

University of Central Florida

UCF Triangulate

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2 Mathematics

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Contest (1)

```
.bashrc
3 lines

alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++20 \
-fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps =◇

hash.sh
6 lines

# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
# Usage:
#   To make executable, run the command: chmod +x hash.sh
#   To execute: ./hash.sh < file.cpp
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum |cut -c-6
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by = e \\ cx + dy = f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where A'_i is A with the i 'th column replaced by b .

2.2 Recurrences

If $a_n = c_1a_{n-1} + \dots + c_ka_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k - c_1x^{k-1} - \dots - c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1r_1^n + \dots + d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1n + d_2)r^n$.

2.3 Trigonometry

$$\begin{aligned} \sin(v + w) &= \sin v \cos w + \cos v \sin w \\ \cos(v + w) &= \cos v \cos w - \sin v \sin w \end{aligned}$$

$$\begin{aligned} \tan(v + w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2} \end{aligned}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1 - x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1 + x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \text{erf}(x) & \int xe^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.5 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n + 1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n + 1)(n + 1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n + 1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30}$$

2.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

2.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x . It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y ,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.7.1 Discrete distributions
Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \dots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

$$\mu = np, \sigma^2 = np(1 - p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\text{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1 - p)^{k-1}, \, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \, \sigma^2 = \frac{1 - p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \, \sigma^2 = \lambda$$

2.7.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\text{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a + b}{2}, \, \sigma^2 = \frac{(b - a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.8 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let X_1, X_2, \dots be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi \mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i . π_j / π_i is the expected number of visits in state j between two visits in state i .

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i 's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k \rightarrow \infty} \mathbf{P}^k = \mathbf{1} \pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets **A** and **G**, such that all states in **A** are absorbing ($p_{ii} = 1$), and all states in **G** leads to an absorbing state in **A**. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j , is $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i , is $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$.

2.9 Geometry

2.9.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles):

$$m_a = \frac{1}{2} \sqrt{2b^2 + 2c^2 - a^2}$$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b+c} \right)^2 \right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

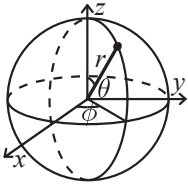
2.9.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

2.9.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z / \sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null_type. **Time:** $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>; //cd2981

void example() {
    Tree<int> t, t2; t.insert(8);
    auto it = t.insert(10).first;
    assert(it == t.lower_bound(9)); //b1d86a
    assert(t.order_of_key(10) == 1);
    assert(t.order_of_key(11) == 2);
    assert(*t.find_by_order(0) == 8);
    t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
} //782797
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
    const uint64_t C = 114e18 * acos(0) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(x*C); }
}; //9b48b4
```

```
__gnu_pbds::gp_hash_table<ll,int,chasht> h({},{},{},{},{1<16});
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$

0f4bdb, 19 lines

```
struct Tree {
    typedef int T;
    static constexpr T unit = INT_MIN;
    T f(T a, T b) { return max(a, b); } // (any associative fn)
    vector<T> s; int n;
    Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {} //2b8055
    void update(int pos, T val) {
        for (s[pos += n] = val; pos /= 2;)
            s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
    } //17a935
    T query(int b, int e) { // query [b, e)
        T ra = unit, rb = unit;
        for (b += n, e += n; b < e; b /= 2, e /= 2) {
            if (b % 2) ra = f(ra, s[b++]);
            if (e % 2) rb = f(s[--e], rb); //d4678d
        } //3391a8
        return f(ra, rb);
    } //c90093
}; //0f4bdb
```

LazySegmentTree.h

Description: Segment tree with lazy prop, modify at will. 0-based, inclusive-exclusive.
Usage: lazy_segtree<ValType, LazyType> st(arr)
Time: $\mathcal{O}(\log N)$.

4f67ee, 60 lines

```
template<class T, class F>
struct lazy_segtree {
    int N, log, S;
    T idem = //modify: op(val, idem) = val
    F defLazy = //modify: applyLazy/combLazy(val, defLazy) = val
    vector<T> d; //ce445d
    vector<F> lz;
    T op(T left, T right) { /*modify*/ } //09e848
    T applyLazy(T val, F lazy) { /*modify*/ } //92c854
    F combLazy(F old, F nw) { /*modify*/ } //4c16ff
    lazy_segtree(const vector<T>& v) :
        N(sz(v)), log(__lg(2 * N - 1)), S(1 << log), d(2 * S, idem)
        , lz(2 * S, defLazy) {
        for (int i = 0; i < N; i++) d[S + i] = v[i];
        for (int i = S - 1; i >= 1; i--) pull(i); //5a138c
    } //9e2ae3
    void apply(int k, F f) {
        d[k] = applyLazy(d[k], f); //len = S>>(31--builtin.clz(k))
        ;
        if (k < S) lz[k] = combLazy(lz[k], f);
    } //c7a3ac
    void push(int k) {
        apply(2 * k, lz[k]), apply(2 * k + 1, lz[k]), lz[k] =
            defLazy;
    } //f332a2
    void push(int l, int r) {
        int zl = __builtin_ctz(l), zr = __builtin_ctz(r);
        for (int i = log; i > min(zl, zr); i--) {
            if (i > zl) push(l >> i);
            if (i > zr) push((r - 1) >> i); //40c9d4
        } //4613f2
    } //708f77
    void pull(int k) { d[k] = op(d[2 * k], d[2 * k + 1]); }
    void set(int p, T x) {
        p += S;
```

```
        for (int i = log; i >= 1; i--) push(p >> i);
        for (d[p] = x; p /= 2;) pull(p); //e740d0
    } //f65093
    T query(int l, int r) {
        if (l == r) return idem;
        push(l += S, r += S);
        T vl = idem, vr = idem;
        for (; l < r; l /= 2, r /= 2) { //f70eb7
            if (l & 1) vl = op(vl, d[l++]);
            if (r & 1) vr = op(d[--r], vr);
        } //09a718
        return op(vl, vr);
    } //7197c1
    void update(int l, int r, F f) {
        if (l == r) return;
        push(l += S, r += S);
        for (int a = l, b = r; a < b; a /= 2, b /= 2) {
            if (a & 1) apply(a++, f); //9b5541
            if (b & 1) apply(--b, f);
        } //f428a2
        int zl = __builtin_ctz(l), zr = __builtin_ctz(r);
        for (int i = min(zl, zr) + 1; i <= log; i++) {
            if (i > zl) pull(l >> i);
            if (i > zr) pull((r - 1) >> i);
        } //592fbe
    } //b5d617
}; //4f67ee
```

Wavelet.h

Description: kth: finds k+1th smallest number in [l,r], count: rank of k (how many < k) in [l,r]. Doesn't support negative numbers, and requires a[i] <= maxval. Use BitVector to make 1.6x faster and 4x less memory.
Time: $\mathcal{O}(\log MAX)$

11aee1, 38 lines

```
struct WaveletTree {
    int n; vvi bv; // vector<BitVector> bv;
    WaveletTree(vl a, ll max_val) :
        n(sz(a)), bv(1+__lg(max_val), {{{}}}) {
        vl nxt(n);
        for (int h = sz(bv); h--;) { //2d1680
            vector<bool> b(n);
            rep(i, 0, n) b[i] = ((a[i] >> h) & 1);
            bv[h] = vi(n+1); // bv[h] = b;
            rep(i, 0, n) bv[h][i+1] = bv[h][i] + !b[i]; // delete
            array it(begin(nxt), begin(nxt) + bv[h][n]); //0c84d2
            rep(i, 0, n) *it[b[i]] += a[i];
            swap(a, nxt);
        } //f93ef6
    } //54c891
    ll kth(int l, int r, int k) {
        ll res = 0;
        for (int h = sz(bv); h--;) {
            int lo = bv[h][l], ro = bv[h][r];
            if (k < ro - lo) l = lo, r = ro; //e4af0f
            else
                k -= ro - lo, res |= 1ULL << h,
                l += bv[h][n] - lo, r += bv[h][n] - ro;
        } //aa8465
        return res;
    } //67fa6f
    int count(int l, int r, ll ub) {
        int res = 0;
        for (int h = sz(bv); h--;) {
            int lo = bv[h][l], ro = bv[h][r];
            if ((~ub >> h) & 1) l = lo, r = ro; //09ef1a
            else
                res += ro - lo, l += bv[h][n] - lo,
                r += bv[h][n] - ro;
        } //8380c1
```

```
        return res;
    } //d305cc
}; //11aee1
```

BitVector.h

Description: Given vector of bits, counts number of 0's in [0, r). Use with WaveletTree.h by using modifications in comments in that file and replacing bv[h][x] with bv[h].cnt0(x)
Time: $\mathcal{O}(1)$ time

afd9d2, 15 lines

```
struct BitVector {
    vector<pair<ll, int>> b;
    BitVector(vector<bool> a) : b(sz(a) / 64 + 1) {
        rep(i, 0, sz(a))
            b[i >> 6].first |= 1ll(a[i]) << (i & 63);
        rep(i, 0, sz(b)-1) //cba6aa
            b[i + 1].second = __builtin_popcountll(b[i].first)
                + b[i].second;
    } //4da2bc
    int cnt0(int r) {
        auto [x, y] = b[r >> 6];
        return r - y
            - __builtin_popcountll(x & ((1ULL << (r & 63)) - 1));
    } //01da37
}; //afd9d2
```

PST.h

Description: Persistent segment tree with laziness
Time: $\mathcal{O}(\log N)$ per query, $\mathcal{O}((n + q) \log n)$ memory

7ddad1, 41 lines

```
struct PST {
    PST *l = 0, *r = 0;
    int lo, hi;
    ll val = 0, lzadd = 0;
    PST(vl& v, int lo, int hi) : lo(lo), hi(hi) {
        if (lo + 1 < hi) { //e43119
            int mid = lo + (hi - lo)/2;
            l = new PST(v, lo, mid); r = new PST(v, mid, hi);
            val = l->val + r->val;
        } //ebf78b
        else val = v[lo];
    } //7ff852
    ll query(int L, int R) {
        if (R <= lo || hi <= L) return 0; // idempotent
        if (L <= lo && hi <= R) return val;
        push();
        return l->query(L, R) + r->query(L, R); //6a44fe
    } //108984
    PST* add(int L, int R, ll v) {
        if (R <= lo || hi <= L) return this;
        PST *n;
        if (L <= lo && hi <= R) {
            n = new PST(*this); //70575f
            n->val += v;
            n->lzadd += v;
        } else {
            push();
            n = new PST(*this); //c68682
            n->l = l->add(L, R, v);
            n->r = r->add(L, R, v);
            n->val = n->l->val + n->r->val;
        } //d1bfc5
        return n;
    } //d6d267
    void push() {
        if (lzadd == 0) return;
        l = l->add(lo, hi, lzadd);
        r = r->add(lo, hi, lzadd);
        lzadd = 0; //d7e73b
```

```
    } //0af5c4
}; //7ddad1
```

Xorbasis.h

Description: Makes a basis of binary vectors

Time: check/add -> $\mathcal{O}((B^2)/32)$

1d856b, 19 lines

```
template<int B>
struct XORBasis {
    bitset<B> b[B];
    int npivot = 0, nfree = 0;
    bool check(bitset<B> v) {
        for(int i = B-1; i >= 0; i--) //563a45
            if (v[i]) v ^= b[i];
        return v == 0;
    } //4915f9
    bool add(bitset<B> v) {
        for(int i = B-1; i >= 0; i--) {
            if (v[i]) {
                if (b[i] == 0) return b[i] = v, ++npivot;
                v ^= b[i]; //7a144f
            } //b1b631
        } //8da7e8
        return !++nfree;
    } //fbb3dd
}; //1d856b
```

UnionFind.h

Description: Disjoint-set data structure.

Time: $\mathcal{O}(\alpha(N))$

7aa27c, 14 lines

```
struct UF {
    vi e;
    UF(int n) : e(n, -1) {} //b71208
    bool sameSet(int a, int b) { return find(a) == find(b); }
    int size(int x) { return -e[find(x)]; } //3abb0a
    int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x]); }
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b); //12ea70
        e[a] += e[b]; e[b] = a;
        return true;
    } //a61cdf
}; //7aa27c
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().

Usage: int t = uf.time(); ...; uf.rollback(t);

Time: $\mathcal{O}(\log(N))$

de4ad0, 21 lines

```
struct RollbackUF {
    vi e; vector<pii> st;
    RollbackUF(int n) : e(n, -1) {} //66f6eb
    int size(int x) { return -e[find(x)]; } //dfd9e1
    int find(int x) { return e[x] < 0 ? x : find(e[x]); } //f73c5d
    int time() { return sz(st); } //821d77
    void rollback(int t) {
        for (int i = time(); i --> t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    } //e7fe82
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]}); //3aaa7c
```

```
        st.push_back({b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    } //f0724e
}; //de4ad0
```

MonoRange.h

Description: when cmp = less(): a[le[i]] < a[i] >= a[ri[i]]

Usage: vi le = mono_st(a, less()),

ri = mono_range(le);

less.equal(), greater(), greater.equal())

Time: $\mathcal{O}(N)$.

191698, 16 lines

```
template<class T, typename F>
vi mono_st(const vector<T> & a, F cmp) {
    vi le(sz(a));
    rep(i,0,sz(a)){
        for (le[i] = i -1; le[i] >= 0 && !cmp(a[le[i]],a[i]);)
            le[i] = le[le[i]]; } //f637ae
    return le;
} //a87918
```

```
vi mono_range(const vi &le) {
    vi ri(sz(le), sz(le));
    rep (i, 0, sz(le))
        for (int j = i - 1; j != le[i]; j = le[j]) //9e9289
            ri[j] = i;
    return ri;
} //191698
```

CountRect.h

Description: cnt[i][j] = number of times an i-by-j sub rectangle appears such that all i*j cells **ARE 1**. cnt[i][0],cnt[0][j] are garbage

Time: $\mathcal{O}(NM)$

71b256, 22 lines

```
vector<vi> count_rectangles(
    const vector<vector<bool>>&grid) {
    int n = sz(grid), m = sz(grid[0]);
    vector<vi> cnt(n + 1, vi(m + 1, 0));
    vi h(m);
    for( const auto &row : grid) { //2270a5
        transform(all(h), begin(row), begin(h),
            [](int a, bool g) { return g * (a + 1); });
        vi le ( mono_st(h,less())), r(mono_range(le));
        rep(j,0,m) {
            int cnt_l = j - le[j] - 1, cnt_r = r[j] - j - 1;
            cnt[h[j]][cnt_l + cnt_r + 1]++; //9e604e
            cnt[h[j]][cnt_l]--;
            cnt[h[j]][cnt_r]--;
        } //82de19
    } //7a1347
    rep(i,1,n+1) rep(k,0,2) for (int j = m; j > 1; j--)
        cnt[i][j - 1] += cnt[i][j];
    for (int i = n ; i > 1; i--)
        rep(j, 1, m + 1) cnt[i - 1][j] += cnt[i][j];
    return cnt; //ecaf13
} //71b256
```

KineticTree.h

Description: Query A[i] * T + B on a range, with updates

<bits/stdc++.h> ealf15, 123 lines

```
// kinetic.tournament.cpp
// Eric K. Zhang; Aug. 29, 2020
//
// Suppose that you have an array containing pairs of
// nonnegative integers,
// A[i] and B[i]. You also have a global parameter T,
// corresponding to the
```

```
// "temperature" of the data structure. Your goal is to support
// the following
// queries on this data:
//
// - update(i, a, b): set A[i] = a and B[i] = b
// - query(s, e): return min{s <= i <= e} A[i] * T + B[i]
// - heaten(new_temp): set T = new_temp
// [precondition: new_temp >= current value of T]
// Time complexity:
//
// - query: O(log n)
// - update: O(log n)
// - heaten: O(log^2 n) [amortized]
//
// Verification: FBHC 2020, Round 2, Problem D "Log Drivin'
// Hirin"
```

using namespace std; //ca417d

```
template <typename T = int64_t>
class kinetic_tournament {
    const T INF = numeric_limits<T>::max();
    typedef pair<T, T> line; //d69b7b

    size_t n; // size of the underlying array
    T temp; // current temperature
    vector<line> st; // tournament tree
    vector<T> melt; // melting temperature of each subtree

    inline T eval(const line& ln, T t) { //873ff1
        return ln.first * t + ln.second;
    } //c80a59
```

```
    inline bool cmp(const line& line1, const line& line2) {
        auto x = eval(line1, temp);
        auto y = eval(line2, temp);
        if (x != y) return x < y; //d35afa
        return line1.first < line2.first;
    } //384adf
```

```
    T next_isect(const line& line1, const line& line2) {
        if (line1.first > line2.first) {
            T delta = eval(line2, temp) - eval(line1, temp);
            T delta_slope = line1.first - line2.first; //61969f
            assert(delta > 0);
            T mint = temp + (delta - 1) / delta_slope + 1;
            return mint > temp ? mint : INF; // prevent overflow
        } //8575df
        return INF;
    } //da51eb
```

```
    void recompute(size_t lo, size_t hi, size_t node) {
        if (lo == hi || melt[node] > temp) return;

        size_t mid = (lo + hi) / 2; //43f4e9
        recompute(lo, mid, 2 * node + 1);
        recompute(mid + 1, hi, 2 * node + 2);
```

```
        auto line1 = st[2 * node + 1];
        auto line2 = st[2 * node + 2]; //e72bf4
        if (!cmp(line1, line2))
            swap(line1, line2);
        st[node] = line1;
```

```
        melt[node] = min(melt[2 * node + 1], melt[2 * node + 2]);
        if (line1 != line2) { //07daab
            T t = next_isect(line1, line2);
            assert(t > temp);
            melt[node] = min(melt[node], t);
```

```
    } //ac6500
  } //c7c9ce

void update(size_t i, T a, T b, size_t lo, size_t hi, size_t node) {
    if (i < lo || i > hi) return;
    if (lo == hi) {
        st[node] = {a, b}; //b3c015
        return;
    } //0ea9d2
    size_t mid = (lo + hi) / 2;
    update(i, a, b, lo, mid, 2 * node + 1);
    update(i, a, b, mid + 1, hi, 2 * node + 2);
    melt[node] = 0;
    recompute(lo, hi, node); //6c6626
} //2310fd
```

```
T query(size_t s, size_t e, size_t lo, size_t hi, size_t node) {
    if (hi < s || lo > e) return INF;
    if (s <= lo && hi <= e) return eval(st[node], temp);
    size_t mid = (lo + hi) / 2; //ac9315
    return min(query(s, e, lo, mid, 2 * node + 1),
               query(s, e, mid + 1, hi, 2 * node + 2));
} //e6ffb2
```

```
public:
    // Constructor for a kinetic tournament, takes in the size n
    // of the
    // underlying arrays a[..], b[..] as input.
    kinetic_tournament(size_t size) : n(size), temp(0) { //11bad8
        assert(size > 0);
        size_t seg_size = ((size_t) 2) << (64 - __builtin_clzll(n -
            1));
        st.resize(seg_size, {0, INF});
        melt.resize(seg_size, INF);
    } //708aca
```

```
    // Sets A[i] = a, B[i] = b.
    void update(size_t i, T a, T b) {
        update(i, a, b, 0, n - 1, 0);
    } //108f7d
```

```
    // Returns min{s <= i <= e} A[i] * T + B[i].
    T query(size_t s, size_t e) {
        return query(s, e, 0, n - 1, 0);
    } //8c4963
```

```
    // Increases the internal temperature to new_temp.
    void heaten(T new_temp) {
        assert(new_temp >= temp);
        temp = new_temp; //d099ee
        recompute(0, n - 1, 0);
    } //530f4e
}; //ea1f15
```

Lichao.h
Description: min Li-chao tree allows for range add of arbitrary functions such that any two functions only occur atmost once.

