}][y - v_{n-1}]) \tag{23}$$

while ignoring the contribution of all other three terms in Eq. 11. The formula we derived Eq.21 is exactly

$$Cov_n(x,y) = W_n E_n([x - \mu_n][y - v_n]) \tag{24}$$

2 COVARIANCE OF TWO BATCH MEANS

Let n pair of random variables X_i,Y_j sampled from two different distribution correlated only when $i = j$. And $\bar{X} = \frac{1}{n}\sum X_i$, and $\bar{Y} = \frac{1}{n}\sum Y_j$. Then:

$$Cov(\bar{X},\bar{Y}) = Cov(\frac{1}{n}\sum X_i, \frac{1}{n}\sum Y_j) \tag{25}$$

$$= \frac{1}{n^2}\sum_i \sum_j Cov(X_i, Y_j) \tag{26}$$

Since X_i,Y_j are correlated only when $i = j$,

$$Cov(\bar{X},\bar{Y}) = \frac{1}{n^{\cancel{2}}} \cdot \cancel{n} Cov(X,Y) \tag{27}$$

3 CV COEFFICIENT AND RESIDUAL FUNCTION SELECTION

In this section we provide an example of how we arrived at the CV coefficient the updating function, and the residual function for variance tracking if CV requires Monte Carlo sampling (Note that the purpose of having a variance estimation is for real-time adaptive sampling. The temporal variance should have minimal impact on it. Otherwise, the sample count will be greatly increased. Since it's real-time rendering, we also cannot afford a lot of textures to store history information to approximate best coefficient).

3.1 Scenario setup

We have a spatial-temporal variant function to integrate $f(y,t) = h(t) \cdot 1/(1+y)$, with the control variable as $g(y,t) = h(t) \cdot (1-y)$, $\mathcal{D} \in [0,1]$. Then $G(t) = \int_{\mathcal{D}} g(y,t)dy = 0.5 \cdot h(t)$. For simplicity, 100 frame times are performed and a fixed number of samples 16 spp are used for the correlated Monte Carlo sampling of f and g to get the estimation $\bar{f}(t)$ and $\bar{g}(t)$. The sample count is fixed to help understand how our method helps to mitigate temporal variance. We have the following 5 configurations for the residual and CV coefficient estimation.

(1) $Res_{t,\bar{f}}$. No CV is applied. The CV coefficient is 0. Therefore $Res_{t,\bar{f}} = \bar{f}(t)$. This configuration demonstrates the impact of temporal variance.
(2) $Res_{t,SF}$. We estimate the CV with single frame covariance between f and g. Then it is used as the CV coefficient $\alpha_{t-1,SF}$ for the next frame to calculate the residual as Res(t) = $\bar{f}(t)$-$\alpha_{t-1,SF} \cdot \bar{g}(t)$.
(3) $Res_{t,SF+EMA}$. After estimating $\alpha_{t-1,SF}$, exponential moving average is used to get a better estimation of the coefficient over time. Then $\alpha_{t-1,SF+EMA}$ are used to estimate the residual.
(4) $Res_{t,EMCM}$. The covariance matrix $\mathbf{\Sigma}_t$ between $\bar{f}(t)$ and $\bar{g}(t)$ is updated with EMCM over time with a weighting factor $\eta_{cov} = 0.05$. Then the CV coefficient is $\alpha_{t-1,EMCM}$ based on $\mathbf{\Sigma}_{t-1}$.
(5) $\widehat{Res}_{t,EMCM}$. The CV coefficient is the same as $\alpha_{t-1,EMCM}$. However, the residual is calculated as $\widehat{Res}_{t,EMCM} = \bar{f}(t) - \alpha_{t-1,EMCM}G(t)$. The reason we use $G(t)$ instead of $\bar{g}(y)$ is that CV are used to deal with temporal variance, but should not affect the variance estimation of spatial variance for adaptive sampling (please see the result section for a better understanding).

With this configuration, we monitor the exponential moving variance on the residual with $\eta_{res} = 0.2$. In the meanwhile, we also have the ground truth configuration Res_{GT}, which is to apply the optimal CV coefficient. The coefficient is estimated beforehand with large sample counts as ground truth.

