Programming Languages and Computation

Week 11: The Halting Problem & Reductions

* 1. (Trick question.) Is it decidable whether God exists?

** 2. Is the predicate

$$\mathsf{LUCKY}_{127} = \{\ \ulcorner S \urcorner \mid \text{ running } S \text{ on input 1 runs for at least 127 computational steps }\}$$

decidable? [Hint: if it is, describe a program that decides it. Think simply, write informally, and do not let the syntactic poverty of While confine you.]

** 3. Prove that the set

$$\mathsf{Zero} = \{\ \ulcorner S \urcorner \mid [\![\mathtt{S}]\!]_{\mathtt{x}}(0)\downarrow\}$$

is semi-decidable. [Hint: As above, think simply, write informally, and do not let the syntactic poverty of While confine you.]

*** 4. Prove that if the predicates U and V are semi-decidable, then so is $U \cup V$. [Hint: use simulations.]

*** 5. Suppose we have a way of encoding every DFA M as a natural number $\delta(M) \in \mathbb{N}$.
Is the predicate

$$\mathsf{EMPTY} = \{\delta(M) \mid L(M) = \emptyset\}$$

decidable? [Hint: if it is, describe a program that decides it. Think simply, write informally, and do not let the syntactic poverty of While confine you.]

** 6. Show that if $f : U \lesssim V$ and $g : V \lesssim W$ then $g \circ f : U \lesssim W$.

**** 7. Prove that the set

$$\mathsf{Zero} = \{\ \ulcorner S \urcorner \mid [\![\mathtt{S}]\!]_{\mathtt{x}}(0)\downarrow\}$$

is undecidable by reduction from HALT.

**** 8. [Trick question.] Is the predicate

$$V = \{\ulcorner S \urcorner \mid \forall n \in \mathbb{N}.\ [\![\mathtt{S}]\!]_{\mathtt{x}}(n)\downarrow\}$$

from the last lecture semi-decidable? Why or why not? Discuss only, do not prove.