STAT535: Statistical Computing **Fall 2020**

Lab 3: Casino

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Objective Many real-world contracts are written such that a sequence of uncertain future events may benefit one party or the other. Building models of how these contracts might turn out is a useful way to evaluate whether they are something that the parties should agree to. A small-scale model of a contract is a casino game. Indeed, statistics and probability have a long history of connections to gambling. Note: We are in no way advocating gambling here. We're only using these games because they provide a simple, well understood set of rules that we can build computational ideas around.

1. **Coins** The goal of this game is to decide if the result of a sequence of coin tosses is fair or not.
   (a) Write a function that simulates a sequence of coin flips. Decide what the parameters of the program should be and what the returned value should be. At a minimum the probability of heads should be one of the parameters.
   (b) Use the function to simulate many sequences and make some histogram plots of some summary statistics. For example, plot the number of heads in the sequences. For each of these summary statistics write a function that takes a list of sequences and produces a list of statistics. Each of these functions is a *feature generator*.
   (c) Generate tow sets of sequences (and summary statistics) one using a fair coin and the other using a biased coin. Fit a (linear or logistics) regression relating the feature vector you generated to the fair/not fair outcome.
   (d) On your own try to generate a sequence of heads/tails that is from a fair coin. How might you write a program to decide whether the sequence is sufficiently close to fair?

2. **Dice** This game is a simple one where you will try to guess the outcome of the dice.
   (a) Write a function that simulates the sum of two size-sided dice. Write another function that takes the outcome and reports the probability of that outcome if the dice were fair.
   (b) The odds of an event is the inverse of the probability of the event. So, if we were dealing with a fair coin, the odds of heads would be 2 to 1. The odds are also the payout for a unit bet on the outcome. So, if you had an odds of 2 to 1 for heads and you bet one dollar, what would your expected return be?
   (c) When the probabilities corresponding to the odds of all of the events sum to 1, then you have a game where everyone comes out even in the long run. But, when you look at the odds for any game, the corresponding probabilities sum to something greater than 1. What is going on? Read about the *house edge* in roulette https://en.wikipedia.org/wiki/Roulette.
   (d) Look at the odds for each of the events for your dice game. Now, suppose you are working on behalf of the house and the house wants to make a 1 percent profit on the game in the long run. How might you adjust the odds to ensure this outcome?

3. **Game of your choice**

   Write a program for a game of your own design. Describe he house advantage if any. Once the game is designed, try to come up with a strategy for playing the game the eliminates the house advantage and returns a positive profit in the long run.