λ

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UCF Lambda

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Contest (1)

.bashrc — 3 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++20 \
  -fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps = <>
```

hash.sh — 6 lines

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
# Usage:
#   To make executable, run the command: chmod +x hash.sh
#   To execute: ./hash.sh < file.cpp
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

Mathematics (2)

2.1 Equations

$$ax^2+bx+c=0 \Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The extremum is given by $x=-b/2a$.

$$\begin{aligned}ax+by=e\\cx+dy=f\end{aligned} \Rightarrow \begin{aligned}x=\frac{ed-bf}{ad-bc}\\y=\frac{af-ec}{ad-bc}\end{aligned}$$

In general, given an equation $Ax=b$, the solution to a variable x_i is given by

$$x_i=\frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n=c_1a_{n-1}+\cdots+c_ka_{n-k}$, and $r_1,\ldots,r_k$ are distinct roots of $x^k-c_1x^{k-1}-\cdots-c_k$, there are $d_1,\ldots,d_k$ s.t.

$$a_n=d_1r_1^n+\cdots+d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n=(d_1n+d_2)r^n$.

2.3 Trigonometry

$$\sin(v+w)=\sin v\cos w+\cos v\sin w$$
$$\cos(v+w)=\cos v\cos w-\sin v\sin w$$

$$\tan(v+w)=\frac{\tan v+\tan w}{1-\tan v\tan w}$$
$$\sin v+\sin w=2\sin\frac{v+w}{2}\cos\frac{v-w}{2}$$
$$\cos v+\cos w=2\cos\frac{v+w}{2}\cos\frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2=(V-W)\tan(v+w)/2$$

where V,W are lengths of sides opposite angles v,w.

$$a\cos x+b\sin x=r\cos(x-\phi)$$
$$a\sin x+b\cos x=r\sin(x+\phi)$$

where $r=\sqrt{a^2+b^2}, \phi=\operatorname{atan2}(b,a)$.

2.4 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x=-\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x=1+\tan^2 x \qquad \frac{d}{dx}\arctan x=\frac{1}{1+x^2}$$
$$\int\tan ax=-\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax=\frac{\sin ax-ax\cos ax}{a^2}$$
$$\int e^{-x^2}=\frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx=\frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx=[F(x)g(x)]_a^b-\int_a^b F(x)g'(x)dx$$

2.5 Sums

$$c^a+c^{a+1}+\cdots+c^b=\frac{c^{b+1}-c^a}{c-1}, c\neq 1$$

$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$
$$1^2+2^2+3^2+\cdots+n^2=\frac{n(2n+1)(n+1)}{6}$$
$$1^3+2^3+3^3+\cdots+n^3=\frac{n^2(n+1)^2}{4}$$
$$1^4+2^4+3^4+\cdots+n^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.6 Series

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots,\ (-\infty<x<\infty)$$
$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots,\ (-1<x\leq 1)$$
$$\sqrt{1+x}=1+\frac{x}{2}-\frac{x^2}{8}+\frac{2x^3}{32}-\frac{5x^4}{128}+\ldots,\ (-1\leq x\leq 1)$$
$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots,\ (-\infty<x<\infty)$$
$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots,\ (-\infty<x<\infty)$$

2.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu=\mathbb{E}(X)=\sum_x xp_X(x)$ and variance $\sigma^2=V(X)=\mathbb{E}(X^2)-(\mathbb{E}(X))^2=\sum_x(x-\mathbb{E}(X))^2p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX+bY)=a\mathbb{E}(X)+b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX+bY)=a^2V(X)+b^2V(Y).$$

2.7.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n,p)$, $n=1,2,\ldots,\ 0\leq p\leq 1$.

$$p(k)=\binom{n}{k}p^k(1-p)^{n-k}$$

$$\mu=np,\ \sigma^2=np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \le p \le 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \dots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.7.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.8 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \dots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki} t_k$.

2.9 Geometry

2.9.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

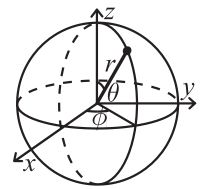
2.9.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.9.3 Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \mathrm{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \mathrm{atan2}(y, x) \end{aligned}$$

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

782797, 16 lines

```cpp
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>; //cd2981

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9)); //b1d86a
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
} //782797
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```cpp
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
}; //9b48b4
```

```
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16});
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$

0f4bdb, 19 lines

```
struct Tree {
  typedef int T;
  static constexpr T unit = INT_MIN;
  T f(T a, T b) { return max(a, b); } // (any associative fn)
  vector<T> s; int n;
  Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {} //2b8055
  void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;)
      s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
  } //17a935
  T query(int b, int e) { // query [b, e)
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
      if (b % 2) ra = f(ra, s[b++]);
      if (e % 2) rb = f(s[--e], rb); //d4678d
    } //3391a8
    return f(ra, rb);
  } //c90093
}; //0f4bdb
```

LazySegmentTree.h

Description: Segment tree with lazy prop, modify at will. 0-based, inclusive-exclusive.
Usage: `lazy_segtree<ValType,LazyType> st(arr)`
Time: $\mathcal{O}(\log N)$.

4f67ee, 60 lines

```
template<class T, class F>
struct lazy_segtree {
  int N, log, S;
  T idem = //modify: op(val,idem) = val
  F defLazy = //modify: applyLazy/combLazy(val,defLazy) = val
  vector<T> d; //ce445d
  vector<F> lz;
  T op(T left, T right) { /*modify*/ } //09e848
  T applyLazy(T val, F lazy) { /*modify*/ } //92c854
  F combLazy(F old, F nw) { /*modify*/ } //4c16ff
  lazy_segtree(const vector<T>& v):
    N(sz(v)), log(__lg(2 * N - 1)), S(1 << log), d(2 * S, idem)
      ,
    lz(S, defLazy) {
    for (int i = 0; i < N; i++) d[S + i] = v[i];
    for (int i = S - 1; i >= 1; i--) pull(i); //5a138c
  } //9e2ae3
  void apply(int k, F f) {
    d[k] = applyLazy(d[k], f); //len = S>>(31-__builtin_clz(k))
      ;
    if (k < S) lz[k] = combLazy(lz[k],f);
  } //c7a3ac
  void push(int k) {
    apply(2 * k, lz[k]), apply(2 * k + 1, lz[k]), lz[k] =
        defLazy;
  } //f332a2
  void push(int l, int r) {
    int zl = __builtin_ctz(l), zr = __builtin_ctz(r);
    for (int i = log; i > min(zl, zr); i--) {
      if (i > zl) push(l >> i);
      if (i > zr) push((r - 1) >> i); //40c9d4
    } //4613f2
  } //708f77
  void pull(int k) { d[k] = op(d[2 * k], d[2 * k + 1]); }
  void set(int p, T x) {
    p += S;
    for (int i = log; i >= 1; i--) push(p >> i);
    for (d[p] = x; p /= 2;) pull(p); //e740d0
  } //f65093
  T query(int l, int r) {
    if (l == r) return idem;
    push(l += S, r += S);
    T vl = idem, vr = idem;
    for (; l < r; l /= 2, r /= 2) { //f70eb7
      if (l & 1) vl = op(vl, d[l++]);
      if (r & 1) vr = op(d[--r], vr);
    } //09a718
    return op(vl, vr);
  } //7197c1
  void update(int l, int r, F f) {
    if (l == r) return;
    push(l += S, r += S);
    for (int a = l, b = r; a < b; a /= 2, b /= 2) {
      if (a & 1) apply(a++, f); //9b5541
      if (b & 1) apply(--b, f);
    } //f428a2
    int zl = __builtin_ctz(l), zr = __builtin_ctz(r);
    for (int i = min(zl, zr) + 1; i <= log; i++) {
      if (i > zl) pull(l >> i);
      if (i > zr) pull((r - 1) >> i);
    } //592fbe
  } //b5d617
}; //4f67ee
```

Wavelet.h

Description: kth: finds k+1th smallest number in [l,r), count: rank of k (how many < k) in [l,r). Doesn't support negative numbers, and requires a[i] <= maxval. Use BitVector to make 1.6x faster and 4x less memory.
Time: $\mathcal{O}(\log MAX)$

11aee1, 38 lines

```
struct WaveletTree {
  int n; vvi bv; // vector<BitVector> bv;
  WaveletTree(vl a, ll max_val):
    n(sz(a)), bv(1+__lg(max_val), {{}}) {
    vl nxt(n);
    for (int h = sz(bv); h--;) { //2d1680
      vector<bool> b(n);
      rep(i, 0, n) b[i] = ((a[i] >> h) & 1);
      bv[h] = vi(n+1); // bv[h] = b;
      rep(i, 0, n) bv[h][i+1] = bv[h][i] + !b[i]; // delete
      array it{begin(nxt), begin(nxt) + bv[h][n]}; //0c84d2
      rep(i, 0, n) *it[b[i]]++ = a[i];
      swap(a, nxt);
    } //f93ef6
  } //54c891
  ll kth(int l, int r, int k) {
    ll res = 0;
    for (int h = sz(bv); h--;) {
      int l0 = bv[h][l], r0 = bv[h][r];
      if (k < r0 - l0) l = l0, r = r0; //e4af0f
      else
        k -= r0 - l0, res |= 1ULL << h,
          l += bv[h][n] - l0, r += bv[h][n] - r0;
    } //aa8465
    return res;
  } //67fa6f
  int count(int l, int r, ll ub) {
    int res = 0;
    for (int h = sz(bv); h--;) {
      int l0 = bv[h][l], r0 = bv[h][r];
      if ((~ub >> h) & 1) l = l0, r = r0; //09ef1a
      else
        res += r0 - l0, l += bv[h][n] - l0,
          r += bv[h][n] - r0;
    } //8380c1
    return res;
  } //d305cc
}; //11aee1
```

BitVector.h

Description: Given vector of bits, counts number of 0's in [0, r). Use with WaveletTree.h by using modifications in comments in that file and replacing bv[h][x] with bv[h].cnt0(x)
Time: $\mathcal{O}(1)$ time

afd9d2, 15 lines

```
struct BitVector {
  vector<pair<ll, int>> b;
  BitVector(vector<bool> a): b(sz(a) / 64 + 1) {
    rep(i, 0, sz(a))
      b[i >> 6].first |= ll(a[i]) << (i & 63);
    rep(i, 0, sz(b)-1) //cba6aa
      b[i + 1].second = __builtin_popcountll(b[i].first)
        + b[i].second;
  } //4da2bc
  int cnt0(int r) {
    auto [x, y] = b[r >> 6];
    return r - y
      - __builtin_popcountll(x & ((1ULL << (r & 63)) - 1));
  } //01da37
}; //afd9d2
```

