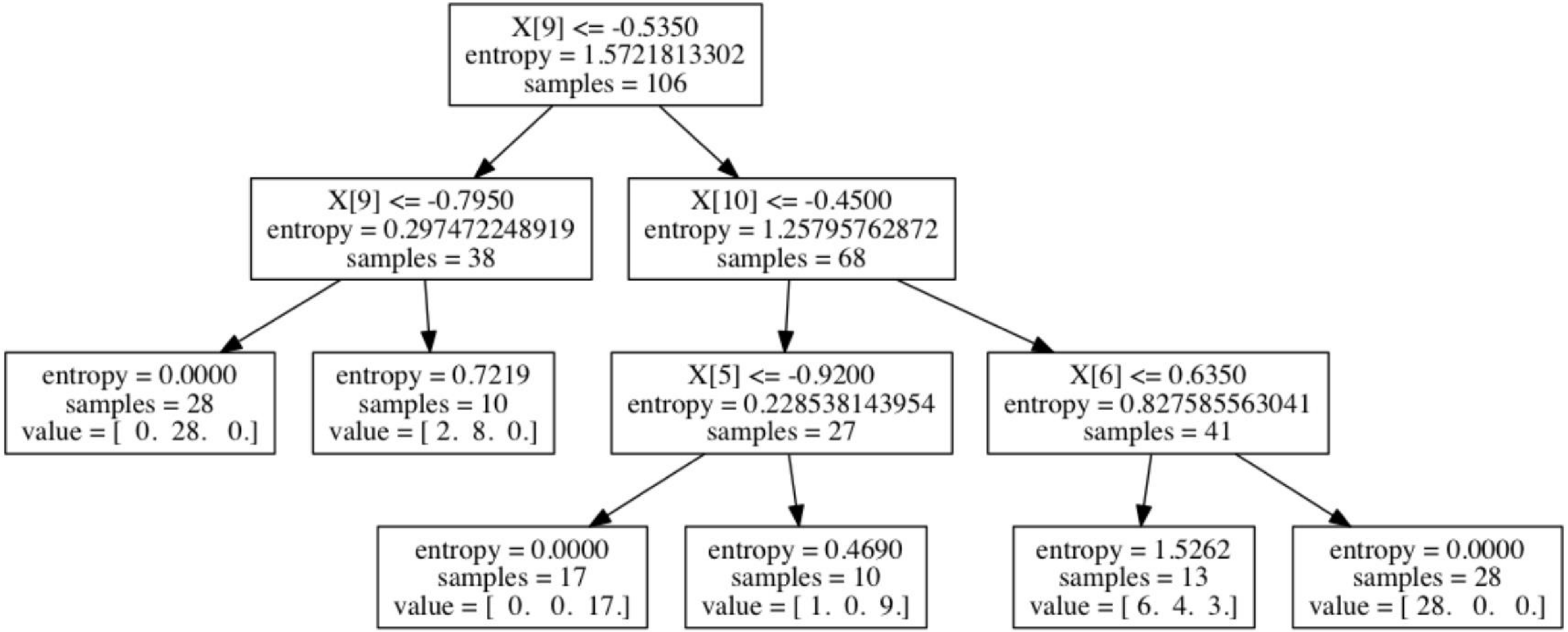
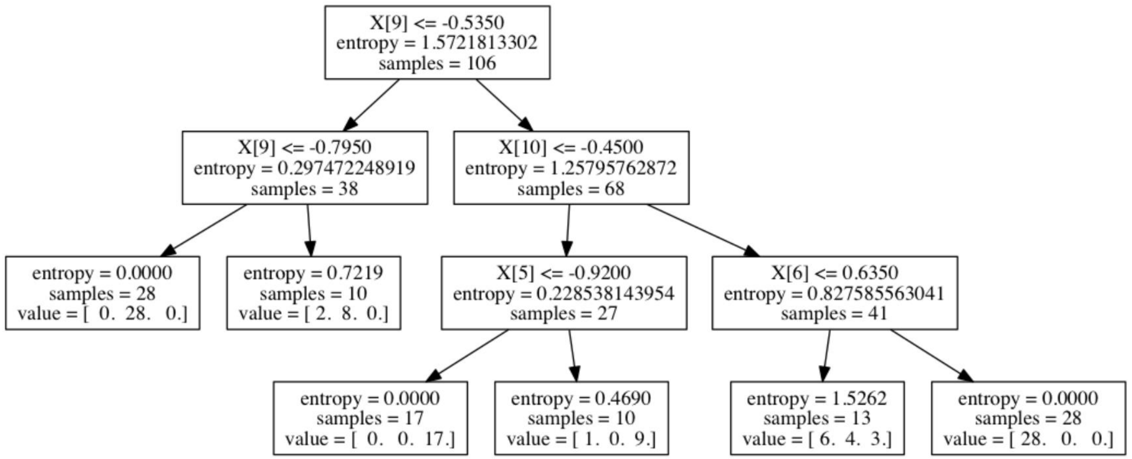
decision tree models