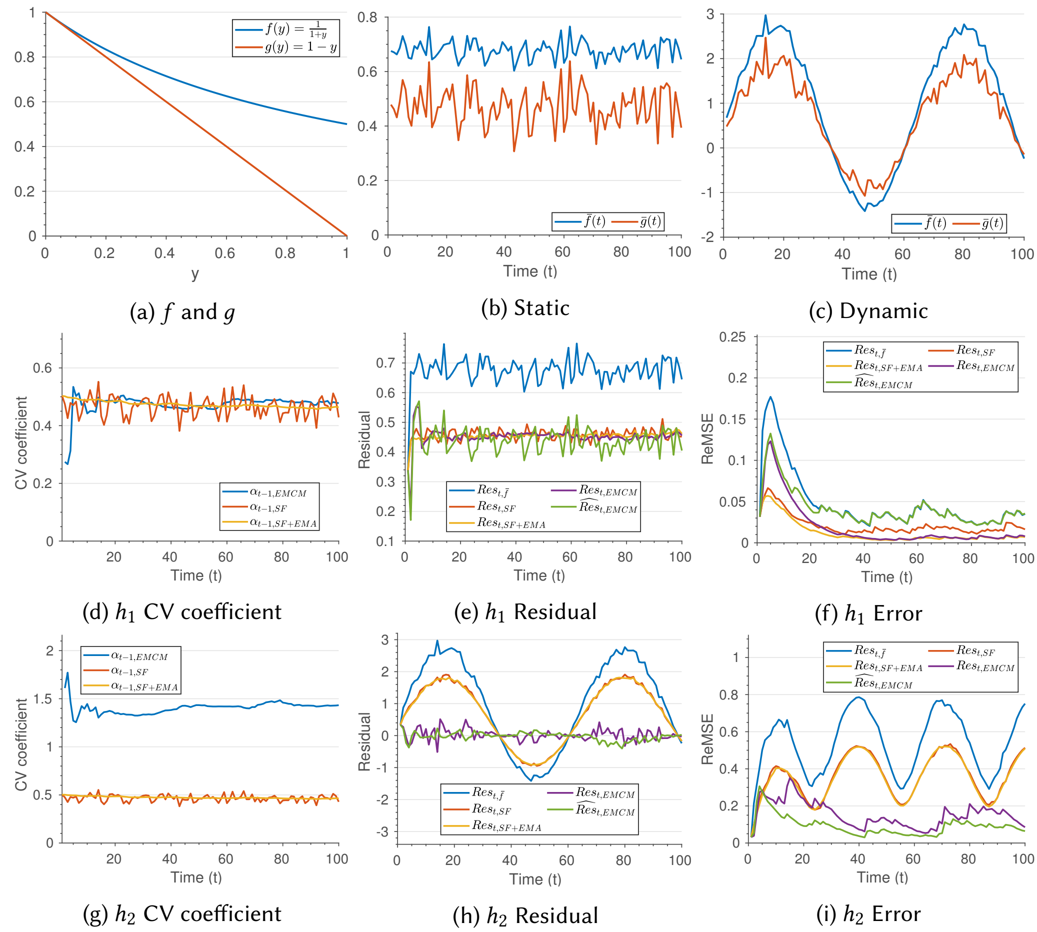
We use two temporal function, $f_1(y,t)$ with $h_1(t) = 1$ and $f_2(y,t)$ with $h_2(t) = 1 + 3 * sin(\frac{t}{10})$ to simulate both static and dynamic scenarios.

	$Res_{t,\bar{f}}$	$Res_{t,SF}$	$Res_{t,SF+EMA}$	$Res_{t,EMCM}$	$\widehat{Res}_{t,EMCM}$	Res_{GT}
Static ($h = h_1$)	0.0325	0.0160	0.0058	0.0070	**0.0324**	0.0055
Dynamic ($h = h_2$)	0.5585	0.3797	0.3742	0.1295	**0.0732**	0.1243

Table 1. The ReMSE of five configurations and the groud truth Res_{GT}. $\widehat{Res}_{t,EMCM}$ is the best choice based on our selection criteria: 1) Similar ReMSE to $Res_{t,\bar{f}}$ in static scene, 2)mitigating temporal variance in dynamic scene, yet the ReMSE is no less than the static scene counterpart of $\widehat{Res}_{t,EMCM}$ or $Res_{t,\bar{f}}$.