PST.h

Description: Persistent segment tree with laziness
Time: $\mathcal{O}(\log N)$ per query, $\mathcal{O}((n+q)\log n)$ memory

7ddad1, 41 lines

```
struct PST {
  PST *l = 0, *r = 0;
  int lo, hi;
  ll val = 0, lzadd = 0;
  PST(vl& v, int lo, int hi) : lo(lo), hi(hi) {
    if (lo + 1 < hi) { //e43119
      int mid = lo + (hi - lo)/2;
      l = new PST(v, lo, mid); r = new PST(v, mid, hi);
      val = l->val + r->val;
    } //ebf78b
    else val = v[lo];
  } //7ff852
  ll query(int L, int R) {
    if (R <= lo || hi <= L) return 0; // idempotent
    if (L <= lo && hi <= R) return val;
    push();
    return l->query(L, R) + r->query(L, R); //6a44fe
  } //108984
  PST* add(int L, int R, ll v) {
    if (R <= lo || hi <= L) return this;
    PST *n;
    if (L <= lo && hi <= R) {
      n = new PST(*this); //70575f
      n->val += v;
      n->lzadd += v;
    } else {
      push();
      n = new PST(*this); //c68682
      n->l = l->add(L, R, v);
      n->r = r->add(L, R, v);
      n->val = n->l->val + n->r->val;
    } //d1bfc5
    return n;
  } //d6d267
  void push() {
    if(lzadd == 0) return;
    l = l->add(lo, hi, lzadd);
    r = r->add(lo, hi, lzadd);
    lzadd = 0; //d7e73b
```

```
  } //0af5c4
}; //7ddad1
```

Xorbasis.h

Description: Makes a basis of binary vectors
Time: check/add -> $\mathcal{O}\left((B^2)/32\right)$

1d856b, 19 lines

```
template<int B>
struct XORBasis {
  bitset<B> b[B];
  int npivot = 0, nfree = 0;
  bool check(bitset<B> v) {
    for(int i = B-1; i >= 0; i--) //563a45
      if (v[i]) v ^= b[i];
    return v == 0;
  } //4915f9
  bool add(bitset<B> v) {
    for(int i = B-1; i >= 0; i--) {
      if (v[i]) {
        if (b[i] == 0) return b[i] = v, ++npivot;
        v ^= b[i]; //7a144f
      } //b1b631
    } //8da7e8
    return !++nfree;
  } //fbb3dd
}; //1d856b
```

UnionFind.h

Description: Disjoint-set data structure.
Time: $\mathcal{O}(\alpha(N))$

7aa27c, 14 lines

```
struct UF {
  vi e;
  UF(int n) : e(n, -1) {} //b71208
  bool sameSet(int a, int b) { return find(a) == find(b); }
  int size(int x) { return -e[find(x)]; } //3abb0a
  int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x]); }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b); //12ea70
    e[a] += e[b]; e[b] = a;
    return true;
  } //a61cdf
}; //7aa27c
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$

de4ad0, 21 lines

```
struct RollbackUF {
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {} //66f6eb
  int size(int x) { return -e[find(x)]; } //dfd9e1
  int find(int x) { return e[x] < 0 ? x : find(e[x]); } //
      f73c5d
  int time() { return sz(st); } //821d77
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  } //e7fe82
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]}); //3aaa7c
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  } //f0724e
}; //de4ad0
```

MonoRange.h

Description: when cmp = less(): a[le[i]] < a[i] >= a[ri[i]]
Usage: vi le = mono_st(a, less()),
ri = mono_range(le);
less_equal(), greater(), greater_equal()
Time: $\mathcal{O}(N)$.

191698, 16 lines

```
template<class T, typename F>
vi mono_st(const vector<T> & a, F cmp) {
  vi le(sz(a));
  rep(i,0,sz(a)){
    for (le[i] = i -1; le[i] >= 0 && !cmp(a[le[i]],a[i]);)
        le[i] = le[le[i]]; } //f637ae
  return le;
} //a87918

vi mono_range(const vi &le) {
  vi ri(sz(le), sz(le));
  rep (i, 0, sz(le))
    for (int j = i - 1; j != le[i]; j = le[j]) //9e9289
    ri[j] = i;
  return ri;
} //191698
```

CountRect.h

Description: cnt[i][j] = number of times an i-by-j sub rectangle appears such that all i*j cells **ARE 1**. cnt[i][0],cnt[0][j] are garbage
Time: $\mathcal{O}(NM)$

71b256, 22 lines

```
vector<vi> count_rectangles(
  const vector<vector<bool>>&grid) {
    int n = sz(grid), m = sz(grid[0]);
    vector<vi> cnt(n + 1, vi(m + 1, 0));
    vi h(m);
    for( const auto &row : grid) { //2270a5
        transform(all(h), begin(row), begin(h),
        [](int a, bool g) { return g * (a + 1); });
        vi le ( mono_st(h,less())), r(mono_range(le));
        rep(j,0,m) {
            int cnt_l = j - le[j] - 1, cnt_r = r[j] - j - 1;
            cnt[h[j]][cnt_l + cnt_r + 1]++; //9e604e
            cnt[h[j]][cnt_l]--;
            cnt[h[j]][cnt_r]--;
        } //82de19
    } //7a1347
    rep(i,1,n+1) rep(k,0,2) for (int j = m; j > 1; j--)
        cnt[i][j - 1] += cnt[i][j];
    for (int i = n ; i > 1; i--)
        rep(j, 1, m + 1) cnt[i - 1][j] += cnt[i][j];
    return cnt; //eca1f3
} //71b256
```

KineticTree.h

Description: Query A[i] * T + B on a range, with updates
<bits/stdc++.h>

ea1f15, 123 lines

```
// kinetic_tournament.cpp
// Eric K. Zhang; Aug. 29, 2020
//
// Suppose that you have an array containing pairs of
     nonnegative integers,
// A[i] and B[i]. You also have a global parameter T,
     corresponding to the
// "temperature" of the data structure. Your goal is to support
     the following
// queries on this data:
//
//   - update(i, a, b): set A[i] = a and B[i] = b
//   - query(s, e): return min{s <= i <= e} A[i] * T + B[i]
//   - heaten(new_temp): set T = new_temp
//         [precondition: new_temp >= current value of T]
// Time complexity:
//
//   - query: O(log n)
//   - update: O(log n)
//   - heaten: O(log^2 n)  [amortized]
//
// Verification: FBHC 2020, Round 2, Problem D "Log Drivin'
    Hirin'"

using namespace std; //ca417d

template <typename T = int64_t>
class kinetic_tournament {
  const T INF = numeric_limits<T>::max();
  typedef pair<T, T> line; //d69b7b

  size_t n;          // size of the underlying array
  T temp;            // current temperature
  vector<line> st;   // tournament tree
  vector<T> melt;    // melting temperature of each subtree

  inline T eval(const line& ln, T t) { //873ff1
    return ln.first * t + ln.second;
  } //c80a59

  inline bool cmp(const line& line1, const line& line2) {
    auto x = eval(line1, temp);
    auto y = eval(line2, temp);
    if (x != y) return x < y; //d35afa
    return line1.first < line2.first;
  } //384adf

  T next_isect(const line& line1, const line& line2) {
    if (line1.first > line2.first) {
      T delta = eval(line2, temp) - eval(line1, temp);
      T delta_slope = line1.first - line2.first; //61969f
      assert(delta > 0);
      T mint = temp + (delta - 1) / delta_slope + 1;
      return mint > temp ? mint : INF;  // prevent overflow
    } //8575df
    return INF;
  } //da51eb

  void recompute(size_t lo, size_t hi, size_t node) {
    if (lo == hi || melt[node] > temp) return;

    size_t mid = (lo + hi) / 2; //43f4e9
    recompute(lo, mid, 2 * node + 1);
    recompute(mid + 1, hi, 2 * node + 2);

    auto line1 = st[2 * node + 1];
    auto line2 = st[2 * node + 2]; //e72bf4
    if (!cmp(line1, line2))
      swap(line1, line2);
    st[node] = line1;

    melt[node] = min(melt[2 * node + 1], melt[2 * node + 2]);
    if (line1 != line2) { //07daab
      T t = next_isect(line1, line2);
      assert(t > temp);
      melt[node] = min(melt[node], t);
```

```
    } //ae6500
  } //c7c9ce

  void update(size_t i, T a, T b, size_t lo, size_t hi, size_t
      node) {
    if (i < lo || i > hi) return;
    if (lo == hi) {
      st[node] = {a, b}; //b3c015
      return;
    } //0ea9d2
    size_t mid = (lo + hi) / 2;
    update(i, a, b, lo, mid, 2 * node + 1);
    update(i, a, b, mid + 1, hi, 2 * node + 2);
    melt[node] = 0;
    recompute(lo, hi, node); //6c6626
  } //2310fd

  T query(size_t s, size_t e, size_t lo, size_t hi, size_t node
      ) {
    if (hi < s || lo > e) return INF;
    if (s <= lo && hi <= e) return eval(st[node], temp);
    size_t mid = (lo + hi) / 2; //ac9315
    return min(query(s, e, lo, mid, 2 * node + 1),
      query(s, e, mid + 1, hi, 2 * node + 2));
  } //e6ffb2

public:
  // Constructor for a kinetic tournament, takes in the size n
      of the
  // underlying arrays a[..], b[..] as input.
  kinetic_tournament(size_t size) : n(size), temp(0) { //11bad8
    assert(size > 0);
    size_t seg_size = ((size_t) 2) << (64 - __builtin_clzll(n -
        1));
    st.resize(seg_size, {0, INF});
    melt.resize(seg_size, INF);
  } //708aca

  // Sets A[i] = a, B[i] = b.
  void update(size_t i, T a, T b) {
    update(i, a, b, 0, n - 1, 0);
  } //108f7d

  // Returns min{s <= i <= e} A[i] * T + B[i].
  T query(size_t s, size_t e) {
    return query(s, e, 0, n - 1, 0);
  } //8c4963

