

University of Central Florida

UCF Triangulate

Benjamin Prins, Brian Barak, Thomas Meeks

1	Contest	1
2	Mathematics	1
3	Data structures	2
4	Geometry	5
5	Graphs	9
6	Numerical Methods	16
7	Number theory	20
8	Combinatorial	22
9	Strings	22
10	Various	24

Contest (1)

```
.bashrc
3 lines
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++20 \
-fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps =◇
```

```
hash.sh
6 lines
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
# Usage:
#   To make executable, run the command: chmod +x hash.sh
#   To execute: ./hash.sh < file.cpp
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum |cut -c-6
```

Mathematics (2)

2.1 Recurrences

If $a_n = c_1a_{n-1} + \dots + c_ka_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k - c_1x^{k-1} - \dots - c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1r_1^n + \dots + d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1n + d_2)r^n$.

2.2 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.3 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx} \tan x = 1 + \tan^2 x \qquad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$
$$\int \tan ax = -\frac{\ln |\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) \qquad \int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.4 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n + 1)}{2}$$
$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n + 1)(n + 1)}{6}$$
$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n + 1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30}$$

2.5 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$
$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

2.5.1 Discrete distributions
Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\operatorname{Bin}(n, p)$, $n = 1, 2, \dots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1 - p)^{n - k}$$
$$\mu = np, \sigma^2 = np(1 - p)$$

$\operatorname{Bin}(n, p)$ is approximately $\operatorname{Po}(np)$ for small p .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\operatorname{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1 - p)^{k - 1}, k = 1, 2, \dots$$
$$\mu = \frac{1}{p}, \sigma^2 = \frac{1 - p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\operatorname{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$
$$\mu = \lambda, \sigma^2 = \lambda$$

2.5.2 Continuous distributions
Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\operatorname{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b - a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$
$$\mu = \frac{a + b}{2}, \sigma^2 = \frac{(b - a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$
$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.6 Geometry

2.6.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b + c} \right)^2 \right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

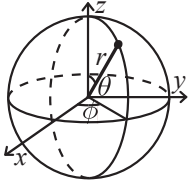
2.6.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

2.6.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z / \sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

Data structures (3)

```
OrderStatisticTree.h
Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null_type.
Time: O(log N)
782797, 16 lines

#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>; //cd2981

void example() {
    Tree<int> t, t2; t.insert(8);
    auto it = t.insert(10).first;
    assert(it == t.lower_bound(9)); //b1d86a
    assert(t.order_of_key(10) == 1);
    assert(t.order_of_key(11) == 2);
    assert(*t.find_by_order(0) == 8);
    t.join(t2); // assuming T< T2 or T> T2, merge t2 into t
} //782797
```

```
HashMap.h
Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).
d77092, 7 lines

#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
    const uint64_t C = 11(4e18 * acos(0)) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(x*C); }
}; //9b48b4
__gnu_pbds::gp_hash_table<ll, int, chash> h({}, {}, {}, {}, {}, {1<<16});
```

```
SegmentTree.h
Description: Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.
Time: O(log N)
0f4bdb, 19 lines

struct Tree {
    typedef int T;
    static constexpr T unit = INT_MIN;
    T f(T a, T b) { return max(a, b); } // (any associative fn)
    vector<T> s; int n;
    Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {} //2b8055
    void update(int pos, T val) {
        for (s[pos += n] = val; pos /= 2;)
            s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
    }
};
```

```
} //17a935
T query(int b, int e) { // query [b, e)
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
        if (b % 2) ra = f(ra, s[b++]);
        if (e % 2) rb = f(s[--e], rb); //d4678d
    } //3391a8
    return f(ra, rb);
} //c90093
}; //0f4bdb
```

```
LazySegmentTree.h
Description: Segment tree with lazy prop, modify at will. 0-based, inclusive-exclusive.
Usage: lazy_segtree<ValType, LazyType> st(arr)
Time: O(log N).
```

```
4f67ee, 60 lines

template<class T, class F>
struct lazy_segtree {
    int N, log, S;
    T idem = //modify: op(val, idem) = val
    F defLazy = //modify: applyLazy/combLazy(val, defLazy) = val
    vector<T> d; //ce445d
    vector<F> lz;
    T op(T left, T right) { //modify*/ } //09e848
    T applyLazy(T val, F lazy) { //modify*/ } //92c854
    F combLazy(F old, F nw) { //modify*/ } //4c16ff
    lazy_segtree(const vector<T>& v) :
        N(sz(v)), log(__lg(2 * N - 1)), S(1 << log), d(2 * S, idem),
        lz(S, defLazy) {
        for (int i = 0; i < N; i++) d[S + i] = v[i];
        for (int i = S - 1; i >= 1; i--) pull(i); //5a138c
    } //9e2ae3
    void apply(int k, F f) {
        d[k] = applyLazy(d[k], f); //len= S>>(31--builtin.clz(k));
        if (k < S) lz[k] = combLazy(lz[k], f);
    } //c7a3ac
    void push(int k) {
        apply(2 * k, lz[k]), apply(2 * k + 1, lz[k]), lz[k] =
            defLazy;
    } //f332a2
    void push(int l, int r) {
        int zl = __builtin_ctz(l), zr = __builtin_ctz(r);
        for (int i = log; i > min(zl, zr); i--) {
            if (i > zl) push(1 >> i);
            if (i > zr) push((r - 1) >> i); //40c9d4
        } //4613f2
    } //708f77
    void pull(int k) { d[k] = op(d[2 * k], d[2 * k + 1]); }
    void set(int p, T x) {
        p += S;
        for (int i = log; i >= 1; i--) push(p >> i);
        for (d[p] = x; p /= 2;) pull(p); //e740d0
    } //f65093
    T query(int l, int r) {
        if (l == r) return idem;
        push(1 += S, r += S);
        T vl = idem, vr = idem;
        for (; l < r; l /= 2, r /= 2) { //f70eb7
            if (l & 1) vl = op(vl, d[l++]);
            if (r & 1) vr = op(d[--r], vr);
        } //09a718
        return op(vl, vr);
    } //7197c1
    void update(int l, int r, F f) {
        if (l == r) return;
        push(1 += S, r += S);
        for (int a = l, b = r; a < b; a /= 2, b /= 2) {
```

```
        if (a & 1) apply(a++, f); //9b5541
        if (b & 1) apply(--b, f);
    } //f428a2
    int zl = __builtin_ctz(l), zr = __builtin_ctz(r);
    for (int i = min(zl, zr) + 1; i <= log; i++) {
        if (i > zl) pull(1 >> i);
        if (i > zr) pull((r - 1) >> i);
    } //592fbe
    } //b5d617
}; //4f67ee
```

Wavelet.h

Description: kth: finds k+1th smallest number in [l,r], count: rank of k (how many < k) in [l,r]. Doesn't support negative numbers, and requires a[i] <= maxval. Use BitVector to make 1.6x faster and 4x less memory.
Time: $\mathcal{O}(\log MAX)$

```
struct WaveletTree {
    int n; vvi bv; // vector<BitVector> bv;
    WaveletTree(vl a, ll max_val):
        n(sz(a)), bv(1+__lg(max_val), {{{}}}) {
        vl nxt(n);
        for (int h = sz(bv); h--;) { //2d1680
            vector<bool> b(n);
            rep(i, 0, n) b[i] = ((a[i] >> h) & 1);
            bv[h] = vi(n+1); // bv[h] = b;
            rep(i, 0, n) bv[h][i+1] = bv[h][i] + !b[i]; // delete
            array it{begin(nxt), begin(nxt) + bv[h][n]}; //0c84d2
            rep(i, 0, n) *it[b[i]]++ = a[i];
            swap(a, nxt);
        } //f93ef6
    } //54c891
    ll kth(int l, int r, int k) {
        ll res = 0;
        for (int h = sz(bv); h--;) {
            int l0 = bv[h][l], r0 = bv[h][r];
            if (k < r0 - l0) l = l0, r = r0; //e4af0f
            else
                k -= r0 - l0, res |= 1ULL << h,
                l += bv[h][n] - l0, r += bv[h][n] - r0;
        } //aa8465
        return res;
    } //67fa6f
    int count(int l, int r, ll ub) {
        int res = 0;
        for (int h = sz(bv); h--;) {
            int l0 = bv[h][l], r0 = bv[h][r];
            if ((~ub >> h) & 1) l = l0, r = r0; //09ef1a
            else
                res += r0 - l0, l += bv[h][n] - l0,
                r += bv[h][n] - r0;
        } //8380c1
        return res;
    } //d305cc
}; //11aee1
```

BitVector.h

Description: Given vector of bits, counts number of 0's in [0, r). Use with WaveletTree.h by using modifications in comments in that file and replacing bv[h][x] with bv[h].cnt0(x)
Time: $\mathcal{O}(1)$ time

```
struct BitVector {
    vector<pair<ll, int>> b;
    BitVector(vector<bool> a): b(sz(a) / 64 + 1) {
        rep(i, 0, sz(a))
            b[i >> 6].first |= 1ll(a[i]) << (i & 63);
        rep(i, 0, sz(b)-1) //cba6aa
            b[i + 1].second = __builtin_popcountll(b[i].first)
```

```
        + b[i].second;
    } //4da2bc
    int cnt0(int r) {
        auto [x, y] = b[r >> 6];
        return r - y
            - __builtin_popcountll(x & ((1ULL << (r & 63)) - 1));
    } //01da37
}; //afd9d2
```

PST.h

Description: Persistent segment tree with laziness
Time: $\mathcal{O}(\log N)$ per query, $\mathcal{O}((n + q) \log n)$ memory

```
struct PST {
    PST *l = 0, *r = 0;
    int lo, hi;
    ll val = 0, lzadd = 0;
    PST(vl& v, int lo, int hi) : lo(lo), hi(hi) {
        if (lo + 1 < hi) { //e43119
            int mid = lo + (hi - lo)/2;
            l = new PST(v, lo, mid); r = new PST(v, mid, hi);
            val = l->val + r->val;
        } //ebf78b
        else val = v[lo];
    } //7ff852
    ll query(int L, int R) {
        if (R <= lo || hi <= L) return 0; // idempotent
        if (L <= lo && hi <= R) return val;
        push();
        return l->query(L, R) + r->query(L, R); //6a44fe
    } //108984
    PST* add(int L, int R, ll v) {
        if (R <= lo || hi <= L) return this;
        PST *n;
        if (L <= lo && hi <= R) {
            n = new PST(*this); //70575f
            n->val += v;
            n->lzadd += v;
        } else {
            push();
            n = new PST(*this); //c68682
            n->l = l->add(L, R, v);
            n->r = r->add(L, R, v);
            n->val = n->l->val + n->r->val;
        } //d1bfc5
        return n;
    } //d6d267
    void push() {
        if (lzadd == 0) return;
        l = l->add(lo, hi, lzadd);
        r = r->add(lo, hi, lzadd);
        lzadd = 0; //d7e73b
    } //0af5c4
}; //7ddad1
```

Xorbasis.h

Description: Makes a basis of binary vectors
Time: check/add -> $\mathcal{O}((B^2)/32)$

```
template<int B>
struct XORBasis {
    bitset<B> b[B];
    int npivot = 0, nfree = 0;
    bool check(bitset<B> v) {
        for(int i = B-1; i >= 0; i--) //563a45
            if (v[i]) v ^= b[i];
        return v == 0;
    } //4915f9
    bool add(bitset<B> v) {
```

```
        for(int i = B-1; i >= 0; i--) {
            if (v[i]) {
                if (b[i] == 0) return b[i] = v, ++npivot;
                v ^= b[i]; //7a144f
            } //b1b631
        } //8da7e8
        return !++nfree;
    } //fbb3dd
}; //1d856b
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback(). and add path compression at the commented line
Usage: int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$

```
struct RollbackUF {
    vi e; vector<pii> st;
    RollbackUF(int n) : e(n, -1) {} //66f6eb
    int size(int x) { return -e[find(x)]; } //if no rollback
    int find(int x) { return e[x] < 0 ? x : *e[x] ==*/find(e[x]);
    }
    int time() { return sz(st); } //821d77
    void rollback(int t) {
        for (int i = time(); i --> t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    } //e7fe82
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]}); //3aaa7c
        st.push_back({b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    } //f0724e
}; //de4ad0
```

MonoRange.h

Description: when cmp = less(): a[le[i]] < a[i] >= a[ri[i]]
Usage: vi le = mono.st(a, less()),
ri = mono.range(le);
less.equal(), greater(), greater.equal()
Time: $\mathcal{O}(N)$.

```
template<class T, typename F>
vi mono_st(const vector<T> & a, F cmp) {
    vi le(sz(a));
    rep(i, 0, sz(a)){
        for (le[i] = i - 1; le[i] >= 0 && !cmp(a[le[i]], a[i]);)
            le[i] = le[le[i]]; } //f637ae
        return le;
    } //a87918
```

```
vi mono_range(const vi &le) {
    vi ri(sz(le), sz(le));
    rep (i, 0, sz(le))
        for (int j = i - 1; j != le[i]; j = le[j]) //9e9289
            ri[j] = i;
    return ri;
} //191698
```

CountRect.h

Description: cnt[i][j] = number of times an i-by-j sub rectangle appears such that all i*j cells ARE 1. cnt[i][0],cnt[0][j] are garbage
Time: $\mathcal{O}(NM)$

```
191698, 16 lines
71b256, 22 lines
```

UCF

```
vector<vi> count_rectangles(  
    const vector<vector<bool>>&grid) {  
        int n = sz(grid), m = sz(grid[0]);  
        vector<vi> cnt(n + 1, vi(m + 1, 0));  
        vi h(m);  
        for( const auto &row : grid) { //2270a5  
            transform(all(h), begin(row), begin(h),  
                [](int a, bool g) { return g * (a + 1); });  
            vi le ( mono_st(h,less()), r(mono_range(le));  
            rep(j,0,m) {  
                int cnt_l = j - le[j] - 1, cnt_r = r[j] - j - 1;  
                cnt[h[j]][cnt_l + cnt_r + 1]++; //9e604e  
                cnt[h[j]][cnt_l]--;  
                cnt[h[j]][cnt_r]--;  
            } //82de19  
        } //7a1347  
        rep(i,1,n+1) rep(k,0,2) for (int j = m; j > 1; j--)  
            cnt[i][j - 1] += cnt[i][j];  
        for (int i = n; i > 1; i--)  
            rep(j, 1, m + 1) cnt[i - 1][j] += cnt[i][j];  
        return cnt; //eca1f3  
    } //71b256
```

Lichao.h
Description: min Li-chao tree allows for range add of arbitrary functions such that any two functions only occur atmost once.
Usage: inc-inc, implicit, works with negative indices, O(log(n)) query
flip signs in update and modify query to change to max

```
struct func {  
    ll A,B;  
    func(ll A, ll B): A(A), B(B) {} //4cab61  
    ll operator()(ll x) { return (A*x + B); } //e8aa73  
}; //b3c3bf  
const func idem = {0,(ll)8e18}; //idempotent, change for max  
struct node {  
    int lo, md, hi;  
    func f = idem;  
    node *left = nullptr, *right = nullptr; //12341d  
    node(int lo, int hi): lo(lo), hi(hi), md(lo+(hi-lo)/2) {}  
    void check(){  
        if(left) return;  
        left = new node(lo,md);  
        right = new node(md+1,hi); //b79f4d  
    } //edfaa5  
    void update(func e) { //flip signs for max  
        if(e.md) < f(md)) swap(e, f);  
        if(lo == hi) return;  
        if(e(lo) > f(lo) && e(hi) > f(hi)) return;  
        check(); //cf8828  
        if(e(lo) < f(lo)) left->update(e);  
        else right->update(e);  
    } //fd3fd  
    void rangeUpdate(int L, int R, func e) { ///[  
        if(R < lo || hi < L) return;  
        if(L <= lo && hi <= R) return update(e);  
        check();  
        left->rangeUpdate(L, R, e); //44440a  
        right->rangeUpdate(L, R,e);  
    } //02b2a9  
    ll query(int x) { //change to max if needed  
        if(lo == hi) return f(x); check();  
        if(x <= md) return min(f(x), left->query(x));  
        return min(f(x), right->query(x));  
    } //66991a  
}; //1eac23
```

Lichao LineContainer Treap PQupdate FenwickTree

LineContainer.h
Description: Container where you can add lines of the form $kx+m$, and query maximum values at points x . Useful for dynamic programming (“convex hull trick”).
Time: $\mathcal{O}(\log N)$

```
struct Line {  
    mutable ll k, m, p;  
    bool operator<(const Line& o) const { return k < o.k; }  
    bool operator<(ll x) const { return p < x; } //0dcc67  
}; //7e3ecf  
  
struct LineContainer : multiset<Line, less<>> {  
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)  
    static const ll inf = LLONG_MAX;  
    ll div(ll a, ll b) { // floored division  
        return a / b - ((a ^ b) < 0 && a % b); } //fa88a2  
    bool isect(iterator x, iterator y) {  
        if (y == end()) return x->p = inf, 0;  
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;  
        else x->p = div(y->m - x->m, x->k - y->k);  
        return x->p >= y->p; //846095  
    } //31f5a2  
    void add(ll k, ll m) {  
        auto z = insert({k, m, 0}), y = z++, x = y;  
        while (isect(y, z)) z = erase(z);  
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));  
        while ((y = x) != begin() && (--x)->p >= y->p) //6dc2b6  
            isect(x, erase(y));  
    } //4e2c33  
    ll query(ll x) {  
        assert(!empty());  
        auto l = *lower_bound(x);  
        return l.k * x + l.m;  
    } //5a0881  
}; //8ec1c7
```

Treap.h
Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

```
typedef struct Node {  
    Node *l = 0, *r = 0;  
    int val, y, c;  
    Node(int val) : val(val), y(rand()) { pull(); } //28152f  
    void pull(); void push();  
} *np;  
int cnt(np n) { return n ? n->c : 0; } //c83ac4  
void Node::pull() { c = cnt(l) + cnt(r) + 1; } //42405c  
void Node::push() {} //193a22  
np nl, nr;  
pair<np, np> split(np x, int i, np &l = nl, np &r = nr) {  
    if (!x) return {l = r = 0, 0}; //dcdcd  
    x->push(); // VVV "cnt(x->l)" => "x->val" for lower_bound  
    if (i <= cnt(x->l)) split(x->l, i, l, x->l), r = x;  
    else split(x->r, i - cnt(x->l) - 1, x->r, r), l = x;  
    x->pull(); // ^^ and "i - cnt(x->l) - 1" => "i"  
    return {l, r}; //b9fc00  
} //dd2863  
auto merge(np l, np r, np &x = nl) {  
    if (!l || !r) return x = l ? r;  
    l->push(), r->push();  
    if (l->y < r->y) merge(l, r->l, r->l), x = r;  
    else merge(l->r, r, l->r), x = l; //7af065  
    x->pull();  
    return x;  
} //cceed  
template<class F> void each(np n, F f) {
```

```
    if (n) { n->push(); each(n->l, f); f(n->val); each(n->r, f);  
    }  
} //069e4f
```

PQupdate.h
Description: T: value/update type. DS: Stores T. Same semantics as std::priority_queue.
Time: $\mathcal{O}(U \log N)$.