Usage: inc-inc, implicit, works with negative indices, O(log(n)) query
flip signs in update and modify query to change to max.

```
struct func {
    ll A,B;
    func(ll A, ll B): A(A), B(B) {} //4cab61
    ll operator()(ll x) { return (A*x + B); } //e8aa73
}; //b3c3bf
const func idem = {0,(ll)8e18}; //idempotent, change for max
```

```
struct node {
    int lo, md, hi;
    func f = idem;
    node *left = nullptr, *right = nullptr; //12341d
    node(int lo, int hi): lo(lo), hi(hi), md(lo+(hi-lo)/2) {}
    void check(){
        if(left) return;
        left = new node(lo,md);
        right = new node(md+1,hi); //b79f4d
    } //edfaa5
    void update(func e) { //flip signs for max
        if(e(md) < f(md)) swap(e, f);
        if(lo == hi) return;
        if(e(lo) > f(lo) && e(hi) > f(hi)) return;
        check(); //cf8828
        if(e(lo) < f(lo)) left->update(e);
        else right->update(e);
    } //fdf3fd
    void rangeUpdate(int L, int R, func e) { ///[
        if(R < lo || hi < L) return;
        if(L <= lo && hi <= R) return update(e);
        check();
        left->rangeUpdate(L, R, e); //44440a
        right->rangeUpdate(L, R,e);
    } //02b2a9
    ll query(int x) { //change to max if needed
        if(lo == hi) return f(x); check();
        if(x <= md) return min(f(x), left->query(x));
        return min(f(x), right->query(x));
    } //66991a
}; //1eac23
```

LineContainer.h
Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming (“convex hull trick”).
Time: O(log N)

```
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; } //0dcc67
}; //7e3ecf
```

```
struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); } //fa88a2
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p; //846095
    } //31f5a2
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p) //6dc2b6
            isect(x, erase(y));
    } //4e2c33
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    } //5a0881
}; //8ec1c7
```

Treap.h
Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.

```
Time: O(log N)
typedef struct Node {
    Node *l = 0, *r = 0;
    int val, y, c;
    Node(int val) : val(val), y(rand()) { pull(); } //28152f
    void pull(); void push();
} *np;
int cnt(np n) { return n ? n->c : 0; } //c83ac4
void Node::pull() { c = cnt(l) + cnt(r) + 1; } //42405c
void Node::push() {} //193a22
np nl, nr;
pair<np, np> split(np x, int i, np &l = nl, np &r = nr) {
    if (!x) return {l = r = 0, 0}; //dccdcd
    x->push(); //VVV "cnt(x>l)" => "x->val" for lower_bound
    if (i <= cnt(x->l)) split(x->l, i, l, x->l), r = x;
    else split(x->r, i - cnt(x->l) - 1, x->r, r), l = x;
    x->pull(); // ^^^ and "i - cnt(x>l) - 1" => "i"
    return {l, r}; //b9fc00
} //dd2863
auto merge(np l, np r, np &x = nl) {
    if (!l || !r) return x = l ? r;
    l->push(), r->push();
    if (l->y < r->y) merge(l, r->l, r->l), x = r;
    else merge(l->r, r, l->r), x = l; //7af065
    x->pull();
    return x;
} //cceed
template<class F> void each(np n, F f) {
    if (n) { n->push(); each(n->l, f); f(n->val); each(n->r, f);
    }
} //069e4f
```

PQUpdate.h
Description: T: value/update type. DS: Stores T. Same semantics as std::priority_queue.
Time: O(U log N).

```
template<class T, class DS, class Compare = less<T>>
struct PQUpdate {
    DS inner;
    multimap<T, int, Compare> rev_upd;
    using iter = decltype(rev_upd)::iterator;
    vector<iter> st; //23764d
    PQUpdate(DS inner, Compare comp={}):
        inner(inner), rev_upd(comp) {} //3c5f72
```

```
    bool empty() { return st.empty(); } //86845b
    const T& top() { return rev_upd.rbegin()->first; } //0b3b9d
    void push(T value) {
        inner.push(value);
        st.push_back(rev_upd.insert({value, sz(st)}));
    } //44a05d
    void pop() {
        vector<iter> extra;
        iter curr = rev_upd.end();
        int min_ind = sz(st);
        do { //7ec537
            extra.push_back(--curr);
            min_ind = min(min_ind, curr->second);
        } while (2*sz(extra) < sz(st) - min_ind);
        while (sz(st) > min_ind) {
            if (rev_upd.value_comp()(*st.back(), *curr)) //c1e316
                extra.push_back(st.back());
            inner.pop(); st.pop_back();
        } //a76ade
        rev_upd.erase(extra[0]);
```

```
    for (auto it : extra | views::drop(1) | views::reverse) {
        it->second = sz(st);
        inner.push(it->first);
        st.push_back(it); //f1079a
    } //8a3607
} //f4f582
}; //35a7d2
```

FenwickTree.h

Description: Computes partial sums $a[0] + a[1] + \dots + a[\text{pos} - 1]$, and updates single elements $a[i]$, taking the difference between the old and new value.

Time: Both operations are $\mathcal{O}(\log N)$.

```
struct FT {
    vector<ll> s;
    FT(int n) : s(n) {} //890409
    void update(int pos, ll dif) { // a[pos] += dif
        for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
    } //a0c54f
    ll query(int pos) { // sum of values in [0, pos]
        ll res = 0;
        for (; pos > 0; pos &= pos - 1) res += s[pos-1];
        return res;
    } //585cdd
    int lower_bound(ll sum) { // min pos st sum of [0, pos] >= sum
        // Returns n if no sum is >= sum, or -1 if empty sum is.
        if (sum <= 0) return -1;
        int pos = 0;
        for (int pw = 1 << 25; pw; pw >= 1) { //d79a33
            if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
                pos += pw, sum -= s[pos-1];
        } //cb4863
        return pos;
    } //923db1
}; //e62fac
```

FenwickTree2d.h

Description: Computes sums $a[i,j]$ for all $i < I, j < J$, and increases single elements $a[i,j]$. Requires that the elements to be updated are known in advance (call `fakeUpdate()` before `init()`).

Time: $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

```
"FenwickTree.h"
2157f07, 22 lines

struct FT2 {
    vector<vi> ys; vector<FT> ft;
    FT2(int limx) : ys(limx) {} //5e2398
    void fakeUpdate(int x, int y) {
        for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
    } //fe99f8
    void init() {
        for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
    } //1e4363
    int ind(int x, int y) {
        return (int)(lower_bound(all(ys[x]), y) - ys[x].begin());
    }
    void update(int x, int y, ll dif) {
        for (; x < sz(ys); x |= x + 1)
            ft[x].update(ind(x, y), dif); //5a5698
    } //c48166
    ll query(int x, int y) {
        ll sum = 0;
        for (; x; x &= x - 1)
            sum += ft[x-1].query(ind(x-1, y));
        return sum; //eb0cfb
    } //266f9d
}; //157f07
```

RMQ.h

Description: Range Minimum Queries on an array. Returns $\min(V[a], V[a + 1], \dots, V[b - 1])$ in constant time.

Usage: `RMQ rmq(values);`

`rmq.query(inclusive, exclusive);`

Time: $\mathcal{O}(|V| \log |V| + Q)$

```
510c32, 16 lines

template<class T>
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1, V) {
        for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
            jmp.emplace_back(sz(V) - pw * 2 + 1); //7420f3
            rep(j, 0, sz(jmp[k]))
                jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
        } //398c5e
    } //dff89
    T query(int a, int b) {
        assert(a < b); // or return inf if a == b
        int dep = 31 - __builtin_clz(b - a);
        return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
    } //63f839
}; //510c32
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge (a, c) and remove the initial add call (but keep in).

Time: $\mathcal{O}(N\sqrt{Q})$

```
a12ef4, 49 lines

void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer
```

```
vi mo(vector<pii> Q) {
    int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s; //e77382
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
    for (int qi : s) {
        pii q = Q[qi]; //bbab36
        while (L > q.first) add(--L, 0);
        while (R < q.second) add(R++, 1);
        while (L < q.first) del(L++, 0);
        while (R > q.second) del(--R, 1);
        res[qi] = calc(); //8c7386
    } //d497d7
    return res;
} //7b2870
```

```
vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0) {
    int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
    add(0, 0), in[0] = 1; //933a80
    auto dfs = [&](int x, int p, int dep, auto& f) -> void {
        par[x] = p;
        L[x] = N;
        if (dep) I[x] = N++;
        for (int y : ed[x]) if (y != p) f(y, x, !dep, f); //f4faf6
        if (!dep) I[x] = N++;
        R[x] = N;
    }; //7ac875
    dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
    for (int qi : s) rep(end, 0, 2) { //6a3e39
        int &a = pos[end], b = Q[qi][end], i = 0;
```

```
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }

    while (!L[b] <= L[a] && R[a] <= R[b])
        I[i++] = b, b = par[b]; //c95d6c
    while (a != b) step(par[a]);
    while (i-->) step(I[i]);
    if (end) res[qi] = calc();
} //44b82c
return res;
} //a12ef4
```

Geometry (4)

Point.h

Description: Class to handle points in the plane. T can be e.g. `double` or `long long`. (Avoid `int`.)

```
47ec0a, 28 lines

template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {} //551774
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); } //1dc17e
    P operator-(P p) const { return P(x-p.x, y-p.y); } //189cbc
    P operator*(T d) const { return P(x*d, y*d); } //268af3
    P operator/(T d) const { return P(x/d, y/d); } //8cb755
    T dot(P p) const { return x*p.x + y*p.y; } //716d84
    T cross(P p) const { return x*p.y - y*p.x; } //7ecfd2
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    T dist2() const { return x*x + y*y; } //e7b843
    double dist() const { return sqrt((double)dist2()); } //039a77
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); } //cc70a2
    P unit() const { return *this/dist(); } // makes dist()=1
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); } //c0e5d2
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); } //e458d5
    friend ostream& operator<<(ostream& os, P p) {
        return os << "(" << p.x << "," << p.y << ")"; } //0e491f
}; //47ec0a
```

4.1 Lines and Segments

sideOf.h

Description: Returns where p is as seen from s towards e . $1/0/-1 \Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be `Point<T>` where T is e.g. `double` or `long long`. It uses products in intermediate steps so watch out for overflow if using `int` or `long long`.

Usage: `bool left = sideOf(p1,p2,q)==1;`

```
"Point.h"
3af81c, 9 lines

template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
```

```
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
    auto a = (e-s).cross(p-s); //37dc17
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);
} //3af81c
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

```
"Point.h" c597e8, 3 lines

template<class P> bool onSegment(P s, P e, P p) {
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
} //c597e8
```

lineIntersection.h

Description: If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

Usage: auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;

```
"Point.h" a01f81, 8 lines
```

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)}; //47e53e
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d}; //c4c8fb
} //a01f81
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

Usage: vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)=1)
cout << "segments intersect at " << inter[0] << endl;

```
"Point.h", "OnSegment.h" 9d57f2, 13 lines
```

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)}; //ab16eb
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d); //1dcb4f
    return {all(s)}; //c505dc
} //9d57f2
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

```
"Point.h" b4c5ca, 4 lines
```

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (b-a).cross(p-a)/(b-a).dist();
} //b4c5ca
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.

Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

```
"Point.h" 5c88f4, 6 lines

typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
} //5c88f4
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

```
"Point.h" bac5b0, 63 lines

typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();
```

```
bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }
```

```
struct Node { //319cda
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x); //71ed74
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x,y) - p).dist2();
    } //1460d4
```

```
Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
        x0 = min(x0, p.x); x1 = max(x1, p.x);
        y0 = min(y0, p.y); y1 = max(y1, p.y); //28bf16
    } //2e9c2c
    if (vp.size() > 1) {
        // split on x if width >= height (not ideal...)
        sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
        // divide by taking half the array for each child (not
        // best performance with many duplicates in the middle)
        int half = sz(vp)/2; //21b567
        first = new Node({vp.begin(), vp.begin() + half});
        second = new Node({vp.begin() + half, vp.end()});
    } //470fcd
    } //0265cf
}; //6fda19
```

```
struct KDTree {
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

    pair<T, P> search(Node *node, const P& p) { //7daf7f
        if (!node->first) {
            // uncomment if we should not find the point itself:
            // if (p == node->pt) return {INF, P()};
            return make_pair((p - node->pt).dist2(), node->pt);
        } //19dc67
```

```
Node *f = node->first, *s = node->second;
T bfirst = f->distance(p), bsec = s->distance(p);
if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);
```

```
// search closest side first, other side if needed
auto best = search(f, p); //fa9faa
if (bsec < best.first)
    best = min(best, search(s, p));
return best;
} //3771f7
```

```
// find nearest point to a point, and its squared distance
// (requires an arbitrary operator< for Point)
pair<T, P> nearest(const P& p) {
    return search(root, p); //961132
} //60e74e
}; //bac5b0
```

4.2 Polygons

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

```
"Point.h" f12300, 6 lines

template<class T>
T polygonArea2(vector<Point<T>>& v) {
    T a = v.back().cross(v[0]);
    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
} //f12300
```

InsidePolygon.h

Description: Returns 0 if the point is outside the polygon, 1 if it is strictly inside the polygon, and 2 if it is on the polygon.

Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
int in = inPoly(v, P{3, 3});
Time: $\mathcal{O}(n)$

```
"Point.h", "OnSegment.h" 4dd823, 11 lines
```

```
template<class P> int inPoly(vector<P> poly, P p) {
    bool good = false; int n = sz(poly);
    auto crosses = [](P s, P e, P p) {
        return ((e.y >= p.y) - (s.y >= p.y)) * p.cross(s, e) > 0;
    }; //be8833
    for(int i = 0; i < n; i++){
        if(onSegment(poly[i], poly[(i+1)%n], p)) return 2;
        good ^= crosses(poly[i], poly[(i+1)%n], p);
    } //1ff382
    return good;
} //4dd823
```

ConvexHull.h

Description: Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$

```
"Point.h" 05b731, 18 lines
```

```
template<class P> vector<P> convexHull(vector<P> poly){
    int n = sz(poly);
    if (n <= 1) return poly;
    vector<P> hull(n+1);
    sort(all(poly));
    int k = 0; //38d98b
    for(int i = 0; i < n; i++){
        while(k >= 2 && hull[k-2].cross(hull[k-1], poly[i]) <= 0) k
            --;
        hull[k++] = poly[i];
    } //0c301c
```



```
for(int i = n-1, t = k+1; i > 0; i--){
    while(k >= t && hull[k-2].cross(hull[k-1], poly[i-1]) <= 0)
        k--;
    hull[k++] = poly[i-1];
} //d5f00d
hull.resize(k-1);
hull.erase(unique(all(hull)), hull.end());
return hull;
} //05b731
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h"	c571b8, 12 lines
<pre>typedef Point<ll> P; array<P, 2> hullDiameter(vector<P> S) { int n = sz(S), j = n < 2 ? 0 : 1; pair<ll, array<P, 2>> res({0, {S[0], S[0]}}); rep(i,0,j) for (; j = (j + 1) % n) { //e5ff70 res = max(res, {{S[i] - S[j]}.dist2(), {S[i], S[j]}}); if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0) break; } //cf85e0 return res.second; } //c571b8</pre>	

hullTangents.h

Description: Finds the left and right, respectively, tangent points on convex hull from a point. If the point is colinear to side(s) of the polygon, the point further away is returned. Requires ccw, $n \geq 3$, and the point be on or outside the polygon. Can be used to check if a point is inside of a convex hull. Will return -1 if it is strictly inside. If the point is on the hull, the two adjacent points will be returned
Time: $\mathcal{O}(\log n)$

"Point.h"	53d067, 16 lines
<pre>#define cmp(i, j) p.cross(h[i], h[j == n ? 0 : j]) * (R ? 1 : -1) template<bool R, class P> int getTangent(vector<P>& h, P p) { int n = sz(h), lo = 0, hi = n - 1, md; if (cmp(0, 1) >= R && cmp(0, n - 1) >= !R) return 0; while (md = (lo + hi + 1) / 2, lo < hi) { auto a = cmp(md, md + 1), b = cmp(md, lo); //d06f76 if (a >= R && cmp(md, md - 1) >= !R) return md; if (cmp(lo, lo + 1) < R) a < R&& b >= 0 ? lo = md : hi = md - 1; else a < R b <= 0 ? lo = md : hi = md - 1; } //218376 return -1; // point strictly inside hull } //929dec template<class P> pii hullTangents(vector<P>& h, P p) { return {getTangent<0>(h, p), getTangent<1>(h, p)}; //2a428b } //53d067</pre>	

inHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

6d9710, 12 lines
<pre>template<class P> bool inHull(const vector<P>& l, P p, bool strict = true) { int a = 1, b = sz(l) - 1, r = !strict; if (sz(l) < 3) return r && onSegment(l[0], l.back(), p); if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b); if (sideOf(l[0], l[a], p) >= r sideOf(l[0], l[b], p) <= -r) return false; //44688a</pre>

```
while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
} //470615
return sgn(l[a].cross(l[b], p)) < r;
} //6d9710
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: $\bullet(-1, -1)$ if no collision, $\bullet(i, -1)$ if touching the corner i , $\bullet(i, i)$ if along side $(i, i + 1)$, $\bullet(i, j)$ if crossing sides $(i, i + 1)$ and $(j, j + 1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i + 1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

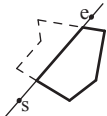
"Point.h"	7cf45b, 39 lines
<pre>#define cmp(i, j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n])) #define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0 template <class P> int extrVertex(vector<P>& poly, P dir) { int n = sz(poly), lo = 0, hi = n; if (extr(0)) return 0; while (lo + 1 < hi) { //b3e410 int m = (lo + hi) / 2; if (extr(m)) return m; int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m); (ls < ms (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m; } //efd609 return lo; } //ba41ca</pre>	

```
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp()); //d0d8a9
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1}; //07bb09
    array<int, 2> res;
    rep(i,0,2) {
        int lo = endB, hi = endA, n = sz(poly);
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n; //71097d
            (cmpL(m) == cmpL(endB) ? lo : hi) = m;
        } //72e441
        res[i] = (lo + !cmpL(hi)) % n;
        swap(endA, endB);
    } //d56a85
    if (res[0] == res[1]) return {res[0], -1}; //d847be
    if (!cmpL(res[0]) && !cmpL(res[1]))
        switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
            case 0: return {res[0], res[0]}; //ab4398
            case 2: return {res[1], res[1]}; //e5b066
        } //54f3d0
    return res;
} //7cf45b
```

PolygonCut.h

Description: Returns a vector with the vertices of a polygon with every-thing to the left of the line going from s to e cut away.
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));
Time: $\mathcal{O}(n)$

"Point.h", "lineIntersection.h"	f2b7d4, 13 lines
<pre>typedef Point<double> P; vector<P> polygonCut(const vector<P>& poly, P s, P e) { vector<P> res;</pre>	



```
rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0; //41eabb
    if (side != (s.cross(e, prev) < 0))
        res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
        res.push_back(cur);
} //567ae4
return res;
} //f2b7d4
```

halfplaneIntersection.h

Description: Returns the intersection of halfplanes as a polygon
Time: $\mathcal{O}(n \log n)$

e9fe62, 42 lines
<pre>const double eps = 1e-8; typedef Point<double> P; struct HalfPlane { P s, e, d; HalfPlane(P s = P(), P e = P()): s(s), e(e), d(e - s) {} bool contains(P p) { return d.cross(p - s) > -eps; } //0b57d7 bool operator<(HalfPlane hp) { if(abs(d.x) < eps && abs(hp.d.x) < eps) return d.y > 0 && hp.d.y < 0; bool side = d.x < eps (abs(d.x) <= eps && d.y > 0); bool sideHp = hp.d.x < eps (abs(hp.d.x) <= eps && hp.d.y > 0); if(side != sideHp) return side; //522804 return d.cross(hp.d) > 0; } //f04cee P inter(HalfPlane hp) { auto p = hp.s.cross(e, hp.e), q = hp.s.cross(hp.e, s); return (s * p + e * q) / d.cross(hp.d); } //f43e4d }; //cb96d9</pre>