  // Increases the internal temperature to new_temp.
  void heaten(T new_temp) {
    assert(new_temp >= temp);
    temp = new_temp; //d099ee
    recompute(0, n - 1, 0);
  } //530f4e
}; //ea1f15
```

Lichao.h

Description: min Li-chao tree allows for range add of arbitary functions such that any two functions only occur atmost once.
Usage: `inc-inc, implicit, works with negative indices, O(log(n)) query`
`flip signs in update and modify query to change to max.`

1eac23, 37 lines

```
struct func {
    ll A,B;
    func(ll A, ll B): A(A), B(B) {} //4cab61
    ll operator()(ll x) { return (A*x + B); } //e8aa73
}; //b3c3bf
const func idem = {0,(ll)8e18}; //idempotent, change for max
struct node {
    int lo, md, hi;
    func f = idem;
    node *left = nullptr, *right = nullptr; //12341d
    node(int lo, int hi): lo(lo), hi(hi), md(lo+(hi-lo)/2) {}
    void check(){
        if(left) return;
        left = new node(lo,md);
        right = new node(md+1,hi); //b79f4d
    } //edfaa5
    void update(func e) { //flip signs for max
        if(e(md) < f(md)) swap(e, f);
        if(lo == hi) return;
        if(e(lo) > f(lo) && e(hi) > f(hi)) return;
        check(); //cf8828
        if(e(lo) < f(lo)) left->update(e);
        else right->update(e);
    } //fdf3fd
    void rangeUpdate(int L, int R, func e) { //[]
        if(R < lo || hi < L) return;
        if(L <= lo && hi <= R) return update(e);
        check();
        left->rangeUpdate(L, R, e); //44440a
        right->rangeUpdate(L, R,e);
    } //02b2a9
    ll query(int x) { //change to max if needed
        if(lo == hi) return f(x); check();
        if(x <= md) return min(f(x), left->query(x));
        return min(f(x), right->query(x));
    } //66991a
}; //1eac23
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; } //0dcc67
}; //7e3ecf

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); } //fa88a2
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p; //846095
  } //31f5a2
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p) //6dc2b6
      isect(x, erase(y));
  } //4e2c33
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  } //5a0881
}; //8ec1c7
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

635edf, 41 lines

```
struct node {
  int val, prior, sz = 1;
  node *left = nullptr, *right = nullptr;
  node(int val = 0): val(val), prior(rand()) {} //96a0e3
}; //a6e97c

int getSz(node *cur) { return cur ? cur->sz : 0; } //717b67
void recalc(node *cur) { cur->sz = getSz(cur->left) + getSz(cur
    ->right) + 1; }

pair<node*, node*> split(node *cur, int v) {
  if(!cur) return {nullptr, nullptr}; //2decad
  node *left, *right;
  if(getSz(cur->left) >= v) {
    right = cur;
    auto [L, R] = split(cur->left, v);
    left = L, right->left = R; //d01f57
    recalc(right);
  } //2d647f
  else {
    left = cur;
    auto [L, R] = split(cur->right, v - getSz(cur->left) - 1);
    left->right = L, right = R;
    recalc(left); //7f88e8
  } //e8ea2b
  return {left, right}; //77f3d8
} //0a24d2

node* merge(node *t1, node *t2) {
  if(!t1 || !t2) return t1 ? t1 : t2;
  node *res;
  if(t1->prior > t2->prior) { //9a5f42
    res = t1;
    res->right = merge(t1->right, t2);
  } //26c3b7
  else {
    res = t2;
    res->left = merge(t1, t2->left);
  } //91db68
  recalc(res);
  return res;
} //635edf
```

PQupdate.h

Description: T: value/update type. DS: Stores T. Same semantics as std::priority_queue.
Time: $\mathcal{O}(U \log N)$.

35a7d2, 36 lines

```
template<class T, class DS, class Compare = less<T>>
struct PQUpdate {
  DS inner;
  multimap<T, int, Compare> rev_upd;
  using iter = decltype(rev_upd)::iterator;
  vector<iter> st; //23764d
  PQUpdate(DS inner, Compare comp={}):
    inner(inner), rev_upd(comp) {} //3c5f72

  bool empty() { return st.empty(); } //86845b
  const T& top() { return rev_upd.rbegin()->first; } //0b3b9d
  void push(T value) {
    inner.push(value);
    st.push_back(rev_upd.insert({value, sz(st)}));
  } //44a05d
  void pop() {
    vector<iter> extra;
```

```
    iter curr = rev_upd.end();
    int min_ind = sz(st);
    do { //7ec537
      extra.push_back(--curr);
      min_ind = min(min_ind, curr->second);
    } while (2*sz(extra) < sz(st) - min_ind);
    while (sz(st) > min_ind) {
      if (rev_upd.value_comp()(*st.back(), *curr)) //c1e316
        extra.push_back(st.back());
      inner.pop(); st.pop_back();
    } //a76ade
    rev_upd.erase(extra[0]);
    for (auto it : extra | views::drop(1) | views::reverse) {
      it->second = sz(st);
      inner.push(it->first);
      st.push_back(it); //f1079a
    } //8a3607
  } //f4f582
}; //35a7d2
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.

Time: Both operations are $\mathcal{O}(\log N)$.

e62fac, 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {} //890409
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
  } //a0c54f
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  } //585cdd
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) { //d79a33
      if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    } //cb4863
    return pos;
  } //923db1
}; //e62fac
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).

Time: $\mathcal{O}\left(\log^2 N\right)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" 157f07, 22 lines

```
struct FT2 {
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {} //5e2398
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
  } //fe99f8
  void init() {
    for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
  } //1e4363
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()); }
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif); //5a5698
  } //c48166
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum; //eb0cfb
  } //266f9d
}; //157f07
```

RMQ.h

Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.

Usage: `RMQ rmq(values);`
`rmq.query(inclusive, exclusive);`

Time: $\mathcal{O}(|V|\log|V| + Q)$

510c32, 16 lines

```
template<class T>
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
      jmp.emplace_back(sz(V) - pw * 2 + 1); //7420f3
      rep(j,0,sz(jmp[k]))
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    } //398c5e
  } //dffd89
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  } //63f839
}; //510c32
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change `step` to add/remove the edge (a, c) and remove the initial `add` call (but keep `in`).

Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

a12ef4, 49 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s; //e77382
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) {
    pii q = Q[qi]; //bbab36
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc(); //8c7386
  } //d497d7
  return res;
} //7b2870

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0){
  int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1; //933a80
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f); //f4faf6
    if (!dep) I[x] = N++;
    R[x] = N;
  }; //7ac875
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) rep(end,0,2) { //6a3e39
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b]; //c95d6c
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  } //44b82c
  return res;
} //a12ef4
```

Geometry (4)

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {} //551774
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); } //1dc17e
  P operator-(P p) const { return P(x-p.x, y-p.y); } //189cbc
  P operator*(T d) const { return P(x*d, y*d); } //268af3
  P operator/(T d) const { return P(x/d, y/d); } //8cb755
  T dot(P p) const { return x*p.x + y*p.y; } //716d84
  T cross(P p) const { return x*p.y - y*p.x; } //7ecfd2
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; } //e7b843
  double dist() const { return sqrt((double)dist2()); } //039
      a77
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); } //cc70a2
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); } //c0e5d2
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); } //e458d5
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; } //0e491f
}; //47ec0a
```

4.1 Lines and Segments

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 9 lines

```
template<class P>
```

```
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s); //37dc17
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
} //3af81c
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
} //c597e8
```

lineIntersection.h

Description: If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

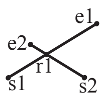
Usage:
```
auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```

"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)}; //47e53e
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d}; //c4c8fb
} //a01f81
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



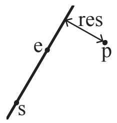
Usage:
```
vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
```

"Point.h", "OnSegment.h" 9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)}; //ab16eb
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d); //1dcb4f
  return {all(s)}; //c505dc
} //9d57f2
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

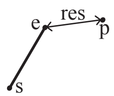


"Point.h" b4c5ca, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (b-a).cross(p-a)/(b-a).dist();
} //b4c5ca
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



Usage:
```
Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
```

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
} //5c88f4
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node { //319cda
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x); //71ed74
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  } //1460d4

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y); //28bf16
    } //2e9c2c
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2; //21b567
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    } //470fcd
  } //0265cf
}; //6fda19

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) { //7daf7f
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    } //19dc67

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p); //fa9faa
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  } //3771f7

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p); //961132
  } //60e74e
}; //bac5b0
```

4.2 Polygons

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
} //f12300
```

InsidePolygon.h

Description: Returns 0 if the point is outside the polygon, 1 if it is strictly inside the polygon, and 2 if it is on the polygon.

Usage:
```
vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
int in = inPoly(v, P{3, 3});
```

Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h" 4dd823, 11 lines

```
template<class P> int inPoly(vector<P> poly, P p) {
  bool good = false; int n = sz(poly);
  auto crosses = [](P s, P e, P p) {
    return ((e.y >= p.y) - (s.y >= p.y)) * p.cross(s, e) > 0;
  }; //be8833
  for(int i = 0; i < n; i++){
    if(onSegment(poly[i], poly[(i+1)%n], p)) return 2;
    good ^= crosses(poly[i], poly[(i+1)%n], p);
  } //1ff382
  return good;
} //4dd823
```

ConvexHull.h

Description: Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$

"Point.h" 05b731, 18 lines

```
template<class P> vector<P> convexHull(vector<P> poly){
```

```
  int n = sz(poly);
  if (n <= 1) return poly;
  vector<P> hull(n+1);
  sort(all(poly));
  int k = 0; //38d98b
  for(int i = 0; i < n; i++){
    while(k >= 2 && hull[k-2].cross(hull[k-1], poly[i]) <= 0) k--;
    hull[k++] = poly[i];
  } //0c301c
  for(int i = n-1, t = k+1; i > 0; i--){
    while(k >= t && hull[k-2].cross(hull[k-1], poly[i-1]) <= 0) k--;
    hull[k++] = poly[i-1];
  } //d5f00d
  hull.resize(k-1);
  hull.erase(unique(all(hull)), hull.end());
  return hull;
} //05b731
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) { //e5ff70
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    } //cf85e0
  return res.second;
} //c571b8
```

hullTangents.h

Description: Finds the left and right, respectively, tangent points on convex hull from a point. If the point is colinear to side(s) of the polygon, the point further away is returned. Requires ccw, $n \geq 3$, and the point be on or outside the polygon. Can be used to check if a point is inside of a convex hull. Will return -1 if it is strictly inside. If the point is on the hull, the two adjacent points will be returned
Time: $\mathcal{O}(\log n)$

"Point.h" 53d067, 16 lines