$$G(D,x) = \mathrm{Entropy}(D) - \sum_{v \in x} \frac{|D_v|}{|D|} \mathrm{Entropy}(D_v)$$



decision tree models



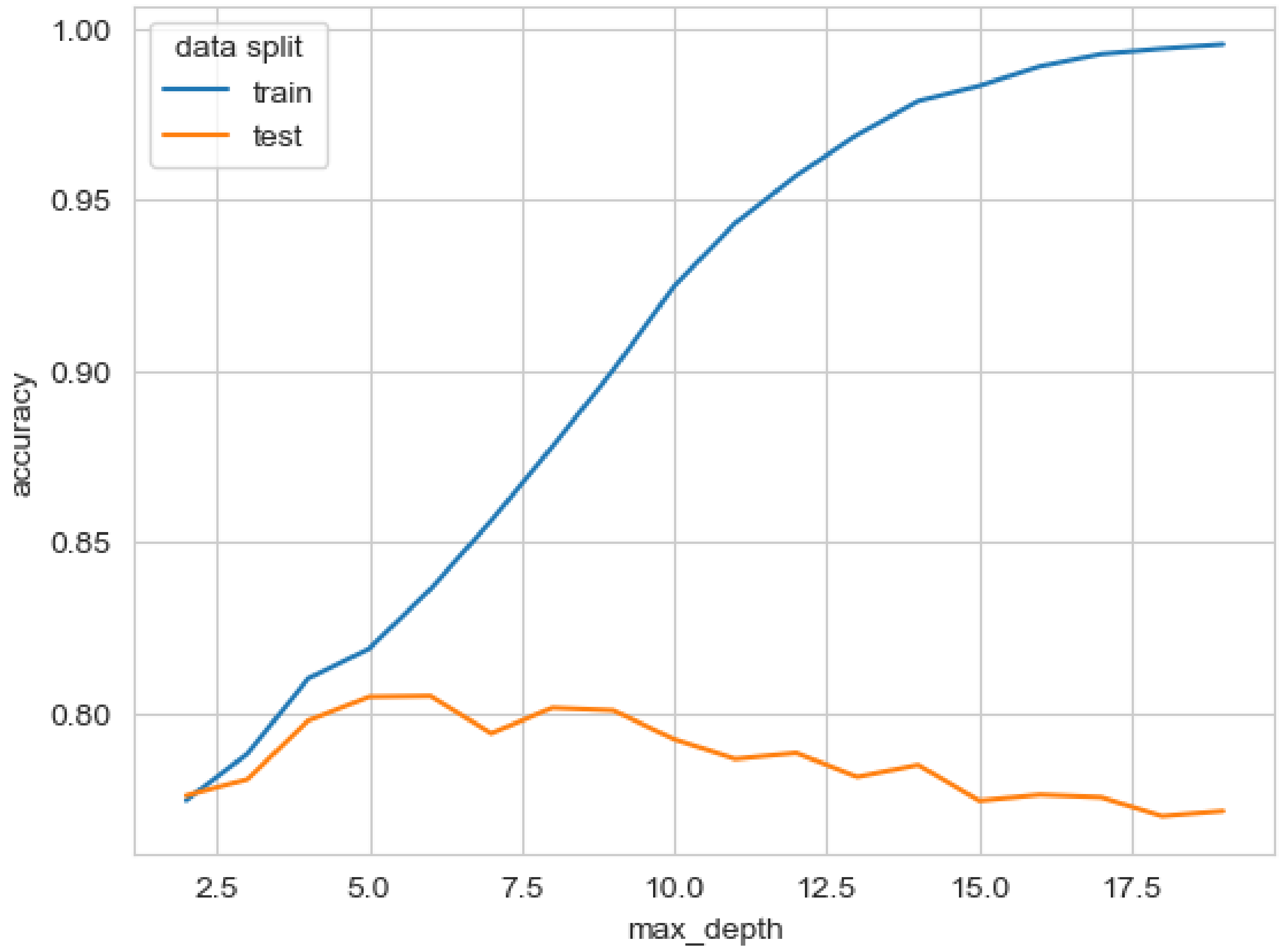
advantages:

ease of interpretation
handles continuous and discrete features
invariant to monotone transformation of features
variable selection automated
low bias (deep trees)

disadvantages:

high variance
overfitting

model selection

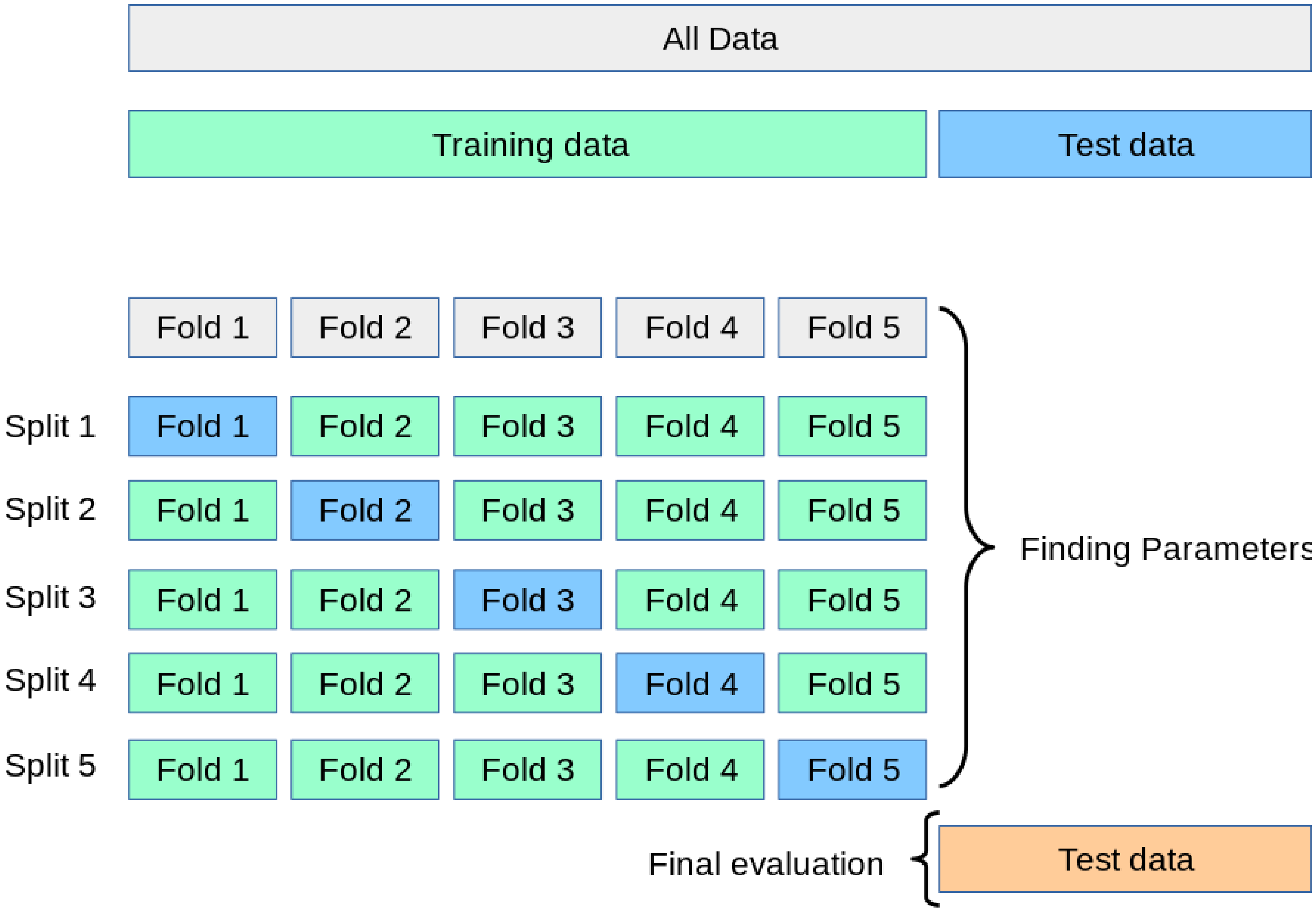


k-fold cross-validation

StatQuest,

https://www.youtube.com/@statquest

5-fold cross-validation