3.2 Result

Static scenarios. Fig. 1a shows the spatial function of f and g when h = h_1. Fig. 1b shows the corresponding MC sampling result at each frame time. Fig. 1d shows the estimated optimal coefficient for different configurations. The optimal coefficient based on a MC sampling of 100k samples has $\alpha^* \approx 0.477$. Fig. 1e shows the corresponding residual after applying CV. Fig. 1f shows the root exponential mean square error (ReMSE), which is applying a root operation on the exponential moving variance. Although it is straight forward that CV can reduce the variance estimation, however, please note that the reason we apply CV is to remove temporal dimension variance, it should not affect the spatial variance estimation, otherwise, we could underestimate the number of samples in static scenes. Since our application is in sample domain instead of shading domain, by utilizing a good configuration like $Res_{t,SF+EMA}$ or $Res_{t,EMCM}$ will lead to under-sampling in the shading domain as the constant term $\alpha \cdot G(t)$ is not added in shading domain. Because of this, $\widehat{Res}_{t,EMCM}$ is the best choice as it has almost the same ReMSE as $Res_{t,\bar{f}}$, which is not applying CV. Table 1 shows the average ReMSE of frame from 25 to 100 (to not count cold start).
Dynamic scenarios. With $h = h_2$, the problem is different as temporal variance is introduced. Fig. 1c shows the corresponding MC sampling result at each frame time. We want to minimize the temporal variance. The CV coefficient should be derived from the covariance matrix of temporal domain t instead of spatial domain y as $Cov(\bar{f}_2(t), \bar{g}_2(t))$. With 1024 samples per frame, we estimate



(a) f and g (b) Static (c) Dynamic

(d) h_1 CV coefficient (e) h_1 Residual (f) h_1 Error

(g) h_2 CV coefficient (h) h_2 Residual (i) h_2 Error

Fig. 1. The CV coefficient and variance under different configuration with 16 samples per frame time.

the optimal CV coefficient $\alpha^* \approx 1.388$. Fig. 1g shows the run-time estimation of the coefficient. Fig. 1h shows the residual of all configurations. Fig.1i shows the ReMSE. Here we could observe that the spatial coefficient $\alpha_{t-1,SF}$ and $\alpha_{t-1,SF+EMA}$ can reduce the variance but not the temporal variance. With $Res_{t,EMCM}$ and $\widehat{Res}_{t,EMCM}$, most of the temporal variance has been removed. Table 1 shows the mean ReMSE. It seems both are viable for our adaptive sampling purpose. However, $Res_{t,EMCM}$ causes under sampling during static scene. Therefore, the configuration that is suitable for our use is $\widehat{Res}_{t,EMCM}$.

In a summary, the CV coefficient is updated with the EMCM. In side the residual function, $G(t)$ instead of $\bar{g}(y)$ is selected to allow CV deal with temporal variance, but do not affect the estimation of spatial variance for adaptive sampling.

4 THEORETICAL FOUNDATION

In this section, we provide the theoretical foundation for:

(1) why the control variates updating function works based on EMCM.

(2) the relationship among CV coefficient, the expectation and variance of CV residual under different lighting condition.
(3) time-variant CV coefficient with in-frame constant control variable.