```
#define cmp(i, j) p.cross(h[i], h[j == n ? 0 : j]) * (R ? 1 : -1)
template<bool R, class P> int getTangent(vector<P>& h, P p) {
  int n = sz(h), lo = 0, hi = n - 1, md;
  if (cmp(0, 1) >= R && cmp(0, n - 1) >= !R) return 0;
  while (md = (lo + hi + 1) / 2, lo < hi) {
    auto a = cmp(md, md + 1), b = cmp(md, lo); //d06f76
    if (a >= R && cmp(md, md - 1) >= !R) return md;
    if (cmp(lo, lo + 1) < R)
      a < R&& b >= 0 ? lo = md : hi = md - 1;
    else a < R || b <= 0 ? lo = md : hi = md - 1;
  } //218376
  return -1; // point strictly inside hull
} //929dec
template<class P> pii hullTangents(vector<P>& h, P p) {
  return {getTangent<0>(h, p), getTangent<1>(h, p)}; //2a428b
} //53d067
```

inHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

6d9710, 12 lines

```
template<class P> bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false; //44688a
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  } //470615
  return sgn(l[a].cross(l[b], p)) < r;
} //6d9710
```

LineHullIntersection.h

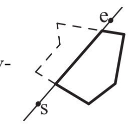
Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) { //b3e410
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  } //efd609
  return lo;
} //ba41ca

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp()); //d0d8a9
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1}; //07bb09
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n; //71097d
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    } //72e441
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  } //d56a85
  if (res[0] == res[1]) return {res[0], -1}; //d847be
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]}; //ab4398
      case 2: return {res[1], res[1]}; //e5b066
    } //54f3d0
  return res;
} //7cf45b
```

PolygonCut.h



Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" f2b7d4, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0; //41eabb
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  } //567ae4
  return res;
} //f2b7d4
```

halfplaneIntersection.h

Description: Returns the intersection of halfplanes as a polygon
Time: $\mathcal{O}(n \log n)$

e9fe62, 42 lines

```
const double eps = 1e-8;
typedef Point<double> P;
struct HalfPlane {
  P s, e, d;
  HalfPlane(P s = P(), P e = P()): s(s), e(e), d(e - s) {}
  bool contains(P p) { return d.cross(p - s) > -eps; } //0b57d7
  bool operator<(HalfPlane hp) {
    if(abs(d.x) < eps && abs(hp.d.x) < eps)
      return d.y > 0 && hp.d.y < 0;
    bool side = d.x < eps || (abs(d.x) <= eps && d.y > 0);
    bool sideHp = hp.d.x < eps || (abs(hp.d.x) <= eps && hp.d.y > 0);
    if(side != sideHp) return side; //522804
    return d.cross(hp.d) > 0;
  } //f04cee
  P inter(HalfPlane hp) {
    auto p = hp.s.cross(e, hp.e), q = hp.s.cross(hp.e, s);
    return (s * p + e * q) / d.cross(hp.d);
  } //f43e4d
}; //cb96d9

vector<P> hpIntersection(vector<HalfPlane> hps) {
  sort(all(hps));
  int n = sz(hps), l = 1, r = 0;
  vector<HalfPlane> dq(n+1); //7f023a
  rep(i, 0, n) {
    while(l < r && !hps[i].contains(dq[r].inter(dq[r-1]))) r--;
    while(l < r && !hps[i].contains(dq[l].inter(dq[l+1]))) l++;
    dq[++r] = hps[i];
    if(l < r && abs(dq[r].d.cross(dq[r-1].d)) < eps) { //6aee5e
      if(dq[r].d.dot(dq[r-1].d) < 0) return {}; //2605d9
      if(dq[--r].contains(hps[i].s)) dq[r] = hps[i];
    } //575960
  } //d8f849
  while(l < r - 1 && !dq[l].contains(dq[r].inter(dq[r-1]))) r--;
  while(l < r - 1 && !dq[r].contains(dq[l].inter(dq[l+1]))) l++;
  if(l > r - 2) return {}; //5ca32f
  vector<P> poly;
  rep(i, l, r)
    poly.push_back(dq[i].inter(dq[i+1]));
```

```
  poly.push_back(dq[r].inter(dq[l]));
  return poly; //0b254d
} //e9fe62
```

centerOfMass.h

Description: Returns the center of mass for a polygon.
Memory: $\mathcal{O}(1)$
Time: $\mathcal{O}(n)$

ccce20, 8 lines

```
template<class P> P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  } //938654
  return res / A / 3;
} //ccce20
```

minkowskiSum.h

Description: Returns the minkowski sum of a set of convex polygons
Time: $\mathcal{O}(n \log n)$

6a76f5, 20 lines

```
#define side(p) (p.x > 0 || (p.x == 0 && p.y > 0))
template<class P>
vector<P> convolve(vector<vector<P>> &polys){
  P init; vector<P> dir;
  for(auto poly: polys) {
    int n = sz(poly); //aee8e7
    if(n) init = init + poly[0];
    if(n < 2) continue;
    rep(i, 0, n) dir.push_back(poly[(i+1)%n] - poly[i]);
  } //98f301
  if(size(dir) == 0) return { init }; //b85ac7
  stable_sort(all(dir), [&](P a, P b)->bool {
    if(side(a) != side(b)) return side(a);
    return a.cross(b) > 0;
  });
  vector<P> sum; P cur = init; //03ea38
  rep(i, 0, sz(dir))
    sum.push_back(cur), cur = cur + dir[i];
  return sum;
} //6a76f5
```

PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}\left(N^2\right)$, where N is the total number of points

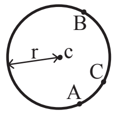
"Point.h", "sideOf.h" 3931c6, 33 lines

```
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
    P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
    vector<pair<double, int>> segs = {{0, 0}, {1, 0}}; //e9da64
    rep(j,0,sz(poly)) if (i != j) {
      rep(u,0,sz(poly[j])) {
        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
        if (sc != sd) { //ac826b
          double sa = C.cross(D, A), sb = C.cross(D, B);
          if (min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
        } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
          segs.emplace_back(rat(C - A, B - A), 1); //a4636e
          segs.emplace_back(rat(D - A, B - A), -1);
        } //67520d
      } //c4b419
    } //a1900f
    sort(all(segs));
    for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
    double sum = 0;
    int cnt = segs[0].second;
    rep(j,1,sz(segs)) { //317ef1
      if (!cnt) sum += segs[j].first - segs[j - 1].first;
      cnt += segs[j].second;
    } //d3398f
    ret += A.cross(B) * sum;
  } //6f2b4e
  return ret / 2;
} //3931c6
```

4.3 Circles

circumcircle.h

Description:



The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
} //032e3d
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
} //1caa3a
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 9 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
  if (h2 < 0) return {}; //64a27f
  if (h2 == 0) return {p}; //3d9ab3
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h}; //3b1a3f
} //e0cfba
```

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; } //7e53c0
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per}; //3dd318
  return true;
} //84d6d3
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

"../../content/geometry/Point.h" a1ee63, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p; //c0445a
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2; //1b08d3
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  }; //6470ed
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
} //a1ee63
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {}; //c18727
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  } //41b560
  if (h2 == 0) out.pop_back();
  return out;
} //b0153d
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0; //5e7038
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]); //931d7a
        r = (o - ps[i]).dist();
      } //7cd516
    } //03da47
  } //bfac59
  return {o, r}; //5ebee7
```

```
} //09dd0a
```

4.4 3D Geometry

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

01f8f7, 33 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const { //9e2218
    return tie(x, y, z) < tie(p.x, p.y, p.z); } //af5a46
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); } //fa5b42
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); } //1ee29d
  P operator/(T d) const { return P(x/d, y/d, z/d); } //660667
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; } //d7cc17
  P cross(R a, R b) const { return (a-*this).cross(b-*this); }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  } //7f1984
  T dist2() const { return x*x + y*y + z*z; } //061c10
  double dist() const { return sqrt((double)dist2()); } //
      f7bfe3
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); } //f3fa7c
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); } //e107dc
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  } //83bd4d
}; //01f8f7
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" faa885, 36 lines

```
typedef Point3D<double> P3;
const double eps = 1e-6;

struct F { int a, b, c; }; //4fccf3

vector<F> hull3d(vector<P3> &p) {
  int n = sz(p);
  if(n < 3) return {}; //1262ac
  vector<F> faces;

  vvi active(n, vi(n, false));

  auto add_face = [&](int a, int b, int c) -> void { //cbed44
    faces.push_back({a, b, c});
    active[a][b] = active[b][c] = active[c][a] = true;
  }; //55d48b

  add_face(0, 1, 2);
  add_face(0, 2, 1);
  rep(i, 3, n) { //7dcd92
    vector<array<int, 3>> new_faces;
    for(auto [a, b, c]: faces)
      if((p[i] - p[a]).dot(p[a].cross(p[b], p[c])) > eps)
        active[a][b] = active[b][c] = active[c][a] = false;
      else new_faces.push_back({a, b, c}); //77474c
    faces.clear();
    for(auto f: new_faces)
      rep(j, 0, 3) if(!active[f[(j+1)%3]][f[j]])
        add_face(f[(j+1)%3], f[j], i);
    for(auto [a, b, c]: new_faces) //9deb86
      faces.push_back({a, b, c});
  } //978258

  return faces;
} //faa885
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz); //819384
  return radius*2*asin(d/2);
} //611f07
```

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
} //3058c3
```

4.5 Miscellaneous

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}}; //db620d
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0}; //484ee7
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p); //afb942
  } //a4382b
  return ret.second;
} //ac41a6
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad { //4dcdd0
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; } //001543
  Q& r() { return rot->rot; } //9a9030
  Q prev() { return rot->o->rot; } //a6d183
  Q next() { return r()->prev(); } //c2bc3a
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2; //4e353f
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
} //cae086
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest; //e2ee6e
  return r;
} //25ccf6
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
} //0ef350
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q; //f4703d
} //eef885

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() }; //d46520
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  } //60c127

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2; //4dbbd2
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A); //a3dcbe
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \ //5b7586
```

