Innlevering 1 Mat104

1) a) $f(x) = x - \sqrt{2x-4} = 2$

$$-\sqrt{2x-4} = 2-x$$

$$\sqrt{2x-4} = x-2$$

$$\sqrt{2(x-2)} = x-2$$

$$2(x-2) = (x-2)^2$$

$$2x-4 = x^2-4x+4$$

$$0 = x^2-6x+8$$

$$0 = (x-4)(x-2)$$

$$x-4=0 \Leftrightarrow x=4$$

$$x-2=0 \Leftrightarrow x=2$$

$$\therefore\ x=2 \wedge x=4$$

b) $f(x) = x - \sqrt{2x-4} < 2 \quad / -2 + \sqrt{2x-4}$

$$x-2 < \sqrt{2x-4}$$

$$(x-2)^2 < 2x-4$$

$$x^2 - 4x + 4 < 2x - 4$$

$$x^2 - 6x + 8 < 0$$

$$(x-2)(x-4) < 0$$

$$x - 4 < 0 \to \underline{x < 4}$$

$\sqrt{2x-4} \notin \mathbb{R}$ når $x < 2$ $\wedge$ $f(2) = 2$

så $x > 2$

$$x > 2 \wedge x < 4 \Leftrightarrow \underline{x \in \langle 2, 4 \rangle}$$

c) $x < 2 \Rightarrow \sqrt{2x-4} \notin \mathbb{R}$

$\therefore$ $\underline{D_f = [2, \infty\rangle}$

Made with Goodnotes

d) $\frac{d}{dx}\left(x - \sqrt{2x-4}\right)$

$= \frac{d}{dx}\left(x - (2x-4)^{\frac{1}{2}}\right)$

$= \frac{d}{dx} x + \frac{d}{dx} -(2x-4)^{\frac{1}{2}}$

$= 1 + \left(-\frac{1}{2}\sqrt{2x-4}^{-1} \cdot 2\right)$

$= 1 - \frac{\cancel{2}}{\cancel{2}\sqrt{2x-4}} = 1 - \frac{1}{\sqrt{2x-4}}$

$1 - \frac{1}{\sqrt{2x-4}} = 0$

$1 = \frac{1}{\sqrt{2x-4}} \quad / \cdot \sqrt{2x-4}$

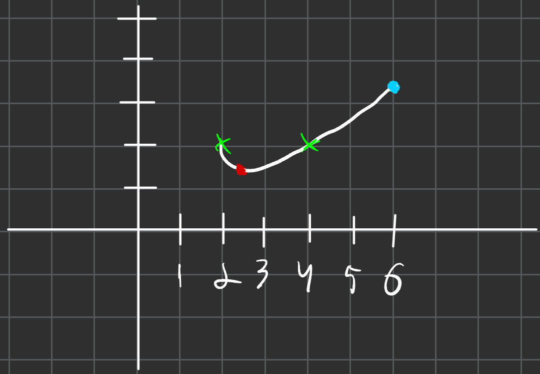
$\sqrt{2x-4} = 1$

$2x - 4 = 1$

$2x = 5$

$x = \frac{5}{2}$ $\quad f\left(\frac{5}{2}\right) = \frac{5}{2} - \sqrt{5-4}$

$= \frac{3}{2}$



$f(6) = 6 - \sqrt{12-4}$

$= 6 - \sqrt{8}$

$= 6 - \sqrt{2^3}$

≈ 3.17

e) $V_f \approx \left[\frac{3}{2}, 3.17\right] \approx [1.5, 3.2]$

f) <u>Nei</u> $f(2) = f(4)$

2) a) $\frac{a(a^2-b^2)}{a-b} = \frac{a(a+b)\cancel{(a-b)}}{\cancel{a-b}} = \underline{a^2+ab}$

b) $\frac{(x-1)^2+(x+1)^2-4x}{(x+1)^2}$

$= \frac{x^2-\cancel{2x}+1+x^2+\cancel{2x}+1+4x}{(x+1)^2} = \frac{2x^2+4x+2}{(x+1)^2}$

$= \frac{2(x^2+2x+1)}{(x+1)^2} = \frac{2\cancel{(x+1)(x+1)}}{\cancel{(x+1)(x+1)}}$

$= \underline{2}$

c) $\frac{a^2(a^2b)^3}{(b^3)^2 \cdot a} = \frac{a^8 \cdot b^3}{a \cdot b^6}$

$= \frac{a^7}{b^3} = \underline{a^7 b^{-3}}$

d) $\dfrac{3^{-1} \cdot a^{-1} \cdot b}{6^{-1} \cdot (a^{-2} \cdot b^{3})^{2}} = \dfrac{6\,b}{3\,a \cdot a^{-4} \cdot b^{6}}$