4.1 Updating function

Generally, if we have three random variables T, F and G, where $E(TG)$ is a known constant, then the unbiased Monte Carlo estimator for TF,

$$\langle TF \rangle = \alpha E(TG) + \langle TF - \alpha \cdot TG \rangle, \tag{28}$$

~~have~~ has the minimal variance based on the control variates concept where the optimal control variates coefficient is

$$\alpha^* = \frac{Cov(TF, TG)}{Var(TG)}, \tag{29}$$

which can also be derived with the partial derivates of $Var(\langle TF \rangle)$ according to α as

$$V[\langle TF \rangle_*] = Var(\langle TF \rangle) + \alpha^2 \cdot Var(\langle TG \rangle) - 2\alpha Cov(TF, TG) \tag{30}$$

In subsurface scattering, T is the temporal intensity changing term. TF is the joint random variable of temporal intensity and the subsurface scattering function, while TG the one of temporal intensity change and the pre-integration irradiance function. Please note that T might not be independent from F and G, and the point of interest is *per pixel*. Namely, T, F, and G are functions on pixel level. In the adaptive sampling framework, they are scalar.

4.1.1 Time-invariant Scenario. In time-invariant scenario, lighting intensity is temporally stable ($E(T)$ is constant and $Var(T) = 0$). Therefore, Eq. 29 is simplified to

$$\alpha_s^* = \frac{Cov(F, G)}{Var(G)}, \tag{31}$$

which indicates we can use any method to use temporal samples to improve the estimate of CV coefficient. Note that Eq. 31 and Eq. 29 is only true when E(TG) can be derived analytically. Otherwise, the formulation is more complex [Rousselle et al. 2016]. As illustrated in our experimental example shown in Fig. 1d, applying EMCM to estimate CV coefficient ($\alpha_{t-1,EMCM}$) is equivalent to applying EMA on single frame estimation ($\alpha_{t-1,SF+EMA}$). They all try to converge to $\alpha^* \approx 0.477$.

4.1.2 Time-variant Scenario. However, when the lighting is not temporally stable, the optimal stable CV coefficient by Eq. 31 does not hold as a valid alternative as α^*. We have to fall back to the original Eq. 29 instead. The temporal information in T has to be used for a good estimation. This is why $\alpha_{t-1,SF+EMA}$ fails the purpose. What it does is to estimate the in-frame CV coefficient α_s^*. Because inside each frame, the time-invariant property holds. It only uses temporal history to improve the α_s^* estimation instead of considering the temporal random variable T, the temporal unstable intensity. Due to the time-variant feature of T, we are unable to know the future of $E(TG)$ in real-time rendering beforehand. However EMCM provides the capability to focus most recent changes to estimate temporal unstable CV coefficient, leading to a different coefficient. For example, the one as shown in Fig. 1g ($\alpha^* \approx 1.388$), where $\int_{\mathcal{D}} \frac{1}{1+y} - \alpha^*(1-y)dy = -8.53 \times 10^{-4}$, and the variance remains resistant to temporal change (Fig. 1i).

4.2 CV Coefficient, Mean and Variance of CV Residual

To provide a better understanding of the effect of an additional temporal term on CV (the relationship among α^*, the variance and mean of CV residual), we provide an analysis of a general case in real-time rendering where T is independent from F and G. This assumption is universal in real-time rendering where nearly all types of lights have an intensity term and there is one major light changing intensity in subsurface scattering range. As long as it is still subsurface scattering, the intensity change will not cause the change of the subsurface scattering and the pre-integrated irradiance function when the history is properly projected.

Generally in statistics, if we have three random variables T, F, G and a constant parameter α, where T is independent from both F and G. We then have the variance function for the residual as

$$Var(TF - \alpha TG) = Var(T(F - \alpha G)) \tag{32}$$

$$= E(T)^2 Var(F - \alpha G) + Var(T) Var(F - \alpha G) + Var(T) E^2(F - \alpha G) \tag{33}$$

4.2.1 Stable lighting. During stable lighting ($E(T)^2 \gg Var(T)$), the intensity $E(T)$ becomes constant and the variance term zero. Then Eq. 33 simplifies to

$$Var_{E(T)^2 \gg Var(T)}(TF - \alpha TG) = E(T)^2 Var(F - \alpha G) \tag{34}$$

where the only variance contribution is $Var(F - \alpha G)$. Note that the time-invariant part of CV residual, $E(F - \alpha G)$, is masked by $Var(T) = 0$. Therefore normally, CV coefficient does not care the expectation of the residual $E(F - \alpha G)$. It can be any value. The CV coefficient is just to reduce the variance part $Var(F - \alpha G)$ of the residual.

