Wireless Receivers: Algorithms and Architectures	Assignment 10
Fall Semester 2016/2017	Out: 2016-11-29
Telecommunications Circuits Laboratory	Due: 2016-12-23
EPF Lausanne	$Revision: 2.2$

1 Orthogonal Frequency Division Multiplexing

Wideband single carrier systems require very complicated equalization and channel estimation schemes due to frequency selective fading. Orthogonal frequency division multiplexing (OFDM) is a way to convert a wideband channel into many intersymbol interference free narrowband channels, for which equalization and channel estimation are straightforward.

1.1 The Concept of OFDM

Imagine that we want to transmit a data stream, consisting of complex symbols, at a symbol rate R. In a single carrier system (as we have discussed up to now), the data symbols would simply be transmitted one after the other, using the whole available bandwidth B, which is approximately equal to the desired symbol rate R.
In a multicarrier system, the data stream is split into N parallel streams. Each stream is transmitted over a narrow frequency band of bandwidth B/N, at a rate R/N. Figure **??** illustrates the different concepts in the time-frequency plane. The left subfigure shows a singlecarrier system, where the symbol duration is $T = 1/R$, i.e. shorter than $1\,\mu\text{s}$ in modern high-speed systems. Since the length of the channel impulse response in typical outdoor scenarios is in the order of several microseconds, intersymbol interference between many consecutive symbols arises. The right subfigure shows an OFDM transmission with $N = 8$ subcarriers.[1] The data symbols of the OFDM system are denoted as $a[n, m]$, where n is the OFDM symbol index, and m is the subcarrier index. The symbol duration is now $T = N/R$, i.e. N times longer than in the singlecarrier system. The OFDM symbols are separated by a guard interval. If this interval is longer than the channel impulse response (due to the longer symbol duration, this is now possible without introducing too much overhead), the system is free of intersymbol interference.

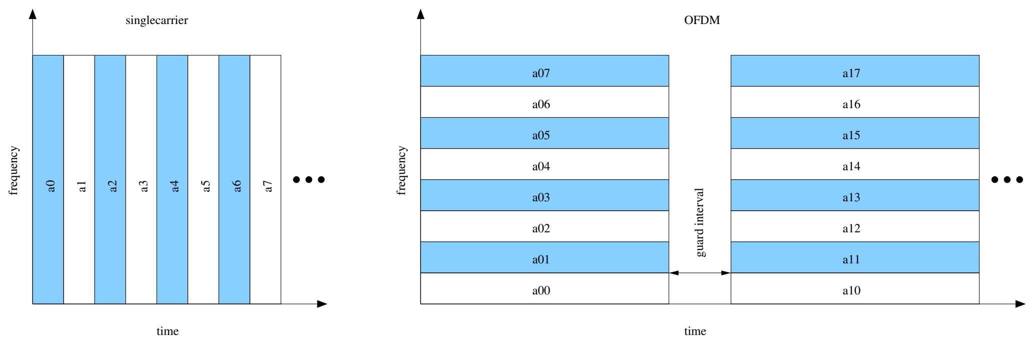


Figure 1: Comparison of singlecarrier and multicarrier (OFDM) transmission

[1] In practice, the number of subcarriers is usually between 128 and 8196.

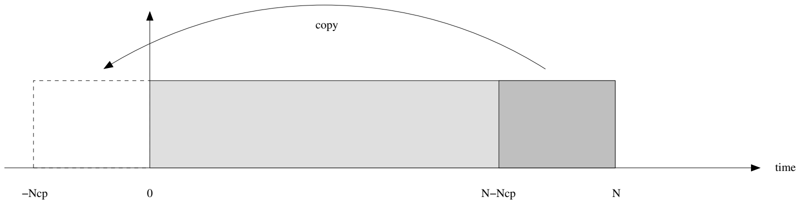


Figure 2: Cyclic Prefix

1.2 Cyclic Prefix

What do we transmit during this guard interval? One option is to simply transmit nothing. Most systems, however, use a so-called *cyclic prefix*, which means that the last N_{cp} samples of each OFDM symbol are copied and inserted in front of the symbol, as illustrated in Figure **??**. The reason of this cyclic extension has to do with the properties of the discrete Fourier transform (DFT), which is used in digital processors. Remember that our aim is to transform the received signal, which is a convolution of the transmitted signal with the channel impulse response, into the frequency domain, and then compensate for the influence of the channel by dividing the received signal spectrum by the channel transfer function.

