Session 1 - Trend/Cycle Decomposition

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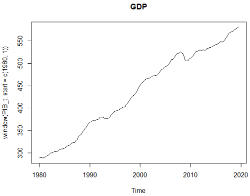
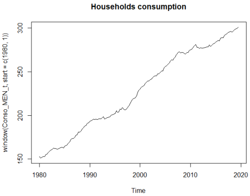
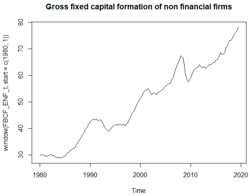
Motivations

Distinguish in the economic analysis between long term evolutions (trend) and short term developments (cycle)

- Long term: structural evolutions, growth theories...
- Short term: cyclical developments, position in the business cycle (output gap)







Motivations

More "operational"purposes:

- Evaluation of cyclical-adjusted variables: structural deficit...
- Conduct of cyclical economic policies: monetary policy (Taylor rule), fiscal policy...

Macroeconomic modelisation:

- Some models (implicitely) use "detrended"variables

Aim of the decomposition

Let us consider Y_t an economic variable (GDP, consumption, investment...) and y_t the logarithm of Y_t.

We want to find y_t^* (trend) and $\tilde{y}_t$ (cycle) such as:

$$y_t = y_t^* + \tilde{y}_t$$

where $\tilde{y}_t$ is a mean-zero stationary process.

Why using y_t (logarithm) and not Y_t ?

- the change in the ln of a variable is almost equal to its growth rate: $\Delta y_t = \ln(\frac{Y_t}{Y_{t-1}}) \simeq \frac{Y_t}{Y_{t-1}} - 1$
- the decomposition is more likely to be valid on y_t than on Y_t

Plan of the session

1. Simple case of a linear deterministic trend
2. Linear deterministic trends with break points
3. No trend
4. HP filter
5. Kalman filter
6. Illustration: calculating the output gap
7. References

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Simple (but mostly theoretical) case of a linear deterministic trend

Assume that Y_t has a linear deterministic trend:

$$y_t = a + gt + u_t$$

with u_t stationary.

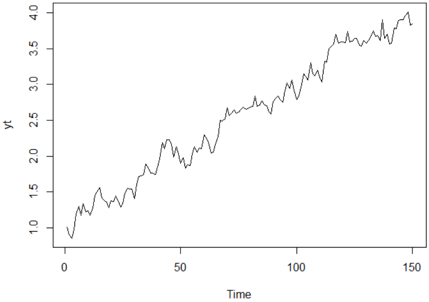
Coefficients a and g can be estimated with an OLS estimator and we have therefore:

- the trend: $y_t^* = \hat{a} + \hat{g}\,t$ (fitted model)
- the cyclical component: $\tilde{y}_t = y_t - y_t^* = \tilde{u}_t$ (residuals of the OLS regression)

Note the likely auto-correlation of the residuals: Newey-West procedure to estimate the covariance matrix of residuals.

Simple (but mostly theoretical) case of a linear deterministic trend

Illustration (150 observations):



Note the value of R^2

Simple (but mostly theoretical) case of a linear deterministic trend

Illustration (150 observations):

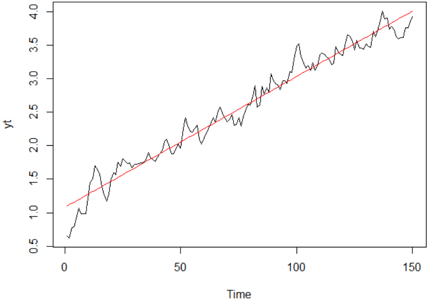
OLS regression: $y_t = a + gt + u_t$

	y_t
t	0.020*** (0.0003)
Constant	1.078*** (0.027)
R^2	0.963
Adjusted R^2	0.963

Simple (but mostly theoretical) case of a linear deterministic trend

Illustration (150 observations):

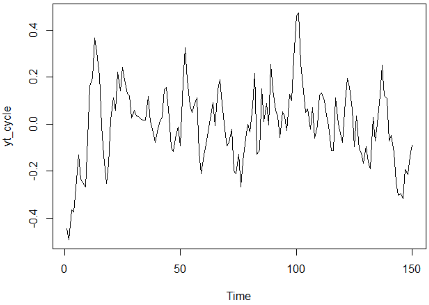
We define the trend: $y_t^* = 1.078 + 0.02t$



Simple (but mostly theoretical) case of a deterministic trend

Illustration (150 observations):