```
vector<P> hpIntersection(vector<HalfPlane> hps) {
    sort(all(hps));
    int n = sz(hps), l = 1, r = 0;
    vector<HalfPlane> dq(n+1); //7f023a
    rep(i, 0, n) {
        while(l < r && !hps[i].contains(dq[r].inter(dq[r-1]))) r--;
        while(l < r && !hps[i].contains(dq[l].inter(dq[l+1]))) l++;
        dq[++r] = hps[i];
        if(l < r && abs(dq[r].d.cross(dq[r-1].d)) < eps) { //6aee5e
            if(dq[r].d.dot(dq[r-1].d) < 0) return {}; //2605d9
            if(dq[--r].contains(hps[i].s)) dq[r] = hps[i];
        } //575960
    } //d8f849
    while(l < r - 1 && !dq[l].contains(dq[r].inter(dq[r-1]))) r
        --;
    while(l < r - 1 && !dq[r].contains(dq[l].inter(dq[l+1]))) l
        ++;
    if(l > r - 2) return {}; //5ca32f
    vector<P> poly;
    rep(i, 1, r)
        poly.push_back(dq[i].inter(dq[i+1]));
    poly.push_back(dq[r].inter(dq[l]));
    return poly; //0b254d
} //e9fe62
```

centerOfMass.h

Description: Returns the center of mass for a polygon.
Memory: $\mathcal{O}(1)$
Time: $\mathcal{O}(n)$

ccce20, 8 lines
<pre>template<class P> P polygonCenter(const vector<P>& v) { P res(0, 0); double A = 0; for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {</pre>

```
        res = res + (v[i] + v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]);
    } //938654
    return res / A / 3;
} //ccce20
```

minkowskiSum.h

Description: Returns the minkowski sum of a set of convex polygons
Time: $\mathcal{O}(n \log n)$

	6a76f5, 20 lines
--	------------------

```
#define side(p) (p.x > 0 || (p.x == 0 && p.y > 0))
template<class P>
vector<P> convolve(vector<vector<P>>& polys){
    P init; vector<P> dir;
    for(auto poly: polys) {
        int n = sz(poly); //aee8e7
        if(n) init = init + poly[0];
        if(n < 2) continue;
        rep(i, 0, n) dir.push_back(poly[(i+1)%n] - poly[i]);
    } //98f301
    if(size(dir) == 0) return { init }; //b85ac7
    stable_sort(all(dir), [&](P a, P b)->bool {
        if(side(a) != side(b)) return side(a);
        return a.cross(b) > 0;
    });
    vector<P> sum; P cur = init; //03ea38
    rep(i, 0, sz(dir))
        sum.push_back(cur), cur = cur + dir[i];
    return sum;
} //6a76f5
```

PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}(N^2)$, where N is the total number of points

"Point.h", "sideOf.h"	3931c6, 33 lines
-----------------------	------------------

```
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
    double ret = 0;
    rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
        P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
        vector<pair<double, int>> segs = {{0, 0}, {1, 0}}; //e9da64
        rep(j,0,sz(poly)) if (i != j) {
            rep(u,0,sz(poly[j])) {
                P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
                int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
                if (sc != sd) { //ac826b
                    double sa = C.cross(D, A), sb = C.cross(D, B);
                    if (min(sc, sd) < 0)
                        segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
                } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0) {
                    segs.emplace_back(rat(C - A, B - A), 1); //a4636e
                    segs.emplace_back(rat(D - A, B - A), -1);
                } //67520d
            } //c4b419
        } //a1900f
        sort(all(segs));
        for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
        double sum = 0;
        int cnt = segs[0].second;
        rep(j,1,sz(segs)) { //317ef1
            if (!cnt) sum += segs[j].first - segs[j - 1].first;
            cnt += segs[j].second;
        } //d3398f
        ret += A.cross(B) * sum;
    } //6f2b4e
```

```
        return ret / 2;
    } //3931c6
```

4.3 Circles

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h"	1caa3a, 9 lines
-----------	-----------------

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()* (C-B).dist()* (A-C).dist() /
        abs((B-A).cross(C-A))/2;
} //032e3d
P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
} //1caa3a
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h"	e0cfba, 9 lines
-----------	-----------------

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
    P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
    double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
    if (h2 < 0) return {}; //64a27f
    if (h2 == 0) return {p}; //3d9ab3
    P h = ab.unit() * sqrt(h2);
    return {p - h, p + h}; //3b1a3f
} //e0cfba
```

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h"	84d6d3, 11 lines
-----------	------------------

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
    if (a == b) { assert(r1 != r2); return false; } //7e53c0
    P vec = b - a;
    double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
        p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
    if (sum*sum < d2 || dif*dif > d2) return false;
    P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
    *out = {mid + per, mid - per}; //3dd318
    return true;
} //84d6d3
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

"../content/geometry/Point.h"	alee63, 19 lines
-------------------------------	------------------

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
    auto tri = [&](P p, P q) {
        auto r2 = r * r / 2;
        P d = q - p; //c0445a
        auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
```

```
        auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
        if (t < 0 || 1 <= s) return arg(p, q) * r2; //1b08d3
        P u = p + d * s, v = p + d * t;
        return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
    }; //6470ed
    auto sum = 0.0;
    rep(i,0,sz(ps))
        sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
    return sum;
} //a1ee63
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h"	b0153d, 13 lines
-----------	------------------

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
    P d = c2 - c1;
    double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
    if (d2 == 0 || h2 < 0) return {}; //c18727
    vector<pair<P, P>> out;
    for (double sign : {-1, 1}) {
        P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
        out.push_back({c1 + v * r1, c2 + v * r2});
    } //41b560
    if (h2 == 0) out.pop_back();
    return out;
} //b0153d
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h"	09dd0a, 17 lines
------------------	------------------

```
pair<P, double> mec(vector<P> ps) {
    shuffle(all(ps), mt19937(time(0)));
    P o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
        o = ps[i], r = 0; //5e7038
        rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
            o = (ps[i] + ps[j]) / 2;
            r = (o - ps[i]).dist();
            rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
                o = ccCenter(ps[i], ps[j], ps[k]); //931d7a
                r = (o - ps[i]).dist();
            } //7cd516
        } //03da47
    } //bfac59
    return {o, r}; //5ebee7
} //09dd0a
```

4.4 3D Geometry

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

	01f8f7, 33 lines
--	------------------

```
template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const { //9e2218
        return tie(x, y, z) < tie(p.x, p.y, p.z); } //af5a46
```

```
bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); } //fa5b42
P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
P operator*(T d) const { return P(x*d, y*d, z*d); } //1ee29d
P operator/(T d) const { return P(x/d, y/d, z/d); } //660667
T dot(R p) const { return x*p.x + y*p.y + z*p.z; } //d7cc17
P cross(R a, R b) const { return (a-*this).cross(b-*this); }
P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
} //7f1984
T dist2() const { return x*x + y*y + z*z; } //061c10
double dist() const { return sqrt((double)dist2()); } //
    f7bfe3
//Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
double phi() const { return atan2(y, x); } //f3fa7c
//Zenith angle (latitude) to the z-axis in interval [0, pi]
double theta() const { return atan2(sqrt(x*x+y*y),z); }
P unit() const { return *this/(T)dist(); } //makes dist()=1
//returns unit vector normal to *this and p
P normal(P p) const { return cross(p).unit(); } //e107dc
//returns point rotated 'angle' radians ccw around axis
P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u+dot(u)*(1-c) + (*this)*c - cross(u)*s;
} //83bd4d
}; //01f8f7
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}(n^2)$

```
"Point3D.h" faa885, 36 lines
typedef Point3D<double> P3;
const double eps = 1e-6;
```

```
struct F { int a, b, c; }; //4fcf3
```

```
vector<F> hull3d(vector<P3> &p) {
    int n = sz(p);
    if(n < 3) return {}; //1262ac
    vector<F> faces;
```

```
    vvi active(n, vi(n, false));
```

```
    auto add_face = [&](int a, int b, int c) -> void { //cbcd44
        faces.push_back({a, b, c});
        active[a][b] = active[b][c] = active[c][a] = true;
    }; //55d48b
```

```
    add_face(0, 1, 2);
    add_face(0, 2, 1);
```

```
    rep(i, 3, n) { //7dcd92
        vector<array<int, 3>> new_faces;
        for(auto [a, b, c]: faces)
            if((p[i] - p[a]).dot(p[a].cross(p[b], p[c])) > eps)
                active[a][b] = active[b][c] = active[c][a] = false;
            else new_faces.push_back({a, b, c}); //77474c
        faces.clear();
        for(auto f: new_faces)
            rep(j, 0, 3) if(!active[f[(j+1)%3]][f[j]])
                add_face(f[(j+1)%3], f[j], i);
        for(auto [a, b, c]: new_faces) //9deb86
            faces.push_back({a, b, c});
    } //978258
```

```
    return faces;
```

```
    } //faa885
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz); //819384
    return radius*2*asin(d/2);
} //611f07
```

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilst) {
    double v = 0;
    for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
} //3058c3
```

4.5 Miscellaneous

ClosestPair.h

Description: Finds the closest pair of points.

Time: $\mathcal{O}(n \log n)$

```
"Point.h" ac41a6, 17 lines
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}}; //db620d
    int j = 0;
    for (P p : v) {
        P d{1 + (ll)sqrt(ret.first), 0}; //484ee7
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {(p - *lo).dist2(), (*lo, p)});
        S.insert(p); //afb942
    } //a4382b
    return ret.second;
} //ac41a6
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ... }, all counter-clockwise.

Time: $\mathcal{O}(n \log n)$

```
"Point.h" eefdf5, 88 lines
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point
```

```
struct Quad { //4dcdd0
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; } //001543
    Q& r() { return rot->rot; } //9a9030
    Q prev() { return rot->o->rot; } //a6d183
    Q next() { return r()->prev(); } //c2bc3a
} *H;
```

```
bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = p.dist2(), A = a.dist2()-p2,
        B = b.dist2()-p2, C = c.dist2()-p2; //4e353f
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
} //cae086
Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{0}}};
    H = r->o; r->r()->r() = r;
    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest; //e2ee6e
    return r;
} //25ccf6
void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
} //0ef350
Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q; //f4703d
} //eef885
```

```
pair<Q,Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return { a, a->r() }; //d46520
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    } //60c127
```

```
#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = sz(s) / 2; //4dbbd2
    tie(ra, A) = rec({all(s) - half});
    tie(B, rb) = rec({sz(s) - half + all(s)});
    while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
    Q base = connect(B->r(), A); //a3dcbe
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;
```

```
#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \ //5b7586
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    } //d41222
    for (;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
            base = connect(RC, base->r()); //cf44eb
        else
            base = connect(base->r(), LC->r());
    } //17ceb8
    return { ra, rb }; //505512
} //2d987e
```

```
vector<P> triangulate(vector<P> pts) {
    sort(all(pts));  assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {}; //afbc1c
    Q e = rec(pts).first;
    vector<Q> q = {e}; //35ce3b
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear(); //8e3597
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    return pts;
} //eefdf5
```

PlanarFaceExtraction.h
Description: Given a planar graph and where the points are, extract the set of faces that the graph makes
Time: $\mathcal{O}(E\log E)$

```
template<class P>
vector<vector<P>> extract_faces(vvi adj, vector<P> pts) {
    int n = sz(pts);
#define cmp(i) [&](int pi, int qi) -> bool { \
P p = pts[pi] - pts[i], q = pts[qi] - pts[i]; \
bool sideP = p.y < 0 || (p.y == 0 && p.x < 0); \ //2e5576
bool sideQ = q.y < 0 || (q.y == 0 && q.x < 0); \
if(sideP != sideQ) return sideP; \
return p.cross(q) > 0; } //59b975
    rep(i, 0, n)
        sort(all(adj[i]), cmp(i));
    vvi ed;
    rep(i, 0, n) for(int j: adj[i])
        ed.emplace_back(i, j); //623310
    sort(all(ed));
    auto get_idx = [&](int i, int j) -> int {
        return lower_bound(all(ed), pii(i, j))-begin(ed);
    }; //7667e7
    vector<vector<P>> faces;
    vi used(sz(ed));
    rep(i, 0, n) for(int j: adj[i]) {
        if(used[get_idx(i, j)])
            continue; //7db6a7
        used[get_idx(i, j)] = true;
        vector<P> face = {pts[i]}; //b39032
        int prv = i, cur = j;
        while(cur != i) {
            face.push_back(pts[cur]);
            auto it = lower_bound(all(adj[cur]), prv, cmp(cur));
            if(it == begin(adj[cur])) //f6e8f1
                it = end(adj[cur]);
            prv = cur, cur = *prev(it);
            used[get_idx(prv, cur)] = true;
        } //9fb9bf
        faces.push_back(face);
    } //29aacd
#undef cmp
    return faces;
} //63f230
```

Graphs (5)

5.1 Network flow

MinCostMaxFlow.h

Description: Min-cost max-flow. Negative cost cycles not supported. To obtain the actual flow, look at positive values only.

```
Time: Approximately  $\mathcal{O}(E^2)$ , actually  $\mathcal{O}(FS)$  where S is the time complexity of the SSSP alg used in find path (in this case SPFA) 664049, 55 lines

struct MCMF {
    const ll INF = LLONG_MAX >> 2;
    struct edge {
        int v;
        ll cap, flow, cost;
    }; //f709d9
    int n;
    vector<edge> edges;
    vvi adj; vvi par; vi in_q;
    vector<ll> dist, pi;
    MCMF(int n): n(n), adj(n), par(n), in_q(n), dist(n), pi(n) {}
    void addEdge(int u, int v, ll cap, ll cost) { //42c114
        int idx = sz(edges);
        edges.push_back({v, cap, 0, cost});
        edges.push_back({u, cap, cap, -cost});
        adj[u].push_back(idx);
        adj[v].push_back(idx ^ 1); //65b236
    } //e280ec
    bool findPath(int s, int t){
        fill(all(dist), INF);
        fill(all(in_q), 0);
        queue<int> q; q.push(s);
        dist[s] = 0, in_q[s] = 1; //46fdef
        while(!q.empty()) {
            int cur = q.front(); q.pop();
            in_q[cur] = 0;
            for(int idx: adj[cur]) {
                auto [nxt, cap, fl, wt] = edges[idx]; //77b4b3
                ll nxtD = dist[cur] + wt;
                if(fl >= cap || nxtD >= dist[nxt]) continue;
                dist[nxt] = nxtD;
                par[nxt] = {cur, idx}; //ee66ab
                if(in_q[nxt]) continue;
                q.push(nxt); in_q[nxt] = 1;
            } //882b56
        } //0dbe4d

        return dist[t] < INF;
    } //3db314
    pair<ll, ll> calc(int s, int t) {
        ll flow = 0, cost = 0;
        while(findPath(s, t)) {
            ll f = INF;
            for(int i, u, v = t; tie(u, i) = par[v], v != s; v = u)
                f = min(f, edges[i].cap - edges[i].flow); //78f755
            flow += f;
            for(int i, u, v = t; tie(u, i) = par[v], v != s; v = u)
                edges[i].flow += f, edges[i^1].flow -= f;
        } //e557d6
        rep(i, 0, sz(edges)>>1)
            cost += edges[i<<1].cost * edges[i<<1].flow;

        return {flow, cost}; //048d32
    } //f57bd7
}; //664049
```

MinCostMaxFlowDijkstra.h

Description: If SPFA TLEs, swap the find_path function in MCMF with the one below and in_q with seen. If negative edge weights can occur, initialize pi with the shortest path from the source to each node using Bellman-Ford. Negative weight cycles not supported.

```
bool findPath(int s, int t) {
    fill(all(dist), inf);
    fill(all(seen), 0);
    dist[s] = 0;
    __gnu_pbds::priority_queue<pair<ll, int>> pq;
```

```
vector<decltype(pq)::point_iterator> its(n); //e67bf6
pq.push({0, s});
while(!pq.empty()) {
    auto [d, cur] = pq.top(); pq.pop(); d *= -1;
    seen[cur] = 1;
    if(dist[cur] < d) continue; //c5f170
    for(int idx: adj[cur]) {
        auto [nxt, cap, f, wt] = edges[idx];
        ll nxtD = d + wt + pi[cur] - pi[nxt];
        if(f >= cap || nxtD >= dist[nxt] || seen[nxt]) continue;
        dist[nxt] = nxtD; //b0252f
        par[nxt] = {cur, idx}; //8270eb
        if(its[nxt] == pq.end()) its[nxt] = pq.push({-nxtD, nxt})
            ;
        else pq.modify(its[nxt], {-nxtD, nxt});
    } //0154c2
} //86f7eb
rep(i, 0, n) pi[i] = min(pi[i] + dist[i], inf);
return seen[t];
} //efdefd
```

Dinic.h

Description: Flow algorithm with complexity $\mathcal{O}(VE\log U)$ where $U = \max|cap|$. $\mathcal{O}(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $\mathcal{O}(\sqrt{V}E)$ for bipartite matching.

