Mat 104 O2

1) a) $\lim_{x \to 1} \frac{2x^2-3x+1}{x-1}$ $\quad x=1 \Rightarrow \frac{0}{0}$, L'hopital gjelder

$$\Rightarrow \frac{\frac{d}{dx}\Big|_{x=1} 2x^2-3x+1}{\frac{d}{dx}\Big|_{x=1} x-1} = \frac{4(1)-3}{1} = \underline{\underline{1}}$$

b) $\lim_{x \to \infty} \frac{x^2 \cancel{(x^3+1)}}{(x^3+1)^{\cancel{2}}} = \lim_{x \to \infty} \frac{x^2}{x^3+1}$

$f(x) = x^2 \quad g(x) = x^3+1$

$f'(x) = 2x \quad g'(x) = 3x^2$

$f''(x) = 2 \quad g''(x) = 6x$

$\frac{2}{6x} \Rightarrow 0$ når $x \to \infty$

$$\lim_{x \to \infty} \frac{x^2}{x^3+1} = \underline{\underline{0}}$$

c) $\lim\limits_{x\to\infty} \dfrac{2x^3+x-1}{-3x^3-2x+2}$ $\begin{matrix}\div x^3 \\ \div x^3\end{matrix}$

$= \lim\limits_{x\to\infty} \dfrac{2+x^{-2}-x^{-3}}{-3+2x^{-2}-2x^{-3}} = \underline{\underline{-\dfrac{2}{3}}}$

$x\to\infty \Rightarrow x^{n}\to 0$ for $n<0$

2) a) $f(x) = \dfrac{3x^5}{\sin(2x)}$

$\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$\dfrac{d}{dx}f(x) = \dfrac{15x^4\sin(2x) - 2\cos(2x\cdot 3x^5}{\sin^2(2x)}$

$= \boxed{\dfrac{15x^4\cdot\sin(2x) - 6x^5\cos(2x)}{\sin^2(2x)}}$

b) $g(x) = \sqrt{\cos^2(3\pi x)} = \left(\cos^2(3\pi x)\right)^{\frac{1}{2}}$

$$= \frac{1}{2}\frac{1}{\sqrt{\cos^2(3\pi x)}} \cdot \frac{d}{dx}\left[\cos^2(3\pi x)\right]$$

$\frac{d}{dx}\left[\cos^2(3\pi x)\right]$

$= 2\cos(3\pi x) \times (-\sin(3\pi x)) \cdot 3\pi$

$= -6\pi \cos(3\pi x)\sin(3\pi x)$

$$g'(x) = \boxed{-\frac{3\pi\cos(3\pi x)\sin(3\pi x)}{\sqrt{\cos^2(3\pi x)}}}$$

$$= -3\pi \tan(3\pi x)\left|\cos(3\pi x)\right|$$

$$= -\frac{3i\pi\left(e^{-3i\pi x} - 3^{3i\pi x}\right)\sqrt{\left(e^{-3i\pi x} + e^{3i\pi x}\right)^2}}{2\left(e^{-3i\pi x} + e^{3i\pi x}\right)}$$

:)

c) $h(x) = 3x e^{4x^2-x}$

$= 3e^{4x^2-x} + (3x)(8x-1)(e^{4x^2-x})$

$= \boxed{e^{4x^2-x}(24x^2-3x+3)}$

d) $i(x) = 4x^4 - 3x^2 + \cos x$

$\frac{d}{dx} i(x) = \boxed{16x^3 - 6x - \sin x}$

e) $j(x) = \dfrac{\cos(2x) - \sin x}{x^3}$

$j'(x) = \boxed{\dfrac{(-2\sin(2x) - \cos x)\, x^3 - 3x^2(\cos(2x) - \sin x)}{x^6}}$

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f) $k(x) = \cos(x)(3x^2 + \ln x)$