```
    Q t = e->dir; \
    splice(e, e->prev()); \
    splice(e->r(), e->r()->prev()); \
    e->o = H; H = e; e = t; \
  } //d41222
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r()); //cf44eb
    else
      base = connect(base->r(), LC->r());
  } //17ceb8
  return { ra, rb }; //505512
} //2d987e

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {}; //afbc1c
  Q e = rec(pts).first;
  vector<Q> q = {e}; //35ce3b
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear(); //8e3597
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
} //eefdf5
```

PlanarFaceExtraction.h

Description: Given a planar graph and where the points are, extract the set of faces that the graph makes

Time: $\mathcal{O}(ElogE)$ 63f230, 39 lines

```
template<class P>
vector<vector<P>> extract_faces(vvi adj, vector<P> pts) {
  int n = sz(pts);
  #define cmp(i) [&](int pi, int qi) -> bool { \
    P p = pts[pi] - pts[i], q = pts[qi] - pts[i]; \
    bool sideP = p.y < 0 || (p.y == 0 && p.x < 0); \ //2e5576
    bool sideQ = q.y < 0 || (q.y == 0 && q.x < 0); \
    if(sideP != sideQ) return sideP; \
    return p.cross(q) > 0; } //59b975
  rep(i, 0, n)
    sort(all(adj[i]), cmp(i));
  vii ed;
  rep(i, 0, n) for(int j: adj[i])
    ed.emplace_back(i, j); //623310
  sort(all(ed));
  auto get_idx = [&](int i, int j) -> int {
    return lower_bound(all(ed), pii(i, j))-begin(ed);
  }; //7667e7
  vector<vector<P>> faces;
  vi used(sz(ed));
  rep(i, 0, n) for(int j: adj[i]) {
    if(used[get_idx(i, j)])
      continue; //7db6a7
    used[get_idx(i, j)] = true;
    vector<P> face = {pts[i]}; //b39032
    int prv = i, cur = j;
    while(cur != i) {
      face.push_back(pts[cur]);
      auto it = lower_bound(all(adj[cur]), prv, cmp(cur));
      if(it == begin(adj[cur])) //f6e8f1
        it = end(adj[cur]);
      prv = cur, cur = *prev(it);
      used[get_idx(prv, cur)] = true;
    } //9fb9bf
    faces.push_back(face);
  } //29aacd
  #undef cmp
  return faces;
} //63f230
```

Graphs (5)

5.1 Network flow

MinCostMaxFlow.h

Description: Min-cost max-flow. Negative cost cycles not supported. To obtain the actual flow, look at positive values only.

Time: Approximately $\mathcal{O}(E^2)$, actually $\mathcal{O}(FS)$ where S is the time complexity of the SSSP alg used in find path (in this case SPFA) 664049, 55 lines

```
struct MCMF {
  const ll INF = LLONG_MAX >> 2;
  struct edge {
    int v;
    ll cap, flow, cost;
  }; //f709d9
  int n;
  vector<edge> edges;
  vvi adj; vii par; vi in_q;
  vector<ll> dist, pi;
  MCMF(int n): n(n), adj(n), par(n), in_q(n), dist(n), pi(n) {}
  void addEdge(int u, int v, ll cap, ll cost) { //42c114
    int idx = sz(edges);
    edges.push_back({v, cap, 0, cost});
    edges.push_back({u, cap, cap, -cost});
    adj[u].push_back(idx);
    adj[v].push_back(idx ^ 1); //65b236
  } //e280ec
  bool findPath(int s, int t) {
    fill(all(dist), INF);
    fill(all(in_q), 0);
    queue<int> q; q.push(s);
    dist[s] = 0, in_q[s] = 1; //46fdef
    while(!q.empty()) {
      int cur = q.front(); q.pop();
      in_q[cur] = 0;
      for(int idx: adj[cur]) {
        auto [nxt, cap, fl, wt] = edges[idx]; //77b4b3
        ll nxtD = dist[cur] + wt;
        if(fl >= cap || nxtD >= dist[nxt]) continue;
        dist[nxt] = nxtD;
        par[nxt] = {cur, idx}; //ee66ab
        if(in_q[nxt]) continue;
        q.push(nxt); in_q[nxt] = 1;
      } //882b56
    } //0dbe4d

    return dist[t] < INF;
  } //3db314
  pair<ll, ll> calc(int s, int t) {
    ll flow = 0, cost = 0;
    while(findPath(s, t)) {
      ll f = INF;
      for(int i, u, v = t; tie(u, i) = par[v], v != s; v = u)
        f = min(f, edges[i].cap - edges[i].flow); //78f755
      flow += f;
      for(int i, u, v = t; tie(u, i) = par[v], v != s; v = u)
        edges[i].flow += f, edges[i^1].flow -= f;
    } //e557d6
    rep(i, 0, sz(edges)>>1)
      cost += edges[i<<1].cost * edges[i<<1].flow;
    return {flow, cost}; //048d32
  } //f57bd7
}; //664049
```

MinCostMaxFlowDijkstra.h

Description: If SPFA TLEs, swap the find_path function in MCMF with the one below and in_q with seen. If negative edge weights can occur, initialize pi with the shortest path from the source to each node using Bellman-Ford. Negative weight cycles not supported. efdefd, 24 lines

```
bool findPath(int s, int t) {
  fill(all(dist), inf);
  fill(all(seen), 0);
  dist[s] = 0;
  __gnu_pbds::priority_queue<pair<ll, int>> pq;
  vector<decltype(pq)::point_iterator> its(n); //e67bf6
  pq.push({0, s});
  while(!pq.empty()) {
    auto [d, cur] = pq.top(); pq.pop(); d *= -1;
    seen[cur] = 1;
    if(dist[cur] < d) continue; //c5f170
    for(int idx: adj[cur]) {
      auto [nxt, cap, f, wt] = edges[idx];
      ll nxtD = d + wt + pi[cur] - pi[nxt];
      if(f >= cap || nxtD >= dist[nxt] || seen[nxt]) continue;
      dist[nxt] = nxtD; //b0252f
      par[nxt] = {cur, idx}; //8270eb
      if(its[nxt] == pq.end()) its[nxt] = pq.push({-nxtD, nxt})
          ;
      else pq.modify(its[nxt], {-nxtD, nxt});
    } //0154c2
  } //86f7eb
  rep(i, 0, n) pi[i] = min(pi[i] + dist[i], inf);
  return seen[t];
} //efdefd
```

Dinic.h

Description: Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching. d7f0f1, 42 lines

```
struct Dinic {
  struct Edge {
    int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, 0LL); } // if you need flows
  }; //9d5927
  vi lvl, ptr, q;
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {} //fdd5b9
  void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].push_back({b, sz(adj[b]), c, c});
    adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
  } //a45d7e
  ll dfs(int v, int t, ll f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < sz(adj[v]); i++) {
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1) //591b8b
        if (ll p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;
        } //d3bb27
    } //f4fbea
    return 0;
  } //72048c
  ll calc(int s, int t) {
    ll flow = 0; q[0] = s;
    rep(L,0,31) do { // 'int L=30' maybe faster for random data
```

```
    lvl = ptr = vi(sz(q));
    int qi = 0, qe = lvl[s] = 1; //5d9371
    while (qi < qe && !lvl[t]) {
      int v = q[qi++];
      for (Edge e : adj[v])
        if (!lvl[e.to] && e.c >> (30 - L))
          q[qe++] = e.to, lvl[e.to] = lvl[v] + 1; //0d5640
    } //16dd6b
    while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
  } while (lvl[t]);
  return flow;
} //2b90e4
bool leftOfMinCut(int a) { return lvl[a] != 0; } //761cc4
}; //d7f0f1
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}}; //81f955
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i}; //f640ab
  rep(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio. queue
      w[t] = INT_MIN; //c98135
      s = t, t = max_element(all(w)) - w.begin();
      rep(i,0,n) w[i] += mat[t][i];
    } //8c07c9
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN; //e25d4b
  } //076888
  return best;
} //8b0e19
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
Time: $\mathcal{O}(V)$ Flow Computations

"Dinic.h" e2b333, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
  vector<Edge> tree;
  vi par(N);
  rep(i,1,N) {
    Dinic D(N); //53565e
    for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
    tree.push_back({i, par[i], D.calc(i, par[i])});
    rep(j,i+1,N)
      if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
  } //c14544
  return tree;
} //e2b333
```

MatroidIntersection.h

Description: Given two matroids, finds the largest common independent set. For the color and graph matroids, this would be the largest forest where no two edges are the same color. A matroid has 3 functions
- check(int x): returns if current matroid can add x without becoming dependent
- add(int x): adds an element to the matroid (guaranteed to never make it dependent)
- clear(): sets the matroid to the empty matroid
The matroid is given an int representing the element, and is expected to convert it (e.g: the color or the endpoints) Pass the matroid with more expensive add/clear operations to M1.
Time: $R^2 N(M2.add + M1.check + M2.check) + R^3 M1.add + R^2 M1.clear + RNM2.clear$

"../data-structures/UnionFind.h" 9812a7, 60 lines

