A Fast Labeled Multi-Bernoulli Filter Using Belief Propagation Supplementary Material

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This manuscript provides a pseudocode for the belief propagation (BP) based labeled multi-Bernoulli (BP-LMB) filter proposed in the corresponding publication, 'A Fast Labeled Multi-Bernoulli Filter Using Belief Propagation' by the same authors. The manuscript further provides a brief introduction to the framework of BP, derives the BP-based data association algorithm proposed in the same publication and provides a comparison of the marginal association probabilities used in the labeled multi-Bernoulli (LMB) filter with the approximate marginal association probabilities used in the BP-LMB filter.

1 Pseudocode of BP-LMB Filter

A pseudocode of the BP-LMB filter is depicted in Algorithm 1. Note that the lines 3-10 describe the generation scheme of new Bernoulli components used in the simulation part in [1, Sec. VII]. In this scheme, one new Bernoulli component is generated for each measurement at the previous time $k-1$ that is considered not to be generated by an already existing object. However, for different applications other generation schemes might be more suitable. In fact, a different Bernoulli generation scheme can be accommodated in the BP-LMB filter by replacing the lines 3-10 by the corresponding scheme.

2 A Short Review of Belief Propagation

In this section, we briefly review the methodological basis of factor graphs and the corresponding framework of BP. As an illustrating example, we consider the problem of estimating the parameter vectors $\mathsf{x}^{(j)}$, $j \in \{1, ..., J\}$ given a measurement vector $\boldsymbol{z}$. For obtaining an estimate of $\mathsf{x}^{(j)}$, most Bayesian methods require the posterior pdf $f(\boldsymbol{x}^{(j)}|\boldsymbol{z})$ which is a marginal pdf of the joint posterior pdf $f(\boldsymbol{x}|\boldsymbol{z})$ with $\mathsf{x} = [\mathsf{x}^{(1)\mathrm{T}} \ldots \mathsf{x}^{(J)\mathrm{T}}]^{\mathrm{T}}$. However, often direct marginalization is computationally expensive or infeasible. A fast marginalization can be conducted, if $f(\boldsymbol{x}|\boldsymbol{z})$ factorizes according

$$f(\boldsymbol{x}|\boldsymbol{z}) \propto \prod_{q=1}^{Q} \psi_q(\boldsymbol{x}^{(q)}). \tag{1}$$

Here, each argument $\boldsymbol{x}^{(q)}$ comprises certain parameter vectors $\boldsymbol{x}^{(j)}$. In general, the factors $\psi_q(\boldsymbol{x}^{(q)})$ also depend on $\boldsymbol{z}$.

The factorization structure (1) can be represented by a graphical model known as factor graph. As an example, for $\mathsf{x} = [\mathsf{x}^{(1)\mathrm{T}}\mathsf{x}^{(2)\mathrm{T}}]^{\mathrm{T}}$, the factor graph representing the factorization $f(\boldsymbol{x}|\boldsymbol{z}) \propto \psi_1(\boldsymbol{x}^{(1)})\psi_2(\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)})\psi_3(\boldsymbol{x}^{(2)})$ is depicted in Fig 1. In a factor graph, each parameter variable $\boldsymbol{x}^{(j)}$ is represented by a variable node and each factor $\psi_q(\cdot)$ by a factor node. Variable node "$\boldsymbol{x}^{(j)}$" and factor

