

University of Central Florida

# UCF Triangulate

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Contest (1)

```
.bashrc
3 lines
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++20 \
-fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps =◇
```

```
hash.sh
6 lines
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
# Usage:
#   To make executable, run the command: chmod +x hash.sh
#   To execute: ./hash.sh < file.cpp
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum |cut -c-6
```

Mathematics (2)

2.1 Recurrences

If  $a_n = c_1a_{n-1} + \dots + c_ka_{n-k}$ , and  $r_1, \dots, r_k$  are distinct roots of  $x^k - c_1x^{k-1} - \dots - c_k$ , there are  $d_1, \dots, d_k$  s.t.

$$a_n = d_1r_1^n + \dots + d_kr_k^n.$$

Non-distinct roots  $r$  become polynomial factors, e.g.  $a_n = (d_1n + d_2)r^n$ .

2.2 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where  $V, W$  are lengths of sides opposite angles  $v, w$ .

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where  $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$ .

2.3 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx} \tan x = 1 + \tan^2 x \qquad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$
$$\int \tan ax = -\frac{\ln |\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) \qquad \int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.4 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n + 1)}{2}$$
$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n + 1)(n + 1)}{6}$$
$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n + 1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30}$$

2.5 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$
$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

2.5.1 Discrete distributions  
Binomial distribution

The number of successes in  $n$  independent yes/no experiments, each which yields success with probability  $p$  is  $\operatorname{Bin}(n, p)$ ,  $n = 1, 2, \dots$ ,  $0 \leq p \leq 1$ .

$$p(k) = \binom{n}{k} p^k (1 - p)^{n - k}$$
$$\mu = np, \sigma^2 = np(1 - p)$$

$\operatorname{Bin}(n, p)$  is approximately  $\operatorname{Po}(np)$  for small  $p$ .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability  $p$  is  $\operatorname{Fs}(p)$ ,  $0 \leq p \leq 1$ .

$$p(k) = p(1 - p)^{k - 1}, k = 1, 2, \dots$$
$$\mu = \frac{1}{p}, \sigma^2 = \frac{1 - p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time  $t$  if these events occur with a known average rate  $\kappa$  and independently of the time since the last event is  $\operatorname{Po}(\lambda)$ ,  $\lambda = t\kappa$ .

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$
$$\mu = \lambda, \sigma^2 = \lambda$$

2.5.2 Continuous distributions  
Uniform distribution

If the probability density function is constant between  $a$  and  $b$  and 0 elsewhere it is  $\operatorname{U}(a, b)$ ,  $a < b$ .

$$f(x) = \begin{cases} \frac{1}{b - a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$
$$\mu = \frac{a + b}{2}, \sigma^2 = \frac{(b - a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is  $\text{Exp}(\lambda)$ ,  $\lambda > 0$ .

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$
$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean  $\mu$  and variance  $\sigma^2$  are well described by  $\mathcal{N}(\mu, \sigma^2)$ ,  $\sigma > 0$ .

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If  $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$  then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.6 Geometry

2.6.1 Triangles

Side lengths:  $a, b, c$

Semiperimeter:  $p = \frac{a + b + c}{2}$

Area:  $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius:  $R = \frac{abc}{4A}$

Inradius:  $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles):

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[ 1 - \left( \frac{a}{b+c} \right)^2 \right]}$$

Law of sines:  $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines:  $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents:  $\frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$

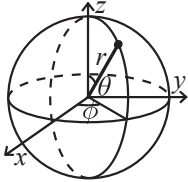
2.6.2 Quadrilaterals

With side lengths  $a, b, c, d$ , diagonals  $e, f$ , diagonals angle  $\theta$ , area  $A$  and magic flux  $F = b^2 + d^2 - a^2 - c^2$ :

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is  $180^\circ$ ,  $ef = ac + bd$ , and  $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ .

2.6.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z / \sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

Data structures (3)

OrderStatisticTree.h  
**Description:** A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null\_type.  
**Time:**  $\mathcal{O}(\log N)$

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
```

```
template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
tree_order_statistics_node_update>; //cd2981
```

```
void example() {
    Tree<int> t, t2; t.insert(8);
    auto it = t.insert(10).first;
    assert(it == t.lower_bound(9)); //b1d86a
    assert(t.order_of_key(10) == 1);
    assert(t.order_of_key(11) == 2);
    assert(*t.find_by_order(0) == 8);
    t.join(t2); // assuming T< T2 or T> T2, merge t2 into t
} //782797
```

HashMap.h  
**Description:** Hash map with mostly the same API as unordered\_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
    const uint64_t C = 11(4e18 * acos(0)) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(x*C); }
}; //9b48b4
__gnu_pbds::gp_hash_table<ll,int,chash> h({}, {}, {}, {}, {}, {1<=16});
```

SegmentTree.h  
**Description:** Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.  
**Time:**  $\mathcal{O}(\log N)$

```
struct Tree {
    typedef int T;
    static constexpr T unit = INT_MIN;
    T f(T a, T b) { return max(a, b); } // (any associative fn)
    vector<T> s; int n;
    Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {} //2b8055
    void update(int pos, T val) {
        for (s[pos += n] = val; pos /= 2;)
            s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
    }
};
```

```
} //17a935
T query(int b, int e) { // query [b, e)
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
        if (b % 2) ra = f(ra, s[b++]);
        if (e % 2) rb = f(s[--e], rb); //d4678d
    } //3391a8
    return f(ra, rb);
} //c90093
}; //0f4bdb
```

LazySegmentTree.h  
**Description:** Segment tree with lazy prop, modify at will. 0-based, inclusive-exclusive.  
**Usage:** lazy\_segtree<ValType, LazyType> st(arr)  
**Time:**  $\mathcal{O}(\log N)$ .

```
template<class T, class F>
struct lazy_segtree {
    int N, log, S;
    T idem = //modify: op(val, idem) = val
    F defLazy = //modify: applyLazy/combLazy(val, defLazy) = val
    vector<T> d; //ce445d
    vector<F> lz;
    T op(T left, T right) { //modify*/ } //09e848
    T applyLazy(T val, F lazy) { //modify*/ } //92c854
    F combLazy(F old, F nw) { //modify*/ } //4c16ff
    lazy_segtree(const vector<T>& v):
        N(sz(v)), log(__lg(2 * N - 1)), S(1 << log), d(2 * S, idem),
        lz(S, defLazy) {
        for (int i = 0; i < N; i++) d[S + i] = v[i];
        for (int i = S - 1; i >= 1; i--) pull(i); //5a138c
    } //9e2ae3
    void apply(int k, F f) {
        d[k] = applyLazy(d[k], f); //len= S>>(31--builtin.clz(k));
        if (k < S) lz[k] = combLazy(lz[k], f);
    } //c7a3ac
    void push(int k) {
        apply(2 * k, lz[k]), apply(2 * k + 1, lz[k]), lz[k] =
            defLazy;
    } //f332a2
    void push(int l, int r) {
        int zl = __builtin_ctz(l), zr = __builtin_ctz(r);
        for (int i = log; i > min(zl, zr); i--) {
            if (i > zl) push(1 >> i);
            if (i > zr) push((r - 1) >> i); //40c9d4
        } //4613f2
    } //708f77
    void pull(int k) { d[k] = op(d[2 * k], d[2 * k + 1]); }
    void set(int p, T x) {
        p += S;
        for (int i = log; i >= 1; i--) push(p >> i);
        for (d[p] = x; p /= 2;) pull(p); //e740d0
    } //f65093
    T query(int l, int r) {
        if (l == r) return idem;
        push(1 += S, r += S);
        T vl = idem, vr = idem;
        for (; l < r; l /= 2, r /= 2) { //f70eb7
            if (l & 1) vl = op(vl, d[l++]);
            if (r & 1) vr = op(d[--r], vr);
        } //09a718
        return op(vl, vr);
    } //7197c1
    void update(int l, int r, F f) {
        if (l == r) return;
        push(1 += S, r += S);
        for (int a = l, b = r; a < b; a /= 2, b /= 2) {
```

```
        if (a & 1) apply(a++, f); //9b5541
        if (b & 1) apply(--b, f);
    } //f428a2
    int zl = __builtin_ctz(l), zr = __builtin_ctz(r);
    for (int i = min(zl, zr) + 1; i <= log; i++) {
        if (i > zl) pull(1 >> i);
        if (i > zr) pull((r - 1) >> i);
    } //592fbe
    } //b5d617
}; //4f67ee
```

Wavelet.h

**Description:** kth: finds k+1th smallest number in [l,r], count: rank of k (how many < k) in [l,r]. Doesn't support negative numbers, and requires a[i] <= maxval. Use BitVector to make 1.6x faster and 4x less memory.  
**Time:**  $\mathcal{O}(\log MAX)$

<pre>struct WaveletTree {     int n; vvi bv; // vector&lt;BitVector&gt; bv;     WaveletTree(vl a, ll max_val):         n(sz(a)), bv(1+__lg(max_val), {{{}}}) {         vl nxt(n);         for (int h = sz(bv); h--;) { //2d1680             vector&lt;bool&gt; b(n);             rep(i, 0, n) b[i] = ((a[i] &gt;&gt; h) &amp; 1);             bv[h] = vi(n+1); // bv[h] = b;             rep(i, 0, n) bv[h][i+1] = bv[h][i] + !b[i]; // delete             array it{begin(nxt), begin(nxt) + bv[h][n]}; //0c84d2             rep(i, 0, n) *it[b[i]]++ = a[i];             swap(a, nxt);         } //f93ef6     } //54c891     ll kth(int l, int r, int k) {         ll res = 0;         for (int h = sz(bv); h--;) {             int l0 = bv[h][l], r0 = bv[h][r];             if (k &lt; r0 - l0) l = l0, r = r0; //e4af0f             else                 k -= r0 - l0, res  = 1ULL &lt;&lt; h,                 l += bv[h][n] - l0, r += bv[h][n] - r0;         } //aa8465         return res;     } //67fa6f     int count(int l, int r, ll ub) {         int res = 0;         for (int h = sz(bv); h--;) {             int l0 = bv[h][l], r0 = bv[h][r];             if ((~ub &gt;&gt; h) &amp; 1) l = l0, r = r0; //09ef1a             else                 res += r0 - l0, l += bv[h][n] - l0,                 r += bv[h][n] - r0;         } //8380c1         return res;     } //d305cc }; //11aee1</pre>	11aee1, 38 lines
---	------------------

BitVector.h

**Description:** Given vector of bits, counts number of 0's in [0, r). Use with WaveletTree.h by using modifications in comments in that file and replacing bv[h][x] with bv[h].cnt0(x)  
**Time:**  $\mathcal{O}(1)$  time

<pre>struct BitVector {     vector&lt;pair&lt;ll, int&gt;&gt; b;     BitVector(vector&lt;bool&gt; a): b(sz(a) / 64 + 1) {         rep(i, 0, sz(a))             b[i &gt;&gt; 6].first  = 1ll(a[i]) &lt;&lt; (i &amp; 63);         rep(i, 0, sz(b)-1) //cba6aa             b[i + 1].second = __builtin_popcountll(b[i].first)</pre>	afd9d2, 15 lines
---	------------------

```
        + b[i].second;
    } //4da2bc
    int cnt0(int r) {
        auto [x, y] = b[r >> 6];
        return r - y
            - __builtin_popcountll(x & ((1ULL << (r & 63)) - 1));
    } //01da37
}; //afd9d2
```

PST.h

**Description:** Persistent segment tree with laziness

**Time:**  $\mathcal{O}(\log N)$  per query,  $\mathcal{O}((n + q) \log n)$  memory

<pre>struct PST {     PST *l = 0, *r = 0;     int lo, hi;     ll val = 0, lzadd = 0;     PST(vl&amp; v, int lo, int hi) : lo(lo), hi(hi) {         if (lo + 1 &lt; hi) { //e43119             int mid = lo + (hi - lo)/2;             l = new PST(v, lo, mid); r = new PST(v, mid, hi);             val = l-&gt;val + r-&gt;val;         } //ebf78b         else val = v[lo];     } //7ff852     ll query(int L, int R) {         if (R &lt;= lo    hi &lt;= L) return 0; // idempotent         if (L &lt;= lo &amp;&amp; hi &lt;= R) return val;         push();         return l-&gt;query(L, R) + r-&gt;query(L, R); //6a44fe     } //108984     PST* add(int L, int R, ll v) {         if (R &lt;= lo    hi &lt;= L) return this;         PST *n;         if (L &lt;= lo &amp;&amp; hi &lt;= R) {             n = new PST(*this); //70575f             n-&gt;val += v;             n-&gt;lzadd += v;         } else {             push();             n = new PST(*this); //c68682             n-&gt;l = l-&gt;add(L, R, v);             n-&gt;r = r-&gt;add(L, R, v);             n-&gt;val = n-&gt;l-&gt;val + n-&gt;r-&gt;val;         } //d1bfc5         return n;     } //d6d267     void push() {         if (lzadd == 0) return;         l = l-&gt;add(lo, hi, lzadd);         r = r-&gt;add(lo, hi, lzadd);         lzadd = 0; //d7e73b     } //0af5c4 }; //7ddad1</pre>	7ddad1, 41 lines
--	------------------

Xorbasis.h

**Description:** Makes a basis of binary vectors

**Time:** check/add ->  $\mathcal{O}((B^2)/32)$

<pre>template&lt;int B&gt; struct XORBasis {     bitset&lt;B&gt; b[B];     int npivot = 0, nfree = 0;     bool check(bitset&lt;B&gt; v) {         for(int i = B-1; i &gt;= 0; i--) //563a45             if (v[i]) v ^= b[i];         return v == 0;     } //4915f9     bool add(bitset&lt;B&gt; v) {</pre>	1d856b, 19 lines
--	------------------

```
    for(int i = B-1; i >= 0; i--) {
        if (v[i]) {
            if (b[i] == 0) return b[i] = v, ++npivot;
            v ^= b[i]; //7a144f
        } //b1b631
    } //8da7e8
    return !++nfree;
} //fbb3dd
}; //1d856b
```

UnionFindRollback.h

**Description:** Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback(). and add path compression at the commented line

**Usage:** int t = uf.time(); ...; uf.rollback(t);

**Time:**  $\mathcal{O}(\log(N))$

<pre>struct RollbackUF {     vi e; vector&lt;pii&gt; st;     RollbackUF(int n) : e(n, -1) {} //66f6eb     int size(int x) { return -e[find(x)]; } //if no rollback     int find(int x) { return e[x] &lt; 0 ? x : *e[x] ==*/find(e[x]);     }     int time() { return sz(st); } //821d77     void rollback(int t) {         for (int i = time(); i --&gt; t;)             e[st[i].first] = st[i].second;         st.resize(t);     } //e7fe82     bool join(int a, int b) {         a = find(a), b = find(b);         if (a == b) return false;         if (e[a] &gt; e[b]) swap(a, b);         st.push_back({a, e[a]}); //3aaa7c         st.push_back({b, e[b]});         e[a] += e[b]; e[b] = a;         return true;     } //f0724e }; //de4ad0</pre>	de4ad0, 21 lines
--	------------------

MonoRange.h

**Description:** when cmp = less(): a[le[i]] < a[i] >= a[ri[i]]

**Usage:** vi le = mono.st(a, less()),

ri = mono.range(le);

less.equal(), greater(), greater.equal()

**Time:**  $\mathcal{O}(N)$ .

<pre>template&lt;class T, typename F&gt; vi mono_st(const vector&lt;T&gt; &amp; a, F cmp) {     vi le(sz(a));     rep(i, 0, sz(a)){         for (le[i] = i -1; le[i] &gt;= 0 &amp;&amp; !cmp(a[le[i]], a[i]);)             le[i] = le[le[i]]; } //f637ae     return le; } //a87918</pre>	191698, 16 lines
--	------------------

```
vi mono_range(const vi &le) {
    vi ri(sz(le), sz(le));
    rep (i, 0, sz(le))
        for (int j = i - 1; j != le[i]; j = le[j]) //9e9289
            ri[j] = i;
    return ri;
} //191698
```

CountRect.h

**Description:** cnt[i][j] = number of times an i-by-j sub rectangle appears such that all i\*j cells ARE 1. cnt[i][0],cnt[0][j] are garbage

**Time:**  $\mathcal{O}(NM)$

	71b256, 22 lines
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```
vector<vi> count_rectangles(  
    const vector<vector<bool>>&grid) {  
        int n = sz(grid), m = sz(grid[0]);  
        vector<vi> cnt(n + 1, vi(m + 1, 0));  
        vi h(m);  
        for( const auto &row : grid) { //2270a5  
            transform(all(h), begin(row), begin(h),  
                [](int a, bool g) { return g * (a + 1); });  
            vi le ( mono_st(h,less())), r(mono_range(le));  
            rep(j,0,m) {  
                int cnt_l = j - le[j] - 1, cnt_r = r[j] - j - 1;  
                cnt[h[j]][cnt_l + cnt_r + 1]++; //9e604e  
                cnt[h[j]][cnt_l]--;  
                cnt[h[j]][cnt_r]--;  
            } //82de19  
        } //7a1347  
        rep(i,1,n+1) rep(k,0,2) for (int j = m; j > 1; j--)  
            cnt[i][j - 1] += cnt[i][j];  
        for (int i = n; i > 1; i--)  
            rep(j, 1, m + 1) cnt[i - 1][j] += cnt[i][j];  
        return cnt; //eca1f3  
    } //71b256
```

Lichao.h  
**Description:** min Li-chao tree allows for range add of arbitrary functions such that any two functions only occur atmost once.  
**Usage:** inc-inc, implicit, works with negative indices, O(log(n)) query  
flip signs in update and modify query to change to max

```
struct func {  
    ll A,B;  
    func(ll A, ll B): A(A), B(B) {} //4cab61  
    ll operator()(ll x) { return (A*x + B); } //e8aa73  
}; //b3c3bf  
const func idem = {0,(ll)8e18}; //idempotent, change for max  
struct node {  
    int lo, md, hi;  
    func f = idem;  
    node *left = nullptr, *right = nullptr; //12341d  
    node(int lo, int hi): lo(lo), hi(hi), md(lo+(hi-lo)/2) {}  
    void check(){  
        if(left) return;  
        left = new node(lo,md);  
        right = new node(md+1,hi); //b79f4d  
    } //edfaa5  
    void update(func e) { //flip signs for max  
        if(e.md) < f(md)) swap(e, f);  
        if(lo == hi) return;  
        if(e(lo) > f(lo) && e(hi) > f(hi)) return;  
        check(); //cf8828  
        if(e(lo) < f(lo)) left->update(e);  
        else right->update(e);  
    } //fd3fd  
    void rangeUpdate(int L, int R, func e) { ///[]  
        if(R < lo || hi < L) return;  
        if(L <= lo && hi <= R) return update(e);  
        check();  
        left->rangeUpdate(L, R, e); //44440a  
        right->rangeUpdate(L, R,e);  
    } //02b2a9  
    ll query(int x) { //change to max if needed  
        if(lo == hi) return f(x); check();  
        if(x <= md) return min(f(x), left->query(x));  
        return min(f(x), right->query(x));  
    } //66991a  
}; //1eac23
```

Lichao LineContainer Treap PQupdate FenwickTree

LineContainer.h  
**Description:** Container where you can add lines of the form  $kx+m$ , and query maximum values at points  $x$ . Useful for dynamic programming (“convex hull trick”).  
**Time:**  $\mathcal{O}(\log N)$

```
struct Line {  
    mutable ll k, m, p;  
    bool operator<(const Line& o) const { return k < o.k; }  
    bool operator<(ll x) const { return p < x; } //0dcc67  
}; //7e3ecf  
  
struct LineContainer : multiset<Line, less<>> {  
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)  
    static const ll inf = LLONG_MAX;  
    ll div(ll a, ll b) { // floored division  
        return a / b - ((a ^ b) < 0 && a % b); } //fa88a2  
    bool isect(iterator x, iterator y) {  
        if (y == end()) return x->p = inf, 0;  
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;  
        else x->p = div(y->m - x->m, x->k - y->k);  
        return x->p >= y->p; //846095  
    } //31f5a2  
    void add(ll k, ll m) {  
        auto z = insert({k, m, 0}), y = z++, x = y;  
        while (isect(y, z)) z = erase(z);  
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));  
        while ((y = x) != begin() && (--x)->p >= y->p) //6dc2b6  
            isect(x, erase(y));  
    } //4e2c33  
    ll query(ll x) {  
        assert(!empty());  
        auto l = *lower_bound(x);  
        return l.k * x + l.m;  
    } //5a0881  
}; //8ec1c7
```

Treap.h  
**Description:** A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.  
**Time:**  $\mathcal{O}(\log N)$

```
typedef struct Node {  
    Node *l = 0, *r = 0;  
    int val, y, c;  
    Node(int val) : val(val), y(rand()) { pull(); } //28152f  
    void pull(); void push();  
} *np;  
int cnt(np n) { return n ? n->c : 0; } //c83ac4  
void Node::pull() { c = cnt(l) + cnt(r) + 1; } //42405c  
void Node::push() {} //193a22  
np nl, nr;  
pair<np, np> split(np x, int i, np &l = nl, np &r = nr) {  
    if (!x) return {l = r = 0, 0}; //dccdcd  
    x->push(); // VVV "cnt(x->l)" => "x->val" for lower_bound  
    if (i <= cnt(x->l)) split(x->l, i, l, x->l), r = x;  
    else split(x->r, i - cnt(x->l) - 1, x->r, r), l = x;  
    x->pull(); // ^^ and "i - cnt(x->l) - 1" => "i"  
    return {l, r}; //b9fc00  
} //dd2863  
auto merge(np l, np r, np &x = nl) {  
    if (!l || !r) return x = l ? r;  
    l->push(), r->push();  
    if (l->y < r->y) merge(l, r->l, r->l), x = r;  
    else merge(l->r, r, l->r), x = l; //7af065  
    x->pull();  
    return x;  
} //cceed  
template<class F> void each(np n, F f) {
```

4

```
    if (n) { n->push(); each(n->l, f); f(n->val); each(n->r, f);  
    }  
} //069e4f
```

PQUpdate.h  
**Description:** T: value/update type. DS: Stores T. Same semantics as std::priority\_queue.  
**Time:**  $\mathcal{O}(U \log N)$ .