```
template<class T, class DS, class Compare = less<T>>  
struct PQUpdate {  
    DS inner;  
    multimap<T, int, Compare> rev_upd;  
    using iter = decltype(rev_upd)::iterator;  
    vector<iter> st; //23764d  
    PQUpdate(DS inner, Compare comp={}) :  
        inner(inner), rev_upd(comp) {} //3c5f72  
  
    bool empty() { return st.empty(); } //86845b  
    const T& top() { return rev_upd.rbegin()->first; } //0b3b9d  
    void push(T value) {  
        inner.push(value);  
        st.push_back(rev_upd.insert({value, sz(st)}));  
    } //44a05d  
    void pop() {  
        vector<iter> extra;  
        iter curr = rev_upd.end();  
        int min_ind = sz(st);  
        do { //7ec537  
            extra.push_back(--curr);  
            min_ind = min(min_ind, curr->second);  
        } while (2*sz(extra) < sz(st) - min_ind);  
        while (sz(st) > min_ind) {  
            if (rev_upd.value_comp()(*st.back(), *curr)) //c1e316  
                extra.push_back(st.back());  
            inner.pop(); st.pop_back();  
        } //a76ade  
        rev_upd.erase(extra[0]);  
        for (auto it : extra | views::drop(1) | views::reverse) {  
            it->second = sz(st);  
            inner.push(it->first);  
            st.push_back(it); //ff1079a  
        } //8a3607  
    } //f4f582  
}; //35a7d2
```

FenwickTree.h
Description: Computes partial sums $a[0] + a[1] + \dots + a[pos - 1]$, and updates single elements $a[i]$, taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

```
struct FT {  
    vector<ll> s;  
    FT(int n) : s(n) {} //890409  
    void update(int pos, ll dif) { // a[pos] += dif  
        for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;  
    } //a0c54f  
    ll query(int pos) { // sum of values in [0, pos)  
        ll res = 0;  
        for (; pos > 0; pos &= pos - 1) res += s[pos-1];  
        return res;  
    } //585cdd  
    int lower_bound(ll sum) { // min pos st sum of [0, pos] >= sum  
        // Returns n if no sum is >= sum, or -1 if empty sum is.  
        if (sum <= 0) return -1;  
        int pos = 0;  
        for (int pw = 1 << 25; pw; pw >= 1) { //d79a33  
            if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
```

```
        pos += pw, sum -= s[pos-1];
    } //cb4863
    return pos;
} //923db1
}; //e62fac
```

FenwickTree2d.h

Description: Computes sums $a[i,j]$ for all $i < I, j < J$, and increases single elements $a[i,j]$. Requires that the elements to be updated are known in advance (call `fakeUpdate()` before `init()`).

Time: $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h"	157f07, 22 lines
-----------------	------------------

```
struct FT2 {
    vector<vi> ys; vector<FT> ft;
    FT2(int limx) : ys(limx) {} //5e2398
    void fakeUpdate(int x, int y) {
        for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
    } //fe99f8
    void init() {
        for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
    } //1e4363
    int ind(int x, int y) {
        return (int) (lower_bound(all(ys[x]), y) - ys[x].begin());
    }
    void update(int x, int y, ll dif) {
        for (; x < sz(ys); x |= x + 1)
            ft[x].update(ind(x, y), dif); //5a5698
    } //c48166
    ll query(int x, int y) {
        ll sum = 0;
        for (; x; x = x - 1)
            sum += ft[x-1].query(ind(x-1, y));
        return sum; //eb0cfb
    } //266f9d
}; //157f07
```

RMQ.h

Description: Range Minimum Queries on an array. Returns $\min(V[a], V[a+1], \dots, V[b-1])$ in constant time.

Usage: `RMQ rmq(values);`
`rmq.query(inclusive, exclusive);`

Time: $\mathcal{O}(|V| \log |V| + Q)$

	510c32, 16 lines
--	------------------

```
template<class T>
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1, V) {
        for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
            jmp.emplace_back(sz(V) - pw * 2 + 1); //7420f3
            rep(j, 0, sz(jmp[k]))
                jmp[k][j] = min(jmp[k-1][j], jmp[k-1][j+pw]);
        } //398c5e
    } //dfd89
    T query(int a, int b) {
        assert(a < b); // or return inf if a == b
        int dep = 31 - __builtin_clz(b - a);
        return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
    } //63f839
}; //510c32
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge (a, c) and remove the initial add call (but keep in).

Time: $\mathcal{O}(N\sqrt{Q})$

	a12ef4, 49 lines
--	------------------

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer
```

```
vi mo(vector<pii> Q) {
    int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s; //e77382
#define K(x) pii(x.first/blk, x.second ^ ~(x.first/blk & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
    for (int qi : s) {
        pii q = Q[qi]; //bbab36
        while (L > q.first) add(--L, 0);
        while (R < q.second) add(R++, 1);
        while (L < q.first) del(L++, 0);
        while (R > q.second) del(--R, 1);
        res[qi] = calc(); //8c7386
    } //d497d7
    return res;
} //7b2870
```

```
vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0) {
    int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
    add(0, 0), in[0] = 1; //933a80
    auto dfs = [&](int x, int p, int dep, auto& f) -> void {
        par[x] = p;
        L[x] = N;
        if (dep) I[x] = N++;
        for (int y : ed[x]) if (y != p) f(y, x, !dep, f); //f4faf6
        if (!dep) I[x] = N++;
        R[x] = N;
    }; //7ac875
    dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ ~(I[x[0]] / blk & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
    for (int qi : s) rep(end, 0, 2) { //6a3e39
        int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                    else { add(c, end); in[c] = 1; } a = c; }
        while (!(L[b] <= L[a] && R[a] <= R[b]))
            I[i++] = b, b = par[b]; //c95d6c
        while (a != b) step(par[a]);
        while (i--) step(I[i]);
        if (end) res[qi] = calc();
    } //44b82c
    return res;
} //a12ef4
```

Geometry (4)

4.1 Lines and Segments

sideOf.h

Description: Returns where p is as seen from s towards e . $1/0/-1 \Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be `Point<T>` where T is e.g. `double` or `long long`. It uses products in intermediate steps so watch out for overflow if using `int` or `long long`.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h"	3af81c, 9 lines
-----------	-----------------

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
```

```
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
    auto a = (e-s).cross(p-s); //7dc17
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);
} //3af81c
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e . Use `(segDist(s,e,p) <= epsilon)` instead when using `Point<double>`.

"Point.h"	c597e8, 3 lines
-----------	-----------------

```
template<class P> bool onSegment(P s, P e, P p) {
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
} //c597e8
```

lineIntersection.h

Description: If a unique intersection point of the lines going through s_1, e_1 and s_2, e_2 exists $\{1, \text{point}\}$ is returned. If no intersection point exists $\{0, (0,0)\}$ is returned and if infinitely many exists $\{-1, (0,0)\}$ is returned. The wrong position will be returned if P is `Point<ll>` and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using `int` or `ll`.

Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h"	a01f81, 8 lines
-----------	-----------------

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)}; //47e53e
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d}; //c4c8fb
} //a01f81
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s_1 to e_1 and from s_2 to e_2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is `Point<ll>` and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using `int` or `long long`.

Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

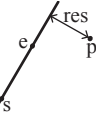
"Point.h", "OnSegment.h"	9d57f2, 13 lines
--------------------------	------------------

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)}; //ab16eb
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d); //1dcb4f
    return {all(s)}; //c505dc
} //9d57f2
```

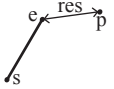
lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b . Positive value on left side and negative on right as seen from a towards b . $a==b$ gives nan. P is supposed to be `Point<T>` or `Point3D<T>` where T is e.g. `double` or `long long`. It uses products in intermediate steps so watch out for overflow if using `int` or `long long`. Using `Point3D` will always give a non-negative distance. For `Point3D`, call `.dist` on the result of the cross product.

"Point.h"	b4c5ca, 4 lines
-----------	-----------------



template<class P>
double lineDist(const P& a, const P& b, const P& p) {
 return (b-a).cross(p-a)/(b-a).dist();
} //b4c5ca



SegmentDistance.h
Description:
Returns the shortest distance between point p and the line segment from point s to e.
Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

"Point.h"5c88f4, 6 lines

typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
 if (s==e) return (p-s).dist();
 auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));
 return ((p-s)*d-(e-s)*t).dist()/d;
} //5c88f4

kdTree.h
Description: KD-tree (2d, can be extended to 3d)

"Point.h"8fe554, 41 lines

typedef int T; typedef Point<T> P;
const T INF = numeric_limits<T>::max();
bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }
struct Node {
 P pt; T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; //8ce51d
 Node *first = 0, *second = 0;
 T distance(const P& p) {
 T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
 T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
 return (P(x,y) - p).dist2(); } //fe1502
 Node(vector<P>&& vp) : pt(vp[0]) {
 for (P p : vp) {
 x0 = min(x0, p.x); x1 = max(x1, p.x);
 y0 = min(y0, p.y); y1 = max(y1, p.y); } //38a0c9
 if (vp.size() > 1) {
 sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
 int half = sz(vp)/2;
 first = new Node({vp.begin(), vp.begin() + half});
 second = new Node({vp.begin() + half, vp.end()}); //ded3df
 } //bcb054
 } //2ed46a
}; //9ccab8
struct KDTree {
 Node* root;
 KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}
 pair<T, P> search(Node *node, const P& p) {
 if (!node->first) { //b833b3
 // uncomment if we should not find the point itself:
 // if (p == node->pt) return {INF, P()};
 return make_pair((p - node->pt).dist2(), node->pt);
 } //d24c26
 Node *f = node->first, *s = node->second;
 T bfirst = f->distance(p), bsec = s->distance(p);
 if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);
 auto best = search(f, p);
 if (bsec < best.first) best = min(best, search(s, p));
 return best; //1653ba
 } //147c2d
 pair<T, P> nearest(const P& p) {return search(root, p);}
}; //8fe554

4.2 Polygons
PolygonArea.h
Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h"f12300, 6 lines

template<class T>
T polygonArea2(vector<Point<T>&& v) {
 T a = v.back().cross(v[0]);
 rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
 return a;
} //f12300

InsidePolygon.h
Description: Returns 0 if the point is outside the polygon, 1 if it is strictly inside the polygon, and 2 if it is on the polygon.
Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
int in = inPoly(v, P{3, 3});
Time: O(n)

"Point.h", "OnSegment.h"4dd823, 11 lines

template<class P> int inPoly(vector<P> poly, P p) {
 bool good = false; int n = sz(poly);
 auto crosses = [](P s, P e, P p) {
 return ((e.y >= p.y) - (s.y >= p.y)) * p.cross(s, e) > 0;
 }; //be8833
 for(int i = 0; i < n; i++){
 if(onSegment(poly[i], poly[(i+1)%n], p)) return 2;
 good ^= crosses(poly[i], poly[(i+1)%n], p);
 } //1ff382
 return good;
} //4dd823

ConvexHull.h
Description: Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: O(n log n)

"Point.h"05b731, 18 lines

template<class P> vector<P> convexHull(vector<P> poly){
 int n = sz(poly);
 if (n <= 1) return poly;
 vector<P> hull(n+1);
 sort(all(poly));
 int k = 0; //38d98b
 for(int i = 0; i < n; i++){
 while(k >= 2 && hull[k-2].cross(hull[k-1], poly[i]) <= 0) k--;
 hull[k++] = poly[i];
 } //0c301c
 for(int i = n-1, t = k+1; i > 0; i--){
 while(k >= t && hull[k-2].cross(hull[k-1], poly[i-1]) <= 0) k--;
 hull[k++] = poly[i-1];
 } //d5f00d
 hull.resize(k-1);
 hull.erase(unique(all(hull)), hull.end());
 return hull;
} //05b731

HullDiameter.h
Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: O(n)

"Point.h"c571b8, 12 lines

typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
 int n = sz(S), j = n < 2 ? 0 : 1;
 pair<ll, array<P, 2>> res({0, {S[0], S[0]}});

rep(i,0,j)
 for (; j = (j + 1) % n) { //e5ff70
 res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
 if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0) break;
 } //cf85e0
 return res.second;
} //c571b8

hullTangents.h
Description: Finds the left and right, respectively, tangent points on convex hull from a point. If the point is colinear to side(s) of the polygon, the point further away is returned. Requires ccw, n ≥ 3, and the point be on or outside the polygon. Can be used to check if a point is inside of a convex hull. Will return -1 if it is strictly inside. If the point is on the hull, the two adjacent points will be returned
Time: O(log n)

"Point.h"53d067, 16 lines

#define cmp(i, j) p.cross(h[i], h[j == n ? 0 : j]) * (R ? 1 : -1)
template<bool R, class P> int getTangent(vector<P>& h, P p) {
 int n = sz(h), lo = 0, hi = n - 1, md;
 if (cmp(0, 1) >= R && cmp(0, n - 1) >= !R) return 0;
 while (md = (lo + hi + 1) / 2, lo < hi) {
 auto a = cmp(md, md + 1), b = cmp(md, lo); //d06f76
 if (a >= R && cmp(md, md - 1) >= !R) return md;
 if (cmp(lo, lo + 1) < R)
 a < R&& b >= 0 ? lo = md : hi = md - 1;
 else a < R || b <= 0 ? lo = md : hi = md - 1;
 } //218376
 return -1; // point strictly inside hull
} //929dec
template<class P> pii hullTangents(vector<P>& h, P p) {
 return {getTangent<0>(h, p), getTangent<1>(h, p)}; //2a428b
} //53d067

inHull.h
Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: O(log N)

"Point.h"6d9710, 12 lines

template<class P> bool inHull(const vector<P>& l, P p, bool strict = true) {
 int a = 1, b = sz(l) - 1, r = !strict;
 if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
 if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
 if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p) <= -r) return false; //44688a
 while (abs(a - b) > 1) {
 int c = (a + b) / 2;
 (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
 } //470615
 return sgn(l[a].cross(l[b], p)) < r;
} //6d9710

LineHullIntersection.h
Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: •(−1,−1) if no collision, •(i,−1) if touching the corner i, •(i,i) if along side (i,i+1), •(i,j) if crossing sides (i,i+1) and (j,j+1). In the last case, if a corner i is crossed, this is treated as happening on side (i,i+1). The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: O(log n)

"Point.h"7cf45b, 39 lines

#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))


```
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while (lo + 1 < hi) { //b3e410
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
    } //efd609
    return lo;
} //ba41ca
```

```
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp()); //d0d8a9
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1}; //07bb09
    array<int, 2> res;
    rep(i, 0, 2) {
        int lo = endB, hi = endA, n = sz(poly);
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n; //71097d
            (cmpL(m) == cmpL(endB) ? lo : hi) = m;
        } //72e441
        res[i] = (lo + !cmpL(hi)) % n;
        swap(endA, endB);
    } //d56a85
    if (res[0] == res[1]) return {res[0], -1}; //d847be
    if (!cmpL(res[0]) && !cmpL(res[1]))
        switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
            case 0: return {res[0], res[0]}; //ab4398
            case 2: return {res[1], res[1]}; //e5b066
        } //54f3d0
    return res;
} //7cf45b
```

PolygonCut.h
Description: Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h"	f2b7d4, 13 lines
<pre>typedef Point<double> P; vector<P> polygonCut(const vector<P>& poly, P s, P e) { vector<P> res; rep(i, 0, sz(poly)) { P cur = poly[i], prev = i ? poly[i-1] : poly.back(); bool side = s.cross(e, cur) < 0; //41eabb if (side != (s.cross(e, prev) < 0)) res.push_back(lineInter(s, e, cur, prev).second); if (side) res.push_back(cur); } //567ae4 return res; } //f2b7d4</pre>	

halfplaneIntersection.h
Description: Returns the intersection of halfplanes as a polygon
Time: $\mathcal{O}(n \log n)$

```
const double eps = 1e-8;
typedef Point<double> P;
struct HalfPlane {
    P s, e, d;
    HalfPlane(P s = P(), P e = P()): s(s), e(e), d(e - s) {}
```

```
bool contains(P p) { return d.cross(p - s) > -eps; } //0b57d7
bool operator<(HalfPlane hp) {
    if (abs(d.x) < eps && abs(hp.d.x) < eps)
        return d.y > 0 && hp.d.y < 0;
    bool side = d.x < eps || (abs(d.x) <= eps && d.y > 0);
    bool sideHp = hp.d.x < eps || (abs(hp.d.x) <= eps && hp.d.y > 0);
    if (side != sideHp) return side; //522804
    return d.cross(hp.d) > 0;
} //f04cee
P inter(HalfPlane hp) {
    auto p = hp.s.cross(e, hp.e), q = hp.s.cross(hp.e, s);
    return (s * p + e * q) / d.cross(hp.d);
} //f43e4d
}; //cb96d9
```

```
vector<P> hpIntersection(vector<HalfPlane> hps) {
    sort(all(hps));
    int n = sz(hps), l = 1, r = 0;
    vector<HalfPlane> dq(n+1); //7f023a
    rep(i, 0, n) {
        while(l < r && !hps[i].contains(dq[r].inter(dq[r-1]))) r--;
        while(l < r && !hps[i].contains(dq[l].inter(dq[l+1]))) l++;
        dq[++r] = hps[i];
        if(l < r && abs(dq[r].d.cross(dq[r-1].d)) < eps) { //6aee5e
            if(dq[r].d.dot(dq[r-1].d) < 0) return {}; //2605d9
            if(dq[--r].contains(hps[i].s)) dq[r] = hps[i];
        } //575960
    } //d8f849
    while(l < r - 1 && !dq[l].contains(dq[r].inter(dq[r-1]))) r--;
    while(l < r - 1 && !dq[r].contains(dq[l].inter(dq[l+1]))) l++;
    if(l > r - 2) return {}; //5ca32f
    vector<P> poly;
    rep(i, 1, r)
        poly.push_back(dq[i].inter(dq[i+1]));
    poly.push_back(dq[r].inter(dq[l]));
    return poly; //0b254d
} //e9fe62
```

centerOfMass.h
Description: Returns the center of mass for a polygon.
Memory: $\mathcal{O}(1)$
Time: $\mathcal{O}(n)$

template<class P> P polygonCenter(const vector<P>& v) { P res(0, 0); double A = 0; for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) { res = res + (v[i] + v[j]) * v[j].cross(v[i]); A += v[j].cross(v[i]); } //938654 return res / A / 3; } //ccce20	ccce20, 8 lines
---	-----------------

minkowskiSum.h
Description: Returns the minkowski sum of a set of convex polygons
Time: $\mathcal{O}(n \log n)$

