Team Work

TEAM WORK INSTRUCTIONS

Dear group,

1. Collect time series data of prices of **TWO** **risky financial assets** (preferably individual stocks). The time series should have **daily** frequency. Indicate the most detailed information about your source. These two time series should be used to answer **ALL** the questions of this report.

2. **100% coincidence** of the selected time series between groups **is not allowed**. Moreover, you cannot select time series discussed during the lectures or in the lecture notes. To avoid these situations, send the name/description, and source of your two time series to **nsobreira@iseg.ulisboa.pt** until **3rd May and wait for approval**. The first group that sends all this information wins the exclusive right to analyze that time series. So it is recommended that the group sends this information as soon as possible.

3. The written report has a **maximum** of **35 pages** (not counting appendices, table of contents, index and bibliographic references). Hence all relevant figures and tables should be prepared as an Appendix and cited in the written report.

4. The report should be written in a standard format.

5. The delivery date is until **22nd May 10:00 AM**. Send the written report to **nsobreira@iseg.ulisboa.pt** with the name **Group XX - Team Work**.

6. Any case of **plagiarism** is **strictly forbidden** and will be **dealt with** in accordance with ISEG Masters' regulations.

GARCH models and Value-at-Risk

1 Introduction

Any investment in financial markets has several types of risks. These may be summarized in four main types: operational risk, credit risk, liquidity risk and market risk.

This exercise deals with market risk measurement. The market risk has to do with daily stock price flutuations that are due to general factors affecting the financial markets. Given its importance in finance, there is a vast amount of research on issues related with market risk measurement. Popular methodologies to measure market risk include sensitivity analysis, scenario analysis, stress testing and downside risk measures.

The objective of this exercise is to obtain the Value-at-Risk (VaR) for **TWO** financial assets. The VaR is a downside market risk measure widely used in financial institutions. The econometric approach should be used to compute the VaR. The results obtained will be compared with other available methodologies. With the econometric approach, we make use of return and volatility forecasts of a given ARMA-GARCH model to obtain the VaR.

The outline of this exercise sheet is as follows. Section 2 introduces VaR under a probabilistic framework. Section 3 discusses VaR calculation from an econometric approach. Sections 2 and 3 rely heavily on Chapter 7 of Tsay (2005). In Section 4 you may find the description of the exercise.

2 Value-at-Risk[1]

The VaR is a measure of maximal potential change in the value of an asset (or portfolio of assets) over a period of time for a given probability. Given that the VaR is a measure of market risk, we are mainly concerned with negative changes, that is, losses. Hence, the VaR represents the maximal potential loss with probability $1-p$ over a specified time horizon h, that is, between periods t and $t+h$. In other words, the asset holder expects to incur in a loss greater or equal than the VaR with probability p (p is a small number) over the time horizon h.

[1]The following text introduces Value-at-Risk from the perspective of the holder of a **long** financial position. It is straightforward to modify it for the **short** financial position's perspective.

Example 1. Bank A holds a long financial position of 10 Million Euros in stocks of firm B. Assume that the daily log returns of firm B follow a normal distribution with mean zero and standard deviation 2.2%. Obtain the 5% VaR or the VaR at 95% confidence level.

Answer: We have that $r_t \sim N\left(\mu = 0, \sigma^2 = 2.2^2\%\right)$ where r_t denotes the log returns of firm B. Hence, the log returns do not drop more than $1.645 \times 2.2\% = 3.619\%$ with 95% probability over a 1 day period:

$$
\begin{aligned}
&P\left(r_t \geq a\right) = 0.95 \Leftrightarrow P\left(\underbrace{\frac{r_t}{2.2}}_{\sim N(0,1)} \geq \frac{a}{2.2}\right) = 0.95 \Leftrightarrow \frac{a}{2.2} = -1.645 \\
&\Leftrightarrow a = -1.645 \times 2.2 = -3.619\% \qquad (1)
\end{aligned}
$$