```
template<class T, class DS, class Compare = less<T>>  
struct PQUpdate {  
    DS inner;  
    multimap<T, int, Compare> rev_upd;  
    using iter = decltype(rev_upd)::iterator;  
    vector<iter> st; //23764d  
    PQUpdate(DS inner, Compare comp={}) :  
        inner(inner), rev_upd(comp) {} //3c5f72  
  
    bool empty() { return st.empty(); } //86845b  
    const T& top() { return rev_upd.rbegin()->first; } //0b3b9d  
    void push(T value) {  
        inner.push(value);  
        st.push_back(rev_upd.insert({value, sz(st)}));  
    } //44a05d  
    void pop() {  
        vector<iter> extra;  
        iter curr = rev_upd.end();  
        int min_ind = sz(st);  
        do { //7ec537  
            extra.push_back(--curr);  
            min_ind = min(min_ind, curr->second);  
        } while (2*sz(extra) < sz(st) - min_ind);  
        while (sz(st) > min_ind) {  
            if (rev_upd.value_comp()(*st.back(), *curr)) //c1e316  
                extra.push_back(st.back());  
            inner.pop(); st.pop_back();  
        } //a76ade  
        rev_upd.erase(extra[0]);  
        for (auto it : extra | views::drop(1) | views::reverse) {  
            it->second = sz(st);  
            inner.push(it->first);  
            st.push_back(it); //ff1079a  
        } //8a3607  
    } //f4f582  
}; //35a7d2
```

FenwickTree.h  
**Description:** Computes partial sums  $a[0] + a[1] + \dots + a[\text{pos} - 1]$ , and updates single elements  $a[i]$ , taking the difference between the old and new value.  
**Time:** Both operations are  $\mathcal{O}(\log N)$ .

```
struct FT {  
    vector<ll> s;  
    FT(int n) : s(n) {} //890409  
    void update(int pos, ll dif) { // a[pos] += dif  
        for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;  
    } //a0c54f  
    ll query(int pos) { // sum of values in [0, pos)  
        ll res = 0;  
        for (; pos > 0; pos &= pos - 1) res += s[pos-1];  
        return res;  
    } //585cdd  
    int lower_bound(ll sum) { // min pos st sum of [0, pos] >= sum  
        // Returns n if no sum is >= sum, or -1 if empty sum is.  
        if (sum <= 0) return -1;  
        int pos = 0;  
        for (int pw = 1 << 25; pw; pw >= 1) { //d79a33  
            if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
```

```
        pos += pw, sum -= s[pos-1];
    } //cb4863
    return pos;
} //923db1
}; //e62fac
```

### FenwickTree2d.h

**Description:** Computes sums a[i,j] for all i<L, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).

**Time:**  $\mathcal{O}(\log^2 N)$ . (Use persistent segment trees for  $\mathcal{O}(\log N)$ .)

"FenwickTree.h"	157f07, 22 lines
<pre>struct FT2 {     vector&lt;vi&gt; ys; vector&lt;FT&gt; ft;     FT2(int limx) : ys(limx) {} //5e2398     void fakeUpdate(int x, int y) {         for (; x &lt; sz(ys); x  = x + 1) ys[x].push_back(y);     } //fe99f8     void init() {         for (vi&amp; v : ys) sort(all(v)), ft.emplace_back(sz(v));     } //1e4363     int ind(int x, int y) {         return (int) (lower_bound(all(ys[x]), y) - ys[x].begin());     }     void update(int x, int y, ll dif) {         for (; x &lt; sz(ys); x  = x + 1)             ft[x].update(ind(x, y), dif); //5a5698     } //c48166     ll query(int x, int y) {         ll sum = 0;         for (; x; x &amp;= x - 1)             sum += ft[x-1].query(ind(x-1, y));         return sum; //eb0cfb     } //266f9d }; //157f07</pre>	

### RMQ.h

**Description:** Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.

**Usage:** RMQ rmq(values);

rmq.query(inclusive, exclusive);

**Time:**  $\mathcal{O}(|V|\log|V| + Q)$

template<class T>	510c32, 16 lines
<pre>struct RMQ {     vector&lt;vector&lt;T&gt;&gt; jmp;     RMQ(const vector&lt;T&gt;&amp; V) : jmp(1, V) {         for (int pw = 1, k = 1; pw * 2 &lt;= sz(V); pw *= 2, ++k) {             jmp.emplace_back(sz(V) - pw * 2 + 1); //7420f3             rep(j, 0, sz(jmp[k]))                 jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);         } //398c5e     } //dff89     T query(int a, int b) {         assert(a &lt; b); // or return inf if a == b         int dep = 31 - __builtin_clz(b - a);         return min(jmp[dep][a], jmp[dep][b - (1 &lt;&lt; dep)]);     } //63f839 }; //510c32</pre>	

### MoQueries.h

**Description:** Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge (a, c) and remove the initial add call (but keep in).

**Time:**  $\mathcal{O}(N\sqrt{Q})$

	a12ef4, 49 lines
<pre>void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1) void del(int ind, int end) { ... } // remove a[ind] int calc() { ... } // compute current answer</pre>	

<pre>vi mo(vector&lt;pii&gt; Q) {     int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)     vi s(sz(Q)), res = s; //e77382 #define K(x) pii(x.first/blk, x.second ^ ~(x.first/blk &amp; 1))     iota(all(s), 0);     sort(all(s), [&amp;](int s, int t){ return K(Q[s]) &lt; K(Q[t]); });     for (int qi : s) {         pii q = Q[qi]; //bbab36         while (L &gt; q.first) add(--L, 0);         while (R &lt; q.second) add(R++, 1);         while (L &lt; q.first) del(L++, 0);         while (R &gt; q.second) del(--R, 1);         res[qi] = calc(); //8c7386     } //d497d7     return res; } //7b2870</pre>	
--	--

<pre>vi moTree(vector&lt;array&lt;int, 2&gt;&gt; Q, vector&lt;vi&gt;&amp; ed, int root=0) {     int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)     vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);     add(0, 0), in[0] = 1; //933a80     auto dfs = [&amp;](int x, int p, int dep, auto&amp; f) -&gt; void {         par[x] = p;         L[x] = N;         if (dep) I[x] = N++;         for (int y : ed[x]) if (y != p) f(y, x, !dep, f); //f4faf6         if (!dep) I[x] = N++;         R[x] = N;     }; //7ac875     dfs(root, -1, 0, dfs); #define K(x) pii(I[x[0]] / blk, I[x[1]] ^ ~(I[x[0]] / blk &amp; 1))     iota(all(s), 0);     sort(all(s), [&amp;](int s, int t){ return K(Q[s]) &lt; K(Q[t]); });     for (int qi : s) rep(end, 0, 2) { //6a3e39         int &amp;a = pos[end], b = Q[qi][end], i = 0; #define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \                     else { add(c, end); in[c] = 1; } a = c; }         while (!(L[b] &lt;= L[a] &amp;&amp; R[a] &lt;= R[b]))             I[i++] = b, b = par[b]; //c95d6c         while (a != b) step(par[a]);         while (i--) step(I[i]);         if (end) res[qi] = calc();     } //44b82c     return res; } //a12ef4</pre>	
---	--

## Geometry (4)

### Point.h

**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

	47ec0a, 28 lines
<pre>template &lt;class T&gt; int sgn(T x) { return (x &gt; 0) - (x &lt; 0); } template&lt;class T&gt; struct Point {     typedef Point P;     T x, y;     explicit Point(T x=0, T y=0) : x(x), y(y) {} //551774     bool operator&lt;(P p) const { return tie(x,y) &lt; tie(p.x,p.y); }     bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }     P operator+(P p) const { return P(x+p.x, y+p.y); } //1dc17e     P operator-(P p) const { return P(x-p.x, y-p.y); } //189cbc     P operator*(T d) const { return P(x*d, y*d); } //268af3     P operator/(T d) const { return P(x/d, y/d); } //8cb755     T dot(P p) const { return x*p.x + y*p.y; } //716d84     T cross(P p) const { return x*p.y - y*p.x; } //7ecfd2     T cross(P a, P b) const { return (a-*this).cross(b-*this); }</pre>	

<pre>T dist2() const { return x*x + y*y; } //e7b843 double dist() const { return sqrt((double)dist2()); } // angle to x-axis in interval [-pi, pi] double angle() const { return atan2(y, x); } //cc70a2 P unit() const { return *this/dist(); } // makes dist()==1 P perp() const { return P(-y, x); } // rotates +90 degrees P normal() const { return perp().unit(); } //c0e5d2 // returns point rotated 'a' radians ccw around the origin P rotate(double a) const {     return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); } //e458d5 friend ostream&amp; operator&lt;&lt;(ostream&amp; os, P p) {     return os &lt;&lt; "(" &lt;&lt; p.x &lt;&lt; ", " &lt;&lt; p.y &lt;&lt; ")"; } //0e491f }; //47ec0a</pre>	
---	--

## 4.1 Lines and Segments

### sideOf.h

**Description:** Returns where  $p$  is as seen from  $s$  towards  $e$ .  $1/0/-1 \Leftrightarrow$  left/on line/right. If the optional argument  $eps$  is given 0 is returned if  $p$  is within distance  $eps$  from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

**Usage:** bool left = sideOf(p1,p2,q)==1;

"Point.h"	3af81c, 9 lines
<pre>template&lt;class P&gt; int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }</pre>	
<pre>template&lt;class P&gt; int sideOf(const P&amp; s, const P&amp; e, const P&amp; p, double eps) {     auto a = (e-s).cross(p-s); //37dc17     double l = (e-s).dist()*eps;     return (a &gt; l) - (a &lt; -l); } //3af81c</pre>	

### OnSegment.h

**Description:** Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

"Point.h"	c597e8, 3 lines
<pre>template&lt;class P&gt; bool onSegment(P s, P e, P p) {     return p.cross(s, e) == 0 &amp;&amp; (s - p).dot(e - p) &lt;= 0; } //c597e8</pre>	

### lineIntersection.h

**Description:** If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

**Usage:** auto res = lineInter(s1,e1,s2,e2);

if (res.first == 1)

cout << "intersection point at " << res.second << endl;

"Point.h"	a01f81, 8 lines
<pre>template&lt;class P&gt; pair&lt;int, P&gt; lineInter(P s1, P e1, P s2, P e2) {     auto d = (e1 - s1).cross(e2 - s2);     if (d == 0) // if parallel         return {- (s1.cross(e1, s2) == 0), P(0, 0)}; //47e53e     auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);     return {1, (s1 * p + e1 * q) / d}; //c4c8fb } //a01f81</pre>	

### SegmentIntersection.h

Description:

If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

Usage: vector<P> inter = segInter(s1,e1,s2,e2);

if (sz(inter)==1)

cout << "segments intersect at " << inter[0] << endl;

"Point.h", "OnSegment.h"

9d57f2, 13 lines

template<class P> vector<P> segInter(P a, P b, P c, P d) {  
 auto oa = c.cross(d, a), ob = c.cross(d, b),  
 oc = a.cross(b, c), od = a.cross(b, d);  
 // Checks if intersection is single non-endpoint point.  
 if (sgn(oa) \* sgn(ob) < 0 && sgn(oc) \* sgn(od) < 0)  
 return {(a \* ob - b \* oa) / (ob - oa)}; //ab16eb  
 set<P> s;  
 if (onSegment(c, d, a)) s.insert(a);  
 if (onSegment(c, d, b)) s.insert(b);  
 if (onSegment(a, b, c)) s.insert(c);  
 if (onSegment(a, b, d)) s.insert(d); //1dcb4f  
 return {all(s)}; //c505dc  
} //9d57f2

lineDistance.h

Description:

Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

"Point.h"

b4c5ca, 4 lines

template<class P>  
double lineDist(const P& a, const P& b, const P& p) {  
 return (b-a).cross(p-a) / (b-a).dist();  
} //b4c5ca

SegmentDistance.h

Description:

Returns the shortest distance between point p and the line segment from point s to e.

Usage: Point<double> a, b(2,2), p(1,1);

bool onSegment = segDist(a,b,p) < 1e-10;

"Point.h"

5c88f4, 6 lines

typedef Point<double> P;  
double segDist(P& s, P& e, P& p) {  
 if (s==e) return (p-s).dist();  
 auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));  
 return ((p-s)\*d-(e-s)\*t).dist() / d;  
} //5c88f4

kdTree.h

Description:

KD-tree (2d, can be extended to 3d)

"Point.h"

bac5b0, 63 lines

typedef long long T;  
typedef Point<T> P;  
const T INF = numeric\_limits<T>::max();  
  
bool on\_x(const P& a, const P& b) { return a.x < b.x; }  
bool on\_y(const P& a, const P& b) { return a.y < b.y; }

struct Node { //319cda  
 P pt; // if this is a leaf, the single point in it  
 T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds  
 Node \*first = 0, \*second = 0;  
  
 T distance(const P& p) { // min squared distance to a point  
 T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x); //71ed74  
 T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);  
 return (P(x,y) - p).dist2();  
 } //1460d4  
  
 Node(vector<P>&& vp) : pt(vp[0]) {  
 for (P p : vp) {  
 x0 = min(x0, p.x); x1 = max(x1, p.x);  
 y0 = min(y0, p.y); y1 = max(y1, p.y); //28bf16  
 } //2e9c2c  
 if (vp.size() > 1) {  
 // split on x if width >= height (not ideal...)  
 sort(all(vp), x1 - x0 >= y1 - y0 ? on\_x : on\_y);  
 // divide by taking half the array for each child (not  
 // best performance with many duplicates in the middle)  
 int half = sz(vp)/2; //21b567  
 first = new Node({vp.begin(), vp.begin() + half});  
 second = new Node({vp.begin() + half, vp.end()});  
 } //470fcd  
 } //0265cf  
}; //6fda19

struct KDTree {  
 Node\* root;  
 KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}  
  
 pair<T, P> search(Node \*node, const P& p) { //7daf7f  
 if (!node->first) {  
 // uncomment if we should not find the point itself:  
 // if (p == node->pt) return {INF, P()};  
 return make\_pair((p - node->pt).dist2(), node->pt);  
 } //19dc67  
  
 Node \*f = node->first, \*s = node->second;  
 T bfirst = f->distance(p), bsec = s->distance(p);  
 if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);  
  
 // search closest side first, other side if needed  
 auto best = search(f, p); //fa9faa  
 if (bsec < best.first)  
 best = min(best, search(s, p));  
 return best;  
 } //3771f7  
  
 // find nearest point to a point, and its squared distance  
 // (requires an arbitrary operator< for Point)  
 pair<T, P> nearest(const P& p) {  
 return search(root, p); //961132  
 } //60e74e  
}; //bac5b0

4.2 Polygons

PolygonArea.h

Description:

Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h"

f12300, 6 lines

template<class T>  
T polygonArea2(vector<Point<T>>& v) {  
 T a = v.back().cross(v[0]);  
 rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);  
 return a;  
} //f12300

InsidePolygon.h

Description:

Returns 0 if the point is outside the polygon, 1 if it is strictly inside the polygon, and 2 if it is on the polygon.

Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};

int in = inPoly(v, P{3, 3});

Time:  $\mathcal{O}(n)$

"Point.h", "OnSegment.h"

4dd823, 11 lines

template<class P> int inPoly(vector<P> poly, P p) {  
 bool good = false; int n = sz(poly);  
 auto crosses = [](P s, P e, P p) {  
 return ((e.y >= p.y) - (s.y >= p.y)) \* p.cross(s, e) > 0;  
 }; //be8833  
 for(int i = 0; i < n; i++){  
 if(onSegment(poly[i], poly[(i+1)%n], p)) return 2;  
 good ^= crosses(poly[i], poly[(i+1)%n], p);  
 } //1ff382  
 return good;  
} //4dd823

ConvexHull.h

Description:

Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time:  $\mathcal{O}(n \log n)$

"Point.h"

05b731, 18 lines

template<class P> vector<P> convexHull(vector<P> poly){  
 int n = sz(poly);  
 if (n <= 1) return poly;  
 vector<P> hull(n+1);  
 sort(all(poly));  
 int k = 0; //38d98b  
 for(int i = 0; i < n; i++){  
 while(k >= 2 && hull[k-2].cross(hull[k-1], poly[i]) <= 0) k--;  
 hull[k++] = poly[i];  
 } //0c301c  
 for(int i = n-1, t = k+1; i > 0; i--){  
 while(k >= t && hull[k-2].cross(hull[k-1], poly[i-1]) <= 0) k--;  
 hull[k++] = poly[i-1];  
 } //d5f00d  
 hull.resize(k-1);  
 hull.erase(unique(all(hull)), hull.end());  
 return hull;  
} //05b731

HullDiameter.h

Description:

Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

Time:  $\mathcal{O}(n)$

"Point.h"

c571b8, 12 lines

typedef Point<ll> P;  
array<P, 2> hullDiameter(vector<P> S) {  
 int n = sz(S), j = n < 2 ? 0 : 1;  
 pair<ll, array<P, 2>> res({0, {S[0], S[0]}});  
 rep(i,0,j)  
 for (; j = (j + 1) % n) { //e5ff70  
 res = max(res, {{S[i] - S[j]}.dist2(), {S[i], S[j]}});  
 if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)  
 break;  
 } //cf85e0  
 return res.second;  
} //c571b8

hullTangents.h



**Description:** Finds the left and right, respectively, tangent points on convex hull from a point. If the point is colinear to side(s) of the polygon, the point further away is returned. Requires ccw,  $n \geq 3$ , and the point be on or outside the polygon. Can be used to check if a point is inside of a convex hull. Will return -1 if it is strictly inside. If the point is on the hull, the two adjacent points will be returned  
**Time:**  $\mathcal{O}(\log n)$

"Point.h"	53d067, 16 lines
<pre>#define cmp(i, j) p.cross(h[i], h[j == n ? 0 : j]) * (R ? 1 : -1) template&lt;bool R, class P&gt; int getTangent(vector&lt;P&gt;&amp; h, P p) {     int n = sz(h), lo = 0, hi = n - 1, md;     if (cmp(0, 1) &gt;= R &amp;&amp; cmp(0, n - 1) &gt;= !R) return 0;     while (md = (lo + hi + 1) / 2, lo &lt; hi) {         auto a = cmp(md, md + 1), b = cmp(md, lo); //d06f76         if (a &gt;= R &amp;&amp; cmp(md, md - 1) &gt;= !R) return md;         if (cmp(lo, lo + 1) &lt; R)             a &lt; R&amp;&amp; b &gt;= 0 ? lo = md : hi = md - 1;         else a &lt; R    b &lt;= 0 ? lo = md : hi = md - 1;     } //218376     return -1; // point strictly inside hull } //929dec template&lt;class P&gt; pii hullTangents(vector&lt;P&gt;&amp; h, P p) {     return {getTangent&lt;0&gt;(h, p), getTangent&lt;1&gt;(h, p)}; //2a428b } //53d067</pre>	

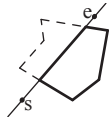
inHull.h  
**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.  
**Time:**  $\mathcal{O}(\log N)$

	6d9710, 12 lines
<pre>template&lt;class P&gt; bool inHull(const vector&lt;P&gt;&amp; l, P p, bool     strict = true) {     int a = 1, b = sz(l) - 1, r = !strict;     if (sz(l) &lt; 3) return r &amp;&amp; onSegment(l[0], l.back(), p);     if (sideOf(l[0], l[a], l[b]) &gt; 0) swap(a, b);     if (sideOf(l[0], l[a], p) &gt;= r    sideOf(l[0], l[b], p) &lt;= -r)         return false; //44688a     while (abs(a - b) &gt; 1) {         int c = (a + b) / 2;         (sideOf(l[0], l[c], p) &gt; 0 ? b : a) = c;     } //470615     return sgn(l[a].cross(l[b], p)) &lt; r; } //6d9710</pre>	

LineHullIntersection.h  
**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon:  $\bullet(-1, -1)$  if no collision,  $\bullet(i, -1)$  if touching the corner  $i$ ,  $\bullet(i, i)$  if along side  $(i, i + 1)$ ,  $\bullet(i, j)$  if crossing sides  $(i, i + 1)$  and  $(j, j + 1)$ . In the last case, if a corner  $i$  is crossed, this is treated as happening on side  $(i, i + 1)$ . The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.  
**Time:**  $\mathcal{O}(\log n)$

"Point.h"	7cf45b, 39 lines
<pre>#define cmp(i, j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n])) #define extr(i) cmp(i + 1, i) &gt;= 0 &amp;&amp; cmp(i, i - 1 + n) &lt; 0 template&lt;class P&gt; int extrVertex(vector&lt;P&gt;&amp; poly, P dir) {     int n = sz(poly), lo = 0, hi = n;     if (extr(0)) return 0;     while (lo + 1 &lt; hi) { //b3e410         int m = (lo + hi) / 2;         if (extr(m)) return m;         int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);         (ls &lt; ms    (ls == ms &amp;&amp; ls == cmp(lo, m)) ? hi : lo) = m;     } //efd609</pre>	

<pre>        return lo;     } //ba41ca  #define cmpL(i) sgn(a.cross(poly[i], b)) template&lt;class P&gt; array&lt;int, 2&gt; lineHull(P a, P b, vector&lt;P&gt;&amp; poly) {     int endA = extrVertex(poly, (a - b).perp()); //d0d8a9     int endB = extrVertex(poly, (b - a).perp());     if (cmpL(endA) &lt; 0    cmpL(endB) &gt; 0)         return {-1, -1}; //07bb09     array&lt;int, 2&gt; res;     rep(i, 0, 2) {         int lo = endB, hi = endA, n = sz(poly);         while ((lo + 1) % n != hi) {             int m = ((lo + hi + (lo &lt; hi ? 0 : n)) / 2) % n; //71097d             (cmpL(m) == cmpL(endB) ? lo : hi) = m;         } //72e441         res[i] = (lo + !cmpL(hi)) % n;         swap(endA, endB);     } //d56a85     if (res[0] == res[1]) return {res[0], -1}; //d847be     if (!cmpL(res[0]) &amp;&amp; !cmpL(res[1])) {         switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {             case 0: return {res[0], res[0]}; //ab4398             case 2: return {res[1], res[1]}; //e5b066         } //54f3d0         return res;     } //7cf45b</pre>	
--	--

PolygonCut.h  
**Description:**  
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.  
**Usage:** vector<P> p = ...;  
p = polygonCut(p, P(0,0), P(1,0));  
  
**Time:**  $\mathcal{O}(n)$

"Point.h", "lineIntersection.h"	f2b7d4, 13 lines
<pre>typedef Point&lt;double&gt; P; vector&lt;P&gt; polygonCut(const vector&lt;P&gt;&amp; poly, P s, P e) {     vector&lt;P&gt; res;     rep(i, 0, sz(poly)) {         P cur = poly[i], prev = i ? poly[i - 1] : poly.back();         bool side = s.cross(e, cur) &lt; 0; //41eabb         if (side != (s.cross(e, prev) &lt; 0))             res.push_back(lineInter(s, e, cur, prev).second);         if (side)             res.push_back(cur);     } //567ae4     return res; } //f2b7d4</pre>	

halfplaneIntersection.h  
**Description:** Returns the intersection of halfplanes as a polygon  
**Time:**  $\mathcal{O}(n \log n)$

	e9fe62, 42 lines
<pre>const double eps = 1e-8; typedef Point&lt;double&gt; P; struct HalfPlane {     P s, e, d;     HalfPlane(P s = P(), P e = P()): s(s), e(e), d(e - s) {}     bool contains(P p) { return d.cross(p - s) &gt; -eps; } //0b57d7     bool operator&lt;(HalfPlane hp) {         if (abs(d.x) &lt; eps &amp;&amp; abs(hp.d.x) &lt; eps)             return d.y &gt; 0 &amp;&amp; hp.d.y &lt; 0;         bool side = d.x &lt; eps    (abs(d.x) &lt;= eps &amp;&amp; d.y &gt; 0);         bool sideHp = hp.d.x &lt; eps    (abs(hp.d.x) &lt;= eps &amp;&amp; hp.d.y &gt; 0);         if (side != sideHp) return side; //522804         return d.cross(hp.d) &gt; 0;     } //f04cee</pre>	

<pre>    P inter(HalfPlane hp) {         auto p = hp.s.cross(e, hp.e), q = hp.s.cross(hp.e, s);         return (s * p + e * q) / d.cross(hp.d);     } //f43e4d }; //cb96d9  vector&lt;P&gt; hpIntersection(vector&lt;HalfPlane&gt; hps) {     sort(all(hps));     int n = sz(hps), l = 1, r = 0;     vector&lt;HalfPlane&gt; dq(n + 1); //7f023a     rep(i, 0, n) {         while (l &lt; r &amp;&amp; !hps[i].contains(dq[r].inter(dq[r - 1]))) r--;         while (l &lt; r &amp;&amp; !hps[i].contains(dq[l].inter(dq[l + 1]))) l++;         dq[++r] = hps[i];         if (l &lt; r &amp;&amp; abs(dq[r].d.cross(dq[r - 1].d)) &lt; eps) { //6aee5e             if (dq[r].d.dot(dq[r - 1].d) &lt; 0) return {}; //2605d9             if (dq[--r].contains(hps[i].s)) dq[r] = hps[i];         } //575960     } //d8f849     while (l &lt; r - 1 &amp;&amp; !dq[l].contains(dq[r].inter(dq[r - 1]))) r--;     while (l &lt; r - 1 &amp;&amp; !dq[r].contains(dq[l].inter(dq[l + 1]))) l++;     if (l &gt; r - 2) return {}; //5ca32f     vector&lt;P&gt; poly;     rep(i, 1, r)         poly.push_back(dq[i].inter(dq[i + 1]));     poly.push_back(dq[r].inter(dq[l]));     return poly; //0b254d } //e9fe62</pre>	
---	--

centerOfMass.h  
**Description:** Returns the center of mass for a polygon.  
**Memory:**  $\mathcal{O}(1)$   
**Time:**  $\mathcal{O}(n)$

	ccce20, 8 lines
<pre>template&lt;class P&gt; P polygonCenter(const vector&lt;P&gt;&amp; v) {     P res(0, 0); double A = 0;     for (int i = 0, j = sz(v) - 1; i &lt; sz(v); j = i++) {         res = res + (v[i] + v[j]) * v[j].cross(v[i]);         A += v[j].cross(v[i]);     } //938654     return res / A / 3; } //ccce20</pre>	

minkowskiSum.h  
**Description:** Returns the minkowski sum of a set of convex polygons  
**Time:**  $\mathcal{O}(n \log n)$

	6a76f5, 20 lines
<pre>#define side(p) (p.x &gt; 0    (p.x == 0 &amp;&amp; p.y &gt; 0)) template&lt;class P&gt; vector&lt;P&gt; convolve(vector&lt;vector&lt;P&gt;&gt; &amp;polys) {     P init; vector&lt;P&gt; dir;     for (auto poly: polys) {         int n = sz(poly); //aee8e7         if (n) init = init + poly[0];         if (n &lt; 2) continue;         rep(i, 0, n) dir.push_back(poly[(i + 1) % n] - poly[i]);     } //98f301     if (size(dir) == 0) return { init }; //b85ac7     stable_sort(all(dir), [&amp;](P a, P b) -&gt; bool {         if (side(a) != side(b)) return side(a);         return a.cross(b) &gt; 0;     });     vector&lt;P&gt; sum; P cur = init; //03ea38     rep(i, 0, sz(dir))         sum.push_back(cur), cur = cur + dir[i];     return sum; } //6a76f5</pre>	



PolygonUnion.h

**Description:** Calculates the area of the union of  $n$  polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)

**Time:**  $\mathcal{O}(N^2)$ , where  $N$  is the total number of points

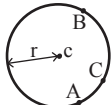
"Point.h", "sideOf.h"	3931c6, 33 lines
<pre>typedef Point&lt;double&gt; P; double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; } double polyUnion(vector&lt;vector&lt;P&gt;&gt;&amp; poly) {     double ret = 0;     rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {         P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];         vector&lt;pair&lt;double, int&gt;&gt; segs = {{0, 0}, {1, 0}}; //e9da64         rep(j,0,sz(poly)) if (i != j) {             rep(u,0,sz(poly[j])) {                 P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];                 int sc = sideOf(A, B, C), sd = sideOf(A, B, D);                 if (sc != sd) { //ac826b                     double sa = C.cross(D, A), sb = C.cross(D, B);                     if (min(sc, sd) &lt; 0)                         segs.emplace_back(sa / (sa - sb), sgn(sc - sd));                 } else if (!sc &amp;&amp; !sd &amp;&amp; j&lt;i &amp;&amp; sgn((B-A).dot(D-C))&gt;0) {                     segs.emplace_back(rat(C - A, B - A), 1); //a4636e                     segs.emplace_back(rat(D - A, B - A), -1);                 } //67520d             } //c4b419         } //a1900f         sort(all(segs));         for (auto&amp; s : segs) s.first = min(max(s.first, 0.0), 1.0);         double sum = 0;         int cnt = segs[0].second;         rep(j,1,sz(segs)) { //317ef1             if (!cnt) sum += segs[j].first - segs[j - 1].first;             cnt += segs[j].second;         } //d3398f         ret += A.cross(B) * sum;     } //6f2b4e     return ret / 2; } //3931c6</pre>	

4.3 Circles

circumcircle.h

**Description:**

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h"	1caa3a, 9 lines
<pre>typedef Point&lt;double&gt; P; double ccRadius(const P&amp; A, const P&amp; B, const P&amp; C) {     return (B-A).dist()*(C-B).dist()*(A-C).dist() /         abs((B-A).cross(C-A))/2; } //032e3d P ccCenter(const P&amp; A, const P&amp; B, const P&amp; C) {     P b = C-A, c = B-A;     return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2; } //1caa3a</pre>	

CircleLine.h

**Description:** Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h"	e0cfba, 9 lines
<pre>template&lt;class P&gt; vector&lt;P&gt; circleLine(P c, double r, P a, P b) {     P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();</pre>	

<pre>double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2(); if (h2 &lt; 0) return {}; //64a27f if (h2 == 0) return {p}; //3d9ab3 P h = ab.unit() * sqrt(h2); return {p - h, p + h}; //3b1a3f } //e0cfba</pre>	
---	--

CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h"	84d6d3, 11 lines
<pre>typedef Point&lt;double&gt; P; bool circleInter(P a,P b,double r1,double r2,pair&lt;P, P&gt;* out) {     if (a == b) { assert(r1 != r2); return false; } //7e53c0     P vec = b - a;     double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;     if (sum*sum &lt; d2    dif*dif &gt; d2) return false;     P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);     *out = {mid + per, mid - per}; //3dd318     return true; } //84d6d3</pre>	

CirclePolygonIntersection.h

**Description:** Returns the area of the intersection of a circle with a ccw polygon.

**Time:**  $\mathcal{O}(n)$

"../content/geometry/Point.h"	alee63, 19 lines
<pre>typedef Point&lt;double&gt; P; #define arg(p, q) atan2(p.cross(q), p.dot(q)) double circlePoly(P c, double r, vector&lt;P&gt; ps) {     auto tri = [&amp;](P p, P q) {         auto r2 = r * r / 2;         P d = q - p; //c0445a         auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();         auto det = a * a - b;         if (det &lt;= 0) return arg(p, q) * r2;         auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));         if (t &lt; 0    1 &lt;= s) return arg(p, q) * r2; //1b08d3         P u = p + d * s, v = p + d * t;         return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;     }; //6470ed     auto sum = 0.0;     rep(i,0,sz(ps))         sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);     return sum; } //alee63</pre>	

CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h"	b0153d, 13 lines
<pre>template&lt;class P&gt; vector&lt;pair&lt;P, P&gt;&gt; tangents(P c1, double r1, P c2, double r2) {     P d = c2 - c1;     double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;     if (d2 == 0    h2 &lt; 0) return {}; //c18727     vector&lt;pair&lt;P, P&gt;&gt; out;     for (double sign : {-1, 1}) {         P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;         out.push_back({c1 + v * r1, c2 + v * r2});     } //41b560</pre>	

<pre>if (h2 == 0) out.pop_back(); return out; } //b0153d</pre>	
--	--

MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points.

**Time:** expected  $\mathcal{O}(n)$

"circumcircle.h"	09dd0a, 17 lines
<pre>pair&lt;P, double&gt; mec(vector&lt;P&gt; ps) {     shuffle(all(ps), mt19937(time(0)));     P o = ps[0];     double r = 0, EPS = 1 + 1e-8;     rep(i,0,sz(ps)) if ((o - ps[i]).dist() &gt; r * EPS) {         o = ps[i], r = 0; //5e7038         rep(j,0,i) if ((o - ps[j]).dist() &gt; r * EPS) {             o = (ps[i] + ps[j]) / 2;             r = (o - ps[i]).dist();             rep(k,0,j) if ((o - ps[k]).dist() &gt; r * EPS) {                 o = ccCenter(ps[i], ps[j], ps[k]); //931d7a                 r = (o - ps[i]).dist();             } //7cd516         } //03da47     } //bfac59     return {o, r}; //5ebee7 } //09dd0a</pre>	

4.4 3D Geometry

Point3D.h

**Description:** Class to handle points in 3D space. T can be e.g. double or long long.

		01f8f7, 33 lines
<pre>template&lt;class T&gt; struct Point3D {     typedef Point3D P;     typedef const P&amp; R;     T x, y, z;     explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}     bool operator&lt;(R p) const { //9e2218         return tie(x, y, z) &lt; tie(p.x, p.y, p.z); } //af5a46     bool operator==(R p) const {         return tie(x, y, z) == tie(p.x, p.y, p.z); } //fa5b42     P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }     P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }     P operator*(T d) const { return P(x*d, y*d, z*d); } //1ee29d     P operator/(T d) const { return P(x/d, y/d, z/d); } //660667     T dot(R p) const { return x*p.x + y*p.y + z*p.z; } //d7cc17     P cross(R a, R b) const { return (a-*this).cross(b-*this); }     P cross(R p) const {         return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);     } //7f1984     T dist2() const { return x*x + y*y + z*z; } //061c10     double dist() const { return sqrt((double)dist2()); }     //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]     double phi() const { return atan2(y, x); } //f3fa7c     //Zenith angle (latitude) to the z-axis in interval [0, pi]     double theta() const { return atan2(sqrt(x*x+y*y),z); }     P unit() const { return *this/(T)dist(); } //makes dist()=1     //returns unit vector normal to *this and p     P normal(P p) const { return cross(p).unit(); } //e107dc     //returns point rotated 'angle' radians ccw around axis     P rotate(double angle, P axis) const {         double s = sin(angle), c = cos(angle); P u = axis.unit();         return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;     } //83bd4d }; //01f8f7</pre>		

3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. \*No four points must be coplanar\*, or else random results will be returned. All faces will point outwards.  
**Time:**  $\mathcal{O}(n^2)$

"Point3D.h"faa885, 36 lines