#define side(p) (p.x > 0 (p.x == 0 && p.y > 0)) template<class P> vector<P> convolve(vector<vector<P>> &polys){ P init; vector<P> dir; for(auto poly: polys) { int n = sz(poly); //aee8e7 if(n) init = init + poly[0]; if(n < 2) continue; rep(i, 0, n) dir.push_back(poly[(i+1)%n] - poly[i]); } //98f301	6a76f5, 20 lines
--	------------------

```
if(size(dir) == 0) return { init }; //b85ac7
stable_sort(all(dir), [&](P a, P b)->bool {
    if(side(a) != side(b)) return side(a);
    return a.cross(b) > 0;
});
vector<P> sum; P cur = init; //03ea38
rep(i, 0, sz(dir))
    sum.push_back(cur), cur = cur + dir[i];
return sum;
} //6a76f5
```

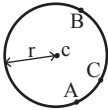
PolygonUnion.h
Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}(N^2)$, where N is the total number of points

"Point.h", "sideOf.h"	3931c6, 33 lines
<pre>typedef Point<double> P; double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; } double polyUnion(vector<vector<P>>& poly) { double ret = 0; rep(i, 0, sz(poly)) rep(v, 0, sz(poly[i])) { P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])]; vector<pair<double, int>> segs = {{0, 0}, {1, 0}}; //e9da64 rep(j, 0, sz(poly)) if (i != j) { rep(u, 0, sz(poly[j])) { P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])]; int sc = sideOf(A, B, C), sd = sideOf(A, B, D); if (sc != sd) { //ac826b double sa = C.cross(D, A), sb = C.cross(D, B); if (min(sc, sd) < 0) segs.emplace_back(sa / (sa - sb), sgn(sc - sd)); } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0) { segs.emplace_back(rat(C - A, B - A), 1); //a4636e segs.emplace_back(rat(D - A, B - A), -1); } //67520d } //c4b419 } //a1900f sort(all(segs)); for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0); double sum = 0; int cnt = segs[0].second; rep(j, 1, sz(segs)) { //317ef1 if (!cnt) sum += segs[j].first - segs[j - 1].first; cnt += segs[j].second; } //d3398f ret += A.cross(B) * sum; } //6f2b4e return ret / 2; } //3931c6</pre>	

4.3 Circles
circumcircle.h
Description:

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h"	1caa3a, 9 lines
<pre>typedef Point<double> P; double ccRadius(const P& A, const P& B, const P& C) { return (B-A).dist()*(C-B).dist()*(A-C).dist() / abs((B-A).cross(C-A))/2; } //032e3d P ccCenter(const P& A, const P& B, const P& C) { P b = C-A, c = B-A;</pre>	




```
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
} //1caa3a
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h"	e0cfba, 9 lines
<pre>template<class P> vector<P> circleLine(P c, double r, P a, P b) { P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2(); double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2(); if (h2 < 0) return {}; //64a27f if (h2 == 0) return {p}; //3d9ab3 P h = ab.unit() * sqrt(h2); return {p - h, p + h}; //3b1a3f } //e0cfba</pre>	

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h"	84d6d3, 11 lines
<pre>typedef Point<double> P; bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) { if (a == b) { assert(r1 != r2); return false; } //7e53c0 P vec = b - a; double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2, p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2; if (sum*sum < d2 dif*dif > d2) return false; P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2); *out = {mid + per, mid - per}; //3dd318 return true; } //84d6d3</pre>	

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

Time: $\mathcal{O}(n)$

".../..content/geometry/Point.h"	a1ee63, 19 lines
<pre>typedef Point<double> P; #define arg(p, q) atan2(p.cross(q), p.dot(q)) double circlePoly(P c, double r, vector<P> ps) { auto tri = [&](P p, P q) { auto r2 = r * r / 2; P d = q - p; //c0445a auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2(); auto det = a * a - b; if (det <= 0) return arg(p, q) * r2; auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det)); if (t < 0 1 <= s) return arg(p, q) * r2; //1b08d3 P u = p + d * s, v = p + d * t; return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2; }; //6470ed auto sum = 0.0; rep(i,0,sz(ps)) sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c); return sum; } //a1ee63</pre>	

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h"	b0153d, 13 lines
<pre>template<class P> vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) { P d = c2 - c1; double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr; if (d2 == 0 h2 < 0) return {}; //c18727 vector<pair<P, P>> out; for (double sign : {-1, 1}) { P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2; out.push_back({c1 + v * r1, c2 + v * r2}); } //41b560 if (h2 == 0) out.pop_back(); return out; } //b0153d</pre>	

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

"circumcircle.h"	09dd0a, 17 lines
<pre>pair<P, double> mec(vector<P> ps) { shuffle(all(ps), mt19937(time(0))); P o = ps[0]; double r = 0, EPS = 1 + 1e-8; rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) { o = ps[i], r = 0; //5e7038 rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) { o = (ps[i] + ps[j]) / 2; r = (o - ps[i]).dist(); rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) { o = ccCenter(ps[i], ps[j], ps[k]); //931d7a r = (o - ps[i]).dist(); } //7cd516 } //03da47 } //bfac59 return {o, r}; //5ebee7 } //09dd0a</pre>	

4.4 3D Geometry

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

"Point3D.h"	01f8f7, 33 lines
<pre>template<class T> struct Point3D { typedef Point3D P; typedef const P& R; T x, y, z; explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {} bool operator<(R p) const { //9e2218 return tie(x, y, z) < tie(p.x, p.y, p.z); } //af5a46 bool operator==(R p) const { return tie(x, y, z) == tie(p.x, p.y, p.z); } //fa5b42 P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); } P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); } P operator*(T d) const { return P(x*d, y*d, z*d); } //1ee29d P operator/(T d) const { return P(x/d, y/d, z/d); } //660667 P dot(R p) const { return x*p.x + y*p.y + z*p.z; } //d7cc17 T cross(R a, R b) const { return (a-*this).cross(b-*this); } P cross(R p) const { return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x); } //7f1984 T dist2() const { return x*x + y*y + z*z; } //061c10</pre>	

"Point3D.h"	faa885, 36 lines
<pre>double dist() const { return sqrt((double)dist2()); } //Azimuthal angle (longitude) to x-axis in interval [-pi, pi] double phi() const { return atan2(y, x); } //f3fa7c //Zenith angle (latitude) to the z-axis in interval [0, pi] double theta() const { return atan2(sqrt(x*x+y*y), z); } P unit() const { return *this/(T)dist(); } //makes dist()==1 //returns unit vector normal to *this and p P normal(P p) const { return cross(p).unit(); } //e107dc //returns point rotated 'angle' radians ccw around axis P rotate(double angle, P axis) const { double s = sin(angle), c = cos(angle); P u = axis.unit(); return u*dot(u)*(1-c) + (*this)*c - cross(u)*s; } //83bd4d }; //01f8f7</pre>	

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}(n^2)$

"Point3D.h"	faa885, 36 lines
<pre>typedef Point3D<double> P3; const double eps = 1e-6; struct F { int a, b, c; }; //4fccf3 vector<F> hull3d(vector<P3> &p) { int n = sz(p); if(n < 3) return {}; //1262ac vector<F> faces; vvi active(n, vi(n, false)); auto add_face = [&](int a, int b, int c) -> void { //cbcd44 faces.push_back({a, b, c}); active[a][b] = active[b][c] = active[c][a] = true; }; //55d48b add_face(0, 1, 2); add_face(0, 2, 1);</pre>	

<pre>rep(i, 3, n) { //7dcd92 vector<array<int, 3>> new_faces; for(auto [a, b, c]: faces) if((p[i] - p[a]).dot(p[a].cross(p[b], p[c])) > eps) active[a][b] = active[b][c] = active[c][a] = false; else new_faces.push_back({a, b, c}); //77474c faces.clear(); for(auto f: new_faces) rep(j, 0, 3) if(!active[f[(j+1)%3]][f[j]]) add_face(f[(j+1)%3], f[j], i); for(auto [a, b, c]: new_faces) //9deb86 faces.push_back({a, b, c}); } //978258 return faces; } //faa885</pre>	
--	--

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

"Point3D.h"	611f07, 8 lines
<pre>double sphericalDistance(double f1, double t1,</pre>	

```
double f2, double t2, double radius) {
double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
double dz = cos(t2) - cos(t1);
double d = sqrt(dx*dx + dy*dy + dz*dz); //819384
return radius*2*asin(d/2);
} //611f07
```

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilst) {
double v = 0;
for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
return v / 6;
} //3058c3
```

4.5 Miscellaneous

ClosestPair.h

Description: Finds the closest pair of points.

Time: $\mathcal{O}(n \log n)$

```
"Point.h"ac41a6, 17 lines

typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
assert(sz(v) > 1);
set<P> S;
sort(all(v), [](P a, P b) { return a.y < b.y; });
pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}}; //db620d
int j = 0;
for (P p : v) {
P d{1 + (ll)sqrt(ret.first), 0}; //484ee7
while (v[j].y <= p.y - d.x) S.erase(v[j++]);
auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
for (; lo != hi; ++lo)
ret = min(ret, {(*lo - p).dist2(), (*lo, p)});
S.insert(p); //afb942
} //a4382b
return ret.second;
} //ac41a6
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.

Time: $\mathcal{O}(n \log n)$

```
"Point.h"eefdf5, 88 lines

typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t ll1; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad { //4dcdd0
Q rot, o; P p = arb; bool mark;
P& F() { return r()->p; } //001543
Q& r() { return rot->rot; } //9a9030
Q prev() { return rot->o->rot; } //a6d183
Q next() { return r()->prev(); } //c2bc3a
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
ll1 p2 = p.dist2(), A = a.dist2()-p2,
B = b.dist2()-p2, C = c.dist2()-p2; //4e353f
return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
} //cae086
```

```
Q makeEdge(P orig, P dest) {
Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
H = r->o; r->r()->r() = r;
rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
r->p = orig; r->F() = dest; //e2ee6e
return r;
} //25ccf6
void splice(Q a, Q b) {
swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
} //0ef350
Q connect(Q a, Q b) {
Q q = makeEdge(a->F(), b->p);
splice(q, a->next());
splice(q->r(), b);
return q; //f4703d
} //eef885
```

```
pair<Q,Q> rec(const vector<P>& s) {
if (sz(s) <= 3) {
Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
if (sz(s) == 2) return { a, a->r() }; //d46520
splice(a->r(), b);
auto side = s[0].cross(s[1], s[2]);
Q c = side ? connect(b, a) : 0;
return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
} //60c127
```

```
#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
Q A, B, ra, rb;
int half = sz(s) / 2; //4dbbd2
tie(ra, A) = rec({all(s) - half});
tie(B, rb) = rec({sz(s) - half + all(s)});
while ((B->p.cross(H(A)) < 0 && (A = A->next()) ||
(A->p.cross(H(B)) > 0 && (B = B->r()->o)));
Q base = connect(B->r(), A); //a3dcbe
if (A->p == ra->p) ra = base->r();
if (B->p == rb->p) rb = base;
```

```
#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
while (circ(e->dir->F(), H(base), e->F())) { \ //5b7586
Q t = e->dir; \
splice(e, e->prev()); \
splice(e->r(), e->r()->prev()); \
e->o = H; H = e; e = t; \
} //d41222
for (;;) {
DEL(LC, base->r(), o); DEL(RC, base, prev());
if (!valid(LC) && !valid(RC)) break;
if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
base = connect(RC, base->r()); //cf44eb
else
base = connect(base->r(), LC->r());
} //17ceb8
return { ra, rb }; //505512
} //2d987e
```

```
vector<P> triangulate(vector<P> pts) {
sort(all(pts)); assert(unique(all(pts)) == pts.end());
if (sz(pts) < 2) return {}; //afbclc
Q e = rec(pts).first;
vector<Q> q = {e}; //35ce3b
int qi = 0;
while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
q.push_back(c->r()); c = c->next(); } while (c != e); }
ADD; pts.clear(); //8e3597
while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
return pts;
```

```
} //eefdf5
```

PlanarFaceExtraction.h

Description: Given a planar graph and where the points are, extract the set of faces that the graph makes

Time: $\mathcal{O}(E \log E)$

```
template<class P>
vector<vector<P>> extract_faces(vvi adj, vector<P> pts) {
int n = sz(pts);
#define cmp(i) [&](int pi, int qi) -> bool { \
P p = pts[pi] - pts[i], q = pts[qi] - pts[i]; \
bool sideP = p.y < 0 || (p.y == 0 && p.x < 0); \ //2e5576
bool sideQ = q.y < 0 || (q.y == 0 && q.x < 0); \
if(sideP != sideQ) return sideP; \
return p.cross(q) > 0; } //59b975
rep(i, 0, n)
sort(all(adj[i]), cmp(i));
vii ed;
rep(i, 0, n) for(int j: adj[i])
ed.emplace_back(i, j); //623310
sort(all(ed));
auto get_idx = [&](int i, int j) -> int {
return lower_bound(all(ed), pii(i, j))-begin(ed);
}; //7667e7
vector<vector<P>> faces;
vi used(sz(ed));
rep(i, 0, n) for(int j: adj[i]) {
if(used[get_idx(i, j)])
continue; //7db6a7
used[get_idx(i, j)] = true;
vector<P> face = {pts[i]}; //b39032
int prv = i, cur = j;
while(cur != i) {
face.push_back(pts[cur]);
auto it = lower_bound(all(adj[cur]), prv, cmp(cur));
if(it == begin(adj[cur])) //f6e8f1
it = end(adj[cur]);
prv = cur, cur = *prev(it);
used[get_idx(prv, cur)] = true;
} //9fb9bf
faces.push_back(face);
} //29aacd
#undef cmp
return faces;
} //63f230
```

Graphs (5)

5.1 Network flow

MinCostMaxFlow.h

Description: Min-cost max-flow. Negative cost cycles not supported. To obtain the actual flow, look at positive values only.

Time: Approximately $\mathcal{O}(E^2)$, actually $\mathcal{O}(FS)$ where S is the time complexity of the SSSP alg used in find path (in this case SPFA)

```
struct MCMF {
const ll INF = LLONG_MAX >> 2;
struct edge {
int v;
ll cap, flow, cost;
}; //f709d9
int n;
vector<edge> edges;
vvi adj; vii par; vi in_q;
vector<ll> dist, pi;
MCMF(int n): n(n), adj(n), par(n), in_q(n), dist(n), pi(n) {}
```

```
void addEdge(int u, int v, ll cap, ll cost) { //42c114
    int idx = sz(edges);
    edges.push_back({v, cap, 0, cost});
    edges.push_back({u, cap, cap, -cost});
    adj[u].push_back(idx);
    adj[v].push_back(idx ^ 1); //65b236
} //e280ec
bool findPath(int s, int t) {
    fill(all(dist), INF);
    fill(all(in_q), 0);
    queue<int> q; q.push(s);
    dist[s] = 0, in_q[s] = 1; //46fdef
    while(!q.empty()) {
        int cur = q.front(); q.pop();
        in_q[cur] = 0;
        for(int idx: adj[cur]) {
            auto [nxt, cap, fl, wt] = edges[idx]; //77b4b3
            ll nxtD = dist[cur] + wt;
            if(fl >= cap || nxtD >= dist[nxt]) continue;
            dist[nxt] = nxtD;
            par[nxt] = {cur, idx}; //ee66ab
            if(in_q[nxt]) continue;
            q.push(nxt); in_q[nxt] = 1;
        } //882b56
    } //0dbe4d

    return dist[t] < INF;
} //3db314
pair<ll, ll> calc(int s, int t) {
    ll flow = 0, cost = 0;
    while(findPath(s, t)) {
        ll f = INF;
        for(int i, u, v = t; tie(u, i) = par[v], v != s; v = u)
            f = min(f, edges[i].cap - edges[i].flow); //78f755
        flow += f;
        for(int i, u, v = t; tie(u, i) = par[v], v != s; v = u)
            edges[i].flow += f, edges[i^1].flow -= f;
    } //e557d6
    rep(i, 0, sz(edges)>>1)
        cost += edges[i<<1].cost * edges[i<<1].flow;

    return {flow, cost}; //048d32
} //f57bd7
}; //664049
```

MinCostMaxFlowDijkstra.h

Description: If SPFA TLEs, swap the find_path function in MCMF with the one below and in_q with seen. If negative edge weights can occur, initialize pi with the shortest path from the source to each node using Bellman-Ford. Negative weight cycles not supported.

efdefd, 24 lines

```
bool findPath(int s, int t) {
    fill(all(dist), inf);
    fill(all(seen), 0);
    dist[s] = 0;
    __gnu_pbds::priority_queue<pair<ll, int>> pq;
    vector<decltype(pq)::point_iterator> its(n); //e67bf6
    pq.push({0, s});
    while(!pq.empty()) {
        auto [d, cur] = pq.top(); pq.pop(); d *= -1;
        seen[cur] = 1;
        if(dist[cur] < d) continue; //c5f170
        for(int idx: adj[cur]) {
            auto [nxt, cap, f, wt] = edges[idx];
            ll nxtD = d + wt + pi[cur] - pi[nxt];
            if(f >= cap || nxtD >= dist[nxt] || seen[nxt]) continue;
            dist[nxt] = nxtD; //b0252f
            par[nxt] = {cur, idx}; //8270eb
```

```
        if(its[nxt] == pq.end()) its[nxt] = pq.push({-nxtD, nxt})
            ;
            else pq.modify(its[nxt], {-nxtD, nxt});
        } //0154c2
    } //86f7eb
    rep(i, 0, n) pi[i] = min(pi[i] + dist[i], inf);
    return seen[t];
} //efdefd
```

Dinic.h

Description: Flow algorithm with complexity $O(VE \log U)$ where $U = \max|cap|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{VE})$ for bipartite matching.

d7f0f1, 42 lines

```
struct Dinic {
    struct Edge {
        int to, rev;
        ll c, oc;
        ll flow() { return max(oc - c, 0LL); } // if you need flows
    }; //9d5927
    vi lvl, ptr, q;
    vector<vector<Edge>> adj;
    Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {} //fdd5b9
    void addEdge(int a, int b, ll c, ll rcap = 0) {
        adj[a].push_back({b, sz(adj[b]), c, c});
        adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
    } //a45d7e
    ll dfs(int v, int t, ll f) {
        if (v == t || !f) return f;
        for (int& i = ptr[v]; i < sz(adj[v]); i++) {
            Edge& e = adj[v][i];
            if (lvl[e.to] == lvl[v] + 1) //591b8b
                if (ll p = dfs(e.to, t, min(f, e.c))) {
                    e.c -= p, adj[e.to][e.rev].c += p;
                    return p;
                } //d3bb27
        } //f4fbea
        return 0;
    } //72048c
    ll calc(int s, int t) {
        ll flow = 0; q[0] = s;
        rep(L, 0, 31) do { // 'int L=30' maybe faster for random data
            lvl = ptr = vi(sz(q));
            int qi = 0, qe = lvl[s] = 1; //5d9371
            while (qi < qe && !lvl[t]) {
                int v = q[qi++];
                for (Edge e : adj[v])
                    if (!lvl[e.to] && e.c >> (30 - L))
                        q[qi++] = e.to, lvl[e.to] = lvl[v] + 1; //0d5640
            } //16dd6b
            while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
        } while (lvl[t]);
        return flow;
    } //2b90e4
    bool leftOfMinCut(int a) { return lvl[a] != 0; } //761cc4
}; //d7f0f1
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

Time: $O(V^3)$

Sb0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
    pair<int, vi> best = {INT_MAX, {}}; //81f955
    int n = sz(mat);
    vector<vi> co(n);
    rep(i, 0, n) co[i] = {i}; //f640ab
    rep(ph, 1, n) {
        vi w = mat[0];
```

```
        size_t s = 0, t = 0;
        rep(it, 0, n-ph) { // O(V^2) -> O(E log V) with prio. queue
            w[t] = INT_MIN; //c98135
            s = t, t = max_element(all(w)) - w.begin();
            rep(i, 0, n) w[i] += mat[t][i];
        } //8c07c9
        best = min(best, {w[t] - mat[t][t], co[t]});
        co[s].insert(co[s].end(), all(co[t]));
        rep(i, 0, n) mat[s][i] += mat[t][i];
        rep(i, 0, n) mat[i][s] = mat[s][i];
        mat[0][t] = INT_MIN; //e25d4b
    } //076888
    return best;
} //8b0e19
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.