And we define the cycle: $\hat{y}_t = y_t - y_t^*$

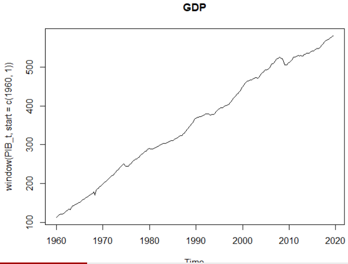


Simple (but mostly theoretical) case of a linear deterministic trend

Very simple case but relatively uncommon in practice

- From a statistical point of view, residuals of the OLS regression are often non-stationary
- It is a strong economic hypothesis to assume the cyclicality of economic growth around a constant level

More realistic hypothesis: several linear deterministic trends with breakpoints



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Linear deterministic trends with break points

Assume that Y_t has two linear deterministic trends around a break point t_1:

$$y_t = a + g_1 t + g_2 (t - t_1) I_{t \geq t_1} + u_t$$

with u_t stationary.

- The OLS regression on a single deterministic trend does not result in a stationary cyclical component
- But two separate OLS regressions, respectively on $t < t_1$ and on $t > t_1$, yields estimators of g_1 and g_2 and then the trend and the cycle components

But how to identify break points ?

- exogenous identification: break points are usually known (oil price shock in 1973, Great Recession in 2008...)
- endogenous identification: estimation of the break points from the data (Chow test, Bai & Perron test...)

Linear deterministic trends with break points

One **specific** break point : H_0 (stable model on $[1, T]$) vs H_1 (one break point in $T_1 \in [1, T]$)

Chow test: under the assumption that errors are i.i.d and from a normal distribution,

- the test statistic

$$F_T^{Ch}(T_1) = \frac{T - 2q - p}{q} \left(\frac{SSR_{1,T} - (SSR_{1,T_1} + SSR_{T_1,T})}{SSR_{1,T_1} + SSR_{T_1,T}} \right)$$

  follows a Fisher distribution with q and $T - 2q - p$ degrees of freedom
- q is the number of coefficients subject to break point
  p is the number of stable coefficients
  $SSR_{1,T}$ is the sum of square residuals for the stable model on $[1, T]$ without break
  SSR_{1,T_1} (resp. $SSR_{T_1,T}$) is the sum of square residuals for the model on $[1, T_1]$ (resp. $[T_1, T]$)

Linear deterministic trends with break points

One **specific** break point : H_0 (stable model on $[1, T]$) vs H_1 (one break point in $T_1 \in [1, T]$)

Asymptotic Wald test, under the assumption that errors are i.i.d:

- the test statistic
$$F(\frac{T_1}{T}) = qF_T^{Ch}(T_1)$$
asymptotically follows a Chi-2 distribution with q degrees of freedom

Linear deterministic trends with break points

Searching one break point : H_0 (stable model on $[1, T]$) vs H_1 (one break point in an unknown $T_1 \in [1, T]$)

SupF statistic:

- $$SupF = \max_{T_{min} \dots T_{max}} F(\frac{T_1}{T})$$

  which means that the sum of squared residuals of the model with break points (that is $SSR_{1,T_1} + SSR_{T_1,T}$) is minimum
- if *SupF* is "high enough", H_0 is rejected and T_1 is such as $F(\frac{T_1}{T}) = SupF$
- the asymptotic *SupF* distribution is non standard (Andrews, 1993)

Linear deterministic trends with break points

Searching m multiple break points (m is knwon, points are unknown)

$SupF(m)$ statistic (Bai & Perron, 1998a and b):

- generalization of the $SupF$ statistic:

$$SupF(m) = \max_{(T_1,\ldots,T_m)} F(m)$$

  where $F(m)$ is the Wald statistic for asymptotically testing H_0 (stable model on $[1, T]$) vs H_1 (m known break points in $T_1 \ldots T_m$)
- the $SupF(m)$ follows a non-standard distribution, documented in Bai & Perron (1998)

$SupF(m+1|m)$ statistic (Bai & Perron, 1998a and b):

- testing H_0 (m unknown break points) vs H_1 (one additional break point)

Linear deterministic trends with break points

Searching some break points (m and dates are unknwon)

Sequential *SupF* tests (Bai & Perron, 1998a and b):