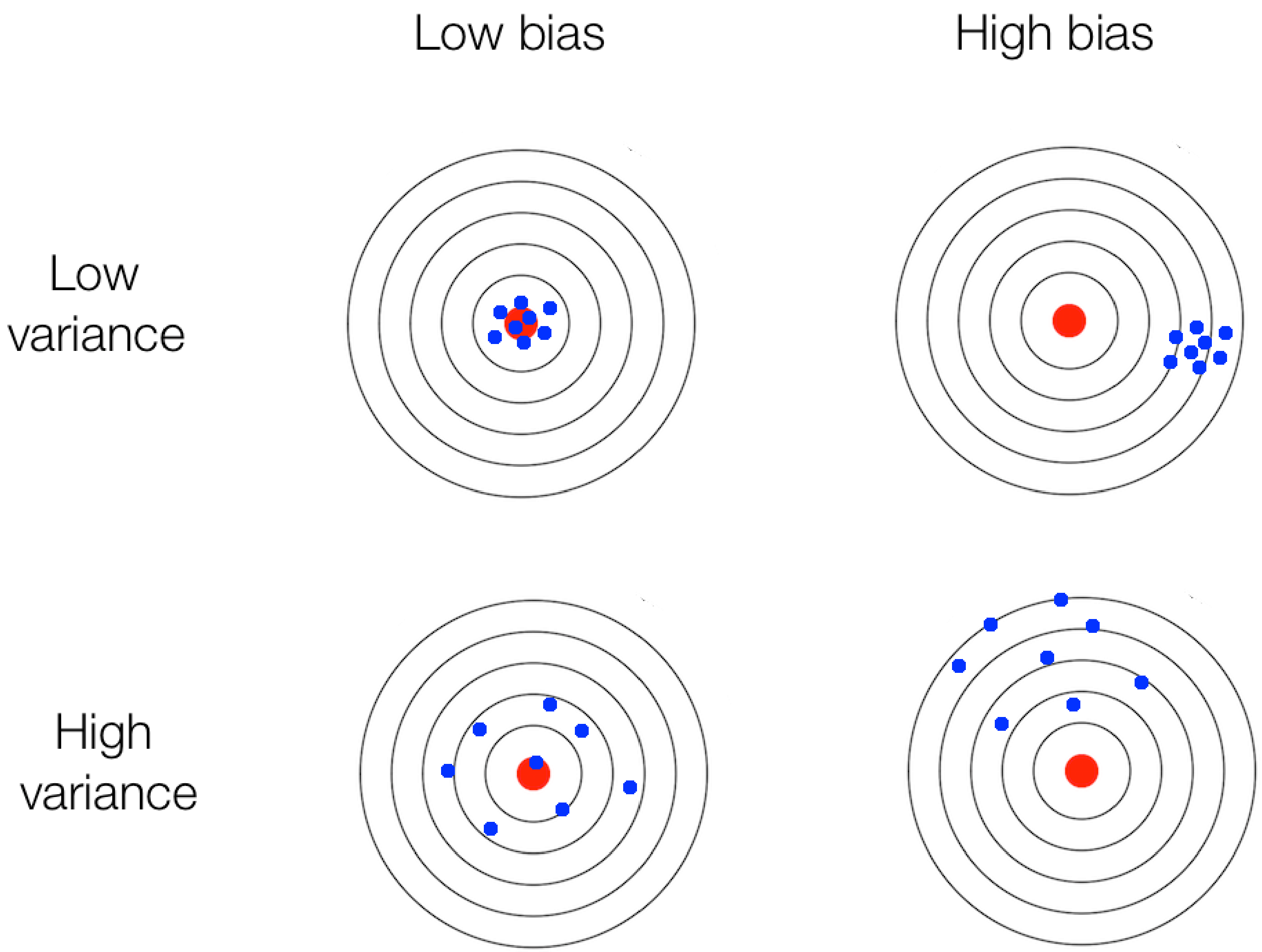


bias <> variance

Error due to bias

Error due to variance

Irreducible error



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