4.2.2 Dynamic lighting. With T being time-variant. $Var(T)$ and $E(T)^2$ can be both non-zero and uncontrollable by our algorithm. The only controllable terms in our algorithm are the expectation and variance of the redisual term, $Var(F - \alpha G)$ and $E(F - \alpha G)$. Moreover, the variance term $Var(T)$ could be *arbitrarily large* in game like a continuous gunshots, lightning, and rhythmic lighting in nightclub ($Var(T) \gg E(T)^2$), then

$$V_{Var(T) \gg E(T)^2}(TF - \alpha TG) = Var(T)(Var(F - \alpha G) + E(F - \alpha G)^2) \tag{35}$$

$$= Var(T) E((F - \alpha G)^2) \tag{36}$$

Since Eq. 30 derived the optimal CV coefficient solution to minimize the total variance analytically, an alternative representattion based on Eq. 36 is

$$\alpha^* = \operatorname*{argmin}_{\alpha} E((F - \alpha G)^2), \tag{37}$$

where $Var(T)$ is removed as it does not affect the optimization. This formulation is least square estimation. When $\alpha = \alpha^*$, we have $E((F - \alpha G)^2)$ minimized.

4.3 In-frame constant control variable

We try to resolve the temporal stability issue in *real-time* adaptive sampling. Therefore being real-time is important. We cannot afford samples to sample both TF and TG random variables if they all need to access memory, especially if the memory access pattern is incoherent. Monte Carlo sampling makes it worse. Because of this, TG is accessed as cheap as possible without Monte Carlo sampling, we use the analytic solution (e.g., $G(t) = 0.5 \cdot h(t)$). Namely, we have the following assumption :

$$TG = E(G)T \text{ and } Var(G) = 0. \tag{38}$$

4.3.1 Stable lighting. With the given assumption in Eq. 38, the time-invariant CV coefficient updating formula (Eq. 31) does not hold anymore and the estimator is

$$\langle TF \rangle = TF - \alpha \cdot \cancel{(TG - E(TG))}, \tag{39}$$

where α is not meaningful and can be any number. If T is still independent, the residual variance is

$$Var(TF - \alpha TG) = Var(\langle TF \rangle) = E(T)^2 Var(F). \tag{40}$$

4.3.2 Dynamic lighting. During dynamic lighting, the optimal CV coefficient Eq. 29 simplifies to

$$\alpha^* = \frac{Cov(TF, T)}{E(G)Var(T)}, \tag{41}$$

which means, even with a constant in-frame control variable, CV is still valid and working in temporal domain. This is key to resolve the temporal stability issue in real-time adaptive sampling.

T is independent. If we can further assume that F and T are independent, Eq. 41 can be as simple as

$$\alpha^* = \frac{E(F)}{E(G)}. \tag{42}$$

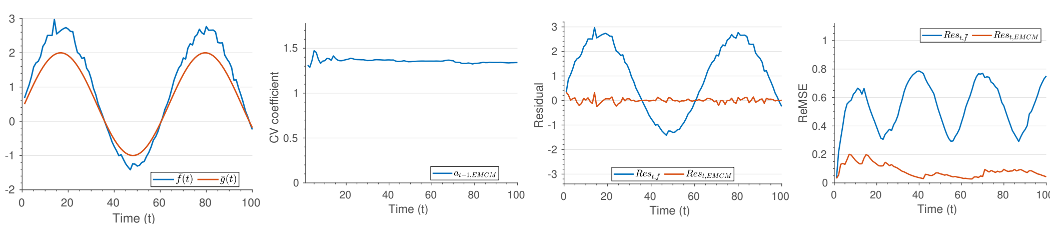
With the optimal CV coefficient, the time-invariant part of the residual becomes zero as

$$E(F - \alpha^* G) = 0. \tag{43}$$

More specifically, $\int f(y) - \alpha^* g(y) dy = 0$, and the variance Eq. 33 is simplified to

$$Var(TF - \alpha TG) = (Var(T) + E(T)^2)Var(F). \tag{44}$$

To help the understanding of this concept, we repeated the same experiment shown in Section 3.1. In this example the F random variable $f(y) = \frac{1}{1+y}$ and the G random varriable $g(y) = 1 - y$ is independent from the time-variant variable T, $h_2(t)$. The in-frame constant control variate uses $TG = E(G)T = 0.5 \cdot h_2(t)$ directly.



