My Experience With the Subject of Mathematics

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Keeping track of progress is one of the most rewarding aspects of learning. There is truly no better feeling than looking back at a problem that once seemed incomprehensible and realizing that, with time and effort, it now makes sense. This progression, from confusion to understanding, is what makes learning so satisfying. With hindsight, one can really appreciate the struggles and hiccups along the way.

1 Early Memories (2008)

I remember two children arguing about who knew the highest number, ”100 is highest”, ”1000 is higher” and so on, not realizing that no matter what the other came up with you could just retort with ”+1”.

2 Primary and Middle School (2010–2018)

- Standard Norwegian curriculum.
- Basic algebra, equations, and problem-solving techniques.

10th Grade Oral Examination

In 10th grade we were tasked to hold a short informal presentation before the teacher explaining some problem and how to solve it mathematically. My interest in more abstract concepts showed itself when, where the others had opted to explain how to find the amount of planks needed for a triangular floor with an area A, or similar problems, I had chosen to tell my own version of Hilbert's Hotel using a warehouse as a surrogate. The teacher told me that it was not relevant to the subject and I should have done a different problem.

I think it's a shame that more abstract mathematics is not revealed or hinted to in earlier education. While I understand why this is the case, I still feel that a lot of people would appreciate the subject more if its profound beauty was more lavishly on display.

3 High School (VGS) (2020–2023)

- Learned basic calculus, including differentiation and integration up to integration by parts.
- Had some probability and statistics which, in the manner taught at that stage did not engage me as much as I think it could have.
- I recall some trigonometry.
- No formal linear algebra coursework at this stage.

4 Programming and Linear Algebra (2022–2023)

- Developed a strong interest in programming.
- Learned fundamental linear algebra concepts:
  - Vectors and their properties
  - Dot and cross products
  - Matrix operations

3D Renderer on the CPU

My interest in linear algebra resulted in two 3D rendering engines written from scratch, in JavaScript and later Java.

5 University and Discrete Mathematics (2024)

5.1 First Semester (Spring 2024)

- Began my Bachelor's in IT (Specializing in Machine Learning Engineering and Artificial Intelligence).
- Took Discrete Mathematics 1, which deepened my interest in mathematics.

First Proof

I can't remember it exactly, and I remember having "proofs" in high-school, but I regard these sorts of problems in set-theory to be the first proofs I ever did. Let A, B, C be sets. Prove that if $a \in A$ and $a \in C$, then $a \in B \cup (C \cap A)$. We are given that:

1. $a \in A$
2. $a \in C$

We need to show that $a \in B \cup (C \cap A)$.

- Since $a \in C$ and $a \in A$, by the definition of intersection, we conclude that $a \in C \cap A$.
- By the definition of union, since $a \in C \cap A$, it follows that $a \in B \cup (C \cap A)$, regardless of whether $a \in B$.

Thus, $a \in B \cup (C \cap A)$, completing the proof. $\square$

Return to Hilbert's Hotel

I spent a lot of time jumping forward in my discrete math book, by Susanna S. Epp, and looking at what was next. During one of these times I came across a familiar problem. Prove that $|\mathbb{N}| = |\mathbb{Z}|$ Two sets have the same cardinality if there exists a bijection (a one-to-one and onto function) between them. We define a function $f : \mathbb{N} \to \mathbb{Z}$ as follows:

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ -\frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

Injectivity: Suppose $f(n_1) = f(n_2)$. We consider two cases: 1. If both n_1 and n_2 are even, then $f(n_1) = \frac{n_1}{2}$ and $f(n_2) = \frac{n_2}{2}$, which implies $\frac{n_1}{2} = \frac{n_2}{2}$, so $n_1 = n_2$. 2. If both n_1 and n_2 are odd, then $f(n_1) = -\frac{n_1+1}{2}$ and $f(n_2) = -\frac{n_2+1}{2}$, which implies $\frac{n_1+1}{2} = \frac{n_2+1}{2}$, so $n_1 = n_2$. Thus, f is injective. Surjectivity: For any integer $z \in \mathbb{Z}$, we need to find some $n \in \mathbb{N}$ such that $f(n) = z$. 1. If $z \geq 0$, we choose $n = 2z$. Then, $f(n) = f(2z) = \frac{2z}{2} = z$. 2. If $z < 0$, we choose $n = -2z - 1$. Then, $f(n) = f(-2z-1) = -\frac{(-2z-1)+1}{2} = -\frac{-2z}{2} = z$. Thus, f is surjective. Since f is both injective and surjective, it is a bijection, proving that $|\mathbb{N}| = |\mathbb{Z}|$. $\square$

Instead of having the magic of the infinite hotel swept away when presented in this rigorous fashion, I found the concept even prettier. This line of reasoning was so elegant, and proofs with functions remain among my favourite problems.