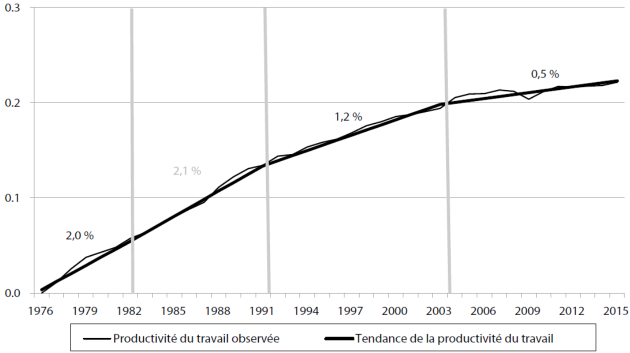
- Test of H_0 (stable model) vs H_1 (one break point)
- If stability is rejected, test of H_0 (one break point) vs H_1 (two break points) with the $SupF(m+1|m)$ statistic...
- ... until H_0 (m break points) is not rejected

Linear deterministic trends with break points

Example from Cette, Corde et Lecat (2017):

- use of Bai & Perron test to identify some deterministic trends in the evolution of the labour productivity between 1976 and 2015
- $y_t = \alpha + \sum_{k=0}^{m} \beta_k (t - T_k) I_{t \geq T_k} + \gamma TUC_t + u_t$

D – Productivité du travail par employé (PTN)
en log, base 0 en 1976, taux de croissance annuel moyen tendanciel en % (méthode Bai et Perron avec TUC)



Linear deterministic trends with break points

To sum up:

- To test m break points in $T_1 ... T_m$:
  - Chow statistic F^{Ch} (errors i.i.d and from a normal distribution)
  - Wald statistic F in an asymptotic case (errors i.i.d)
  - Newey-West procedure when errors are correlated
- To search m break points:
  - if $m = 1$, $SupF$ statistic
  - if $m > 1$, $SupF(m)$ statistic (Bai & Perron (1998))
- To search some break points:
  - sequential $SupF$ tests (Bai & Perron (1998))
  - BIC criteria

More information: Le Bihan (2004)

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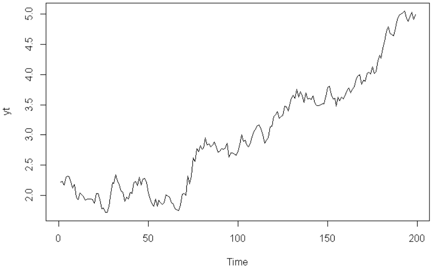
No trends

So far we have assumed that the residuals of the OLS regressions were stationary.

What happens if it is not the case? Previous methods are not valid anymore and may result in spurious results!

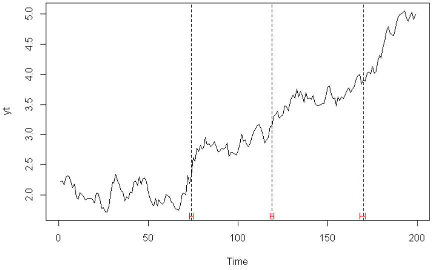
Example with the following evolution of y_t:



No trends

y_t seems to present several deterministic trends:

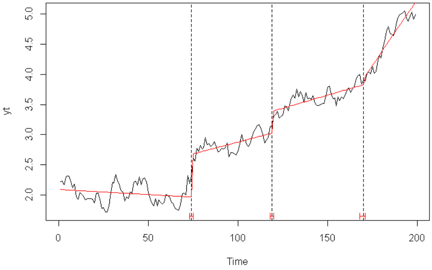
- we use the Bai & Perron test...



No trends

y_t seems to present several deterministic trends:

- we use the Bai & Perron test...
- ... and we identify the deterministic trend on each segment



No trends

Actually y_t is a random walk with drift:

$$y_t - y_{t-1} = g + \varepsilon_t$$

with ε_t a white noise. In other words, y_t presents a stochastic trend:

$$y_t = gt + y_0 + \sum_{\tau=1}^{t} \varepsilon_\tau$$

As a consequence, identifying break points and deterministic trends like above is not valid

- in practice it is difficult to discriminate between series with deterministic trends and series with stochastic trend
- when studying macroeconomic series, important to test the stationarity of series or of regression residuals

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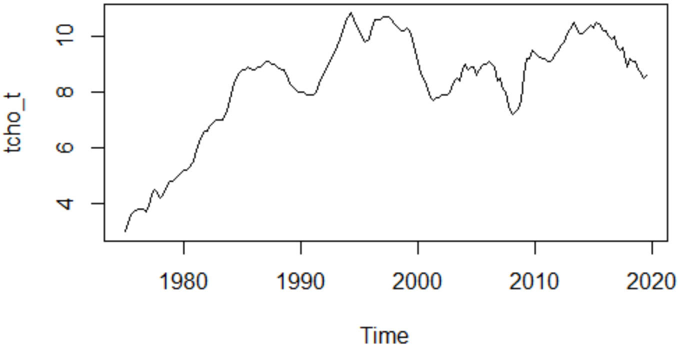
Trend/cycle decomposition using filters: HP filter

How to identify a trend and a cyclical component in time series which do not present some visible linear trends ?