Algorithm 1 BP-LMB Filter Algorithm

Initialization at time $k\!=\!0$: $\{(r_0^{(l)}, s^{(l)}(\boldsymbol{x}_0))\}_{l\in\mathbb{L}_0}$;
for $k \geq 1$ **do**
  Prediction Step:
  if $k \geq 2$ **then**
    Determine $\mathbb{L}_k^{\mathrm{B}*}$ using $\hat{p}(b_{k-1}^{(m)}\!=\!0)$, $m \in \{1,\ldots,M_{k-1}\}$, according to [1, Sec. VII-A];
    for $l \in \mathbb{L}_k^{\mathrm{B}*}$ **do**
      Create new Bernoulli component $(r_{\mathrm{B},k}^{(l)}, s_{\mathrm{B}}^{(l)}(\boldsymbol{x}_k))$ with $r_{\mathrm{B},k}^{(l)}$ according to [1, Sec. VII-A] and $s_{\mathrm{B}}^{(l)}(\boldsymbol{x}_k)$ according to [1, Eq. (28)], respectively;
    end for
  else
    $\mathbb{L}_k^{\mathrm{B}*} = \emptyset$;
  end if
  for $l \in \mathbb{L}_{k-1}^{*}$ **do**
    Calculate $r_{k|k-1}^{(l)}$ and $s_{k|k-1}^{(l)}(\boldsymbol{x}_k)$ according to the equations in [2];
  end for
  Determine $\{(r_{k|k-1}^{(l)}, s_{k|k-1}^{(l)}(\boldsymbol{x}_k))\}_{l\in\mathbb{L}_k^*}$ with $\mathbb{L}_k^* = \mathbb{L}_{k-1}^* \cup \mathbb{L}_k^{\mathrm{B}*}$ as $\{(r_{k|k-1}^{(l)}, s_{k|k-1}^{(l)}(\boldsymbol{x}_k))\}_{l\in\mathbb{L}_{k-1}^*} \cup \{(r_{k|k-1}^{(l)}, s_{k|k-1}^{(l)}(\boldsymbol{x}_k))\}_{l\in\mathbb{L}_k^{\mathrm{B}*}}$, where for $l \in \mathbb{L}_k^{\mathrm{B}*}$, $r_{k|k-1}^{(l)} \triangleq r_{\mathrm{B},k}^{(l)}$ and $s_{k|k-1}^{(l)}(\boldsymbol{x}_k) \triangleq s_{\mathrm{B}}^{(l)}(\boldsymbol{x}_k)$;
  Update Step:
  for $l \in \mathbb{L}_k^*$ **do**
    Calculate $\beta_k^{(l,m)}$ for $m \in \{0,\ldots,M_k\}$ as $\beta_k^{(l,m)} = r_{k|k-1}^{(l)}\eta^{(l,m)}$ with $\eta^{(l,m)}$ given in [1, Sec. IV] and for $m=-1$ as $\beta_k^{(l,-1)} = 1 - r_{k|k-1}^{(l)}$, respectively;
    Calculate the spatial pdfs $s^{(l,m)}(\boldsymbol{x}_k)$ for $m \in \{0,\ldots,M_k\}$ according to [1, Eq. (4)];
  end for
  Initialize $\nu_k^{[0](m\to l)}\!=\!1$;
  for $i = 1 : I$ **do**
    Calculate $\zeta_k^{[i](l\to m)}$ according to [1, Eq. (23)];
    Calculate $\nu_k^{[i](m\to l)}$ according to [1, Eq. (24)];
  end for
  Calculate the approximate marginal association probabilities $\hat{p}(a_k^{(l)})$ and $\hat{p}(b_k^{(m)}\!=\!0)$ according to [1, Eq. (25)] and [1, Eq. (26)];
  Calculate the updated existence probabilities $r_k^{(l)}$ and spatial pdfs $s^{(l)}(\boldsymbol{x}_k)$ for $l\!\in\!\mathbb{L}_k^*$ according to [1, Eq. (17)] and [1, Eq. (18)], respectively;
  Object Detection, State Estimation and Pruning:
  An object with $l\!\in\!\mathbb{L}_k^*$ is detected, if $r_k^{(l)}$ is larger than some threshold γ_{D}. For each detected object, an estimate is computed according to $\hat{\boldsymbol{x}}_k = \int \boldsymbol{x}_k s_k^{(l)}(\boldsymbol{x}_k)\mathrm{d}\boldsymbol{x}_k$;
  Prune all Bernoulli components with $r_k^{(l)} < \gamma_{\mathrm{P}}$ for $l \in \mathbb{L}_k^*$;
end for

node "ψ_q" are adjacent, i.e., connected by an edge, if the variable $\boldsymbol{x}^{(j)}$ is an argument of the factor $\psi(\cdot)$, i.e., part of $\boldsymbol{x}^{(q)}$.