5.2 Winter Break (December 2024)

- Completed a Calculus II Udemy course in less than a week , eager to feel qualified to move on to more exotic territory.
- Explored Real Analysis, learning about:
  - Limits and their formal definition
  - Convergence proofs

5.3 Second Semester (Spring 2025)

- Introduced to Abstract Algebra through algorithms professor, covering:
  - Isomorphisms
  - Cosets and Kernels
  - Groups, Rings and Fields
- Started exploring Enumerative Combinatorics, fascinated by formula derivations and counting puzzles.
- Stumbled upon a more relaxed course in Differential Equations, finally taking the time to learn some.

First Proof in Abstract Algebra

Prove that $\mathbb{R}-\{1\}$ under the operation $\star$, with $a,b \in \mathbb{R}-\{1\} \Rightarrow a\star b = a+b-ab$, forms a group.

1) $a, b \in \mathbb{R}-\{1\} \Rightarrow a \neq 1 \neq b$

   Now let's assume $\star$ is not a binary operation on $\mathbb{R}-\{1\}$,

   meaning we can get $a \star b = 1 \notin \mathbb{R}-\{1\}$

   $a \star b = 1 \Leftrightarrow a+b-ab = 1 \Leftrightarrow a+b = 1+ab$

   Take the partial derivative with respect to a on both sides to get

   $\dfrac{\partial}{\partial a}[a+b] = \dfrac{\partial}{\partial a}[1+ab]$

   yielding $1 = b$ which contradicts $b \neq 1$

   Conversely,

   $\dfrac{\partial}{\partial b}[a+b] = \dfrac{\partial}{\partial a}[1+ab]$

   $1 = a$

   $\therefore a \star b = 1 \Leftrightarrow a = 1 \vee b = 1$

   Thus $a, b \in \mathbb{R}-\{1\}$ gives $a \star b \neq 1$,

   $\Rightarrow a \star b \in \mathbb{R}-\{1\}$

2) $(a \star b) \star c = (a+b-ab) \star c = (a+b-ab) + c - (a+b-ab)\cdot c$

   $= a+b+c-ab-ac-bc+abc = a + (b+c-bc) - a\cdot(b+c-bc)$

   $= a \star (b \star c)$, $\therefore \star$ is associative on $\mathbb{R}-\{1\}$

3) Let the identity $e = 0$, $a \star e = a + 0 - a\cdot 0 = a$

4) For this to be a group we need that $\forall a \in \mathbb{R}-\{1\}$

   $\exists a^{-1}$ such that $a \star a^{-1} = e = 0$

   Let $x = a^{-1}$, $a \star x = a + x - ax$

   $a + x - ax = 0$

   $x - ax = -a$

   $x(1-a) = -a$

   $x = -\dfrac{a}{1-a} = a^{-1} \because 1 - a \neq 0 + \forall a \in \mathbb{R}-\{1\} \quad \square$

Thus, we have shown closure, associativity, the existence of an identity and inverses, making $\mathbb{R}-\{1\}$ a group under $\star$. **Note** that this is not a solid proof, just how I solved the problem first time around. In part 1), using partial derivatives is not really a valid way to show closure under $\star$, it should instead be done algebraically, or at least that is what chatGPT tells me. Which I find a real shame since using partial derivatives where it doesn't really belong is much more fun. Who knows maybe it is a valid approach and chatGPT doesn't know what it is talking about. Regardless, I think this is a very fun problem.

6 Long Term Goals

- Take extra courses in mathematics during my undergraduate studies to gain a broader understanding of mathematics as a whole, with deeper insight into my favourite areas.
- Educate myself further in mathematics and theoretical computer science.