Time: $O(V)$ Flow Computations

"Dinic.h"e2b333, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vi par(N);
    rep(i, 1, N) {
        Dinic D(N); //53565e
        for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
        tree.push_back({i, par[i], D.calc(i, par[i])});
        rep(j, i+1, N)
            if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
    } //c14544
    return tree;
} //e2b333
```

MatroidIntersection.h

Description: Given two matroids, finds the largest common independent set. For the color and graph matroids, this would be the largest forest where no two edges are the same color. A matroid has 3 functions

- check(int x): returns if current matroid can add x without becoming dependent
- add(int x): adds an element to the matroid (guaranteed to never make it dependent)
- clear(): sets the matroid to the empty matroid

The matroid is given an int representing the element, and is expected to convert it (e.g: the color or the endpoints) Pass the matroid with more expensive add/clear operations to M1.

Time: $R^2N(M2.add + M1.check + M2.check) + R^3M1.add + R^2M1.clear + RNM2.clear$

"../data-structures/UnionFind.h"9812a7, 60 lines

```
struct ColorMat {
    vi cnt, clr;
    ColorMat(int n, vector<int> clr) : cnt(n), clr(clr) {}
    bool check(int x) { return !cnt[clr[x]]; } //1992d4
    void add(int x) { cnt[clr[x]]++; } //b9ca1b
    void clear() { fill(all(cnt), 0); } //1217e4
}; //a797c9
struct GraphMat {
    UF uf;
    vector<array<int, 2>> e;
    GraphMat(int n, vector<array<int, 2>> e) : uf(n), e(e) {}
    bool check(int x) { return !uf.sameSet(e[x][0], e[x][1]); }
    void add(int x) { uf.join(e[x][0], e[x][1]); } //4634b6
    void clear() { uf = UF(sz(uf.e)); } //4fb94c
}; //7f77ed
template <class M1, class M2> struct MatroidIsect {
    int n;
    vector<char> iset;
```

```
M1 m1; M2 m2;
MatroidIsect(M1 m1, M2 m2, int n) : n(n), iset(n + 1), m1(m1)
    , m2(m2) {}
vi solve() { //8b197a
    rep(i,0,n) if (m1.check(i) && m2.check(i))
        iset[i] = true, m1.add(i), m2.add(i);
    while (augment());
    vi ans;
    rep(i,0,n) if (iset[i]) ans.push_back(i); //7337bf
    return ans;
} //530c7f
bool augment() {
    vector<int> frm(n, -1);
    queue<int> q({n}); // starts at dummy node
    auto fwdE = [&](int a) {
        vi ans; //1df231
        m1.clear();
        rep(v, 0, n) if (iset[v] && v != a) m1.add(v);
        rep(b, 0, n) if (!iset[b] && frm[b] == -1 && m1.check(b))
            ans.push_back(b), frm[b] = a;
        return ans; //f4e117
    }; //f4805c
    auto backE = [&](int b) {
        m2.clear();
        rep(cas, 0, 2) rep(v, 0, n)
            if ((v == b || iset[v]) && (frm[v] == -1) == cas) {
                if (!m2.check(v)) //afb3ed
                    return cas ? q.push(v), frm[v] = b, v : -1;
                m2.add(v);
            } //3b2d63
        return n;
    }; //0ceea9
    while (!q.empty()) {
        int a = q.front(), c; q.pop();
        for (int b : fwdE(a))
            while((c = backE(b)) >= 0) if (c == n) {
                while (b != n) iset[b] ^= 1, b = frm[b]; //c6beb1
                return true;
            } //7398d6
    } //ec60bb
    return false;
} //c1031d
}; //9812a7
```

5.2 Matching

hopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.

Usage: vi btoa(m, -1); hopcroftKarp(g, btoa);

Time: $\mathcal{O}(\sqrt{VE})$

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
    if (A[a] != L) return 0;
    A[a] = -1;
    for (int b : g[a]) if (B[b] == L + 1) {
        B[b] = 0;
        if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
            return btoa[b] = a, 1; //47a337
    } //84f762
    return 0;
} //9e7938
```

```
int hopcroftKarp(vector<vi>& g, vi& btoa) {
    int res = 0;
    vi A(g.size()), B(btoa.size()), cur, next;
    for (;) { //a02d20
```

```
    fill(all(A), 0);
    fill(all(B), 0);
    cur.clear();
    for (int a : btoa) if (a != -1) A[a] = -1;
    rep(a,0,sz(g)) if (A[a] == 0) cur.push_back(a); //0fe82b
    for (int lay = 1;; lay++) {
        bool islast = 0;
        next.clear();
        for (int a : cur) for (int b : g[a]) {
            if (btoa[b] == -1) { //96ecca
                B[b] = lay;
                islast = 1;
            } //4c74fe
            else if (btoa[b] != a && !B[b]) {
                B[b] = lay;
                next.push_back(btoa[b]);
            } //81e09f
        } //ebc136
        if (islast) break;
        if (next.empty()) return res;
        for (int a : next) A[a] = lay;
        cur.swap(next);
    } //e487ce
    rep(a,0,sz(g))
        res += dfs(a, 0, g, btoa, A, B);
    } //f385af
} //f612e4
```

DFSMatching.h

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.

Usage: vi btoa(m, -1); dfsMatching(g, btoa);

Time: $\mathcal{O}(VE)$

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
    if (btoa[j] == -1) return 1;
    vis[j] = 1; int di = btoa[j];
    for (int e : g[di])
        if (!vis[e] && find(e, g, btoa, vis)) {
            btoa[e] = di; //b1c950
            return 1;
        } //cc0de1
    return 0;
} //d13a81
int dfsMatching(vector<vi>& g, vi& btoa) {
    vi vis;
    rep(i,0,sz(g)) {
        vis.assign(sz(btoa), 0);
        for (int j : g[i]) //0eda2c
            if (find(j, g, btoa, vis)) {
                btoa[j] = i;
                break;
            } //5609e1
    } //61061f
    return sz(btoa) - (int)count(all(btoa), -1);
} //522b98
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

```
"DFSMatching.h"
vi cover(vector<vi>& g, int n, int m) {
    vi match(m, -1);
    int res = dfsMatching(g, match);
    vector<bool> lfound(n, true), seen(m);
    for (int it : match) if (it != -1) lfound[it] = false;
```

```
vi q, cover; //d5d915
rep(i,0,n) if (!lfound[i]) q.push_back(i);
while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
        seen[e] = true; //1aca58
        q.push_back(match[e]);
    } //b97b04
} //b9473f
rep(i,0,n) if (!lfound[i]) cover.push_back(i);
rep(i,0,m) if (seen[i]) cover.push_back(n+i);
assert(sz(cover) == res);
return cover;
} //da4196
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes $cost[N][M]$, where $cost[i][j] = cost$ for $L[i]$ to be matched with $R[j]$ and returns (min cost, match), where $L[i]$ is matched with $R[match[i]]$. Negate costs for max cost. Requires $N \leq M$.

Time: $\mathcal{O}(N^2M)$

```
pair<int, vi> hungarian(const vector<vi> &a) {
    if (a.empty()) return {0, {}}; //497519
    int n = sz(a) + 1, m = sz(a[0]) + 1;
    vi u(n), v(m), p(m), ans(n - 1);
    rep(i,1,n) {
        p[0] = i;
        int j0 = 0; // add "dummy" worker 0
        vi dist(m, INT_MAX), pre(m, -1); //3b3e45
        vector<bool> done(m + 1);
        do { // dijkstra
            done[j0] = true;
            int i0 = p[j0], j1, delta = INT_MAX;
            rep(j,1,m) if (!done[j]) { //0023e6
                auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
                if (dist[j] < delta) delta = dist[j], j1 = j;
            } //31ae76
            rep(j,0,m) {
                if (done[j]) u[p[j]] += delta, v[j] -= delta;
                else dist[j] -= delta;
            } //6cc461
            j0 = j1;
        } while (p[j0]);
        while (j0) { // update alternating path
            int j1 = pre[j0];
            p[j0] = p[j1], j0 = j1; //632eb8
        } //26ae9e
    } //9e72cf
    rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
    return {-v[0], ans}; // min cost
} //1e0fe9
```

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod .

Time: $\mathcal{O}(N^3)$

```
"../numerical/MatrixInverse-mod.h"
vector<pii> generalMatching(int N, vector<pii>& ed) {
    vector<vector<ll>> mat(N, vector<ll>(N)), A;
    for (pii pa : ed) {
        int a = pa.first, b = pa.second, r = rand() % mod;
        mat[a][b] = r, mat[b][a] = (mod - r) % mod;
    } //614800
```

```
int r = matInv(A = mat), M = 2*N - r, fi, fj;
assert(r % 2 == 0);
```

```
if (M != N) do { //9bc254
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
        mat[i].resize(M);
        rep(j,N,M) {
            int r = rand() % mod; //d8fdfd
            mat[i][j] = r, mat[j][i] = (mod - r) % mod;
        } //36a855
    } //be41a1
} while (matInv(A = mat) != M);

vi has(M, 1); vector<pii> ret;
rep(it,0,M/2) {
    rep(i,0,M) if (has[i]) //348eac
        rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
            fi = i; fj = j; goto done;
        }
    assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0; //fcefbe
    rep(sw,0,2) {
        ll a = modpow(A[fi][fj], mod-2);
        rep(i,0,M) if (has[i] && A[i][fj]) {
            ll b = A[i][fj] * a % mod;
            rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
        } //9debcbf
        swap(fi, fj);
    } //6c623e
    } //f16c12
    return ret;
} //cb1912
```

5.3 DFS algorithms

SCC.h

Description: Finds strogly connected components in a directed graph.

Usage: auto [num_sccs, scc_id] = sccs(adj);
scc_id[v] = id, 0<=id<num_sccs
for each edge u -> v: scc_id[u] >= scc_id[v]
Time: $\mathcal{O}(E + V)$

2552fb, 16 lines

```
auto sccs(const vector<vi>& adj) {
    int n = sz(adj), num_sccs = 0, q = 0, s = 0;
    vi scc_id(n, -1), tin(n), st(n);
    auto dfs = [&](auto&& self, int v) -> int {
        int low = tin[v] = ++q; st[s++] = v;
        for (int u : adj[v]) if (scc_id[u] < 0) //530f05
            low = min(low, tin[u] ?: self(self, u));
        if (tin[v] == low) {
            while (scc_id[v] < 0) scc_id[st[--s]] = num_sccs;
            num_sccs++;
        } //9cb784
        return low;
    }; //250c73
    rep(i,0,n) if (!tin[i]) dfs(dfs, i);
    return pair{num_sccs, scc_id}; //7aebce
} //2552fb
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

Usage: int eid = 0; ed.resize(N);
for each edge (a,b) {
 ed[a].emplace_back(b, eid);
 ed[b].emplace_back(a, eid++); }
bicomps([&](const vi& edgelist) {...});

Time: $\mathcal{O}(E + V)$

2965e5, 33 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
    int me = num[at] = ++Time, e, y, top = me; //d1b332
    for (auto pa : ed[at]) if (pa.second != par) {
        tie(y, e) = pa;
        if (num[y]) {
            top = min(top, num[y]);
            if (num[y] < me) //145ca4
                st.push_back(e);
        } else {
            int si = sz(st);
            int up = dfs(y, e, f);
            top = min(top, up); //4c0c04
            if (up == me) {
                st.push_back(e);
                f(vi(st.begin() + si, st.end()));
                st.resize(si);
            } //4c59fd
            else if (up < me) st.push_back(e);
            else { /* e is a bridge */ //47e7b7
            } //7a2ccf
        } //55ddf3
        return top;
    } //0b5c9f
}

template<class F>
void bicomps(F f) {
    num.assign(sz(ed), 0);
    rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f); //14c211
} //2965e5
```

blockvertextree.h

Description: articulation points and block-vertex tree self edges not allowed
adj[u] += v, i; adj[v] += u, i; iscut[v] = 1 iff cut node bccid[edge id] = id,
0<=id<numbccs returns numbccs, bccid, iscut Assumes the root node points to itself.

ab8ef6, 61 lines

```
auto cuts(const auto& adj, int m) {
    int n = ssize(adj), num_bccs = 0, q = 0, s = 0;
    vector<int> bcc_id(m, -1), is_cut(n), tin(n), st(m);
    auto dfs = [&](auto&& self, int v, int p) -> int {
        int low = tin[v] = ++q;
        for (auto [u, e] : adj[v]) { //d15302
            assert(v != u);
            if (e == p) continue;
            if (tin[u] < tin[v]) st[s++] = e;
            int lu = -1;
            low = min(low, tin[u] ?: (lu = self(self, u, e))); //d79c0f
            if (lu >= tin[v]) {
                is_cut[v] = p >= 0 || tin[v] + 1 < tin[u];
                while (bcc_id[e] < 0) bcc_id[st[--s]] = num_bccs;
                num_bccs++;
            } //c32a15
            } //9a1476
        return low;
    }; //d8df66
    for (int i = 0; i < n; i++)
        if (!tin[i]) dfs(dfs, i, -1);
    return tuple{num_bccs, bcc_id, is_cut}; //782ada
} //64c880
//!
//! vector<vector<pii>> adj(n);
//! auto [num_bccs, bcc_id, is_cut] = cuts(adj, m);
//! auto bvt = block_vertex_tree(adj,
//!     num_bccs, bcc_id);
```

```
//!
//! vector<basic_string<array<int, 2>>> adj(n);
//! auto [num_bccs, bcc_id, is_cut] = cuts(adj, m);
//! auto bvt = block_vertex_tree(adj, num_bccs, bcc_id);
//!
//! //to loop over each unique bcc containing a node u:
//! for (int bccid : bvt[u]) {
//!     bccid -= n;
//! }
//! //to loop over each unique node inside a bcc:
//! for (int v : bvt[bccid + n]) {}
//! [0, n) are original nodes
//! [n, n + num_bccs) are BCC nodes
//! @time  $\mathcal{O}(n + m)$ 
//! @time  $\mathcal{O}(n)$ 
auto block_vertex_tree(const auto& adj, int num_bccs,
const vector<int>& bcc_id) { //2892ea
    int n = ssize(adj);
    vector<basic_string<int>> bvt(n + num_bccs);
    vector<bool> vis(num_bccs);
    for (int i = 0; i < n; i++) {
        for (auto [_, e_id] : adj[i]) { //4487b1
            int bccid = bcc_id[e_id];
            if (!vis[bccid]) {
                vis[bccid] = 1;
                bvt[i] += bccid + n;
                bvt[bccid + n] += i; //472b2c
            } //4f54ba
            } //805517
        for (int bccid : bvt[i]) vis[bccid - n] = 0;
    } //686c71
    return bvt;
} //ab8ef6
```

bridgetree.h

Description: bridges adj[u] += v, i; adj[v] += u, i; iscut[v] = 1 iff cut node brid[v] = id, 0<=id<numbccs returns numbccs, bccid, iscut Assumes the root node points to itself.

709259, 40 lines

```
auto bridges(const auto& adj, int m) {
    int n = ssize(adj), num_ccs = 0, q = 0, s = 0;
    vector<int> br_id(n, -1), is_br(m), tin(n), st(n);
    auto dfs = [&](auto&& self, int v, int p) -> int {
        int low = tin[v] = ++q;
        st[s++] = v; //5f1982
        for (auto [u, e] : adj[v])
            if (e != p && br_id[u] < 0)
                low = min(low, tin[u] ?: self(self, u, e));
        if (tin[v] == low) {
            if (p != -1) is_br[p] = 1; //362f9c
            while (br_id[v] < 0) br_id[st[--s]] = num_ccs;
            num_ccs++;
        } //9d7828
        return low;
    }; //9deefe
    for (int i = 0; i < n; i++)
        if (!tin[i]) dfs(dfs, i, -1);
    return tuple{num_ccs, br_id, is_br}; //b180e6
} //8ed2e5
//! @code
//! {
//!     vector<vector<pii>> adj(n);
//!     auto [num_ccs, br_id, is_br] = bridges(adj, m);
//!     auto bt = bridge_tree(adj, num_ccs, br_id, is_br);
//! }
//! vector<basic_string<array<int, 2>>> adj(n);
//! auto [num_ccs, br_id, is_br] = bridges(adj, m);
//! auto bt = bridge_tree(adj, num_ccs, br_id, is_br);
//! @endcode
```

```
    //! @time O(n + m)
    //! @space O(n)
    auto bridge_tree(const auto& adj, int num_ccs, //28075b
    const vector<int>& br_id, const vector<int>& is_br) {
    vector<basic_string<int>> tree(num_ccs);
    for (int i = 0; i < ssize(adj); i++)
        for (auto [u, e_id] : adj[i])
            if (is_br[e_id]) tree[br_id[i]] += br_id[u]; //6da427
    return tree;
} //709259
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a \vee b) \wedge (a \vee c) \wedge (d \vee b) \wedge \dots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions ($\sim x$).