BP is based on a factor graph and aims at computing the marginal association pdfs $f(\boldsymbol{x}^{(j)}|\boldsymbol{z})$ in a fast way. For each node, certain messages are calculated, each of which is passed to one of the adjacent nodes. For each variable node, the messages are functions of the corresponding variable. More specifically, consider a variable node "$\boldsymbol{x}^{(j)}$" and an adjacent factor node "ψ_q", i.e., the variable $\boldsymbol{x}^{(j)}$ is part of the argument $\boldsymbol{x}^{(q)}$ of $\psi_q(\boldsymbol{x}^{(q)})$. The neighborhood set $\mathcal{N}_q$ is defined as the set of indices j of all variable nodes "$\boldsymbol{x}^{(j)}$" that are adjacent to the factor node "ψ_q". Then, the message passed from factor node "ψ_q" to variable node "$\boldsymbol{x}^{(j)}$" is given by

$$\zeta^{(\psi_q\to\boldsymbol{x}^{(j)})}(\boldsymbol{x}^{(j)}) = \int \psi_q(\boldsymbol{x}^{(q)}) \prod_{j'\in\mathcal{N}_q\setminus\{j\}} \eta^{(\boldsymbol{x}^{(j')}\to\psi_q)}(\boldsymbol{x}^{(j')})\mathrm{d}\boldsymbol{x}^{(\sim j)}, \tag{2}$$

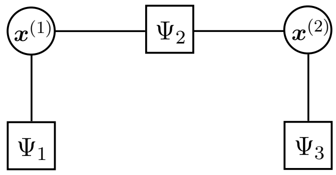

Figure 1: Factor graph representing the factorization of the joint posterior pdf $f(\boldsymbol{x}|\boldsymbol{z}) \propto \psi_1(\boldsymbol{x}^{(1)})\psi_2(\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)})\psi_3(\boldsymbol{x}^{(2)})$. Variable nodes are depicted as circles, factor nodes as squares, respectively.

where $\eta^{(\boldsymbol{x}^{(j')} \to \psi_q)}(\boldsymbol{x}^{(j')})$ is the message passed from variable node "$\boldsymbol{x}^{(j')}$" to factor node "ψ_q" and $\int \ldots \mathrm{d}\boldsymbol{x}^{(\sim j)}$ denotes integration with respect to all vectors $\boldsymbol{x}^{(j')} \in \mathcal{N}_q$ except $\boldsymbol{x}^{(j)}$. For example, the message passed from factor node "ψ_2" to variable node "$\boldsymbol{x}^{(2)}$" in Fig. 1 is $\zeta^{(\psi_2 \to \boldsymbol{x}^{(2)})}(\boldsymbol{x}^{(2)}) = \int \psi_2(\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)})\eta^{(\boldsymbol{x}^{(1)} \to \psi_2)}(\boldsymbol{x}_1)\mathrm{d}\boldsymbol{x}^{(1)}$.

Furthermore, the message $\eta^{(\boldsymbol{x}^{(j)} \to \psi_q)}(\boldsymbol{x}^{(j)})$ passed from variable node "$\boldsymbol{x}^{(j)}$" to factor node "ψ_q" is given by the product of the messages passed to variable node "$\boldsymbol{x}^{(j)}$" from all adjacent factor nodes except "ψ_q",i.e.,

$$\eta^{(\boldsymbol{x}^{(j)} \to \psi_q)}(\boldsymbol{x}^{(j)}) = \prod_{q' \in \mathcal{N}_j \setminus \{q\}} \zeta^{(\psi_{q'} \to \boldsymbol{x}^{(j)})}(\boldsymbol{x}^{(j)}), \tag{3}$$

where the neighborhood set $\mathcal{N}_j$ comprise the set of all indices q of factor nodes "ψ_q" that are adjacent to variable node "$\boldsymbol{x}^{(j)}$". For example, in Fig. 1, the message passed from variable node "$\boldsymbol{x}^{(2)}$" to factor node "ψ_3" is $\eta^{(\boldsymbol{x}^{(2)} \to \psi_3)}(\boldsymbol{x}^{(2)}) = \zeta^{(\psi_2 \to \boldsymbol{x}^{(2)})}(\boldsymbol{x}^{(2)})$. BP is started at variable nodes with only one edge (which pass a constant message) and/or factor nodes with only one edge (which pass the corresponding factor). Note that BP can also be applied to factorizations involving discrete variables by replacing in (2) integration with summation.