```
typedef Point3D<double> P3;
const double eps = 1e-6;
```

```
struct F { int a, b, c; }; //4fccf3
```

```
vector<F> hull3d(vector<P3> &p) {
    int n = sz(p);
    if(n < 3) return {}; //1262ac
    vector<F> faces;
```

```
    vvi active(n, vi(n, false));
```

```
    auto add_face = [&](int a, int b, int c) -> void { //cbcd44
        faces.push_back({a, b, c});
        active[a][b] = active[b][c] = active[c][a] = true;
    }; //55d48b
```

```
    add_face(0, 1, 2);
    add_face(0, 2, 1);
```

```
    rep(i, 3, n) { //7dcd92
        vector<array<int, 3>> new_faces;
        for(auto [a, b, c]: faces)
            if((p[i] - p[a]).dot(p[a].cross(p[b], p[c])) > eps)
                active[a][b] = active[b][c] = active[c][a] = false;
            else new_faces.push_back({a, b, c}); //77474c
        faces.clear();
        for(auto f: new_faces)
            rep(j, 0, 3) if(!active[f[(j+1)%3]][f[j]])
                add_face(f[(j+1)%3], f[j], i);
        for(auto [a, b, c]: new_faces) //9deb86
            faces.push_back({a, b, c});
    } //978258
```

```
    return faces;
} //faa885
```

### sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 ( $\phi_1$ ) and f2 ( $\phi_2$ ) from x axis and zenith angles (latitude) t1 ( $\theta_1$ ) and t2 ( $\theta_2$ ) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx\*radius is then the difference between the two points in the x direction and d\*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz); //819384
    return radius*2*asin(d/2);
} //611f07
```

### PolyhedronVolume.h

**Description:** Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilst) {
    double v = 0;
    for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
```

```
    return v / 6;
} //3058c3
```

## 4.5 Miscellaneous

### ClosestPair.h

**Description:** Finds the closest pair of points.

**Time:**  $\mathcal{O}(n \log n)$

"Point.h"ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}}; //db620d
    int j = 0;
    for (P p : v) {
        P d(1 + (ll)sqrt(ret.first), 0); //484ee7
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {(lo - p).dist2(), {lo, p}});
        S.insert(p); //afb942
    } //a4382b
    return ret.second;
} //ac41a6
```

### FastDelaunay.h

**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.

**Time:**  $\mathcal{O}(n \log n)$

"Point.h"eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point
```

```
struct Quad { //4dcdd0
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; } //001543
    Q& r() { return rot->rot; } //9a9030
    Q prev() { return rot->o->rot; } //a6d183
    Q next() { return r()->prev(); } //c2bc3a
} *H;
```

```
bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = p.dist2(), A = a.dist2()-p2,
        B = b.dist2()-p2, C = c.dist2()-p2; //4e353f
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
} //cae086
Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{0}}}};
    H = r->o; r->r()->r() = r;
    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest; //e2ee6e
    return r;
} //25ccf6
void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
} //0ef350
Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q; //f4703d
} //eef885
```

```
pair<Q,Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return { a, a->r() }; //d46520
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    } //60c127
```

```
#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
Q A, B, ra, rb;
int half = sz(s) / 2; //4dbbd2
tie(ra, A) = rec({all(s) - half});
tie(B, rb) = rec({sz(s) - half + all(s)});
while ((B->p.cross(H(A)) < 0 && (A = A->next()) ||
    (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
Q base = connect(B->r(), A); //a3dcbe
if (A->p == ra->p) ra = base->r();
if (B->p == rb->p) rb = base;
```

```
#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \ //5b7586
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    } //d41222
for (;;) {
    DEL(LC, base->r(), o); DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
        base = connect(RC, base->r()); //cf44eb
    else
        base = connect(base->r(), LC->r());
} //17ceb8
return { ra, rb }; //505512
} //2d987e
```

```
vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {}; //afbclc
    Q e = rec(pts).first;
    vector<Q> q = {e}; //35ce3b
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear(); //8e3597
    while (qi < sz(q)) if (!(e = q[qi++]->mark) ADD;
    return pts;
} //eefdf5
```

### PlanarFaceExtraction.h

**Description:** Given a planar graph and where the points are, extract the set of faces that the graph makes

**Time:**  $\mathcal{O}(E \log E)$

63f230, 39 lines

```
template<class P>
vector<vector<P>> extract_faces(vvi adj, vector<P> pts) {
    int n = sz(pts);
    #define cmp(i) [&](int pi, int qi) -> bool { \
        P p = pts[pi] - pts[i], q = pts[qi] - pts[i]; \
        bool sideP = p.y < 0 || (p.y == 0 && p.x < 0); \ //2e5576
        bool sideQ = q.y < 0 || (q.y == 0 && q.x < 0); \
        if(sideP != sideQ) return sideP; \
        return p.cross(q) > 0; } //59b975
```

```
rep(i, 0, n)
    sort(all(adj[i]), cmp(i));
vii ed;
rep(i, 0, n) for(int j: adj[i])
    ed.emplace_back(i, j); //623310
sort(all(ed));
auto get_idx = [&](int i, int j) -> int {
    return lower_bound(all(ed), pii(i, j))-begin(ed);
}; //7667e7
vector<vector<P>> faces;
vi used(sz(ed));
rep(i, 0, n) for(int j: adj[i]) {
    if(used[get_idx(i, j)])
        continue; //7db6a7
    used[get_idx(i, j)] = true;
    vector<P> face = {pts[i]}; //b39032
    int prv = i, cur = j;
    while(cur != i) {
        face.push_back(pts[cur]);
        auto it = lower_bound(all(adj[cur]), prv, cmp(cur));
        if(it == begin(adj[cur])) //f6e8f1
            it = end(adj[cur]);
        prv = cur, cur = *prev(it);
        used[get_idx(prv, cur)] = true;
    } //9fb9bf
    faces.push_back(face);
} //29aacd
#undef cmp
return faces;
} //63f230
```

## Graphs (5)

### 5.1 Network flow

#### MinCostMaxFlow.h

**Description:** Min-cost max-flow. Negative cost cycles not supported. To obtain the actual flow, look at positive values only.  
**Time:** Approximately  $\mathcal{O}(E^2)$ , actually  $\mathcal{O}(FS)$  where  $S$  is the time complexity of the SSSP alg used in find path (in this case SPFA) 664049, 55 lines

```
struct MCMF {
    const ll INF = LLONG_MAX >> 2;
    struct edge {
        int v;
        ll cap, flow, cost;
    }; //f709d9
    int n;
    vector<edge> edges;
    vvi adj; vii par; vi in_q;
    vector<ll> dist, pi;
    MCMF(int n): n(n), adj(n), par(n), in_q(n), dist(n), pi(n) {}
    void addEdge(int u, int v, ll cap, ll cost) { //42c114
        int idx = sz(edges);
        edges.push_back({v, cap, 0, cost});
        edges.push_back({u, cap, cap, -cost});
        adj[u].push_back(idx);
        adj[v].push_back(idx ^ 1); //65b236
    } //e280ec
    bool findPath(int s, int t) {
        fill(all(dist), INF);
        fill(all(in_q), 0);
        queue<int> q; q.push(s);
        dist[s] = 0, in_q[s] = 1; //46fdef
        while(!q.empty()) {
            int cur = q.front(); q.pop();
            in_q[cur] = 0;
            for(int idx: adj[cur]) {
```