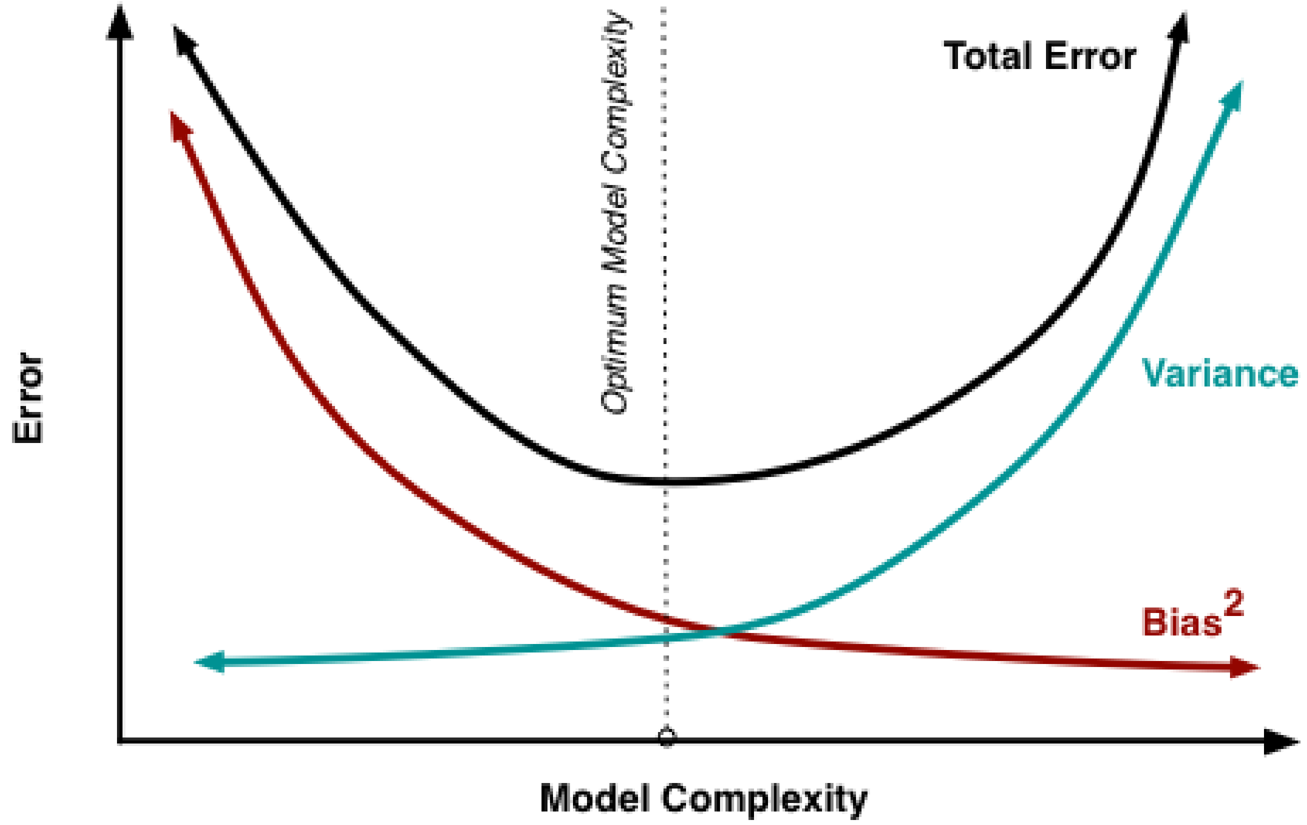
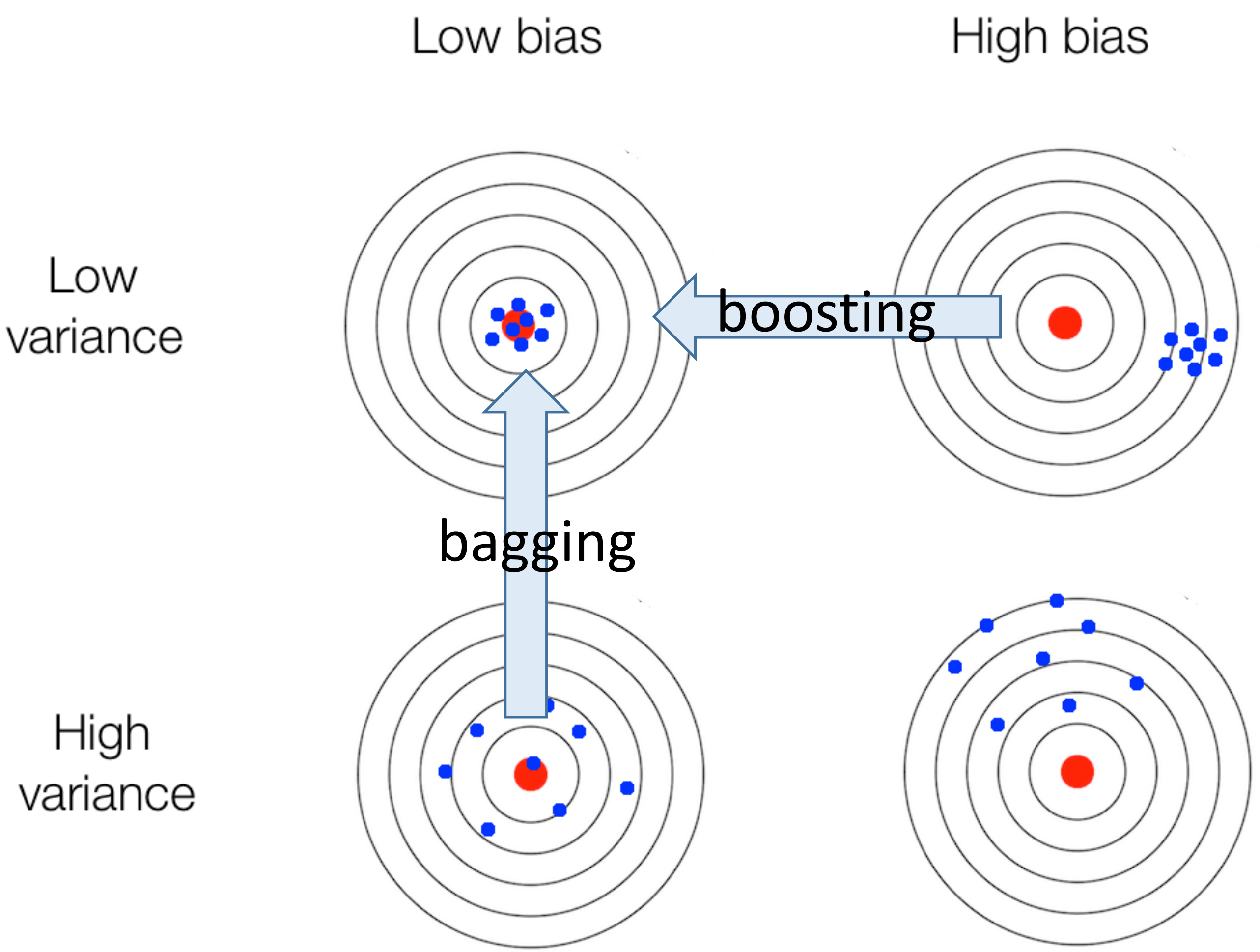


bagging <> boosting



bagging

- Simulate the notion of different train set samples:

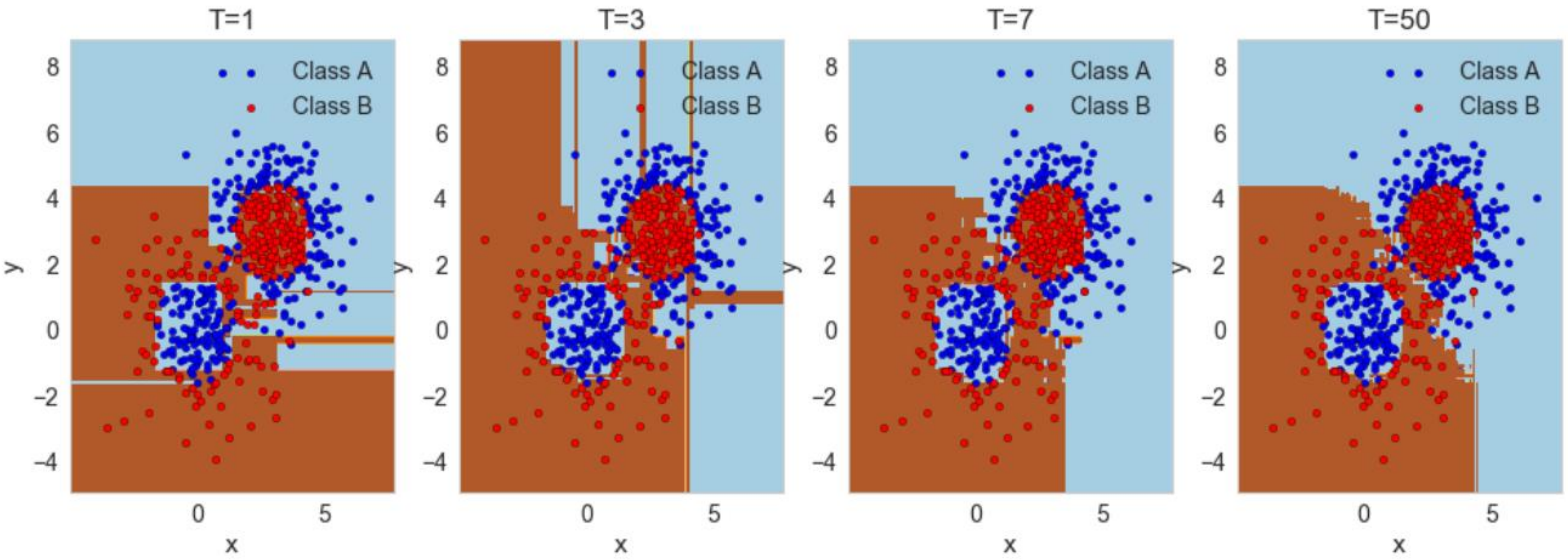
    1. sample data points from train set to create new train set
    2. fit low bias high variance model on new train set
    3. repeat steps (1) and (2) *T* times
    4. average the predictions of the *T* models

$$\hat{f}(x,\theta) = \frac{1}{T}\sum_{t=1}^{T} f_t(x,\theta)$$

bagging: Random Forest

- Train set contains *n* data points with *m* features.
- Construct *T* low bias high variance decision trees by following these steps:
  1. Sample ***n*** data points at random **with replacement** from the train set.
  2. At each node, select *h*<<*m* features at random and compute the best split using only these *h* features.
  3. Each tree is grown to the largest extent possible. There is no pruning or early stopping.
- Step 3 ensures that the bagged models are low bias by learning deep complex decision trees.

bagging: Random Forest



boosting

reduce the bias of a high bias low variance model

turning an ensemble of weak learners into a strong learner

the meta-model is additive, i.e. Adaboost:

$$\hat{f}(x, \theta) = \sum_{t=1}^{T} \alpha_t f_t(x, \theta)$$

boosting: Adaboost

$$\boxed{y_i \in \{-1, +1\}}$$

Initialize the weights $D_1(i) = 1/n, i = 1, 2, \ldots, n.$

For $t = 1$ to T:

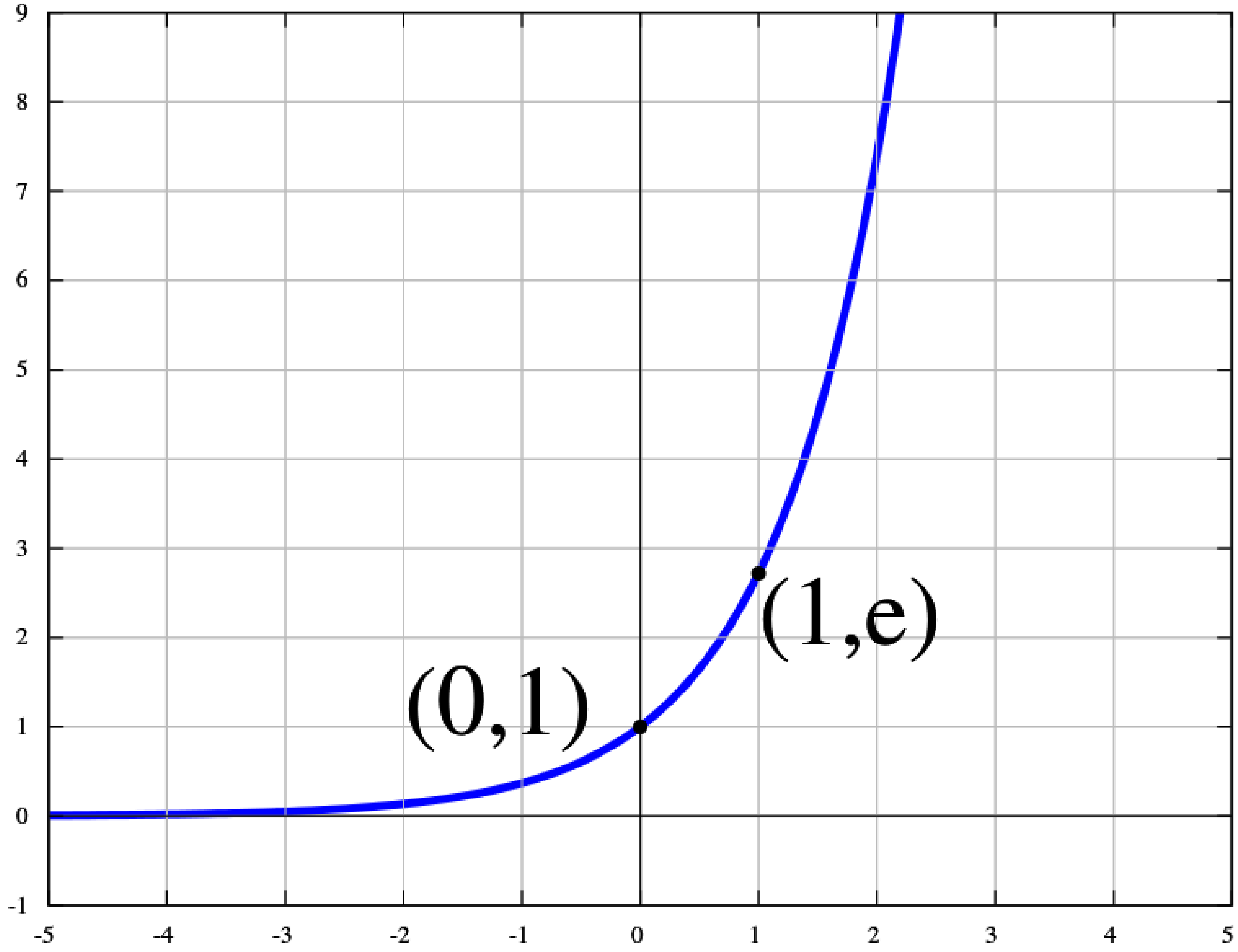
1. Fit a weak classifier $f_t(x, \theta)$ to the trainset data using weights $D_1(i)$.
2. Set $\alpha_t = \frac{1}{2} ln(\frac{1-error}{error})$.
3. Update weights:

$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i f_t(x_i, \theta))}{Z_t},$$

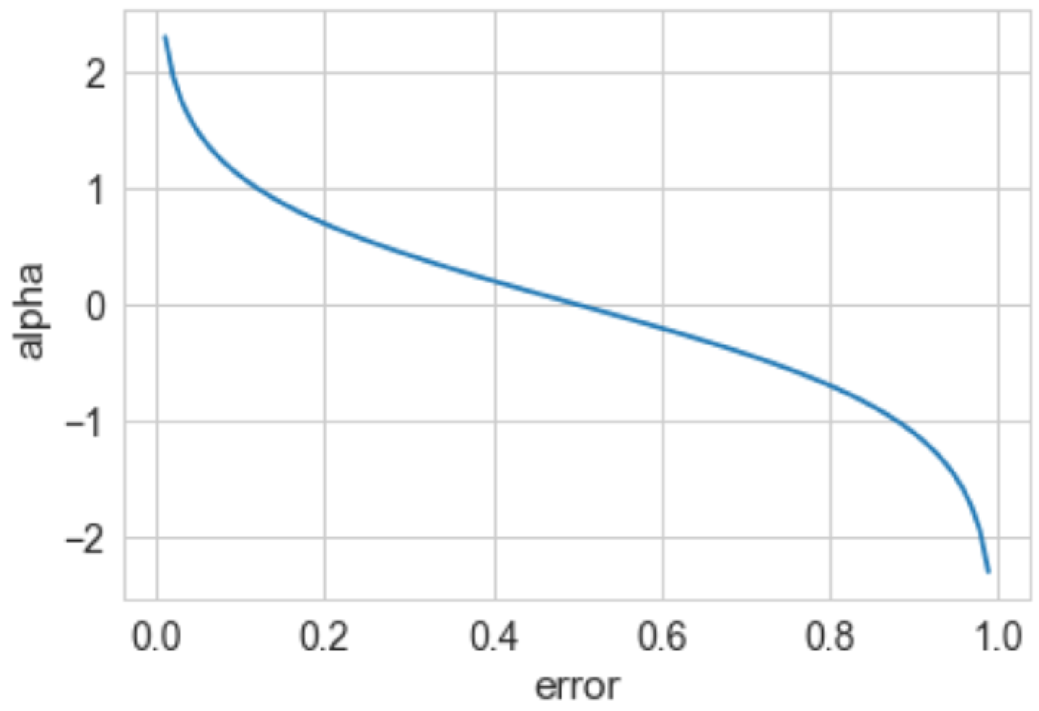
where Z_t is a normalizing factor that makes sure that $\sum D_{t+1}(i) = 1$.