Alternatively:

$$
\begin{aligned}
&P\left(r_t \leq a\right) = 0.05 \Leftrightarrow P\left(\underbrace{\frac{r_t}{2.2}}_{\sim N(0,1)} \leq \frac{a}{2.2}\right) = 0.05 \Leftrightarrow \frac{a}{2.2} = -1.645 \\
&\Leftrightarrow a = -1.645 \times 2.2 = -3.619\% \qquad (2)
\end{aligned}
$$

Notice that -1.645 is the 5% quantile of the standard normal distribution. Given that the VaR is interpreted as a measure of maximal potential loss, the VaR of the log returns is the quantity 3.619% (and not -3.619%).

The VaR is usually not presented in log returns but in cash amount. In this example, the 5% VaR or VaR at 95% confidence level is obtained as:

$$VaR_{5\%} = \text{Value in Euros } \times VaR(\text{log returns}) = 10.000.000€ \times 0.03619 = 361900€$$

We conclude that we have a 1-day maximal potential loss of $361900€$ with 95% probability. In other words, with only 5% probability we have a loss of $361900€$ or higher if we hold stocks of firm B for 1 day.

We now present the general VaR definition. Suppose we are at period t and interested in measuring the risk of a financial position for the next h periods. Let $V(t)$ and $V(t+h)$ be the asset values of the financial position at periods t and $t+h$. The change of value for this financial position from t to $t+h$ is then given by:

$$\Delta V(h) = V(t+h) - V(t)$$

Moreover, given that $\Delta V(h)$ is a random variable, we also introduce the cumulative distribution function of $\Delta V(h)$ which we denote as F_h. Formally, $F_h(a) = P\left(\Delta V(h) \leq a\right),\ a \in \mathbb{R}$. We assume that F_h is a symmetric distribution function.

Now we can define the VaR of a long financial position over time horizon h with probability p, $0 < p < 1$, as the $(1-p)^{th}$ quantile of the distribution F_h:

$$P\left(\Delta V(h) \leq -VaR\right) = p \Leftrightarrow P\left(\Delta V(h) \geq VaR\right) = p \Leftrightarrow F_h\left(VaR\right) = 1 - p$$

<u>Remarks:</u>

1. The VaR depends on the specified probability p and time horizon h. To highlight this dependence we will denote the $100p\%$ VaR as $VaR_{100p\%}$. In the example, $p = 0.05$ and $h = 1$.

2. The VaR is measured in cash value (Euros, dollars,...). Usually we start by assuming a distribution for the log returns, for example, a Normal or a t-student distribution. The VaR is initially obtained over the quantiles of the distribution of log returns (see equations (1) or (2), for example). Given that log returns are approximately equal to percentage changes in the value of the financial position, we obtain the VaR of the financial position in cash value as:

$$VaR = \text{Value of the financial position} \times VaR(\text{of log returns})$$

   In the example we have Value of the financial position $= 10.000.000$€ and $VaR(\text{of log returns}) = 0.03619$.

3 VaR and ARMA-GARCH models

Financial log returns either exhibit no serial correlation or autocorrelations of small magnitude. In general, time dependence of log returns emerges at higher moments (variance). In particular, an important stylized fact of log returns is volatility clustering. A class of models very popular for being capable to address volatility clustering is the ARMA(p,q)-GARCH(m,n) class:

$$r_t = \mu_t + \varepsilon_t,\ \varepsilon_t = \sigma_t z_t,\ z_t \overset{iid}{\sim} (0,1)$$
$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{n} \beta_j \sigma_{t-j}^2$$

GARCH models enjoy such popularity because they are able to mimic the most important regularities of financial time series with reasonable accuracy. Moreover, it is possible to capture other empirically relevant features of the data with simple variants of the standard GARCH model and/or the inclusion of additional explanatory variables.