For example, the quarterly unumployment rate in France since 1975



Trend/cycle decomposition using filters: HP filter

Hodrick-Prescott filter (HP filter, 1980) :

- agnostic approach
- the trend component as "a very smooth series that does not differ too much from the observed series"

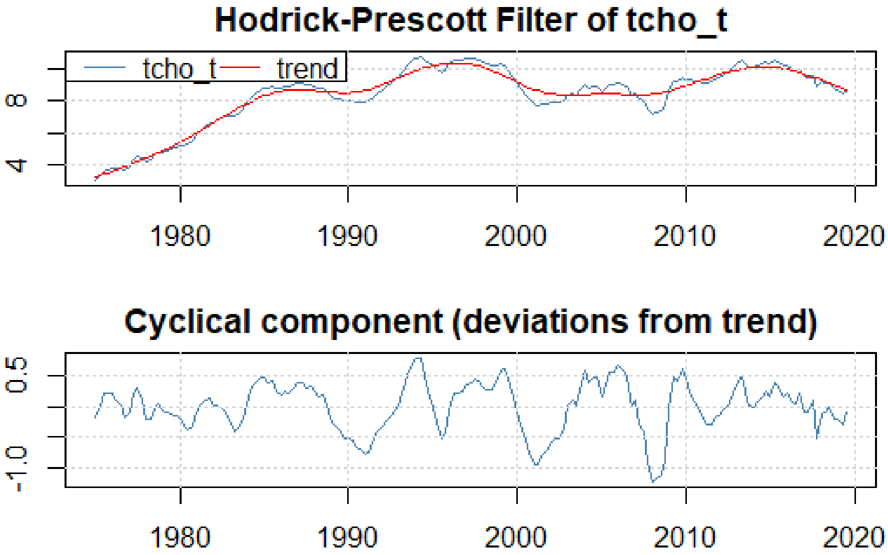
Find y_t^* and $\tilde{y}_t = y_t - y_t^*$ such as

$$\min \left[\sum_{t=1}^{T} (y_t - y_t^*)^2 + \lambda \sum_{t=3}^{T} (\Delta y_t^* - \Delta y_{t-1}^*)^2 \right]$$

where $\lambda > 0$ (smoothing parameter) :

- $\lambda = 0$: $\tilde{y}_t = 0$ and $y_t^* = y_t$ (no smoothing)
- $\lambda = +\infty$: $y_t^* = gt$ is a linear trend and $\tilde{y}_t = y_t - gt$ (maximal smoothing)
- a usual value for quaterly data is $\lambda = 1600$

Trend/cycle decomposition using filters: HP filter



Trend/cycle decomposition using filters: why "filters"?

The HP filter can be interpreted in a frequency approach.

Let us consider

- y_t a stationary time series
- $\gamma(\tau) = E(y_t - E(y_t))E(y_{t+\tau} - E(y_{t+\tau}))$: autocovariance between y_t and $y_{t+\tau}$ for $\tau \in]-\infty, +\infty[$
- with the assumption that $\sum_{-\infty}^{+\infty} |\gamma(\tau)|^2 < \infty$

For $\omega \in [-\pi, +\pi]$, we define the spectral density (or spectrum) of y_t:

$$s_y(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{+\infty} \gamma(\tau) e^{-i\omega\tau}$$

It is the Fourier transform of the autocovariances $\gamma(\tau)$.

For example, a time series with a cycle duration of p units of time will have a spectral density with a remarkable point in the frequency $\omega = \frac{2\pi}{p}$

Trend/cycle decomposition using filters: why "filters"?

