Oblig 2

1 a)

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad \underline{15} \quad \underline{20} \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad \underline{21} \quad 35 \quad \underline{35} \quad 21 \quad 7 \quad 1 \\
1 \quad 8 \quad 28 \quad \underline{56} \quad \underline{70} \quad 56 \quad 28 \quad 8 \quad 1
\end{array}
$$

$n = 0$

$n = 1$

$\vdots$

$n = 8$

b) $\binom{6}{2} \to$ Rad $n = 6 \to 15 = \cancel{3} \cdot \cancel{5}$

$\binom{7}{4} \to 35 = \cancel{7 \cdot 5}$

$\binom{8}{3} \to 56 = 8 \cdot 7 = \cancel{4} \cdot \cancel{2} \cdot \cancel{7}$

$\binom{6}{3} \to 20 = \cancel{4} \cdot \cancel{5}$

$\binom{7}{2} \to 21 = \cancel{3} \cdot \cancel{7}$

$\binom{8}{4} \to 70 = \cancel{2} \cdot \cancel{5} \cdot \cancel{7}$

1 c) $\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = V.S$

$\binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1} = H.S$

$$V.S = \frac{(n-1)!\ n!\ (n+1)!}{(k-1)!\,(n-1-k+1)!\ (k+1)!\,(n-k-1)!\ k!\,(n-k)!}$$

$$= \frac{(n-1)!\ n\cdot(n-1)!\ (n+1)\cdot n\cdot(n-1)!}{(k-1)!\ (n-k)!\ (k+1)!\ (n-k-1)!\ k!\,(n-k)!}$$

$$= \left.\frac{(n+1)\cdot 2n\cdot 3(n-1)!}{(k-1)!\,(n-k-1)!\ (n-k)!\,(k+1)!\ k!\,(n-k)!}\right\}$$

$$H.S = \frac{(n-1)!\ n!\ (n+1)!}{k!\,(n-1-k)!\ (k-1)!\,(n-k+1)!\ (k+1)!\,(n-k-1)!}$$

$$= \frac{(n-1)!\ n!\ (n+1)!}{(n-k)!\,k!\ (n-k-1)!\ (k-1)!\ (n-k+1)\,(n-k-1)!\ (k+1)!}$$

$$= \left.\frac{(n+1)\cdot 2n\cdot 3(n-1)!}{(k-1)!\ (n-k-1)!\,(n-k)!\,(k+1)!\ k!\ (n-k)!}\right\}$$

$V.S = H.S$

1 d

1 9 36 84 126 ...

1 10 45 120 210 252 ...

1 11 55 165 330 462 ...

For n=11 ta sum av de over.

For n=9 merk at 1 må være først neste blir 10-1=9 deretter 45-9=36, 120-36=84, etc. Når vi når midten, bruk symetrien av trekanten å skriv resten.

2 a) $\binom{48}{6} = \frac{48!}{6!\,(42)!} = \frac{1}{6!} \prod_{n=43}^{48} n$

$= \frac{1}{6!} \; 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43$

$= \frac{1}{6!} \; (6 \cdot 8) \cdot 47 \cdot (2 \cdot 23) \cdot (5 \cdot 9) \cdot (4 \cdot 11) \cdot 43$

$= \frac{\cancel{6} \cdot 8 \cdot 47 \cdot \cancel{2} \cdot 23 \cdot \cancel{5} \cdot \cancel{9} \cdot \cancel{4} \cdot 11 \cdot 43}{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$

$= \frac{8 \cdot 47 \cdot 23 \cdot 3 \cdot 11 \cdot 43}{1}$

Viss vi antar 52 veker i året har vi: ~~[illegible]~~ 520 rekker i året.
Vi kan berre være sikker på at vi vinner etter at alle muligheter er testet.

$\Rightarrow \frac{\binom{48}{6}}{520} \approx 23600$

b) $(x+y)^7 = \sum_{n=0}^{7} \binom{7}{n} x^{7-n} y^n$

Via pascal's trekant:

$x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

$\equiv x^7 + y^7 \pmod 7$ (Sant for alle primtal)

c) $(x+2y)^{13} = \sum_{n=0}^{13} \binom{13}{n} x^{\cancel{13}-n} \cancel{*} 2y^n$

$= 2 \sum_{n=0}^{13} \binom{13}{n} x^{13-n} y^n$

$x^5 y^8$ $\quad 13-8=5 \rightarrow$ Kolonne 8 fra høyre i pascal's ved $n=13$

$\rightarrow \binom{13}{5}$

Koeffisienten blir da

$2\binom{13}{5} = \underline{2574}$

13 primtal så 0 i $\mathbb{Z}_{13}$

d) $\sum_{i=0}^{10} \binom{10}{i} 4^i = \sum_{i=0}^{10} \binom{10}{i} 0^{10-i} 4^i$