```
                auto [nxt, cap, f, wt] = edges[idx]; //77b4b3
                ll nxtD = dist[cur] + wt;
                if(f1 >= cap || nxtD >= dist[nxt]) continue;
                dist[nxt] = nxtD;
                par[nxt] = {cur, idx}; //ee66ab
                if(in_q[nxt]) continue;
                q.push(nxt); in_q[nxt] = 1;
            } //882b56
        } //0dbe4d

        return dist[t] < INF;
    } //3db314
    pair<ll, ll> calc(int s, int t) {
        ll flow = 0, cost = 0;
        while(findPath(s, t)) {
            ll f = INF;
            for(int i, u, v = t; tie(u, i) = par[v], v != s; v = u)
                f = min(f, edges[i].cap - edges[i].flow); //78f755
            flow += f;
            for(int i, u, v = t; tie(u, i) = par[v], v != s; v = u)
                edges[i].flow += f, edges[i^1].flow -= f;
        } //e557d6
        rep(i, 0, sz(edges)>>1)
            cost += edges[i<<1].cost * edges[i<<1].flow;

        return {flow, cost}; //048d32
    } //f57bd7
}; //664049
```

#### MinCostMaxFlowDijkstra.h

**Description:** If SPFA TLEs, swap the find\_path function in MCMF with the one below and in\_q with seen. If negative edge weights can occur, initialize pi with the shortest path from the source to each node using Bellman-Ford. Negative weight cycles not supported. efdefd, 24 lines

```
bool findPath(int s, int t) {
    fill(all(dist), inf);
    fill(all(seen), 0);
    dist[s] = 0;
    __gnu_pbds::priority_queue<pair<ll, int>> pq;
    vector<decltype(pq)::point_iterator> its(n); //e67bf6
    pq.push({0, s});
    while(!pq.empty()) {
        auto [d, cur] = pq.top(); pq.pop(); d *= -1;
        seen[cur] = 1;
        if(dist[cur] < d) continue; //c5f170
        for(int idx: adj[cur]) {
            auto [nxt, cap, f, wt] = edges[idx];
            ll nxtD = d + wt + pi[cur] - pi[nxt];
            if(f >= cap || nxtD >= dist[nxt] || seen[nxt]) continue;
            dist[nxt] = nxtD; //b0252f
            par[nxt] = {cur, idx}; //8270eb
            if(its[nxt] == pq.end()) its[nxt] = pq.push({-nxtD, nxt});
            ;
            else pq.modify(its[nxt], {-nxtD, nxt});
        } //0154c2
    } //86f7eb
    rep(i, 0, n) pi[i] = min(pi[i] + dist[i], inf);
    return seen[t];
} //efdefd
```

#### Dinic.h

**Description:** Flow algorithm with complexity  $\mathcal{O}(VE \log U)$  where  $U = \max|cap|$ .  $\mathcal{O}(\min(E^{1/2}, V^{2/3})E)$  if  $U = 1$ ;  $\mathcal{O}(\sqrt{V}E)$  for bipartite matching. d7f0f1, 42 lines

```
struct Dinic {
    struct Edge {
        int to, rev;
```

```
        ll c, oc;
        ll flow() { return max(oc - c, 0LL); } // if you need flows
    }; //9d5927
    vi lvl, ptr, q;
    vector<vector<Edge>> adj;
    Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {} //fd5b9
    void addEdge(int a, int b, ll c, ll rcap = 0) {
        adj[a].push_back({b, sz(adj[b]), c, c});
        adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
    } //a45d7e
    ll dfs(int v, int t, ll f) {
        if (v == t || !f) return f;
        for (int& i = ptr[v]; i < sz(adj[v]); i++) {
            Edge& e = adj[v][i];
            if (lvl[e.to] == lvl[v] + 1) //591b8b
                if (ll p = dfs(e.to, t, min(f, e.c))) {
                    e.c -= p, adj[e.to][e.rev].c += p;
                    return p;
                } //d3bb27
        } //f4fba
        return 0;
    } //72048c
    ll calc(int s, int t) {
        ll flow = 0; q[0] = s;
        rep(L, 0, 31) do { // 'int L=30' maybe faster for random data
            lvl = ptr = vi(sz(q));
            int qi = 0, qe = lvl[s] = 1; //5d9371
            while (qi < qe && !lvl[t]) {
                int v = q[qi++];
                for (Edge e : adj[v])
                    if (!lvl[e.to] && e.c >> (30 - L))
                        q[qi++] = e.to, lvl[e.to] = lvl[v] + 1; //0d5640
            } //16dd6b
            while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
        } while (lvl[t]);
        return flow;
    } //2b90e4
    bool leftOfMinCut(int a) { return lvl[a] != 0; } //761cc4
}; //d7f0f1
```

#### GlobalMinCut.h

**Description:** Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.  
**Time:**  $\mathcal{O}(V^3)$  8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
    pair<int, vi> best = {INT_MAX, {}}; //81f955
    int n = sz(mat);
    vector<vi> co(n);
    rep(i, 0, n) co[i] = {i}; //f640ab
    rep(ph, 1, n) {
        vi w = mat[0];
        size_t s = 0, t = 0;
        rep(it, 0, n-ph) { //  $\mathcal{O}(V^2) \rightarrow \mathcal{O}(E \log V)$  with prio. queue
            w[t] = INT_MIN; //c98135
            s = t, t = max_element(all(w)) - w.begin();
            rep(i, 0, n) w[i] += mat[t][i];
        } //8c07c9
        best = min(best, {w[t] - mat[t][t], co[t]});
        co[s].insert(co[s].end(), all(co[t]));
        rep(i, 0, n) mat[s][i] += mat[t][i];
        rep(i, 0, n) mat[i][s] = mat[i][t];
        mat[0][t] = INT_MIN; //e25d4b
    } //076888
    return best;
} //8b0e19
```

GomoryHu.h  
**Description:** Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.  
**Time:**  $\mathcal{O}(V)$  Flow Computations

```
"Dinic.h"                                     e2b333, 13 lines

typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vi par(N);
    rep(i,1,N) {
        Dinic D(N); //53565e
        for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
        tree.push_back({i, par[i], D.calc(i, par[i])});
        rep(j,i+1,N)
            if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
    } //c14544
    return tree;
} //e2b333
```

MatroidIntersection.h  
**Description:** Given two matroids, finds the largest common independent set. For the color and graph matroids, this would be the largest forest where no two edges are the same color. A matroid has 3 functions  
- check(int x): returns if current matroid can add x without becoming dependent  
- add(int x): adds an element to the matroid (guaranteed to never make it dependent)  
- clear(): sets the matroid to the empty matroid  
The matroid is given an int representing the element, and is expected to convert it (e.g: the color or the endpoints) Pass the matroid with more expensive add/clear operations to M1.  
**Time:**  $R^2N(M2.add+M1.check+M2.check)+R^3M1.add+R^2M1.clear+RMN2.clear$

```
"../data-structures/UnionFind.h"             9812a7, 60 lines

struct ColorMat {
    vi cnt, clr;
    ColorMat(int n, vector<int> clr) : cnt(n), clr(clr) {}
    bool check(int x) { return !cnt[clr[x]]; } //1992d4
    void add(int x) { cnt[clr[x]]++; } //b9ca1b
    void clear() { fill(all(cnt), 0); } //1217e4
}; //a797c9

struct GraphMat {
    UF uf;
    vector<array<int, 2>> e;
    GraphMat(int n, vector<array<int, 2>> e) : uf(n), e(e) {}
    bool check(int x) { return !uf.sameSet(e[x][0], e[x][1]); }
    void add(int x) { uf.join(e[x][0], e[x][1]); } //4634b6
    void clear() { uf = UF(sz(uf.e)); } //4fb94c
}; //7f77ed

template <class M1, class M2> struct MatroidIsect {
    int n;
    vector<char> iset;
    M1 m1; M2 m2;
    MatroidIsect(M1 m1, M2 m2, int n) : n(n), iset(n + 1), m1(m1), m2(m2) {}

    vi solve() { //8b197a
        rep(i,0,n) if (m1.check(i) && m2.check(i))
            iset[i] = true, m1.add(i), m2.add(i);
        while (augment());
        vi ans;
        rep(i,0,n) if (iset[i]) ans.push_back(i); //7337bf
        return ans;
    } //530c7f
    bool augment() {
        vector<int> frm(n, -1);
        queue<int> q({n}); // starts at dummy node
        auto fwdE = [&](int a) {
```

```
vi ans; //1df231
m1.clear();
rep(v, 0, n) if (iset[v] && v != a) m1.add(v);
rep(b, 0, n) if (!iset[b] && frm[b] == -1 && m1.check(b))
    ans.push_back(b), frm[b] = a;
return ans; //f4e117
}; //f4805c
auto backE = [&](int b) {
    m2.clear();
    rep(cas, 0, 2) rep(v, 0, n)
        if ((v == b || iset[v]) && (frm[v] == -1) == cas) {
            if (!m2.check(v)) //afb3ed
                return cas ? q.push(v), frm[v] = b, v : -1;
            m2.add(v);
        } //3b2d63
    return n;
}; //0ceea9
while (!q.empty()) {
    int a = q.front(), c; q.pop();
    for (int b : fwdE(a))
        while((c = backE(b)) >= 0) if (c == n) {
            while (b != n) iset[b] ^= 1, b = frm[b]; //c6beb1
            return true;
        } //7398d6
    } //ec60bb
    return false;
} //c1031d
} //9812a7
```

## 5.2 Matching

hopcroftKarp.h  
**Description:** Fast bipartite matching algorithm. Graph  $g$  should be a list of neighbors of the left partition, and  $btoa$  should be a vector full of -1's of the same size as the right partition. Returns the size of the matching.  $btoa[i]$  will be the match for vertex  $i$  on the right side, or  $-1$  if it's not matched.  
**Usage:** vi btoa(m, -1); hopcroftKarp(g, btoa);  
**Time:**  $\mathcal{O}(\sqrt{VE})$

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
    if (A[a] != L) return 0;
    A[a] = -1;
    for (int b : g[a]) if (B[b] == L + 1) {
        B[b] = 0;
        if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
            return btoa[b] = a, 1; //47a337
    } //84f762
    return 0;
} //9e7938
```

```
int hopcroftKarp(vector<vi>& g, vi& btoa) {
    int res = 0;
    vi A(g.size()), B(btoa.size()), cur, next;
    for (;;) { //a02d20
        fill(all(A), 0);
        fill(all(B), 0);
        cur.clear();
        for (int a : btoa) if (a != -1) A[a] = -1;
        rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a); //0fe82b
        for (int lay = 1;; lay++) {
            bool islast = 0;
            next.clear();
            for (int a : cur) for (int b : g[a]) {
                if (btoa[b] == -1) { //96ecca
                    B[b] = lay;
                    islast = 1;
                } //4c74fe
                else if (btoa[b] != a && !B[b]) {
                    B[b] = lay;
```

```
                next.push_back(btoa[b]);
            } //81e09f
        } //ebc136
        if (islast) break;
        if (next.empty()) return res;
        for (int a : next) A[a] = lay;
        cur.swap(next);
    } //e487ce
    rep(a,0,sz(g))
        res += dfs(a, 0, g, btoa, A, B);
    } //f385af
} //f612e4
```

DFSMatching.h  
**Description:** Simple bipartite matching algorithm. Graph  $g$  should be a list of neighbors of the left partition, and  $btoa$  should be a vector full of -1's of the same size as the right partition. Returns the size of the matching.  $btoa[i]$  will be the match for vertex  $i$  on the right side, or  $-1$  if it's not matched.  
**Usage:** vi btoa(m, -1); dfsMatching(g, btoa);  
**Time:**  $\mathcal{O}(VE)$

```
522b98, 22 lines

bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
    if (btoa[j] == -1) return 1;
    vis[j] = 1; int di = btoa[j];
    for (int e : g[di])
        if (!vis[e] && find(e, g, btoa, vis)) {
            btoa[e] = di; //b1c950
            return 1;
        } //cc0de1
    return 0;
} //d13a81
int dfsMatching(vector<vi>& g, vi& btoa) {
    vi vis;
    rep(i,0,sz(g)) {
        vis.assign(sz(btoa), 0);
        for (int j : g[i]) //0eda2c
            if (find(j, g, btoa, vis)) {
                btoa[j] = i;
                break;
            } //5609e1
    } //61061f
    return sz(btoa) - (int)count(all(btoa), -1);
} //522b98
```

MinimumVertexCover.h  
**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

```
"DFSMatching.h"                             da4196, 20 lines

vi cover(vector<vi>& g, int n, int m) {
    vi match(m, -1);
    int res = dfsMatching(g, match);
    vector<bool> lfound(n, true), seen(m);
    for (int it : match) if (it != -1) lfound[it] = false;
    vi q, cover; //d5d915
    rep(i,0,n) if (lfound[i]) q.push_back(i);
    while (!q.empty()) {
        int i = q.back(); q.pop_back();
        lfound[i] = 1;
        for (int e : g[i]) if (!seen[e] && match[e] != -1) {
            seen[e] = true; //1aca58
            q.push_back(match[e]);
        } //b97b04
    } //b9473f
    rep(i,0,n) if (!lfound[i]) cover.push_back(i);
    rep(i,0,m) if (seen[i]) cover.push_back(n+i);
    assert(sz(cover) == res);
    return cover;
} //da4196
```

WeightedMatching.h

**Description:** Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires  $N \leq M$ .  
**Time:**  $\mathcal{O}(N^2M)$

```
1e0fe9, 31 lines
pair<int, vi> hungarian(const vector<vi> &a) {
    if (a.empty()) return {0, {}}; //497519
    int n = sz(a) + 1, m = sz(a[0]) + 1;
    vi u(n), v(m), p(m), ans(n - 1);
    rep(i,1,n) {
        p[0] = i;
        int j0 = 0; // add "dummy" worker 0
        vi dist(m, INT_MAX), pre(m, -1); //3b3e45
        vector<bool> done(m + 1);
        do { // dijkstra
            done[j0] = true;
            int i0 = p[j0], j1, delta = INT_MAX;
            rep(j,1,m) if (!done[j]) { //0023e6
                auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
                if (dist[j] < delta) delta = dist[j], j1 = j;
            } //31ae76
            rep(j,0,m) {
                if (done[j]) u[p[j]] += delta, v[j] -= delta;
                else dist[j] -= delta;
            } //6cc461
            j0 = j1;
        } while (p[j0]);
        while (j0) { // update alternating path
            int j1 = pre[j0];
            p[j0] = p[j1], j0 = j1; //632eb8
        } //26ae9e
    } //9e72cf
    rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
    return {-v[0], ans}; // min cost
} //1e0fe9
```

GeneralMatching.h

**Description:** Matching for general graphs. Fails with probability  $N/mod$ .  
**Time:**  $\mathcal{O}(N^3)$

```
cb1912, 40 lines
vector<pii> generalMatching(int N, vector<pii>& ed) {
    vector<vector<ll>> mat(N, vector<ll>(N)), A;
    for (pii pa : ed) {
        int a = pa.first, b = pa.second, r = rand() % mod;
        mat[a][b] = r, mat[b][a] = (mod - r) % mod;
    } //614800

    int r = matInv(A = mat), M = 2*N - r, fi, fj;
    assert(r % 2 == 0);

    if (M != N) do { //9bc254
        mat.resize(M, vector<ll>(M));
        rep(i,0,N) {
            mat[i].resize(M);
            rep(j,N,M) {
                int r = rand() % mod; //d8dfd
                mat[i][j] = r, mat[j][i] = (mod - r) % mod;
            } //36a855
        } //be41a1
    } while (matInv(A = mat) != M);

    vi has(M, 1); vector<pii> ret;
    rep(it,0,M/2) {
        rep(i,0,M) if (has[i]) //348eac
            rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
```

```
fi = i; fj = j; goto done;
    } assert(0); done;
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0; //fcefbe
    rep(sw,0,2) {
        ll a = modpow(A[fi][fj], mod-2);
        rep(i,0,M) if (has[i] && A[i][fj]) {
            ll b = A[i][fj] * a % mod;
            rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
        } //9debcbf
        swap(fi, fj);
    } //6c623e
    } //f16c12
    return ret;
} //cb1912
```

5.3 DFS algorithms

SCC.h

**Description:** Finds strogly connected components in a directed graph.  
**Usage:** auto [num\_sccs, scc\_id] = sccs(adj);  
scc\_id[v] = id, 0<=id<num\_sccs  
for each edge u -> v: scc\_id[u] >= scc\_id[v]  
**Time:**  $\mathcal{O}(E + V)$

```
2552fb, 16 lines
auto sccs(const vector<vi>& adj) {
    int n = sz(adj), num_sccs = 0, q = 0, s = 0;
    vi scc_id(n, -1), tin(n), st(n);
    auto dfs = [&](auto&& self, int v) -> int {
        int low = tin[v] = ++q; st[s++] = v;
        for (int u : adj[v]) if (scc_id[u] < 0) //530f05
            low = min(low, tin[u] ?: self(self, u));
        if (tin[v] == low) {
            while (scc_id[v] < 0) scc_id[st[--s]] = num_sccs;
            num_sccs++;
        } //9cb784
        return low;
    }; //250c73
    rep(i,0,n) if (!tin[i]) dfs(dfs, i);
    return pair{num_sccs, scc_id}; //7aebce
} //2552fb
```

BiconnectedComponents.h

**Description:** Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.  
**Usage:** int eid = 0; ed.resize(N);  
for each edge (a,b) {  
ed[a].emplace\_back(b, eid);  
ed[b].emplace\_back(a, eid++); }  
bicomps([&](const vi& edgelist) {...});  
**Time:**  $\mathcal{O}(E + V)$

```
2965e5, 33 lines
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
    int me = num[at] = ++Time, e, y, top = me; //d1b332
    for (auto pa : ed[at]) if (pa.second != par) {
        tie(y, e) = pa;
        if (num[y]) {
            top = min(top, num[y]);
            if (num[y] < me) //145ca4
                st.push_back(e);
        } else {
            int si = sz(st);
            int up = dfs(y, e, f);
```

```
top = min(top, up); //4c0c04
    if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
    } //4c59fd
    else if (up < me) st.push_back(e);
    else { /* e is a bridge */ } //47e7b7
    } //7a2ccf
    } //55ddf3
    return top;
} //0b5c9f
```

```
template<class F>
void bicomps(F f) {
    num.assign(sz(ed), 0);
    rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f); //14c211
} //2965e5
```

blockvertex

**Description:** articulation points and block-vertex tree self edges not allowed  
adj[u] += v, i; adj[v] += u, i; iscut[v] = 1 iff cut node bccid[edge id] = id, 0<=id<numbccs returns numbccs, bccid, iscut Assumes the root node points to itself.

