

Inlevering 1 Mat 104

1) a) $f(x) = x - \sqrt{2x-4} = 2$

$$-\sqrt{2x-4} = 2-x$$

$$\sqrt{2x-4} = x-2$$

$$\sqrt{2(x-2)} = x-2$$

$$2(x-2) = (x-2)^2$$

$$2x-4 = x^2 - 4x + 4$$

$$0 = x^2 - 6x + 8$$

$$0 = (x-4)(x-2)$$

$$x-4 = 0 \Leftrightarrow x=4$$

$$x-2 = 0 \Leftrightarrow x=2$$

$$\therefore x=2 \text{ en } x=4$$

$$b) f(x) = x - \sqrt{2x-4} < 2 \quad / -2 + \sqrt{2x-4}$$

$$x-2 < \sqrt{2x-4}$$

$$(x-2)^2 < 2x-4$$

$$x^2 - 4x + 4 < 2x - 4$$

$$x^2 - 6x + 8 < 0$$

$$(x-2)(x-4) < 0$$

$$x-4 < 0 \rightarrow x < 4$$

$$-\sqrt{2x-4} \notin \mathbb{R} \text{ nur } x < 2 \quad f(2)=2$$

$$\text{sonst } x > 2$$

$$x > 2 \wedge x < 4 \Leftrightarrow x \in (2, 4)$$

$$c) x < 2 \Rightarrow \sqrt{2x-4} \notin \mathbb{R}$$

$$\therefore D_f = [2, \infty)$$

$$J) \frac{d}{dx} \left(x - \sqrt{2x-4} \right)$$

$$= \frac{d}{dx} \left(x - (2x-4)^{\frac{1}{2}} \right)$$

$$= \frac{d}{dx} x + \frac{d}{dx} -(2x-4)^{\frac{1}{2}}$$

$$= 1 + \left(-\frac{1}{2} (2x-4)^{-\frac{1}{2}} \cdot 2 \right)$$

$$= 1 - \frac{1}{\sqrt{2x-4}} = 1 - \frac{1}{\sqrt{2x-4}}$$

$$1 - \frac{1}{\sqrt{2x-4}} = 0$$

$$1 = \frac{1}{\sqrt{2x-4}} \quad / \times \sqrt{2x-4}$$

$$\sqrt{2x-4} = 1$$

$$2x-4 = 1$$

$$2x = 5$$

$$x = \underline{\frac{5}{2}}$$

$$f\left(\frac{5}{2}\right) = \frac{5}{2} - \sqrt{5-4}$$

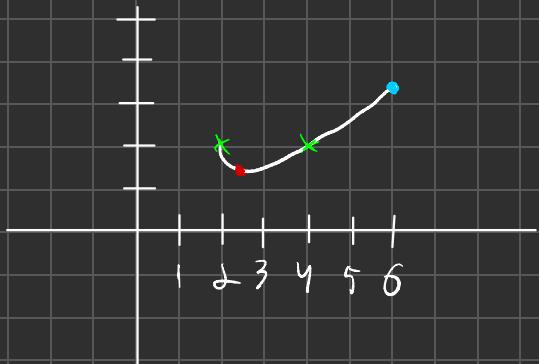
$$= \underline{\frac{3}{2}}$$

$$f(6) = 6 - \sqrt{12-4}$$

$$= 6 - \sqrt{8}$$

$$= 6 - \sqrt{8^3}$$

$$\approx 3.17$$



$$e) V_f \approx \left[\frac{3}{2}, 3.17 \right] \approx [1.5, 3.2]$$

$$f) \text{ Nei } f(2) = f(4)$$

$$J) a) \frac{a(a^2 - b^2)}{a-b} = \frac{a(a+b)(a-b)}{a-b} = \frac{a^2 + ab}{a}$$

$$b) \frac{(x-1)^2 + (x+1)^2 + 4x}{(x+1)^2}$$

$$= \frac{x^2 - 2x + 1 + x^2 + 2x + 1 + 4x}{(x+1)^2} = \frac{2x^2 + 4x + 2}{(x+1)^2}$$
$$= \frac{2(x^2 + 2x + 1)}{(x+1)^2} = \frac{2(x+1)(x+1)}{(x+1)(x+1)}$$

$$= 2$$

$$c) \frac{a^2 (a^2 b)^3}{(b^3)^2 \cdot a} = \frac{a^8 \cdot b^3}{a \cdot b^6}$$
$$= \frac{a^7}{b^3} = \frac{a^7 b^3}{b^3}$$

$$\text{J) } \frac{\bar{3}^1 \cdot \bar{a}^1 \cdot b}{\bar{6}^1 \cdot (\bar{a}^2 \cdot \bar{b}^3)^2} = \frac{6^6}{3^6 a^4 b^6}$$

$$= 2 \cdot \frac{b^4 a^4}{a^6 b^6} = 2 \cdot \frac{a^3}{b^5} = \underline{2 a^3 b^{-5}}$$