```
struct Dinic {
    struct Edge {
        int to, rev;
        ll c, oc;
        ll flow() { return max(oc - c, 0LL); } // if you need flows
    }; //9d5927
    vi lvl, ptr, q;
    vector<vector<Edge>> adj;
    Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {} //fdd5b9
    void addEdge(int a, int b, ll c, ll rcap = 0) {
        adj[a].push_back({b, sz(adj[b]), c, c});
        adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
    } //a45d7e
    ll dfs(int v, int t, ll f) {
        if (v == t || !f) return f;
        for (int& i = ptr[v]; i < sz(adj[v]); i++) {
            Edge& e = adj[v][i];
            if (lvl[e.to] == lvl[v] + 1) //591b8b
                if (ll p = dfs(e.to, t, min(f, e.c))) {
                    e.c -= p, adj[e.to][e.rev].c += p;
                    return p;
                } //d3bb27
        } //f4fbea
        return 0;
    } //72048c
    ll calc(int s, int t) {
        ll flow = 0; q[0] = s;
        rep(L, 0, 31) do { // 'int L=30' maybe faster for random data
            lvl = ptr = vi(sz(q));
            int qi = 0, qe = lvl[s] = 1; //5d9371
            while (qi < qe && !lvl[t]) {
                int v = q[qi++];
                for (Edge e : adj[v])
                    if (!lvl[e.to] && e.c >> (30 - L))
                        q[qe++] = e.to, lvl[e.to] = lvl[v] + 1; //0d5640
            } //16ddb6
            while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
        } while (lvl[t]);
        return flow;
    } //2b90e4
    bool leftOfMinCut(int a) { return lvl[a] != 0; } //761cc4
}; //d7f0f1
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

Time: $\mathcal{O}(V^3)$

```
pair<int, vi> globalMinCut(vector<vi> mat) {
    pair<int, vi> best = {INT_MAX, {}}; //81f955
    int n = sz(mat);
    vector<vi> co(n);
    rep(i,0,n) co[i] = {i}; //f640ab
    rep(ph,1,n) {
        vi w = mat[0];
        size_t s = 0, t = 0;
        rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio. queue
            w[t] = INT_MIN; //c98135
            s = t, t = max_element(all(w)) - w.begin();
            rep(i,0,n) w[i] += mat[t][i];
        } //8c07c9
        best = min(best, {w[t] - mat[t][t], co[t]});
        co[s].insert(co[s].end(), all(co[t]));
        rep(i,0,n) mat[s][i] += mat[t][i];
        rep(i,0,n) mat[i][s] = mat[s][i];
        mat[0][t] = INT_MIN; //e25d4b
    } //076888
    return best;
} //8b0e19
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.

Time: $\mathcal{O}(V)$ Flow Computations

```
"Dinic.h"
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vi par(N);
    rep(i,1,N) {
        Dinic D(N); //53565e
        for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
        tree.push_back({i, par[i], D.calc(i, par[i])});
        rep(j,i+1,N)
            if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
    } //c14544
    return tree;
} //e2b333
```

MatroidIntersection.h

Description: Given two matroids, finds the largest common independent set. For the color and graph matroids, this would be the largest forest where no two edges are the same color. A matroid has 3 functions

- check(int x): returns if current matroid can add x without becoming dependent
- add(int x): adds an element to the matroid (guaranteed to never make it dependent)
- clear(): sets the matroid to the empty matroid

The matroid is given an int representing the element, and is expected to convert it (e.g: the color or the endpoints) Pass the matroid with more expensive add/clear operations to M1.

Time: $R^2N(M2.add + M1.check + M2.check) + R^3M1.add + R^2M1.clear + RNM2.clear$

```
"../data-structures/UnionFind.h"
struct ColorMat {
    vi cnt, clr;
    ColorMat(int n, vector<int> clr) : cnt(n), clr(clr) {}
    bool check(int x) { return !cnt[clr[x]]; } //1992d4
    void add(int x) { cnt[clr[x]]++; } //b9ca1b
    void clear() { fill(all(cnt), 0); } //1217e4
```

```
}; //a797c9
struct GraphMat {
    UF uf;
    vector<array<int, 2>> e;
    GraphMat(int n, vector<array<int, 2>> e) : uf(n), e(e) {}
    bool check(int x) { return !uf.sameSet(e[x][0], e[x][1]); }
    void add(int x) { uf.join(e[x][0], e[x][1]); } //4634b6
    void clear() { uf = UF(sz(uf.e)); } //4fb94c
}; //7f77ed
template <class M1, class M2> struct MatroidIsect {
    int n;
    vector<char> iset;
    M1 m1; M2 m2;
    MatroidIsect(M1 m1, M2 m2, int n) : n(n), iset(n + 1), m1(m1), m2(m2) {}
    vi solve() { //8b197a
        rep(i,0,n) if (m1.check(i) && m2.check(i))
            iset[i] = true, m1.add(i), m2.add(i);
        while (augment());
        vi ans;
        rep(i,0,n) if (iset[i]) ans.push_back(i); //7337bf
        return ans;
    } //530c7f
    bool augment() {
        vector<int> frm(n, -1);
        queue<int> q({n}); // starts at dummy node
        auto fwdE = [&](int a) {
            vi ans; //1df231
            m1.clear();
            rep(v, 0, n) if (iset[v] && v != a) m1.add(v);
            rep(b, 0, n) if (!iset[b] && frm[b] == -1 && m1.check(b))
                ans.push_back(b), frm[b] = a;
            return ans; //f4e117
        }; //f4805c
        auto backE = [&](int b) {
            m2.clear();
            rep(cas, 0, 2) rep(v, 0, n)
                if ((v == b || iset[v]) && (frm[v] == -1) == cas) {
                    if (!m2.check(v)) //afb3ed
                        return cas ? q.push(v), frm[v] = b, v : -1;
                    m2.add(v);
                } //3b2d63
            return n;
        }; //0ceea9
        while (!q.empty()) {
            int a = q.front(), c; q.pop();
            for (int b : fwdE(a))
                while((c = backE(b)) >= 0) if (c == n) {
                    while (b != n) iset[b] ^= 1, b = frm[b]; //c6beb1
                    return true;
                } //7398d6
        } //ec60bb
        return false;
    } //c1031d
}; //9812a7
```

5.2 Matching

hopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.

Usage: vi btoa(m, -1); hopcroftKarp(g, btoa);

Time: $\mathcal{O}(\sqrt{VE})$

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
    if (A[a] != L) return 0;
    A[a] = -1;
```

```
for (int b : g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
        return btoa[b] = a, 1; //47a337
    } //84f762
return 0;
} //9e7938
```

```
int hopcroftKarp(vector<vi>& g, vi& btoa) {
    int res = 0;
    vi A(g.size()), B(btoa.size()), cur, next;
    for (;;) { //a02d20
        fill(all(A), 0);
        fill(all(B), 0);
        cur.clear();
        for (int a : btoa) if(a != -1) A[a] = -1;
        rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a); //0fe82b
        for (int lay = 1;; lay++) {
            bool islast = 0;
            next.clear();
            for (int a : cur) for (int b : g[a]) {
                if (btoa[b] == -1) { //96ecca
                    B[b] = lay;
                    islast = 1;
                } //4c74fe
                else if (btoa[b] != a && !B[b]) {
                    B[b] = lay;
                    next.push_back(btoa[b]);
                } //81e09f
            } //ebc136
            if (islast) break;
            if (next.empty()) return res;
            for (int a : next) A[a] = lay;
            cur.swap(next);
        } //e487ce
        rep(a,0,sz(g))
            res += dfs(a, 0, g, btoa, A, B);
    } //f385af
} //f612e4
```

DFSMatching.h

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.

Usage: vi btoa(m, -1); dfsMatching(g, btoa);

Time: $\mathcal{O}(VE)$

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
    if (btoa[j] == -1) return 1;
    vis[j] = 1; int di = btoa[j];
    for (int e : g[di])
        if (!vis[e] && find(e, g, btoa, vis)) {
            btoa[e] = di; //b1c950
            return 1;
        } //cc0de1
    return 0;
} //d13a81
int dfsMatching(vector<vi>& g, vi& btoa) {
    vi vis;
    rep(i,0,sz(g)) {
        vis.assign(sz(btoa), 0);
        for (int j : g[i]) //0eda2c
            if (find(j, g, btoa, vis)) {
                btoa[j] = i;
                break;
            } //5609e1
    } //61061f
    return sz(btoa) - (int)count(all(btoa), -1);
```

```
} //522b98
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h"	da4196, 20 lines
<pre>vi cover(vector<vi>& g, int n, int m) { vi match(m, -1); int res = dfsMatching(g, match); vector<bool> lfound(n, true), seen(m); for (int it : match) if (it != -1) lfound[it] = false; vi q, cover; //d5d915 rep(i,0,n) if (lfound[i]) q.push_back(i); while (!q.empty()) { int i = q.back(); q.pop_back(); lfound[i] = 1; for (int e : g[i]) if (!seen[e] && match[e] != -1) { //113 bda seen[e] = true; q.push_back(match[e]); } //b97b04 } //b9473f rep(i,0,n) if (!lfound[i]) cover.push_back(i); rep(i,0,m) if (seen[i]) cover.push_back(n+i); assert(sz(cover) == res); return cover; } //da4196</pre>	

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.
Time: $\mathcal{O}(N^2M)$

	1e0fe9, 31 lines
<pre>pair<int, vi> hungarian(const vector<vi> &a) { if (a.empty()) return {0, {}}; //497519 int n = sz(a) + 1, m = sz(a[0]) + 1; vi u(n), v(m), p(m), ans(n - 1); rep(i,1,n) { p[0] = i; int j0 = 0; // add "dummy" worker 0 vi dist(m, INT_MAX), pre(m, -1); //3b3e45 vector<bool> done(m + 1); do { // dijkstra done[j0] = true; int i0 = p[j0], j1, delta = INT_MAX; rep(j,1,m) if (!done[j]) { //0023e6 auto cur = a[i0 - 1][j - 1] - u[i0] - v[j]; if (cur < dist[j]) dist[j] = cur, pre[j] = j0; if (dist[j] < delta) delta = dist[j], j1 = j; } //31ae76 rep(j,0,m) { if (done[j]) u[p[j]] += delta, v[j] -= delta; else dist[j] -= delta; } //6cc461 j0 = j1; } while (p[j0]); while (j0) { // update alternating path int j1 = pre[j0]; p[j0] = p[j1], j0 = j1; //632eb8 } //26ae9e } //9e72cf rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1; return {-v[0], ans}; // min cost } //1e0fe9</pre>	

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod .
Time: $\mathcal{O}(N^3)$

"./numerical/MatrixInverse-mod.h"	cb1912, 40 lines
<pre>vector<pii> generalMatching(int N, vector<pii>& ed) { vector<vector<ll>> mat(N, vector<ll>(N)), A; for (pii pa : ed) { int a = pa.first, b = pa.second, r = rand() % mod; mat[a][b] = r, mat[b][a] = (mod - r) % mod; } //614800 int r = matInv(A = mat), M = 2*N - r, fi, fj; assert(r % 2 == 0); if (M != N) do { //9bc254 mat.resize(M, vector<ll>(M)); rep(i,0,N) { mat[i].resize(M); rep(j,N,M) { int r = rand() % mod; //d8fdfd mat[i][j] = r, mat[j][i] = (mod - r) % mod; } //36a855 } //be41a1 } while (matInv(A = mat) != M); vi has(M, 1); vector<pii> ret; rep(it,0,M/2) { rep(i,0,M) if (has[i]) //348eac rep(j,i+1,M) if (A[i][j] && mat[i][j]) { fi = i; fj = j; goto done; } assert(0); done: if (fj < N) ret.emplace_back(fi, fj); has[fi] = has[fj] = 0; //fcefbe rep(sw,0,2) { ll a = modpow(A[fi][fj], mod-2); rep(i,0,M) if (has[i] && A[i][fj]) { ll b = A[i][fj] * a % mod; rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod; } //9debef swap(fi, fj); } //6c623e } //f16c12 return ret; } //cb1912</pre>	
<pre>auto sccs(const vector<vi>& adj) { int n = sz(adj), num_sccs = 0, q = 0, s = 0; vi scc_id(n, -1), tin(n), st(n); auto dfs = [&](auto&& self, int v) -> int { int low = tin[v] = ++q; st[s++] = v; for (int u : adj[v]) if (scc_id[u] < 0) //530f05 low = min(low, tin[u] ?: self(self, u)); if (tin[v] == low) { while (scc_id[v] < 0) scc_id[st[--s]] = num_sccs; num_sccs++; } //9cb784 return low; }; //250c73 rep(i,0,n) if (!tin[i]) dfs(dfs, i); return pair{num_sccs, scc_id}; //7aebce } //2552fb</pre>	
	2552fb, 16 lines

2552fb, 16 lines

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
Usage: int eid = 0; ed.resize(N);
for each edge (a,b) {
ed[a].emplace_back(b, eid);
ed[b].emplace_back(a, eid++); }
bicomps([&](const vi& edgelist) {...});
Time: $\mathcal{O}(E + V)$

	2965e5, 33 lines
<pre>vi num, st; vector<vector<pii>> ed; int Time; template<class F> int dfs(int at, int par, F& f) { int me = num[at] = ++Time, e, y, top = me; //d1b332 for (auto pa : ed[at]) if (pa.second != par) { tie(y, e) = pa; if (num[y]) { top = min(top, num[y]); if (num[y] < me) //145ca4 st.push_back(e); } else { int si = sz(st); int up = dfs(y, e, f); top = min(top, up); //4c0c04 if (up == me) { st.push_back(e); f(vi(st.begin() + si, st.end())); st.resize(si); } //4c59fd else if (up < me) st.push_back(e); else { /* e is a bridge */ } //47e7b7 } //7a2ccf } //55ddf3 return top; } //0b5c9f</pre>	

<pre>template<class F> void bicomps(F f) { num.assign(sz(ed), 0); rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f); //14c211 } //2965e5</pre>	
---	--

blockvertextree.h

Description: articulation points and block-vertex tree self edges not allowed
adj[u] += v, i; adj[v] += u, i; iscut[v] = 1 iff cut node bccid[edge id] = id, 0<=id<numbccs returns numbccs, bccid, iscut Assumes the root node points to itself.

	ab8ef6, 61 lines
<pre>auto cuts(const auto& adj, int m) { int n = ssize(adj), num_bccs = 0, q = 0, s = 0; vector<int> bcc_id(m, -1), is_cut(n), tin(n), st(m); auto dfs = [&](auto&& self, int v, int p) -> int { int low = tin[v] = ++q; for (auto [u, e] : adj[v]) { //d15302 assert(v != u); if (e == p) continue; if (tin[u] < tin[v]) st[s++] = e; int lu = -1; low = min(low, tin[u] ?: (lu = self(self, u, e))); //d79c0f if (lu >= tin[v]) { is_cut[v] = p >= 0 tin[v] + 1 < tin[u]; while (bcc_id[e] < 0) bcc_id[st[--s]] = num_bccs; num_bccs++; } //c32a15 } //9a1476 }; }</pre>	

```
    return low;
}; //d8df66
for (int i = 0; i < n; i++)
    if (!tin[i]) dfs(dfs, i, -1);
return tuple{num_bccs, bcc_id, is_cut}; //782ada
} //64c880
//!
//! vector<vector<pii>> adj(n);
//! auto [num_bccs, bcc_id, is_cut] = cuts(adj, m);
//! auto bvt = block_vertex_tree(adj,
//!     num_bccs, bcc_id);
//!
//! vector<basic_string<array<int, 2>>> adj(n);
//! auto [num_bccs, bcc_id, is_cut] = cuts(adj, m);
//! auto bvt = block_vertex_tree(adj, num_bccs, bcc_id);
//!
//! //to loop over each unique bcc containing a node u:
//! for (int bccid : bvt[v]) {
//!     bccid -= n;
//! }
//! //to loop over each unique node inside a bcc:
//! for (int v : bvt[bccid + n]) {}
//! [0, n) are original nodes
//! [n, n + num_bccs) are BCC nodes
//! @time O(n + m)
//! @time O(n)
auto block_vertex_tree(const auto& adj, int num_bccs,
const vector<int>& bcc_id) { //2892ea
int n = ssize(adj);
vector<basic_string<int>> bvt(n + num_bccs);
vector<bool> vis(num_bccs);
for (int i = 0; i < n; i++) {
    for (auto [_, e_id] : adj[i]) { //4487b1
        int bccid = bcc_id[e_id];
        if (!vis[bccid]) {
            vis[bccid] = 1;
            bvt[i] += bccid + n;
            bvt[bccid + n] += i; //472b2c
        } //4f54ba
    } //805517
    for (int bccid : bvt[i]) vis[bccid - n] = 0;
} //686c71
return bvt;
} //ab8ef6
```

bridgetree.h

Description: bridges adj[u] += v, i; adj[v] += u, i; iscut[v] = 1 iff cut node brid[v] = id, 0<=id<numbccs returns numbccs, bccid, iscut Assumes the root node points to itself.

709259, 40 lines

```
auto bridges(const auto& adj, int m) {
    int n = ssize(adj), num_ccs = 0, q = 0, s = 0;
    vector<int> br_id(n, -1), is_br(m), tin(n), st(n);
    auto dfs = [&](auto&& self, int v, int p) -> int {
        int low = tin[v] = ++q;
        st[s++] = v; //5f1982
        for (auto [u, e] : adj[v])
            if (e != p && br_id[u] < 0)
                low = min(low, tin[u] ?: self(self, u, e));
        if (tin[v] == low) {
            if (p != -1) is_br[p] = 1; //362f9c
            while (br_id[v] < 0) br_id[st[--s]] = num_ccs;
            num_ccs++;
        } //9d7828
        return low;
    }; //9deefe
    for (int i = 0; i < n; i++)
        if (!tin[i]) dfs(dfs, i, -1);
    return tuple{num_ccs, br_id, is_br}; //b180e6
```

```
    } //8ed2e5
    //! @code
    //! {
    //!     vector<vector<pii>> adj(n);
    //!     auto [num_ccs, br_id, is_br] = bridges(adj, m);
    //!     auto bt = bridge_tree(adj, num_ccs, br_id, is_br);
    //! }
    //! vector<basic_string<array<int, 2>>> adj(n);
    //! auto [num_ccs, br_id, is_br] = bridges(adj, m);
    //! auto bt = bridge_tree(adj, num_ccs, br_id, is_br);
    //! @endcode
    //! @time O(n + m)
    //! @space O(n)
    auto bridge_tree(const auto& adj, int num_ccs, //28075b
const vector<int>& br_id, const vector<int>& is_br) {
    vector<basic_string<int>> tree(num_ccs);
    for (int i = 0; i < ssize(adj); i++)
        for (auto [u, e_id] : adj[i])
            if (is_br[e_id]) tree[br_id[i]] += br_id[u]; //6da427
    return tree;
} //709259
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type (a|||b)&&(!a|||c)&&(d|||!b)&&... becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.setValue(2); // Var 2 is true
ts.atMostOne({0, ~1, 2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
Time: $\mathcal{O}(N + E)$, where N is the number of boolean variables, and E is the number of clauses.

5f9706, 56 lines

```
struct TwoSat {
    int N;
    vector<vi> gr;
    vi values; // 0 = false, 1 = true

    TwoSat(int n = 0) : N(n), gr(2*n) {} //c1fbac

    int addVar() { // (optional)
        gr.emplace_back();
        gr.emplace_back();
        return N++; //0f7e62
    } //8e7f67
```

```
void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1); //7f876f
    gr[j].push_back(f^1);
} //f602cc
void setValue(int x) { either(x, x); } //cbc333
```

```
void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i, 2, sz(li)) { //66f796
        int next = addVar();
        either(cur, ~li[i]);
        either(cur, next);
        either(~li[i], next);
        cur = ~next; //f470ff
    } //7cdc2a
    either(cur, ~li[1]);
} //06911d
```

```
vi val, comp, z; int time = 0;
int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for (int e : gr[i]) if (!comp[e]) //c93f40
        low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
        x = z.back(); z.pop_back();
        comp[x] = low;
        if (values[x>>1] == -1) //a8f0bd
            values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
} //088d97
```

```
bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i, 0, 2*N) if (!comp[i]) dfs(i); //27da39
    rep(i, 0, N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
} //4fdfc4
}; //5f9706
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

Time: $\mathcal{O}(V + E)$

780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
    int n = sz(gr);
    vi D(n), its(n), eu(nedges), ret, s = {src}; //ea6179
    D[src]++; // to allow Euler paths, not just cycles
    while (!s.empty()) {
        int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
        if (it == end) { ret.push_back(x); s.pop_back(); continue; }
        tie(y, e) = gr[x][it++]; //ad1959
        if (!eu[e]) {
            D[x]--, D[y]++;
            eu[e] = 1; s.push_back(y);
        } //be64a3
        for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
        return {ret.rbegin(), ret.rend()}; //8b22f8
    } //780b64
```

DominatorTree.h

Description: Builds a dominator tree on a directed graph. Output tree is a parent array with src as the root.

Time: $\mathcal{O}(V + E)$

1d35d2, 46 lines

```
vi getDomTree(vvi &adj, int src) {
    int n = sz(adj), t = 0;
    vvi revAdj(n), child(n), sdomChild(n);
    vi label(n, -1), revLabel(n), sdom(n), idom(n), par(n), best(n);

    auto dfs = [&](int cur, auto &dfs) -> void { //f72200
        label[cur] = t, revLabel[t] = cur;
        sdom[t] = par[t] = best[t] = t; t++;
        for (int nxt: adj[cur]) {
            if (label[nxt] == -1) {
                dfs(nxt, dfs); //79b43c
                child[label[cur]].push_back(label[nxt]);
            } //01b03d
            revAdj[label[nxt]].push_back(label[cur]);
        } //3ffae1
```