```
ab8ef6, 61 lines
auto cuts(const auto& adj, int m) {
    int n = ssize(adj), num_bccs = 0, q = 0, s = 0;
    vector<int> bcc_id(m, -1), is_cut(n), tin(n), st(m);
    auto dfs = [&](auto&& self, int v, int p) -> int {
        int low = tin[v] = ++q;
        for (auto [u, e] : adj[v]) { //d15302
            assert(v != u);
            if (e == p) continue;
            if (tin[u] < tin[v]) st[s++] = e;
            int lu = -1;
            low = min(low, tin[u] ?: (lu = self(self, u, e))); //d79c0f
            if (lu >= tin[v]) {
                is_cut[v] = p >= 0 || tin[v] + 1 < tin[u];
                while (bcc_id[e] < 0) bcc_id[st[--s]] = num_bccs;
                num_bccs++;
            } //c32a15
        } //9a1476
        return low;
    }; //d8df66
    for (int i = 0; i < n; i++)
        if (!tin[i]) dfs(dfs, i, -1);
    return tuple{num_bccs, bcc_id, is_cut}; //782ada
} //64c880
//!
//! vector<vector<pii>> adj(n);
//! auto [num_bccs, bcc_id, is_cut] = cuts(adj, m);
//! auto bvt = block_vertex_tree(adj,
//!     num_bccs, bcc_id);
//!
//! vector<basic_string<array<int, 2>>> adj(n);
//! auto [num_bccs, bcc_id, is_cut] = cuts(adj, m);
//! auto bvt = block_vertex_tree(adj, num_bccs, bcc_id);
//!
//! //to loop over each unique bcc containing a node u:
//! for (int bccid : bvt[v]) {
//!     bccid -= n;
//! }
//! //to loop over each unique node inside a bcc:
//! for (int v : bvt[bccid + n]) { }
//! [0, n) are original nodes
//! [n, n + num_bccs) are BCC nodes
//! @time O(n + m)
//! @time O(n)
auto block_vertex_tree(const auto& adj, int num_bccs,
```

UCF	
<pre> const vector&lt;int&gt;&amp; bcc_id) { //2892ea int n = ssize(adj); vector&lt;basic_string&lt;int&gt;&gt; bvt(n + num_bccs); vector&lt;bool&gt; vis(num_bccs); for (int i = 0; i &lt; n; i++) {     for (auto [_, e_id] : adj[i]) { //4487b1         int bccid = bcc_id[e_id];         if (!vis[bccid]) {             vis[bccid] = 1;             bvt[i] += bccid + n;             bvt[bccid + n] += i; //472b2c         } //4f54ba     } //805517     for (int bccid : bvt[i]) vis[bccid - n] = 0; } //686c71 return bvt; } //ab8ef6 </pre>	
<p>bridgetree.h</p> <p><b>Description:</b> bridges adj[u] += v, i; adj[v] += u, i; iscut[v] = 1 iff cut node brid[v] = id, 0&lt;=id&lt;numbccs returns numbccs, bccid, iscut Assumes the root node points to itself.</p>	709259, 40 lines

<pre> auto bridges(const auto&amp; adj, int m) {     int n = ssize(adj), num_ccs = 0, q = 0, s = 0;     vector&lt;int&gt; br_id(n, -1), is_br(m), tin(n), st(n);     auto dfs = [&amp;](auto&amp;&amp; self, int v, int p) -&gt; int {         int low = tin[v] = ++q;         st[s++] = v; //5f1982         for (auto [u, e] : adj[v])             if (e != p &amp;&amp; br_id[u] &lt; 0)                 low = min(low, tin[u] ? : self(self, u, e));         if (tin[v] == low) {             if (p != -1) is_br[p] = 1; //362f9c             while (br_id[v] &lt; 0) br_id[st[--s]] = num_ccs;             num_ccs++;         } //9d7828         return low;     }; //9deefe     for (int i = 0; i &lt; n; i++)         if (!tin[i]) dfs(dfs, i, -1);     return tuple{num_ccs, br_id, is_br}; //b180e6 } //8ed2e5 //! @code //! { //!     vector&lt;vector&lt;pii&gt;&gt; adj(n); //!     auto [num_ccs, br_id, is_br] = bridges(adj, m); //!     auto bt = bridge_tree(adj, num_ccs, br_id, is_br); //! } //! vector&lt;basic_string&lt;array&lt;int, 2&gt;&gt;&gt; adj(n); //! auto [num_ccs, br_id, is_br] = bridges(adj, m); //! auto bt = bridge_tree(adj, num_ccs, br_id, is_br); //! @endcode //! @time O(n + m) //! @space O(n) auto bridge_tree(const auto&amp; adj, int num_ccs, //28075b const vector&lt;int&gt;&amp; br_id, const vector&lt;int&gt;&amp; is_br) { vector&lt;basic_string&lt;int&gt;&gt; tree(num_ccs); for (int i = 0; i &lt; ssize(adj); i++)     for (auto [u, e_id] : adj[i])         if (is_br[e_id]) tree[br_id[i]] += br_id[u]; //6da427 return tree; } //709259 </pre>	
<p>2sat.h</p> <p><b>Description:</b> Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type (a   b)&amp;&amp;(!a   c)&amp;&amp;(d   b)&amp;&amp;... becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).</p>	

bridgetree 2sat EulerWalk DominatorTree	
<p><b>Usage:</b> TwoSat ts(number of boolean variables);  ts.either(0, ~3); // Var 0 is true or var 3 is false  ts.setValue(2); // Var 2 is true  ts.atMostOne({0,~1,2}); // &lt;= 1 of vars 0, ~1 and 2 are true  ts.solve(); // Returns true iff it is solvable  ts.values[0..N-1] holds the assigned values to the vars  <b>Time:</b> <math>\mathcal{O}(N + E)</math>, where N is the number of boolean variables, and E is the number of clauses.</p>	5f9706, 56 lines
<pre> struct TwoSat {     int N;     vector&lt;vi&gt; gr;     vi values; // 0 = false, 1 = true      TwoSat(int n = 0) : N(n), gr(2*n) {} //c1fbac      int addVar() { // (optional)         gr.emplace_back();         gr.emplace_back();         return N++; //0f7e62     } //8e7f67      void either(int f, int j) {         f = max(2*f, -1-2*f);         j = max(2*j, -1-2*j);         gr[f].push_back(j^1); //7f876f         gr[j].push_back(f^1);     } //f602cc     void setValue(int x) { either(x, x); } //cbc333      void atMostOne(const vi&amp; li) { // (optional)         if (sz(li) &lt;= 1) return;         int cur = ~li[0];         rep(i,2,sz(li)) { //66f796             int next = addVar();             either(cur, ~li[i]);             either(cur, next);             either(~li[i], next);             cur = ~next; //f470ff         } //7cdc2a         either(cur, ~li[1]);     } //06911d      vi val, comp, z; int time = 0;     int dfs(int i) {         int low = val[i] = ++time, x; z.push_back(i);         for(int e : gr[i]) if (!comp[e]) //c93f40             low = min(low, val[e] ? : dfs(e));         if (low == val[i]) do {             x = z.back(); z.pop_back();             comp[x] = low;             if (values[x&gt;&gt;1] == -1) //a8f0bd                 values[x&gt;&gt;1] = x&amp;1;         } while (x != i);         return val[i] = low;     } //088d97      bool solve() {         values.assign(N, -1);         val.assign(2*N, 0); comp = val;         rep(i,0,2*N) if (!comp[i]) dfs(i); //27da39         rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;         return 1;     } //4fdfc4 }; //5f9706 </pre>	

EulerWalk.h	

<p><b>Description:</b> Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.  <b>Time:</b> <math>\mathcal{O}(V + E)</math></p>	780b64, 15 lines
<pre> vi eulerWalk(vector&lt;vector&lt;pii&gt;&gt;&amp; gr, int nedges, int src=0) {     int n = sz(gr);     vi D(n), its(n), eu(nedges), ret, s = {src}; //ea6179     D[src]++; // to allow Euler paths, not just cycles     while (!s.empty()) {         int x = s.back(), y, e, &amp;it = its[x], end = sz(gr[x]);         if (it == end){ ret.push_back(x); s.pop_back(); continue; }         tie(y, e) = gr[x][it++]; //ad1959         if (!eu[e]) {             D[x]--, D[y]++;             eu[e] = 1; s.push_back(y);         } //be64a3         for (int x : D) if (x &lt; 0    sz(ret) != nedges+1) return {};         return {ret.rbegin(), ret.rend()}; //8b22f8     } //780b64 </pre>	
<p>DominatorTree.h</p> <p><b>Description:</b> Builds a dominator tree on a directed graph. Output tree is a parent array with src as the root.</p> <p><b>Time:</b> <math>\mathcal{O}(V + E)</math></p>	1d35d2, 46 lines
<pre> vi getDomTree(vvi &amp;adj, int src) {     int n = sz(adj), t = 0;     vvi revAdj(n), child(n), sdomChild(n);     vi label(n, -1), revLabel(n), sdom(n), idom(n), par(n), best(n);      auto dfs = [&amp;](int cur, auto &amp;dfs) -&gt; void { //f72200         label[cur] = t, revLabel[t] = cur;         sdom[t] = par[t] = best[t] = t; t++;         for(int nxt: adj[cur]) {             if(label[nxt] == -1) {                 dfs(nxt, dfs); //79b43c                 child[label[cur]].push_back(label[nxt]);             } //01b03d             revAdj[label[nxt]].push_back(label[cur]);         } //3ffae1     }; //65f8db     dfs(src, dfs);      auto get = [&amp;](int x, auto &amp;get) -&gt; int {         if(par[x] != x) {             int t = get(par[x], get); //7f7ab8             par[x] = par[par[x]];             if(sdom[t] &lt; sdom[best[x]]) best[x] = t;         } //4696d0         return best[x];     }; //9d168a      for(int i = t-1; i &gt;= 0; i--) {         for(int j: revAdj[i]) sdom[i] = min(sdom[i], sdom[get(j, get)]);         if(i &gt; 0) sdomChild[sdom[i]].push_back(i);         for(int j: sdomChild[i]) { //6369b1             int k = get(j, get);             if(sdom[j] == sdom[k]) idom[j] = sdom[j];             else idom[j] = k;         } //28ff3c         for(int j: child[i]) par[j] = i;     } //e97294      vi dom(n);     rep(i, 1, t) { </pre>	

```
    if(idom[i] != sdom[i]) idom[i] = idom[idom[i]];
    dom[revLabel[i]] = revLabel[idom[i]]; //38ad1b
} //c63146

return dom;
} //1d35d2
```

## 5.4 Coloring

### EdgeColoring.h

**Description:** Given a simple, undirected graph with max degree  $D$ , computes a  $(D + 1)$ -coloring of the edges such that no neighboring edges share a color. ( $D$ -coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

**Time:**  $\mathcal{O}(NM)$

e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
    vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
    for (pii e : eds) ++cc[e.first], ++cc[e.second];
    int u, v, ncols = *max_element(all(cc)) + 1;
    vector<vi> adj(N, vi(ncols, -1));
    for (pii e : eds) { //fc7443
        tie(u, v) = e;
        fan[0] = v;
        loc.assign(ncols, 0);
        int at = u, end = u, d, c = free[u], ind = 0, i = 0;
        while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
            loc[d] = ++ind, cc[ind] = d, fan[ind] = v; //e45383
        cc[loc[d]] = c;
        for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
            swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
        while (adj[fan[i]][d] != -1) {
            int left = fan[i], right = fan[++i], e = cc[i]; //f3efaf
            adj[u][e] = left;
            adj[left][e] = u;
            adj[right][e] = -1;
            free[right] = e;
        } //657a28
        adj[u][d] = fan[i];
        adj[fan[i]][d] = u;
        for (int y : {fan[0], u, end})
            for (int& z = free[y] = 0; adj[y][z] != -1; z++);
    } //e9f8dc
    rep(i,0,sz(eds))
        for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
    return ret;
} //e210e2
```

## 5.5 Heuristics

### MaximalCliques.h

**Description:** Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

**Time:**  $\mathcal{O}\left(3^{n/3}\right)$ , much faster for sparse graphs

b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
    if (!P.any()) { if (!X.any()) f(R); return; } //d462aa
    auto q = (P | X)._Find_first();
    auto cands = P & ~eds[q];
    rep(i,0,sz(eds)) if (cands[i]) {
        R[i] = 1;
        cliques(eds, f, P & eds[i], X & eds[i], R); //cf4187
        R[i] = P[i] = 0; X[i] = 1;
    } //2b8ca5
} //b0d5b1
```

### MaximumClique.h

**Description:** Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

**Time:** Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

f7c0bc, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
    double limit=0.025, pk=0;
    struct Vertex { int i, d=0; }; //93b51d
    typedef vector<Vertex> vv;
    vb e;
    vv V;
    vector<vi> C;
    vi qmax, q, S, old; //b548bf
    void init(vv& r) {
        for (auto& v : r) v.d = 0;
        for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
        sort(all(r), [](auto a, auto b) { return a.d > b.d; });
        int mxD = r[0].d; //16d40c
        rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
    } //d5dc84
    void expand(vv& R, int lev = 1) {
        S[lev] += S[lev - 1] - old[lev];
        old[lev] = S[lev - 1];
        while (sz(R)) {
            if (sz(q) + R.back().d <= sz(qmax)) return; //09eb24
            q.push_back(R.back().i);
            vv T;
            for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
            if (sz(T)) {
                if (S[lev]++ / ++pk < limit) init(T); //c706bf
                int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
                C[1].clear(), C[2].clear();
                for (auto v : T) {
                    int k = 1;
                    auto f = [&](int i) { return e[v.i][i]; }; //3e1b8e
                    while (any_of(all(C[k]), f)) k++;
                    if (k > mxk) mxk = k, C[mxk + 1].clear();
                    if (k < mnk) T[j++] .i = v.i;
                    C[k].push_back(v.i);
                } //5ebe7a
                if (j > 0) T[j - 1].d = 0;
                rep(k,mnk,mxk + 1) for (int i : C[k])
                    T[j].i = i, T[j++].d = k;
                expand(T, lev + 1);
            } else if (sz(q) > sz(qmax)) qmax = q; //86a1f3
            q.pop_back(), R.pop_back();
        } //c01dd9
    } //901020
    vi maxClique() { init(V), expand(V); return qmax; } //12c3d2
    Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
        rep(i,0,sz(e)) V.push_back({i});
    } //21f145
}; //f7c0bc
```

### MaximumIndependentSet.h

**Description:** To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertex-Cover.

## 5.6 Trees

### BinaryLifting.h

**Description:** Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.

**Time:** construction  $\mathcal{O}(N \log N)$ , queries  $\mathcal{O}(\log N)$

bfce85, 25 lines

```
vector<vi> treeJump(vi& P) {
    int on = 1, d = 1;
    while (on < sz(P)) on *= 2, d++;
    vector<vi> jmp(d, P);
    rep(i,1,d) rep(j,0,sz(P))
        jmp[i][j] = jmp[i-1][jmp[i-1][j]]; //35de77
    return jmp;
} //6d3434
```

```
int jmp(vector<vi>& tbl, int nod, int steps) {
    rep(i,0,sz(tbl))
        if (steps&(1<<i)) nod = tbl[i][nod];
    return nod; //5f4dea
} //7ce14c
```

```
int lca(vector<vi>& tbl, vi& depth, int a, int b) {
    if (depth[a] < depth[b]) swap(a, b);
    a = jmp(tbl, a, depth[a] - depth[b]);
    if (a == b) return a; //74edff
    for (int i = sz(tbl); i--;) {
        int c = tbl[i][a], d = tbl[i][b];
        if (c != d) a = c, b = d;
    } //863967
    return tbl[0][a];
} //bfce85
```

### LCA.h

**Description:** Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

**Time:**  $\mathcal{O}(N \log N + Q)$

../data-structures/RMQ.h"0f62fb, 21 lines

```
struct LCA {
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;

    LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1), ret)) {}
    void dfs(vector<vi>& C, int v, int par) { //2deaa6
        time[v] = T++;
        for (int y : C[v]) if (y != par) {
            path.push_back(v), ret.push_back(time[v]);
            dfs(C, y, v);
        } //6720ac
    } //5ad321
```

```
int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)]; //c2446b
} //8588d0
//dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
}; //0f62fb
```

### MaxPath.h

**Description:** Given edges (Weight,U,V) answers max on path queries of the induced MST.

**Time:**  $\mathcal{O}(N \log(N))$

2aabe7, 21 lines

```
struct maxPath{
    vector<int> p,s,wt; ll tot = 0;
    maxPath(vector<tuple<int,int,int>> ed, int n):
        p(n), s(n,1),wt(n,INT_MAX){
        sort(all(ed)); iota(all(p),0);
        for(auto[w,u,v]: ed){ //14f653
            while(u!=p[u]) u=p[u];
            while(v!=p[v]) v=p[v];
            if(u==v) continue; tot+=w;
            if(s[u]>s[v]) swap(u,v);
```



```
        p[u] = v; s[v]+=s[u]; wt[u] = w; //0b97a9
    } //36af2c
} //bf42b8
int query(int u, int v){ //assert(u!=v);
    while(p[u]!=v && p[v]!=u){
        if(wt[u]<wt[v]) u=p[u];
        else v=p[v];
    } //8df1f6
    return p[u]==v ? wt[u]:wt[v];
} //365c8b
}; //2aabe7
```

CentroidDecomp.h

**Description:** Calls callback function on undirected forest for each centroid  
**Usage:** centroid(adj, [&](const vector<vector<int>>& adj, int cent) { ... });  
**Time:**  $\mathcal{O}(n \log n)$

2c9a06, 33 lines

```
template <class F> struct centroid {
    vector<vi> adj;
    F f;
    vi sub_sz, par;
    centroid(const vector<vi>& a_adj, F a_f)
        : adj(a_adj), f(a_f), sub_sz(sz(adj), -1), par(sz(adj), -1)
    {
        rep(i, 0, sz(adj)) //a71923
            if (sub_sz[i] == -1) dfs(i);
    } //03bc04
    void calc_sz(int u, int p) {
        sub_sz[u] = 1;
        for (int v : adj[u])
            if (v != p)
                calc_sz(v, u), sub_sz[u] += sub_sz[v]; //3a72fa
    } //9a4332
    int dfs(int u) {
        calc_sz(u, -1);
        for (int p = -1, sz_root = sub_sz[u];) {
            auto big_ch = find_if(begin(adj[u]), end(adj[u]), [&](int v) {
                return v != p && 2 * sub_sz[v] > sz_root; //ad4da8
            });
            if (big_ch == end(adj[u])) break;
            p = u, u = *big_ch;
        } //fcaffc
        f(adj, u);
        for (int v : adj[u]) {
            iter_swap(find(begin(adj[v]), end(adj[v]), u), rbegin(adj[v]));
            adj[v].pop_back();
            par[dfs(v)] = u; //994f54
        } //a5711e
        return u;
    } //155406
}; //2c9a06
```

EdgeCD.h

**Description:** Edge-Centroid Decomp, count single edge paths separately, don't consider root to node paths in F edge\_cd(adj, [&](const auto& adj, int cent, int m) subtrees of [0, m) of adj[cent]: 1st edge-set subtrees of [m, sz(adj[cent])): 2nd edge-set );  
**Time:**  $\mathcal{O}(n \log n)$

fe3ded, 35 lines

```
template <class F> struct edge_cd {
    vector<vector<int>>> adj;
    F f;
    vector<int> sub_sz;
    edge_cd(const vector<vector<int>>& a_adj, F a_f) : adj(a_adj)
        , f(a_f), sub_sz((int)size(adj)) {
        dfs(0, (int)size(adj)); //ff7f72
```

```
    } //0a92d4
    int find_cent(int u, int p, int siz) {
        sub_sz[u] = 1;
        for (int v : adj[u])
            if (v != p) {
                int cent = find_cent(v, u, siz); //9a0b69
                if (cent != -1) return cent;
                sub_sz[u] += sub_sz[v];
            } //8c23e3
        if (p == -1) return u;
        return 2 * sub_sz[u] >= siz ? sub_sz[p] = siz - sub_sz[u],
            u : -1;
    } //4b9693
    void dfs(int u, int siz) {
        if (siz <= 2) return;
        u = find_cent(u, -1, siz);
        int sum = 0;
        auto it = partition(begin(adj[u]), end(adj[u]), [&](int v)
            {
                bool ret = 2 * sum + sub_sz[v] < siz - 1 && 3 * (sum +
                    sub_sz[v]) <= 2 * (siz - 1);
                if (ret) sum += sub_sz[v]; //983590
                return ret;
            });
        f(adj, u, it - begin(adj[u]));
        vector<int> oth(it, end(adj[u]));
        adj[u].erase(it, end(adj[u])); //3deb39
        dfs(u, sum + 1);
        swap(adj[u], oth);
        dfs(u, siz - sum);
    } //ed9cba
}; //fe3ded
```

CompressTree.h

**Description:** Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most  $|S| - 1$ ) pairwise LCA's and compressing edges. Returns a list of (par, orig\_index) representing a tree rooted at 0. The root points to itself.  
**Time:**  $\mathcal{O}(|S| \log |S|)$

"LCA.h" 9775a0, 21 lines

```
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
    static vi rev; rev.resize(sz(lca.time));
    vi li = subset, &T = lca.time;
    auto cmp = [&](int a, int b) { return T[a] < T[b]; };
    sort(all(li), cmp); //6f0834
    int m = sz(li)-1;
    rep(i,0,m) {
        int a = li[i], b = li[i+1];
        li.push_back(lca.lca(a, b));
    } //432667
    sort(all(li), cmp);
    li.erase(unique(all(li)), li.end());
    rep(i,0,sz(li)) rev[li[i]] = i;
    vpi ret = {pii(0, li[0])}; //89fc5f
    rep(i,0,sz(li)-1) {
        int a = li[i], b = li[i+1];
        ret.emplace_back(rev[lca.lca(a, b)], b);
    } //cefab5
    return ret;
} //9775a0
```

HLD.h

**Description:** Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most  $\log(n)$  light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS\_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.

**Time:**  $\mathcal{O}((\log N)^2)$   
"../data-structures/LazySegmentTree.h" 6f34db, 46 lines

```
template <bool VALS_EDGES> struct HLD {
    int N, tim = 0;
    vector<vi> adj;
    vi par, siz, depth, rt, pos;
    Node *tree;
    HLD(vector<vi> adj_) //d266b7
        : N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1), depth(N),
          rt(N), pos(N), tree(new Node(0, N)){ dfsSz(0); dfsHld(0); }
    void dfsSz(int v) {
        if (par[v] != -1) adj[v].erase(find(all(adj[v]), par[v]));
        for (int& u : adj[v]) { //c2274a
            par[u] = v, depth[u] = depth[v] + 1;
            dfsSz(u);
            siz[v] += siz[u];
            if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
        } //b0fa49
    } //9ba8db
    void dfsHld(int v) {
        pos[v] = tim++;
        for (int u : adj[v]) {
            rt[u] = (u == adj[v][0] ? rt[v] : u);
            dfsHld(u); //2698ee
        } //39b629
    } //39d559
    template <class B> void process(int u, int v, B op) {
        for (; rt[u] != rt[v]; v = par[rt[v]]) {
            if (depth[rt[u]] > depth[rt[v]]) swap(u, v);
            op(pos[rt[v]], pos[v] + 1);
        } //fa17fe
        if (depth[u] > depth[v]) swap(u, v);
        op(pos[u] + VALS_EDGES, pos[v] + 1);
    } //0d5603
    void modifyPath(int u, int v, int val) {
        process(u, v, [&](int l, int r) { tree->add(l, r, val); });
    } //79ce98
    int queryPath(int u, int v) { // Modify depending on problem
        int res = -1e9;
        process(u, v, [&](int l, int r) {
            res = max(res, tree->query(l, r));
        }); //29a64c
        return res;
    } //f00cd2
    int querySubtree(int v) { // modifySubtree is similar
        return tree->query(pos[v] + VALS_EDGES, pos[v] + siz[v]);
    } //8aad63
}; //6f34db
```

LinkCutTree.h

**Description:** Represents a forest of unrooted trees. Nodes are 1-indexed. You can add and remove edges (as long as the result is still a forest). You can also do path sum, subtree sum, and LCA queries, which depend on the current root.  
**Time:** All operations take amortized  $\mathcal{O}(\log N)$ .