```
struct ColorMat {
  vi cnt, clr;
  ColorMat(int n, vector<int> clr) : cnt(n), clr(clr) {}
  bool check(int x) { return !cnt[clr[x]]; } //1992d4
  void add(int x) { cnt[clr[x]]++; } //b9ca1b
  void clear() { fill(all(cnt), 0); } //1217e4
}; //a797c9
struct GraphMat {
  UF uf;
  vector<array<int, 2>> e;
  GraphMat(int n, vector<array<int, 2>> e) : uf(n), e(e) {}
  bool check(int x) { return !uf.sameSet(e[x][0], e[x][1]); }
  void add(int x) { uf.join(e[x][0], e[x][1]); } //4634b6
  void clear() { uf = UF(sz(uf.e)); } //4fb94c
}; //7f77ed
template <class M1, class M2> struct MatroidIsect {
  int n;
  vector<char> iset;
  M1 m1; M2 m2;
  MatroidIsect(M1 m1, M2 m2, int n) : n(n), iset(n + 1), m1(m1)
      , m2(m2) {}
  vi solve() { //8b197a
    rep(i,0,n) if (m1.check(i) && m2.check(i))
      iset[i] = true, m1.add(i), m2.add(i);
    while (augment());
    vi ans;
    rep(i,0,n) if (iset[i]) ans.push_back(i); //7337bf
    return ans;
  } //530c7f
  bool augment() {
    vector<int> frm(n, -1);
    queue<int> q({n}); // starts at dummy node
    auto fwdE = [&](int a) {
      vi ans; //1df231
      m1.clear();
      rep(v, 0, n) if (iset[v] && v != a) m1.add(v);
      rep(b, 0, n) if (!iset[b] && frm[b] == -1 && m1.check(b))
        ans.push_back(b), frm[b] = a;
      return ans; //f4e117
    }; //f4805c
    auto backE = [&](int b) {
      m2.clear();
      rep(cas, 0, 2) rep(v, 0, n)
        if ((v == b || iset[v]) && (frm[v] == -1) == cas) {
          if (!m2.check(v)) //afb3ed
            return cas ? q.push(v), frm[v] = b, v : -1;
          m2.add(v);
        } //3b2d63
      return n;
    }; //0ceea9
    while (!q.empty()) {
      int a = q.front(), c; q.pop();
      for (int b : fwdE(a))
        while((c = backE(b)) >= 0) if (c == n) {
          while (b != n) iset[b] ^= 1, b = frm[b]; //c6beb1
          return true;
        } //7398d6
    } //ec60bb
    return false;
  } //c1031d
}; //9812a7
```

5.2 Matching

hopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); hopcroftKarp(g, btoa);`
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

f612e4, 42 lines

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
  if (A[a] != L) return 0;
  A[a] = -1;
  for (int b : g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
      return btoa[b] = a, 1; //47a337
  } //84f762
  return 0;
} //9e7938

int hopcroftKarp(vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) { //a02d20
    fill(all(A), 0);
    fill(all(B), 0);
    cur.clear();
    for (int a : btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a); //0fe82b
    for (int lay = 1;; lay++) {
      bool islast = 0;
      next.clear();
      for (int a : cur) for (int b : g[a]) {
        if (btoa[b] == -1) { //96ecca
          B[b] = lay;
          islast = 1;
        } //4c74fe
        else if (btoa[b] != a && !B[b]) {
          B[b] = lay;
          next.push_back(btoa[b]);
        } //81e09f
      } //ebc136
      if (islast) break;
      if (next.empty()) return res;
      for (int a : next) A[a] = lay;
      cur.swap(next);
    } //e487ce
    rep(a,0,sz(g))
      res += dfs(a, 0, g, btoa, A, B);
  } //f385af
} //f612e4
```

DFSMatching.h

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); dfsMatching(g, btoa);`
Time: $\mathcal{O}(VE)$

522b98, 22 lines

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
```

```
  if (btoa[j] == -1) return 1;
  vis[j] = 1; int di = btoa[j];
  for (int e : g[di])
    if (!vis[e] && find(e, g, btoa, vis)) {
      btoa[e] = di; //b1c950
      return 1;
    } //cc0de1
  return 0;
} //d13a81
int dfsMatching(vector<vi>& g, vi& btoa) {
  vi vis;
  rep(i,0,sz(g)) {
    vis.assign(sz(btoa), 0);
    for (int j : g[i]) //0eda2c
      if (find(j, g, btoa, vis)) {
        btoa[j] = i;
        break;
      } //5609e1
  } //61061f
  return sz(btoa) - (int)count(all(btoa), -1);
} //522b98
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" da4196, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = dfsMatching(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover; //d5d915
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) { //113
        bda
      seen[e] = true;
      q.push_back(match[e]);
    } //b97b04
  } //b9473f
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
} //da4196
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \le M$.
Time: $\mathcal{O}(N^2M)$

1e0fe9, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}}; //497519
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1); //3b3e45
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      rep(j,1,m) if (!done[j]) { //0023e6
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      } //31ae76
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      } //6cc461
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1; //632eb8
    } //26ae9e
  } //9e72cf
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
} //1e0fe9
```

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod.
Time: $\mathcal{O}(N^3)$

"../numerical/MatrixInverse-mod.h" cb1912, 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  for (pii pa : ed) {
    int a = pa.first, b = pa.second, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  } //614800

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);

  if (M != N) do { //9bc254
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
      mat[i].resize(M);
      rep(j,N,M) {
        int r = rand() % mod; //d8fdfd
        mat[i][j] = r, mat[j][i] = (mod - r) % mod;
      } //36a855
    } //be41a1
  } while (matInv(A = mat) != M);

  vi has(M, 1); vector<pii> ret;
  rep(it,0,M/2) {
    rep(i,0,M) if (has[i]) //348eac
      rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;
    } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0; //fcefbe
    rep(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);
      rep(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
      } //9debcf
      swap(fi,fj);
    } //6c623e
  } //f16c12
  return ret;
} //cb1912
```

5.3 DFS algorithms

SCC.h

Description: Finds strogly connected components in a directed graph.
Usage: `auto [num_sccs, scc_id] = sccs(adj);`
`scc_id[v] = id, 0<=id<num_sccs`
`for each edge u -> v: scc_id[u] >= scc_id[v]`
Time: $\mathcal{O}(E+V)$

2552fb, 16 lines

```
auto sccs(const vector<vi>& adj) {
  int n = sz(adj), num_sccs = 0, q = 0, s = 0;
  vi scc_id(n, -1), tin(n), st(n);
  auto dfs = [&](auto&& self, int v) -> int {
    int low = tin[v] = ++q; st[s++] = v;
    for (int u : adj[v]) if (scc_id[u] < 0) //530f05
        low = min(low, tin[u] ?: self(self, u));
    if (tin[v] == low) {
      while (scc_id[v] < 0) scc_id[st[--s]] = num_sccs;
      num_sccs++;
    } //9cb784
    return low;
  }; //250c73
  rep(i,0,n) if (!tin[i]) dfs(dfs, i);
  return pair{num_sccs, scc_id}; //7aebce
} //2552fb
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
Usage: `int eid = 0; ed.resize(N);`
`for each edge (a,b) {`
`ed[a].emplace_back(b, eid);`
`ed[b].emplace_back(a, eid++); }`
`bicomps([&](const vi& edgelist) {...});`
Time: $\mathcal{O}(E+V)$

2965e5, 33 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
  int me = num[at] = ++Time, e, y, top = me; //d1b332
  for (auto pa : ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me) //145ca4
        st.push_back(e);
    } else {
      int si = sz(st);
      int up = dfs(y, e, f);
      top = min(top, up); //4c0c04
      if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      } //4c59fd
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ } //47e7b7
    } //7a2ccf
  } //55ddf3
  return top;
} //0b5c9f

template<class F>
void bicomps(F f) {
  num.assign(sz(ed), 0);
```

```
  rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f); //14c211
} //2965e5
```

blockvertextree.h

Description: articulation points and block-vertex tree self edges not allowed adj[u] += v, i; adj[v] += u, i; iscut[v] = 1 iff cut node bccid[edge id] = id, 0<=id<numbccs returns numbccs, bccid, iscut Assumes the root node points to itself.

ab8ef6, 61 lines

```
auto cuts(const auto& adj, int m) {
  int n = ssize(adj), num_bccs = 0, q = 0, s = 0;
  vector<int> bcc_id(m, -1), is_cut(n), tin(n), st(m);
  auto dfs = [&](auto&& self, int v, int p) -> int {
    int low = tin[v] = ++q;
    for (auto [u, e] : adj[v]) { //d15302
    assert(v != u);
    if (e == p) continue;
    if (tin[u] < tin[v]) st[s++] = e;
    int lu = -1;
    low = min(low, tin[u] ?: (lu = self(self, u, e))); //d79c0f
    if (lu >= tin[v]) {
      is_cut[v] = p >= 0 || tin[v] + 1 < tin[u];
      while (bcc_id[e] < 0) bcc_id[st[--s]] = num_bccs;
      num_bccs++;
    } //c32a15
    } //9a1476
    return low;
  }; //d8df66
  for (int i = 0; i < n; i++)
    if (!tin[i]) dfs(dfs, i, -1);
  return tuple{num_bccs, bcc_id, is_cut}; //782ada
  } //64c880
  //!
  //! vector<vector<pii>> adj(n);
  //! auto [num_bccs, bcc_id, is_cut] = cuts(adj, m);
  //! auto bvt = block_vertex_tree(adj,
  //!   num_bccs, bcc_id);
  //!
  //! vector<basic_string<array<int, 2>>> adj(n);
  //! auto [num_bccs, bcc_id, is_cut] = cuts(adj, m);
  //! auto bvt = block_vertex_tree(adj, num_bccs, bcc_id);
  //!
  //! //to loop over each unique bcc containing a node u:
  //! for (int bccid : bvt[v]) {
  //!     bccid -= n;
  //! }
  //! //to loop over each unique node inside a bcc:
  //! for (int v : bvt[bccid + n]) {}
  //! [0, n) are original nodes
  //! [n, n + num_bccs) are BCC nodes
  //! @time O(n + m)
  //! @time O(n)
  auto block_vertex_tree(const auto& adj, int num_bccs,
  const vector<int>& bcc_id) { //2892ea
  int n = ssize(adj);
  vector<basic_string<int>> bvt(n + num_bccs);
  vector<bool> vis(num_bccs);
  for (int i = 0; i < n; i++) {
    for (auto [_, e_id] : adj[i]) { //4487b1
    int bccid = bcc_id[e_id];
    if (!vis[bccid]) {
      vis[bccid] = 1;
      bvt[i] += bccid + n;
      bvt[bccid + n] += i; //472b2c
    } //4f54ba
    } //805517
    for (int bccid : bvt[i]) vis[bccid - n] = 0;
  } //686c71
  return bvt;
  } //ab8ef6
```

bridgetree.h

Description: bridges adj[u] += v, i; adj[v] += u, i; iscut[v] = 1 iff cut node brid[v] = id, 0<=id<numbccs returns numbccs, bccid, iscut Assumes the root node points to itself.

