Professor Dave's Differential Equations

Thobias K. Høivik

March 4, 2025

Classification of Differential Equations

Algebraic functions such as $x^2 + 4x + 4 = 0$ have an unknown variable which we are solving for. Differential equations such as $\frac{dy}{dx} = x^2$, on the other hand, have an unknown function. When we find indefinite integrals, we are technically finding the function of a differential equation: $\frac{dy}{dx} = x^2 \Rightarrow y = \int x^2 dx = \frac{x^3}{3} + C$.

Terminology Pertaining to the Classification of Differential Equations

1. y is the dependent variable and x the independent variable in the differential equation above.

2. **ODE:** An ordinary differential equation contains only **one** independent variable, for instance:
$$\frac{dy}{dx} + xy = \tan x$$

3. **PDE:** A partial differential equation contains more than one independent variable, for instance:
$$\frac{\partial y}{\partial x} - 2\frac{\partial y}{\partial t} = t^2 y$$

4. Linear differential equations are DE's where the dependent variables, and it's derivatives, are alone (no exponent or function acting on y or it's derivatives):
$$\frac{dx}{dy} + y \sin x = 0$$

5. Nonlinear differential equations are DE's not explicitly satisfying the conditions that make a DE linear:
$$\frac{dy}{dx} + y^2 = 0$$

6. A homogenous differntial equation is a DE where there are no terms with just the independent variable or constant:

$$\frac{dy}{dx} + y = 0$$

7. A nonhomegnous differential equation has terms with just the independent variable or a constant:

$$\frac{dy}{dx} + y = x \vee \frac{dy}{dx} + y = 1$$

8. The order of a DE is the highest derivative present:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x$$

has order = 2.

9. The degree is the exponent on the highest derivative present:

$$\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} = x$$

would have order = 2 and degree = 3.

10. Autonomous DE's are DE's where the independent variable only appears as a derivative:

$$\frac{dy}{dx} + y = 0$$

has no term with x other than the derivative.

11. Nonautonomous have the independent variable appearing in other terms:

$$\frac{dy}{dx} + xy = 0$$

Solving Differential Equations

We want to solve for the dependent variable y. Getting y as a function of x would be an **explicit solution**.
If we can't express y in terms of x we would call it an **implicit solution**.
Consider

$$y = \frac{x^3}{3} + C$$

the solution to a differential equation. This solution is what we would call a general solution as it contains the constant term $+C$. Boundary conditions tell us information about the solution. A common type of boundary condition is an initial condition, for example given $y(0) = 1$ we could find the value for C in the genral solution above. This number of boundary conditions required to reach a particular solution is equal to the order of the DE. First derivative yields one C term, second two C terms and so on.