After all messages have been passed as described above, for each variable node "$\boldsymbol{x}^{(j)}$", a belief $\tilde{f}(\boldsymbol{x}^{(j)})$ is calculated as the product of all incoming messages (passed from all adjacent factor nodes) followed by a normalization such that $\int \tilde{f}(\boldsymbol{x}^{(j)})\mathrm{d}\boldsymbol{x}^{(j)} = 1$. For example, in Fig. 1,

$$\tilde{f}(\boldsymbol{x}^{(2)}) \propto \zeta^{(\psi_2 \to \boldsymbol{x}^{(2)})}(\boldsymbol{x}^{(2)})\zeta^{(\psi_3 \to \boldsymbol{x}^{(2)})}(\boldsymbol{x}^{(2)}). \tag{4}$$

If the factor graph is a tree, then the obtained belief $\tilde{f}(\boldsymbol{x}^{(j)})$ is exactly equal to the marginal posterior pdf $f(\boldsymbol{x}^{(j)}|\boldsymbol{z})$. For factor graphs with loops, BP can be applied in an iterative manner, and the beliefs $\tilde{f}(\boldsymbol{x}^{(j)})$ are only approximations of the respective marginal posterior pdfs $f(\boldsymbol{x}^{(j)}|\boldsymbol{z})$. In these iterative "loopy BP" schemes, there is no canonical order in which the messages should be calculated, and different orders may lead to different beliefs.

3 Derivation of BP-based Data Association Algorithm

In this section, we derive the BP-based data association algorithm, more precisely the equations [1, Eq. (23)] and [1, Eq. (24)], presented in [1, Sec. VI-A]. This is done by running BP on the factor graph depicted in [1, Fig. 1]. In fact, messages are sent between the factor nodes "$\Psi_{l,m}\big(a_k^{(l)}, b_k^{(m)}\big)$" and the variable nodes "$a_k^{(l)}$" and "$b_k^{(m)}$". More specifically, at message passing iteration $i \in \{1, \ldots, I\}$, a message $\zeta^{[i](\Psi_{l,m} \to b_k^{(m)})}(b_k^{(m)})$ is passed from "$\Psi_{l,m}\big(a_k^{(l)}, b_k^{(m)}\big)$" to "$b_k^{(m)}$" and another message $\nu^{[i](\Psi_{l,m} \to a_k^{(l)})}(a_k^{(l)})$ from "$\Psi_{l,m}\big(a_k^{(l)}, b_k^{(m)}\big)$" to "$a_k^{(l)}$", respectively. Since each factor node is just connected to only two variable nodes, an outgoing message from such a factor node is obtained according to (3), where the incoming message from the corresponding variable node is obtained according to (2). By further replacing the integral operator by the sum operator, one obtains

$$\zeta^{[i](\Psi_{l,m}\to b_k^{(m)})}(b_k^{(m)}) = \sum_{a_k^{(l)}=-1}^{M_k} \beta_k^{(l,a_k^{(l)})}\,\Psi_{l,m}(a_k^{(l)}, b_k^{(m)}) \prod_{\substack{m'=1\\ m'\neq m}}^{M_k} \nu^{[i-1](\Psi_{l,m'}\to a_k^{(l)})}(a_k^{(l)}) \tag{5}$$

and

$$\nu^{[i](\Psi_{l,m}\to a_k^{(l)})}(a_k^{(l)}) = \sum_{b_k^{(m)}\in(0\cup\mathbb{L}_k^*)} \Psi_{l,m}(a_k^{(l)}, b_k^{(m)}) \prod_{l'\in\mathbb{L}_k^*\setminus\{l\}} \zeta^{[i](\Psi_{l',m}\to, b_k^{(m)})}(b_k^{(m)}) \tag{6}$$

for $l\in\mathbb{L}_k^*$ and $m\in\{1,\ldots,M_k\}$. Following [3], the vector-valued messages (5) and (6) can be simplified to scalar ones. Because of the admissibility constraint $\Psi_{l,m}\big(a_k^{(l)}, b_k^{(m)}\big)$ each message comprises only two different values. In fact, for $\zeta^{[i](\Psi_{l,m}\to b_k^{(m)})}(b_k^{(m)})$ we get

$$\zeta^{[i](\Psi_{l,m}\to b_k^{(m)})}(b_k^{(m)}) = \begin{cases} \zeta_{l,m} & , b_k^{(m)} = l \\ \\ \zeta'_{l,m} & , b_k^{(m)} \neq l, \end{cases} \tag{7}$$