The ARMA-GARCH model can be used to calculate the VaR in the following way. Suppose that $\varepsilon_t \sim N(0,1)$. In that case, the distribution of r_{T+1} conditional on the information available until moment T is:

$$r_{T+1}|I_T \sim N\left(r_{T+1|T}, \sigma_{T+1|T}^2\right)$$

where $r_{T+1|T}$ and $\sigma_{T+1|T}$ are the one-step ahead forecasts of the return and variance, respectively. Then, the $100p\%$ VaR for the log returns is:

$$VaR_{100p\%} = \widehat{r}_{T+1|T} + z_{1-p}\widehat{\sigma}_{T+1|T}$$

where z_{1-p} is the $(1-p)$th quantile of the standard normal distribution. For example, the 5% VaR is:

$$VaR_{5\%} = \widehat{r}_{T+1|T} + 1.645\widehat{\sigma}_{T+1|T}$$

On the other hand, if one assumes that the error term follows a Student-t distribution, $\varepsilon_t \sim t_\upsilon$, we have that:

$$VaR_{100p\%} = \widehat{r}_{T+1|T} + \frac{t_\upsilon\left(1-p\right)\widehat{\sigma}_{T+1|T}}{\sqrt{\upsilon/\left(\upsilon-2\right)}}$$

where $t_\upsilon(1-p)$ is the $(1-p)$th quantile of Student t-distribution with υ degrees of freedom.

Consider now the multiperiod VaR. From the properties of log returns, we have that the log return from time $T+1$ until $T+k$ (inclusive) is:

$$r_T[k] = r_{T+1} + r_{T+2} + \ldots + r_{T+K}$$

where $r_T[k]$ denotes the $k-$period log return. The following proposition summarizes the multiperiod VaR calculations with ARMA-GARCH models:

Proposition 1. *The multiperiod VaR with ARMA-GARCH models is obtained as follows:*

1. *The $k-$period log return forecast is:*

   $$r_T[k] = r_{T+1|T} + r_{T+2|T} + \ldots + r_{T+k|T}$$

   where $r_{T+s|T}$ is the $s-$steps ahead forecast at origin T.

2. *The forecasting error of $r_T[k]$ is given by:*

   $$\varepsilon_T[k] = \varepsilon_{T+k} + (1+\psi_1)\varepsilon_{T+k-1} + \ldots + (1+\psi_1+\ldots+\psi_{k-1})\varepsilon_{T+1}$$

   where ψ_i is the i^{th} coefficient of the MA representation of the selected ARMA model for the log returns.

3. *The volatility forecast for the $k-$period log return is:*

   $$Var(\varepsilon_T[k]|I_T) = \sigma^2_{T+k|T} + (1+\psi_1)^2\sigma^2_{T+k-1|T} + \ldots + (1+\psi_1+\ldots+\psi_{k-1})^2\sigma^2_{T+1|T}$$

   where $\widehat{\sigma}^2_{T+s|T}$ is the $s-$steps ahead volatility forecast at origin T.

4. *If we assume that $\varepsilon_t \sim N(0,1)$ then:*

   $$r_T[k]|I_T \sim N(r_T[k], Var(\varepsilon_T[k]|I_T))$$

   The $100p\%$ VaR for the $k-$period log returns is constructed as:

   $$VaR_{100p\%} = \widehat{r}_T[k] + z_{1-p}\sqrt{\widehat{Var}(\varepsilon_T[k]|I_T)}$$

5. *Similarly, we obtain the $100p\%$ multiperiod VaR assuming that $\varepsilon_t \sim t_\upsilon$:*

   $$VaR_{100p\%} = \widehat{r}_T[k] + \frac{t_\upsilon(1-p)\sqrt{\widehat{Var}(\varepsilon_T[k]|I_T)}}{\sqrt{\upsilon/(\upsilon-2)}}$$

4 Exercise

Question 1 (13 (or 15) points)

(a) Discuss the time series History: when do you observe a higher volatility? What were the most relevant events that affected the time series behaviour?

From now on, consider time series data from the **last 5 years** and answer the following questions. Yet, if you find problems in finding a well specified ARIMA-GARCH model feel free to use a different sample period (last 4 or 6 years, for example). Please mention the sample period in the written report and justify your choice.