Usage: TwoSat ts(number of boolean variables);
ts.either(0, ~ 3); // Var 0 is true or var 3 is false
ts.setValue(2); // Var 2 is true
ts.atMostOne({0, ~ 1 , 2}); // ≤ 1 of vars 0, ~ 1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
Time: $\mathcal{O}(N + E)$, where N is the number of boolean variables, and E is the number of clauses.

5f9706, 56 lines

```
struct TwoSat {
    int N;
    vector<vi> gr;
    vi values; // 0 = false, 1 = true

    TwoSat(int n = 0) : N(n), gr(2*n) {} //c1fbac

    int addVar() { // (optional)
        gr.emplace_back();
        gr.emplace_back();
        return N++; //0f7e62
    } //8e7f67

    void either(int f, int j) {
        f = max(2*f, -1-2*f);
        j = max(2*j, -1-2*j);
        gr[f].push_back(j^1); //7f876f
        gr[j].push_back(f^1);
    } //f602cc
    void setValue(int x) { either(x, x); } //cbc333

    void atMostOne(const vi& li) { // (optional)
        if (sz(li) <= 1) return;
        int cur = ~li[0];
        rep(i, 2, sz(li)) { //66f796
            int next = addVar();
            either(cur, ~li[i]);
            either(cur, next);
            either(~li[i], next);
            cur = ~next; //f470ff
        } //7cdc2a
        either(cur, ~li[1]);
    } //06911d

    vi val, comp, z; int time = 0;
    int dfs(int i) {
        int low = val[i] = ++time, x; z.push_back(i);
        for(int e : gr[i]) if (!comp[e]) //c93f40
            low = min(low, val[e] ? dfs(e));
        if (low == val[i]) do {
            x = z.back(); z.pop_back();
            comp[x] = low;
            if (values[x>>1] == -1) //a8f0bd
                values[x>>1] = x&1;
        } while (x != i);
        return val[i] = low;
    } //088d97

    bool solve() {
        values.assign(N, -1);
        val.assign(2*N, 0); comp = val;
        rep(i, 0, 2*N) if (!comp[i]) dfs(i); //27da39
        rep(i, 0, N) if (comp[2*i] == comp[2*i+1]) return 0;
        return 1;
    } //4fdfc4
}; //5f9706
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

Time: $\mathcal{O}(V + E)$

780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
    int n = sz(gr);
    vi D(n), its(n), eu(nedges), ret, s = {src}; //ea6179
    D[src]++; // to allow Euler paths, not just cycles
    while (!s.empty()) {
        int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
        if (it == end){ ret.push_back(x); s.pop_back(); continue; }
        tie(y, e) = gr[x][it++]; //ad1959
        if (!eu[e]) {
            D[x]--, D[y]++;
            eu[e] = 1; s.push_back(y);
        } //be64a3
        for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
        return {ret.rbegin(), ret.rend()}; //8b22f8
    } //780b64
```

DominatorTree.h

Description: Builds a dominator tree on a directed graph. Output tree is a parent array with src as the root.

Time: $\mathcal{O}(V + E)$

1d35d2, 46 lines

```
vi getDomTree(vvi &adj, int src) {
    int n = sz(adj), t = 0;
    vvi revAdj(n), child(n), sdomChild(n);
    vi label(n, -1), revLabel(n), sdom(n), idom(n), par(n), best(n);

    auto dfs = [&](int cur, auto &dfs) -> void { //f72200
        label[cur] = t, revLabel[t] = cur;
        sdom[t] = par[t] = best[t] = t; t++;
        for(int nxt: adj[cur]) {
            if(label[nxt] == -1) {
                dfs(nxt, dfs); //79b43c
                child[label[cur]].push_back(label[nxt]);
            } //01b03d
            revAdj[label[nxt]].push_back(label[cur]);
        } //3ffa21
    }; //65f8db
    dfs(src, dfs);

    auto get = [&](int x, auto &get) -> int {
        if(par[x] != x) {
            int t = get(par[x], get); //7f7ab8
            par[x] = par[par[x]];
            if(sdom[t] < sdom[best[x]]) best[x] = t;
        } //4696d0
        return best[x];
    }; //9d168a
```

```
for(int i = t-1; i >= 0; i--) {
    for(int j: revAdj[i]) sdom[i] = min(sdom[i], sdom[get(j, get)]);
    if(i > 0) sdomChild[sdom[i]].push_back(i);
    for(int j: sdomChild[i]) { //6369b1
        int k = get(j, get);
        if(sdom[j] == sdom[k]) idom[j] = sdom[j];
        else idom[j] = k;
    } //28ff3c
    for(int j: child[i]) par[j] = i;
} //e97294

vi dom(n);
rep(i, 1, t) {
    if(idom[i] != sdom[i]) idom[i] = idom[idom[i]];
    dom[revLabel[i]] = revLabel[idom[i]]; //38ad1b
} //c63146

return dom;
} //1d35d2
```

5.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D , computes a $(D + 1)$ -coloring of the edges such that no neighboring edges share a color. (D -coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

Time: $\mathcal{O}(NM)$

e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
    vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
    for (pii e : eds) ++cc[e.first], ++cc[e.second];
    int u, v, ncols = *max_element(all(cc)) + 1;
    vector<vi> adj(N, vi(ncols, -1));
    for (pii e : eds) { //fc7443
        tie(u, v) = e;
        fan[0] = v;
        loc.assign(ncols, 0);
        int at = u, end = u, d, c = free[u], ind = 0, i = 0;
        while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
            loc[d] = ++ind, cc[ind] = d, fan[ind] = v; //e45383
        cc[loc[d]] = c;
        for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
            swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
        while (adj[fan[i]][d] != -1) {
            int left = fan[i], right = fan[++i], e = cc[i]; //f3efaf
            adj[u][e] = left;
            adj[left][e] = u;
            adj[right][e] = -1;
            free[right] = e;
        } //657a28
        adj[u][d] = fan[i];
        adj[fan[i]][d] = u;
        for (int y : {fan[0], u, end})
            for (int& z = free[y] = 0; adj[y][z] != -1; z++);
    } //e9f8dc
    rep(i, 0, sz(eds))
        for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
    return ret;
} //e210e2
```

5.5 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs	b0d5b1, 12 lines
typedef bitset<128> B; template < class F> void cliques(vector& eds, F f, B P = ~B(), B X={}, B R={}) { if (!P.any()) { if (!X.any()) f(R); return ; } //d462aa auto q = (P X)._Find_first(); auto cand = P & ~eds[q]; rep(i,0,sz(eds)) if (cands[i]) { R[i] = 1; cliques(eds, f, P & eds[i], X & eds[i], R); //cf4187 R[i] = P[i] = 0; X[i] = 1; } //2b8ca5 } //b0d5b1	

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

typedef vector<bitset<200>> vb; struct Maxclique { double limit=0.025, pk=0; struct Vertex { int i, d=0; }; //93b51d typedef vector<Vertex> vv; vb e; vv V; vector<vi> C; vi qmax, q, S, old; //b548bf void init(vv& r) { for (auto & v : r) v.d = 0; for (auto & v : r) for (auto j : r) v.d += e[v.i][j.i]; sort(all(r), [](auto a, auto b) { return a.d > b.d; }); int mxD = r[0].d; //16d40c rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1; } //d5dc84 void expand(vv& R, int lev = 1) { S[lev] += S[lev - 1] - old[lev]; old[lev] = S[lev - 1]; while (sz(R)) { if (sz(q) + R.back().d <= sz(qmax)) return ; //09eb24 q.push_back(R.back().i); vv T; for (auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i}); if (sz(T)) { if (S[lev]++ / ++pk < limit) init(T); //c706bf int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1); C[l].clear(), C[2].clear(); for (auto v : T) { int k = 1; auto f = [&](int i) { return e[v.i][i]; }; //3e1b8e while (any_of(all(C[k]), f)) k++; if (k > mxk) mxk = k, C[mxk + 1].clear(); if (k < mnk) T[j++] .i = v.i; C[k].push_back(v.i); } //5ebe7a if (j > 0) T[j - 1].d = 0; rep(k,mnk,mxk + 1) for (int i : C[k]) T[j].i = i, T[j++].d = k; expand(T, lev + 1); } else if (sz(q) > sz(qmax)) qmax = q; //86a1f3 q.pop_back(), R.pop_back(); } //c01dd9 } //901020 vi maxClique() { init(V), expand(V); return qmax; } //12c3d2 Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) { rep(i,0,sz(e)) V.push_back({i}); }	f7c0bc, 49 lines
--	------------------

} //21f145 }; //f7c0bc	
---------------------------	--

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertex-Cover.

5.6 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

vector<vi> treeJump(vi& P) { int on = 1, d = 1; while (on < sz(P)) on *= 2, d++; vector<vi> jmp(d, P); rep(i,1,d) rep(j,0,sz(P)) jmp[i][j] = jmp[i-1][jmp[i-1][j]]; //35de77 return jmp; } //6d3434 int jmp(vector<vi>& tbl, int nod, int steps){ rep(i,0,sz(tbl)) if (steps&(1<<i)) nod = tbl[i][nod]; return nod; //5f4dea } //7ce14c int lca(vector<vi>& tbl, vi& depth, int a, int b) { if (depth[a] < depth[b]) swap(a, b); a = jmp(tbl, a, depth[a] - depth[b]); if (a == b) return a; //74edff for (int i = sz(tbl); i--;) { int c = tbl[i][a], d = tbl[i][b]; if (c != d) a = c, b = d; } //863967 return tbl[0][a]; } //bfce85 LCA.h Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected. Time: $\mathcal{O}(N \log N + Q)$	bfce85, 25 lines
struct LCA { int T = 0; vi time, path, ret; RMQ< int > rmq; LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1), ret)) {} void dfs(vector<vi>& C, int v, int par) { //2deaa6 time[v] = T++; for (int y : C[v]) if (y != par) { path.push_back(v), ret.push_back(time[v]); dfs(C, y, v); } //6720ac } //5ad321 int lca(int a, int b) { if (a == b) return a; tie(a, b) = minmax(time[a], time[b]); return path[rmq.query(a, b)]; //c2446b } //8588d0 //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];} }; //0f62fb	0f62fb, 21 lines

MaxPath.h

Description: Given edges (Weight,U,V) answers max on path queries of the induced MST.
Time: $\mathcal{O}(N \log(N))$

struct maxPath{ vector< int > p,s,wt; ll tot = 0; maxPath(vector<tuple< int,int,int >> ed, int n): p(n), s(n,1),wt(n,INT_MAX){ sort(all(ed)); iota(all(p),0); for (auto [w,u,v]: ed){ //14f653 while (u!=p[u]) u=p[u]; while (v!=p[v]) v=p[v]; if (u==v) continue ; tot+=w; if (s[u]>s[v]) swap(u,v); p[u] = v; s[v]+=s[u]; wt[u] = w; //0b97a9 } //36af2c } //bf42b8 int query(int u, int v){ //assert(u!=v); while (p[u]!=v && p[v]!=u) { if (wt[u]<wt[v]) u=p[u]; else v=p[v]; } //8df1f6 return p[u]==v ? wt[u]:wt[v]; } //365c8b }; //2aabe7	2aabe7, 21 lines
---	------------------

CentroidDecomp.h

Description: Calls callback function on undirected forest for each centroid
Usage: centroid(adj, [&](const vector<vector<**int**>>& adj, **int** cent) { ... });
Time: $\mathcal{O}(n \log n)$

template < class F> struct centroid { vector<vi> adj; F f; vi sub_sz, par; centroid(const vector<vi>& a_adj, F a_f) : adj(a_adj), f(a_f), sub_sz(sz(adj), -1), par(sz(adj), -1) { rep(i, 0, sz(adj)) //a71923 if (sub_sz[i] == -1) dfs(i); } //03bc04 void calc_sz(int u, int p) { sub_sz[u] = 1; for (int v : adj[u]) if (v != p) calc_sz(v, u), sub_sz[u] += sub_sz[v]; //3a72fa } //9a4332 int dfs(int u) { calc_sz(u, -1); for (int p = -1, sz_root = sub_sz[u];;) { auto big_ch = find_if(begin(adj[u]), end(adj[u]), [&](int v) { return v != p && 2 * sub_sz[v] > sz_root; //ad4da8 }); if (big_ch == end(adj[u])) break ; p = u, u = *big_ch; } //fcaffc f(adj, u); for (int v : adj[u]) { iter_swap(find(begin(adj[v]), end(adj[v]), u), rbegin(adj[v])); adj[v].pop_back(); par[dfs(v)] = u; //994f54 } //a5711e return u; } //155406 }; //2c9a06	2c9a06, 33 lines
--	------------------

UCF	
EdgeCD.h	
Description: Edge-Centroid Decomp, count single edge paths separately, don't consider root to node paths in F edge_cd(adj, [&](const auto& adj, int cent, int m) subtrees of [0, m) of adj[cent]: 1st edge-set subtrees of [m, sz(adj[cent])): 2nd edge-set);	
Time: $\mathcal{O}(n \log n)$	fe3ded, 35 lines

```
template <class F> struct edge_cd {
    vector<vector<int>>> adj;
    F f;
    vector<int> sub_sz;
    edge_cd(const vector<vector<int>>>& a_adj, F a_f) : adj(a_adj)
        , f(a_f), sub_sz((int)size(adj)) {
        dfs(0, (int)size(adj)); //ff7f72
    } //0a92d4
    int find_cent(int u, int p, int siz) {
        sub_sz[u] = 1;
        for (int v : adj[u])
            if (v != p) {
                int cent = find_cent(v, u, siz); //9a0b69
                if (cent != -1) return cent;
                sub_sz[u] += sub_sz[v];
            } //8c23e3
        if (p == -1) return u;
        return 2 * sub_sz[u] >= siz ? sub_sz[p] = siz - sub_sz[u],
            u : -1;
    } //4b9693
    void dfs(int u, int siz) {
        if (siz <= 2) return;
        u = find_cent(u, -1, siz);
        int sum = 0;
        auto it = partition(begin(adj[u]), end(adj[u]), [&](int v)
            {
                bool ret = 2 * sum + sub_sz[v] < siz - 1 && 3 * (sum +
                    sub_sz[v]) <= 2 * (siz - 1);
                if (ret) sum += sub_sz[v]; //983590
                return ret;
            });
        f(adj, u, it - begin(adj[u]));
        vector<int> oth(it, end(adj[u]));
        adj[u].erase(it, end(adj[u])); //3deb39
        dfs(u, sum + 1);
        swap(adj[u], oth);
        dfs(u, siz - sum);
    } //ed9cba
}; //fe3ded
```

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.

Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h"	9775a0, 21 lines
typedef vector<pair<int, int>> vpi;	
vpi compressTree(LCA& lca, const vi& subset) {	
static vi rev; rev.resize(sz(lca.time));	
vi li = subset, &T = lca.time;	
auto cmp = [&](int a, int b) { return T[a] < T[b]; };	
sort(all(li), cmp); //6f0834	
int m = sz(li)-1;	
rep(i,0,m) {	
int a = li[i], b = li[i+1];	
li.push_back(lca.lca(a, b));	
} //432667	
sort(all(li), cmp);	
li.erase(unique(all(li)), li.end());	
rep(i,0,sz(li)) rev[li[i]] = i;	
vpi ret = {pii(0, li[0])}; //89fc5f	

EdgeCD CompressTree HLD LinkCutTree	
rep(i,0,sz(li)-1) {	
int a = li[i], b = li[i+1];	
ret.emplace_back(rev[lca.lca(a, b)], b);	
} //cefab5	
return ret;	
} //9775a0	

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most $\log(n)$ light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.

Time: $\mathcal{O}((\log N)^2)$

"./data-structures/LazySegmentTree.h"	6f34db, 46 lines
template <bool VALS_EDGES> struct HLD {	
int N, tim = 0;	
vector<vi> adj;	
vi par, siz, depth, rt, pos;	
Node *tree;	
HLD(vector<vi> adj_) //d266b7	
: N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1), depth(N),	
rt(N),pos(N),tree(new Node(0, N)){ dfsSz(0); dfsHld(0); }	
void dfsSz(int v) {	
if (par[v] != -1) adj[v].erase(find(all(adj[v]), par[v]));	
for (int& u : adj[v]) { //c2274a	
par[u] = v, depth[u] = depth[v] + 1;	
dfsSz(u);	
siz[v] += siz[u];	
if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);	
} //b0fa49	
} //9ba8db	
void dfsHld(int v) {	
pos[v] = tim++;	
for (int u : adj[v]) {	
rt[u] = (u == adj[v][0] ? rt[v] : u);	
dfsHld(u); //2698ee	
} //39b629	
} //39d559	
template <class B> void process(int u, int v, B op) {	
for (; rt[u] != rt[v]; v = par[rt[v]]) {	
if (depth[rt[u]] > depth[rt[v]]) swap(u, v);	
op(pos[rt[v]], pos[v] + 1);	
} //fa17fe	
if (depth[u] > depth[v]) swap(u, v);	
op(pos[u] + VALS_EDGES, pos[v] + 1);	
} //0d5603	
void modifyPath(int u, int v, int val) {	
process(u, v, [&](int l, int r) { tree->add(l, r, val); });	
} //79ce98	
int typePath(int u, int v) { // Modify depending on problem	
int res = -le9;	
process(u, v, [&](int l, int r) {	
res = max(res, tree->query(l, r));	
}); //29a64c	
return res;	
} //f00cd2	
int querySubtree(int v) { // modifySubtree is similar	
return tree->query(pos[v] + VALS_EDGES, pos[v] + siz[v]);	
} //8aad63	
}; //6f34db	

LinkCutTree.h

Description: Represents a forest of unrooted trees. Nodes are 1-indexed. You can add and remove edges (as long as the result is still a forest). You can also do path sum, subtree sum, and LCA queries, which depend on the current root.

Time: All operations take amortized $\mathcal{O}(\log N)$.

	97ef3b, 75 lines
struct SplayTree {	
struct Node {	
int ch[2] = {-1, -1}, p = -1;	
ll self = 0, path = 0; // Path aggregates	
ll sub = 0, vir = 0; // Subtree aggregates	
bool flip = 0; // Lazy tags	
}; //482fc0	
vector<Node> Ts; Node *T;	
SplayTree(int n) : Ts(n+1), T(&Ts[1]) {} //91363a	
void push(int x) {	
if (x == -1 !T[x].flip) return;	
int l = T[x].ch[0], r = T[x].ch[1];	
T[l].flip ^= 1, T[r].flip ^= 1;	
swap(T[x].ch[0], T[x].ch[1]); //0411c8	
T[x].flip = 0; } //f870fe	
void pull(int x) {	
int l = T[x].ch[0], r = T[x].ch[1]; push(l); push(r);	
T[x].path = T[l].path + T[x].self + T[r].path;	
T[x].sub = T[x].vir + T[l].sub + T[r].sub + T[x].self;	
} //672aff	
void set(int x, int d, int y) {	
T[x].ch[d] = y; T[y].p = x; pull(x); } //84c4f7	
void splay(int x) {	
auto dir = [&](int x) {	
int p = T[x].p; if (p == -1) return -1;	
return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 : -1;	
}; //dc48c9	
auto rotate = [&](int x) {	
int y = T[x].p, z = T[y].p, dx = dir(x), dy = dir(y);	
set(y, dx, T[x].ch[!dx]);	
set(x, !dx, y);	
if (~dy) set(z, dy, x); //4108b6	
T[x].p = z; }; //4543f0	
for (push(x); ~dir(x);) {	
int y = T[x].p, z = T[y].p;	
push(z); push(y); push(x);	
int dx = dir(x), dy = dir(y);	
if (~dy) rotate(dx != dy ? x : y); //67e33c	
rotate(x); } //7e3076	
} //6fa8ab	
}; //97d901	
struct LinkCut : SplayTree {	
LinkCut(int n) : SplayTree(n) {} //92d23f	
int access(int x) {	
int u = x, v = -1;	
for (; u != -1; v = u, u = T[u].p) {	
splay(u);	
int& ov = T[u].ch[1]; //f6ee08	
T[u].vir += T[ov].sub;	
T[u].vir -= T[v].sub;	
ov = v; pull(u); } //dc673b	
return splay(x), v; } //533ecb	
void reroot(int x) {	
access(x); T[x].flip ^= 1; push(x); } //dbd108	
void link(int u, int v) {	
reroot(u); access(v);	
T[v].vir += T[u].sub;	
T[u].p = v; pull(v); } //c76755	
void cut(int u, int v) {	
reroot(u); access(v);	
T[v].ch[0] = T[u].p = -1; pull(v); } //33b01d	
bool connected(int u, int v) { return lca(u, v) != -1; }	
// Rooted tree LCA. Returns -1 if u and v arent connected.	

