

# Metropolis-Hastings Sampling

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## Example 1

### Define the Sampling Algorithm

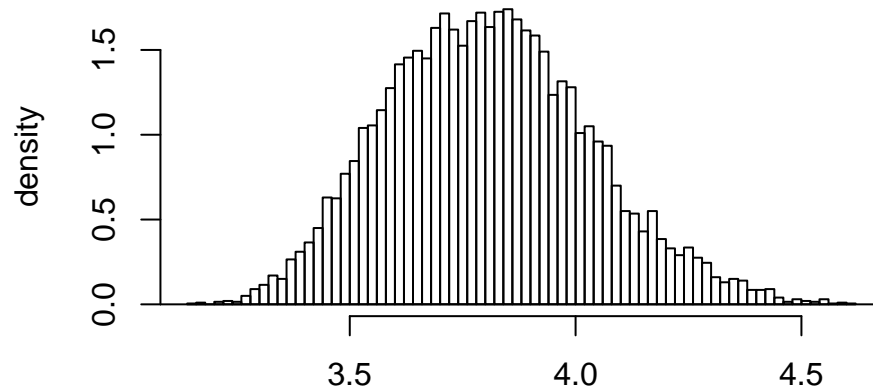
```
target <- function(x){
  sin(x)^2*sin(2*x)^2*dnorm(x)}
metropolis <- function(x, alpha=1, accept=1){
  y = runif(1, x-alpha, x+alpha)
  if (runif(1) > target(y)/target(x)){y = x
    accept = 0}
  return(list('y'=y, 'accept'=accept))}

sammpole_metropolis <- function(T, sample_fun, alpha=1){
  x = rep(3.14,T)
  accept = 0
  for (t in 2:T) {
    out = sample_fun(x[t-1], alpha=alpha)
    x[t] = out$y
    accept = accept+out$accept
  }
  return(list('chain'=x, 'rate'=accept/T))
}
```

e.g. I:  $\alpha=0.1$

```
N = 1e4
out = sammpole_metropolis(N, metropolis, alpha=0.1)
# Sampling results histogram
hist(as.matrix(out$chain), breaks = 100, freq=FALSE,
     main=paste('Alpha: 0.1, Accept Rate:', out$rate), ylab='density', xlab='')
# Approximate original distribution
d = 0.05
x = seq(-5, 5, by=d)
y = x
for (i in seq(length(x))) {y[i]=target(x[i])}
y = y/sum(y*d)
lines(x, y, type='l')
```

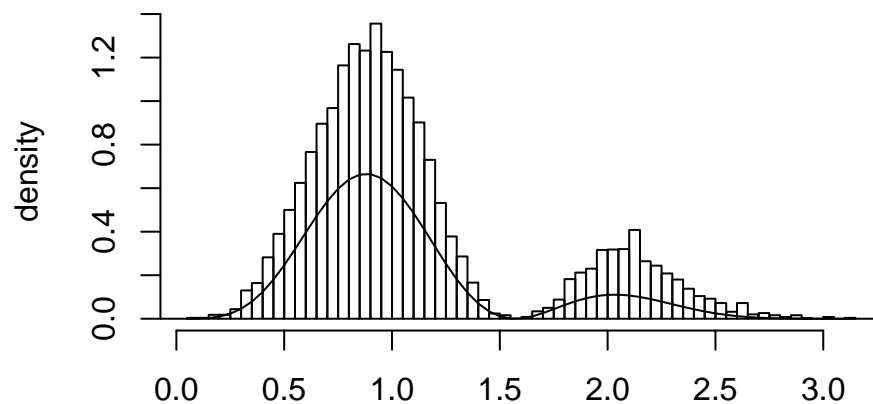
**Alpha: 0.1, Accept Rate: 0.919**



e.g. II:  $\alpha=0.2$

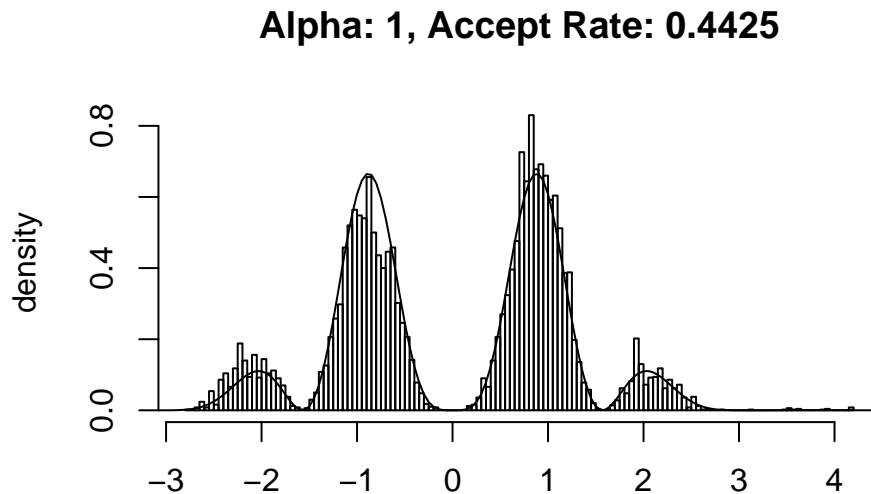
```
out = sammpole_metropolis(N, metropolis, alpha=0.2)
# Sampling results histgraph
hist(as.matrix(out$chain), breaks = 100, freq=FALSE,
     main=paste('Alpha: 0.2, Accept Rate:', out$rate), ylab='density', xlab='')
# Approximate original distribution
lines(x, y, type='l')
```

**Alpha: 0.2, Accept Rate: 0.8499**



e.g. III:  $\alpha=1$

```
out = sammpole_metropolis(N, metropolis, alpha=1)
# Sampling results histgraph
hist(as.matrix(out$chain), breaks = 100, freq=FALSE,
     main=paste('Alpha: 1, Accept Rate:', out$rate), ylab='density', xlab='')
# Approximate original distribution
lines(x, y, type='l')
```



e.g. IV:  $\alpha=10$

```
out = sammpole_metropolis(N, metropolis, alpha=10)
# Sampling results histgraph
hist(as.matrix(out$chain), breaks = 100, freq=FALSE,
     main=paste('Alpha: 10, Accept Rate:', out$rate), ylab='density', xlab='')
# Approximate original distribution
lines(x, y, type='l')
```

**Alpha: 10, Accept Rate: 0.11**