where $\zeta_{l,m} = \beta_k^{(l,m)}\prod_{m'=1,m'\neq m}^{M_k}\nu^{[i-1](\Psi_{l,m'}\to a_k^{(l)})}(m)$ and $\zeta'_{l,m} = \sum_{a_k^{(l)}=-1,a_k^{(l)}\neq m}^{M_k}\beta_k^{(l,a_k^{(l)})}$ $\times\nu^{[i-1](\Psi_{l,m'}\to a_k^{(l)})}(a_k^{(l)})$. Since we are free to normalize messages, we define $\zeta'^{[i](\Psi_{l,m}\to, b_k^{(m)})}(b_k^{(m)})$ as the quotient of (7) and $\zeta'_{l,m}$ which leads

$$\zeta'^{[i](\Psi_{l,m}\to b_k^{(m)})}(l) = \zeta_{l,m}/\zeta'_{l,m} \tag{8}$$

and to $\zeta'^{[i](\Psi_{l,m}\to b_k^{(m)})}(b_k^{(m)}) = 1$ for $b_k^{(m)}\neq l$. In a similar way, we can define the "normalized" message $\nu'^{[i](\Psi_{l,m}\to a_k^{(l)})}(a_k^{(l)})$ as

$$\nu'^{[i](\Psi_{l,m}\to a_k^{(l)})}(m) = \nu_{l,m}/\nu'_{l,m} \tag{9}$$

with $\nu_{l,m} = \prod_{l'\in\mathbb{L}_k^*\setminus\{l\}}\zeta^{[i](\Psi_{l',m}\to, b_k^{(m)})}(l)$ and $\nu'_{l,m} = \sum_{b_k^{(m)}\in(0\cup\mathbb{L}_k^*), b_k^{(m)}\neq l}\prod_{l'\in\mathbb{L}_k^*\setminus\{l\}}\zeta^{[i](\Psi_{l',m}\to, b_k^{(m)})}(b_k^{(m)})$ and $\nu'^{[i](\Psi_{l,m}\to a_k^{(l)})}(a_k^{(l)}) = 1$ for $a_k^{(l)}\neq m$. Next, we define the message $\bar{\zeta}^{[i](\Psi_{l,m}\to b_k^{(m)})}(b_k^{(m)})$ as the quotient of the quantities $\bar{\zeta}_{l,m}$ and $\bar{\zeta}'_{l,m}$, where both quantities are defined equally as $\zeta_{l,m}$ and $\zeta'_{l,m}$ except that $\nu^{[i-1](\Psi_{l,m'}\to a_k^{(l)})}(m)$ is replaced by their normalized version $\nu'^{[i-1](\Psi_{l,m'}\to a_k^{(l)})}(m)$. Further using the fact that all but one values of $\nu'^{[i-1](\Psi_{l,m'}\to a_k^{(l)})}(m)$ are one, the message $\bar{\zeta}^{[i](\Psi_{l,m}\to b_k^{(m)})}(b_k^{(m)})$ simplifies to

$$\bar{\zeta}^{[i](\Psi_{l,m}\to b_k^{(m)})}(l) = \frac{\beta_k^{(l,m)}}{\beta_k^{(l,-1)} + \beta_k^{(l,0)} + \sum_{\substack{m'=1\\ m'\neq m}}^{M_k}\beta_k^{(l,m')}\nu^{[i-1](\Psi_{l,m'}\to a_k^{(l)})}(m)} \tag{10}$$

and to $\bar{\zeta}^{[i](\Psi_{l,m}\to b_k^{(m)})}(b_k^{(m)}) = 1$ for $b_k^{(m)}\neq l$. An analogous approach results in the message

$$\bar{\nu}^{[i](\Psi_{l,m}\to a_k^{(l)})}(m) = \frac{1}{1+\sum_{l'\in\mathbb{L}_k^*\setminus\{l\}}\zeta_k^{[i](l'\to m)}(l)} \tag{11}$$

and $\bar{\nu}^{[i](\Psi_{l,m}\to a_k^{(l)})}(a_k^{(l)}) = 1$ for $a_k^{(l)}\neq m$. Since both messages $\bar{\zeta}^{[i](\Psi_{l,m}\to b_k^{(m)})}(b_k^{(m)})$ and $\bar{\nu}^{[i](\Psi_{l,m}\to a_k^{(l)})}(a_k^{(l)})$ just consist of one value that is different from one, we define the scalar-valued messages $\zeta_k^{[i](l\to m)}$ and $\nu_k^{[i](m\to l)}$ which contain the same information than the vector-valued messages (5) and (6). Theses