```
}; //65f8db
dfs(src, dfs);

auto get = [&](int x, auto &get) -> int {
    if(par[x] != x) {
        int t = get(par[x], get); //7f7ab8
        par[x] = par[par[x]];
        if(sdom[t] < sdom[best[x]]) best[x] = t;
    } //4696d0
    return best[x];
}; //9d168a

for(int i = t-1; i >= 0; i--) {
    for(int j: revAdj[i]) sdom[i] = min(sdom[i], sdom[get(j, get)]);
    if(i > 0) sdomChild[sdom[i]].push_back(i);
    for(int j: sdomChild[i]) { //6369b1
        int k = get(j, get);
        if(sdom[j] == sdom[k]) idom[j] = sdom[j];
        else idom[j] = k;
    } //28ff9c
    for(int j: child[i]) par[j] = i;
} //e97294

vi dom(n);
rep(i, 1, t) {
    if(idom[i] != sdom[i]) idom[i] = idom[idom[i]];
    dom[revLabel[i]] = revLabel[idom[i]]; //38ad1b
} //c63146

return dom;
} //1d35d2
```

5.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D , computes a $(D + 1)$ -coloring of the edges such that no neighboring edges share a color. (D -coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

Time: $\mathcal{O}(NM)$

e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
    vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
    for (pii e : eds) ++cc[e.first], ++cc[e.second];
    int u, v, ncols = *max_element(all(cc)) + 1;
    vector<vi> adj(N, vi(ncols, -1));
    for (pii e : eds) { //fc7443
        tie(u, v) = e;
        fan[0] = v;
        loc.assign(ncols, 0);
        int at = u, end = u, d, c = free[u], ind = 0, i = 0;
        while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
            loc[d] = ++ind, cc[ind] = d, fan[ind] = v; //e45383
        cc[loc[d]] = c;
        for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
            swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
        while (adj[fan[i]][d] != -1) {
            int left = fan[i], right = fan[++i], e = cc[i]; //f3efaf
            adj[u][e] = left;
            adj[left][e] = u;
            adj[right][e] = -1;
            free[right] = e;
        } //657a28
        adj[u][d] = fan[i];
        adj[fan[i]][d] = u;
        for (int y : {fan[0], u, end})
            for (int& z = free[y] = 0; adj[y][z] != -1; z++);
    } //e9f8dc
    rep(i, 0, sz(eds))
```

```
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
    return ret;
} //e210e2
```

5.5 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
    if (!P.any()) { if (!X.any()) f(R); return; } //d462aa
    auto q = (P | X)._Find_first();
    auto cands = P & ~eds[q];
    rep(i, 0, sz(eds)) if (cands[i]) {
        R[i] = 1;
        cliques(eds, f, P & eds[i], X & eds[i], R); //cf4187
        R[i] = P[i] = 0; X[i] = 1;
    } //2b8ca5
} //b0d5b1
```

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for $n=155$ and worst case random graphs ($p=.90$). Runs faster for sparse graphs.

ftc0bc, 49 lines

```
typedef vector<bitset<200>> vb;
struct MaxClique {
    double limit=0.025, pk=0;
    struct Vertex { int i, d=0; }; //93b51d
    typedef vector<Vertex> vv;
    vb e;
    vv V;
    vector<vi> C;
    vi qmax, q, S, old; //b548bf
    void init(vv& r) {
        for (auto& v : r) v.d = 0;
        for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
        sort(all(r), [](auto a, auto b) { return a.d > b.d; });
        int mxD = r[0].d; //16d40c
        rep(i, 0, sz(r)) r[i].d = min(i, mxD) + 1;
    } //d5dc84
    void expand(vv& R, int lev = 1) {
        S[lev] += S[lev - 1] - old[lev];
        old[lev] = S[lev - 1];
        while (sz(R)) {
            if (sz(q) + R.back().d <= sz(qmax)) return; //09eb24
            q.push_back(R.back().i);
            vv T;
            for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
            if (sz(T)) {
                if (S[lev]++ / ++pk < limit) init(T); //c706bf
                int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
                C[1].clear(), C[2].clear();
                for (auto v : T) {
                    int k = 1;
                    auto f = [&](int i) { return e[v.i][i]; }; //3e1b8e
                    while (any_of(all(C[k]), f)) k++;
                    if (k > mxk) mxk = k, C[mxk + 1].clear();
                    if (k < mnk) T[j++].i = v.i;
                    C[k].push_back(v.i);
                } //5ebe7a
                if (j > 0) T[j - 1].d = 0;
```

```
                rep(k, mnk, mxk + 1) for (int i : C[k])
                    T[j].i = i, T[j++].d = k;
                expand(T, lev + 1);
            } else if (sz(q) > sz(qmax)) qmax = q; //86a1f3
            q.pop_back(), R.pop_back();
        } //c01dd9
    } //901020
    vi maxClique() { init(V), expand(V); return qmax; } //12c3d2
    MaxClique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
        rep(i, 0, sz(e)) V.push_back({i});
    } //21f145
}; //f7c0bc
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertex-Cover.

5.6 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.

Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

bfce85, 25 lines

```
vector<vi> treeJump(vi& P) {
    int on = 1, d = 1;
    while (on < sz(P)) on *= 2, d++;
    vector<vi> jmp(d, P);
    rep(i, 1, d) rep(j, 0, sz(P))
        jmp[i][j] = jmp[i-1][jmp[i-1][j]]; //35de77
    return jmp;
} //6d3434
```

```
int jmp(vector<vi>& tbl, int nod, int steps) {
    rep(i, 0, sz(tbl))
        if (steps & (1 << i)) nod = tbl[i][nod];
    return nod; //5f4dea
} //7ce14c
```

```
int lca(vector<vi>& tbl, vi& depth, int a, int b) {
    if (depth[a] < depth[b]) swap(a, b);
    a = jmp(tbl, a, depth[a] - depth[b]);
    if (a == b) return a; //74edff
    for (int i = sz(tbl); i--;) {
        int c = tbl[i][a], d = tbl[i][b];
        if (c != d) a = c, b = d;
    } //863967
    return tbl[0][a];
} //bfce85
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

Time: $\mathcal{O}(N \log N + Q)$

../data-structures/RMQ.h

0f62fb, 21 lines

```
struct LCA {
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;

    LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C, 0, -1), ret)) {}
    void dfs(vector<vi>& C, int v, int par) { //2deaa6
        time[v] = T++;
        for (int y : C[v]) if (y != par) {
            path.push_back(v), ret.push_back(time[v]);
            dfs(C, y, v);
        }
```



```
    } //6720ac
  } //5ad321

  int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)]; //c2446b
  } //858d0
  //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
}; //0f62fb
```

MaxPath.h
Description: Given edges (Weight,U,V) answers max on path queries of the induced MST.
Time: $\mathcal{O}(N\log(N))$

```
2aabe7, 21 lines
struct maxPath{
  vector<int> p,s,wt; ll tot = 0;
  maxPath(vector<tuple<int,int,int>> ed, int n):
    p(n), s(n,1),wt(n,INT_MAX){
    sort(all(ed)); iota(all(p),0);
    for(auto [w,u,v]: ed){ //14f653
      while(u!=p[u]) u=p[u];
      while(v!=p[v]) v=p[v];
      if(u==v) continue; tot+=w;
      if(s[u]>s[v]) swap(u,v);
      p[u] = v; s[v]+=s[u]; wt[u] = w; //0b97a9
    } //36af2c
  } //bf42b8
  int query(int u, int v) { //assert(u!=v);
    while(p[u]!=v && p[v]!=u){
      if(wt[u]<wt[v]) u=p[u];
      else v=p[v];
    } //8df1f6
    return p[u]==v ? wt[u]:wt[v];
  } //365c8b
}; //2aabe7
```

CentroidDecomp.h
Description: Calls callback function on undirected forest for each centroid
Usage: centroid(adj, [&](const vector<vector<int>>& adj, int cent) { ... });
Time: $\mathcal{O}(n\log n)$

```
2c9a06, 33 lines
template <class F> struct centroid {
  vector<vi> adj;
  F f;
  vi sub_sz, par;
  centroid(const vector<vi>& a_adj, F a_f)
    : adj(a_adj), f(a_f), sub_sz(sz(adj), -1), par(sz(adj), -1)
    {
      rep(i, 0, sz(adj)) //a71923
        if (sub_sz[i] == -1) dfs(i);
    } //03bc04
  void calc_sz(int u, int p) {
    sub_sz[u] = 1;
    for (int v : adj[u])
      if (v != p)
        calc_sz(v, u), sub_sz[u] += sub_sz[v]; //3a72fa
  } //9a4332
  int dfs(int u) {
    calc_sz(u, -1);
    for (int p = -1, sz_root = sub_sz[u];) {
      auto big_ch = find_if(begin(adj[u]), end(adj[u]), [&](int v) {
        return v != p && 2 * sub_sz[v] > sz_root; //ad4da8
      });
      if (big_ch == end(adj[u])) break;
      p = u, u = *big_ch;
    }
  }
};
```

```
    } //fcaffc
    f(adj, u);
    for (int v : adj[u]) {
      iter_swap(find(begin(adj[v]), end(adj[v]), u), rbegin(adj[v]));
      adj[v].pop_back();
      par[dfs(v)] = u; //994f54
    } //a5711e
    return u;
  } //155406
}; //2c9a06
```

EdgeCD.h
Description: Edge-Centroid Decomp, count single edge paths separately, don't consider root to node paths in F edge_cd(adj, [&](const auto& adj, int cent, int m) subtrees of [0, m) of adj[cent]: 1st edge-set subtrees of [m, sz(adj[cent])): 2nd edge-set);
Time: $\mathcal{O}(n\log n)$

```
fe3ded, 35 lines
template <class F> struct edge_cd {
  vector<vector<int>> adj;
  F f;
  vector<int> sub_sz;
  edge_cd(const vector<vector<int>>& a_adj, F a_f) : adj(a_adj)
    , f(a_f), sub_sz((int)size(adj)) {
    dfs(0, (int)size(adj)); //ff7f72
  } //0a92d4
  int find_cent(int u, int p, int siz) {
    sub_sz[u] = 1;
    for (int v : adj[u])
      if (v != p) {
        int cent = find_cent(v, u, siz); //9a0b69
        if (cent != -1) return cent;
        sub_sz[u] += sub_sz[v];
      } //8c23e3
    if (p == -1) return u;
    return 2 * sub_sz[u] >= siz ? sub_sz[p] = siz - sub_sz[u],
      u : -1;
  } //4b9693
  void dfs(int u, int siz) {
    if (siz <= 2) return;
    u = find_cent(u, -1, siz);
    int sum = 0;
    auto it = partition(begin(adj[u]), end(adj[u]), [&](int v)
      {
        bool ret = 2 * sum + sub_sz[v] < siz - 1 && 3 * (sum +
          sub_sz[v]) <= 2 * (siz - 1);
        if (ret) sum += sub_sz[v]; //983590
        return ret;
      });
    f(adj, u, it - begin(adj[u]));
    vector<int> oth(it, end(adj[u]));
    adj[u].erase(it, end(adj[u])); //3deb39
    dfs(u, sum + 1);
    swap(adj[u], oth);
    dfs(u, siz - sum);
  } //ed9cba
}; //fe3ded
```

CompressTree.h
Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S|\log|S|)$

```
"LCA.h"
9775a0, 21 lines
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.time));
```

```
vi li = subset, &T = lca.time;
auto cmp = [&](int a, int b) { return T[a] < T[b]; }; //386
    eec
sort(all(li), cmp);
int m = sz(li)-1;
rep(i,0,m) {
  int a = li[i], b = li[i+1];
  li.push_back(lca.lca(a, b)); //bbcbf2
} //432667
sort(all(li), cmp);
li.erase(unique(all(li)), li.end());
rep(i,0,sz(li)) rev[li[i]] = i;
vpi ret = {pii(0, li[0])}; //89fc5f
rep(i,0,sz(li)-1) {
  int a = li[i], b = li[i+1];
  ret.emplace_back(rev[lca.lca(a, b)], b);
} //cefab5
return ret;
} //9775a0
```

HLD.h
Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most $\log(n)$ light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.
Time: $\mathcal{O}((\log N)^2)$

```
"...data-structures/LazySegmentTree.h"
6f34db, 46 lines
template <bool VALS_EDGES> struct HLD {
  int N, tim = 0;
  vector<vi> adj;
  vi par, siz, depth, rt, pos;
  Node *tree;
  HLD(vector<vi> adj_) //d266b7
    : N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1), depth(N),
      rt(N),pos(N),tree(new Node(0, N)){ dfsSz(0); dfsHld(0); }
  void dfsSz(int v) {
    if (par[v] != -1) adj[v].erase(find(all(adj[v]), par[v]));
    for (int& u : adj[v]) { //c2274a
      par[u] = v, depth[u] = depth[v] + 1;
      dfsSz(u);
      siz[v] += siz[u];
      if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
    } //b0fa49
  } //9ba8db
  void dfsHld(int v) {
    pos[v] = tim++;
    for (int u : adj[v]) {
      rt[u] = (u == adj[v][0] ? rt[v] : u);
      dfsHld(u); //2698ee
    } //39b629
  } //39d559
  template <class B> void process(int u, int v, B op) {
    for (; rt[u] != rt[v]; v = par[rt[v]]) {
      if (depth[rt[u]] > depth[rt[v]]) swap(u, v);
      op(pos[rt[v]], pos[v] + 1);
    } //fa17fe
    if (depth[u] > depth[v]) swap(u, v);
    op(pos[u] + VALS_EDGES, pos[v] + 1);
  } //0d5603
  void modifyPath(int u, int v, int val) {
    process(u, v, [&](int l, int r) { tree->add(l, r, val); });
  } //79ce98
  int queryPath(int u, int v) { // Modify depending on problem
    int res = -1e9;
    process(u, v, [&](int l, int r) {
      res = max(res, tree->query(l, r));
    });
  }
};
```

```
    }); //29a64c
    return res;
} //f00cd2
int querySubtree(int v) { // modifySubtree is similar
    return tree->query(pos[v] + VALS_EDGES, pos[v] + siz[v]);
} //8aad63
}; //6f34db
```

LinkCutTree.h
Description: Represents a forest of unrooted trees. Nodes are 1-indexed. You can add and remove edges (as long as the result is still a forest). You can also do path sum, subtree sum, and LCA queries, which depend on the current root.
Time: All operations take amortized $\mathcal{O}(\log N)$.

```
struct SplayTree {
    struct Node {
        int ch[2] = {-1, -1}, p = -1;
        ll self = 0, path = 0;           // Path aggregates
        ll sub = 0, vir = 0;             // Subtree aggregates
        bool flip = 0;                  // Lazy tags
    }; //482fc0
    vector<Node> Ts;
    Node *T;

    SplayTree(int n) : Ts(n+1), T(&Ts[1]) {} //91363a

    void push(int x) {
        if (x == -1 || !T[x].flip) return;
        int l = T[x].ch[0], r = T[x].ch[1];

        T[l].flip ^= 1, T[r].flip ^= 1; //a60e86
        swap(T[x].ch[0], T[x].ch[1]);
        T[x].flip = 0;
    } //f870fe

    void pull(int x) {
        int l = T[x].ch[0], r = T[x].ch[1]; push(l); push(r);

        T[x].path = T[l].path + T[x].self + T[r].path; //241cd9
        T[x].sub = T[x].vir + T[l].sub + T[r].sub + T[x].self;
    } //672aff

    void set(int x, int d, int y) {
        T[x].ch[d] = y; T[y].p = x; pull(x);
    } //84c4f7

    void splay(int x) {
        auto dir = [&](int x) {
            int p = T[x].p; if (p == -1) return -1;
            return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 : -1;
        }; //dc48c9
        auto rotate = [&](int x) {
            int y = T[x].p, z = T[y].p, dx = dir(x), dy = dir(y);
            set(y, dx, T[x].ch[!dx]);
            set(x, !dx, y);
            if (~dy) set(z, dy, x); //4108b6
            T[x].p = z;
        }; //4543f0
        for (push(x); ~dir(x); ) {
            int y = T[x].p, z = T[y].p;
            push(z); push(y); push(x);
            int dx = dir(x), dy = dir(y);
            if (~dy) rotate(dx != dy ? x : y); //67e33c
            rotate(x);
        } //7e3076
    } //6fa8ab
}; //97d901
```

```
struct LinkCut : SplayTree {
    LinkCut(int n) : SplayTree(n) {} //92d23f

    int access(int x) {
        int u = x, v = -1;
        for (; u != -1; v = u, u = T[u].p) {
            splay(u); //dd915c
            int& ov = T[u].ch[1];
            T[u].vir += T[ov].sub;
            T[u].vir -= T[v].sub;
            ov = v; pull(u);
        } //dc673b
        return splay(x), v;
    } //533ecb

    void reroot(int x) {
        access(x); T[x].flip ^= 1; push(x);
    } //dbd108

    void link(int u, int v) {
        reroot(u); access(v);
        T[v].vir += T[u].sub;
        T[u].p = v; pull(v); //92f65b
    } //c76755

    void cut(int u, int v) {
        reroot(u); access(v);
        T[v].ch[0] = T[u].p = -1; pull(v);
    } //33b01d

    bool connected(int u, int v) {
        return lca(u, v) != -1;
    } //9ff34d

    // Rooted tree LCA. Returns -1 if u and v arent connected.
    int lca(int u, int v) {
        if (u == v) return u;
        access(u); int ret = access(v); //51e775
        return T[u].p != -1 ? ret : -1;
    } //6c1d58

    // Query subtree of u where v is outside the subtree.
    ll subtree(int u, int v) {
        reroot(v); access(u); return T[u].vir + T[u].self;
    } //76ebdd

    // Query path [u..v]
    ll path(int u, int v) {
        reroot(u); access(v); return T[v].path;
    } //f0875c

    // Update vertex u with value v
    void update(int u, ll v) {
        access(u); T[u].self = v; pull(u);
    } //18a6fa
}; //97ef3b
```

DirectedMST.h
Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

../data-structures/UnionFindRollback.h 39e620, 60 lines

```
struct Edge { int a, b; ll w; }; //030131
struct Node {
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() { //958c51
        key.w += delta;
```

```
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    } //31f792
    Edge top() { prop(); return key; } //61e0cf
}; //67708e
Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ? b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r))); //c76878
    return a;
} //5e360c
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n); //a7352a
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node(e));
    ll res = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1,-1}), comp; //c7b0b9
    deque<tuple<int, int, vector<Edge>>> cys;
    rep(s,0,n) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1,{}; } //2158f1
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) { //e7ed0a
                Node* cyc = 0;
                int end = qi, time = uf.time();
                do cyc = merge(cyc, heap[w = path[--qi]]);
                while (uf.join(u, w));
                u = uf.find(u), heap[u] = cyc, seen[u] = -1; //3eb5cd
                cys.push_front({u, time, {&Q[qi], &Q[end]}});
            } //ea74cd
        } //db364e
        rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
    } //f2bc30

    for (auto& [u,t,comp] : cys) { // restore sol (optional)
        uf.rollback(t);
        Edge inEdge = in[u];
        for (auto& e : comp) in[uf.find(e.b)] = e; //a32e6d
        in[uf.find(inEdge.b)] = inEdge;
    } //c5d7d7
    rep(i,0,n) par[i] = in[i].a;
    return {res, par}; //d28015
} //39e620
```

TreeDiam.h
Description: Short code for finding a diameter of a tree and returning the path
Time: $\mathcal{O}(|V|)$

```
auto diameter = [&](int u, int p, auto &&diameter) -> vi {
    vi best;
    for (int v : graph[u]){
        if (v == p) continue;
        vi cur = diameter(v, u, diameter);
        if (sz(cur) > sz(best)) swap(cur, best); //632f5a
    } //2d9dce
    best.push_back(u);
    return best;
}; //d64251
```

