FINA4380 Stock Return Model

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1 Our model

We model return of stock i as

$$r_{it} = \beta_{i0t} + \beta_{i1t}PC_{1t} + \beta_{i2t}PC_{2t} + ... + \beta_{int}PC_{nt} + \varepsilon_{it}$$

where PC_i is the value of the ith principal component at time t obtained by PCA, β_{it} are state variables that follow random walk and we assume that $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_i^2)$ and $Cov(\varepsilon_{it}, \varepsilon_{jt}) = 0$ for $i \neq j$.

2 Principal Components

We select the first n (not necessarily 5) principal components such that the proportion of variance explained exceeds 0.8.
We assume that

$$PC_{it} = \mu + \sum_{j=1}^{p} a_{ij} y_{t-j} + \sum_{j=1}^{q} b_{ij} \varepsilon_{t-j} + \varepsilon_{it}$$

$$\varepsilon_{it} = \sigma_{it}\epsilon_{it}, \quad \sigma_{it}^2 = \alpha_{i0} + \alpha_{i1}\varepsilon_{i,t-1}^2 + \beta_{i1}\sigma_{i,t-1}^2$$

$$\epsilon_{it} \overset{iid}{\sim} \mathcal{N}(0,1)$$

And the correlation matrix between error terms (and hence stock return) is given by DCC.

3 Betas

We assume that betas change over time and can be modeled by

$$\beta_{it} = I\beta_{it} + \epsilon_{it}$$

$$where \quad \epsilon_{it} \sim N(0, \mathbf{Q})$$

We obtain the forecast for β and $\mathbf{Q}$ by Kalman filter and for simplicity we assume that $Cov(\beta_{it}, \beta_{jt} = \mathbf{0})$

4 Residual of Returns

We use GARCH(1, 1) model to estimate the variance of residual of returns ε_{it}. i.e.,

$$\varepsilon_{it} = \sigma_{it}\epsilon_{it}, \quad \sigma_{it}^2 = \alpha_{i0} + \alpha_{i1}\varepsilon_{i,t-1}^2 + \beta_{i1}\sigma_{i,t-1}^2$$

$$\epsilon_{it} \overset{iid}{\sim} \mathcal{N}(0,1)$$

and the variance is given by σ_{it}^2.

5 Covariance matrix of stock return

We have

$$\begin{aligned}
Var(r_p) = Var(\beta_{p0t}) \\
&+ 2\sum_{i=1}^{n} \mathbb{E}(PC_{it})Cov(\beta_{p0t}, \beta_{pit}) \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{n}(\mathbb{E}(\beta_{pit})\mathbb{E}(\beta_{pjt})Cov(PC_{it}, PC_{jt}) \\
&\qquad + \mathbb{E}(PC_{it})\mathbb{E}(PC_{jt})Cov(\beta_{pit}, \beta_{pjt}) \\
&\qquad + Cov(\beta_{p0t}, \beta_{pit})Cov(PC_{it}, PC_{jt})
\end{aligned}$$

And for all $p \neq q$,

$$Cov(r_p, r_q) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathbb{E}(\beta_{pit})\mathbb{E}(\beta_{qjt})Cov(PC_{it}, PC_{jt})$$