The trend component y_t^* of the HP filter applied to y_t is a linear combination of observations y_t:

$$y_t^* = \sum_{\tau=-r}^{s} c_\tau y_{\tau-j} = C(L)y_t$$

Indeed, the minimization program of the HP filter can be written in a matrix form:

$$\min_{y^f} (y - Hy^f)'(y - Hy^f) + \lambda(Qy^f)'(Qy^f)$$

where:

- $y = (y_T, y_{T-1}, ..., y_1)'$ (size $T \times 1$)
- $y^f = (y_T^f, y_{T-1}^f, ..., y_1^f, y_0^f, y_{-1}^f)'$ (size $T + 2 \times 1$)
- $H = [I_T, 0]$ (size $T \times T + 2$)
- Q is a $T \times T + 2$ matrix with the vector $(1, -2, 1)$ along the diagonal

And the solution of the program is:

$$y^* = (H'H + \lambda Q'Q)^{-1}H'y$$

Trend/cycle decomposition using filters: why "filters"?

The spectral density of y_t^* is then:

$$\begin{aligned} s_{y^*}(\omega) = C(e^{-i\omega})C(e^{i\omega})s_y(\omega) &= |C(e^{i\omega}|^2 s_y(\omega) \\ &= G(\omega)^2 s_y(\omega) \end{aligned}$$

$G(\omega)^2$ is called the squared gain of the filter: it amplifies some frequencies and attenuates some others ("filter").

For the HP filter:

- the square gain is: $G(\omega) = \left(1 + \left(\frac{\sin(\omega/2)}{\sin(\omega_0/2)}\right)^4\right)^{-1}$
  where $\omega_0 = 2arcsin\left(\frac{1}{2\lambda^{1/4}}\right)$ (depending on λ)
- ω_0 is such as $G(\omega_0) = 50\%$ (gain higher than 50% for $\omega \leq \omega_0$)
- $\lambda = 1600$ implies 50% gain for a 40-quarter cycle (10 years)

Trend/cycle decomposition using filters: the HP filter in debate

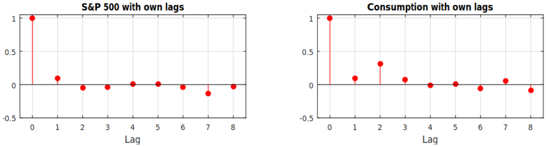
The HP filter is widely used by "institutional"macroeconomists (central banks, international economic agencies, government administration...).

But it has been subject to criticism in academic work over many years. See Hamilton: "Why You Should Never Use the Hodrick-Prescott Filter", 2018. More particularly:

1. the cyclical component may present some dynamics that have no basis in the underlying data generating process (e.g, HP filtering of a random walk)

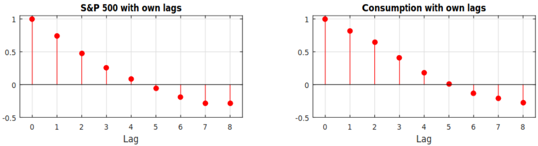
Trend/cycle decomposition using filters: the HP filter in debate

Autocorrelations for log growth rate of stock prices (end-of-quarter value for S&P 500) and for log growth rate of real consumption spending



$\Rightarrow$ Like random walks!

Autocorrelations for HP cyclical components:



$\Rightarrow$ HP cyclical components are predictable, which is purely an artefact

Trend/cycle decomposition using filters: the HP filter in debate

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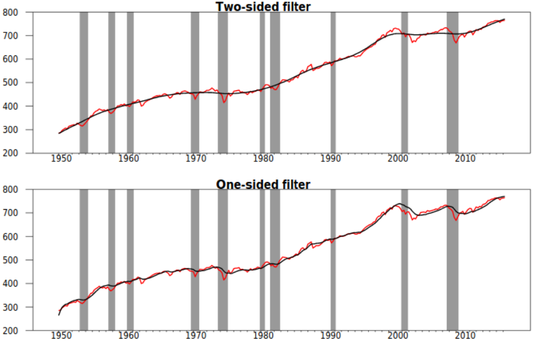
1. the cyclical component may present some dynamics that have no basis in the underlying data generating process (e.g, HP filtering of a random walk)
2. end values of the sample are less robust than those in the middle (because HP trend and cycle are a function of past and future realizations)

Trend/cycle decomposition using filters: the HP filter in debate

Comparison of one-sided (only past realizations) and two-sided HP filters (past and future realizations)



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1. the cyclical component may present some dynamics that have no basis in the underlying data generating process (e.g, HP filtering of a random walk)
2. end values of the sample are less robust than those in the middle (because HP trend and cycle are a function of past and future realizations)
3. the common practice of $\lambda = 1600$ is not always justified

Trend/cycle decomposition using filters: the HP filter as a controversial method

However, other recent papers suggest a "rehabilitation"of the HP filter.