```
int lca(int u, int v) {
    if (u == v) return u;
    access(u); int ret = access(v); //51e775
    return T[u].p != -1 ? ret : -1; } //6c1d58
// Query subtree of u where v is outside the subtree.
ll subtree(int u, int v) {
    reroot(v); access(u); return T[u].vir + T[u].self; }
ll path(int u, int v) { // Query path [u..v]
    reroot(u); access(v); return T[v].path; } //f0875c
void update(int u, ll v) { //Update vert u with val v
    access(u); T[u].self = v; pull(u); } //18a6fa
}; //97ef3b
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

Time: $\mathcal{O}(E \log V)$

"/data-structures/UnionFindRollback.h"6dec70, 50 lines

```
struct Edge { int a, b; ll w; }; //030131
struct Node {
    Edge key;
    Node* l, *r;
    ll delta;
    void prop() { //958c51
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0; } //31f792
    Edge top() { return prop(), key; } }; //316fcc
Node* merge(Node* a, Node* b) {
    if (!a || !b) return a ? b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r))); //235473
    return a; } //db045f
void pop(Node*& a) { a->prop(), a = merge(a->l, a->r); }
pair<ll, vi> dmst(int n, int r, vector<Edge*>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
    ll res = 0; //4920c6
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1, -1}), comp;
    deque<tuple<int, int, vector<Edge*>>> cyscs;
    rep (s, 0, n) { //253912
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1, {}}; //d9c649
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) { //c1afd9
                Node* cyc = 0;
                int end = qi, time = uf.time();
                do cyc = merge(cyc, heap[w = path[--qi]]);
                while (uf.join(u, w));
                u = uf.find(u), heap[u] = cyc, seen[u] = -1; //ecalab
                cyscs.push_front({u, time, {&Q[qi], &Q[end]}}); } }
            rep (i, 0, qi) in[uf.find(Q[i].b)] = Q[i]; } //e4099a
        for (auto& [u, t, comp] : cyscs) { // restore sol (optional)
            uf.rollback(t);
            Edge inEdge = in[u];
            for (auto& e : comp) in[uf.find(e.b)] = e;
            in[uf.find(inEdge.b)] = inEdge; } //c4fe60
        rep (i, 0, n) par[i] = in[i].a;
        return {res, par}; } //6dec70
```

TreeDiam.h

Description: Short code for finding a diameter of a tree and returning the path

Time: $\mathcal{O}(|V|)$

d64251, 13 lines

```
auto diameter = [&](int u, int p, auto &&diameter) -> vi {
    vi best;
    for (int v : graph[u]){
        if (v == p) continue;
        vi cur = diameter(v, u, diameter);
        if (sz(cur) > sz(best)) swap(cur, best); //632f5a
    } //2d9dce
    best.push_back(u);
    return best;
}; //d64251
//vi diam = diameter(0, -1, diameter);
//diam = diameter(diam[0], -1, diameter);
//number of nodes on diam is diam.size()
```

Numerical Methods (6)

6.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val += x) += a[i];
        return val; //3743d7
    } //f7a37b
    void diff() {
        rep(i,1,sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    } //d447a3
    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
        a.pop_back();
    } //43bc43
}; //c9b7b0
```

PolyRoots.h

Description: Finds the real roots to a polynomial.

Usage: polyRoots({{2,-3,1}},-1e9,1e9) // solve $x^2-3x+2 = 0$

Time: $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h"b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; } //a63eaa
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = polyRoots(der, xmin, xmax);
    dr.push_back(xmin-1); //31d1fe
    dr.push_back(xmax+1);
    sort(all(dr));
    rep(i,0,sz(dr)-1) {
        double l = dr[i], h = dr[i+1];
        bool sign = p(l) > 0; //2748c8
        if (sign ^ (p(h) > 0)) {
            rep(it,0,60) { // while (h - l > 1e-8)
                double m = (l + h) / 2, f = p(m);
                if ((f <= 0) ^ sign) l = m;
                else h = m; //8da3ef
            } //4f1379
            ret.push_back((l + h) / 2);
        } //1c9b1d
```

```
    } //d5f24e
```

```
    return ret;
```

```
}; //b00bfe
```

PolyInterpolate.h

Description: Given n points $(x[i], y[i])$, computes an $n-1$ -degree polynomial p that passes through them: $p(x) = a[0] * x^0 + \dots + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.

Time: $\mathcal{O}(n^2)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k,0,n-1) rep(i,k+1,n)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0; temp[0] = 1; //ca948d
    rep(k,0,n) rep(i,0,n) {
        res[i] += y[k] * temp[i];
        swap(last, temp[i]);
        temp[i] -= last * x[k];
    } //8c43d1
    return res;
}; //08bf48
```

BerlekampMassey.h

Description: Recovers any n -order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.

Usage: berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}

Time: $\mathcal{O}(N^2)$

"/number-theory/ModPow.h"96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
    int n = sz(s), L = 0, m = 0;
    vector<ll> C(n), B(n), T;
    C[0] = B[0] = 1;

    ll b = 1; //b7979b
    rep(i,0,n) { ++m;
        ll d = s[i] % mod;
        rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C; ll coef = d * modpow(b, mod-2) % mod; //b3b877
        rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
        if (2 * L > i) continue;
        L = i + 1 - L; B = T; b = d; m = 0;
    } //3dc38b
```

```
C.resize(L + 1); C.erase(C.begin());
for (ll& x : C) x = (mod - x) % mod;
return C;
}; //96548b
```

LinearRecurrence.h

Description: Generates the k 'th term of an n -order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots \geq n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp-Massey.

Usage: linearRec({0, 1}, {1, 1}, k) // k 'th Fibonacci number

Time: $\mathcal{O}(n^2 \log k)$

f4e444, 21 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);
    auto combine = [&](Poly a, Poly b) {
        Poly res(n * 2 + 1);
        rep(i,0,n+1) rep(j,0,n+1) //935f2e
            res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
        for (int i = 2 * n; i > n; --i) rep(j,0,n)
```

```
        res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res; //6c07b9
}; //da80a6
Poly pol(n + 1), e(pol); pol[0] = e[1] = 1;
for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
} //b658e4
ll res = 0;
rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
return res;
} //f4e444
```

6.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a,b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is ϵ *ps*. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.

Usage: double func(double x) { return 4+x+.3*x*x; }
double xmin = gss(-1000,1000,func);
Time: $\mathcal{O}(\log((b-a)/\epsilon))$

```
double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5)-1)/2, eps = 1e-7;
    double x1 = b - r*(b-a), x2 = a + r*(b-a);
    double f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1; //012afe
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        } //821619
    return a;
} //31d45b
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions

typedef array<double, 2> P;

```
template<class F> pair<double, P> hillClimb(P start, F f) {
    pair<double, P> cur(f(start), start);
    for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
        rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) { //1a21bb
            P p = cur.second;
            p[0] += dx*jmp;
            p[1] += dy*jmp;
            cur = min(cur, make_pair(f(p), p));
        } //93215a
    } //523260
    return cur;
} //8eeef
```

IntegrateAdaptiveTyler.h

Description: Gets area under a curve

```
#define approx(a, b) (b-a) / 6 * (f(a) + 4 * f((a+b) / 2) + f(b))

template<class F>
ld adapt (F &f, ld a, ld b, ld A, int iters) {
    ld m = (a+b) / 2;
    ld A1 = approx(a, m), A2 = approx(m, b); //a97d86
    if(!iters && (abs(A1 + A2 - A) < eps || b-a < eps))
```

```
        return A;
    ld left = adapt(f, a, m, A1, max(iters-1, 0));
    ld right = adapt(f, m, b, A2, max(iters-1, 0));
    return left + right; //d787ca
} //f68b38
```

```
template<class F>
ld integrate(F f, ld a, ld b, int iters = 0) {
    return adapt(f, a, b, approx(a, b), iters);
} //e7beba
```

RungeKutta4.h

Description: Numerically approximates the solution to a system of Differential Equations

```
template<class F, class T>
T solveSystem(F f, T x, ld time, int iters) {
    double h = time / iters;
    for(int iter = 0; iter < iters; iter++) {
        T k1 = f(x);
        A k2 = f(x + 0.5 * h * k1); //6adf94
        A k3 = f(x + 0.5 * h * k2);
        A k4 = f(x + h * k3);
        x = x + h / 6.0 * (k1 + 2.0 * k2 + 2.0 * k3 + k4);
    } //004f46
    return x;
} //25c1ac
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b, x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vvd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

```
typedef double T; // long double, Rational, double + modP>...
typedef vector<T> vd;
typedef vector<vd> vvd;
```

const T eps = 1e-8, inf = 1./0;
#define MP make_pair //20f308
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

```
struct LPSolver {
    int m, n;
    vi N, B; //a8b98c
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j]; //a00ca8
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; } //03bb56
        N[n] = -1; D[m+1][n] = 1;
    } //dcadf8
```

```
void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
        T *b = D[i].data(), inv2 = b[s] * inv; //a86c76
        rep(j,0,n+2) b[j] -= a[j] * inv2;
        b[s] = a[s] * inv2;
    } //df792b
```

```
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
} //193de8
```

```
bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
        int s = -1; //8b65cd
        rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
        if (D[x][s] >= -eps) return true;
        int r = -1;
        rep(i,0,m) {
            if (D[i][s] <= eps) continue; //f65882
            if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                < MP(D[r][n+1] / D[r][s], B[r])) r = i;
        } //170720
        if (r == -1) return false;
        pivot(r, s);
    } //d81c2f
} //62b7d3
```

```
T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) { //dc34d7
        pivot(r, n);
        if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
        rep(i,0,m) if (B[i] == -1) {
            int s = 0;
            rep(j,1,n+1) ltj(D[i]); //db9144
            pivot(i, s);
        } //213eb8
    } //36d5c1
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
} //bc3870
}; //aa8530
```

6.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

```
double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1; //c6c8fd
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k]; //979b9a
        } //ebf330
    } //aa3042
    return res;
} //bd5cec
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
```

```
int n = sz(a); ll ans = 1;
rep(i,0,n) {
    rep(j,i+1,n) {
        while (a[j][i] != 0) { // gcd step
            ll t = a[i][i] / a[j][i]; //155e04
            if (t) rep(k,i,n)
                a[i][k] = (a[i][k] - a[j][k] * t) % mod;
            swap(a[i], a[j]);
            ans *= -1;
        } //3e9488
    } //7effce
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
} //666fb0
return (ans + mod) % mod;
} //3313dc
```

SolveLinear.h
Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2m)$

```
typedef vector<double> vd;
const double eps = 1e-12;
```

```
int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m); //61ac86
    vi col(m); iota(all(col), 0);
```

```
    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m) //9bbd0f
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j,i,n) if (fabs(b[j]) > eps) return -1;
            break; //b9eea0
        } //e8dea5
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i]; //bc2598
        rep(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k,i+1,m) A[j][k] -= fac*A[i][k];
        } //34df26
        rank++;
    } //66cd8f
```

```
    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i]; //9d7b80
        rep(j,0,i) b[j] -= A[j][i] * b[i];
    } //55ec26
    return rank; // (multiple solutions if rank < m)
} //44c9ab
```

SolveLinear2.h
Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

```
"SolveLinear.h"
08e495, 7 lines

rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
```

```
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i]; //46800e
fail;; } //08e495
```

SolveLinearBinary.h
Description: Solves $Ax = b$ over \mathbb{F}_2 . If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b .
Time: $\mathcal{O}(n^2m)$

```
typedef bitset<1000> bs;
```

```
int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0); //b3f2a0
    rep(i,0,n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
        if (br == n) {
            rep(j,i,n) if(b[j]) return -1;
            break; //4a27f9
        } //84b30e
        int bc = (int)A[br]._Find_next(i-1);
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) if (A[j][i] != A[j][bc]) { //31f207
            A[j].flip(i); A[j].flip(bc);
        } //bf5e08
        rep(j,i+1,n) if (A[j][i]) {
            b[j] ^= b[i];
            A[j] ^= A[i];
        } //0837c3
        rank++;
    } //4de1ff
```

```
    x = bs();
    for (int i = rank; i--;) {
        if (!b[i]) continue;
        x[col[i]] = 1; //c2244c
        rep(j,0,i) b[j] ^= A[j][i];
    } //fe12f5
    return rank; // (multiple solutions if rank < m)
} //fa2d7a
```

MatrixInverse.h
Description: Invert matrix A . Returns rank; result is stored in A unless singular ($\text{rank} < n$). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of $A \pmod p$, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;
```

```
    rep(i,0,n) { //8ece41
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i; //baa3bb
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i]; //59c017
        rep(j,i+1,n) {
            double f = A[j][i] / v;
```

```
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -= f*tmp[i][k]; //293c3d
        } //4b5802
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    } //cd352a
```

```
    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        double v = A[j][i];
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    } //fd4d51
```

```
    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
    return n;
} //ebfff6
```

MatrixInverse-mod.h
Description: Invert matrix A modulo a prime. Returns rank; result is stored in A unless singular ($\text{rank} < n$). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of $A \pmod p$, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

```
".../number-theory/ModPow.h"
a6f68f, 36 lines

int matInv(vector<vector<ll>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<ll>> tmp(n, vector<ll>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;
```

```
    rep(i,0,n) { //4c70b5
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n) if (A[j][k]) {
            r = j; c = k; goto found;
        } //670a88
        return i;
    }
found:
```

```
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]); //f483b9
    ll v = modpow(A[i][i], mod - 2);
    rep(j,i+1,n) {
        ll f = A[j][i] * v % mod;
        A[j][i] = 0;
        rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod;
        rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
    } //3af408
    rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
    rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
    A[i][i] = 1;
} //b5fe9f
```

```
    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        ll v = A[j][i];
        rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
    } //597dbe
```

```
    rep(i,0,n) rep(j,0,n)
        A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod
            : 0);
    return n;
} //a6f68f
```

JacobianMatrix.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, \, 1 \leq i \leq n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, \dots, -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.