$= -\sin(x)(3x^2 + \ln x) + \cos(x)\left(6x + \frac{1}{x}\right)$

$$= \boxed{\cos(x)\left(\frac{6x^2+1}{x}\right) - \sin(x)(3x^2 + \ln(x))}$$

g) $l(x) = \cos(e^{x^2})$

$$\frac{d}{dx}\left[e^{x^2}\right] = 2xe^{x^2}$$

$$l'(x) = \boxed{-\sin(e^{x^2}) \cdot 2xe^{x^2}}$$

3) a) $\dfrac{\ln(ab^2) - \ln(a^2b)}{\ln(\frac{a}{b})}$

$$= \frac{\ln(a) + 2\ln(b) - (\ln(a^2) + \ln(b))}{\ln(a) - \ln(b)}$$

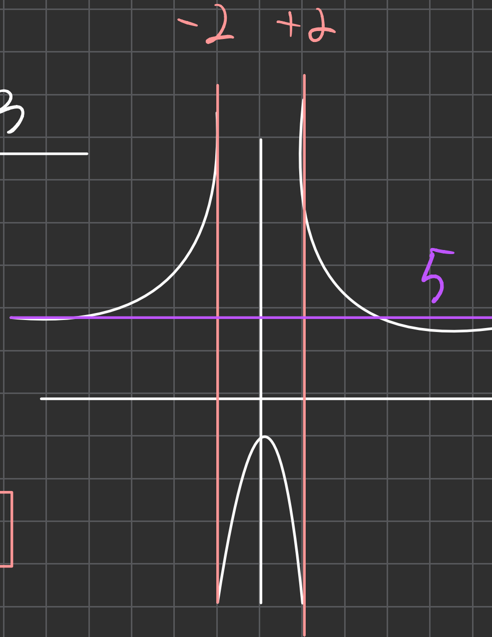
$$= \frac{\cancel{\ln(a)} + \cancel{2}\ln(b) - \cancel{2}\ln(a) - \cancel{\ln(b)}}{\ln(a) - \ln(b)}$$

$$= \frac{-\ln(a) + \ln(b)}{\ln(a) - \ln(b)} = \frac{-(\cancel{\ln(a) - \ln(b)})}{\cancel{\ln(a) - \ln(b)}}$$

$$= \boxed{-1}$$

b) $\ln(e^{4x}) - \dfrac{3}{\ln(e^{\frac{3}{4}})} = 4x - \dfrac{3}{\frac{3}{4}}$

$= 4x - 4 = \boxed{4(x-1)}$



4) a) $f(x) = \dfrac{5x^2 - 7x + 3}{x^2 - 4}$

Vertikale asymptoter:

Når $x^2 - 4 = 0$

$x^2 = 4$

$x = \boxed{\pm 2}$

Horisontale asymptoter:

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to +\infty} f(x)$

$= \lim\limits_{x \to \infty} \dfrac{5x^2 - 7x + 3}{x^2 - 4} = \dfrac{\frac{d^2}{d^2x} \Big| 5x^2 - 7x + 3}{x^2 - 4}$

$= \dfrac{10}{2} = \boxed{5}$

b) $f(x) = \dfrac{x^2 + 3x}{x^2 + 2x - 3}$

Vertikal asymptote:

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3 \Rightarrow f(x) = \frac{0}{0}, \therefore x \neq -3$$

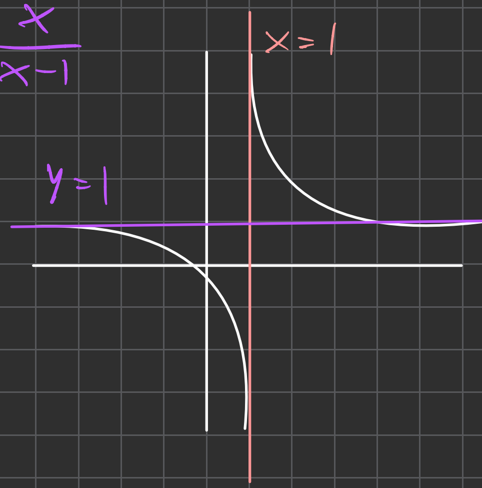
$$x = \boxed{1}$$

Horisontal asymptote:

$$\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = \lim_{x \to \infty} \frac{x^2 + 3x}{x^2 + 2x - 3}$$

$$= \lim_{x \to \infty} \frac{x\cancel{(x+3)}}{(x-1)\cancel{(x+3)}} = \lim_{x \to \infty} \frac{x}{x-1}$$

$$= \frac{\frac{d}{dx}\big| x}{\frac{d}{dx}\big| x-1} = \frac{1}{1} = \boxed{1}$$



c) $f(x) = \dfrac{ax^2 - 5}{x^2 + b}$

$y = 2 \quad x = 4$

Vertikal asymptote:

$x = 4$

$x^2 = 16 \quad \boxed{b = -16}$

Med $x^2 = 16$ får vi også $x = -4$

Horisontal asymptote:

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} f(x) = 2$$

$$\lim_{x \to \infty} \frac{ax^2 - 5}{x^2 - 16} = \frac{\frac{d^2}{d^2x} \left| ax^2 - 5 \right.}{\frac{d^2}{d^2x} \left| x^2 - 16 \right.}$$

$$= \frac{2a}{2} = a \qquad \boxed{a = 2}$$

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5) a/b)

$$f(x)\begin{cases} 2x^2-3x+a, & x\leq 1 \\ \dfrac{x^2+2x-3}{x-1}, & x>1 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$

$$\lim_{x\to 1^+} \frac{x^2+2x-3}{x-1} = \lim_{x\to 1^+} \frac{(x+3)\cancel{(x-1)}}{\cancel{x-1}}$$

$$=(1+3)=4$$

$$\lim_{x\to 1^-} 2x^2-3x+a = 2-3+a = a-1$$

$$a-1=4 \quad \therefore \boxed{a=5}$$

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Bevis for at $f(x)$ er kontinuerlig ved $x=1$.

$$f(1) = 2-3+5 = 4$$

$$\forall \varepsilon > 0 \ \exists \delta > 0 \text{ s.t } 0 < |x-1| < \delta \Rightarrow |f(x) - 4| < \varepsilon$$

La $\varepsilon > 0$ og velg $\delta = \varepsilon$.

Anta $|x-1| < \delta$.

For $x > 1$ har vi:

$$\left|\frac{x^2+2x-3}{x-1} - 4\right| = |x+3-4|$$

$= |x-1|$ så når $|x-1| < \delta$

$\Rightarrow |x-1| < \varepsilon$

For $x \leq 1$:

$$|2x^2-3x+5-4| = |2x^2-3x+1|$$

$x=1 \Rightarrow |2x^2-3x+1| = |x-1| = 0$

$x=0 \Rightarrow |2x^2-3x+1| = |x-1| = -1$

$\frac{d}{dx}\left[2x^2-3x+1\right] = 4x-3 \quad 4x-3=0 \quad 4x=3$

$x = \frac{3}{4} \qquad |f(\frac{3}{4})| = \frac{1}{8} < |\frac{3}{4}-1| = \frac{1}{4}$

Siden $|x-1| = |2x^2-5x+1|$ ved $x=0 \wedge x=1$

og ekstremalpunktet $|f(\frac{3}{4})| < |\frac{3}{4}-1|$

kan vi konkludere at for x mindre enn og nærme 1 har vi: $|f(x)-f(1)| \leq |x-1| < \delta = \varepsilon \quad \square$

$$\left|\frac{x^2+2x-3}{x-1} - 4\right|$$

$$\frac{x^2+2x-3}{x-1}$$

$= x+3$

$|x+3-4| = |x-1|$

$\varepsilon = \delta$

$|2x^2-3x+5-4|$

$= |2x^2-3x-1|$

$x=1 \Rightarrow f(x) = x-1$

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6) a) $K(t) = 150 \cdot \left(\frac{1}{2}\right)^{\frac{t}{4.25}}$

Koffein etter t timer vil være gitt som en eksponential funksjon der start mengden delest på 2 hver 4.25 timer.