Phillips & Shi (2019): "Boosting the Hodrick-Prescott Filter"

- "Since the cyclical component may often retain trend elements, we feed the data into the filter again to clean the leftover elements."

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Trend/cycle decomposition using filters: the Kalman filter

A less agnostic and more structural approach: the Kalman filter

- observations $(y_t)_{1 \dots T}$ and an unobserved variable $(y_t^*)_{1 \dots T}$
- state-space representation: a state equation (dynamic of the unobserved variable) and an observation equation (link between the unobserved variable and observations)
- recursively calculating the best forecast of the unobserved variable given past and present observations
- refining the estimation of the unobserved variable by calculating its best estimate given *all* observations
- if parameters of the state-space representation are unknown, estimating them with log-likelihood maximization (resulting from the previous step)

Trend/cycle decomposition using filters: the Kalman filter

State-space representation (multivariate processes):

- state equation:

$$Y_{t+1}^* = FY_t^* + V_{t+1}$$

  where Y_t^* (dimension $r \times 1$) are unobserved variables at t and V_{t+1} are innovations at t

- observation equation:

$$Y_t = A'X_t + H'Y_t^* + W_t$$

  where Y_t ($n \times 1$) are observations at t, X_t ($k \times 1$) are exogenous or predetermined variables (e.g Y_{t-1}) and W_t are measurement errors

- $V_t \sim WN(0, Q)$, $W_t \sim WN(0, R)$ and V_t and W_t are uncorrelated
- Y_1^* uncorrelated with any V_t or W_t
- F ($r \times r$) is the transition matrix, A' ($n \times k$) and H' ($n \times r$) are measurement matrices

Assume that F, A, H, Q and R are known.

The Kalman filter is a recursive method to calculate:

- the best forecast of the state vector on the basis of data observed through date t:
$$\hat{Y}^*_{t+1|t} = \hat{E}(Y^*_{t+1}|\mathcal{Y}_t)$$
where $\mathcal{Y}_t = (Y'_t, Y'_{t-1}, ..., Y'_1, X'_t, X'_{t-1}, ..., X'_1)'$
- the associated mean squared error (MSE) matrix:

$$\mathrm{P}_{t+1|t} = E[(Y^*_{t+1|t} - \hat{Y}^*_{t+1|t})(Y^*_{t+1|t} - \hat{Y}^*_{t+1|t})']$$

Trend/cycle decomposition using filters: the Kalman filter

Step 1: Initializing the recursion

$$\hat{Y}^*_{1|0} = E(Y^*_1)$$

and

$$\mathrm{P}_{1|0} = E[(Y^*_1 - E(Y^*_1))(Y^*_1 - E(Y^*_1))']$$

$\hat{Y}^*_{1|0}$ can thus represent the best guest of the analyst (his prior) and $\mathrm{P}_{1|0}$ the confidence in this guess.

Trend/cycle decomposition using filters: the Kalman filter

Step 2: Forecasting Y^*_{t+1} and calculating the MSE matrix

Given $\hat{Y}^*_{1|0}$, $\hat{Y}^*_{2|1}$, ..., $\hat{Y}^*_{t|t-1}$ and $\mathrm{P}_{1|0}$, $\mathrm{P}_{2|1}$, ..., $\mathrm{P}_{t|t-1}$, how to calculate $\hat{Y}^*_{t+1|t}$ and $\mathrm{P}_{t+1|t}$?

We have:

- $\hat{Y}^*_{t|t} = \hat{Y}^*_{t|t-1} + \mathrm{P}_{t|t-1} H (H' \mathrm{P}_{t|t-1} H + R)^{-1} (Y_t - A' X_t - H' \hat{Y}^*_{t|t-1})$
- $\hat{Y}^*_{t+1|t} = F \hat{Y}^*_{t|t}$

and

- $\mathrm{P}_{t|t} = \mathrm{P}_{t|t-1} - \mathrm{P}_{t|t-1} H (H' \mathrm{P}_{t|t-1} H + R)^{-1} H' \mathrm{P}_{t|t-1}$
- $\mathrm{P}_{t+1|t} = F \mathrm{P}_{t|t} F' + Q$

Trend/cycle decomposition using filters: the Kalman filter

Step 3: Smoothing

Rather than forecasting Y_t^* given $\mathcal{Y}_t$, why not using all the observations, i.e calculating $\hat{Y}^*_{t|T} = \hat{E}(Y_t^*|\mathcal{Y}_T)$?