(viss vi definerer $0^0 := 1$)

Via binominske teorem:

$(0+4)^{10} = \underline{4^{10}}$

3 a) $(00)^* (01)^* (10)^* (11)^*$

eller

$(00|01|10|11)^*$

eller

$[(0|1)(0|1)]^*$

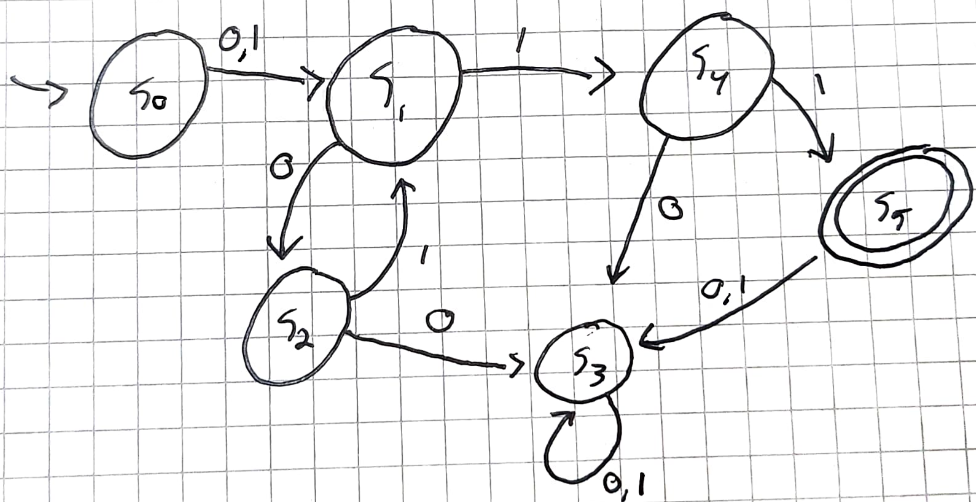
b) $(0|10)^* 0$

c) $1^* 0 (1^* 0 1^* 0 1^*)^*$

4 $(0|1)(01)^* 11$

a) 011, 111, 00111, 10111, 0010111, 1010111

b)



4 c) a bb   a a bbb   a abab ... ab bb
b bb   b a bbb   b abab ... abbb

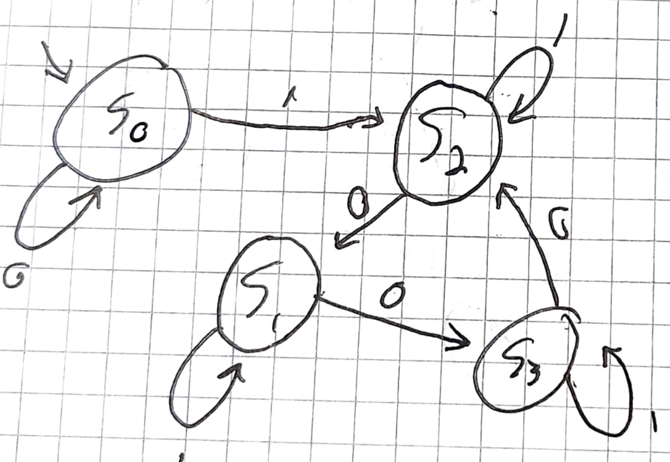
$(a|b)$ $\underbrace{abab \ldots ab}_{(ab)^*}$ bb

$(a|b)(ab)^* bb$

5 a)
i. s_0, s_1, s_2
ii. $\{0,1\}$
iii. s_0
iv. s_2
v)

	0	1
s_0	s_0	s_1
s_1	s_2	s_1
s_2	s_2	s_2

b)



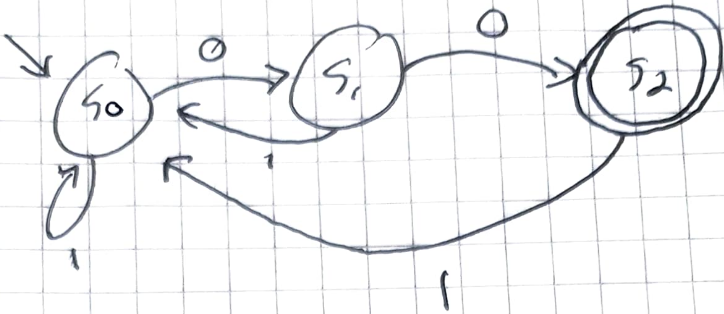
Antar at visst dette var en FSM vil s_0 være ~~start~~ siden ingen går til s_0.

5

c) i. S_1

ii. $S3$

d) i.



ii. $1^*00^*1(0|1)^*$

1010   1011