Computability and Complexity

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1 Introduction

The following will be my notes for computability and complexity, which I shall hopefully be taking at the University of Oslo during the spring of 2026 (Course-code: IN2080). If this document is renamed to IN2080 - Computability and Complexity, then I was allowed to take the course. Like with the rest of my electives at UiO, I have to learn everything by myself which will undoubtedly manifest in my notes being a bit scattered at times. If you are someone who stumbled over these notes keep this in mind.

Furthermore I have dabbled in some automata theory earlier so while these notes will be heavily Sipser-flavoured, I will not go very into detail on DFA's.

2 Nondeterministic Finite Automata

Definition 2.1. *A nondeterministic finite automaton N is a 5-tuple*

$$(Q, \Sigma, \delta, q_0, F),$$

the same as a DFA, except:

$$\begin{aligned} \delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(A) &= \{R : R \subseteq Q\}, \\ \Sigma_\varepsilon &= \Sigma \cup \{\varepsilon\} \text{ and} \\ \mathcal{P}(Q) &= \text{the powerset of } Q \end{aligned}$$

We say that an NFA N accepts a string w if at least one computation path reads all of w and ends in a state in F.

This is a bit difficult to grasp with the definition alone so we look at an example.

Example 2.1. *Consider an NFA over $\Sigma = \{a, b\}$ which accepts strings ending in ab constructed as follows:*

- *States:* $Q = \{q_0, q_1, q_2\}$
- *Start:* q_0
- *Accepting:* q_2

with the δ defined such that

- $q_0 \overset{a}{\mapsto} q_0$
- $q_0 \overset{b}{\mapsto} q_0$
- $q_0 \overset{a}{\mapsto} q_1$
- $q_1 \overset{b}{\mapsto} q_2$

Take $w = aab$. From q_0 with the middle symbol a we got to q_0 or q_1. From q_1 we go to q_2 so there is one branch where we end in an accept state so w gets accepted.

Theorem 2.1. *If an NFA recognizes a language A, then A is regular.*

Proof. Let the NFA $N = (Q, \Sigma, \delta, q_0, F)$ recognize A, construct the DFA $N' = (Q', \Sigma', \delta', q_0', F')$ with

- $Q' = \mathcal{P}(Q), R \in Q'$
- $\delta'(R, a) = \{q : q \in \delta(r, a), r \in R\}$
- $q_0' = \{q_0\}$
- $F' = \{R \in Q' : R \cap F \neq \emptyset\}$

At least one computation path in N ends in an accept state. We have all possible computations $R \in Q'$. At least one R accepts w and $R \cap F'$ is nonempty so N' accepts w. □