Given $\hat{Y}^*_{T|T}$ from the previous step, we have:

- $\hat{Y}^*_{t|T} = \hat{Y}^*_{t|t} + J_t(\hat{Y}^*_{t+1|T} - \hat{Y}^*_{t+1|t})$
- $\mathrm{P}_{t|T} = \mathrm{P}_{t|t} + J_t \left[-E((\hat{Y}^*_{t+1|T})(\hat{Y}^*_{t+1|T})') + E((\hat{Y}^*_{t+1|t})(\hat{Y}^*_{t+1|t})') \right] J_t^{-1}$

where $J_t = (\mathrm{P}_{t|t})F'(\mathrm{P}_{t+1|t})^{-1}$

Trend/cycle decomposition using filters: the Kalman filter

Step 4: Estimating the parameters of the state-space representation

Until now, we assumed that F, A, H, Q and R were known (summarized in the Θ parameter). What happens if it is not the case ?

Assuming that $V_t \sim \mathcal{N}(0, Q)$ and $W_t \sim \mathcal{N}(0, R)$:

- for any Θ and from step 2, it is possible to calculate recursively the distribution of Y_t conditional on $(X_t, \mathcal{Y}_{t-1})$:

$$Y_t | X_t, \mathcal{Y}_{t-1} \sim \mathcal{N}(A'_\Theta X_t + H'_\Theta \hat{y}^*_{t+1|t}, H'_\Theta \mathrm{P}_{t|t-1} H_\Theta + R_\Theta)$$

- which gives the sample log likelihood:

$$\mathcal{L}(\theta) = \sum_{t=1}^{T} \ln f^{\Theta}_{\mathrm{Y}_t | \mathrm{X}_t, \mathcal{Y}_{t-1}}(Y_t | X_t, \mathcal{Y}_{t-1})$$

- so we can estimate the true Θ_0 by maximizing the log likelihood $\mathcal{L}(\theta)$

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An illustration: estimation of output gap

A typical exercise of trend/cycle decomposition is the estimation of potential GDP

- **potential GDP** is the level of GDP in the absence of any supply/demand imbalances in the economy
- the **output gap (OG)** is the (relative) difference between GDP and potential GDP:

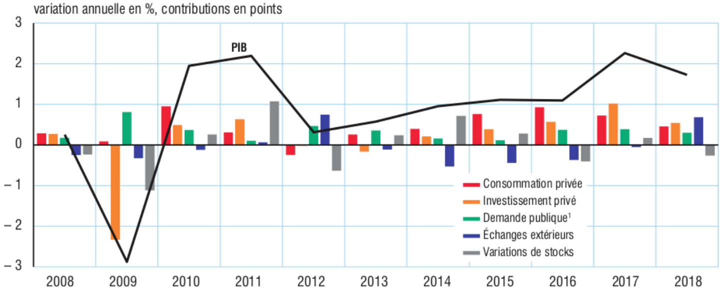
  $$OG_t = \frac{Y_t - Y_t^*}{Y_t^*}$$

  where Y_t is the GDP at time t and Y_t^* is the potential GDP
- **potential growth** is the growth of potential GDP
- OG denotes short-term fluctuations (due to demand shocks) while potential GDP denotes medium-term dynamics (due to supply shocks)

An illustration: estimation of output gap

Frequent use of OG in economic analyses. For example:

- to evaluate the position of the economy in the business cycle
  - 2017-2018: catching up or overheating?



1. Y compris institutions sans but lucratif au service des ménages.
Champ : France.
Source : Insee, comptes nationaux, base 2014.

An illustration: estimation of output gap

Frequent use of OG in economic analyses. For example:

- to evaluate government budgetary position, with the notion of structural deficit
  - LOI No. 2018-32 du 22 janvier 2018 de programmation des finances publiques pour les années 2018 à 2022:

	2017	2018	2019	2020	2021	2022
Solde structurel	- 2,2	- 2,1	- 1,9	- 1,6	- 1,2	- 0,8
Ajustement structurel	0,3	0,1	0,3	0,3	0,4	0,4

An illustration: estimation of output gap

Potential GDP refers to a hypothetical and unobservable state of the economy, hence the difficulty to measure it.

Two/Three kinds of methods:

- **structural** methods: based on a theoretical framework
  - (+) decomposition of potential GDP according to its drivers ; forecasting
  - ($-$) some lack of robustness due to theoretical assumptions
- **statistical** methods: direct extraction from data
  - (+) more robustness because there is no theoretical framework
  - ($-$) no "story"about what drives GDP potential ; no forecasting
- **semi-structural** methods: mix of structural and statistical methods

An illustration: estimation of output gap

The most standard structural method is based on a Cobb-Douglas function of the effective (and potential) production (e.g. European Commission method).