(b) Obtain the log returns and plot the time series graph of prices and log returns. Obtain the correlogram and descriptive statistics for prices and log returns. Comment on the distributional and dynamical properties of the data (mean, volatility, skewness, kurtosis, stationarity, outliers,. . .). Which stylized facts about financial returns you observe in the data?

(c) Apply the Augmented Dickey-Fuller (ADF) test to the **log prices**. Describe in detail the conclusions of the test.

(d) Perform a simplified Box-Jenkins analysis to find an ARMA model (AR, MA or mixed) that best describes the **log returns**. Justify your choice.

(e) Obtain the residuals of the ARMA model estimated in item (e). Test for the presence of ARCH effects. Analyze the time series properties of the squared residuals and use that information to propose ARMA-GARCH models for the log returns. Justify in detail your options.

(f) Evaluate the adequacy of the proposed models with the appropriate diagnostic checking procedures. Select the ARMA-GARCH model that best fits the dynamic properties of the log returns. Justify your choice.

(g) Write explicitly the estimated equation in the standard mathematical ARMA-GARCH form. Approximate your results to 3 decimal places.

(h) Redo the **previous 3 items** on the basis of the t-distribution for the error term. Do you find a better goodness-of-fit than in item (e)?

(i) Choose the estimation sample and the forecast sample for each time series and obtain the static and dynamic forecasts for both the log returns and volatility. Use the two preferred GARCH specifications (the models should be different for the volatility equation). Compare your results and comment.

(j) For only one of the time series, construct a table with the values obtained for the 1^{st} and 2^{nd} forecast obtained with the two models (the models should be different for the volatility equation). Now explain and show exactly how these values were obtained. Solve this exercise both for the dynamic and static forecasts of the volatility (you do not need to show how the forecasts of r_t were obtained).

(k) Do you find evidence that the log returns are on average different from zero? Do you find evidence for leverage effects? Do you find evidence of risk premia? Estimate the appropriate models and perform the necessary formal statistical tests to answer these questions.

Question 2 (7 points)

The objective of this exercise is to obtain VaR on the basis of different assumptions regarding the ARMA-GARCH process for the log returns. Use both financial time series to answer the following question.

(a) Suppose that a given firm holds a long financial position of 10 Million Euros in stocks of the financial asset under analysis. Obtain the VaR for the next trading day and the next 5 trading days using:

   (i) the unconditional moments of an assumed normal distribution (historical mean and standard deviation)

   (ii) a GARCH(1,1) with only a constant in the conditional mean function and normal errors. Hint: for an ARMA(0,0) model, it is possible to show that $\psi_i = 0$ for every i.

   (iii) an AR(1)-GARCH(1,1) model with normal errors. Hint: for a stationary AR(1) model, it is possible to show that $\psi_i = \phi_1^i$ for every i.

   (iv) an AR(1)-GARCH(1,1) model with Student-t innovations

   (v) Another ARMA-GARCH specification choosen by the group. Here you should choose either a constant or an AR(1) conditional mean function. For the GARCH specification you may choose a GARCH(m,n) for any m and n with either normal or student-t innovations. You may also explore extensions of the standard GARCH model (GARCH-in-mean, T-GARCH, EGARCH,...). Justify your option.

   Explain in detail your calculations. Warning: answers that only present table(s) with numbers will be ignored. It is required a proper explanation of how the values were obtained in the written report.

   Use 90%, 95% and 99% confidence levels. Compare the VaR estimates obtained for different models. Comment your findings. What happens to the VaR when the time horizon increases? What happens to the VaR when we change to Student-t innovations? What explains these results? Comment any other differences that you might find relevant.

(b) What difficulties/problems a professional portfolio manager may find with the use of the econometric approach to obtain the VaR. Motivate your answer.

References

Tsay, R. S. (2005). Analysis of Financial Time Series, Wiley.