709259, 40 lines

```
auto bridges(const auto& adj, int m) {
  int n = ssize(adj), num_ccs = 0, q = 0, s = 0;
  vector<int> br_id(n, -1), is_br(m), tin(n), st(n);
  auto dfs = [&](auto&& self, int v, int p) -> int {
    int low = tin[v] = ++q;
    st[s++] = v; //5f1982
    for (auto [u, e] : adj[v])
    if (e != p && br_id[u] < 0)
      low = min(low, tin[u] ?: self(self, u, e));
    if (tin[v] == low) {
    if (p != -1) is_br[p] = 1; //362f9c
    while (br_id[v] < 0) br_id[st[--s]] = num_ccs;
    num_ccs++;
    } //9d7828
    return low;
  }; //9deefe
  for (int i = 0; i < n; i++)
    if (!tin[i]) dfs(dfs, i, -1);
  return tuple{num_ccs, br_id, is_br}; //b180e6
  } //8ed2e5
  //! @code
  //!   {
  //!     vector<vector<pii>> adj(n);
  //!     auto [num_ccs, br_id, is_br] = bridges(adj, m);
  //!     auto bt = bridge_tree(adj, num_ccs, br_id, is_br);
  //!   }
  //!   vector<basic_string<array<int, 2>>> adj(n);
  //!   auto [num_ccs, br_id, is_br] = bridges(adj, m);
  //!   auto bt = bridge_tree(adj, num_ccs, br_id, is_br);
  //! @endcode
  //! @time O(n + m)
  //! @space O(n)
  auto bridge_tree(const auto& adj, int num_ccs, //28075b
  const vector<int>& br_id, const vector<int>& is_br) {
  vector<basic_string<int>> tree(num_ccs);
  for (int i = 0; i < ssize(adj); i++)
    for (auto [u, e_id] : adj[i])
    if (is_br[e_id]) tree[br_id[i]] += br_id[u]; //6da427
  return tree;
  } //709259
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a\|\|b)\&\&(!a\|\|c)\&\&(d\|\|!b)\&\&...$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage:
```
TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.setValue(2); // Var 2 is true
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
```

Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

5f9706, 56 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {} //c1fbac

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++; //0f7e62
  } //8e7f67

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1); //7f876f
    gr[j].push_back(f^1);
  } //f602cc
  void setValue(int x) { either(x, x); } //cbc333

  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) { //66f796
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next; //f470ff
    } //7cdc2a
    either(cur, ~li[1]);
  } //06911d

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e]) //c93f40
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1) //a8f0bd
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  } //088d97

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i); //27da39
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  } //4fdfc4
}; //5f9706
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

Time: $\mathcal{O}(V+E)$

780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src}; //ea6179
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
    if (it == end){ ret.push_back(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++]; //ad1959
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
```

```
  }} //be64a3
  for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
  return {ret.rbegin(), ret.rend()}; //8b22f8
} //780b64
```

DominatorTree.h

Description: Builds a dominator tree on a directed graph. Output tree is a parent array with *src* as the root.
Time: $\mathcal{O}(V+E)$ 1d35d2, 46 lines

```
vi getDomTree(vvi &adj, int src) {
  int n = sz(adj), t = 0;
  vvi revAdj(n), child(n), sdomChild(n);
  vi label(n, -1), revLabel(n), sdom(n), idom(n), par(n), best(
      n);

  auto dfs = [&](int cur, auto &dfs) -> void { //f72200
    label[cur] = t, revLabel[t] = cur;
    sdom[t] = par[t] = best[t] = t; t++;
    for(int nxt: adj[cur]) {
      if(label[nxt] == -1) {
        dfs(nxt, dfs); //79b43c
        child[label[cur]].push_back(label[nxt]);
      } //01b03d
      revAdj[label[nxt]].push_back(label[cur]);
    } //3ffae1
  }; //65f8db
  dfs(src, dfs);

  auto get = [&](int x, auto &get) -> int {
    if(par[x] != x) {
      int t = get(par[x], get); //7f7ab8
      par[x] = par[par[x]];
      if(sdom[t] < sdom[best[x]]) best[x] = t;
    } //4696d0
    return best[x];
  }; //9d168a

  for(int i = t-1; i >= 0; i--) {
    for(int j: revAdj[i]) sdom[i] = min(sdom[i], sdom[get(j,
        get)]);
    if(i > 0) sdomChild[sdom[i]].push_back(i);
    for(int j: sdomChild[i]) { //6369b1
      int k = get(j, get);
      if(sdom[j] == sdom[k]) idom[j] = sdom[j];
      else idom[j] = k;
    } //28ff3c
    for(int j: child[i]) par[j] = i;
  } //e97294

  vi dom(n);
  rep(i, 1, t) {
    if(idom[i] != sdom[i]) idom[i] = idom[idom[i]];
    dom[revLabel[i]] = revLabel[idom[i]]; //38ad1b
  } //c63146

  return dom;
} //1d35d2
```

5.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$ e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) { //fc7443
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v; //e45383
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i]; //f3efaf
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    } //657a28
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  } //e9f8dc
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
  return ret;
} //e210e2
```

5.5 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; } //d462aa
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R); //cf4187
    R[i] = P[i] = 0; X[i] = 1;
  } //2b8ca5
} //b0d5b1
```

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs. f7c0bc, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; }; //93b51d
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old; //b548bf
  void init(vv& r) {
    for (auto& v : r) v.d = 0;
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d; //16d40c
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  } //d5dc84
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return; //09eb24
      q.push_back(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T); //c706bf
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; }; //3e1b8e
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        } //5ebe7a
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q; //86a1f3
      q.pop_back(), R.pop_back();
    } //c01dd9
  } //901020
  vi maxClique() { init(V), expand(V); return qmax; } //12c3d2
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  } //21f145
}; //f7c0bc
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

5.6 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N\log N)$, queries $\mathcal{O}(\log N)$ bfce85, 25 lines

```
vector<vi> treeJump(vi& P){
  int on = 1, d = 1;
  while(on < sz(P)) on *= 2, d++;
  vector<vi> jmp(d, P);
  rep(i,1,d) rep(j,0,sz(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]]; //35de77
  return jmp;
} //6d3434

int jmp(vector<vi>& tbl, int nod, int steps){
  rep(i,0,sz(tbl))
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod; //5f4dea
} //7ce14c

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
```

```
  if (a == b) return a; //74edff
  for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
  } //863967
  return tbl[0][a];
} //bfce85
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.
Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" 0f62fb, 21 lines

```
struct LCA {
  int T = 0;
  vi time, path, ret;
  RMQ<int> rmq;

  LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1), ret)) {}
  void dfs(vector<vi>& C, int v, int par) { //2deaa6
    time[v] = T++;
    for (int y : C[v]) if (y != par) {
      path.push_back(v), ret.push_back(time[v]);
      dfs(C, y, v);
    } //6720ac
  } //5ad321

  int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)]; //c2446b
  } //8588d0
  //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
}; //0f62fb
```

MaxPath.h

Description: Given edges (Weight,U,V) answers max on path queries of the induced MST.
Time: $\mathcal{O}(Nlog(N))$

2aabe7, 21 lines

```
struct maxPath{
    vector<int> p,s,wt; ll tot = 0;
    maxPath(vector<tuple<int,int,int>> ed, int n):
        p(n), s(n,1),wt(n,INT_MAX){
        sort(all(ed)); iota(all(p),0);
        for(auto[w,u,v]: ed){ //14f653
            while(u!=p[u]) u=p[u];
            while(v!=p[v]) v=p[v];
            if(u==v) continue; tot+=w;
            if(s[u]>s[v]) swap(u,v);
            p[u] = v; s[v]+=s[u]; wt[u] = w; //0b97a9
        } //36af2c
    } //bf42b8
    int query(int u, int v){ //assert(u!=v);
        while(p[u]!=v && p[v]!=u){
            if(wt[u]<wt[v]) u=p[u];
            else v=p[v];
        } //8df1f6
        return p[u]==v ? wt[u]:wt[v];
    } //365c8b
}; //2aabe7
```

CentroidDecomp.h

Description: Calls callback function on undirected forest for each centroid
Usage: `centroid(adj, [&](const vector<vector<int>>& adj, int cent) { ... });`
Time: $\mathcal{O}(n \log n)$

2c9a06, 33 lines

```
template <class F> struct centroid {
  vector<vi> adj;
  F f;
  vi sub_sz, par;
  centroid(const vector<vi>& a_adj, F a_f)
    : adj(a_adj), f(a_f), sub_sz(sz(adj), -1), par(sz(adj), -1)
        {
    rep(i, 0, sz(adj)) //a71923
      if (sub_sz[i] == -1) dfs(i);
  } //03bc04
  void calc_sz(int u, int p) {
    sub_sz[u] = 1;
    for (int v : adj[u])
      if (v != p)
        calc_sz(v, u), sub_sz[u] += sub_sz[v]; //3a72fa
  } //9a4332
  int dfs(int u) {
    calc_sz(u, -1);
    for (int p = -1, sz_root = sub_sz[u];;) {
      auto big_ch = find_if(begin(adj[u]), end(adj[u]), [&](int
          v) {
        return v != p && 2 * sub_sz[v] > sz_root; //ad4da8
      });
      if (big_ch == end(adj[u])) break;
      p = u, u = *big_ch;
    } //fcaffc
    f(adj, u);
    for (int v : adj[u]) {
      iter_swap(find(begin(adj[v]), end(adj[v]), u), rbegin(adj
          [v]));
      adj[v].pop_back();
      par[dfs(v)] = u; //994f54
    } //a5711e
    return u;
  } //155406
}; //2c9a06
```

EdgeCD.h

Description: Edge-Centroid Decomp, count single edge paths separately, don't consider root to node paths in F edge_cd(adj, [&](const auto& adj, int cent, int m) subtrees of [0, m) of adj[cent]: 1st edge-set subtrees of [m, sz(adj[cent])): 2nd edge-set);
Time: $\mathcal{O}(n \log n)$

fe3ded, 35 lines

```
template <class F> struct edge_cd {
  vector<vector<int>> adj;
  F f;
  vector<int> sub_sz;
  edge_cd(const vector<vector<int>>& a_adj, F a_f) : adj(a_adj)
      , f(a_f), sub_sz((int)size(adj)) {
    dfs(0, (int)size(adj)); //ff7f72
  } //0a92d4
  int find_cent(int u, int p, int siz) {
    sub_sz[u] = 1;
    for (int v : adj[u])
      if (v != p) {
        int cent = find_cent(v, u, siz); //9a0b69
        if (cent != -1) return cent;
        sub_sz[u] += sub_sz[v];
      } //8c23e3
    if (p == -1) return u;
    return 2 * sub_sz[u] >= siz ? sub_sz[p] = siz - sub_sz[u],
        u : -1;
  } //4b9693
  void dfs(int u, int siz) {
    if (siz <= 2) return;
    u = find_cent(u, -1, siz);
    int sum = 0;
    auto it = partition(begin(adj[u]), end(adj[u]), [&](int v)
        {
      bool ret = 2 * sum + sub_sz[v] < siz - 1 && 3 * (sum +
          sub_sz[v]) <= 2 * (siz - 1);
      if (ret) sum += sub_sz[v]; //983590
      return ret;
    });
    f(adj, u, it - begin(adj[u]));
    vector<int> oth(it, end(adj[u]));
    adj[u].erase(it, end(adj[u])); //3deb39
    dfs(u, sum + 1);
    swap(adj[u], oth);
    dfs(u, siz - sum);
  } //ed9cba
}; //fe3ded
```

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 9775a0, 21 lines