$p(a_{60}^{(l^{(1)})} = m)$	exact	$I = 1$	$I = 2$	$I = 5$
$p(a_{60}^{(l^{(1)})} = -1)$	0.5126	0.3637	0.5015	0.5126
$p(a_{60}^{(l^{(1)})} = 0)$	0.0928	0.0659	0.0908	0.0928
$p(a_{60}^{(l^{(1)})} = 1)$	1.87×10^{-5}	0.007	2.67×10^{-5}	1.87×10^{-5}
$p(a_{60}^{(l^{(1)})} = 2)$	0	0	0	0
$p(a_{60}^{(l^{(1)})} = 3)$	6.20×10^{-12}	2.77×10^{-9}	1.67×10^{-10}	6.20×10^{-5}
$p(a_{60}^{(l^{(1)})} = 4)$	0.3945	0.5633	0.4077	0.3945
$p(a_{60}^{(l^{(1)})} = 5)$	2.09×10^{-6}	1.6×10^{-4}	2.04×10^{-6}	2.08×10^{-6}

Table 1: Comparison of the exact marginal probabilities $p(a_{60}^{(l_1)})$ and the approximate marginal probabilities $\hat{p}(a_{60}^{(l_1)})$ for $I = \{1, 2, 5\}$ BP iterations at time $k = 60$.

messages are given by the expressions [1, Eq. (23)] and [1, Eq. (24)], respectively.

4 Comparison of Exact and Approximate Marginal Association Probabilities

In this section, we present a comparison of the exact marginal probabilities used in the original LMB filter and the approximate marginal probabilities used in the proposed BP-LMB filter. For the matter of comparison, we used a scenario comprising seven Bernoulli components and five measurements. This scenario was obtained by running the BP-LMB filter on the simulation scenario described in [1, Sec. VII-A] and extracting the sensor measurements and Bernoulli components employed by the BP-LMB filter after the prediction step at time $k = 60$. That is, the Bernoulli components are parametrized by $\big\{\big(r_{60|59}^{(l)}, s_{60|59}^{(l)}(\boldsymbol{x}_{60})\big)\big\}_{l \in \mathbb{L}_{60}^*}$ with different existence probabilities $r_{60|59}^{(l)}$, spatial pdfs $s_{60|59}^{(l)}(\boldsymbol{x}_{60})$ and corresponding label set $\mathbb{L}_{60}^* = \{l^{(1)}, \ldots, l^{(7)}\}$. Furthermore, the measurements are given by $Z_{60} = \{\boldsymbol{z}_{60}^{(1)}, \ldots, \boldsymbol{z}_{60}^{(5)}\}$. The simulation parameters and the parameters used in the BP-LMB filter were chosen equally to [1, Sec. VII-A] except that the number of objects and the clutter rate were both set to five. This was necessary because the calculation of the exact marginal probabilities becomes infeasible for a higher number of objects/clutter. Note that, although the number of objects and clutter measurement was reduced to five, the scenario is still challenging as the objects are in close proximity around $k = 60$.

Given the parameters of the Bernoulli components and the measurements, we calculated the exact marginal probabilities according to [1, Eq. 14] and the approximate marginal probabilities by using the equations [1, Eq. 23], [1, Eq. 24] and [1, Eq. 25]. The results for $p(a_{60}^{(l^{(1)})})$ are depicted in Table 1. Whereas the values of the approximate marginal probabilities deviate from its exact values for $I = \{1, 2\}$ BP iterations, they are almost identical for $I = 5$ iterations. A similar behavior was observed for the remaining marginal association probabilities $p(a_{60}^{(l)})$, $l \in \mathbb{L}_{60}^* \setminus \{l^{(1)}\}$, and also for all marginal probabilities at different times k. Hence, this shows that for the chosen scenario the approximate marginal association probabilities converge to its exact values within a few BP iterations. However, as the association problem becomes more difficult, i.e., it consists of a higher number of (close) objects and clutter measurements, the BP algorithm needs also a higher number of I iterations to converge. This is the reason why we set $I = 20$ for the simulations conducted in [1, Sec. VII-A].

References

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