$= 2\dfrac{b\,a^{4}}{a\,b^{6}} = 2\dfrac{a^{3}}{b^{5}} = \underline{2a^{3}b^{-5}}$

e) $\dfrac{(x^{2} \cdot y)^{3} \cdot (x^{-1} \cdot y^{2})^{-1}}{(2x \cdot y^{3})^{2}} = \dfrac{x^{6}\,y^{3}}{4(x^{2} \cdot y^{6} \cdot x^{-1} \cdot y^{2}}$

$= \dfrac{x^{6}\,y^{3}}{4\,x \cdot y^{8}} = \dfrac{1}{4}\,x^{5}\,y^{-5}$

$= \underline{\dfrac{1}{4}\left(\dfrac{x}{y}\right)^{5}}$

Made with Goodnotes

3) $\displaystyle\sum_{j=1}^{2}\sum_{i=1}^{j} i\cdot j$

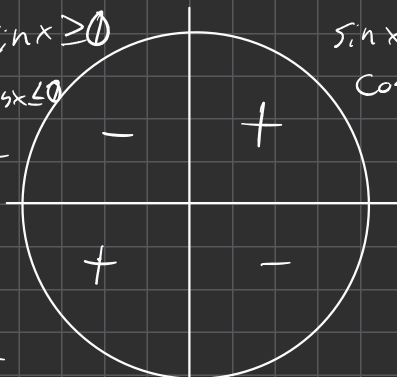
$j=1:\ 1\cdot 1 = \underline{1}$

$j=2:\ 2\cdot 1 + 2\cdot 2 = \underline{6}$

$\underline{\underline{7}}$

4) $\sin x \geq 0$, $\cos x \leq 0$

$\tan x = \frac{+}{-} = -$



$\sin x \geq 0$, $\cos x \geq 0$

$\tan x = \frac{\sin x}{\cos x}$ $\quad \frac{+}{+} \to +$

$\sin x \leq 0$, $\cos x \leq 0$

$\frac{-}{-} = +$

b) $\tan\theta > 0 \Leftrightarrow \frac{\sin\theta}{\cos\theta} > 0$

$2\sin\theta = \tan\theta$

$2\sin\theta = \frac{\sin\theta}{\cos\theta}$

$2 = \frac{1}{\cos\theta} = \sec\theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \arccos\frac{1}{2} = \underline{\underline{\frac{\pi}{3}}}$ eller $\underline{\underline{\frac{4\pi}{3}}}$

c) $\tan\theta < 0$

$$\Rightarrow 2\sin\theta = \frac{-\sin\theta}{\cos\theta} = \frac{\sin\theta}{-\cos\theta}$$

$$\Rightarrow 2 = -\frac{1}{\cos\theta}$$

$$2\cos\theta = -1$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \arccos\left(-\frac{1}{2}\right) = \frac{\pi}{3} + \frac{\pi}{3} = \underline{\underline{\frac{2\pi}{3}}}$$

eller $\frac{2}{3}\pi + \frac{3}{3}\pi = \underline{\underline{\frac{5}{3}\pi}}$

Made with Goodnotes

5) a) $5 \cdot 10^8$ kg

$V = f(h) = 50m \cdot 300m \cdot h_m$

$= 15000\,m^3 \cdot h$

b) $V_{ann} = 1000\,kg/m^3$

$5 \cdot 10^8$ kg

Volum av $5 \cdot 10^8$ kg vann

$$= \frac{5 \cdot 10^8 \cancel{kg} \cdot m^3}{10^3 \cancel{kg}} = 5 \cdot 10^5\,m^3$$

