## FINA4380 Stock Return Model

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### 1 Our model

We model return of stock i as

$$r_i = \beta_{i0} + \beta_{i1}PC_1 + \beta_{i2}PC_2 + \beta_{i3}PC_3 + \beta_{i4}PC_4 + \beta_{i5}PC_5 + \varepsilon_t$$

where  $PC_i$  is the ith principal component obtained by PCA and  $\beta_i$  is state variable that follows random walk.

## 2 Principal Components

We select the first n (not necessarily 5) principal components such that the proportion of variance explained exceeds 0.9. We assume that

$$y_{it} = \mu + \sum_{i=1}^{p} a_{ij} y_{t-j} + \sum_{i=1}^{q} b_{ij} \varepsilon_{t-j} + \varepsilon_{it}$$

$$\varepsilon_{it} = \sigma_{it}\epsilon_{it}, \quad \sigma_{it}^2 = \alpha_{i0} + \sum_{j=1}^m \alpha_{ij}\varepsilon_{it}^2 + \sum_{j=1}^s \beta_{ij}\sigma_{i,t-j}^2$$

epsilon

And the correlation matrix between error terms (and hence stock return) is given by DCC.

#### 3 Betas

We assume that betas change over time and can be modeled by

$$\beta_{\mathbf{t}} = I\beta_{\mathbf{t}} + \epsilon_{\mathbf{t}}$$

where 
$$\epsilon_{\mathbf{t}} \sim N(0, \mathbf{Q})$$

We obtain the forecast for  $\beta$  and  $\mathbf{Q}$  by Kalman filter.

# 4 Covariance matrix of stock return

when m = n,

Cov