

FINA4380 Stock Return Model

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1 Our model

We model return of stock i as

$$r_{it} = \beta_{i0t} + \beta_{i1t}PC_{1t} + \beta_{i2t}PC_{2t} + \dots + \beta_{int}PC_{nt} + \varepsilon_{it}$$

where PC_i is the value of the i th principal component at time t obtained by PCA, β_{it} are state variables that follow random walk and we assume that $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_i^2)$ and $Cov(\varepsilon_{it}, \varepsilon_{jt}) = 0$ for $i \neq j$.

2 Principal Components

We select the first n (not necessarily 5) principal components such that the proportion of variance explained exceeds 0.8.

We assume that

$$PC_{it} = \mu + \sum_{j=1}^p a_{ij}y_{t-j} + \sum_{j=1}^q b_{ij}\varepsilon_{t-j} + \varepsilon_{it}$$
$$\varepsilon_{it} = \sigma_{it}\epsilon_{it}, \quad \sigma_{it}^2 = \alpha_{i0} + \alpha_{i1}\varepsilon_{i,t-1}^2 + \beta_{i1}\sigma_{i,t-1}^2$$
$$\epsilon_{it} \stackrel{iid}{\sim} \mathcal{N}(0, 1)$$

And the correlation matrix between error terms (and hence stock return) is given by DCC.

3 Betas

We assume that betas change over time and can be modeled by

$$\beta_{it} = I\beta_{it} + \epsilon_{it}$$

$$\text{where } \epsilon_{it} \sim N(0, \mathbf{Q})$$

We obtain the forecast for β and \mathbf{Q} by Kalman filter and for simplicity we assume that $Cov(\beta_{it}, \beta_{jt}) = \mathbf{0}$

4 Residual of Returns

We use GARCH(1, 1) model to estimate the variance of residual of returns ε_{it} .
i.e.,

$$\varepsilon_{it} = \sigma_{it}\epsilon_{it}, \quad \sigma_{it}^2 = \alpha_{i0} + \alpha_{i1}\varepsilon_{i,t-1}^2 + \beta_{i1}\sigma_{i,t-1}^2$$

$$\epsilon_{it} \stackrel{iid}{\sim} \mathcal{N}(0, 1)$$

and the variance is given by σ_{it}^2 .

5 Covariance matrix of stock return

We have

$$\begin{aligned} Var(r_{pt}) &= Var(\beta_{p0t}) \\ &+ 2 \sum_{i=1}^n \mathbb{E}(PC_{it}) Cov(\beta_{p0t}, \beta_{pit}) \\ &+ \sum_{i=1}^n \sum_{j=1}^n (\mathbb{E}(\beta_{pit}) \mathbb{E}(\beta_{pjt}) Cov(PC_{it}, PC_{jt}) \\ &\quad + \mathbb{E}(PC_{it}) \mathbb{E}(PC_{jt}) Cov(\beta_{pit}, \beta_{pjt}) \\ &\quad + Cov(\beta_{p0t}, \beta_{pit}) Cov(PC_{it}, PC_{jt})) \\ &+ Var(\varepsilon_{pt}) \end{aligned}$$

And for all $p \neq q$,

$$Cov(r_{pt}, r_{qt}) = \sum_{i=1}^n \sum_{j=1}^n \mathbb{E}(\beta_{pit}) \mathbb{E}(\beta_{qjt}) Cov(PC_{it}, PC_{jt})$$