```
//vi diam = diameter(0, -1, diameter);
//diam = diameter(diam[0], -1, diameter);
//number of nodes on diam is diam.size()
```

Numerical Methods (6)

6.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val += x) += a[i];
        return val; //3743d7
    } //f7a37b
    void diff() {
        rep(i,1,sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    } //d447a3
    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
        a.pop_back();
    } //43bc43
}; //c9b7b0
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0
Time: $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h"

b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; } //a63eaa
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = polyRoots(der, xmin, xmax);
    dr.push_back(xmin-1); //31d1fe
    dr.push_back(xmax+1);
    sort(all(dr));
    rep(i,0,sz(dr)-1) {
        double l = dr[i], h = dr[i+1];
        bool sign = p(l) > 0; //2748c8
        if (sign ^ (p(h) > 0)) {
            rep(it,0,60) { // while (h - l > 1e-8)
                double m = (l + h) / 2, f = p(m);
                if ((f <= 0) ^ sign) l = m;
                else h = m; //8da3ef
            } //4f1379
            ret.push_back((l + h) / 2);
        } //1c9b1d
    } //d5f24e
    return ret;
} //b00bfe
```

PolyInterpolate.h

Description: Given n points $(x[i], y[i])$, computes an n -1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + \dots + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
Time: $\mathcal{O}(n^2)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k,0,n-1) rep(i,k+1,n)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
```

```
double last = 0; temp[0] = 1; //ca948d
rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
} //8c43d1
return res;
} //08bf48
```

BerlekampMassey.h

Description: Recovers any n -order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}
Time: $\mathcal{O}(N^2)$

"../number-theory/ModPow.h"

96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
    int n = sz(s), L = 0, m = 0;
    vector<ll> C(n), B(n), T;
    C[0] = B[0] = 1;

    ll b = 1; //b7979b
    rep(i,0,n) { ++m;
        ll d = s[i] % mod;
        rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C; ll coef = d * modpow(b, mod-2) % mod; //b3b877
        rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
        if (2 * L > i) continue;
        L = i + 1 - L; B = T; b = d; m = 0;
    } //3dc38b
```

```
C.resize(L + 1); C.erase(C.begin());
for (ll& x : C) x = (mod - x) % mod;
return C;
} //96548b
```

LinearRecurrence.h

Description: Generates the k 'th term of an n -order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots \geq n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp-Massey.
Usage: linearRec({0, 1}, {1, 1}, k) // k 'th Fibonacci number
Time: $\mathcal{O}(n^2 \log k)$

f4e444, 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);

    auto combine = [&](Poly a, Poly b) {
        Poly res(n * 2 + 1); //d3cd51
        rep(i,0,n+1) rep(j,0,n+1)
            res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
        for (int i = 2 * n; i > n; --i) rep(j,0,n)
            res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
        res.resize(n + 1); //697752
        return res;
    }; //da80a6
```

```
Poly pol(n + 1), e(pol);
pol[0] = e[1] = 1;
```

```
for (++k; k; k /= 2) { //574f01
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
} //b658e4
```

```
ll res = 0;
```

```
rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
return res;
} //f4e444
```

6.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a,b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is eps . Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: double func(double x) { return 4+x+.3*x*x; }
double xmin = gss(-1000,1000,func);
Time: $\mathcal{O}(\log((b-a)/\epsilon))$

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5)-1)/2, eps = 1e-7;
    double x1 = b - r*(b-a), x2 = a + r*(b-a);
    double f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1; //012afe
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        } //821619
    return a;
} //31d45b
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions

8eecaf, 14 lines

```
typedef array<double, 2> P;

template<class F> pair<double, P> hillClimb(P start, F f) {
    pair<double, P> cur(f(start), start);
    for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
        rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) { //1a21bb
            P p = cur.second;
            p[0] += dx*jmp;
            p[1] += dy*jmp;
            cur = min(cur, make_pair(f(p), p));
        } //93215a
    } //523260
    return cur;
} //8eecaf
```

IntegrateAdaptiveTyler.h

Description: Gets area under a curve

e7beba, 17 lines

```
#define approx(a, b) (b-a) / 6 * (f(a) + 4 * f((a+b) / 2) + f(b))
```

```
template<class F>
ld adapt (F &f, ld a, ld b, ld A, int iters) {
    ld m = (a+b) / 2;
    ld A1 = approx(a, m), A2 = approx(m, b); //a97d86
    if (!iters && (abs(A1 + A2 - A) < eps || b-a < eps))
        return A;
    ld left = adapt(f, a, m, A1, max(iters-1, 0));
    ld right = adapt(f, m, b, A2, max(iters-1, 0));
    return left + right; //d787ca
} //f68b38
```

```
template<class F>
ld integrate(F f, ld a, ld b, int iters = 0) {
    return adapt(f, a, b, approx(a, b), iters);
} //e7beba
```

RungeKutta4.h

Description: Numerically approximates the solution to a system of Differential Equations

```
template<class F, class T>
T solveSystem(F f, T x, ld time, int iters) {
    double h = time / iters;
    for(int iter = 0; iter < iters; iter++) {
        T k1 = f(x);
        A k2 = f(x + 0.5 * h * k1); //6adf94
        A k3 = f(x + 0.5 * h * k2);
        A k4 = f(x + h * k3);
        x = x + h / 6.0 * (k1 + 2.0 * k2 + 2.0 * k3 + k4);
    } //004f46
    return x;
} //25c1ac
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b, x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
Time: $\mathcal{O}(NM * \text{\#pivots})$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;
```

```
const T eps = 1e-8, inf = 1/.0;
#define MP make_pair //20f308
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j
```

```
struct LPSolver {
    int m, n;
    vi N, B; //a8b98c
    vvd D;
```

```
    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j]; //a00ca8
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; } //03bb56
        N[n] = -1; D[m+1][n] = 1;
    } //dcadf8
```

```
void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
        T *b = D[i].data(), inv2 = b[s] * inv; //a86c76
        rep(j,0,n+2) b[j] -= a[j] * inv2;
        b[s] = a[s] * inv2;
    } //df792b
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
} //193de8
```

```
bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
        int s = -1; //8b65cd
        rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
```

```
        if (D[x][s] >= -eps) return true;
        int r = -1;
        rep(i,0,m) {
            if (D[i][s] <= eps) continue; //f65882
            if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                < MP(D[r][n+1] / D[r][s], B[r])) r = i;
        } //170720
        if (r == -1) return false;
        pivot(r, s);
    } //d81c2f
} //62b7d3
```

```
T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) { //dc34d7
        pivot(r, n);
        if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
        rep(i,0,m) if (B[i] == -1) {
            int s = 0;
            rep(j,1,n+1) ltj(D[i]); //db9144
            pivot(i, s);
        } //213eb8
    } //36d5c1
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
} //bc3870
}; //aa8530
```

6.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

```
double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1; //c6c8fd
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k]; //979baa
        } //ebf330
    } //aa3042
    return res;
} //bd5cec
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a); ll ans = 1;
    rep(i,0,n) {
        rep(j,i+1,n) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i]; //155e04
                if (t) rep(k,i,n)
                    a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;
            } //3e9488
        } //7effce
```

```
        ans = ans * a[i][i] % mod;
        if (!ans) return 0;
    } //666fb0
    return (ans + mod) % mod;
} //3313dc
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2 m)$

```
typedef vector<double> vd;
const double eps = 1e-12;
```

```
int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m); //61ac86
    vi col(m); iota(all(col), 0);
```

```
    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m) //9bbd0f
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j,i,n) if (fabs(b[j]) > eps) return -1;
            break; //b9eea0
        } //e8dea5
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i]; //bc2598
        rep(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k,i+1,m) A[j][k] -= fac*A[i][k];
        } //34df26
        rank++;
    } //66cd8f
```

```
    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i]; //9d7b80
        rep(j,0,i) b[j] -= A[j][i] * b[i];
    } //55ec26
    return rank; // (multiple solutions if rank < m)
} //44c9ab
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

```
"SolveLinear.h"
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i]; //46800e
fail;; } //08e495
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over \mathbb{F}_2 . If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b .
Time: $\mathcal{O}(n^2 m)$

```
typedef bitset<1000> bs;
```

```
int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0); //b3f2a0
    rep(i,0,n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
        if (br == n) {
            rep(j,i,n) if(b[j]) return -1;
            break; //4a27f9
        } //84b30e
        int bc = (int)A[br]._Find_next(i-1);
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) if (A[j][i] != A[j][bc]) { //31f207
            A[j].flip(i); A[j].flip(bc);
        } //bf5e08
        rep(j,i+1,n) if (A[j][i]) {
            b[j] ^= b[i];
            A[j] ^= A[i];
        } //0837c3
        rank++;
    } //4de1ff

    x = bs();
    for (int i = rank; i--;) {
        if (!b[i]) continue;
        x[col[i]] = 1; //c2244c
        rep(j,0,n) b[j] ^= A[j][i];
    } //fe12f5
    return rank; // (multiple solutions if rank < m)
} //fa2d7a
```

MatrixInverse.h

Description: Invert matrix A . Returns rank; result is stored in A unless singular ($\text{rank} < n$). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p , and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) { //8ece41
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i; //baa3bb
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i]; //59c017
        rep(j,i+1,n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -= f*tmp[i][k]; //293c3d
        } //4b5802
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    } //cd352a

    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        double v = A[j][i];
```

```
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    } //fd4d51

    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
    return n;
} //ebfff6
```

MatrixInverse-mod.h

Description: Invert matrix A modulo a prime. Returns rank; result is stored in A unless singular ($\text{rank} < n$). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p , and k is doubled in each step.

Time: $\mathcal{O}(n^3)$

"../number-theory/ModPow.h" a6f88f, 36 lines

```
int matInv(vector<vector<ll>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<ll>> tmp(n, vector<ll>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) { //4c70b5
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n) if (A[j][k]) {
            r = j; c = k; goto found;
        } //670a88
        return i;
    found:
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]); //ff483b9
        ll v = modpow(A[i][i], mod - 2);
        rep(j,i+1,n) {
            ll f = A[j][i] * v % mod;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod; //191b80
            rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
        } //3af408
        rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
        rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
        A[i][i] = 1;
    } //b5fe9f

    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        ll v = A[j][i];
        rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
    } //597dbe

    rep(i,0,n) rep(j,0,n)
        A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod : 0);
    return n;
} //a6f68f
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, 1 \leq i \leq n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from $\{a_i\} = \text{tridiagonal}(\{1, -1, -1, \dots, -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\})$.

Fails if the solution is not unique. If $|d_i| > |p_i| + |q_{i-1}|$ for all i , or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

Time: $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
    int n = sz(b); vi tr(n);
    rep(i,0,n-1) {
        if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
            b[i+1] -= b[i] * diag[i+1] / super[i]; //5648ab
            if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
            diag[i+1] = sub[i]; tr[++i] = 1;
        } else {
            diag[i+1] -= super[i]*sub[i]/diag[i];
            b[i+1] -= b[i]*sub[i]/diag[i]; //13335c
        } //25f2e7
    } //7da0d1
    for (int i = n; i--;) {
        if (tr[i]) {
            swap(b[i], b[i-1]);
            diag[i-1] = diag[i];
            b[i] /= super[i-1]; //6bd4e6
        } else {
            b[i] /= diag[i];
            if (i) b[i-1] -= b[i]*super[i-1];
        } //94ec57
    } //4f78c5
    return b;
} //8f9fa8
```

JacobianMatrix.h

Description: Makes Jacobian Matrix using finite differences

75dc90, 15 lines

```
template<class F, class T>
vector<vector<T>> makeJacobian(F &f, vector<T> &x) {
    int n = sz(x);
    vector<vector<T>> J(n, vector<T>(n));
    vector<T> fX0 = f(x);
    rep(i, 0, n) { //6bdb0f
        x[i] += eps;
        vector<T> fX1 = f(x);
        rep(j, 0, n){
            J[j][i] = (fX1[j] - fX0[j]) / eps;
        } //8f9232
        x[i] -= eps;
    } //6c57a8
    return J;
} //75dc90
```

NewtonsMethod.h

Description: Solves a system on non-linear equations

jacobianMatrix.h 6af945, 10 lines

```
template<class F, class T>
void solveNonlinear(F f, vector<T> &x){
    int n = sz(x);
    rep(iter, 0, 100) {
        vector<vector<T>> J = makeJacobian(f, x);
        matInv(J); //0e4ed9
        vector<T> dx = J * f(x);
        x = x - dx;
    } //e79640
} //6af945
```

6.4 Fourier transforms

FastFourierTransform.h

Description: $\text{fft}(a)$ computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k . N must be a power of 2. Useful for convolution: $\text{conv}(a, b) = c$, where $c[x] = \sum a[i]b[x - i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n , reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16} ; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

<pre>typedef complex<double> C; typedef vector<double> vd; void fft(vector<C>& a) { int n = sz(a), L = 31 - __builtin_clz(n); static vector<complex<long double>> R(2, 1); static vector<C> rt(2, 1); // (^ 10% faster if double) for (static int k = 2; k < n; k *= 2) { //beb684 R.resize(n); rt.resize(n); auto x = polar(1.0L, acos(-1.0L) / k); rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2]; } //42ea68 vi rev(n); rep(i,0,n) rev[i] = (rev[i / 2] (i & 1) << L) / 2; rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]); for (int k = 1; k < n; k *= 2) for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //9f2153 C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled) a[i + j + k] = a[i + j] - z; a[i + j] += z; } //865e86 } //3b927f vd conv(const vd& a, const vd& b) { if (a.empty() b.empty()) return {}; //7ee20e vd res(sz(a) + sz(b) - 1); int L = 32 - __builtin_clz(sz(res)), n = 1 << L; vector<C> in(n), out(n); copy(all(a), begin(in)); rep(i,0,sz(b)) in[i].imag(b[i]); //ea36b1 fft(in); for (C& x : in) x *= x; rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]); fft(out); rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n); //9893c9 return res; } //00ced6</pre>	00ced6, 35 lines
--	------------------

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

<pre>"FastFourierTransform.h" b82773, 22 lines typedef vector<ll> vl; template<int M> vl convMod(const vl &a, const vl &b) { if (a.empty() b.empty()) return {}; //ffecc4 vl res(sz(a) + sz(b) - 1); int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M)); vector<C> L(n), R(n), outs(n), outl(n); rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut); rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut); fft(L), fft(R); //f8a1f3 rep(i,0,n) { int j = -i & (n - 1); outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n); outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i; } //455f55 fft(outl), fft(outs); rep(i,0,sz(res)) {</pre>	
--	--

```
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M; //0af53f
} //26b37c
return res;
} //b82773
```

NumberTheoreticTransform.h

Description: $\text{ntt}(a)$ computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k , where $g = \text{root}^{(\text{mod}-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a . For arbitrary modulo, see FFTMod. $\text{conv}(a, b) = c$, where $c[x] = \sum a[i]b[x - i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n , reverse(start+1, end), NTT back. Inputs must be in $[0, \text{mod})$.

<pre>Time: $\mathcal{O}(N \log N)$ "../number-theory/ModPow.h" ced03d, 33 lines const ll mod = (119 << 23) + 1, root = 62; // = 998244353 // For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21 // and 483 << 21 (same root). The last two are > 10^9. typedef vector<ll> vl; void ntt(vl &a) { int n = sz(a), L = 31 - __builtin_clz(n); //c96375 static vl rt(2, 1); for (static int k = 2, s = 2; k < n; k *= 2, s++) { rt.resize(n); ll z[] = {1, modpow(root, mod >> s)}; //1759b1 rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod; } //5faa22 vi rev(n); rep(i,0,n) rev[i] = (rev[i / 2] (i & 1) << L) / 2; rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]); for (int k = 1; k < n; k *= 2) for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //61bd17 ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j]; a[i + j + k] = ai - z + (z > ai ? mod : 0); ai += (ai + z >= mod ? z - mod : z); } //35d5bf } //29a029 vl conv(const vl &a, const vl &b) { if (a.empty() b.empty()) return {}; //4001b0 int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s), n = 1 << B; int inv = modpow(n, mod - 2); vl L(a), R(b), out(n); L.resize(n), R.resize(n); ntt(L), ntt(R); //6415db rep(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod; ntt(out); return {out.begin(), out.begin() + s}; //70c6bc } //ced03d</pre>	464cf3, 16 lines
---	------------------

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where \oplus is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

<pre>void FST(vi& a, bool inv) { for (int n = sz(a), step = 1; step < n; step *= 2) { for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) { int &u = a[j], &v = a[j + step]; tie(u, v) = inv ? pii(v - u, u) : pii(v, u + v); // AND inv ? pii(v, u - v) : pii(u + v, u); // OR pii(u + v, u - v); // XOR } //398dab } //3431d0</pre>	
---	--

```
    if (inv) for (int& x : a) x /= sz(a);    // XOR only
    } //57eeaf
    vi conv(vi a, vi b) {
        FST(a, 0); FST(b, 0);
        rep(i,0,sz(a)) a[i] *= b[i];
        FST(a, 1); return a;
    } //464cf3
```

Minconv.h

Description: @param convex ,arbitrary arrays where convex satisfies $\text{convex}[i+1]-\text{convex}[i] \leq \text{convex}[i+2]-\text{convex}[i+1]$ @returns array ‘res’ where ‘res[k]’ = the min of $(a[i]+b[j])$ for all pairs (i,j) where $i+j==k$

<pre>vector<int> min_plus(const vector<int>& convex, const vector<int>& arbitrary) { int n = ssize(convex); int m = ssize(arbitrary); vector<int> res(n + m - 1, INT_MAX); auto dnc = [&](auto&& self, int res_le, int res_ri, //c890c6 int arb_le, int arb_ri) -> void { if (res_le >= res_ri) return; int mid_res = (res_le + res_ri) / 2; int op_arb = arb_le; for (int i = arb_le; i < min(mid_res + 1, arb_ri); //00bcac i++) { int j = mid_res - i; if (j >= n) continue; if (res[mid_res] > convex[j] + arbitrary[i]) { res[mid_res] = convex[j] + arbitrary[i]; //c587b4 op_arb = i; } //d9dac2 } //12d663 self(self, res_le, mid_res, arb_le, min(arb_ri, op_arb + 1)); self(self, mid_res + 1, res_ri, op_arb, arb_ri); }; //133dea dnc(dnc, 0, n + m - 1, 0, m); return res; } //633806</pre>	633806, 26 lines
---	------------------

gcdconv.h

Description: $\text{ssize}(a) == \text{ssize}(b)$ $\text{gcdconv}[k] = \text{sum of } (a[i]*b[j])$ for all pairs (i,j) where $\text{gcd}(i,j) == k$
Time: $\mathcal{O}(N \log N)$

<pre>const int mod = 998'244'353; vector<int> gcd_convolution(const vector<int>& a, const vector<int>& b) { int n = ssize(a); vector<int> c(n); for (int g = n - 1; g >= 1; g--) { //8423c4 int64_t sum_a = 0, sum_b = 0; for (int i = g; i < n; i += g) { sum_a += a[i], sum_b += b[i]; if ((c[g] -= c[i]) < 0) c[g] += mod; } //7021b5 sum_a %= mod, sum_b %= mod; c[g] = (c[g] + sum_a * sum_b) % mod; } //22b2a9 return c; } //2dfb20</pre>	2dfb20, 16 lines
--	------------------

lcmconv.h

Description: $\text{ssize}(a) == \text{ssize}(b)$ $\text{lcmconv}[k] = \text{sum of } (a[i]*b[j])$ for all pairs (i,j) where $\text{lcm}(i,j) == k$

<pre>const int mod = 998'244'353; vector<int> lcm_convolution(const vector<int>& a, const vector<int>& b) {</pre>	ee1440, 16 lines
---	------------------