If $|d_i| > |p_i| + |q_{i-1}|$ for all i , or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

Time: $\mathcal{O}(N)$

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
    int n = sz(b); vi tr(n);
    rep(i,0,n-1) {
        if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
            b[i+1] -= b[i] * diag[i+1] / super[i]; //5648ab
            if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
            diag[i+1] = sub[i]; tr[i++] = 1;
        } else {
            diag[i+1] -= super[i]*sub[i]/diag[i];
            b[i+1] -= b[i]*sub[i]/diag[i]; //13335c
        } //25f2e7
    } //7da0d1
    for (int i = n; i--;) {
        if (tr[i]) {
            swap(b[i], b[i-1]);
            diag[i-1] = diag[i];
            b[i] /= super[i-1]; //6bd4e6
        } else {
            b[i] /= diag[i];
            if (i) b[i-1] -= b[i]*super[i-1];
        } //94ec57
    } //4f78c5
    return b;
} //8f9fa8
```

JacobianMatrix.h

Description: Makes Jacobian Matrix using finite differences

```
template<class F, class T>
vector<vector<T>> makeJacobian(F &f, vector<T> &x) {
    int n = sz(x);
    vector<vector<T>> J(n, vector<T>(n));
    vector<T> fx0 = f(x);
    rep(i, 0, n) { //6bdb0f
        x[i] += eps;
        vector<T> fx1 = f(x);
        rep(j, 0, n){
            J[j][i] = (fx1[j] - fx0[j]) / eps;
        } //8f9232
        x[i] -= eps;
    } //6c57a8
    return J;
} //75dc90
```

NewtonsMethod.h

Description: Solves a system on non-linear equations

```
JacobianMatrix.h
6af945, 10 lines

template<class F, class T>
void solveNonlinear(F f, vector<T> &x){
    int n = sz(x);
    rep(iter, 0, 100) {
        vector<vector<T>> J = makeJacobian(f, x);
        matInv(J); //0e4ed9
        vector<T> dx = J * f(x);
        x = x - dx;
    } //e79640
} //6af945
```

6.4 Fourier transforms

FastFourierTransform.h

Description: `fft(a)` computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k . N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by `n`, `reverse(start+1, end)`, FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16} ; higher for random inputs). Otherwise, use NTT/FFTMod. **Time:** $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1); // (^ 10% faster if double)
    for (static int k = 2; k < n; k *= 2) { //beb684
        R.resize(n); rt.resize(n);
        auto x = polar(1.0L, acos(-1.0L) / k);
        rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
    } //42ea68
    vi rev(n);
    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //9f2153
            C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
            a[i + j + k] = a[i + j] - z;
            a[i + j] += z;
        } //865e86
    } //3b927f
    vd conv(const vd& a, const vd& b) {
        if (a.empty() || b.empty()) return {}; //7ee20e
        vd res(sz(a) + sz(b) - 1);
        int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
        vector<C> in(n), out(n);
        copy(all(a), begin(in));
        rep(i,0,sz(b)) in[i].imag(b[i]); //ea36b1
        fft(in);
        for (C& x : in) x *= x;
        rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
        fft(out);
        rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n); //9893c9
        return res;
    } //00ced6
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$. **Time:** $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

```
"FastFourierTransform.h"
b82773, 22 lines

typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
```

```
if (a.empty() || b.empty()) return {}; //ffec4
vl res(sz(a) + sz(b) - 1);
int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
vector<C> L(n), R(n), outs(n), outl(n);
rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
fft(L), fft(R); //f8a1f3
rep(i,0,n) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
} //455f55
fft(outl), fft(outs);
rep(i,0,sz(res)) {
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M; //0af53f
} //26b37c
return res;
} //b82773
```

NumberTheoreticTransform.h

Description: `ntt(a)` computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k , where $g = \text{root}^{(\text{mod}-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a . For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by `n`, `reverse(start+1, end)`, NTT back. Inputs must be in $[0, \text{mod})$. **Time:** $\mathcal{O}(N \log N)$

```
"../number-theory/ModPow.h"
ced03d, 33 lines

const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
    int n = sz(a), L = 31 - __builtin_clz(n); //c96375
    static vl rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)}; //1759b1
        rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
    } //5faa22
    vi rev(n);
    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //61bd17
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        } //35d5bf
    } //29a029
    vl conv(const vl &a, const vl &b) {
        if (a.empty() || b.empty()) return {}; //4001b0
        int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s), n = 1 << B;
        int inv = modpow(n, mod - 2);
        vl L(a), R(b), out(n);
        L.resize(n), R.resize(n);
        ntt(L), ntt(R); //6415db
        rep(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
        ntt(out);
        return {out.begin(), out.begin() + s}; //70c6bc
    } //ced03d
```


FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where \oplus is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

	464cf3, 16 lines
<pre>void FST(vi& a, bool inv) { for (int n = sz(a), step = 1; step < n; step *= 2) { for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) { int &u = a[j], &v = a[j + step]; tie(u, v) = inv ? pii(v - u, u) : pii(v, u + v); // AND inv ? pii(v, u - v) : pii(u + v, u); // OR pii(u + v, u - v); // XOR } //398dab } //3431d0 if (inv) for (int& x : a) x /= sz(a); // XOR only } //57eeaf vi conv(vi a, vi b) { FST(a, 0); FST(b, 0); rep(i,0,sz(a)) a[i] *= b[i]; FST(a, 1); return a; } //464cf3</pre>	

Minconv.h

Description: @param convex,arbitrary arrays where convex satisfies $\text{convex}[i+1]-\text{convex}[i] \leq \text{convex}[i+2]-\text{convex}[i+1]$ @returns array 'res' where 'res[k]' = the min of (a[i]+b[j]) for all pairs (i,j) where i+j==k

	633806, 26 lines
<pre>vector<int> min_plus(const vector<int>& convex, const vector<int>& arbitrary) { int n = ssize(convex); int m = ssize(arbitrary); vector<int> res(n + m - 1, INT_MAX); auto dnc = [&](auto&& self, int res_le, int res_ri, //c890c6 int arb_le, int arb_ri) -> void { if (res_le >= res_ri) return; int mid_res = (res_le + res_ri) / 2; int op_arb = arb_le; for (int i = arb_le; i < min(mid_res + 1, arb_ri); //00bcac i++) { int j = mid_res - i; if (j >= n) continue; if (res[mid_res] > convex[j] + arbitrary[i]) { res[mid_res] = convex[j] + arbitrary[i]; //c587b4 op_arb = i; } //d9dac2 } //12d663 self(self, res_le, mid_res, arb_le, min(arb_ri, op_arb + 1)); self(self, mid_res + 1, res_ri, op_arb, arb_ri); }; //133dea dnc(dnc, 0, n + m - 1, 0, m); return res; } //633806</pre>	

gcdconv.h

Description: $\text{ssize}(a) == \text{ssize}(b)$ $\text{gcdconv}[k] = \text{sum of } (a[i]*b[j])$ for all pairs (i,j) where $\text{gcd}(i,j) == k$
Time: $\mathcal{O}(N \log N)$

	2dfb20, 16 lines
<pre>const int mod = 998'244'353; vector<int> gcd_convolution(const vector<int>& a, const vector<int>& b) { int n = ssize(a); vector<int> c(n); for (int g = n - 1; g >= 1; g--) { //8423c4 int64_t sum_a = 0, sum_b = 0; for (int i = g; i < n; i += g) { sum_a += a[i], sum_b += b[i];</pre>	

<pre> if ((c[g] -= c[i]) < 0) c[g] += mod; } //7021b5 sum_a %= mod, sum_b %= mod; c[g] = (c[g] + sum_a * sum_b) % mod; } //22b2a9 return c; } //2dfb20</pre>	

lcmconv.h

Description: $\text{ssize}(a) == \text{ssize}(b)$ $\text{lcmconv}[k] = \text{sum of } (a[i]*b[j])$ for all pairs (i,j) where $\text{lcm}(i,j) == k$

	ee1440, 16 lines
<pre>const int mod = 998'244'353; vector<int> lcm_convolution(const vector<int>& a, const vector<int>& b) { int n = ssize(a); vector<int64_t> sum_a(n), sum_b(n); vector<int> c(n); //f8bc27 for (int i = 1; i < n; i++) { for (int j = i; j < n; j += i) sum_a[j] += a[i], sum_b[j] += b[i]; sum_a[i] %= mod, sum_b[i] %= mod; c[i] = (c[i] + sum_a[i] * sum_b[i]) % mod; //2c8c40 for (int j = i + i; j < n; j += i) if ((c[j] -= c[i]) < 0) c[j] += mod; } //2b66e9 return c; } //ee1440</pre>	

Number theory (7)

7.1 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes $\text{LIM} \leq \text{mod}$ and that mod is a prime.

	6f684f, 3 lines
<pre>const ll mod = 1000000007, LIM = 200000; ll* inv = new ll[LIM] - 1; inv[1] = 1; rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;</pre>	

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. $\text{modLog}(a,1,m)$ can be used to calculate the order of a .

Time: $\mathcal{O}(\sqrt{m})$

	c040b8, 11 lines
<pre>ll modLog(ll a, ll b, ll m) { ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1; unordered_map<ll, ll> A; while (j <= n && (e = f = e * a % m) != b % m) A[e * b % m] = j++; if (e == b % m) return j; //2d9fb0 if (__gcd(m, e) == __gcd(m, b)) rep(i,2,n+2) if (A.count(e = e * f % m)) return n * i - A[e]; return -1; } //c040b8</pre>	

ModSum.h

Description: Sums of mod'ed arithmetic progressions. $\text{modsum}(to, c, k, m) = \sum_{i=0}^{to-1} (ki + c) \% m$. divsum is similar but for floored division.
Time: $\log(m)$, with a large constant.

	5c5bc5, 16 lines
--	------------------

<pre>typedef unsigned long long ull; ull sumsq(ull to) { return to / 2 * ((to-1) 1); } //6bd037 ull divsum(ull to, ull c, ull k, ull m) {</pre>	
--	--

<pre> ull res = k / m * sumsq(to) + c / m * to; k %= m; c %= m; if (!k) return res; //d4b74d ull to2 = (to * k + c) / m; return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k); } //4a574e</pre>	

<pre>ll modsum(ull to, ll c, ll k, ll m) { c = ((c % m) + m) % m; k = ((k % m) + m) % m; return to * c + k * sumsq(to) - m * divsum(to, c, k, m); } //5c5bc5</pre>	
--	--

ModMulLL.h

Description: Calculate $a \cdot b \pmod c$ (or $a^b \pmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow

	bbbb8f, 11 lines
<pre>typedef unsigned long long ull; ull modmul(ull a, ull b, ull M) { ll ret = a * b - M * ull(1.L / M * a * b); return ret + M * (ret < 0) - M * (ret >= (ll)M); } //a9c350 ull modpow(ull b, ull e, ull mod) { ull ans = 1; for (; e; b = modmul(b, b, mod), e /= 2) if (e & 1) ans = modmul(ans, b, mod); return ans; //6d3d5f } //bbbd8f</pre>	

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h"	19a793, 24 lines
------------	------------------

<pre>ll sqrt(ll a, ll p) { a %= p; if (a < 0) a += p; if (a == 0) return 0; assert(modpow(a, (p-1)/2, p) == 1); // else no solution if (p % 4 == 3) return modpow(a, (p+1)/4, p); // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5 ll s = p - 1, n = 2; //6aa127 int r = 0, m; while (s % 2 == 0) ++r, s /= 2; while (modpow(n, (p - 1) / 2, p) != p - 1) ++n; ll x = modpow(a, (s + 1) / 2, p); //94db39 ll b = modpow(a, s, p), g = modpow(n, s, p); for (; r = m) { ll t = b; for (m = 0; m < r && t != 1; ++m) t = t * t % p; //2d5fcd if (m == 0) return x; ll gs = modpow(g, 1LL << (r - m - 1), p); g = gs * gs % p; x = x * gs % p; b = b * g % p; //198af1 } //ac3137 } //19a793</pre>	
--	--

7.2 Primality

LinearSieve.h

Description: Finds smallest prime factor of each integer
Time: $\mathcal{O}(N)$

	32eecc, 8 lines
<pre>const int LIM = 1000000; vi lp(LIM+1), primes; rep(i, 2, LIM + 1) {</pre>	

```
    if (lp[i] == 0) primes.push_back(lp[i] = i);
    for (int j = 0; j < sz(primes) && i * primes[j] <= LIM &&
        primes[j] <= lp[i]; ++j)
        lp[i * primes[j]] = primes[j]; //91f1b5
} //32eeca
```

CountPrimes.h

Description: Count # primes <= N, can be modified to return sum of primes by setting f(p) = n, ps(n) = nth tri number.

Time: $\mathcal{O}\left(n^{3/4}\right)$

```
11 countprimes(1l n) { //n>0
    vector<1l> divs,dp; 1l sq = sqrtl(n);
    for (1l l = 1, r; l <= n && (r = n / (n / l)); l = r + 1)
        divs.push_back(r);
    auto idx = [&](1l x) -> int {
        return x <= sq ? x - 1 : (sz(divs) - n / x); }; //d740a2
    rep(i,0,sz(divs)) dp.push_back(divs[i]-1);
    for(1l p = 2; p*p <= n; ++p) // ^ ps(divs[i])-1
        if(dp[p-1]!=dp[p-2])
            for(int i = sz(divs)-1; divs[i]>=p*p && i>=0; i--)
                dp[i] -= (dp[idx(divs[i]/p)]-dp[p-2]); // *f(p);
    return dp.back(); //0b539f
} //af82c0
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

Time: 7 times the complexity of $a^b \bmod c$.

```
"ModMulLL.h"
60dcd1, 12 lines

bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ull a : A) { // ^ count trailing zeroes
        ull p = modpow(a%n, d, n), i = s; //29e314
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    } //1fad05
    return 1;
} //60dcd1
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

```
"ModMulLL.h", "MillerRabin.h"
d8d98d, 18 lines

ull pollard(ull n) {
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    auto f = [&](ull x) { return modmul(x, x, n) + i; }; //12dccb
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
        x = f(x), y = f(f(y));
    } //0b4d32
    return __gcd(prd, n);
} //cd2ac3
vector<ull> factor(ull n) {
    if (n == 1) return {}; //6303f2
    if (isPrime(n)) return {n}; //74d420
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), all(r));
    return l;
} //d8d98d
```

7.3 Divisibility

euclid.h

Description: Finds two integers x and y , such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

```
33ba8f, 5 lines

11 euclid(1l a, 1l b, 1l &x, 1l &y) {
    if (!b) return x = 1, y = 0, a;
    1l d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
} //33ba8f
```

CRT.h

Description: Chinese Remainder Theorem.

crt(a, m, b, n) computes x such that $x \equiv a \pmod m, x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \leq x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

```
"euclid.h"
04d93a, 7 lines

11 crt(1l a, 1l m, 1l b, 1l n) {
    if (n > m) swap(a, b), swap(m, n);
    1l x, y, g = euclid(m, n, x, y);
    assert((a - b) % g == 0); // else no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + m*n/g : x; //000521
} //04d93a
```

7.3.1 Bézout’s identity

For $a \neq, b \neq 0$, then $d = \gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: Euler’s ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n . $\phi(1) = 1, p \text{ prime} \Rightarrow \phi(p^k) = (p - 1)p^{k-1}$, $m, n \text{ coprime} \Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1}p_2^{k_2}...p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1}...(p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$. $\sum_{d|n} \phi(d) = n, \sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler’s thm: $a, n \text{ coprime} \Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.

Fermat’s little thm: $p \text{ prime} \Rightarrow a^{p-1} \equiv 1 \pmod p \ \forall a$.

```
const int LIM = 50000000;
int phi[LIM];
```

```
void calculatePhi() {
    rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
    for (int i = 3; i < LIM; i += 2) if(phi[i] == i) //9fb18b
        for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
} //cf7d6d
```

7.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$.

For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. (p_k/q_k) alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞ ; if x is the root of a degree 2 polynomial the a ’s eventually become cyclic.

Time: $\mathcal{O}(\log N)$

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<1l, 1l> approximate(d x, 1l N) {
    1l LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
    for (;;) {
        1l lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
            a = (1l)floor(y), b = min(a, lim), //82cd25
            NP = b*P + LP, NQ = b*Q + LQ;
        if (a > b) {
            // If b > a/2, we have a semi-convergent that gives us a
            // better approximation; if b = a/2, we *may* have one.
            // Return {P, Q} here for a more canonical approximation.
            return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
                make_pair(NP, NQ) : make_pair(P, Q); //3c2b26
        } //451a2f
        if (abs(y = 1/(y - (d)a)) > 3*N) {
            return {NP, NQ}; //32957f
        } //ec2d82
        LP = P; P = NP;
        LQ = Q; Q = NQ;
    } //a15756
} //dd6c5e
```

FracBinarySearch.h

Description: Given f and N , finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: fracBS([f](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}

Time: $\mathcal{O}(\log(N))$

```
struct Frac { 1l p, q; }; //feaca1
```

```
template<class F>
Frac fracBS(F f, 1l N) {
    bool dir = 1, A = 1, B = 1;
    Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
    if (f(lo)) return lo; //7f70d6
    assert(f(hi));
    while (A || B) {
        1l adv = 0, step = 1; // move hi if dir, else lo
        for (int si = 0; step; (step *= 2) >= si) {
            adv += step; //3067db
            Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q}; //306933
            if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
                adv -= step; si = 2;
            } //a40ec9
        } //d35347
        hi.p += lo.p * adv;
        hi.q += lo.q * adv;
        dir = !dir;
        swap(lo, hi);
        A = B; B = !adv; //fc82fe
    } //2c9a8f
    return dir ? hi : lo;
} //27ab3e
```

7.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0, k > 0, m \perp n$, and either m or n even.

7.6 Estimates

$$\sum_{d|n} d = \mathcal{O}(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

7.7 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Computation

```

mu[1] = 1;
rep(i,1,sz(mu)) for(int j =2*i; j<sz(mu); j+=i)
    mu[j]-=mu[i];

```

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g(n/d)$$

Other useful formulas/forms:

$$\sum_{d|n} \mu(d) = [n = 1] \text{ (very useful)}$$

$$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n)g(d)$$

$$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m)g(\lfloor \frac{n}{m} \rfloor)$$

Combinatorial (8)

8.1 Permutations

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.)
 Integer -> permutation can use a lookup table.
Time: $\mathcal{O}(n)$

```

int permToInt(vi& v) {
    int use = 0, i = 0, r = 0;
    for(int x:v) r = r * ++i + __builtin_popcount(use & ~(1<<x)),
                use |= 1 << x;          // (note: minus, not ~!)
    return r;
} //044568

```

044568, 6 lines

8.1.1 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^\infty g_S(n) \frac{x^n}{n!} = \exp \left(\sum_{n \in S} \frac{x^n}{n} \right)$$

8.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

IntPerm multinomial

8.1.3 Burnside’s lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts “configurations” (of some sort) of length n , we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

8.2 Partitions and subsets

8.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2\text{e}5$	$\sim 2\text{e}8$

8.2.2 Lucas’ Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \dots + n_1 p + n_0$ and $m = m_k p^k + \dots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod p$.

8.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \dots k_n!}$.

```

ll multinomial(vi& v) {
    ll c = 1, m = v.empty() ? 1 : v[0];
    rep(i,1,sz(v)) rep(j,0,v[i])
        c = c * ++m / (j+1);
    return c;
} //a0a312

```

a0a312, 6 lines

8.3 General purpose numbers

8.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
 $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^\infty f(i) = \int_m^\infty f(x)dx - \sum_{k=1}^\infty \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$
8.3.2 Stirling numbers of the first kind
Number of permutations on n items with k cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k) x^k = x(x+1) \dots (x+n-1)$$

$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
 $c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$
8.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j :s s.t. $\pi(j) \geq j$, k j :s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

8.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

8.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

8.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i : $n_1 n_2 \dots n_k n^{k-2}$
with degrees d_i : $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

8.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$
$$C_0 = 1, \; C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \; C_{n+1} = \sum C_i C_{n-i}$$
$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

Strings (9)

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

Time: $\mathcal{O}(n)$

d4375c, 16 lines

```
vi pi(const string& s) {
    vi p(sz(s));
    rep(i,1,sz(s)) {
        int g = p[i-1];
        while (g && s[i] != s[g]) g = p[g-1];
        p[i] = g + (s[i] == s[g]); //21a657
    } //6c1f11
    return p;
} //9cb7fc
```

```
vi match(const string& s, const string& pat) {
    vi p = pi(pat + '\0' + s), res;
    rep(i,sz(p)-sz(s),sz(p))
        if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat));
    return res; //c66a2a
} //d4375c
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

Time: $\mathcal{O}(n)$

3ae526, 12 lines

```
vi Z(string S) {
    vi z(sz(S));
    int l = -1, r = -1;
    rep(i,1,sz(S)) {
        z[i] = i >= r ? 0 : min(r - i, z[i - l]);
        while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]]) //fe9318
            z[i]++;
        if (i + z[i] > r)
            l = i, r = i + z[i];
    } //1fcbd4
    return z;
} //3ae526
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).