Now imagine that we have two signals $a[k]$ and $b[k]$, with $k = 0, \ldots, N-1$. Their N-point-DFTs are denoted $A[k]$ and $B[k]$. Now define the two sequences $c[k] = a[k] * b[k]$ and $C[k] = A[k]B[k]$. The sequence $C[k]$ is the DFT of $c[k]$ only if $c[k]$ is the *circular* convolution of $a[k]$ and $b[k]$,

$$c[k] = \sum_{i=0}^{N-1} a[i]b[(k-i) \bmod N]\,, \quad k = 0, \ldots, N-1\,, \tag{1}$$

instead of the *linear* convolution, which would be given as

$$c[k] = \sum_{i=0}^{N-1} a[i]b[k-i]\,, \quad k = 0, \ldots, 2N-2\,. \tag{2}$$

We therefore want the convolution of the transmitted signal with the CIR to be circular, but unfortunately it is linear. However, from (**??**) and (**??**) we see that the circular and linear convolution are identical if one of the input sequences is periodic with period N. This is where the cyclic prefix comes into play. By prepending each signal block of length N with a copy of the last N_{cp} samples of that block, and under the condition that $N_{\text{cp}} \geq L$ (the length of the discrete-time CIR), the convolution of the transmitted signal with the CIR is effectively transformed into a circular one.

1.3 OFDM Transmission

In this section, we will give a step-by-step description of an OFDM system. A block diagram of the transmitter is shown in Figure **??**.

1. The complex data symbols are generated as usual and converted into N parallel streams, as illustrated in Figure **??**. We denote the symbols as $a[n, m]$, where $m = -N/2, \ldots, N/2-1$ indicates the stream, and n is the time index. For simpler notation, we use the vector $\mathbf{a}[n] = (a[n, -N/2], \ldots, a[n, N/2-1])$.

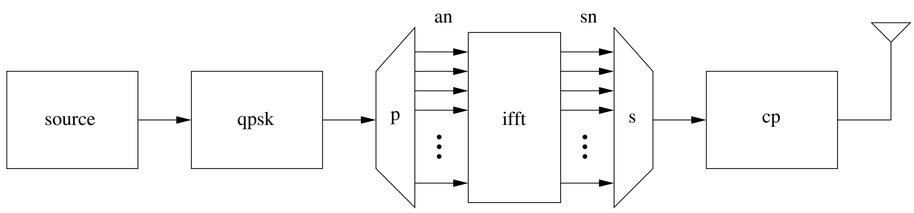


Figure 3: Simple OFDM transmitter

2. The data symbols are transformed into the time domain via an IDFT operation:

$$\mathbf{s}[n] = \mathrm{IDFT}\{\mathbf{a}[n]\}.$$

3. The cyclic prefix is inserted.

4. The resulting sample stream is converted to an analog signal and transmitted as usual. In this exercise we do not consider the D/A and A/D conversion. Instead we combine these converters as well as the physical channel into a discrete-time channel model.

5. The receiver must synchronize itself to the OFDM symbols, i.e. it must be aware at which sample a new OFDM symbol is starting. This is done by the usual frame synchronization.

6. For each time step n, the receiver collects the received samples that belong to the nth OFDM symbol into the vector $\mathbf{r}[n]$. This vector is then converted back into the frequency domain: $\mathbf{z}[n] = \mathrm{DFT}\{\mathbf{r}[n]\}$.

7. The received symbols in the frequency domain can be written as

$$z[n,m] = H[n,m]\,a[n,m] + W[n,m]\,, \tag{3}$$

where $H[n,m]$ is the channel transfer function at time instant n and at the frequency of the subcarrier m, and $W[n,m]$ is additive white Gaussian noise. Note that in this lab, we assume that the channel is constant over time.

In (**??**) we see that in an OFDM system, the data symbols $a[n,m]$ do not interfere with each other.[2] Rather, there is a one-to-one correspondence between transmitted and received symbols. The whole OFDM system, consisting of the IDFT, the cyclic prefix insertion, the channel, the CP removal and the DFT can thus be modeled as N parallel flat fading channels. The receiver removes the influence of the channel by dividing the received symbol by an estimate of the channel. In the present lab, we simply assume $H[n,m] \equiv 1$.

1.4 Synchronization in Single Carrier and OFDM Receivers

In this section, we highlight some differences in the synchronization tasks between the single carrier receiver, which we have discussed in the first labs, and an OFDM receiver.

[2]This statement assumes some idealizations: Perfect frequency synchronization, perfect analog components, and a sufficiently long cyclic prefix. Furthermore, the channel variations must be so slow that the channel can be regarded as constant during one OFDM symbol period.

1. In an OFDM system, we do not have a matched filter. Instead, the receiver uses a simple lowpass filter. This is because, from the receiver's perspective, the combination of the transmit pulse $g(t)$ and the channel $h_c(t)$ looks like an effective transmit pulse

$$g_{\text{eff}}(t) = g(t) * h_c(t) \tag{4}$$

   and hence, a receiver filter $g^*(-t)$ would not be matched to the effective transmit pulse anymore. Instead, the frequency domain equalizer acts as a combined matched filter and channel equalizer.