$$\hat{f}(x, \theta) = \sum_{t=1}^{T} \alpha_t f_t(x, \theta)$$

boosting: Adaboost



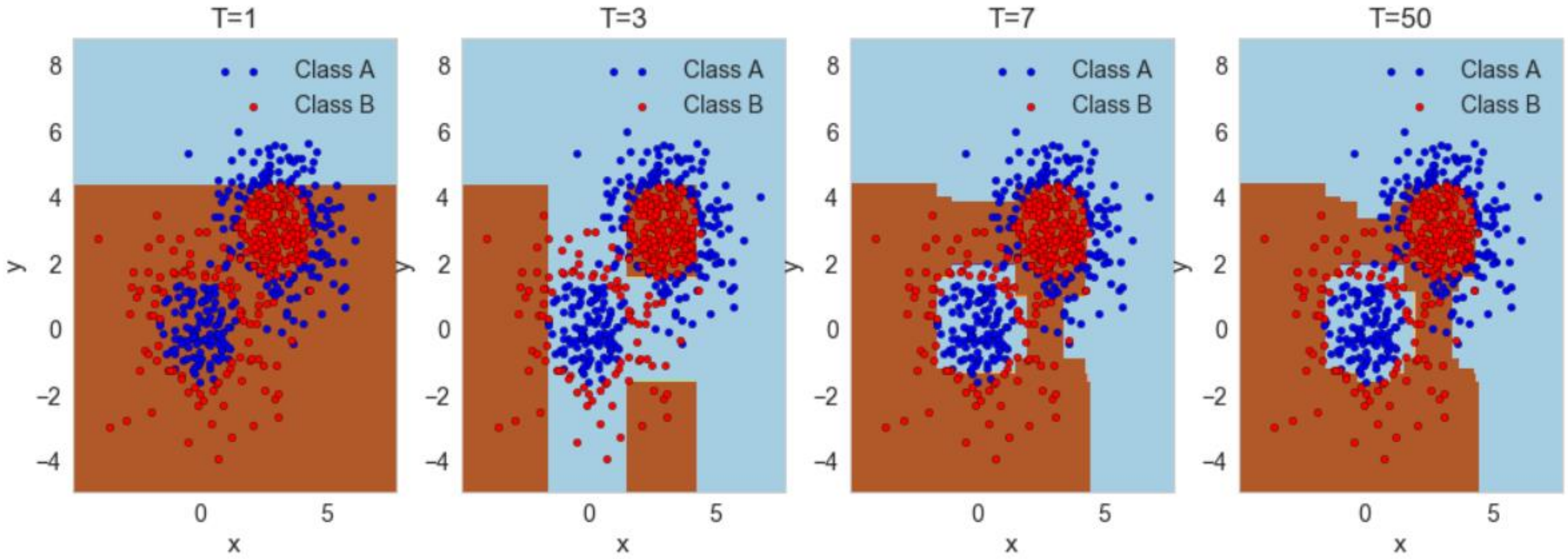
boosting: Adaboost



- The weight of a weak model in the boosted meta-model increases exponentially as the error approaches 0. Better models are given exponentially more weight.
- The weight is zero if the error rate is 0.5. A model with 50% accuracy is no better than random guessing, so it is ignored.
- The weight decreases exponentially as the error approaches 1. A negative weight is given to classifiers with worse than 50% accuracy. "Whatever that classifier says, do the opposite!".

$$\hat{f}(x,\theta) = \sum_{t=1}^{T} \alpha_t f_t(x,\theta)$$

boosting: Adaboost



boosting: Gradient boosting

$$\hat{f}(x,\theta) = \sum_{t=1}^{T} f_t(x,\theta)$$

1. Fit a model $f_1(x,\theta) = y$
2. Fit a model to the **residuals** $h_1(x) = y - f_1(x,\theta)$
3. Create a new model $f_2(x,\theta) = f_1(x,\theta) + h_1(x)$

$$f_0(x,\theta) = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$f_{t+1}(x,\theta) = f_t(x,\theta) + h_t(x)$$

boosting: Gradient boosting