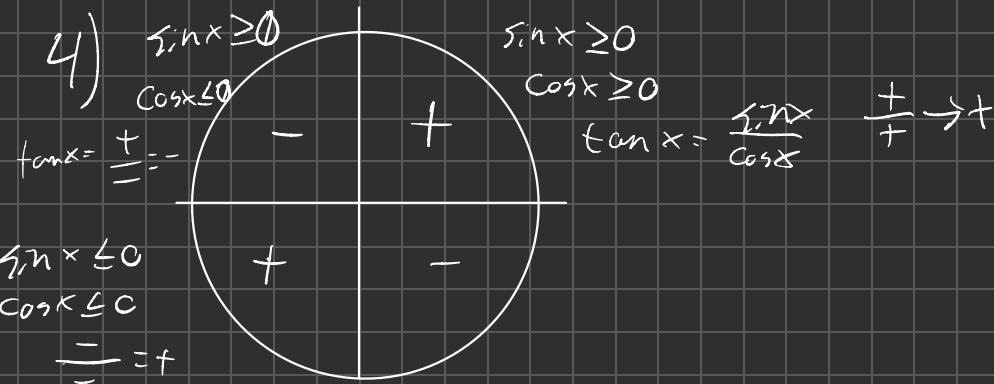
$$\text{e) } \frac{(x^2 \cdot y)^3 \cdot (x^{-1} \cdot y^2)^{-1}}{(2x \cdot y^3)^2} = \frac{x^6 y^3}{4(x^2 \cdot y^6 \cdot x^{-1} \cdot y^{-2})}$$
$$= \frac{x^6 y^3}{4 x \cdot y^8} = \frac{1}{4} x^5 y^{-5}$$
$$= \underline{\frac{1}{4} \left(\frac{x}{y} \right)^5}$$

$$3) \sum_{j=1}^2 \sum_{i=1}^j i \cdot j$$

$$j=1: 1 \cdot 1 = 1$$

$$j=2: 2 \cdot 1 + 2 \cdot 2 = 6$$

F



$$b) \tan \theta > 0 \Leftrightarrow \frac{\sin \theta}{\cos \theta} > 0$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta}$$

$$\lambda = \frac{1}{\cos \theta} = \sec \theta \quad \frac{\pi}{3}$$

$$\frac{1}{\cos \theta} = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \arccos \frac{1}{2} = \frac{\pi}{3} \quad \text{eller} \quad \frac{4\pi}{3}$$

c) $\tan \theta < 0$

$$\Rightarrow \tan \theta = \frac{-\sin \theta}{\cos \theta} = \frac{\sin \theta}{-\cos \theta}$$

$$\Rightarrow \tan \theta = -\frac{1}{\cos \theta}$$

$$\cos \theta = -1$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3} + \frac{\pi}{3} = \frac{5\pi}{3}$$

eller $\frac{2}{3}\pi + \frac{3}{3}\pi = \underline{\underline{\frac{5}{3}\pi}}$

$$5) \quad a) \quad 5 \cdot 10^8 \text{ kg}$$

$$V = f(h) = 50m \cdot 300m \cdot h_m$$
$$= 15000 \text{ m}^3 \cdot h$$

$$b) \quad V_{\text{vorr}} = 1000 \text{ kg/m}^3$$
$$5 \cdot 10^8 \text{ kg}$$

$$\text{Volum av } 5 \cdot 10^8 \text{ kg vorr}$$
$$= \frac{5 \cdot 10^8 \text{ kg} \cdot \text{m}^3}{10^3 \text{ kg}} = 5 \cdot 10^5 \text{ m}^3$$

$$f(h) = 5 \cdot 10^5 \text{ m}^3$$

$$15000 \text{ m}^3 \cdot h = 5 \cdot 10^5 \text{ m}^3$$

$$15 \cdot 10^3 \text{ m}^2 \cdot h_m = 5 \cdot 10^5 \text{ m}^3$$

$$15 \cdot h_m = 5 \cdot 10^2 \text{ m}$$

$$h_m = \frac{5}{15} \cdot 10^2 \text{ m} = \frac{1}{3} \cdot 100 \text{ m}$$

$$h \geq 33.33 \text{ m}$$

$$C) V = f(45) = 675000 \text{ m}^3$$

$$\begin{aligned} M_r &= 1000 \text{ kg/m}^3 \times 675000 \text{ m}^3 \\ &= 10^3 \text{ kg/m}^3 \times 6.75 \times 10^5 \text{ m}^3 = 6.75 \times 10^8 \text{ kg} \end{aligned}$$