2. Timing synchronization is also not needed in an OFDM receiver. This is because, due to the multipath channel, the received signal is composed of many replicas of the transmitted waveform, and there is no longer an optimal sampling instant. Instead, a timing shift of a fraction of the sampling duration would simply translate to a phase shift in the frequency domain and would be compensated by the channel equalizer.

3. For frame synchronisation, you can use a BPSK encoded single carrier preamble. You may take the same sequence as in all previous exercises. As you work in an oversampled system, you need to apply pulse shaping to the preamble (but not to your OFDM symbols). Be careful that you do not destroy your OFDM symbols.

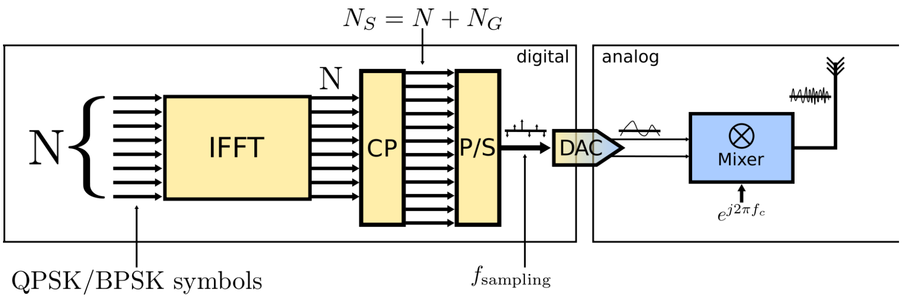
4. One topic that we do not discuss in our lab, but want to mention nonetheless for completeness, is frequency synchronization. The need for frequency synchronization arises because in practical systems, the frequencies of the transmitter and receiver oscillators are never exactly the same, resulting in a frequency shift in the complex baseband model. In multicarrier systems, this problem is more severe than in single carrier systems (of the same total bandwidth) because they use narrow subbands for data transmission, and the same absolute frequency error causes—normalized to the carrier bandwidth—a larger relative frequency error.

2 OFDM for Acoustic Transmission

In the previous section you simulated an OFDM system in MATLAB. In this section, we want to use OFDM modulation with our acoustic transmission system. Due to its inherent capability to deal with frequency-selective channels, OFDM should allow larger bandwidths and, therefore, higher symbol rates. We will now briefly discuss the main differences between your previous OFDM implementation and OFDM for your acoustic link.

2.1 Sampling Rate

As discussed earlier, OFDM modulation can be performed without oversampling. But there is a fixed relationship between the sampling rate f_{sampling} of the DAC, the duration of the OFDM symbol T, the number of subcarriers N, and the baseband bandwidth BW_{BB}. A typical transmitter setup is depicted in the following picture:



T_S is the length of the OFDM symbol T plus the length of the cyclic prefix T_G. In this case:

$$f_{\text{spacing}} = \frac{1}{T} \tag{5}$$

$$f_{\text{sampling}} = \frac{N}{T} \tag{6}$$

$$BW_{BB} = \text{ceil}\left(\frac{N+1}{2}\right) \cdot \frac{1}{T} \tag{7}$$

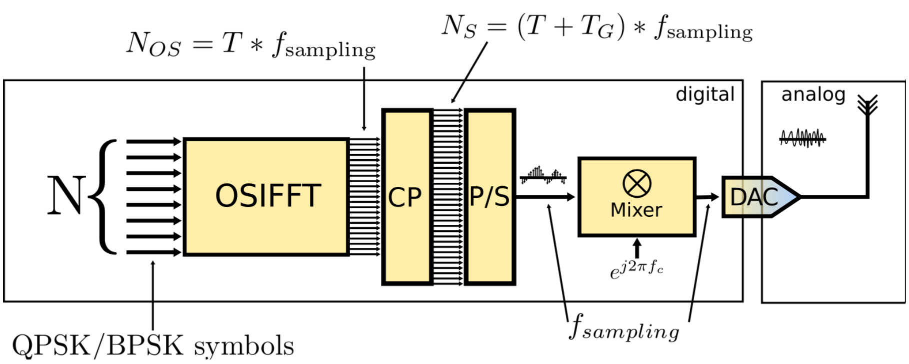
You can see that the sampling rate depends on the number of carriers and their spacing. One way to look at it is that the inverse FFT transforms a continuous spectrum to a time signal. The relation between the size of the spectrum and the sampling rate is determined by the Shannon-Nyquist sampling theorem.

Unfortunately, this does not apply to our setup. The audio transmission system is special in the sense that it is an all-digital transceiver. This means that the upconversion is done in the digital domain. The sampling rate can not be chosen freely. This leads to an unwanted but unavoidable oversampling in the system. There are two approaches to deal with this problem:

1. The IFFT is performed over the desired baseband spectrum only. This leads to a lower sampling rate. Afterwards, the signal must be interpolated to the output sample rate.

