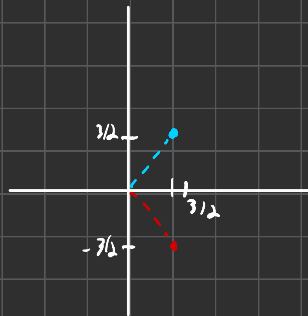
Oblig 4

1) a) i. $z^2 - 3z + 3 = 0$

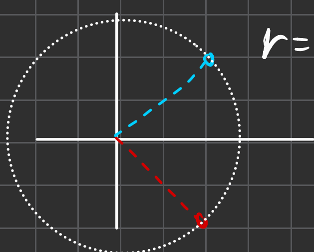
$$z = \frac{3 \pm \sqrt{9-12}}{2} = \frac{3}{2} \pm \frac{\sqrt{-3}}{2} = \frac{3}{2} \pm \frac{3}{2} i$$

$$= \boxed{\frac{3}{2}(1 \pm i)}$$



ii. $z^2 - 4z = -8$

$$z = \frac{4 \pm \sqrt{16-32}}{2} = 2 \pm \frac{\sqrt{-16}}{2} = 2 \pm 2i = \boxed{2(1 \pm i)}$$



$r = \sqrt{8}$

iii. $z^2 + \left(e^{i\pi} - 2e^{\frac{i6\pi}{3}}\right)z - e^{\frac{4}{2}i\pi} = 0$

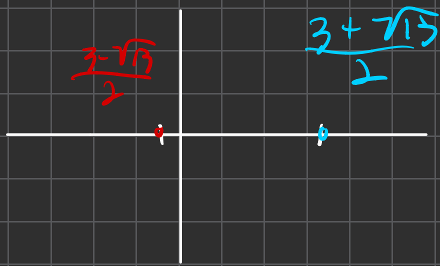
$e^{i\pi} = -1$

$-2e^{2i\pi} = -2(1) = -2$

$e^{\frac{4}{2}i\pi} = e^{2i\pi} = 1$

$\Rightarrow z^2 - 3z - 1 = 0$

$z = \dfrac{3 \pm \sqrt{9+4}}{2} = \dfrac{3}{2} \pm \dfrac{\sqrt{13}}{2} = 3.3, -0.3$



b) i. $z = \sqrt{i}$

$= \sqrt{e^{i\theta}}, \quad \theta = \frac{\pi}{2}$

$\left(e^{i\frac{\pi}{2}}\right)^{\frac{1}{2}} = e^{i\frac{\pi}{4}} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$

$= \boxed{\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}}$

ii. $z = (-3-3i)^8 = \left(re^{i\theta}\right)^8$

$r = \sqrt{9+9} = \sqrt{18}$

$\theta = \tan^{-1}(1)$ = när $\sin\theta = \cos\theta$

$\sin\theta = \cos\theta$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$z^{\frac{1}{8}} = \sqrt{18}\, e^{i\frac{5}{4}\pi} \qquad z = \sqrt{18}^{\,8} e^{i10\pi}$

$= \sqrt{18}^{\,8}\left(\cos(10\pi) + i\sin(\pi 10)\right)$

$\cos(\pi 10) = \cos(5\cdot 2\pi) = \cos(2\pi)$

$= 1 \qquad i\sin(\pi\cdot 10) = 0$

$\boxed{z = 104976 = \sqrt{18}^{\,8}}$

2) $\vec{a}=[1,-3,t]$, $\vec{b}=[2,t,s]$, $\vec{c}=[r,s,1]$

a) $\vec{a} \parallel \vec{b} \Leftrightarrow b = ka,\ k \in \mathbb{R}$

$$\frac{2}{1} = \frac{t}{-3} = \frac{s}{t}$$

$$\boxed{t=-6, \quad s=-12}$$

$$\vec{a} \cdot \vec{c} = 0$$

$$r + (-3s) + t = 0 \quad / \ t=-6,\ s=2t$$

$$\Rightarrow \quad r + 36 - 6 = 0$$

$$\boxed{r = -30}$$

b) $n = a \times c$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -6 \\ -30 & -12 & 1 \end{vmatrix}$$

