Homework for Chapter 11

蒋翌坤 20307100013

11.6

Using software and GEE method, the model is logit $\left[P(Y_{i(k)t})=1\right]=\alpha+\alpha_k+\beta_t$ with $\alpha_1=0$ and $\beta_A=0$

we have the estimates: $\hat{\alpha}=0.33, \hat{\alpha}_1=0.00, \hat{\alpha}_2=0.20, \hat{\alpha}_3=0.09, \hat{\alpha}_4=0.05, \hat{\alpha}_5=-0.22, \hat{\alpha}_6=-0.41, \hat{\beta}_A=0.00, \hat{\beta}_B=-0.10, \hat{\beta}_C=0.35$

We can see that $\hat{\beta}_B<\hat{\beta}_A<\hat{\beta}_C$, so high-dose analgesic has the best treatment effects and low-dose analgesic has the worst treatment effects.

11.33

(a)

The loglinear model (XY, XZ) is in the following form:

$$\log \mu_{ij}(t)=\lambda+\lambda_i^X+\lambda_j^Y+\lambda_t^Z+\lambda_{ij}^{XY}+\lambda_{it}^{XZ}$$

We have $\pi_{j|i}(t)=\frac{\mu_{ij}(t)}{\mu_{i+}(t)}=\frac{\exp\left\{\lambda+\lambda_i^X+\lambda_j^Y+\lambda_t^Z+\lambda_{ij}^{XY}+\lambda_{it}^{XZ}\right\}}{\exp\left\{\lambda+\lambda_i^X+\lambda_t^Z+\lambda_{it}^{XZ}\right\}\sum_j\exp\left\{\lambda_j^Y+\lambda_{ij}^{XY}\right\}}=\frac{\exp\left\{\lambda_j^Y+\lambda_{ij}^{XY}\right\}}{\sum_j\exp\left\{\lambda_j^Y+\lambda_{ij}^{XY}\right\}}$

This does not depend on t, so all transition probabilities are stationary.

(b)

Let the likelihood be

$$L=\left(\prod_{i=1}^{I}\pi_{i0}^{n_{i0}}\right)\left\{\prod_{t=1}^{T}\prod_{i=1}^{I}\left[\prod_{j=1}^{I}\pi_{j|i}(t)^{n_{ij}(t)}\right]\right\}$$

Then,

$$\log L=\sum_{i=1}^{I}n_{i0}\log\pi_{i0}+\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{I}n_{ij}(t)\log\pi_{j|i}=\sum_{i=1}^{I}n_{i0}\log\pi_{i0}+\sum_{i=1}^{I}\sum_{j=1}^{I}n_{ij}\log\pi_{j|i}$$

Note that $\pi_{I|i}=1-\sum_{j=1}^{I-1}\pi_{j|i}$, so for $j\neq I$, we have $\frac{\partial\log L}{\partial\pi_{j|i}}=\frac{n_{ij}}{\pi_{j|i}}-\frac{n_{iI}}{\pi_{I|i}}=0\Rightarrow\hat{\pi}_{j|i}=\frac{n_{ij}}{n_{iI}}\hat{\pi}_{I|i}$

Also note that $\sum_{j=1}^{I}\hat{\pi}_{j|i}=1$, so we have $\hat{\pi}_{I|i}=\frac{n_{iI}}{n_{i+}}$ and $\hat{\pi}_{j|i}=\frac{n_{ij}}{n_{i+}}, i,j=1,\cdots,I$

(c)

We have shown that loglinear model (Y_1Y_2, Y_2Y_3) is equivalent to first order Markov chain. Note that the loglinear model $(Y_1Y_2Y_3)$ is eqivalent to second order Markov chain. So the goodness of fit test is eqivalent to testing first order Markov chain against second order Markov chain.