ETH Zürich
PEACH-Lab

AI-assisted grading UI – testing exam
Basic math

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Name: Student A
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This exam contains 6 pages (including this cover page) and 4 questions. Total of points is 31.
Good luck!

Distribution of Marks

Question	Points	Score
1	8	
2	6	
3	9	
4	8	
Total:	31	

1. (8 points) Consider the function $f(x) = x^3 - 3x^2 - 9x + 5$. Find all the local maximum and local minimum points (both x and y coordinates).

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

critical points: $(f'(x) \stackrel{!}{=} 0)$

$\Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow 3(x^2 - 6x - 9) = 0$

$\Rightarrow 3(x - ?)(x + ?) = 0$

no idea !!

$x_1 = ?$ $x_2 = ?$

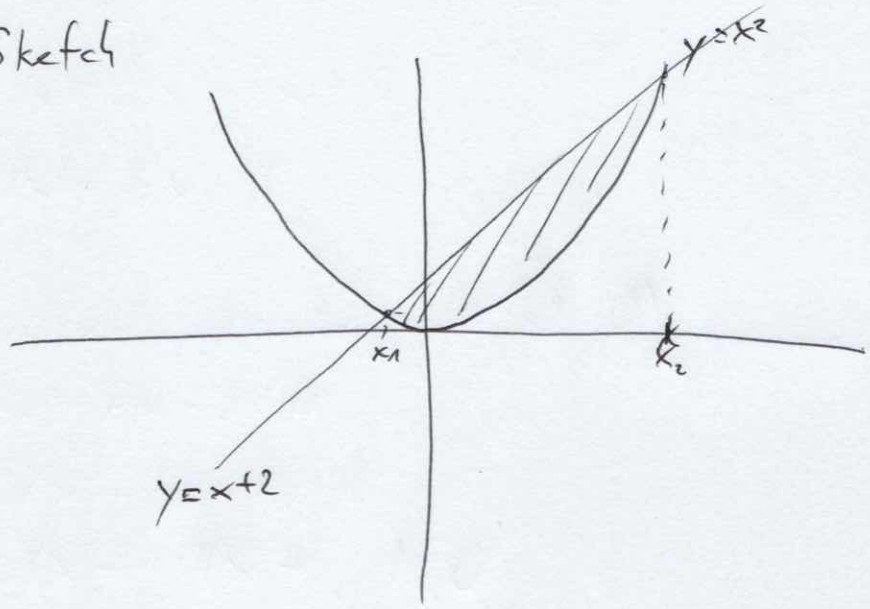
check for min or max

$f''(x_{1,2}) = ??$ $\begin{cases} \text{min if } > 0 \\ \text{max if } < 0 \end{cases}$

Then find y_1 by $f(x_1)$ and find y_2 by $f(x_2)$

2. (6 points) Find the area of the region enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

Sketch



Find intersections x_1 & x_2:

$$x^2 = x + 2$$
$$\Rightarrow x^2 - x - 2 = 0$$
$$\Rightarrow (x+1)(x-2) = 0$$
$$\Rightarrow x_1 = -1 \;,\; x_2 = 2$$

Calc. region:

$$\int_{-1}^{2} x^2\,dx - \int_{-1}^{2} x+2\,dx = \left[\tfrac{1}{3}x^3\right]_{-1}^{2} - \left[\tfrac{1}{2}x^2+2x\right]_{-1}^{2}$$
$$= \tfrac{1}{3}2^3 - \tfrac{1}{3}(-1)^3 - \left(\tfrac{1}{2}2^2 + 2\cdot 2 - \left(\tfrac{1}{2}(-1)^2 + 2\cdot(-1)\right)\right)$$
$$= \tfrac{8}{3} + \tfrac{1}{3} - 2 - 4 + \tfrac{1}{2} - 2$$
$$= 3 - 8 + \tfrac{1}{2} = -4.5$$

3. Consider the following system of linear equations:

$$\begin{cases} x + y + z = 6 \\ 2y + 5z = -4 \\ 2x + 5y - z = 27 \end{cases}$$

(a) (3 points) Write the system in it's augmented matrix form $[A|b]$.

(b) (6 points) Solve the system for x, y and z using Gaussian elimination. Show your steps.

(a)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{pmatrix} \qquad b = \begin{pmatrix} 6 \\ -4 \\ 27 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ (2) & 5 & -1 \end{pmatrix}$$

↖ eliminate but I forgot how ;;

In the end:

$A = \nabla$-form

then solution can be directly calculated by insertion from lower row

4. A factors has two machines, Machine A and Machine B, producing microchips.

   - Machine A produces 60% of the total chips.
   - Machine B produces 40% of the total chips.
   - The defect rate of Machine A is 5% (i.e., 5% of chips from A are defective)
   - The defect rate of Machine B is 2% (i.e., 2% of chips from B are defective)

   (a) (4 points) What is the overall probability that a randomly selected chip from the factory is defective?

   (b) (4 points) A chip is randomly selected and found to be defective. What is the probability that it came from Machine A?

(a) $60\% \cdot 5\% + 40\% \cdot 2\% = 0.6 \cdot 0.05 + 0.4 \cdot 0.02$

$= 0.038$

$= 3.8\%$

(b) $\dfrac{\text{chance defective Machine B}}{\text{chance defective Machine A}}$

$\Rightarrow \dfrac{40\% \cdot 2\%}{60\% \cdot 5\%}$

$\Rightarrow 0.2\overline{6} = 26\%$

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.