Theory of Computation
Lecture 2 (MIT 18.404J)

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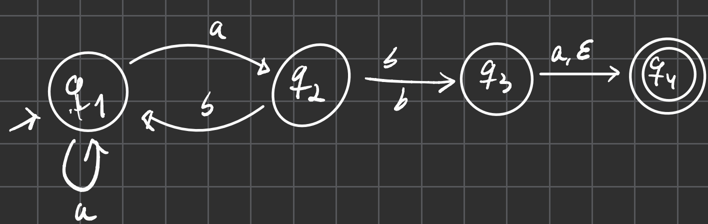
Nondeterministic Finite automata

N_1



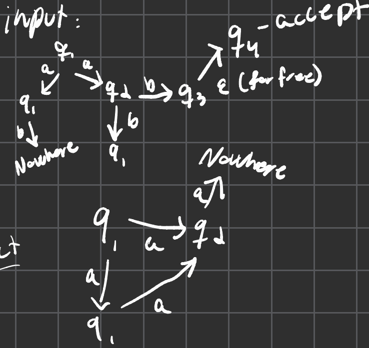
New features:

- multiple paths possible (0, 1 or many at each step)
- ε-transition (free move w/o input)
- Accept inputs if <u>some</u> paths lead to ◎ accept

Example input:

- ab
  - Accept

- aa
  - Reject



While nondeterminism does not correspond to a physical machine we can build, they are useful mathematically.

NFA - Formal defn

Def: A nondeterministic finite automaton N is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, the same as a FA, except:

$$\delta : Q \times \Sigma_\varepsilon \to P(Q) = \{R \mid R \subseteq Q\},$$

$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$, $P(Q)$ the powerset of Q.

Example from N_1:

$$\delta(q_1, a) = \{q_1, q_2\}$$

$$\delta(q_1, b) = \emptyset$$

Ways to think of nondeterminism:

Computational: Parallel threads where accepting any thread leads to accept state.

Mathematical: Tree with branches. Accept if any leaf node is accept.

"Magical": Guess at each step. machine will always find the accept route, if it exists.

Converting NFA → DFA

Thm: If an NFA recognizes A, then A is regular.

Pf: Let NFA $N = (Q, \Sigma, \delta, q_0, F)$ recognize A, Construct DFA $N' = (Q', \Sigma, \delta', q_0', F')$ recognizing A.

$Q' = P(Q)$, $R \in Q'$

$\delta'(R, a) = \{ q \mid q \in \delta(r, a), r \in R \}$

$q_0' = \{ q_0 \}$

$F' = \{ R \in Q' \mid R \cap F \neq \phi \}$

If N accepts w $\forall w \in A$, there exists one computation that leads to acceptance. All possible computations $R \in Q'$. At least one R accepts w and $R \cap F \neq \phi \Rightarrow R \in F'$ so N' accepts w. $\square$

Closure properties again:

Recall Thm: A_1, A_2 regular languages $\Rightarrow A_1 \cup A_2$ is regular.

Pf: Let M_1, M_2 be DFA's, $L(M_1) = A_1$, $L(M_2) = A_2$.
Take q_1, q_2 to be startstates of M_1 and M_2, respectively.

Construct NFA N such that with startstate q_0,
$\delta(q_0, \varepsilon) = \{q_1, q_2\}$
$w \in A_1 \cup A_2 \Rightarrow w \in A_1 \vee w \in A_2$
Since $\delta(q_0, \varepsilon) = \{q_1, q_2\}$ the startstates of M_1 and M_2
M_1 and M_2 will get to read w and one will accept it.
Thus N accepts $w \in A_1 \cup A_2$, making $A_1 \cup A_2$ regular. $\square$