Effective GDP:

$$Y_t = TFP_t K_t^{\alpha} L_t^{1-\alpha}$$

where
TFP_t is the effective total factor productivity
K_t is the capital stock
$L_t = (1 - u_t) T_t POP_t$ is total employment, with u_t the unemployment rate, T_t the participation rate and POP_t the working age population (15-64)

An illustration: estimation of output gap

The most standard structural method is based on a Cobb-Douglas function of the effective (and potential) production.

Potential GDP:

$$Y_t^* = TFP_t^* (K_t^*)^\alpha ((1 - u_t^*) T_t^* POP_t^*)^{1-\alpha}$$

with the following assumption:

- $K_t^* = K_t$ (no room for capital stock utilisation)
- $POP_t^* = POP_t$ (working age population only due to long-run demographic factors)

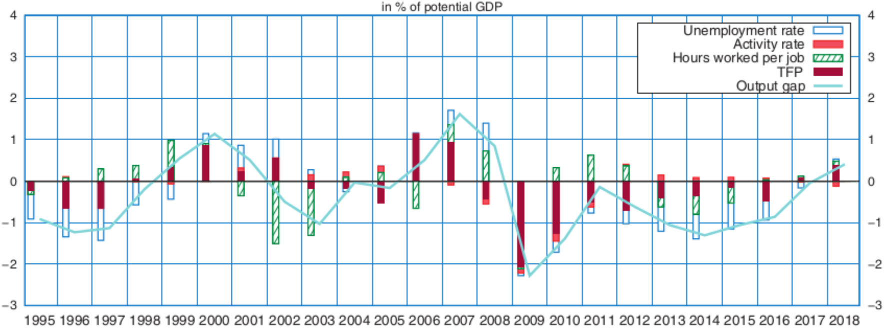
How to calculate TFP^*, u^* and T^* ?

- TFP^* and T^*: HP-filter for example (or more refined methods)
- u^*: NAIRU, so deriving from a Phillips curve

An illustration: estimation of output gap

Example of a structural method applied to calculate French output gap between 1985 and 2018 (Lagouge, Rousset & Virely, 2018)

9 - Contributions (to the output gap for France) estimated using the structural method



Note: the output gap is the one calculated with the structural method

Source: INSEE, Author's calculations

An illustration: estimation of output gap

Another example: a semi-structural method

Estimating the output gap (OG) as a latent variable of two observable indicators:

1. the **production capacity utilisation rate (**TUC**)**
   - calculated from business tendency surveys in industry:"*Give the ratio of your current production to the maximum production attainable if you were to hire additional workers*"
   - TUC is likely to be linked to the *level* of OG
2. the **business climat indicator (**$Climat$**)**
   - synthetic indicator (factor analysis method) of business leaders opinions about past activity, expected activity, general outlook...
   - $Climat$ is likely to be linked to the *variation* of OG

Writing the state-space model and using Kalman filter procedure

An illustration: estimation of output gap

Consider the following state-space model:

- two state variables: OG_t and Δy_t^*
- three observable variables: Δy_t, TUC_t and $Climat_t$

Observation equations:

$$\begin{aligned} \Delta y_t &= \Delta y_t^* + OG_t - OG_{t-1} \\ TUC_t &= TUC_t^{ref} + \alpha OG_t + \varepsilon_{1,t} \\ Climat_t &= 100 + \beta(OG_t - OG_{t-1}) + \varepsilon_{2,t} \end{aligned}$$

with $\varepsilon_{1,t} \sim \mathcal{N}(0, q_1)$ and $\varepsilon_{2,t} \sim \mathcal{N}(0, q_2)$

State equations:

$$\begin{aligned} \Delta y_t^* &= \gamma \Delta y_{t-1}^* + \varepsilon_{y^*,t} \\ OG_t &= \delta OG_{t-1} + \varepsilon_{OG,t} \end{aligned}$$

with $\varepsilon_{y^*,t} \sim \mathcal{N}(0, q_{y^*})$ and $\varepsilon_{OG,t} \sim \mathcal{N}(0, q_{OG})$

An illustration: estimation of output gap

Using Kalman filter procedure with R software:

- `MARSS` package (Multivariate Autoregressive State-Space Modeling)

1. Database

2. Model specification

3. Application of the Kalman filter procedure (ML estimation, Kalman filter and smoother)

4. Results

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