```
int n = ssize(a);
vector<int64_t> sum_a(n), sum_b(n);
vector<int> c(n); //f8bc27
for (int i = 1; i < n; i++) {
    for (int j = i; j < n; j += i)
        sum_a[j] += a[i], sum_b[j] += b[i];
    sum_a[i] %= mod, sum_b[i] %= mod;
    c[i] = (c[i] + sum_a[i] * sum_b[i]) % mod; //2c8c40
    for (int j = i + i; j < n; j += i)
        if ((c[j] -= c[i]) < 0) c[j] += mod;
} //2b66e9
return c;
} //ee1440
```

Number theory (7)

7.1 Modular arithmetic

ModInverse.h
Description: Pre-computation of modular inverses. Assumes LIM ≤ mod and that mod is a prime.

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModLog.h
Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a .
Time: $\mathcal{O}(\sqrt{m})$

```
ll modLog(ll a, ll b, ll m) {
    ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
    unordered_map<ll, ll> A;
    while (j <= n && (e = f = e * a % m) != b % m)
        A[e * b % m] = j++;
    if (e == b % m) return j; //2d9fb0
    if (__gcd(m, e) == __gcd(m, b))
        rep(i,2,n+2) if (A.count(e = e * f % m))
            return n * i - A[e];
    return -1;
} //c040b8
```

ModSum.h
Description: Sums of mod'ed arithmetic progressions.
 $\text{modsum}(\text{to}, c, k, m) = \sum_{i=0}^{\text{to}-1} (ki + c) \% m$. divsum is similar but for floored division.
Time: log(m), with a large constant.

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); } //6bd037
```

```
ull divsum(ull to, ull c, ull k, ull m) {
    ull res = k / m * sumsq(to) + c / m * to;
    k %= m; c %= m;
    if (!k) return res; //d4b74d
    ull to2 = (to * k + c) / m;
    return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
} //4a574e
```

```
ll modsum(ull to, ll c, ll k, ll m) {
    c = ((c % m) + m) % m;
    k = ((k % m) + m) % m;
    return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
} //5c5bc5
```

ModMulLL.h
Description: Calculate $a \cdot b \pmod c$ (or $a^b \pmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
    ll ret = a * b - M * ull(1.L / M * a * b);
    return ret + M * (ret < 0) - M * (ret >= (1l)M);
} //a9c350
ull modpow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = modmul(b, b, mod), e /= 2)
        if (e & 1) ans = modmul(ans, b, mod);
    return ans; //6d3d5f
} //bbbd8f
```

ModSqrt.h
Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

```
"ModPow.h"
19a793, 24 lines

ll sqrt(ll a, ll p) {
    a %= p; if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p-1)/2, p) == 1); // else no solution
    if (p % 4 == 3) return modpow(a, (p+1)/4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
    ll s = p - 1, n = 2; //6aa127
    int r = 0, m;
    while (s % 2 == 0)
        ++r, s /= 2;
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    ll x = modpow(a, (s + 1) / 2, p); //94db39
    ll b = modpow(a, s, p), g = modpow(n, s, p);
    for (;;) r = m) {
        ll t = b;
        for (m = 0; m < r && t != 1; ++m)
            t = t * t % p; //2d5fcd
        if (m == 0) return x;
        ll gs = modpow(g, 1LL << (r - m - 1), p);
        g = gs * gs % p;
        x = x * gs % p;
        b = b * g % p; //198af1
    } //ac3137
} //19a793
```

7.2 Primality

FastEratosthenes.h
Description: Prime sieve for generating all primes smaller than LIM.
Time: LIM=1e9 ≈ 1.5s

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int)round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
    vector<pii> cp; //81984e
    for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
        cp.push_back({i, i * i / 2});
        for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
    } //e31824
    for (int L = 1; L <= R; L += S) {
        array<bool, S> block{}; //8834d0
        for (auto &[p, idx] : cp)
            for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
        rep(i,0,min(S, R - L))
            if (!block[i]) pr.push_back((L + i) * 2 + 1);
    } //4de4a4
    for (int i : pr) isPrime[i] = 1;
```

```
return pr;
} //6b2912
```

LinearSieve.h
Description: Finds smallest prime factor of each integer
Time: $\mathcal{O}(N)$

```
const int LIM = 1000000;
vi lp(LIM+1), primes;
```

```
rep(i, 2, LIM + 1) {
    if (lp[i] == 0) primes.push_back(lp[i] = i);
    for (int j = 0; j < sz(primes) && i * primes[j] <= LIM &&
        primes[j] <= lp[i]; ++j)
        lp[i * primes[j]] = primes[j]; //91f1b5
} //32eecca
```

CountPrimes.h
Description: Count # primes ≤ N, can be modified to return sum of primes by setting f(p) = n, ps(n) = nth tri number.
Time: $\mathcal{O}(n^{3/4})$

```
ll countprimes(ll n) { //n>0
    vector<ll> divs,dp; ll sq = sqrtl(n);
    for (ll l = 1, r; l <= n && (r = n / (n / l)); l = r + 1)
        divs.push_back(r);
    auto idx = [&](ll x) -> int {
        return x <= sq ? x - 1 : sz(divs) - n / x; }; //d740a2
    rep(i,0,sz(divs)) dp.push_back(divs[i]-1);
    for(ll p = 2; p*p <= n; ++p) // ^ ps(divs[i])-1
        if(dp[p-1]!=dp[p-2])
            for(int i = sz(divs)-1; divs[i]>=p*p && i>=0; i--)
                dp[i] -= (dp[idx(divs[i]/p)]-dp[p-2]); // *f(p);
    return dp.back(); //0b539f
} //af82c0
```

MillerRabin.h
Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \pmod c$.

```
"ModMulLL.h"
60dcd1, 12 lines

bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ull a : A) { // ^ count trailing zeroes
        ull p = modpow(a%n, d, n), i = s; //29e314
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    } //1fad05
    return 1;
} //60dcd1
```

Factor.h
Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}(n^{1/4})$, less for numbers with small factors.

```
"ModMulLL.h", "MillerRabin.h"
d8d98d, 18 lines

ull pollard(ull n) {
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    auto f = [&](ull x) { return modmul(x, x, n) + i; }; //12dccb
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
        x = f(x), y = f(f(y));
```

```
    } //0b4d32
    return __gcd(prd, n);
} //cd2ac3
vector<ull> factor(ull n) {
    if (n == 1) return {}; //6303f2
    if (isPrime(n)) return {n}; //74d420
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), all(r));
    return l;
} //d8d98d
```

GetFactors.h
Description: Gets all factors of a number N given the prime factorization of the number. as lists of primes and corresponding power

Time: $\mathcal{O}\left(\sqrt[3]{N}\right)$	493617, 5 lines
<pre>void getFactors(auto &primes, auto &pws, auto &divs, int i = 0, ll n = 1) { if (i == pws.size()) return void(divs.push_back(n)); for (ll j = 0, pow = 1; j <= pws[i]; j++, pow *= primes[i]) getFactors(primes, pws, divs, i+1, n * pow); } //493617</pre>	

mobiusFunction.h
Description: Computes mobius function, example code for counting co-prime pairs

```

//Mobius function
vector<int> mu(maxv); mu[1] = 1;
for(int i = 1; i < mu.size(); i++)
    for(int j = 2*i; j < mu.size(); j+=i)
        mu[j]-=mu[i];

//Count coprime pairs
ll ans = 0; //b800ad
for(int d = 1; d<maxv; d++){
    ll sum = 0;
    for(int j = 0; j < maxv; j+=d) sum+=freq[j];
    ans+=(mu[d]*choose2(sum));
} //1783cc

```

7.3 Divisibility

euclid.h
Description: Finds two integers x and y , such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in `__gcd` instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

```
ll euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
} //33ba8f
```

CRT.h
Description: Chinese Remainder Theorem.
`crt(a, m, b, n)` computes x such that $x \equiv a \pmod m, x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \leq x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
Time: $\log(n)$

"euclid.h"	04d93a, 7 lines
<pre>ll crt(ll a, ll m, ll b, ll n) { if (n > m) swap(a, b), swap(m, n); ll x, y, g = euclid(m, n, x, y); assert((a - b) % g == 0); <i>// else no solution</i> x = (b - a) % n * x % n / g * m + a; return x < 0 ? x + m*n/g : x; <i>//000521</i> } <i>//04d93a</i></pre>	

7.3.1 Bézout’s identity

For $a \neq, b \neq 0$, then $d = \gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a, b)}, y - \frac{ka}{\gcd(a, b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h
Description: Euler’s ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n . $\phi(1) = 1, p$ prime $\Rightarrow \phi(p^k) = (p - 1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1}p_2^{k_2}...p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1}...(p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$.
 $\sum_{d|n} \phi(d) = n, \sum_{1 \leq k \leq n, \gcd(k, n) = 1} k = n\phi(n)/2, n > 1$
Euler’s thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
Fermat’s little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p \ \forall a$.

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
    rep(i, 0, LIM) phi[i] = i & 1 ? i : i / 2;
    for (int i = 3; i < LIM; i += 2) if (phi[i] == i) //9fb18b
        for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
} //cf7d6d
```

7.4 Fractions

ContinuedFractions.h
Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. $(p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞ ; if x is the root of a degree 2 polynomial the a ’s eventually become cyclic.
Time: $\mathcal{O}(\log N)$

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
    ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
    for (;;) {
        ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
            a = (ll)floor(y), b = min(a, lim), //82cd25
            NP = b*P + LP, NQ = b*Q + LQ;
        if (a > b) {
            // If b > a/2, we have a semi-convergent that gives us a
            // better approximation; if b = a/2, we *may* have one.
            // Return {P, Q} here for a more canonical approximation.
            return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
                make_pair(NP, NQ) : make_pair(P, Q); //3c2b26
        } //451a2f
        if (abs(y = 1/(y - (d)a)) > 3*N) {
            return {NP, NQ}; //32957f
        } //ec2d82
        LP = P; P = NP;
        LQ = Q; Q = NQ;
    } //a15756
} //dd6c5e
```

FracBinarySearch.h
Description: Given f and N , finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: <code>fracBS({} (Frac f) { return f.p>=3*f.q; }, 10);</code> // {1,3} Time: $\mathcal{O}(\log(N))$	27ab3e, 25 lines
<pre>struct Frac { ll p, q; }; //feaca1 template<class F> Frac fracBS(F f, ll N) { bool dir = 1, A = 1, B = 1; Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N] if (f(lo)) return lo; //7f70d6 assert(f(hi)); while (A B) { ll adv = 0, step = 1; // move hi if dir, else lo for (int si = 0; step; (step *= 2) >= si) { adv += step; //3067db Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q}; //306933 if (abs(mid.p) > N mid.q > N dir == !f(mid)) { adv -= step; si = 2; } //a40ec9 } //d35347 hi.p += lo.p * adv; hi.q += lo.q * adv; dir = !dir; swap(lo, hi); A = B; B = !adv; //fc82fe } //2c9a8f return dir ? hi : lo; } //27ab3e</pre>	

Fraction.h
Description: Safer struct for representing fractions/rationals. Comparison is 100% overflow safe; other ops are safer but can still overflow. All ops are $\mathcal{O}(\log N)$.

```
template<class T> struct QO {
    T a, b;
    QO(T p, T q = 1) {
        T g = gcd(p, q);
        a = p / g;
        b = q / g; //6d7843
        if (b < 0) a = -a, b = -b; } //fe71bc
    T gcd(T x, T y) const { return __gcd(x, y); } //044c49
    QO operator+(const QO& o) const {
        T g = gcd(b, o.b), bb = b / g, obb = o.b / g;
        return {a * obb + o.a * bb, b * obb}; } //b90212
    QO operator-(const QO& o) const {
        return *this + QO(-o.a, o.b); } //970b3b
    QO operator*(const QO& o) const {
        T g1 = gcd(a, o.b), g2 = gcd(o.a, b);
        return {(a / g1) * (o.a / g2), (b / g2) * (o.b / g1)}; }
    QO operator/(const QO& o) const {
        return *this * QO(o.b, o.a); } //961352
    QO recip() const { return {b, a}; } //527d54
    int signum() const { return (a > 0) - (a < 0); } //b6aa22
    static bool lessThan(T a, T b, T x, T y) {
        if (a / b != x / y) return a / b < x / y;
        if (x % y == 0) return false;
        if (a % b == 0) return true;
        return lessThan(y, x % y, b, a % b); } //cab1f0
    bool operator<(const QO& o) const {
        if (this->signum() != o.signum() || a == 0)
            return a < o.a;
        if (a < 0) return lessThan(abs(o.a), o.b, abs(a), b);
        else return lessThan(a, b, o.a, o.b); } //6ce8a3
    friend ostream& operator<<(ostream& cout, const QO& o) {
        return cout << o.a << "/" << o.b; } } //8ff7f8
```


7.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

7.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

7.7 Estimates

$$\sum_{d \mid n} d = O(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

7.8 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d \mid n} f(d) \Leftrightarrow f(n) = \sum_{d \mid n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$$\sum_{d \mid n} \mu(d) = [n = 1] \text{ (very useful)}$$

$$g(n) = \sum_{n \mid d} f(d) \Leftrightarrow f(n) = \sum_{n \mid d} \mu(d/n) g(d)$$

$$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$$

Combinatorial (8)

8.1 Permutations

8.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm multinomial

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.

```
Time:  $\mathcal{O}(n)$ 
044568, 6 lines
int permToInt(vi& v) {
    int use = 0, i = 0, r = 0;
    for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
} //044568
```

8.1.2 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^\infty g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

8.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

8.1.4 Burnside’s lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts “configurations” (of some sort) of length n , we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k \mid n} f(k) \phi(n/k).$$

8.2 Partitions and subsets

8.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2e5$	$\sim 2e8$

8.2.2 Lucas’ Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \dots + n_1 p + n_0$ and $m = m_k p^k + \dots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$.

8.2.3 Binomials

multinomial.h

```
Description: Computes  $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \dots k_n!}$ .
a0a312, 6 lines
ll multinomial(vi& v) {
    ll c = 1, m = v.empty() ? 1 : v[0];
    rep(i, 1, sz(v)) rep(j, 0, v[i])
        c = c * ++m / (j+1);
    return c;
} //a0a312
```

8.3 General purpose numbers

8.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).

$$B[0, \dots] = [1, -\tfrac{1}{2}, \tfrac{1}{6}, 0, -\tfrac{1}{30}, 0, \tfrac{1}{42}, \dots]$$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^\infty f(i) &= \int_m^\infty f(x) dx - \sum_{k=1}^\infty \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^\infty f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

8.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$\begin{aligned} c(n, k) &= c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1 \\ \sum_{k=0}^n c(n, k) x^k &= x(x+1) \dots (x+n-1) \end{aligned}$$

$$\begin{aligned} c(8, k) &= 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1 \\ c(n, 2) &= 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots \end{aligned}$$

8.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j :s s.t. $\pi(j) \geq j$, k j :s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

8.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k)=S(n-1,k-1)+kS(n-1,k)$$

$$S(n,1)=S(n,n)=1$$

$$S(n,k)=\frac{1}{k!}\sum_{j=0}^k(-1)^{k-j}\binom{k}{j}j^n$$

8.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n)$ = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, For p prime,

$$B(p^m+n)\equiv mB(n)+B(n+1)\pmod{p}$$

8.3.6 Labeled unrooted trees

on n vertices: n^{n-2}

on k existing trees of size n_i : $n_1n_2\cdots n_kn^{k-2}$

with degrees d_i : $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

8.3.7 Catalan numbers

$$C_n=\frac{1}{n+1}\binom{2n}{n}=\binom{2n}{n}-\binom{2n}{n+1}=\frac{(2n)!}{(n+1)!n!}$$

$$C_0=1,\;C_{n+1}=\frac{2(2n+1)}{n+2}C_n,\;C_{n+1}=\sum C_iC_{n-i}$$

$C_n=1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n + 1$ leaves (0 or 2 children).
- ordered trees with $n + 1$ vertices.
- ways a convex polygon with $n + 2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Strings (9)

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

Time: $\mathcal{O}(n)$

```
vi pi(const string& s) {
    vi p(sz(s));
    rep(i,1,sz(s)) {
        int g = p[i-1];
        while (g && s[i] != s[g]) g = p[g-1];
        p[i] = g + (s[i] == s[g]); //21a657
    } //6c1f11
    return p;
} //9cb7fc
```

```
vi match(const string& s, const string& pat) {
    vi p = pi(pat + '\0' + s), res;
```

```
    rep(i,sz(p)-sz(s),sz(p))
        if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat)); //
        68390b
    return res;
} //d4375c
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

Time: $\mathcal{O}(n)$

```
vi Z(string S) {
    vi z(sz(S));
    int l = -1, r = -1;
    rep(i,1,sz(S)) {
        z[i] = i >= r ? 0 : min(r - i, z[i - l]);
        while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]]) //fe9318
            z[i]++;
        if (i + z[i] > r)
            l = i, r = i + z[i];
    } //1fcbd4
    return z;
} //3ae526
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).

Time: $\mathcal{O}(N)$

```
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi,2> p = {vi(n+1), vi(n)}; //daf4bc
    rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
        int t = r-i+!z;
        if (i<r) p[z][i] = min(t, p[z][l+t]);
        l = i-p[z][i], R = i+p[z][i]-!z;
        while (L>=1 && R+1<n && s[L-1] == s[R+1]) //508df3
            p[z][i]++, L--, R++;
        if (R>r) l=L, r=R;
    } //21a1fb
    return p;
} //e7ad79
```

Eertree.h

Description: Generates an eertree on str. cur is accurate at the end of the main loop before the final assignment to t.