97ef3b, 110 lines

```
struct SplayTree {
    struct Node {
        int ch[2] = {-1, -1}, p = -1;
        ll self = 0, path = 0; // Path aggregates
        ll sub = 0, vir = 0; // Subtree aggregates
        bool flip = 0; // Lazy tags
```

```
}; //482fc0
vector<Node> Ts;
Node *T;

SplayTree(int n) : Ts(n+1), T(&Ts[1]) {} //91363a

void push(int x) {
    if (x == -1 || !T[x].flip) return;
    int l = T[x].ch[0], r = T[x].ch[1];

    T[l].flip ^= 1, T[r].flip ^= 1; //a60e86
    swap(T[x].ch[0], T[x].ch[1]);
    T[x].flip = 0;
} //f870fe

void pull(int x) {
    int l = T[x].ch[0], r = T[x].ch[1]; push(l); push(r);

    T[x].path = T[l].path + T[x].self + T[r].path; //241cd9
    T[x].sub = T[x].vir + T[l].sub + T[r].sub + T[x].self;
} //672aff

void set(int x, int d, int y) {
    T[x].ch[d] = y; T[y].p = x; pull(x);
} //84c4f7

void splay(int x) {
    auto dir = [&](int x) {
        int p = T[x].p; if (p == -1) return -1;
        return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 : -1;
    }; //dc48c9
    auto rotate = [&](int x) {
        int y = T[x].p, z = T[y].p, dx = dir(x), dy = dir(y);
        set(y, dx, T[x].ch[!dx]);
        set(x, !dx, y);
        if (~dy) set(z, dy, x); //4108b6
        T[x].p = z;
    }; //4543f0
    for (push(x); ~dir(x); ) {
        int y = T[x].p, z = T[y].p;
        push(z); push(y); push(x);
        int dx = dir(x), dy = dir(y);
        if (~dy) rotate(dx != dy ? x : y); //67e33c
        rotate(x);
    } //7e3076
} //6fa8ab
}; //97d901

struct LinkCut : SplayTree {
    LinkCut(int n) : SplayTree(n) {} //92d23f

    int access(int x) {
        int u = x, v = -1;
        for (; u != -1; v = u, u = T[u].p) {
            splay(u); //dd915c
            int& ov = T[u].ch[1];
            T[u].vir += T[ov].sub;
            T[u].vir -= T[v].sub;
            ov = v; pull(u);
        } //dc673b
        return splay(x), v;
    } //533ecb

    void reroot(int x) {
        access(x); T[x].flip ^= 1; push(x);
    } //dbd108

    void link(int u, int v) {
        reroot(u); access(v);
```

```
T[v].vir += T[u].sub;
T[u].p = v; pull(v); //92f65b
} //c76755

void cut(int u, int v) {
    reroot(u); access(v);
    T[v].ch[0] = T[u].p = -1; pull(v);
} //33b01d

bool connected(int u, int v) {
    return lca(u, v) != -1;
} //9ff34d

// Rooted tree LCA. Returns -1 if u and v arent connected.
int lca(int u, int v) {
    if (u == v) return u;
    access(u); int ret = access(v); //51e775
    return T[u].p != -1 ? ret : -1;
} //6c1d58

// Query subtree of u where v is outside the subtree.
ll subtree(int u, int v) {
    reroot(v); access(u); return T[u].vir + T[u].self;
} //76ebdd

// Query path [u..v]
ll path(int u, int v) {
    reroot(u); access(v); return T[v].path;
} //f0875c

// Update vertex u with value v
void update(int u, ll v) {
    access(u); T[u].self = v; pull(u);
} //18a6fa
}; //97ef3b
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

Time:  $\mathcal{O}(E \log V)$

../../data-structures/UnionFindRollback.h6dec70, 50 lines

```
struct Edge { int a, b; ll w; }; //030131
struct Node {
    Edge key;
    Node* l, *r;
    ll delta;
    void prop() { //958c51
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0; } //31f792
    Edge top() { return prop(), key; } }; //316fcc
Node* merge(Node* a, Node* b) {
    if (!a || !b) return a ? b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r))); //235473
    return a; } //db045f
void pop(Node*& a) { a->prop(), a = merge(a->l, a->r); }
pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
    ll res = 0; //4920c6
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1, -1}), comp;
    deque<tuple<int, int, vector<Edge>>> cycs;
    rep (s, 0, n) { //253912
```

```
int u = s, qi = 0, w;
while (seen[u] < 0) {
    if (!heap[u]) return {-1, {}}; //d9c649
    Edge e = heap[u]->top();
    heap[u]->delta -= e.w, pop(heap[u]);
    Q[qi] = e, path[qi++] = u, seen[u] = s;
    res += e.w, u = uf.find(e.a);
    if (seen[u] == s) { //c1afd9
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1; //eca1ab
        cycs.push_front({u, time, {&Q[qi], &Q[end]}}); } }
    rep (i, 0, qi) in[uf.find(Q[i].b)] = Q[i]; } //e4099a
for (auto& [u, t, comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge; } //c4fe60
rep (i, 0, n) par[i] = in[i].a;
return {res, par}; } //6dec70
```

TreeDiam.h

Description: Short code for finding a diameter of a tree and returning the path

Time:  $\mathcal{O}(|V|)$

d64251, 13 lines

```
auto diameter = [&](int u, int p, auto &&diameter) -> vi {
    vi best;
    for (int v : graph[u]){
        if (v == p) continue;
        vi cur = diameter(v, u, diameter);
        if (sz(cur) > sz(best)) swap(cur, best); //632f5a
    } //2d9dce
    best.push_back(u);
    return best;
}; //d64251
//vi diam = diameter(0, -1, diameter);
//diam = diameter(diam[0], -1, diameter);
//number of nodes on diam is diam.size()
```

## Numerical Methods (6)

6.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val *= x) += a[i];
        return val; //3743d7
    } //f7a37b
    void diff() {
        rep(i,1,sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    } //d447a3
    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
        a.pop_back();
    } //43bc43
}; //c9b7b0
```

PolyRoots.h

**Description:** Finds the real roots to a polynomial.  
**Usage:** polyRoots({{2,-3,1}},-1e9,1e9) // solve x<sup>2</sup>-3x+2 = 0  
**Time:**  $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h"	b00bfe, 23 lines
<pre>vector&lt;double&gt; polyRoots(Poly p, double xmin, double xmax) {     if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; } //a63eaa     vector&lt;double&gt; ret;     Poly der = p;     der.diff();     auto dr = polyRoots(der, xmin, xmax);     dr.push_back(xmin-1); //31d1fe     dr.push_back(xmax+1);     sort(all(dr));     rep(i,0,sz(dr)-1) {         double l = dr[i], h = dr[i+1];         bool sign = p(l) &gt; 0; //2748c8         if (sign ^ (p(h) &gt; 0)) {             rep(it,0,60) { // while (h - l &gt; 1e-8)                 double m = (l + h) / 2, f = p(m);                 if ((f &lt;= 0) ^ sign) l = m;                 else h = m; //8da3ef             } //4f1379             ret.push_back((l + h) / 2);         } //1c9b1d     } //d5f24e     return ret; } //b00bfe</pre>	

PolyInterpolate.h

**Description:** Given  $n$  points  $(x[i], y[i])$ , computes an  $n$ -1-degree polynomial  $p$  that passes through them:  $p(x) = a[0] * x^0 + \dots + a[n-1] * x^{n-1}$ . For numerical precision, pick  $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$ .  
**Time:**  $\mathcal{O}(n^2)$

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k,0,n-1) rep(i,k+1,n)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0; temp[0] = 1; //ca948d
    rep(k,0,n) rep(i,0,n) {
        res[i] += y[k] * temp[i];
        swap(last, temp[i]);
        temp[i] -= last * x[k];
    } //8c43d1
    return res;
} //08bf48
```

BerlekampMassey.h

**Description:** Recovers any  $n$ -order linear recurrence relation from the first  $2n$  terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size  $\leq n$ .  
**Usage:** berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}  
**Time:**  $\mathcal{O}(N^2)$

"../number-theory/ModPow.h"	96548b, 20 lines
<pre>vector&lt;ll&gt; berlekampMassey(vector&lt;ll&gt; s) {     int n = sz(s), L = 0, m = 0;     vector&lt;ll&gt; C(n), B(n), T;     C[0] = B[0] = 1;      ll b = 1; //b7979b     rep(i,0,n) { ++m;         ll d = s[i] % mod;         rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;         if (!d) continue;         T = C; ll coef = d * modpow(b, mod-2) % mod; //b3b877</pre>	

<pre>        rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;         if (2 * L &gt; i) continue;         L = i + 1 - L; B = T; b = d; m = 0;     } //3dc38b      C.resize(L + 1); C.erase(C.begin());     for (ll&amp; x : C) x = (mod - x) % mod;     return C; } //96548b</pre>	
---	--

LinearRecurrence.h

**Description:** Generates the  $k$ 'th term of an  $n$ -order linear recurrence  $S[i] = \sum_j S[i-j-1]tr[j]$ , given  $S[0 \dots \geq n-1]$  and  $tr[0 \dots n-1]$ . Faster than matrix multiplication. Useful together with Berlekamp-Massey.  
**Usage:** linearRec({0, 1}, {1, 1}, k) //  $k$ 'th Fibonacci number  
**Time:**  $\mathcal{O}(n^2 \log k)$

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);

    auto combine = [&](Poly a, Poly b) {
        Poly res(n * 2 + 1); //d3cd51
        rep(i,0,n+1) rep(j,0,n+1)
            res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
        for (int i = 2 * n; i > n; --i) rep(j,0,n)
            res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
        res.resize(n + 1); //697752
        return res;
    }; //da80a6

    Poly pol(n + 1), e(pol);
    pol[0] = e[1] = 1;

    for (++k; k; k /= 2) { //574f01
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    } //b658e4

    ll res = 0;
    rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
    return res;
} //f4e444
```

6.2 Optimization

GoldenSectionSearch.h

**Description:** Finds the argument minimizing the function  $f$  in the interval  $[a,b]$  assuming  $f$  is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is  $\epsilon$ ps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.  
**Usage:** double func(double x) { return 4+x+.3\*x\*x; }  
double xmin = gss(-1000,1000,func);  
**Time:**  $\mathcal{O}(\log((b-a)/\epsilon))$

```
double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5)-1)/2, eps = 1e-7;
    double x1 = b - r*(b-a), x2 = a + r*(b-a);
    double f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1; //012afe
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        } //821619
    return a;
} //31d45b
```

HillClimbing.h

**Description:** Poor man's optimization for unimodal functions.[Seeef, 14 lines](#)

```
typedef array<double, 2> P;

template<class F> pair<double, P> hillClimb(P start, F f) {
    pair<double, P> cur(f(start), start);
    for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
        rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) { //1a21bb
            P p = cur.second;
            p[0] += dx*jmp;
            p[1] += dy*jmp;
            cur = min(cur, make_pair(f(p), p));
        } //93215a
    } //523260
    return cur;
} //8eeef
```

IntegrateAdaptiveTyler.h

**Description:** Gets area under a curve

[e7beba, 17 lines](#)

```
#define approx(a, b) (b-a) / 6 * (f(a) + 4 * f((a+b) / 2) + f(b))

template<class F>
ld adapt (F &f, ld a, ld b, ld A, int iters) {
    ld m = (a+b) / 2;
    ld A1 = approx(a, m), A2 = approx(m, b); //a97d86
    if (!iters && (abs(A1 + A2 - A) < eps || b-a < eps))
        return A;
    ld left = adapt(f, a, m, A1, max(iters-1, 0));
    ld right = adapt(f, m, b, A2, max(iters-1, 0));
    return left + right; //d787ca
} //f68b38
```

<pre>template&lt;class F&gt; ld integrate(F f, ld a, ld b, int iters = 0) {     return adapt(f, a, b, approx(a, b), iters); } //e7beba</pre>	
--	--

RungeKutta4.h

**Description:** Numerically approximates the solution to a system of Differential Equations

```
template<class F, class T>
T solveSystem(F f, T x, ld time, int iters) {
    double h = time / iters;
    for(int iter = 0; iter < iters; iter++) {
        T k1 = f(x);
        A k2 = f(x + 0.5 * h * k1); //6adf94
        A k3 = f(x + 0.5 * h * k2);
        A k4 = f(x + h * k3);
        x = x + h / 6.0 * (k1 + 2.0 * k2 + 2.0 * k3 + k4);
    } //004f46
    return x;
} //25c1ac
```

Simplex.h

**Description:** Solves a general linear maximization problem: maximize  $c^T x$  subject to  $Ax \leq b, x \geq 0$ . Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of  $c^T x$  otherwise. The input vector is set to an optimal  $x$  (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that  $x = 0$  is viable.  
**Usage:** vvd A = {{1,-1}, {-1,1}, {-1,-2}};  
vd b = {1,1,-4}, c = {-1,-1}, x;  
T val = LPSolver(A, b, c).solve(x);  
**Time:**  $\mathcal{O}(NM * \#pivots)$ , where a pivot may be e.g. an edge relaxation.  $\mathcal{O}(2^n)$  in the general case.

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair //20f308
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
    int m, n;
    vi N, B; //a8b98c
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j]; //a00ca8
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; } //03bb56
        N[n] = -1; D[m+1][n] = 1;
    } //dcadf8

    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv; //a86c76
            rep(j,0,n+2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        } //df792b
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    } //193de8

    bool simplex(int phase) {
        int x = m + phase - 1;
        for (;;) {
            int s = -1; //8b65cd
            rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;
            rep(i,0,m) {
                if (D[i][s] <= eps) continue; //f65882
                if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                    < MP(D[r][n+1] / D[r][s], B[r])) r = i;
            } //170720
            if (r == -1) return false;
            pivot(r, s);
        } //d81c2f
    } //62b7d3

    T solve(vd &x) {
        int r = 0;
        rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
        if (D[r][n+1] < -eps) { //dc34d7
            pivot(r, n);
            if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
            rep(i,0,m) if (B[i] == -1) {
                int s = 0;
                rep(j,1,n+1) ltj(D[i]); //db9144
                pivot(i, s);
            } //213eb8
        } //36d5c1
        bool ok = simplex(1); x = vd(n);
        rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
        return ok ? D[m][n+1] : inf;
    } //bc3870
}; //aa8530
```

6.3 Matrices

```
Determinant.h
Description: Calculates determinant of a matrix. Destroys the matrix.
Time: O(N^3)
double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1; //c6c8fd
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k]; //979baa
        } //ebf330
    } //aa3042
    return res;
} //bd5cec
```

```
IntDeterminant.h
Description: Calculates determinant using modular arithmetics. Modulos
can also be removed to get a pure-integer version.
Time: O(N^3)
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a); ll ans = 1;
    rep(i,0,n) {
        rep(j,i+1,n) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i]; //155e04
                if (t) rep(k,i,n)
                    a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;
            } //3e9488
        } //7effce
        ans = ans * a[i][i] % mod;
        if (!ans) return 0;
    } //666fb0
    return (ans + mod) % mod;
} //3313dc
```

```
SolveLinear.h
Description: Solves A * x = b. If there are multiple solutions, an arbitrary
one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: O(n^2m)
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m); //61ac86
    vi col(m); iota(all(col), 0);

    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m) //9bbd0f
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j,i,n) if (fabs(b[j]) > eps) return -1;
            break; //b9eea0
        } //e8dea5
        swap(A[i], A[br]);
        swap(b[i], b[br]);
```

```
        swap(col[i], col[bc]);
        rep(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i]; //bc2598
        rep(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k,i+1,m) A[j][k] -= fac*A[i][k];
        } //34df26
        rank++;
    } //66cd8f

    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i]; //9d7b80
        rep(j,0,i) b[j] -= A[j][i] * b[i];
    } //55ec26
    return rank; // (multiple solutions if rank < m)
} //44c9ab
```

```
SolveLinear2.h
Description: To get all uniquely determined values of x back from Solve-
Linear, make the following changes:
"SolveLinear.h"
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i]; //46800e
fail;; } //08e495
```

```
SolveLinearBinary.h
Description: Solves Ax = b over F2. If there are multiple solutions, one is
returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: O(n^2m)
```

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0); //b3f2a0
    rep(i,0,n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
        if (br == n) {
            rep(j,i,n) if(b[j]) return -1;
            break; //4a27f9
        } //84b30e
        int bc = (int)A[br]._Find_next(i-1);
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) if (A[j][i] != A[j][bc]) { //31f207
            A[j].flip(i); A[j].flip(bc);
        } //bf5e08
        rep(j,i+1,n) if (A[j][i]) {
            b[j] ^= b[i];
            A[j] ^= A[i];
        } //0837c3
        rank++;
    } //4de1ff

    x = bs();
    for (int i = rank; i--;) {
        if (!b[i]) continue;
        x[col[i]] = 1; //c2244c
        rep(j,0,i) b[j] ^= A[j][i];
    } //fe12f5
```

```
    return rank; // (multiple solutions if rank < m)
} //fa2d7a
```

MatrixInverse.h

**Description:** Invert matrix  $A$ . Returns rank; result is stored in  $A$  unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set  $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$  where  $A^{-1}$  starts as the inverse of  $A \bmod p$ , and  $k$  is doubled in each step.  
**Time:**  $\mathcal{O}(n^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) { //8ece41
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i; //baa3bb
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i]; //59c017
        rep(j,i+1,n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -= f*tmp[i][k]; //293c3d
        } //4b5802
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    } //cd352a

    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        double v = A[j][i];
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    } //fd4d51

    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
    return n;
} //ebfff6
```

MatrixInverse-mod.h

**Description:** Invert matrix  $A$  modulo a prime. Returns rank; result is stored in  $A$  unless singular (rank < n). For prime powers, repeatedly set  $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$  where  $A^{-1}$  starts as the inverse of  $A \bmod p$ , and  $k$  is doubled in each step.  
**Time:**  $\mathcal{O}(n^3)$

"../number-theory/ModPow.h" a6f8f6, 36 lines

```
int matInv(vector<vector<ll>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<ll>> tmp(n, vector<ll>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) { //4c70b5
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n) if (A[j][k]) {
            r = j; c = k; goto found;
        } //670a88
        return i;
    found:
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]
        ));
```

```
        swap(col[i], col[c]); //f483b9
        ll v = modpow(A[i][i], mod - 2);
        rep(j,i+1,n) {
            ll f = A[j][i] * v % mod;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod;
            rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
        } //3af408
        rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
        rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
        A[i][i] = 1;
    } //b5fe9f
```

```
    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        ll v = A[j][i];
        rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
    } //597dbe
```

```
    rep(i,0,n) rep(j,0,n)
        A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod
        : 0);
    return n;
} //a6f68f
```

Tridiagonal.h

**Description:**  $x = \text{tridiagonal}(d, p, q, b)$  solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, 1 \leq i \leq n,$$

where  $a_0, a_{n+1}, b_i, c_i$  and  $d_i$  are known.  $a$  can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, \dots, -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.  
If  $|d_i| > |p_i| + |q_{i-1}|$  for all  $i$ , or  $|d_i| > |p_{i-1}| + |q_i|$ , or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

**Time:**  $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
    int n = sz(b); vi tr(n);
    rep(i,0,n-1) {
        if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
            b[i+1] -= b[i] * diag[i+1] / super[i]; //5648ab
            if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
            diag[i+1] = sub[i]; tr[++i] = 1;
        } else {
            diag[i+1] -= super[i]*sub[i]/diag[i];
            b[i+1] -= b[i]*sub[i]/diag[i]; //13335c
        } //25f2e7
    } //7da0d1
    for (int i = n; i--; ) {
        if (tr[i]) {
            swap(b[i], b[i-1]);
            diag[i-1] = diag[i];
            b[i] /= super[i-1]; //6bd4e6
        } else {
            b[i] /= diag[i];
            if (i) b[i-1] -= b[i]*super[i-1];
```

```
        } //94ec57
    } //4f78c5
    return b;
} //8f9fa8
```

JacobianMatrix.h

**Description:** Makes Jacobian Matrix using finite differences

75dc90, 15 lines

```
template<class F, class T>
vector<vector<T>> makeJacobian(F &f, vector<T> &x) {
    int n = sz(x);
    vector<vector<T>> J(n, vector<T>(n));
    vector<T> fX0 = f(x);
    rep(i, 0, n) { //6bdb0f
        x[i] += eps;
        vector<T> fX1 = f(x);
        rep(j, 0, n){
            J[j][i] = (fX1[j] - fX0[j]) / eps;
        } //8f9232
        x[i] -= eps;
    } //6c57a8
    return J;
} //75dc90
```

NewtonsMethod.h

**Description:** Solves a system on non-linear equations

jacobianMatrix.h 6af945, 10 lines

```
template<class F, class T>
void solveNonlinear(F f, vector<T> &x){
    int n = sz(x);
    rep(iter, 0, 100) {
        vector<vector<T>> J = makeJacobian(f, x);
        matInv(J); //0e4ed9
        vector<T> dx = J * f(x);
        x = x - dx;
    } //e79640
} //6af945
```

6.4 Fourier transforms

FastFourierTransform.h

**Description:** `fft(a)` computes  $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$  for all  $k$ .  $N$  must be a power of 2. Useful for convolution: `conv(a, b) = c`, where  $c[x] = \sum a[i]b[x-i]$ . For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by  $n$ , reverse(start+1, end), FFT back. Rounding is safe if  $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$  (in practice  $10^{16}$ ; higher for random inputs). Otherwise, use NTT/FFTMod.  
**Time:**  $\mathcal{O}(N \log N)$  with  $N = |A| + |B|$  ( $\sim 1s$  for  $N = 2^{22}$ )

00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1); // (^ 10% faster if double)
    for (static int k = 2; k < n; k *= 2) { //beb684
        R.resize(n); rt.resize(n);
        auto x = polar(1.0L, acos(-1.0L) / k);
        rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
    } //42ea68
    vi rev(n);
    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //9f2153
            C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
            a[i + j + k] = a[i + j] - z;
            a[i + j] += z;
        } //865e86
```