Så vi vil ha $K(4.25) = \frac{\text{Start}}{2}$

En eksponentiell funksjon for et slikt fenomen gis ved $\text{Start} \times \text{endring}^{t/\text{perioder}}$

Vi starter med 150mg, multipliserer med $\frac{1}{2}$ hver periode = 4.25 timer.

b) $K(t) = 150 \cdot \left(\frac{1}{2}\right)^{t/4.25} = 10$

$$\left(\frac{1}{2}\right)^{t/4.25} = \frac{1}{15}$$

$$\log_2\left(\frac{1}{2}^{t/4.25}\right) = \log_2\left(\frac{1}{15}\right)$$

$$t/4.25 \cdot \log_2\left(\frac{1}{2}\right) = \log_2\left(\frac{1}{15}\right)$$

$$t/4.25 = \frac{\log_2\left(\frac{1}{15}\right)}{\log_2\left(\frac{1}{2}\right)}$$

$$t/4.25 = \frac{-\log_2(15)}{-\log_2(2)} = \log_2(15)$$

$$t = \log_2(15^{4.25}) \approx 16.6$$

$$08:00 + 16.6t = \boxed{00:36}$$

c) $T(t) = K(t+10) + K(t+4) + K(t)$

$= 150 \cdot \left(\frac{1}{2}\right)^{\frac{t+10}{4.25}} + 150 \cdot \left(\frac{1}{2}\right)^{\frac{t+4}{4.25}} + 150 \cdot \left(\frac{1}{2}\right)^{\frac{t}{4.25}}$

$= 150\left(\frac{1}{2}^{\frac{t+10}{4.25}} + \frac{1}{2}^{\frac{t+4}{4.26}} + 150 \cdot \frac{1}{2}^{\frac{t}{4.25}}\right)$

$18:00 \rightarrow 08:00 = 14t$

$T(14) \approx \boxed{26.25}$

7) $f(x) = -\frac{x^5}{8} + 2x^4 - 6x^3 - x^2 + 7x + 2$

$f(1) = -\frac{1}{8} + 2 - 6 - 1 + 7 + 2$

$= -\frac{1}{8} + 4 = 4 - \frac{1}{8} = \frac{7}{6}$

$f(2) = -\frac{2^5}{2^3} + 2^5 - 3 \cdot 2^4 - 2^2 + 2^4$

$= -8$

$f(1) > 0 \wedge f(2) < 0$

f er kontinuerlig så

$\exists c \in [1,2] : f(2) < f(c) = 0 < f(1)$

Made with Goodnotes

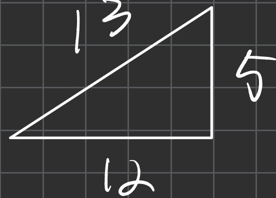
8)

a) $K^2 = 13^2 - 5^2$

$K^2 = 169 - 25$

$K^2 = 144$

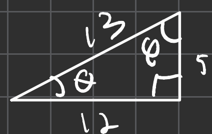
$\sqrt{144} = 12 = K$



b) $\sin\theta = \frac{5}{13}$

$\theta = \arcsin\left(\frac{5}{13}\right) \approx 0.39 \approx \frac{\pi}{8}$

$\frac{\pi}{8} \cdot \frac{180^\circ}{\pi} = 22.5^\circ$



$\theta = 22.5^\circ = \frac{\pi}{8}$

$\varphi = 67.5^\circ = \frac{6}{16}\pi$

c) $H = 12.2$

$\theta = 31.2^\circ \approx \frac{66}{125} \approx \frac{21}{125}\pi$

$\varphi = 180^\circ - 90^\circ - 31.2^\circ = 58.8^\circ \approx 1 \approx \frac{\pi}{3}$

d) $\sin\theta = \frac{K_2}{12.2}$

$12.2 \sin\theta = K_2 = 6.32$

$K_1 = \sqrt{12.2^2 - 6.32^2} = 10.44$