Time: $\mathcal{O}(N)$

e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi,2> p = {vi(n+1), vi(n)}; //daf4bc
    rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
        int t = r-i!+z;
        if (i<r) p[z][i] = min(t, p[z][l+t]);
        int L = i-p[z][i], R = i+p[z][i]-!z;
```

```
        while (L>=1 && R+1<n && s[L-1] == s[R+1]) //508df3
            p[z][i]++, L--, R++;
        if (R>r) l=L, r=R;
    } //21a1fb
    return p;
} //e7ad79
```

Eertree.h

Description: Generates an eertree on str. cur is accurate at the end of the main loop before the final assignment to t.

Time: $\mathcal{O}(|S|)$

288121, 35 lines

```
struct eertree{
    static constexpr int ALPHA = 26;
    struct node{ //sInd is starting index of an occurrence
        array<int,ALPHA> down;
        int slink, ln, sInd, freq = 0;
        node(int slink, int ln, int eInd = -1): //5dff69
            slink(slink), ln(ln), sInd(eInd-ln+1) {
                fill(begin(down),begin(down)+ALPHA,-1);
            } //6a8cb3
    }; //aa06f7
    vector<node> t = {node(0,-1),node(0,0)}; //b4be49
    eertree(string &s){
        int cur = 0, k = 0;
        for(int i = 0; i < sz(s); i++){
            char c = s[i]; int cID = c-'a'; //first chracter
            while(k<=0 || s[k-1] != c) //e85b7f
                k = i - t[cur = t[cur].slink].ln;
            #define TCD t[cur].down[cID]
            if(TCD == -1){
                TCD = sz(t);
                t.emplace_back(-1,t[cur].ln+2,i); //8f1444
                if(t.back().ln > 1){
                    do k = i - t[cur = t[cur].slink].ln;
                    while(k<=0 || s[k-1] != c);
                    t[sz(t)-1].slink = TCD;
                } else t[sz(t)-1].slink = 1; //519576
                cur = sz(t)-1;
            } else cur = TCD;
            t[cur].freq++;
            k = i - t[cur].ln+1;
        } //f67fbd
        for(int i = sz(t)-1; i > 1; i--) //update frequencies
            t[t[i].slink].freq += t[i].freq;
    } //6acbdda
}; //288121
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: rotate(v.begin(), v.begin()+minRotation(v), v.end());

Time: $\mathcal{O}(N)$

d07a42, 8 lines

```
int minRotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b,0,N) rep(k,0,N) {
        if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
        if (s[a+k] > s[b+k]) { a = b; break; } //20f912
    } //b2e25e
    return a;
} //d07a42
```

SuffixArray.h

Description: Builds suffix array for a string. sa[i] is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size n + 1, and sa[0] = n. The lcp array contains longest common prefixes for neighbouring strings in the suffix array: lcp[i] = lcp(sa[i], sa[i-1]), lcp[0] = 0. The input string must not contain any zero bytes.

Time: $\mathcal{O}(n \log n)$

38db9f, 23 lines

```
struct SuffixArray {
    vi sa, lcp;
    SuffixArray(string& s, int lim=256) { // or basic_string<int>
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
        sa = lcp = y, iota(all(sa), 0); //74da6a
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
            p = j, iota(all(y), n - j);
            rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i,0,n) ws[x[i]]++; //499169
            rep(i,1,lim) ws[i] += ws[i - 1];
            for (int i = n; i--; ) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
        } //f30252
        rep(i,1,n) rank[sa[i]] = i;
        for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
            for (k && k--, j = sa[rank[i] - 1];
                s[i + k] == s[j + k]; k++);
    } //22a139
}; //38db9f
```

SuffixArrayQuery.h

Description: Various helper queries for suffix array problems inputs are 0 based input/output is inc-ex

Time: lenlcp: $\mathcal{O}(1)$, cmpsub: $\mathcal{O}(1)$, findstr: $\mathcal{O}(\log(n))$

92e674, 27 lines

```
struct SAQuery{
    SuffixArray sa; RMQ<int> lcp;
    string s; vector<int> sainv;
    SAQuery(SuffixArray sa,string s):sa(sa), lcp(sa.lcp), s(s){
        sainv.resize(sz(s)+1);
        rep(i,0,sz(sa.sa)) sainv[sa.sa[i]] = i; //700e88
    } //859793
    int len_lcp(int u, int v){
        if(u==v) return sz(s)-u;
        auto [l,r] = minmax(sainv[u],sainv[v]);
        return lcp.query(l+1,r+1);
    } //given [] substr of s: s1<s2->-1,s1==s2->0,s1>s2->1
    int cmp_sub(int l1, int r1, int l2, int r2){ //787424
        auto sgn = [](int x){ return (x>0)-(x<0); }; //7a8522
        int len1 = r1-l1+1, len2 = r2-l2+1;
        return len_lcp(l1,l2) < min(len1,len2)
            ? sgn(sainv[l1]-sainv[l2]): sgn(len1-len2);
    } //73a96e
    pair<int, int> find_str(int s_l, int s_r) {
        auto cmp = [&](int i, bool flip) -> bool {
            return flip ^ (len_lcp(i, s_l) < s_r - s_l); }; //67dcf3
        auto it = begin(sa.sa) + sainv[s_l];
        int l=lower_bound(begin(sa.sa),it,0,cmp)-begin(sa.sa);
        int r=lower_bound(it+1,end(sa.sa),1,cmp)-begin(sa.sa);
        return {l, r}; //## -> r-l
    } //102c76
}; //92e674
```

findSubstr.h

Description: returns inc-exclusive range of occurences of needle string in-side suffix array, assumes global sa structure and global s (haystack)

Time: $\mathcal{O}(|ndl| \log(|s|))$

7d1493, 9 lines

```
pair<int,int> find_str(const string &ndl){
    auto le = lower_bound(begin(sa.sa)+1,end(sa.sa), 0,
        [&](int i, int) -> bool { return lexicographical_compare
            (begin(s) + i, end(s), all(ndl)); });
    auto ri = lower_bound(le,end(sa.sa), 0,
        [&](int i, int) -> bool { return mismatch(begin(s) + i,
            end(s), all(ndl)).second == end(ndl);}); //022ea5
```

```
    return {le-begin(sa.sa),ri-begin(sa.sa)}; //[]
} //7d1493
```

SuffixAutomaton.h

Description: Creates a partial DFA (DAG) that accepts all suffixes, with suffix links. One-to-one map between a path from the root and a substring. len is the longest-length substring ending here. pos is the first index in the string matching here. term is whether this node is a terminal (aka a suffix)
Time: construction takes $\mathcal{O}(N \log K)$, where K = Alphabet Size

```
struct st { int len, pos, term; st *link; map<char, st*> next;
};
st *suffixAutomaton(string &str) {
    st *last = new st(), *root = last;
    for(auto c : str) {
        st *p = last, *cur = last = new st{last->len + 1, last->len
        };
        while(p && !p->next.count(c)) //4cd1a8
            p->next[c] = cur, p = p->link;
        if (!p) cur->link = root;
        else {
            st *q = p->next[c];
            if (p->len + 1 == q->len) cur->link = q; //1cc2d6
            else {
                st *clone = new st{p->len+1, q->pos, 0, q->link, q->
                    next};
                for (; p && p->next[c] == q; p = p->link)
                    p->next[c] = clone;
                q->link = cur->link = clone; //08d876
            } //b49887
        } //31bf7e
    } //76ccab
    while(last) last->term = 1, last = last->link;
    return root;
} //1914a9
```

Hashing.h

Description: Self-explanatory methods for string hashing.

```
typedef uint64_t ull;
struct H {
    ull x; H(ull x=0) : x(x) {} //80cf70
    H operator+(H o) { return x + o.x + (x + o.x < x); } //1f9d48
    H operator-(H o) { return *this + ~o.x; } //98ccfa
    H operator*(H o) { auto m = (__uint128_t)x * o.x;
        return H((ull)m) + (ull)(m >> 64); } //4eff44
    ull get() const { return x + !~x; } //f17b1d
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); }
}; //40d284
static const H C = (1l)1e1l+3; // (order ~ 3e9; random also ok)
```

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}

HashInterval.h

Description: Various self-explanatory methods for string hashing.

```
"Hashing.h"
struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
        pw[0] = 1;
        rep(i,0,sz(str))
            ha[i+1] = ha[i] * C + str[i], //c3c119
            pw[i+1] = pw[i] * C;
    } //67307e
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    } //39481a
}; //122649
```

LyndonFactorization.h

Description: A string is called simple (or a Lyndon word), if it is strictly smaller than any of its own nontrivial suffixes. Examples of simple strings are: a , b , ab , aab , abb , $ababb$, $abcd$. It can be shown that a string is simple, if and only if it is strictly smaller than all its nontrivial cyclic shifts. Next, let there be a given string s . The Lyndon factorization of the string s is a factorization $s = w_1w_2 \dots w_k$, where all strings w_i are simple, and they are in non-increasing order $w_1 \geq w_2 \geq \dots \geq w_k$. It can be shown, that for any string such a factorization exists and that it is unique.

Time: $\mathcal{O}(N)$

```
vector<string> duval(string const& s) {
    int n = s.size();
    int i = 0;
    vector<string> factorization;
    while (i < n) {
        int j = i + 1, k = i; //d0372e
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                k++; //8d1eaa
            j++;
        } //cf42b4
        while (i <= k) {
            factorization.push_back(s.substr(i, j - k));
            i += j - k;
        } //46a6db
    } //14171a
    return factorization;
} //0e6ce6
```

Wildcard.h

Description: string matching with wildcards, returns boolean vector of size s-p+1 representing if a match occurs at this start position, wild cards are repented by 0 and can be in s,p or both.

Time: $\mathcal{O}((n+m)\log(n+m))$

```
vector<vl> make_powers(const vl& v) {
    int n = sz(v);
    vector<vl> pws(3, vl(n)); pws[0] = v;
    rep(k,1,3) rep(i,0,n) //mod?
        pws[k][i] = pws[k-1][i]*v[i];
    return pws; //a00fe1
} //10e306
```

```
vector<bool> wildcard_pattern_matching(const vl& s,
const vl& p) {
    int n = sz(s), m = sz(p);
    auto s_pws = make_powers(s), p_pws = make_powers(p);
    for (auto& p_pw : p_pws) reverse(all(p_pw)); //cd7088
    vector<vl> res(3);
    rep(pw_hay,0,3) //ntt
        res[pw_hay] = conv(s_pws[pw_hay], p_pws[2 - pw_hay]);
    vector<bool> mtch(n - m + 1);
    rep(i,0,sz(mtch)){ //890a02
        int id = i + m - 1;
        auto num = res[0][id] - 2 * res[1][id] + res[2][id];
        mtch[i] = !num; //num == 0
    } //934360
    return mtch;
} //b0e86b
```

AhoCorasick-Tyler.h

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.

Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

```
const int ABSIZE = 26;

struct node {
    int nxt[ABSIZE];
    vi ids = {}; //d04adb
    int prv = -1, link = -1;
    char c;
    int linkMemo[ABSIZE];

    node(int prv = -1, char c = '$'): prv(prv), c(c) { //ec9f1e
        fill(all(nxt), -1);
        fill(all(linkMemo), -1);
    } //16055b
}; //432cad
```

vector<node> trie(1);

```
void addWord(string &s, int id) {
    int cur = 0; //aa1bc0
    for(char c: s) {
        int idx = c - 'a';
        if(trie[cur].nxt[idx] == -1) {
            trie[cur].nxt[idx] = sz(trie);
            trie.emplace_back(cur, c); //23b9d2
        } //ba2978
        cur = trie[cur].nxt[idx];
    } //35f152
    trie[cur].ids.push_back(id);
} //1dfc37
```

int getLink(int cur);

```
int calc(int cur, char c) {
    int idx = c - 'a'; //e9a88a
    auto &ret = trie[cur].linkMemo[idx];
    if(ret != -1) return ret;
    if(trie[cur].nxt[idx] != -1)
        return ret = trie[cur].nxt[idx];
    return ret = cur == 0 ? 0 : calc(getLink(cur), c); //1a4276
} //c61f02
```

```
int getLink(int cur) {
    auto &ret = trie[cur].link;
    if(ret != -1) return ret;
    if(cur == 0 || trie[cur].prv == 0) return ret = 0; //be881f
    return ret = calc(getLink(trie[cur].prv), trie[cur].c);
} //647ca9
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).

Time: $\mathcal{O}(\log N)$

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it); //a98b04
    } //381108
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    } //5783d8
    return is.insert(before, {L,R});
} //d57d47
```

```
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second; //51cff5
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
} //edce47
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty).

Time: $\mathcal{O}(N \log N)$

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
    vi S(sz(I)), R;
    iota(all(S), 0);
    sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
    T cur = G.first; //a166e4
    int at = 0;
    while (cur < G.second) { // (A)
        pair<T, int> mx = make_pair(cur, -1);
        while (at < sz(I) && I[S[at]].first <= cur) {
            mx = max(mx, make_pair(I[S[at]].second, S[at])); //201b40
            at++;
        } //470978
        if (mx.second == -1) return {}; //f1e40b
        cur = mx.first;
        R.push_back(mx.second);
    } //cd0c49
    return R;
} //9e9d8d
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.

Usage: constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});

Time: $\mathcal{O}(k \log \frac{n}{k})$

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
    if (p == q) return;
    if (from == to) {
        g(i, to, p);
        i = to; p = q; //a2e0d8
    } else {
        int mid = (from + to) >> 1;
        rec(from, mid, f, g, i, p, f(mid));
        rec(mid+1, to, f, g, i, p, q);
    }
}
```

```
    } //5b694f
    } //69b73b
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
    if (to <= from) return;
    int i = from; auto p = f(i), q = f(to-1);
    rec(from, to-1, f, g, i, p, q); //587254
    g(i, to, q);
} //753a4c
```

10.2 Misc. algorithms

LIS.h

Description: Compute indices for the longest increasing subsequence.

Time: $\mathcal{O}(N \log N)$

```
template<class I> vi lis(const vector<I>& S) {
    if (S.empty()) return {}; //be1376
    vi prev(sz(S));
    typedef pair<I, int> p;
    vector<p> res;
    rep(i,0,sz(S)) {
        // change 0 -> i for longest non-decreasing subsequence
        auto it = lower_bound(all(res), p{S[i], 0}); //f6ef94
        if (it == res.end()) res.emplace_back(), it = res.end()-1;
        *it = {S[i], i}; //26a0a3
        prev[i] = it == res.begin() ? 0 : (it-1)->second;
    } //f2ee22
    int L = sz(res), cur = res.back().second;
    vi ans(L);
    while (L--) ans[L] = cur, cur = prev[cur];
    return ans;
} //2932a0
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.

Time: $\mathcal{O}(N \max(w_i))$

```
int knapsack(vi w, int t) {
    int a = 0, b = 0, x;
    while (b < sz(w) && a + w[b] <= t) a += w[b++];
    if (b == sz(w)) return a;
    int m = *max_element(all(w));
    vi u, v(2*m, -1); //11fd10
    v[a+m-t] = b;
    rep(i,b,sz(w)) {
        u = v;
        rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
        for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x]) //51a6b1
            v[x-w[j]] = max(v[x-w[j]], j);
    } //d2bd39
    for (a = t; v[a+m-t] < 0; a--);
    return a;
} //b20ccc
```

10.3 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i < k < j} (a[i][k] + a[k][j]) + f(i, j)$, where the (minimal) optimal k increases with both i and j , one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j - 1]$ and $p[i + 1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b, c) \leq f(a, d)$ and $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

Time: $\mathcal{O}(N^2)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \leq k < hi(i)} (f(i, k))$ where the (minimal) optimal k increases with i , computes $a[i]$ for $i = L..R - 1$.

Time: $\mathcal{O}((N + (hi - lo)) \log N)$

```
struct DP { // Modify at will:
    int lo(int ind) { return 0; } //ce585d
    int hi(int ind) { return ind; } //f742b2
    ll f(int ind, int k) { return dp[ind][k]; } //29ea0c
    void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

    void rec(int L, int R, int LO, int HI) {
        if (L >= R) return;
        int mid = (L + R) >> 1; //13ddb0
        pair<ll, int> best(LLONG_MAX, LO);
        rep(k, max(LO, lo(mid)), min(HI, hi(mid)))
            best = min(best, make_pair(f(mid, k), k));
        store(mid, best.second, best.first);
        rec(L, mid, LO, best.second+1); //4993b6
        rec(mid+1, R, best.second, HI);
    } //116ea5
    void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
}; //d38d2b
```

10.4 Optimization tricks

__builtin_ia32_ldmxcsr(40896); disables denormals (which make floats 20x slower near their minimum value).

10.4.1 Bit hacks

- $x \ \& \ -x$ is the least bit in x .
- for (int x = m; x;) { --x &= m; ... } loops over all subset masks of m (except m itself).
- $c = x \& -x, \ r = x + c; ((r \wedge x) >> 2) / c \mid r$ is the next number after x with the same number of bits set.
- rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)]; computes all sums of subsets.

10.4.2 Pragmas

- #pragma GCC optimize ("Ofast") will make GCC auto-vectorize loops and optimizes floating points better.
- #pragma GCC target ("avx2") can double performance of vectorized code, but causes crashes on old machines.
- #pragma GCC optimize ("trapv") kills the program on integer overflows (but is really slow).

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.

Usage: ./a.out < input.txt

Time: About 5x as fast as cin/scanf.

```
inline char gc() { // like getchar()
    static char buf[1 << 16];
    static size_t bc, be;
    if (bc >= be) {
        buf[0] = 0, bc = 0;
        be = fread(buf, 1, sizeof(buf), stdin); //bba013
    } //e9a035
```



```
    return buf[bc++]; // returns 0 on EOF
} //0261eb

int readInt() {
    int a, c;
    while ((a = gc()) < 40);
    if (a == '-') return -readInt(); //bc51ee
    while ((c = gc()) >= 48) a = a * 10 + c - 480;
    return a - 48;
} //7b3c70
```