2. The IFFT is performed over the full spectrum given by the output sampling rate, but only the baseband part of the spectrum is modulated.

We propose the use of the second method. You can find adapted versions of the `IFFT` and `FFT` functions on the Moodle. The new `OSIFFT` and `OSFFT` functions transform between time and frequency domain while taking care of the oversampling.



This approach requires that f_{spacing} is a divisor of f_{sampling}. With the following relations:

$$f_{\text{spacing}} = \frac{1}{T} \tag{8}$$

$$BW_{\text{BB}} = \text{ceil}\left(\frac{N+1}{2}\right) \cdot f_{\text{spacing}} \tag{9}$$

$$N_{\text{OS}} = T \cdot f_{\text{sampling}} \tag{10}$$

while f_{sampling} is fixed to e.g. $48\,\text{kHz}$. The `IFFT` and `FFT` functions accept the following parameters:

Y = OSIFFT(X,OS_FACTOR)

1. X - Input vector of length N.
2. OS_FACTOR - Oversampling factor $\frac{f_{\text{sampling}}}{f_{\text{spacing}} \cdot N}$

Y = OSFFT(X,OS_FACTOR)

1. X - Input vector of length $T \cdot f_{\text{sampling}}$.
2. OS_FACTOR - Oversampling factor $\frac{f_{\text{sampling}}}{f_{\text{spacing}} \cdot N}$

2.2 Phase Tracking

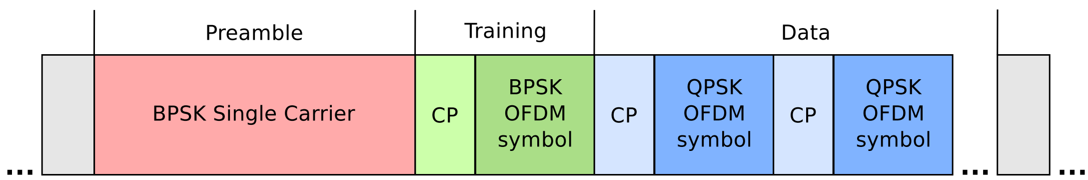
In OFDM each subcarrier has a different phase error and attenuation. To be able to correctly detect the symbols, you need initial estimates of the respective phase errors. You can measure those phase errors by transmitting a known OFDM symbol at the beginning of every frame. It is important to note that you can not use the initial phase estimate provided by the frame synchronization unit, since this is an estimate of the phase offset in the time domain, not in the frequency domain. Afterwards, you can track the phase deviation as in the single carrier system. Remember that you have to do this for each subcarrier independently. This set of phase offsets and magnitudes describes your channel. In other words, it is the estimate of the frequency response of the channel.

2.3 Down Conversion

We supply you a new version of the lowpass filter on the Moodle. The filter cut-off point should be slightly above your BW_{BB}.

2.4 Recommended Frame Structure

The recommended frame structure, which we described previously, looks like this:



3 Your Tasks

1. Implement an acoustic OFDM transmission system. Start with 256 carriers, a spacing of $5\,\mathrm{Hz}$ and a cyclic prefix of half the OFDM symbol length. Transmit the training symbol and one data symbol only. Verify that it works under good channel conditions.

2. Plot the spectrum of your channel (phase and magnitude). Observe how the spectrum changes over time. Do an inverse FFT of the spectrum to estimate the existing delay spread. Calculate the efficiency of your current system. How much does it increase if you reduce the CP to the required length only?

3. How many symbols can you transmit before needing to do continuous phase tracking?

3.1 Hints

- To allow large bandwidth, we recommend to use a carrier frequency of $8\,\mathrm{kHz}$.
- If necessary, the last OFDM symbol of the burst is filled up with random bits that can simply be discarded.
- OFDM can have a high peak-to-average power ratio (PAPR). Thus, the large dynamic range can pose problems, like sound clipping. To get best results, ensure that your preamble and the OFDM symbols have the same energy. Make sure to slightly reduce the volume of the speakers and of the operating system.
- As a starting point, you do not need to implement phase tracking. The training OFDM symbol should be sufficient if you transmit only a small number of symbols.
- In order to verify your code, you can at first bypass the channel by directly using the vector created by the transmitter at the receiver.

Final Presentation and Report

Each student is required to submit a **separate** report. Your report should contain:

1. A brief description of the problem that you are solving and the solution that you used, in your own words.

2. A brief description of your implementation and any problems that you encountered.

3. The results of your implementation of the tasks and any conclusions you were able to draw.

Each group is required to do a 10 minute final presentation of their audio transmission system during the last lecture. The presentation should essentially be a summary of the report using no more than 10 slides.