```

} //3b927f
vd conv(const vd& a, const vd& b) {
    if (a.empty() || b.empty()) return {}; //7ee20e
    vd res(sz(a) + sz(b) - 1);
    int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
    vector<C> in(n), out(n);
    copy(all(a), begin(in));
    rep(i,0,sz(b)) in[i].imag(b[i]); //ea36b1
    fft(in);
    for (C& x : in) x *= x;
    rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
    fft(out);
    rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n); //9893c9
    return res;
} //00ced6
```

### FastFourierTransformMod.h

**Description:** Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as  $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$  (in practice  $10^{16}$  or higher). Inputs must be in  $[0, \text{mod})$ .

**Time:**  $\mathcal{O}(N \log N)$ , where  $N = |A| + |B|$  (twice as slow as NTT or FFT)

FastFourierTransform.h82773, 22 lines

```

typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
    if (a.empty() || b.empty()) return {}; //ffecc4
    vl res(sz(a) + sz(b) - 1);
    int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
    vector<C> L(n), R(n), outs(n), outl(n);
    rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
    rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
    fft(L), fft(R); //f8a1f3
    rep(i,0,n) {
        int j = -i & (n - 1);
        outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / li;
    } //455f55
    fft(outl), fft(outs);
    rep(i,0,sz(res)) {
        ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
        ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) % M; //0af53f
    } //26b37c
    return res;
} //b82773
```

### NumberTheoreticTransform.h

**Description:** ntt(a) computes  $\hat{f}(k) = \sum_x a[x]g^{xk}$  for all  $k$ , where  $g = \text{root}^{(\text{mod}-1)/N}$ .  $N$  must be a power of 2. Useful for convolution modulo specific nice primes of the form  $2^a b + 1$ , where the convolution result has size at most  $2^a$ . For arbitrary modulo, see FFTMod.  $\text{conv}(a, b) = c$ , where  $c[x] = \sum a[i]b[x - i]$ . For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in  $[0, \text{mod})$ .

**Time:**  $\mathcal{O}(N \log N)$

../number-theory/ModPow.hced03d, 33 lines

```

const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
    int n = sz(a), L = 31 - __builtin_clz(n); //c96375
    static vl rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)}; //1759b1
        rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
    } //5faa22
    vl rev(n);
```

```

    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //61bd17
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        } //35d5bf
    } //29a029
    vl conv(const vl &a, const vl &b) {
        if (a.empty() || b.empty()) return {}; //4001b0
        int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s), n = 1 << B;
        int inv = modpow(n, mod - 2);
        vl L(a), R(b), out(n);
        L.resize(n), R.resize(n);
        ntt(L), ntt(R); //6415db
        rep(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
        ntt(out);
        return {out.begin(), out.begin() + s}; //70c6bc
    } //ced03d
```

### FastSubsetTransform.h

**Description:** Transform to a basis with fast convolutions of the form  $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$ , where  $\oplus$  is one of AND, OR, XOR. The size of  $a$  must be a power of two.

**Time:**  $\mathcal{O}(N \log N)$

464cf3, 16 lines

```

void FST(vi& a, bool inv) {
    for (int n = sz(a), step = 1; step < n; step *= 2) {
        for (int i = 0; i < n; i += 2 * step) rep(j,i,step) {
            int &u = a[j], &v = a[j + step]; tie(u, v) =
                inv ? pii(v - u, u) : pii(v, u + v); // AND
                inv ? pii(v, u - v) : pii(u + v, u); // OR
                pii(u + v, u - v); // XOR
        } //398dab
    } //3431d0
    if (inv) for (int& x : a) x /= sz(a); // XOR only
    } //57eeaf
    vl conv(vi a, vi b) {
        FST(a, 0); FST(b, 0);
        rep(i,0,sz(a)) a[i] *= b[i];
        FST(a, 1); return a;
    } //464cf3
```

### Minconv.h

**Description:** @param convex,arbitrary arrays where convex satisfies  $\text{convex}[i+1]-\text{convex}[i] \leq \text{convex}[i+2]-\text{convex}[i+1]$  @returns array 'res' where 'res[k]' = the min of  $(a[i]+b[j])$  for all pairs (i,j) where  $i+j==k$

633806, 26 lines

```

vector<int> min_plus(const vector<int>& convex,
    const vector<int>& arbitrary) {
    int n = ssize(convex);
    int m = ssize(arbitrary);
    vector<int> res(n + m - 1, INT_MAX);
    auto dnc = [&](auto&& self, int res_le, int res_ri, //c890c6
        int arb_le, int arb_ri) -> void {
        if (res_le >= res_ri) return;
        int mid_res = (res_le + res_ri) / 2;
        int op_arb = arb_le;
        for (int i = arb_le; i < min(mid_res + 1, arb_ri); //00bcac
            i++) {
            int j = mid_res - i;
            if (j >= n) continue;
            if (res[mid_res] > convex[j] + arbitrary[i]) {
                res[mid_res] = convex[j] + arbitrary[i]; //c587b4
                op_arb = i;
            } //d9dac2
```

```

        } //12d663
        self(self, res_le, mid_res, arb_le,
            min(arb_ri, op_arb + 1));
        self(self, mid_res + 1, res_ri, op_arb, arb_ri);
    }; //133dea
    dnc(dnc, 0, n + m - 1, 0, m);
    return res;
} //633806
```

### gcdconv.h

**Description:**  $\text{ssize}(a) == \text{ssize}(b)$   $\text{gcdconv}[k] = \text{sum of } (a[i]*b[j])$  for all pairs (i,j) where  $\text{gcd}(i,j)==k$

**Time:**  $\mathcal{O}(N \log N)$

2dfb20, 16 lines

```

const int mod = 998'244'353;
vector<int> gcd_convolution(const vector<int>& a,
    const vector<int>& b) {
    int n = ssize(a);
    vector<int> c(n);
    for (int g = n - 1; g >= 1; g--) { //8423c4
        int64_t sum_a = 0, sum_b = 0;
        for (int i = g; i < n; i += g) {
            sum_a += a[i], sum_b += b[i];
            if ((c[g] -= c[i]) < 0) c[g] += mod;
        } //7021b5
        sum_a %= mod, sum_b %= mod;
        c[g] = (c[g] + sum_a * sum_b) % mod;
    } //22b2a9
    return c;
} //2dfb20
```

### lcmconv.h

**Description:**  $\text{ssize}(a) == \text{ssize}(b)$   $\text{lcmconv}[k] = \text{sum of } (a[i]*b[j])$  for all pairs (i,j) where  $\text{lcm}(i,j)==k$

ee1440, 16 lines

```

const int mod = 998'244'353;
vector<int> lcm_convolution(const vector<int>& a,
    const vector<int>& b) {
    int n = ssize(a);
    vector<int64_t> sum_a(n), sum_b(n);
    vector<int> c(n); //f8bc27
    for (int i = 1; i < n; i++) {
        for (int j = i; j < n; j += i)
            sum_a[j] += a[i], sum_b[j] += b[i];
        sum_a[i] %= mod, sum_b[i] %= mod;
        c[i] = (c[i] + sum_a[i] * sum_b[i]) % mod; //2c8c40
        for (int j = i + i; j < n; j += i)
            if ((c[j] -= c[i]) < 0) c[j] += mod;
    } //2b66e9
    return c;
} //ee1440
```

## Number theory (7)

### 7.1 Modular arithmetic

#### ModInverse.h

**Description:** Pre-computation of modular inverses. Assumes  $\text{LIM} \leq \text{mod}$  and that mod is a prime.

6f684f, 3 lines

```

const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

#### ModLog.h

**Description:** Returns the smallest  $x > 0$  s.t.  $a^x = b \pmod m$ , or  $-1$  if no such  $x$  exists.  $\text{modLog}(a,1,m)$  can be used to calculate the order of  $a$ .

**Time:**  $\mathcal{O}(\sqrt{m})$

c040b8, 11 lines

```
11 modLog(11 a, 11 b, 11 m) {
  11 n = (11) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<11, 11> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j; //2d9fb0
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
} //c040b8
```

ModSum.h

**Description:** Sums of mod'ed arithmetic progressions.

modsum(to, c, k, m) =  $\sum_{i=0}^{to-1} (ki+c)\%m$ . divsum is similar but for floored division.

**Time:** log(*m*), with a large constant.

<pre>typedef unsigned long long ull; ull sumsq(ull to) { return to / 2 * ((to-1)   1); } //6bd037</pre>	5c5bc5, 16 lines
<pre>ull divsum(ull to, ull c, ull k, ull m) {   ull res = k / m * sumsq(to) + c / m * to;   k %= m; c %= m;   if (!k) return res; //d4b74d   ull to2 = (to * k + c) / m;   return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k); } //4a574e</pre>	
<pre>11 modsum(ull to, 11 c, 11 k, 11 m) {   c = ((c % m) + m) % m;   k = ((k % m) + m) % m;   return to * c + k * sumsq(to) - m * divsum(to, c, k, m); } //5c5bc5</pre>	

ModMulLL.h

**Description:** Calculate  $a \cdot b \bmod c$  (or  $a^b \bmod c$ ) for  $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$ .

**Time:**  $\mathcal{O}(1)$  for modmul,  $\mathcal{O}(\log b)$  for modpow

<pre>typedef unsigned long long ull; ull modmul(ull a, ull b, ull M) {   11 ret = a * b - M * ull(1.L / M * a * b);   return ret + M * (ret &lt; 0) - M * (ret &gt;= (11)M); } //a9c350 ull modpow(ull b, ull e, ull mod) {   ull ans = 1;   for (; e; b = modmul(b, b, mod), e /= 2)     if (e &amp; 1) ans = modmul(ans, b, mod);   return ans; //6d3d5f } //bbbd8f</pre>	bbbd8f, 11 lines
---	------------------

ModSqrt.h

**Description:** Tonelli-Shanks algorithm for modular square roots. Finds  $x$  s.t.  $x^2 = a \pmod p$  ( $-x$  gives the other solution).

**Time:**  $\mathcal{O}(\log^2 p)$  worst case,  $\mathcal{O}(\log p)$  for most  $p$

<pre>"ModPow.h"</pre>	19a793, 24 lines
<pre>11 sqrt(11 a, 11 p) {   a %= p; if (a &lt; 0) a += p;   if (a == 0) return 0;   assert(modpow(a, (p-1)/2, p) == 1); // else no solution   if (p % 4 == 3) return modpow(a, (p+1)/4, p);   // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5   11 s = p - 1, n = 2; //6aa127   int r = 0, m;   while (s % 2 == 0)     ++r, s /= 2;   while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;   11 x = modpow(a, (s + 1) / 2, p); //94db39</pre>	

```
11 b = modpow(a, s, p), g = modpow(n, s, p);
for (;;) r = m) {
  11 t = b;
  for (m = 0; m < r && t != 1; ++m)
    t = t * t % p; //2d5fcd
  if (m == 0) return x;
  11 gs = modpow(g, 1LL << (r - m - 1), p);
  g = gs * gs % p;
  x = x * gs % p;
  b = b * g % p; //198af1
} //ac3137
} //19a793
```

## 7.2 Primality

LinearSieve.h

**Description:** Finds smallest prime factor of each integer

**Time:**  $\mathcal{O}(N)$

<pre>const int LIM = 1000000; vi lp(LIM+1), primes;  rep(i, 2, LIM + 1) {   if (lp[i] == 0) primes.push_back(lp[i] = i);   for (int j = 0; j &lt; sz(primes) &amp;&amp; i * primes[j] &lt;= LIM &amp;&amp;     primes[j] &lt;= lp[i]; ++j)     lp[i * primes[j]] = primes[j]; //91f1b5 } //32eeca</pre>	32eeca, 8 lines
---	-----------------

CountPrimes.h

**Description:** Count # primes  $\leq N$ , can be modified to return sum of primes by setting  $f(p) = n$ ,  $ps(n) =$  nth tri number.

**Time:**  $\mathcal{O}\left(n^{3/4}\right)$

<pre>11 countprimes(11 n) { //n&gt;0   vector&lt;11&gt; divs,dp; 11 sq = sqrt1(n);   for (11 l = 1, r; l &lt;= n &amp;&amp; (r = n / (n / l)); l = r + 1)     divs.push_back(r);   auto idx = [&amp;](11 x) -&gt; int {     return x &lt;= sq ? x - 1 : (sz(divs) - n / x); }; //d740a2   rep(i,0,sz(divs)) dp.push_back(divs[i]-1);   for(11 p = 2; p*p &lt;= n; ++p) // ^ ps(divs[i])-1     if(dp[p-1]!=dp[p-2])       for(int i = sz(divs)-1; divs[i]&gt;=p*p &amp;&amp; i&gt;=0; i--)         dp[i] -= (dp[idx(divs[i]/p)]-dp[p-2]); // *f(p);   return dp.back(); //0b539f } //af82c0</pre>	af82c0, 13 lines
--	------------------

MillerRabin.h

**Description:** Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to  $7 \cdot 10^{18}$ ; for larger numbers, use Python and extend A randomly.

**Time:** 7 times the complexity of  $a^b \bmod c$ .

<pre>"ModMulLL.h"</pre>	60dcd1, 12 lines
<pre>bool isPrime(ull n) {   if (n &lt; 2    n % 6 % 4 != 1) return (n   1) == 3;   ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},     s = __builtin_ctzll(n-1), d = n &gt;&gt; s;   for (ull a : A) { // ^ count trailing zeroes     ull p = modpow(a&amp;n, d, n), i = s; //29e314     while (p != 1 &amp;&amp; p != n - 1 &amp;&amp; a % n &amp;&amp; i--)       p = modmul(p, p, n);     if (p != n-1 &amp;&amp; i != s) return 0;   } //1fad05   return 1; } //60dcd1</pre>	

Factor.h

**Description:** Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

**Time:**  $\mathcal{O}\left(n^{1/4}\right)$ , less for numbers with small factors.

<pre>"ModMulLL.h", "MillerRabin.h"</pre>	d8d98d, 18 lines
<pre>ull pollard(ull n) {   ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;   auto f = [&amp;](ull x) { return modmul(x, x, n) + i; }; //12dccb   while (t++ % 40    __gcd(prd, n) == 1) {     if (x == y) x = ++i, y = f(x);     if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;     x = f(x), y = f(f(y));   } //0b4d32   return __gcd(prd, n); } //cd2ac3 vector&lt;ull&gt; factor(ull n) {   if (n == 1) return {}; //6303f2   if (isPrime(n)) return {n}; //74d420   ull x = pollard(n);   auto l = factor(x), r = factor(n / x);   l.insert(l.end(), all(r));   return l; } //d8d98d</pre>	

mobiusFunction.h

**Description:** Computes mobius function, example code for counting co-prime pairs

<pre>//Mobius function vector&lt;int&gt; mu(maxv); mu[1] = 1; for(int i = 1; i &lt; mu.size(); i++)   for(int j = 2*i; j &lt; mu.size(); j+=i)     mu[j]-=mu[i];  //Count coprime pairs 11 ans = 0; //b800ad for(int d = 1; d&lt;maxv; d++){   11 sum = 0;   for(int j = 0; j &lt; maxv; j+=d) sum+=freq[j];   ans+=(mu[d]*choose2(sum)); } //1783cc</pre>	1783cc, 13 lines
--	------------------

## 7.3 Divisibility

euclid.h

**Description:** Finds two integers  $x$  and  $y$ , such that  $ax + by = \gcd(a, b)$ . If you just need gcd, use the built in `__gcd` instead. If  $a$  and  $b$  are coprime, then  $x$  is the inverse of  $a \pmod b$ .

<pre>11 euclid(11 a, 11 b, 11 &amp;x, 11 &amp;y) {   if (!b) return x = 1, y = 0, a;   11 d = euclid(b, a % b, y, x);   return y -= a/b * x, d; } //33ba8f</pre>	33ba8f, 5 lines
--	-----------------

CRT.h

**Description:** Chinese Remainder Theorem.

`crt(a, m, b, n)` computes  $x$  such that  $x \equiv a \pmod m, x \equiv b \pmod n$ . If  $|a| < m$  and  $|b| < n$ ,  $x$  will obey  $0 \leq x < \text{lcm}(m, n)$ . Assumes  $mn < 2^{62}$ .

**Time:** log(*n*)

<pre>"euclid.h"</pre>	04d93a, 7 lines
<pre>11 crt(11 a, 11 m, 11 b, 11 n) {   if (n &gt; m) swap(a, b), swap(m, n);   11 x, y, g = euclid(m, n, x, y);   assert((a - b) % g == 0); // else no solution   x = (b - a) % n * x % n / g * m + a;   return x &lt; 0 ? x + m*n/g : x; //000521 } //04d93a</pre>	



### 7.3.1 Bézout’s identity

For  $a \neq 0, b \neq 0$ , then  $d = \gcd(a, b)$  is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If  $(x, y)$  is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

#### phiFunction.h

**Description:** *Euler’s  $\phi$  function* is defined as  $\phi(n) := \#$  of positive integers  $\leq n$  that are coprime with  $n$ .  $\phi(1) = 1, p$  prime  $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$ ,  $m, n$  coprime  $\Rightarrow \phi(mn) = \phi(m)\phi(n)$ . If  $n = p_1^{k_1}p_2^{k_2}...p_r^{k_r}$  then  $\phi(n) = (p_1-1)p_1^{k_1-1}...(p_r-1)p_r^{k_r-1}$ .  $\phi(n) = n \cdot \prod_{p|n} (1-1/p)$ .  $\sum_{d|n} \phi(d) = n, \sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$   
**Euler’s thm:**  $a, n$  coprime  $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$ .  
**Fermat’s little thm:**  $p$  prime  $\Rightarrow a^{p-1} \equiv 1 \pmod p \ \forall a$ .

```
const int LIM = 5000000;
int phi[LIM];
```

```
void calculatePhi() {
    rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
    for (int i = 3; i < LIM; i += 2) if(phi[i] == i) //9fb18b
        for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
} //cf7d6d
```

## 7.4 Fractions

### ContinuedFractions.h

**Description:** Given  $N$  and a real number  $x \geq 0$ , finds the closest rational approximation  $p/q$  with  $p, q \leq N$ . It will obey  $|p/q - x| \leq 1/qN$ . For consecutive convergents,  $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$ . ( $p_k/q_k$  alternates between  $> x$  and  $< x$ .) If  $x$  is rational,  $y$  eventually becomes  $\infty$ ; if  $x$  is the root of a degree 2 polynomial the  $a$ ’s eventually become cyclic.  
**Time:**  $\mathcal{O}(\log N)$

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
    ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
    for (;;) {
        ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
            a = (ll)floor(y), b = min(a, lim), //82cd25
            NP = b*P + LP, NQ = b*Q + LQ;
        if (a > b) {
            // If b > a/2, we have a semi-convergent that gives us a
            // better approximation; if b = a/2, we *may* have one.
            // Return {P, Q} here for a more canonical approximation.
            return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
                make_pair(NP, NQ) : make_pair(P, Q); //3c2b26
        } //451a2f
        if (abs(y = 1/(y - (d)a)) > 3*N) {
            return {NP, NQ}; //32957f
        } //ec2d82
        LP = P; P = NP;
        LQ = Q; Q = NQ;
    } //a15756
} //dd6c5e
```

### FracBinarySearch.h

**Description:** Given  $f$  and  $N$ , finds the smallest fraction  $p/q \in [0, 1]$  such that  $f(p/q)$  is true, and  $p, q \leq N$ . You may want to throw an exception from  $f$  if it finds an exact solution, in which case  $N$  can be removed.

```
Usage: fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}
Time:  $\mathcal{O}(\log(N))$ 
27ab3e, 25 lines

struct Frac { ll p, q; }; //feaca1

template<class F>
Frac fracBS(F f, ll N) {
    bool dir = 1, A = 1, B = 1;
    Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
    if (f(lo)) return lo; //7f70d6
    assert(f(hi));
    while (A || B) {
        ll adv = 0, step = 1; // move hi if dir, else lo
        for (int si = 0; step; (step *= 2) >= si) {
            adv += step; //3067db
            Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q}; //306933
            if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
                adv -= step; si = 2;
            } //a40ec9
        } //d35347
        hi.p += lo.p * adv;
        hi.q += lo.q * adv;
        dir = !dir;
        swap(lo, hi);
        A = B; B = !adv; //fc82fe
    } //2c9a8f
    return dir ? hi : lo;
} //27ab3e
```

## 7.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with  $m > n > 0, k > 0, m \perp n$ , and either  $m$  or  $n$  even.

## 7.6 Estimates

$$\sum_{d|n} d = \mathcal{O}(n \log \log n).$$

The number of divisors of  $n$  is at most around 100 for  $n < 5e4$ , 500 for  $n < 1e7$ , 2000 for  $n < 1e10$ , 200 000 for  $n < 1e19$ .

## 7.7 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Computation

```
mu[1] = 1;
rep(i,1,sz(mu)) for(int j =2*i; j<sz(mu); j+=i)
    mu[j] -= mu[i];
```

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g(n/d)$$

Other useful formulas/forms:

$$\sum_{d|n} \mu(d) = [n = 1] \text{ (very useful)}$$

$$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n)g(d)$$

$$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m)g(\lfloor \frac{n}{m} \rfloor)$$

# Combinatorial (8)

## 8.1 Permutations

IntPerm.h

**Description:** Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.  
**Time:**  $\mathcal{O}(n)$

```
044568, 6 lines

int permToInt(vi& v) {
    int use = 0, i = 0, r = 0;
    for (int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
} //044568
```

### 8.1.1 Cycles

Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

### 8.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

### 8.1.3 Burnside’s lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

## 8.2 Partitions and subsets

### 8.2.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2e5$	$\sim 2e8$

8.2.2 Lucas’ Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_kp^k + \dots + n_1p + n_0$  and  $m = m_kp^k + \dots + m_1p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

8.2.3 Binomials

multinomial.h

**Description:** Computes  $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1!k_2! \dots k_n!}$ .