(a) Dynamic (b) h_2 CV coefficient (c) h_2 Residual (d) h_2 Error

Fig. 2. The CV coefficient and variance with time independent in-frame constant control variable.

Fig.2 shows the corresponding effect on temporal variance when in-frame constant control variable is used. Based on Eq.42, $\alpha^* \approx 1.386$.

5 ADDITIONAL IMAGES

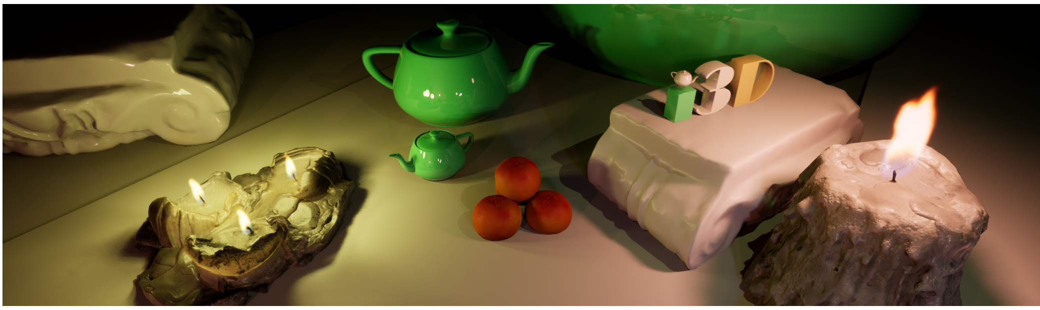
Fig. 3 shows a qualitative comparison between with no subsurface scattering, Separable [Jimenez et al. 2015] and ours.

REFERENCES

Tony Finch. 2009. Incremental calculation of weighted mean and variance. *University of Cambridge* 4, 11-5 (2009), 41–42.

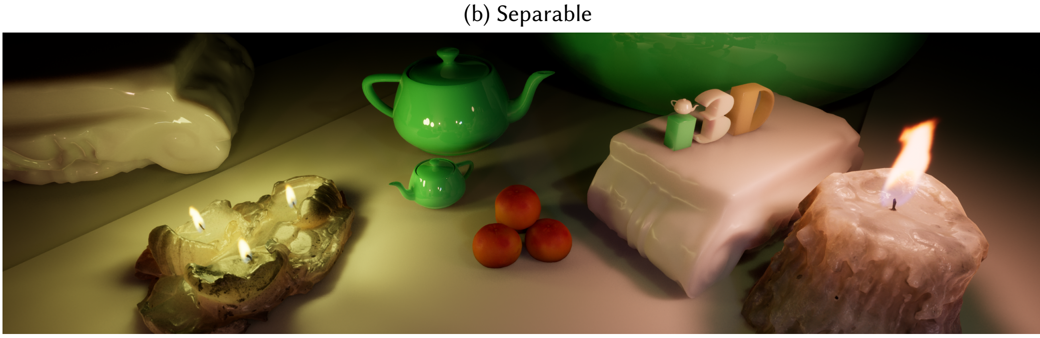
Jorge Jimenez, Károly Zsolnai, Adrian Jarabo, Christian Freude, Thomas Auzinger, Xian-Chun Wu, Javier von der Pahlen, Michael Wimmer, and Diego Gutierrez. 2015. Separable Subsurface Scattering. *Computer Graphics Forum* 34, 6 (Sept. 2015), 188–197.

Fabrice Rousselle, Wojciech Jarosz, and Jan Novák. 2016. Image-space control variates for rendering. *ACM Transactions on Graphics (TOG)* 35, 6 (2016), 1–12.



(a) Without subsurface scattering

(b) Separable



(c) Ours

Fig. 3. Additional qualitative comparison between without subsurface scattering, Separable and Ours.