$$= \hat{i}(-3-72) - \hat{j}(1-180) + \hat{k}(-12-90) = [-75, 179, -102]$$

c) Vi kan velge $-n = [75, -179, 102]$

d) $|\vec{a}| = \sqrt{\sum_{i=1}^{n} a_i^2}$, $\vec{a} = [a_1, a_2, \dots, a_n]$

$|\vec{a}| = \sqrt{1+9+36} = \sqrt{46}$

$|\vec{c}| = \sqrt{900+144+1} = \sqrt{1045}$

e) $|a \times c| = |n| = |-n|$

$= \sqrt{75^2 + 179^2 + 102^2} = \sqrt{48070}$

3) $\begin{cases} x+3y-z=3 \\ x-2y+2z=1 \\ 2x-y+2z=-2 \end{cases}$

a) $\underbrace{\begin{bmatrix} 1 & 3 & -1 \\ 1 & -2 & 2 \\ 2 & -1 & 2 \end{bmatrix}}^{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$

b) $\det A = 1\cdot(-4+2) - 3(2-4) - (-1+4)$

$= -2 + 6 - 3 = 1$

$\det A \neq 0 \Rightarrow$ en løsning.

c) $\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 1 & -2 & 2 & 1 \\ 2 & -1 & 2 & -2 \end{array}\right]$

$R_2 \leftarrow R_2 - R_1$

$\Rightarrow \left|\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & -5 & 3 & -2 \\ 2 & -1 & 2 & -2 \end{array}\right| \xrightarrow{R_3 \leftarrow R_3 - 2R_1} \left|\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & -5 & 3 & -2 \\ 0 & -7 & 4 & -8 \end{array}\right|$

$R_3 \leftarrow R_3 - 2R_1$

$$\left|\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & -5 & 3 & -2 \\ 0 & -7 & 4 & -8 \end{array}\right| \underset{R_2 \leftarrow \frac{1}{-5} R_2}{\Rightarrow} \left|\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & 1 & -\frac{3}{5} & \frac{2}{5} \\ 0 & -7 & 4 & -8 \end{array}\right|$$

$$\Rightarrow \left|\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & 1 & -\frac{3}{5} & \frac{2}{5} \\ 0 & 0 & -\frac{1}{5} & -\frac{52}{10} \end{array}\right| \Rightarrow \left|\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & 1 & -\frac{3}{5} & \frac{2}{5} \\ 0 & 0 & 1 & 26 \end{array}\right|$$

$$\Rightarrow \left|\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 26 \end{array}\right| \Rightarrow \left|\begin{array}{ccc|c} 1 & 0 & 0 & -19 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 26 \end{array}\right|$$

$x = -19 \quad y = 16 \quad z = 26$

d)

$$A^{-1} = \begin{bmatrix} -2 & -5 & 4 \\ 2 & 4 & -3 \\ 3 & 7 & -5 \end{bmatrix}$$

e) $V_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $V_2 = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ $V_3 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$

$$C_1 V_1 + C_2 V_2 + C_3 V_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\det A \neq 0$

$$\begin{bmatrix} 1 & 3 & -1 \\ 1 & -2 & 2 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4) $T : \mathbb{R}^3 \to \mathbb{R}^3$

$$R_x(\theta) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{vmatrix}$$

$$R_x\left(\frac{\pi}{4}\right) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{vmatrix}$$

$$S = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$T = S \cdot R_x\left(\frac{\pi}{4}\right) = \begin{vmatrix} -1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{vmatrix}$$

5) a) $\det \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \end{bmatrix} \overset{?}{=} 0$

$$A = \begin{vmatrix} 3 & 1 & 0 & 1 \\ -1 & -1 & -3 & -2 \\ 2 & 1 & 0 & -1 \end{vmatrix}$$

Nei

4 vektorer i $\mathbb{R}^3$ kan ikkje vore lineært uavhengige.

Minst 1 vektor må vere en lineær kombinasjon av de andre.

De er lineært avhengige.

b) Hvis vi ser på v_1, v_2, v_3:

$$\det \begin{vmatrix} 3 & 1 & 6 \\ -1 & -1 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 3(3) - 6 + 0 = 3 \neq 0$$

De tre første er lineært uavhengige så dimensjonen til spennrommet Span $\{v_1, v_2, v_3, v_4\}$ er 3