```
11 multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i, 1, sz(v)) rep(j, 0, v[i])
    c = c * ++m / (j+1);
  return c;
} //a0a312
```

a0a312, 6 lines

8.3 General purpose numbers

8.3.1 Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).

$$B[0, \dots] = [1, -\tfrac{1}{2}, \tfrac{1}{6}, 0, -\tfrac{1}{30}, 0, \tfrac{1}{42}, \dots]$$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^\infty f(i) &= \int_m^\infty f(x)dx - \sum_{k=1}^\infty \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

8.3.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k) x^k = x(x+1) \dots (x+n-1)$$

$$\begin{aligned} c(8, k) &= 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1 \\ c(n, 2) &= 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots \end{aligned}$$

8.3.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

8.3.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

8.3.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$  For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

8.3.6 Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$

# on  $k$  existing trees of size  $n_i$ :  $n_1n_2 \dots n_k n^{k-2}$

# with degrees  $d_i$ :  $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

8.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

Strings (9)

KMP.h

**Description:** pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

**Time:**  $\mathcal{O}(n)$

vi pi(const string& s) {

vi p(sz(s));

rep(i, 1, sz(s)) {

int g = p[i-1];

while (g && s[i] != s[g]) g = p[g-1];

p[i] = g + (s[i] == s[g]); //21a657

} //6c1f11

return p;

} //9cb7fc

vi match(const string& s, const string& pat) {

vi p = pi(pat + '\0' + s), res;

rep(i, sz(p)-sz(s), sz(p))

if (p[i] == sz(pat)) res.push\_back(i - 2 \* sz(pat));

return res; //c66a2a

} //d4375c

d4375c, 16 lines

Zfunc.h

**Description:** z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

**Time:**  $\mathcal{O}(n)$

vi Z(string S) {

vi z(sz(S));

int l = -1, r = -1;

rep(i, 1, sz(S)) {

z[i] = i >= r ? 0 : min(r - i, z[i - l]);

while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]]) //fe9318

z[i]++;

if (i + z[i] > r)

l = i, r = i + z[i];

} //1fcbd4

return z;

} //3ae526

3ae526, 12 lines

Manacher.h

**Description:** For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).

**Time:**  $\mathcal{O}(N)$

array<vi, 2> manacher(const string& s) {

int n = sz(s);

array<vi, 2> p = {vi(n+1), vi(n)}; //daf4bc

rep(z, 0, 2) for (int i=0, l=0, r=0; i < n; i++) {

int t = r-i+!z;

if (i<r) p[z][i] = min(t, p[z][l+t]);

int L = i-p[z][i], R = i+p[z][i]-!z;

while (L>=1 && R+1<n && s[L-1] == s[R+1]) //508df3

p[z][i]++, L--, R++;

if (R>r) l=L, r=R;

} //21a1fb

return p;

} //e7ad79

e7ad79, 13 lines

Eertree.h

**Description:** Generates an eertree on str. *cur* is accurate at the end of the main loop before the final assignment to *t*.

**Time:**  $\mathcal{O}(|S|)$

struct eertree{

static constexpr int ALPHA = 26;

struct node{ //sInd is starting index of an occurrence

array<int, ALPHA> down;

int slink, ln, sInd, freq = 0;

node(int slink, int ln, int eInd = -1): //5dff69

slink(slink), ln(ln), sInd(eInd-ln+1) {

fill(begin(down), begin(down)+ALPHA, -1);

} //6a8cb3

}; //aa06f7

vector<node> t = {node(0, -1), node(0, 0)}; //b4be49

eertree(string &s){

int cur = 0, k = 0;

for(int i = 0; i < sz(s); i++){

char c = s[i]; int cID = c-'a'; //first chracter

while(k<=0 || s[k-1] != c) //e85b7f

k = i - t[cur] = t[cur].slink.ln;

#define TCD t[cur].down[cID]

if(TCD == -1){

TCD = sz(t);

t.emplace\_back(-1, t[cur].ln+2, i); //8f1444

if(t.back().ln > 1){

do k = i - t[cur] = t[cur].slink.ln;

while(k<=0 || s[k-1] != c);

t[sz(t)-1].slink = TCD;

} else t[sz(t)-1].slink = 1; //519576

cur = sz(t)-1;

} else cur = TCD;

}

288121, 35 lines

```
        t[cur].freq++;
        k = i - t[cur].ln+1;
    } //f67fbd
    for(int i = sz(t)-1; i > 1; i--) //update frequencies
        t[t[i].slink].freq += t[i].freq;
    } //6acbda
}; //288121
```

### MinRotation.h

**Description:** Finds the lexicographically smallest rotation of a string.  
**Usage:** rotate(v.begin(), v.begin()+minRotation(v), v.end());  
**Time:**  $\mathcal{O}(N)$

```
int minRotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b,0,N) rep(k,0,N) {
        if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
        if (s[a+k] > s[b+k]) { a = b; break; } //20f912
    } //b2e25e
    return a;
} //d07a42
```

### SuffixArray.h

**Description:** Builds suffix array for a string. sa[i] is the starting index of the suffix which is  $i$ 'th in the sorted suffix array. The returned vector is of size  $n+1$ , and sa[0] = n. The lcp array contains longest common prefixes for neighbouring strings in the suffix array: lcp[i] = lcp(sa[i], sa[i-1]), lcp[0] = 0. The input string must not contain any zero bytes.  
**Time:**  $\mathcal{O}(n\log n)$

```
struct SuffixArray {
    vi sa, lcp;
    SuffixArray(string& s, int lim=256) { // or basic_string<int>
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
        sa = lcp = y, iota(all(sa), 0); //74da6a
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
            p = j, iota(all(y), n - j);
            rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i,0,n) ws[x[i]]++; //499169
            rep(i,1,lim) ws[i] += ws[i - 1];
            for (int i = n; i--;) sa[--ws[x[i]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
        } //f30252
        rep(i,1,n) rank[sa[i]] = i;
        for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
            for (k && k--, j = sa[rank[i] - 1];
                s[i + k] == s[j + k]; k++);
    } //22a139
}; //38db9f
```

### SuffixArrayQuery.h

**Description:** Various helper queries for suffix array problems inputs are 0 based input/output is inc-ex  
**Time:** lenlcp:  $\mathcal{O}(1)$ , cmpsub:  $\mathcal{O}(1)$ , findstr:  $\mathcal{O}(\log(n))$

```
struct SAQuery{
    SuffixArray sa; RMQ<int> lcp;
    string s; vector<int> sainv;
    SAQuery(SuffixArray sa,string s):sa(sa), lcp(sa.lcp), s(s){
        sainv.resize(sz(s)+1);
        rep(i,0,sz(sa.sa)) sainv[sa.sa[i]] = i; //700e88
    } //859793
    int len_lcp(int u, int v){
        if (u==v) return sz(s)-u;
        auto [l,r] = minmax(sainv[u],sainv[v]);
```

```
        return lcp.query(l+1,r+1);
    } //given [] substr of s: s1<s2->-1,s1==s2->0,s1>s2->1
    int cmp_sub(int l1, int r1, int l2, int r2){ //787424
        auto sgn = [](int x){ return (x>0)-(x<0); }; //7a8522
        int len1 = r1-l1+1, len2 = r2-l2+1;
        return len_lcp(l1,l2) < min(len1,len2)
            ? sgn(sainv[l1]-sainv[l2]): sgn(len1-len2);
    } //73a96e
    pair<int, int> find_str(int s_l, int s_r) {
        auto cmp = [&](int i, bool flip) -> bool {
            return flip ^ (len_lcp(i, s_l) < s_r - s_l); }; //67dcf3
        auto it = begin(sa.sa) + sainv[s_l];
        int l=lower_bound(begin(sa.sa),it,0,cmp)-begin(sa.sa);
        int r=lower_bound(it+1,end(sa.sa),l,cmp)-begin(sa.sa);
        return {l, r}; // # -> r-l
    } //102c76
}; //92e674
```

### findSubstr.h

**Description:** returns inc-exclusive range of occurences of needle string in-side suffix array, assumes global sa structure and global s (haystack)  
**Time:**  $\mathcal{O}(\lvert ndl \rvert \log(\lvert s \rvert))$

```
pair<int,int> find_str(const string &ndl){
    auto le = lower_bound(begin(sa.sa)+1,end(sa.sa), 0,
        [&](int i, int) -> bool { return lexicographical_compare
            (begin(s) + i, end(s), all(ndl)); });
    auto ri = lower_bound(le, end(sa.sa), 0,
        [&](int i, int) -> bool { return mismatch(begin(s) + i,
            end(s), all(ndl)).second == end(ndl);}); //022ea5
    return {le-begin(sa.sa),ri-begin(sa.sa)}; //[]
} //7d1493
```

### SuffixAutomaton.h

**Description:** Creates a partial DFA (DAG) that accepts all suffixes, with suffix links. One-to-one map between a path from the root and a substring. len is the longest-length substring ending here. pos is the first index in the string matching here. term is whether this node is a terminal (aka a suffix)  
**Time:** construction takes  $\mathcal{O}(N \log K)$ , where  $K$  = Alphabet Size

```
struct st { int len, pos, term; st *link; map<char, st*> next;
    };
    st *suffixAutomaton(string &str) {
        st *last = new st(), *root = last;
        for(auto c : str) {
            st *p = last, *cur = last = new st{last->len + 1, last->len
                };
            while(p && !p->next.count(c)) //4cd1a8
                p->next[c] = cur, p = p->link;
            if (!p) cur->link = root;
            else {
                st *q = p->next[c];
                if (p->len + 1 == q->len) cur->link = q; //1cc2d6
                else {
                    st *clone = new st{p->len+1, q->pos, 0, q->link, q->
                        next};
                    for (; p && p->next[c] == q; p = p->link)
                        p->next[c] = clone;
                    q->link = cur->link = clone; //08d876
                } //b49887
            } //31bf7e
        } //76ccab
        while(last) last->term = 1, last = last->link;
        return root;
    } //1914a9
```

### Hashing.h

**Description:** Self-explanatory methods for string hashing.

4b8fa1, 14 lines

```
typedef uint64_t ull;
struct H {
    ull x; H(ull x=0) : x(x) {} //80cf70
    H operator+(H o) { return x + o.x + (x + o.x < x); } //1f9d48
    H operator-(H o) { return *this + ~o.x; } //98ccfa
    H operator*(H o) { auto m = (__uint128_t)x * o.x;
        return H((ull)m) + (ull)(m >> 64); } //4eff44
    ull get() const { return x + !~x; } //f17b1d
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); }
}; //40d284
static const H C = (1l)1e11+3; // (order ~ 3e9; random also ok)
```

H hashString(string& s){H h{}; for(char c:s) h=h\*C+c;return h;}

### HashInterval.h

**Description:** Various self-explanatory methods for string hashing.

```
"Hashing.h"
122649, 12 lines
struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
        pw[0] = 1;
        rep(i,0,sz(str))
            ha[i+1] = ha[i] * C + str[i], //c3c119
            pw[i+1] = pw[i] * C;
    } //67307e
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    } //39481a
}; //122649
```

### LyndonFactorization.h

**Description:** A string is called simple (or a Lyndon word), if it is strictly smaller than any of its own nontrivial suffixes. Examples of simple strings are:  $a, b, ab, aab, abb, ababb, abcd$ . It can be shown that a string is simple, if and only if it is strictly smaller than all its nontrivial cyclic shifts. Next, let there be a given string  $s$ . The Lyndon factorization of the string  $s$  is a factorization  $s = w_1w_2 \dots w_k$ , where all strings  $w_i$  are simple, and they are in non-increasing order  $w_1 \geq w_2 \geq \dots \geq w_k$ . It can be shown, that for any string such a factorization exists and that it is unique.

**Time:**  $\mathcal{O}(N)$

```
vector<string> duval(string const& s) {
    int n = s.size();
    int i = 0;
    vector<string> factorization;
    while (i < n) {
        int j = i + 1, k = i; //d0372e
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                k++; //8d1eaa
            j++;
        } //cf42b4
        while (i <= k) {
            factorization.push_back(s.substr(i, j - k));
            i += j - k;
        } //46a6db
    } //14171a
    return factorization;
} //0e6ce6
```

### Wildcard.h

**Description:** string matching with wildcards, returns boolean vector of size s-p+1 representing if a match occurs at this start position, wild cards are represented by 0 and can be in s,p or both.

**Time:**  $\mathcal{O}((n+m)\log(n+m))$

b0e86b, 24 lines

```
vector<vl> make_powers(const vl& v) {
    int n = sz(v);
    vector<vl> pws(3, vl(n)); pws[0] = v;
    rep(k,1,3) rep(i,0,n) //mod?
        pws[k][i] = pws[k-1][i]*v[i];
    return pws; //a00fe1
} //10e306

vector<bool> wildcard_pattern_matching(const vl& s,
    const vl& p) {
    int n = sz(s), m = sz(p);
    auto s_pws = make_powers(s), p_pws = make_powers(p);
    for (auto& p_pw : p_pws) reverse(all(p_pw)); //cd7088
    vector<vl> res(3);
    rep(pw_hay,0,3) //ntt
        res[pw_hay] = conv(s_pws[pw_hay], p_pws[2 - pw_hay]);
    vector<bool> mtch(n - m + 1);
    rep(i,0,sz(mtch)){ //890a02
        int id = i + m - 1;
        auto num = res[0][id] - 2 * res[1][id] + res[2][id];
        mtch[i] = !num; //num == 0
    } //934360
    return mtch;
} //b0e86b
```

### AhoCorasick-Tyler.h

**Description:** Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to  $N\sqrt{N}$  many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries. **Time:** construction takes  $\mathcal{O}(26N)$ , where  $N$  = sum of length of patterns. find(x) is  $\mathcal{O}(N)$ , where  $N$  = length of x. findAll is  $\mathcal{O}(NM)$ .

```
const int ABSIZE = 26;

struct node {
    int nxt[ABSIZE];
    vi ids = {}; //d04adb
    int prv = -1, link = -1;
    char c;
    int linkMemo[ABSIZE];

    node(int prv = -1, char c = '$'): prv(prv), c(c) { //ec9f1e
        fill(all(nxt), -1);
        fill(all(linkMemo), -1);
    } //16055b
}; //432cad
```

```
vector<node> trie(1);
```

```
void addWord(string &s, int id) {
    int cur = 0; //aa1bc0
    for(char c: s) {
        int idx = c - 'a';
        if(trie[cur].nxt[idx] == -1) {
            trie[cur].nxt[idx] = sz(trie);
            trie.emplace_back(cur, c); //23b9d2
        } //ba2978
        cur = trie[cur].nxt[idx];
    } //35f152
    trie[cur].ids.push_back(id);
} //1dfc37
```

```
int getLink(int cur);
```

```
int calc(int cur, char c) {
    int idx = c - 'a'; //e9a88a
    auto &ret = trie[cur].linkMemo[idx];
    if(ret != -1) return ret;
    if(trie[cur].nxt[idx] != -1)
        return ret = trie[cur].nxt[idx];
    return ret = cur == 0 ? 0 : calc(getLink(cur), c); //1a4276
} //c61f02
```

```
int getLink(int cur) {
    auto &ret = trie[cur].link;
    if(ret != -1) return ret;
    if(cur == 0 || trie[cur].prv == 0) return ret = 0; //be881f
    return ret = calc(getLink(trie[cur].prv), trie[cur].c);
} //647ca9
```

## Various (10)

### 10.1 Intervals

#### IntervalContainer.h

**Description:** Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive). **Time:**  $\mathcal{O}(\log N)$

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it); //a98b04
    } //381108
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    } //5783d8
    return is.insert(before, {L,R});
} //d57d47
```

```
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second; //51cff5
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
} //edce47
```

#### IntervalCover.h

**Description:** Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty). **Time:**  $\mathcal{O}(N \log N)$

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
    vi S(sz(I)), R;
    iota(all(S), 0);
    sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
    T cur = G.first; //a166e4
    int at = 0;
    while (cur < G.second) { // (A)
        pair<T, int> mx = make_pair(cur, -1);
        while (at < sz(I) && I[S[at]].first <= cur) {
```

```
            mx = max(mx, make_pair(I[S[at]].second, S[at])); //201b40
            at++;
        } //470978
        if (mx.second == -1) return {}; //f1e40b
        cur = mx.first;
        R.push_back(mx.second);
    } //cd0c49
    return R;
} //9e9d8d
```

### ConstantIntervals.h

**Description:** Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval. **Usage:** constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...}); **Time:**  $\mathcal{O}(k \log \frac{n}{k})$

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
    if (p == q) return;
    if (from == to) {
        g(i, to, p);
        i = to; p = q; //a2e0d8
    } else {
        int mid = (from + to) >> 1;
        rec(from, mid, f, g, i, p, f(mid));
        rec(mid+1, to, f, g, i, p, q);
    } //5b694f
} //69b73b
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
    if (to <= from) return;
    int i = from; auto p = f(i), q = f(to-1);
    rec(from, to-1, f, g, i, p, q); //587254
    g(i, to, q);
} //753a4c
```

### 10.2 Misc. algorithms

#### LIS.h

**Description:** Compute indices for the longest increasing subsequence. **Time:**  $\mathcal{O}(N \log N)$

```
template<class I> vi lis(const vector<I>& S) {
    if (S.empty()) return {}; //be1376
    vi prev(sz(S));
    typedef pair<I, int> p;
    vector<p> res;
    rep(i,0,sz(S)) {
        // change 0 -> i for longest non-decreasing subsequence
        auto it = lower_bound(all(res), p{S[i], 0}); //f6ef94
        if (it == res.end()) res.emplace_back(), it = res.end()-1;
        *it = {S[i], i}; //26a0a3
        prev[i] = it == res.begin() ? 0 : (it-1)->second;
    } //f2ee22
    int L = sz(res), cur = res.back().second;
    vi ans(L);
    while (L--) ans[L] = cur, cur = prev[cur];
    return ans;
} //2932a0
```

### FastKnapsack.h

**Description:** Given N non-negative integer weights w and a non-negative target t, computes the maximum  $S \leq t$  such that S is the sum of some subset of the weights. **Time:**  $\mathcal{O}(N \max(w_i))$

```
int knapsack(vi w, int t) {
    int a = 0, b = 0, x;
```

```
while (b < sz(w) && a + w[b] <= t) a += w[b++];
if (b == sz(w)) return a;
int m = *max_element(all(w));
vi u, v(2*m, -1); //11fd10
v[a+m-t] = b;
rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m; ) rep(j, max(0,u[x]), v[x]) //51a6b1
        v[x-w[j]] = max(v[x-w[j]], j);
} //d2bd39
for (a = t; v[a+m-t] < 0; a-- )
return a;
} //b20ccc
```

### 10.3    Dynamic programming

**KnuthDP.h**  
**Description:** When doing DP on intervals:  $a[i][j] = \min_{i < k < j} (a[i][k] + a[k][j]) + f(i, j)$ , where the (minimal) optimal  $k$  increases with both  $i$  and  $j$ , one can solve intervals in increasing order of length, and search  $k = p[i][j]$  for  $a[i][j]$  only between  $p[i][j - 1]$  and  $p[i + 1][j]$ . This is known as Knuth DP. Sufficient criteria for this are if  $f(b, c) \leq f(a, d)$  and  $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$  for all  $a \leq b \leq c \leq d$ . Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.  
**Time:**  $O(N^2)$

**DivideAndConquerDP.h**  
**Description:** Given  $a[i] = \min_{lo(i) \leq k < hi(i)} (f(i, k))$  where the (minimal) optimal  $k$  increases with  $i$ , computes  $a[i]$  for  $i = L..R - 1$ .  
**Time:**  $O((N + (hi - lo)) \log N)$

```
struct DP { // Modify at will:
    int lo(int ind) { return 0; } //ce585d
    int hi(int ind) { return ind; } //f742b2
    ll f(int ind, int k) { return dp[ind][k]; } //29ea0c
    void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

    void rec(int L, int R, int LO, int HI) {
        if (L >= R) return;
        int mid = (L + R) >> 1; //13ddb0
        pair<ll, int> best(LLONG_MAX, LO);
        rep(k, max(LO, lo(mid)), min(HI, hi(mid)))
            best = min(best, make_pair(f(mid, k), k));
        store(mid, best.second, best.first);
        rec(L, mid, LO, best.second+1); //4993b6
        rec(mid+1, R, best.second, HI);
    } //116ea5
    void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
}; //d38d2b
```

### 10.4    Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

#### 10.4.1   Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; ((r^x) >> 2)/c` | `r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`

`if (i & 1 << b) D[i] += D[i^(1 << b)];`  
computes all sums of subsets.

#### 10.4.2   Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

**FastInput.h**  
**Description:** Read an integer from stdin. Usage requires your program to pipe in input from file.  
**Usage:** `./a.out < input.txt`  
**Time:** About 5x as fast as `cin/scanf`.

```
inline char gc() { // like getchar()
    static char buf[1 << 16];
    static size_t bc, be;
    if (bc >= be) {
        buf[0] = 0, bc = 0;
        be = fread(buf, 1, sizeof(buf), stdin); //bba013
    } //e9a035
    return buf[bc++]; // returns 0 on EOF
} //0261eb

int readInt() {
    int a, c;
    while ((a = gc()) < 40);
    if (a == '-') return -readInt(); //bc51ee
    while ((c = gc()) >= 48) a = a * 10 + c - 480;
    return a - 48;
} //7b3c70
```