$$\text{La } g(x) = 5 \times 10^4 \text{ kg} + 10^5 \text{ kg} \cdot x$$

$$g(x) = 6.75 \cdot 10^8 \text{ kg}$$

$$5 \times 10^4 + 10^5 \text{ kg} \cdot x = 6.75 \cdot 10^8 \text{ kg}$$

$$10^5 \text{ kg} \cdot x = (6.75 - 5) \cdot 10^8 \text{ kg}$$

$$10^5 \text{ kg} \cdot x = (1.75) 10^8 \text{ kg}$$

$$x = (1.75) 10^{8-5}$$

$$x = 1.75 \cdot 10^3 = \underline{\underline{1750}}$$

$$6) \lim_{x \rightarrow \infty} \frac{3x^2 - 9x - 99}{2x^2 + 5x + 1}$$

Grenseverdien er i formen:

$\frac{\infty}{\infty}$ så vi kan bruke l'Hopital
 Merk at $\lim_{x \rightarrow \infty} \frac{\frac{d}{dx} 3x^2 - 9x - 99}{\frac{d}{dx} 2x^2 + 5x + 1}$

er også i $\frac{\infty}{\infty}$ form så

$$\lim_{x \rightarrow \infty} \frac{3x^2 - 9x - 99}{2x^2 + 5x + 1}$$

$$= \frac{\frac{d}{dx} 3x^2 - 9x - 99}{\frac{d}{dx} 2x^2 + 5x + 1}$$

$$= \frac{(6x - 9)'}{(4x + 5)'} = \frac{6}{4} = \underline{\underline{\frac{3}{2}}}$$

$$b) \lim_{x \rightarrow 3} \frac{3x - 9}{x^2 + 5x + 1} = \frac{9 - 9}{9 + 15 + 1}$$

$$= \frac{0}{15 + 1} = \underline{\underline{0}}$$

$$c) \lim_{x \rightarrow -2} \frac{2x + 4}{5x^2 + 5x - 10}$$

$$= \lim_{x \rightarrow -2} \frac{2(x+2)}{5(x^2 + x - 2)}$$

$$= \lim_{x \rightarrow -2} \frac{2(x+2)}{5(x-1)(x+2)}$$

$$= \frac{2}{5(-3)} = -\frac{2}{15}$$

$$7) f(x) = \begin{cases} 2x + 1, & x < 1 \\ cx^2, & x \geq 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = 2 \cdot 1 + 1 = 3$$

$$\lim_{x \rightarrow 1^+} f(x) = C(1)^2 = C$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$C = \underline{\underline{3}}$$

④

$$f(x) = \begin{cases} 2x+1, & x < 1 \\ 3x^2, & x \geq 1 \end{cases} \quad \text{Beregs for at den er}$$

Kontinuerlig:

$$\text{Case 1: } x < 1 \quad |f(x) - f(1)| < \varepsilon \text{ når } |x-1| < \delta$$

$$|2x+1 - 3| = |2x-2| = 2|x-1|.$$

$$\text{La } \delta = \frac{\varepsilon}{2}. \text{ Når } |x-1| < \delta$$

$$\text{har vi: } 2|x-1| < 2\delta = \varepsilon$$

$$\text{Case 2: } x \geq 1 \quad |f(x) - f(1)| < \varepsilon \text{ når } |x-1| < \delta$$

$$|3x^2 - 3| = 3|x^2 - 1| = 3|x+1||x-1|$$

$$|x-1| < 1 \Rightarrow x \in (0, 2) \text{ og } |x+1| < 3$$

$$\text{Så } 3|x-1||x+1| < 3|x-1|3 = 9|x-1|$$

$$\text{La } \delta = \frac{\varepsilon}{9}. \text{ Når } |x-1| < \delta$$

$$\text{har vi at } 9|x-1| < 9\delta = \varepsilon$$

Konklusion: for $c=3$ er $f(x)$ kontinuerlig
i $x=1$

$$b) f(x) = \begin{cases} \frac{x^2 + 2x - 3}{x-1}, & x \neq 1 \\ c, & x = 1 \end{cases}$$

$$\lim_{x \rightarrow 1} f(x) = c$$

$$\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x-1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+3)}{x-1}$$

$$= 1+3=4 \quad \therefore c = \underline{\underline{4}}$$

For $x \neq 1$ stronger $\forall |f(x) - f(1)| < \varepsilon$ nor $|x-1| < \delta$

$$f(x) = \frac{x^2 + 2x - 3}{x-1} = x+3$$

$$|x+3 - 4| = |x-1|$$

Let $\delta = \varepsilon$. nor $|x-1| < \delta$

have $\forall |x-1| < \delta = \varepsilon$