My Math Notes

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1 Introduction

Definition 1.1 (Sets). *A set is a collection of distinct elements.*

As discussed in definition 1.1, elements must be distinct.

Theorem 1.1 (Sum of evens). *The sum of two even numbers is even.*

Proof. Let the even numbers be $2a$ and $2b$. Their sum is

$$2a + 2b = 2(a + b),$$

which is even. $\square$

As described in theorem 1.1.

Lemma 1.1 (Parity of sum). *If x and y are integers, then $x + y$ is even if and only if x and y have the same parity.*

As seen in lemma 1.1, you are a very nice person.

Proof. Assume x and y are both even or both odd.

- If both even, $x = 2m$, $y = 2n$, then $x + y = 2(m + n)$ is even.
- If both odd, $x = 2m + 1$, $y = 2n + 1$, then

$$x + y = 2m + 1 + 2n + 1 = 2(m + n + 1),$$

also even.

Conversely, if $x + y$ is even and x is even, then y must be even; similarly for odd. $\square$

2 Model Theory

Problem 2.1 (Satisfiability). *Let $\mathfrak{M} \models \phi$. Show that ϕ is satisfiable.*

Proof of Problem 2.1. Since $\mathfrak{M} \models \phi$, by definition ϕ is true in some model (namely $\mathfrak{M}$). Therefore, ϕ is satisfiable.

More explicitly,

$$\phi = \psi \wedge \theta,$$

where ψ and θ are formulas satisfied by $\mathfrak{M}$. Hence ϕ is satisfiable. $\square$

Corollary 2.1 (Consequence of satisfiability). *If ϕ is satisfiable, then $\neg\phi$ is not valid.*

Proof. If $\neg\phi$ were valid, then ϕ would be false in every model. This contradicts the satisfiability of ϕ. □

3 Fourier Analysis

Definition 3.1 (Fourier Transform). *The Fourier transform of a function $f \in L^1(\mathbb{R})$ is defined as*

$$\hat{f}(\xi) = \int_{\mathbb{R}} f(x) e^{-2\pi i x \xi} \, dx.$$

Theorem 3.1 (Fourier Inversion). *If f and $\hat{f}$ are both in $L^1(\mathbb{R})$, then for almost every x,*

$$f(x) = \int_{\mathbb{R}} \hat{f}(\xi) e^{2\pi i x \xi} \, d\xi.$$

Sketch of proof. This theorem follows from Plancherel's theorem and properties of the Fourier transform on Schwartz functions. The full proof is beyond this note. □