$f(h) = 5 \cdot 10^5\,m^3$

$15000\,m^3 \cdot h = 5 \cdot 10^5\,m^3$

$15 \cdot 10^3\,m^2 \cdot h_m = 5 \cdot 10^5\,m^3$

$15 \cdot h_m = 5 \cdot 10^2\,m$

$h_m = \frac{5}{15} \cdot 10^2\,m \quad \frac{1}{3} \cdot 100m$

$h \approx 33{,}33..m$

c) $V = f(45) = 675000\ m^3$

$M_v = 1000\ Kg/m^3 \times 675000\ m^3$

$= 10^3\ Kg/m^3 \times 6.75 \times 10^5\ m^3 = 6{,}75 \times 10^8\ Kg$

La $g(x) = 5 \times 10^8\ Kg + 10^5\ Kg \cdot x$

$g(x) = 6.75 \cdot 10^8\ Kg$

$5 \times 10^8 + 10^5\ Kg \cdot x = 6.75 \cdot 10^8\ Kg$

$10^5\ Kg \cdot x = (6.75 - 5) \cdot 10^8\ Kg$

$10^5\ Kg \cdot x = (1.75)\ 10^8\ Kg$

$x = (1.75)\ 10^{8-5}$

$x = 1.75 \cdot 10^3 = \underline{\underline{1750}}$

6) $\lim_{x\to\infty} \frac{3x^2-9x-99}{2x^2+5x+1}$

Grenseverdien er i formen:

$\frac{\infty}{\infty}$ så vi kan bruke l'Hopital

Merk at $\lim_{x\to\infty} \frac{\frac{d}{dx}3x^2-9x-99}{\frac{d}{dx}2x^2+5x+1}$

er også i $\frac{\infty}{\infty}$ form så

$$\lim_{x\to\infty} \frac{3x^2-9x-99}{2x^2+5x+1}$$

$$= \frac{\frac{d^2}{dx^2}3x^2-9x-99}{\frac{d^2}{dx^2}2x^2+5x+1}$$

$$= \frac{(6x-9)'}{(4x+5)'} = \frac{6}{4} = \underline{\underline{\frac{3}{2}}}$$

b) $\lim_{x \to 3} \frac{3x-9}{x^2+5x+1} = \frac{9-9}{9+15+1}$

$= \frac{0}{9+15+1} = \underline{\underline{0}}$

c) $\lim_{x \to -2} \frac{2x+4}{5x^2+5x-10}$

$= \lim_{x \to -2} \frac{2(x+2)}{5(x^2+x-2)}$

$= \lim_{x \to -2} \frac{2\cancel{(x+2)}}{5(x-1)\cancel{(x+2)}}$

$= \frac{2}{5(-3)} = \underline{\underline{-\frac{2}{15}}}$

7) $f(x) = \begin{cases} 2x+1, & x < 1 \\ cx^2, & x \geq 1 \end{cases}$

$$\lim_{x \to 1^-} f(x) = 2 \cdot 1 + 1 = 3$$

$$\lim_{x \to 1^+} f(x) = c(1)^2 = c$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$\underline{\underline{c = 3}}$$

Made with Goodnotes

$$f(x) = \begin{cases} 2x+1, & x<1 \\ 3x^2, & x \geq 1 \end{cases}$$

Bevis for at den er kontinuerlig:

Case 1: $x<1$ $\quad |f(x)-f(1)| < \varepsilon$ når $|x-1| < \delta$

$$|2x+1-3| = |2x-2| = 2|x-1|.$$

La $\delta = \frac{\varepsilon}{2}$. Når $|x-1| < \delta$

har vi: $2|x-1| < 2\delta = \varepsilon$

Case 2: $x \geq 1$ $\quad |f(x)-f(1)| < \varepsilon$ når $|x-1| < \delta$

$$|3x^2-3| = 3|x^2-1| = 3|x+1| \cdot |x-1|$$

$$|x-1| < 1 \Rightarrow x \in \langle 0,2 \rangle \wedge |x+1| < 3$$

så $3|x-1||x+1| < 3|x-1| 3 = 9|x-1|$

La $\delta = \frac{\varepsilon}{9}$. Når $|x-1| < \delta$

har vi at $9|x-1| < 9\delta = \varepsilon$

Konklusjon: for $c=3$ er $f(x)$ kontinuerlig i $x=1$

b) $f(x) = \begin{cases} \frac{x^2+2x-3}{x-1}, & x \neq 1 \\ C, & x = 1 \end{cases}$

$$\lim_{x \to 1} f(x) = C$$

$$\lim_{x \to 1} \frac{x^2+2x-3}{x-1} = \lim_{x \to 1} \frac{(x-1)(x+3)}{x-1}$$

$$= 1+3 = 4 \quad \therefore C = \underline{\underline{4}}$$

For $x \neq 1$ trenger vi $|f(x) - f(1)| < \varepsilon$ når $|x-1| < \delta$

$$f(x) = \frac{x^2+2x-3}{x-1} = x+3$$

$$|x+3-4| = |x-1|$$

La $\delta = \varepsilon$. Når $|x-1| < \delta$

har vi $|x-1| < \delta = \varepsilon$