My Experience With the Subject of Mathematics

Thobias K. Høivik

Start date: February 27, 2025 — Last updated: March 2, 2025

1 Early Memories (2008)

While I do not think I had any profound interest in mathematics at a young age, I have one distinct memory pertaining to numbers. Some children were competing to count to the highest number, not realizing the fact that they would always be able to add one to get a bigger number, which I remember finding upsetting.

2 Primary and Middle School (2010–2018)

- Standard Norwegian curriculum.
- Basic algebra, equations, and problem-solving techniques.

10th Grade Oral Examination

In 10th grade we were tasked to hold a short informal presentation before the teacher explaining some problem and how to solve it mathematically. My interest in more abstract concepts showed itself when, where the others had opted to explain how to find the amount of planks needed for a triangular floor with an area A, or similar problems, I had chosen to tell my own version of Hilbert's Hotel using a warehouse as a surrogate. The teacher told me that it was not relevant to the subject and I should have done a different problem.

3 High School (VGS) (2020–2023)

- Learned basic calculus, including differentiation and integration up to integration by parts.
- Had some probability and statistics which I found horribly boring and I believe I encountered trigonometry.
- No formal linear algebra coursework at this stage.

4 Programming and Linear Algebra (2022–2023)

- Developed a strong interest in programming.
- Learned fundamental linear algebra concepts:
  - Vectors and their properties
  - Dot and cross products
  - Matrix operations

3D Renderer on the CPU

My interest in linear algebra resulted in two 3D rendering engines written from scratch, in JavaScript and later Java.

5 University and Discrete Mathematics (2024)

5.1 First Semester (Spring 2024)

- Began my Bachelor's in IT (Specializing in Machine Learning Engineering and Artificial Intelligence).
- Took Discrete Mathematics 1, which deepened my interest in mathematics.

First Proof

I can't remember it exactly, and I remember having "proofs" in high-school, but I regard these sorts of problems in set-theory to be the first proofs I ever did. Let A, B, C be sets. Prove that if $a \in A$ and $a \in C$, then $a \in B \cup (C \cap A)$. We are given that:

1. $a \in A$
2. $a \in C$

We need to show that $a \in B \cup (C \cap A)$.

- Since $a \in C$ and $a \in A$, by the definition of intersection, we conclude that $a \in C \cap A$.
- By the definition of union, since $a \in C \cap A$, it follows that $a \in B \cup (C \cap A)$, regardless of whether $a \in B$.

Thus, $a \in B \cup (C \cap A)$, completing the proof. $\square$

Return of Hilbert's Hotel

I spent a lot of time jumping forward in my discrete math book, by Susanna S. Epp, and looking at what was next. During one of these times I came across a familiar problem. Prove that $|\mathbb{N}| = |\mathbb{Z}|$ Two sets have the same cardinality if there exists a bijection (a one-to-one and onto function) between them. We define a function $f : \mathbb{N} \to \mathbb{Z}$ as follows:

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ -\frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

Injectivity: Suppose $f(n_1) = f(n_2)$. We consider two cases: 1. If both n_1 and n_2 are even, then $f(n_1) = \frac{n_1}{2}$ and $f(n_2) = \frac{n_2}{2}$, which implies $\frac{n_1}{2} = \frac{n_2}{2}$, so $n_1 = n_2$. 2. If both n_1 and n_2 are odd, then $f(n_1) = -\frac{n_1+1}{2}$ and $f(n_2) = -\frac{n_2+1}{2}$, which implies $\frac{n_1+1}{2} = \frac{n_2+1}{2}$, so $n_1 = n_2$. Thus, f is injective. Surjectivity: For any integer $z \in \mathbb{Z}$, we need to find some $n \in \mathbb{N}$ such that $f(n) = z$. 1. If $z \geq 0$, we choose $n = 2z$. Then, $f(n) = f(2z) = \frac{2z}{2} = z$. 2. If $z < 0$, we choose $n = -2z - 1$. Then, $f(n) = f(-2z-1) = -\frac{(-2z-1)+1}{2} = -\frac{-2z}{2} = z$. Thus, f is surjective. Since f is both injective and surjective, it is a bijection, proving that $|\mathbb{N}| = |\mathbb{Z}|$. $\square$

5.2 Winter Break (December 2024)

- Completed a Calculus II Udemy course in 4 days.
- Explored Real Analysis, learning about:
  - Limits and their formal definition
  - Convergence proofs

5.3 Second Semester (Spring 2025)

- Developed a connection with my Algorithms professor (PhD in topology and complexity theory).
- Introduced to Abstract Algebra, covering:
  - Isomorphisms
  - Cosets and Kernels
  - Groups, Rings and Fields
- Started exploring Combinatorics, fascinated by formula derivations.

First Proof in Abstract Algebra

Prove that $\mathbb{R}-\{1\}$ under the operation $\star$, with $a, b \in \mathbb{R}-\{1\} \Rightarrow a \star b = a+b-ab$, forms a group.

1) $a, b \in \mathbb{R} - \{1\} \Rightarrow a \neq 1 \neq b$

   Now let's assume $\star$ is not a binary operation on $\mathbb{R} - \{1\}$,

   meaning we can get $a \star b = 1 \notin \mathbb{R} - \{1\}$

   $a \star b = 1 \Leftrightarrow a + b - ab = 1 \Leftrightarrow a + b = 1 + ab$

   Take the partial derivative with respect to a on both sides to get

   $\dfrac{\partial}{\partial a}[a + b] = \dfrac{\partial}{\partial a}[1 + ab]$

   yielding $1 = b$ which contradicts $b \neq 1$

   Conversely,

   $\dfrac{\partial}{\partial b}[a + b] = \dfrac{\partial}{\partial a}[1 + ab]$

   $1 = a$

   $\therefore a \star b = 1 \Leftrightarrow a = 1 \vee b = 1$

   Thus $a, b \in \mathbb{R} - \{1\}$ gives $a \star b \neq 1$,

   furthermore $a \star b \in \mathbb{R} - \{1\}$

2) $(a \star b) \star c = (a + b - ab) \star c = (a + b - ab) + c - (a + b - ab) \cdot c$

   $= a + b + c - ab - ac - bc + abc = a + (b + c - bc) - a \cdot (b + c - bc)$

   $= a \star (b \star c)$, $\therefore \star$ is associative on $\mathbb{R} - \{1\}$

3) Let the identity $e = 0$, $a \star e = a + 0 - a \cdot 0 = a$

4) For this to be a group we need that $\forall a \in \mathbb{R} - \{1\}$

   $\exists a^{-1}$ such that $a \star a^{-1} = e = 0$

   Let $x = a^{-1}$, $a \star x = a + x - ax$

   $a + x - ax = 0$

   $x - ax = -a$

   $x(1 - a) = -a$

   $x = -\dfrac{a}{1 - a} = a^{-1} \because 1 - a \neq 0 + \forall a \in \mathbb{R} - \{1\}$ $\square$

Thus, we have shown closure, associativity, the existence of an identity and inverses, making $\mathbb{R}-\{1\}$ a group under $\star$. **Note** that this is not a solid proof, just how I solved the problem first time around. In part 1), using partial derivatives is not really a valid way to show closure under $\star$, it should instead be done algebraically. Which I find a real shame since using partial derivatives where it doesn't really belong is much more fun.

6 Long Term Goals

- Take extra courses in mathematics during my undergraduate studies to gain a broad skillset in mathematics, and such that I may qualify to further programs in mathematics.
- Educate myself further in mathematics and theoretical computer science and hopefully find myself in the priveleged position of being able to dedicate my life to researching these two areas and their intersection.