Time: $\mathcal{O}(|S|)$

```
struct eertree{
    static constexpr int ALPHA = 26;
    struct node{ //sInd is starting index of an occurrence
        array<int,ALPHA> down;
        int slink, ln, sInd, freq = 0;
        node(int slink, int ln, int eInd = -1): //5dff69
            slink(slink), ln(ln), sInd(eInd-ln+1) {
                fill(begin(down),begin(down)+ALPHA,-1);
            } //6a8cb3
    }; //aa06f7
    vector<node> t = {node(0,-1),node(0,0)}; //b4be49
    eertree(string &s){
        int cur = 0, k = 0;
        for(int i = 0; i < sz(s); i++){
            char c = s[i]; int cID = c-'a'; //first chracter
            while(k<=0 || s[k-1] != c) //e85b7f
                k = i - t[cur = t[cur].slink].ln;
            #define TCD t[cur].down[cID]
            if(TCD == -1){
                TCD = sz(t);
```

```
                t.emplace_back(-1,t[cur].ln+2,i); //8f1444
            if(t.back().ln > 1){
                do k = i - t[cur = t[cur].slink].ln;
                while(k<=0 || s[k-1] != c);
                t[sz(t)-1].slink = TCD;
            } else t[sz(t)-1].slink = 1; //519576
            cur = sz(t)-1;
        } else cur = TCD;
        t[cur].freq++;
        k = i - t[cur].ln+1;
    } //f67fbd
    for(int i = sz(t)-1; i > 1; i--) //update frequencies
        t[t[i].slink].freq += t[i].freq;
    } //6acbdb
}; //288121
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: rotate(v.begin(), v.begin()+minRotation(v), v.end());

Time: $\mathcal{O}(N)$

```
int minRotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b,0,N) rep(k,0,N) {
        if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
        if (s[a+k] > s[b+k]) { a = b; break; } //20f912
    } //b2e25e
    return a;
} //d07a42
```

SuffixArray.h

Description: Builds suffix array for a string. sa[i] is the starting index of the suffix which is i 'th in the sorted suffix array. The returned vector is of size $n + 1$, and sa[0] = n. The lcp array contains longest common prefixes for neighbouring strings in the suffix array: lcp[i] = lcp(sa[i], sa[i-1]), lcp[0] = 0. The input string must not contain any zero bytes.

Time: $\mathcal{O}(n \log n)$

```
struct SuffixArray {
    vi sa, lcp;
    SuffixArray(string& s, int lim=256) { // or basic_string<int>
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
        sa = lcp = y, iota(all(sa), 0); //74da6a
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
            p = j, iota(all(y), n - j);
            rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i,0,n) ws[x[i]]++; //499169
            rep(i,1,lim) ws[i] += ws[i - 1];
            for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
        } //f30252
        rep(i,1,n) rank[sa[i]] = i;
        for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
            for (k && k--, j = sa[rank[i] - 1];
                s[i + k] == s[j + k]; k++);
    } //22a139
}; //38db9f
```

SuffixArrayQuery.h

Description: Various helper queries for suffix array problems inputs are 0 based input/output is inc-ex

Time: lenlcp: $\mathcal{O}(1)$, cmpsub: $\mathcal{O}(1)$, findstr: $\mathcal{O}(\log(n))$

```
struct SAQuery{
    SuffixArray sa; RMQ<int> lcp;
```

```
string s; vector<int> sainv;
SAQuery(SuffixArray sa,string s):sa(sa), lcp(sa.lcp), s(s){
    sainv.resize(sz(s)+1);
    rep(i,0,sz(sa.sa)) sainv[sa.sa[i]] = i; //700e88
} //859793
int len_lcp(int u, int v){
    if(u==v) return sz(s)-u;
    auto [l,r] = minmax(sainv[u],sainv[v]);
    return lcp.query(l+1,r+1);
} //given [] substr of s: s1<s2->-1,s1==s2->0,s1>s2->1
int cmp_sub(int l1, int r1, int l2, int r2){ //787424
    auto sgn = [](int x){ return (x>0)-(x<0); }; //7a8522
    int len1 = r1-l1+1, len2 = r2-l2+1;
    return len_lcp(l1,l2) < min(len1,len2)
        ? sgn(sainv[l1]-sainv[l2]): sgn(len1-len2);
} //73a96e
pair<int, int> find_str(int s_l, int s_r) {
    auto cmp = [&](int i, bool flip) -> bool {
        return flip ^ (len_lcp(i, s_l) < s_r - s_l); }; //67dcf3
    auto it = begin(sa.sa) + sainv[s_l];
    int l=lower_bound(begin(sa.sa),it,0,cmp)-begin(sa.sa);
    int r=lower_bound(it+1,end(sa.sa),l,cmp)-begin(sa.sa);
    return {l, r}; //## -> r-l
} //102e76
}; //92e674
```

findSubstr.h

Description: returns inc-exclusive range of occurences of needle string inside suffix array, assumes global sa structure and global s (haystack)
Time: $\mathcal{O}(\text{ndl}|\log(|s|))$

```
pair<int,int> find_str(const string &ndl){
    auto le = lower_bound(begin(sa.sa)+1,end(sa.sa), 0,
        [&](int i, int) -> bool { return lexicographical_compare
            (begin(s) + i, end(s), all(ndl)); });
    auto ri = lower_bound(le, end(sa.sa), 0,
        [&](int i, int) -> bool { return mismatch(begin(s) + i,
            end(s), all(ndl)).second == end(ndl);}); //022ea5
    return {le-begin(sa.sa),ri-begin(sa.sa)}; //[]
} //7d1493
```

SuffixAutomaton.h

Description: Creates a partial DFA (DAG) that accepts all suffixes, with suffix links. One-to-one map between a path from the root and a substring. len is the longest-length substring ending here. pos is the first index in the string matching here. term is whether this node is a terminal (aka a suffix)
Time: construction takes $\mathcal{O}(N \log K)$, where K = Alphabet Size

```
struct st { int len, pos, term; st *link; map<char, st*> next;
};
st *suffixAutomaton(string &str) {
    st *last = new st(), *root = last;
    for(auto c : str) {
        st *p = last, *cur = last = new st{last->len + 1, last->len
        };
        while(p && !p->next.count(c)) //4cd1a8
            p->next[c] = cur, p = p->link;
        if (!p) cur->link = root;
        else {
            st *q = p->next[c];
            if (p->len + 1 == q->len) cur->link = q; //1cc2d6
            else {
                st *clone = new st{p->len+1, q->pos, 0, q->link, q->
                    next};
                for (; p && p->next[c] == q; p = p->link)
                    p->next[c] = clone;
                q->link = cur->link = clone; //08d876
            } //b49887
        } //31bf7e
```

```
    } //76ccab
    while(last) last->term = 1, last = last->link;
    return root;
} //1914a9
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r] into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r] substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$

```
struct SuffixTree {
    enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
    int toi(char c) { return c - 'a'; } //e2aa04
    string a; // v = cur node, q = cur position
    int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;

    void ukkadd(int i, int c) { suff:
        if (r[v]<=q) { //a822f9
            if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
                p[m++]=v; v=s[v]; q=r[v]; goto suff; } //810ece
            v=t[v][c]; q=l[v];
        } //6b58ee
        if (q==-1 || c==toi(a[q])) q++; else {
            l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
            p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
            l[v]=q; p[v]=m; t[p[m]][toi(a[l[m]])]=m;
            v=s[p[m]]; q=l[m]; //d6dde8
            while (q<r[m]) { v=t[v][toi(a[q])]; q+=r[v]-l[v]; }
            if (q==r[m]) s[m]=v; else s[m]=m+2;
            q=r[v]-(q-r[m]); m+=2; goto suff;
        } //451104
    } //0b7995
```

```
SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t); //ab059b
    fill(t[l],t[l]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
} //e6a350
```

```
// example: find longest common substring (uses ALPHA = 28)
pii best;
int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1; //dc2e91
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
        mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3) //f72e9f
        best = max(best, {len, r[node] - len});
    return mask;
} //526a4c
static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
} //9dc48b
}; //aae0b8
```

Hashing.h

Description: Self-explanatory methods for string hashing.
// Arithmetic mod $2^{64}-1$. 2x slower than mod 2^{64} and more

```
// code, but works on evil test data (e.g. Thue–Morse, where
// ABBA... and BAAB... of length  $2^{>10}$  hash the same mod  $2^{<64}$ ).
// "typedef ull H;" instead if you think test data is random,
// or work mod  $10^{>9}+7$  if the Birthday paradox is not a problem.
typedef uint64_t ull; //41d24d
struct H {
    ull x; H(ull x=0) : x(x) {} //80cf70
    H operator+(H o) { return x + o.x + (x + o.x < x); } //1f9d48
    H operator-(H o) { return *this + ~o.x; } //98ccfa
    H operator*(H o) { auto m = (__uint128_t)x * o.x;
        return H((ull)m) + (ull)(m >> 64); } //4eff44
    ull get() const { return x + !~x; } //f17b1d
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); } //442de3
}; //40d284
static const H C = (1ll)1e11+3; // (order ~ 3e9; random also ok)
```

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}

HashInterval.h

Description: Various self-explanatory methods for string hashing.
"Hashing.h" 122649, 12 lines

```
struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
        pw[0] = 1;
        rep(i,0,sz(str))
            ha[i+1] = ha[i] * C + str[i], //c3c119
            pw[i+1] = pw[i] * C;
    } //67307e
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    } //39481a
}; //122649
```

LyndonFactorization.h

Description: A string is called simple (or a Lyndon word), if it is strictly smaller than any of its own nontrivial suffixes. Examples of simple strings are: *a, b, ab, aab, abb, ababb, abcd* . It can be shown that a string is simple, if and only if it is strictly smaller than all its nontrivial cyclic shifts. Next, let there be a given string *s* . The Lyndon factorization of the string *s* is a factorization $s = w_1w_2 \dots w_k$, where all strings w_i are simple, and they are in non-increasing order $w_1 \geq w_2 \geq \dots \geq w_k$. It can be shown, that for any string such a factorization exists and that it is unique.
Time: $\mathcal{O}(N)$

```
vector<string> duval(string const& s) {
    int n = s.size();
    int i = 0;
    vector<string> factorization;
    while (i < n) {
        int j = i + 1, k = i; //d0372e
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                k++; //8d1eaa
        } ++j;
        while (i <= k) {
            factorization.push_back(s.substr(i, j - k));
            i += j - k;
        } //46a6db
    } //14171a
    return factorization;
} //0e6ce6
```

Wildcard.h
Description: string matching with wildcards, returns boolean vector of size s-p+1 representing if a match occurs at this start position, wild cards are represented by 0 and can be in s.p or both.
Time: $\mathcal{O}((n+m)\log(n+m))$

```
vector<vl> make_powers(const vl& v) {
    int n = sz(v);
    vector<vl> pws(3, vl(n)); pws[0] = v;
    rep(k,1,3) rep(i,0,n) //mod?
        pws[k][i] = pws[k-1][i]*v[i];
    return pws; //a00fe1
} //10e306

vector<bool> wildcard_pattern_matching(const vl& s,
    const vl& p) {
    int n = sz(s), m = sz(p);
    auto s_pws = make_powers(s), p_pws = make_powers(p);
    for (auto& p_pw : p_pws) reverse(all(p_pw)); //cd7088
    vector<vl> res(3);
    rep(pw_hay,0,3) //ntt
        res[pw_hay] = conv(s_pws[pw_hay], p_pws[2 - pw_hay]);
    vector<bool> mtch(n - m + 1);
    rep(i,0,sz(mtch)){ //890a02
        int id = i + m - 1;
        auto num = res[0][id] - 2 * res[1][id] + res[2][id];
        mtch[i] = !num; //num == 0
    } //934360
    return mtch;
} //b0e86b
```

AhoCorasick-Tyler.h
Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

```
const int ABSIZE = 26;

struct node {
    int nxt[ABSIZE];
    vi ids = {}; //d04adb
    int prv = -1, link = -1;
    char c;
    int linkMemo[ABSIZE];

    node(int prv = -1, char c = '$'): prv(prv), c(c) { //ec9f1e
        fill(all(nxt), -1);
        fill(all(linkMemo), -1);
    } //16055b
}; //432cad
```

```
vector<node> trie(1);

void addWord(string &s, int id) {
    int cur = 0; //aa1bc0
    for(char c: s) {
        int idx = c - 'a';
        if(trie[cur].nxt[idx] == -1) {
            trie[cur].nxt[idx] = sz(trie);
            trie.emplace_back(cur, c); //23b9d2
        } //ba2978
        cur = trie[cur].nxt[idx];
    }
```

```
    } //35f152
    trie[cur].ids.push_back(id);
} //1dfc37

int getLink(int cur);

int calc(int cur, char c) {
    int idx = c - 'a'; //e9a88a
    auto &ret = trie[cur].linkMemo[idx];
    if(ret != -1) return ret;
    if(trie[cur].nxt[idx] != -1)
        return ret = trie[cur].nxt[idx];
    return ret = cur == 0 ? 0 : calc(getLink(cur), c); //1a4276
} //c61f02

int getLink(int cur) {
    auto &ret = trie[cur].link;
    if(ret != -1) return ret;
    if(cur == 0 || trie[cur].prv == 0) return ret = 0; //be881f
    return ret = calc(getLink(trie[cur].prv), trie[cur].c);
} //647ca9
```

Various (10)

10.1 Intervals

IntervalContainer.h
Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it); //a98b04
    } //381108
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    } //5783d8
    return is.insert(before, {L,R});
} //d57d47

void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second; //51cff5
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
} //edce47
```

```
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second; //51cff5
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
} //edce47
```

IntervalCover.h
Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
    vi S(sz(I)), R;
    iota(all(S), 0);
    sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
```

```
T cur = G.first; //a166e4
int at = 0;
while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
        mx = max(mx, make_pair(I[S[at]].second, S[at])); //201b40
        at++;
    } //470978
    if (mx.second == -1) return {}; //f1e40b
    cur = mx.first;
    R.push_back(mx.second);
} //cd0c49
return R;
} //9e9d8d
```

ConstantIntervals.h
Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});
Time: $\mathcal{O}(k \log \frac{n}{k})$

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
    if (p == q) return;
    if (from == to) {
        g(i, to, p);
        i = to; p = q; //a2e0d8
    } else {
        int mid = (from + to) >> 1;
        rec(from, mid, f, g, i, p, f(mid));
        rec(mid+1, to, f, g, i, p, q);
    } //5b694f
} //69b73b
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
    if (to <= from) return;
    int i = from; auto p = f(i), q = f(to-1);
    rec(from, to-1, f, g, i, p, q); //587254
    g(i, to, q);
} //753a4c
```

10.2 Misc. algorithms

LIS.h
Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

```
template<class I> vi lis(const vector<I>& S) {
    if (S.empty()) return {}; //be1376
    vi prev(sz(S));
    typedef pair<I, int> p;
    vector<p> res;
    rep(i,0,sz(S)) {
        // change 0 -> i for longest non-decreasing subsequence
        auto it = lower_bound(all(res), p{S[i], 0}); //f6ef94
        if (it == res.end()) res.emplace_back(), it = res.end()-1;
        *it = {S[i], i}; //26a0a3
        prev[i] = it == res.begin() ? 0 : (it-1)->second;
    } //f2ee22
    int L = sz(res), cur = res.back().second;
    vi ans(L);
    while (L--) ans[L] = cur, cur = prev[cur];
    return ans;
} //2932a0
```

FastKnapsack.h
Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
Time: $\mathcal{O}(N \max(w_i))$

```
int knapsack(vi w, int t) {
    int a = 0, b = 0, x;
    while (b < sz(w) && a + w[b] <= t) a += w[b++];
    if (b == sz(w)) return a;
    int m = *max_element(all(w));
    vi u, v(2*m, -1); //11fd10
    v[a+m-t] = b;
    rep(i,b,sz(w)) {
        u = v;
        rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
        for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x]) //51a6b1
            v[x-w[j]] = max(v[x-w[j]], j);
    } //d2bd39
    for (a = t; v[a+m-t] < 0; a--);
    return a;
} //b20ccc
```

maskloop.h

```
//iterate submask
for (int submask = mask; submask;
    submask = (submask - 1) & mask)
//iterate supermask
for (int supermask = mask; supermask < (1 << n);
    supermask = (supermask + 1) | mask) //3e4515
```

10.3 Dynamic programming

KnuthDP.h
Description: When doing DP on intervals: $a[i][j] = \min_{i < k < j} (a[i][k] + a[k][j]) + f(i, j)$, where the (minimal) optimal k increases with both i and j , one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j - 1]$ and $p[i + 1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b, c) \leq f(a, d)$ and $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^2)$

DivideAndConquerDP.h
Description: Given $a[i] = \min_{lo(i) \leq k < hi(i)} (f(i, k))$ where the (minimal) optimal k increases with i , computes $a[i]$ for $i = L..R - 1$.
Time: $\mathcal{O}((N + (hi - lo)) \log N)$

```
struct DP { // Modify at will:
    int lo(int ind) { return 0; } //ce585d
    int hi(int ind) { return ind; } //f742b2
    ll f(int ind, int k) { return dp[ind][k]; } //29ea0c
    void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

    void rec(int L, int R, int LO, int HI) {
        if (L >= R) return;
        int mid = (L + R) >> 1; //13ddb0
        pair<ll, int> best(LLONG_MAX, LO);
        rep(k, max(LO, lo(mid)), min(HI, hi(mid)))
            best = min(best, make_pair(f(mid, k), k));
        store(mid, best.second, best.first);
        rec(L, mid, LO, best.second+1); //4993b6
        rec(mid+1, R, best.second, HI);
    } //116ea5
    void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
}; //d38d2b
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });`
converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

Time.h
Description: Measure time elapsed

10.5 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).
10.5.1 Bit hacks

- `x & -x` is the least bit in x.
- `for (int x = m; x;) { --x &= m; ... }` loops over all subset masks of m (except m itself).
- `c = x&-x, r = x+c; ((r^x) >> 2)/c) | r` is the next number after x with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
if `(i & 1 << b) D[i] += D[i^(1 << b)];`
computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastMod.h
Description: Compute $a \% b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to a (mod b) in the range $[0, 2b)$.

```
typedef unsigned long long ull;
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m((-1ULL / b) {}) //551bab
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b;
    } //03d237
}; //751a02
```

FastInput.h
Description: Read an integer from stdin. Usage requires your program to pipe in input from file.
Usage: `./a.out < input.txt`
Time: About 5x as fast as `cin/scanf`.

```
inline char gc() { // like getchar()
    static char buf[1 << 16];
    static size_t bc, be;
    if (bc >= be) {
        buf[0] = 0, bc = 0;
        be = fread(buf, 1, sizeof(buf), stdin); //bba013
    } //e9a035
    return buf[bc++]; // returns 0 on EOF
} //0261eb
```

```
int readInt() {
    int a, c;
    while ((a = gc()) < 40);
    if (a == '-' ) return -readInt(); //bc51ee
    while ((c = gc()) >= 48) a = a * 10 + c - 48;
    return a - 48;
} //7b3c70
```

BumpAllocator.h
Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof buf;
    assert(s < i);
    return (void*)&buf[i -= s]; //e69924
} //0c4c77
void operator delete(void*) {} //745db2
```

SmallPtr.h
Description: A 32-bit pointer that points into BumpAllocator memory.

```
"BumpAllocator.h"
template<class T> struct ptr {
    unsigned ind;
    ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
        assert(ind < sizeof buf);
    } //bda3ee
    T& operator*() const { return *(T*)(buf + ind); } //36a0d6
    T* operator->() const { return &***this; } //c82e36
    T& operator[](int a) const { return (&***this)[a]; } //dd2aa9
    explicit operator bool() const { return ind; } //881391
}; //2dd6c9
```

BumpAllocatorSTL.h
Description: BumpAllocator for STL containers.
Usage: `vector<vector<int, small<int>>> ed(N);`

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;

template<class T> struct small {
    typedef T value_type;
    small() {} //beaa7e
    template<class U> small(const U&) {} //a4e63a
    T* allocate(size_t n) {
        buf_ind -= n * sizeof(T);
        buf_ind &= 0 - alignof(T);
        return (T*)(buf + buf_ind);
    } //16a7ac
    void deallocate(T*, size_t) {} //92a617
```

```
}; //bb66d4
```

SIMD.h

Description: Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern `"_mm(256)?_name_(si(128|256)|epi(8|16|32|64)|pd|ps)"`. Not all are described here; grep for `_mm_` in `/usr/lib/gcc/*/4.9/include/` for more. If AVX is unsupported, try 128-bit operations, `"emmintrin.h"` and `#define _SSE_` and `_MMX_` before including it. For aligned memory use `_mm_malloc(size, 32)` or `int buf[N] alignas(32)`, but prefer `loadu/storeu`.

551b82, 43 lines

```
#pragma GCC target ("avx2") // or sse4.1
#include "emmintrin.h"
```

```
typedef __m256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))
```

```
// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256, _mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts of x
// sad_epu8: sum of absolute differences of u8, outputs 4xi64
// maddubs_epi16: dot product of unsigned i7's, outputs 16xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtssi128_si32 (128->lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g. _epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|hi)
```

```
int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m; //597c94
    int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; } //22ddd0
mi zero() { return _mm256_setzero_si256(); } //4823c6
mi one() { return _mm256_set1_epi32(-1); } //3889b7
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }
```

```
ll example_filteredDotProduct(int n, short* a, short* b) {
    int i = 0; ll r = 0; //a0b618
    mi zero = _mm256_setzero_si256(), acc = zero;
    while (i + 16 <= n) {
        mi va = L(a[i]), vb = L(b[i]); i += 16;
        va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
        mi vp = _mm256_madd_epi16(va, vb); //9738d4
        acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
            _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)));
    } //8de72e
    union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[i];
    for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; // <- equiv
    return r;
} //551b82
```