ETH Zürich
PEACH-Lab

AI-assisted grading UI – testing exam (rubric)
Basic math

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Name: ______________________________

Student ID: ______________________________

This exam contains 6 pages (including this cover page) and 4 questions. Total of points is 31.
Good luck!

Distribution of Marks

Question	Points	Score
1	8	
2	6	
3	9	
4	8	
Total:	31	

1. (8 points) Consider the function $f(x) = x^3 - 3x^2 - 9x + 5$. Find all the local maximum and local minimum points (both x and y coordinates).

   rubric start

   2 points: For correctly finding the first derivative, $f'(x)$.
   Solution: $f'(x) = 3x^2 - 6x - 9$
   (Award 1 point for $3x^2 - 6x$ and 1 point for -9).

   2 points: For correctly finding the two critical x-values by solving $f'(x) = 0$.
   Solution: $3x^2 - 6x - 9 = 0$
   $3(x^2 - 2x - 3) = 0$
   $3(x - 3)(x + 1) = 0$
   Awards 1 point for $x = 3$, 1 point for $x = -1$.

   2 points: For correctly classifying each critical point.
   Method: Use the 2nd Derivative Test (or first derivative test).$f''(x) = 6x - 6$
   Awards: 1 point: For classifying $x = 3$ as a local minimum (because $f''(3) = 18 - 6 = 12 > 0$).
   1 point: For classifying $x = -1$ as a local maximum (because $f''(-1) = -6 - 6 = -12 < 0$).

   2 points: For finding the correct y-coordinates by plugging the x-values back into the original $f(x)$.
   Awards: 1 point: For the local maximum point: $f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 5 = -1 - 3 + 9 + 5 = 10$.
   Final Point: $(-1, 10)$
   1 point: For the local minimum point: $f(3) = (3)^3 - 3(3)^2 - 9(3) + 5 = 27 - 27 - 27 + 5 = -22$.
   Final Point: $(3, -22)$

2. (6 points) Find the area of the region enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

rubric start

2 points: For finding the correct x-coordinates of the intersection points.
Solution: Set $x^2 = x + 2 \implies x^2 - x - 2 = 0 \implies (x - 2)(x + 1) = 0$
Awards:1 point for $x = 2$, 1 point for $x = -1$.

2 points: For setting up the correct definite integral.
Solution: Area $= \int_{-1}^{2} (\text{upper curve} - \text{lower curve})\, dx = \int_{-1}^{2} ((x + 2) - x^2)\, dx$
Awards: 1 point for the correct integrand: $(x + 2) - x^2$, 1 point for the correct bounds of integration: from -1 to 2.

2 points: For correctly evaluating the integral to find the final area.
Solution: $\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} = \left(\frac{(2)^2}{2} + 2(2) - \frac{(2)^3}{3}\right) - \left(\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right) = \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = \left(6 - \frac{8}{3}\right) - \left(\frac{3}{6} - \frac{12}{6} + \frac{2}{6}\right) = \left(\frac{10}{3}\right) - \left(-\frac{7}{6}\right) = \frac{20}{6} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}$ or 4.5
Awards: 1 point for finding the correct antiderivative: $\frac{x^2}{2} + 2x - \frac{x^3}{3}$, 1 point for the correct final numerical answer: 4.5 or 9/2.

3. Consider the following system of linear equations:

$$\begin{cases} \text{x + y + z = 6} \\ \text{2y + 5z = -4} \\ \text{2x + 5y - z = 27} \end{cases}$$

(a) (3 points) Write the system in it's augmented matrix form $[A|b]$.

(b) (6 points) Solve the system for x, y and z using Gaussian elimination. Show your steps.

rubric start

(a) **3 points:** Awarded for the entirely correct augmented matrix.The points should be distributed 1 point per correct row.

Correct Solution: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & 5 & -4 \\ 2 & 5 & -1 & 27 \end{array}\right]$

1 point for Row 1: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \end{array}\right]$
1 point for Row 2: $\left[\begin{array}{ccc|c} 0 & 2 & 5 & -4 \end{array}\right]$
1 point for Row 3: $\left[\begin{array}{ccc|c} 2 & 5 & -1 & 27 \end{array}\right]$

(b) **2 points:** For correctly applying row operations to get to a valid row-echelon form.

Example steps: $R_3 \to R_3 - 2R_1 \implies \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & 5 & -4 \\ 0 & 3 & -3 & 15 \end{array}\right] R_3 \to R_3/3 \implies \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & 5 & -4 \\ 0 & 1 & -1 & 5 \end{array}\right]$ Swap $R_2, R_3 \implies \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & 5 \\ 0 & 2 & 5 & -4 \end{array}\right] R_3 \to R_3 - 2R_2 \implies \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 7 & -14 \end{array}\right]$ (This is a valid row-echelon form).

1 point: For correctly solving for the first variable (z) from the echelon form.
Solution: $7z = -14 \implies z = -2$

1 point: For correctly back-substituting to find the second variable (y).
Solution: $y - z = 5 \implies y - (-2) = 5 \implies y + 2 = 5 \implies y = 3$

1 point: For correctly back-substituting to find the third variable (x).
Solution: $x + y + z = 6 \implies x + 3 + (-2) = 6 \implies x + 1 = 6 \implies x = 5$

1 point: For clearly stating the final solution as a point or vector.
Solution: $(x, y, z) = (5, 3, -2)$

4. A factors has two machines, Machine A and Machine B, producing microchips.

   - Machine A produces 60% of the total chips.
   - Machine B produces 40% of the total chips.
   - The defect rate of Machine A is 5% (i.e., 5% of chips from A are defective)
   - The defect rate of Machine B is 2% (i.e., 2% of chips from B are defective)

   (a) (4 points) What is the overall probability that a randomly selected chip from the factory is defective?

   (b) (4 points) A chip is randomly selected and found to be defective. What is the probability that it came from Machine A?

   rubric start

   (a) **1 point:** For identifying the correct formula (Law of Total Probability): $P(D) = P(D|A)P(A) + P(D|B)P(B)$
   1 point: For correctly calculating the probability of a defect from A: $P(D \cap A) = 0.05 \times 0.60 = 0.03$
   1 point: For correctly calculating the probability of a defect from B: $P(D \cap B) = 0.02 \times 0.40 = 0.008$
   1 point: For the correct final sum: $P(D) = 0.03 + 0.008 = 0.038$ (or 3.8%)

   (b) **1 point:** For identifying the correct formula (Bayes' Theorem): $P(A|D) = \frac{P(D|A)P(A)}{P(D)}$
   1 point: For correctly identifying the numerator (can be from part a): $P(D|A)P(A) = 0.03$
   1 point: For correctly identifying the denominator (the answer from part a): $P(D) = 0.038$
   1 point: For the correct final calculation: $\frac{0.03}{0.038} \approx 0.789$ (or 78.9%, 30/38, 15/19)

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.