```
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.time));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; }; //386
      eec
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.lca(a, b)); //bbcbf2
  } //432667
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])}; //89fc5f
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
  } //cefab5
  return ret;
} //9775a0
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.
Time: $\mathcal{O}\left((\log N)^2\right)$

"../data-structures/LazySegmentTree.h" 6f34db, 46 lines

```
template <bool VALS_EDGES> struct HLD {
  int N, tim = 0;
  vector<vi> adj;
  vi par, siz, depth, rt, pos;
  Node *tree;
  HLD(vector<vi> adj_) //d266b7
    : N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1), depth(N),
      rt(N),pos(N),tree(new Node(0, N)){ dfsSz(0); dfsHld(0); }
  void dfsSz(int v) {
    if (par[v] != -1) adj[v].erase(find(all(adj[v]), par[v]));
    for (int& u : adj[v]) { //c2274a
      par[u] = v, depth[u] = depth[v] + 1;
```

```
      dfsSz(u);
      siz[v] += siz[u];
      if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
    } //b0fa49
  } //9ba8db
  void dfsHld(int v) {
    pos[v] = tim++;
    for (int u : adj[v]) {
      rt[u] = (u == adj[v][0] ? rt[v] : u);
      dfsHld(u); //2698ee
    } //39b629
  } //39d559
  template <class B> void process(int u, int v, B op) {
    for (; rt[u] != rt[v]; v = par[rt[v]]) {
      if (depth[rt[u]] > depth[rt[v]]) swap(u, v);
      op(pos[rt[v]], pos[v] + 1);
    } //fa17fe
    if (depth[u] > depth[v]) swap(u, v);
    op(pos[u] + VALS_EDGES, pos[v] + 1);
  } //0d5603
  void modifyPath(int u, int v, int val) {
    process(u, v, [&](int l, int r) { tree->add(l, r, val); });
  } //79ce98
  int queryPath(int u, int v) { // Modify depending on problem
    int res = -1e9;
    process(u, v, [&](int l, int r) {
        res = max(res, tree->query(l, r));
    }); //29a64c
    return res;
  } //f00cd2
  int querySubtree(int v) { // modifySubtree is similar
    return tree->query(pos[v] + VALS_EDGES, pos[v] + siz[v]);
  } //8aad63
}; //6f34db
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. Nodes are 1-indexed. You can add and remove edges (as long as the result is still a forest). You can also do path sum, subtree sum, and LCA queries, which depend on the current root.

Time: All operations take amortized $\mathcal{O}(\log N)$.

97ef3b, 110 lines

```
struct SplayTree {
  struct Node {
    int ch[2] = {-1, -1}, p = -1;
    ll self = 0, path = 0;        // Path aggregates
    ll sub = 0, vir = 0;          // Subtree aggregates
    bool flip = 0;                       // Lazy tags
  }; //482fc0
  vector<Node> Ts;
  Node *T;

  SplayTree(int n) : Ts(n+1), T(&Ts[1]) {} //91363a

  void push(int x) {
    if (x == -1 || !T[x].flip) return;
    int l = T[x].ch[0], r = T[x].ch[1];

    T[l].flip ^= 1, T[r].flip ^= 1; //a60e86
    swap(T[x].ch[0], T[x].ch[1]);
    T[x].flip = 0;
  } //f870fe

  void pull(int x) {
    int l = T[x].ch[0], r = T[x].ch[1]; push(l); push(r);

    T[x].path = T[l].path + T[x].self + T[r].path; //241cd9
    T[x].sub = T[x].vir + T[l].sub + T[r].sub + T[x].self;
  } //672aff

  void set(int x, int d, int y) {
    T[x].ch[d] = y; T[y].p = x; pull(x);
  } //84c4f7

  void splay(int x) {
    auto dir = [&](int x) {
      int p = T[x].p; if (p == -1) return -1;
      return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 : -1;
    }; //dc48c9
    auto rotate = [&](int x) {
      int y = T[x].p, z = T[y].p, dx = dir(x), dy = dir(y);
      set(y, dx, T[x].ch[!dx]);
      set(x, !dx, y);
      if (~dy) set(z, dy, x); //4108b6
      T[x].p = z;
    }; //4543f0
    for (push(x); ~dir(x); ) {
      int y = T[x].p, z = T[y].p;
      push(z); push(y); push(x);
      int dx = dir(x), dy = dir(y);
      if (~dy) rotate(dx != dy ? x : y); //67e33c
      rotate(x);
    } //7e3076
  } //6fa8ab
}; //97d901

struct LinkCut : SplayTree {
  LinkCut(int n) : SplayTree(n) {} //92d23f

  int access(int x) {
    int u = x, v = -1;
    for (; u != -1; v = u, u = T[u].p) {
      splay(u); //dd915c
      int& ov = T[u].ch[1];
      T[u].vir += T[ov].sub;
      T[u].vir -= T[v].sub;
      ov = v; pull(u);
    } //dc673b
    return splay(x), v;
  } //533ecb

  void reroot(int x) {
    access(x); T[x].flip ^= 1; push(x);
  } //dbd108

  void link(int u, int v) {
    reroot(u); access(v);
    T[v].vir += T[u].sub;
    T[u].p = v; pull(v); //92f65b
  } //c76755

  void cut(int u, int v) {
    reroot(u); access(v);
    T[v].ch[0] = T[u].p = -1; pull(v);
  } //33b01d

  bool connected(int u, int v) {
    return lca(u, v) != -1;
  } //9ff34d

  // Rooted tree LCA. Returns -1 if u and v arent connected.
  int lca(int u, int v) {
    if (u == v) return u;
    access(u); int ret = access(v); //51e775
    return T[u].p != -1 ? ret : -1;
  } //6c1d58

  // Query subtree of u where v is outside the subtree.
  ll subtree(int u, int v) {
    reroot(v); access(u); return T[u].vir + T[u].self;
  } //76ebdd

  // Query path [u..v]
  ll path(int u, int v) {
    reroot(u); access(v); return T[v].path;
  } //f0875c

  // Update vertex u with value v
  void update(int u, ll v) {
    access(u); T[u].self = v; pull(u);
  } //18a6fa
}; //97ef3b
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620, 60 lines

```
struct Edge { int a, b; ll w; }; //030131
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() { //958c51
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  } //31f792
  Edge top() { prop(); return key; } //61e0cf
}; //67708e
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r))); //c76878
  return a;
} //5e360c
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n); //a7352a
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp; //c7b0b9
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}}; //2158f1
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) { //e7ed0a
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1; //3eb5cd
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      } //ea74cd
    } //db364e
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
```

```
  } //f2bc30

  for (auto& [u,t,comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e; //a32e6d
    in[uf.find(inEdge.b)] = inEdge;
  } //c5d7d7
  rep(i,0,n) par[i] = in[i].a;
  return {res, par}; //d28015
} //39e620
```

TreeDiam.h

Description: Short code for finding a diameter of a tree and returning the path

Time: $\mathcal{O}(|V|)$

d64251, 13 lines

```
auto diameter = [&](int u, int p, auto &&diameter) -> vi {
    vi best;
    for (int v : graph[u]){
        if (v == p) continue;
        vi cur = diameter(v, u, diameter);
        if (sz(cur) > sz(best)) swap(cur, best); //632f5a
    } //2d9dce
    best.push_back(u);
    return best;
}; //d64251
//vi diam = diameter(0, -1, diameter);
//diam = diameter(diam[0], -1, diameter);
//number of nodes on diam is diam.size()
```

Numerical Methods (6)

6.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val; //3743d7
  } //f7a37b
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  } //d447a3
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  } //43bc43
}; //c9b7b0
```

PolyRoots.h

Description: Finds the real roots to a polynomial.

Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`

Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; } //a63eaa
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1); //31d1fe
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0; //2748c8
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m; //8da3ef
      } //4f1379
      ret.push_back((l + h) / 2);
    } //1c9b1d
  } //d5f24e
  return ret;
} //b00bfe
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \ldots n-1$.

Time: $\mathcal{O}\left(n^2\right)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1; //ca948d
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  } //8c43d1
  return res;
} //08bf48
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.

Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`

Time: $\mathcal{O}\left(N^2\right)$

"../number-theory/ModPow.h" 96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1; //b7979b
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod; //b3b877
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  } //3dc38b

  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
} //96548b
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.

Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`

Time: $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1); //d3cd51
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1); //697752
    return res;
  }; //da80a6

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) { //574f01
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  } //b658e4

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
} //f4e444
```

6.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is *eps*. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.

Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`

Time: $\mathcal{O}(\log((b-a)/\epsilon))$

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1; //012afe
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    } //821619
  return a;
} //31d45b
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions

8eeeaf, 14 lines

```
typedef array<double, 2> P;

template<class F> pair<double, P> hillClimb(P start, F f) {
  pair<double, P> cur(f(start), start);
  for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
    rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) { //1a21bb
      P p = cur.second;
```

```
      p[0] += dx*jmp;
      p[1] += dy*jmp;
      cur = min(cur, make_pair(f(p), p));
    } //93215a
  } //523260
  return cur;
} //8eeeaf
```

IntegrateAdaptiveTyler.h

Description: Gets area under a curve

e7beba, 17 lines

```
#define approx(a, b) (b-a) / 6 * (f(a) + 4 * f((a+b) / 2) + f(b
    ))

template<class F>
ld adapt (F &f, ld a, ld b, ld A, int iters) {
  ld m = (a+b) / 2;
  ld A1 = approx(a, m), A2 = approx(m, b); //a97d86
  if(!iters && (abs(A1 + A2 - A) < eps || b-a < eps))
    return A;
  ld left = adapt(f, a, m, A1, max(iters-1, 0));
  ld right = adapt(f, m, b, A2, max(iters-1, 0));
  return left + right; //d787ca
} //f68b38

template<class F>
ld integrate(F f, ld a, ld b, int iters = 0) {
  return adapt(f, a, b, approx(a, b), iters);
} //e7beba
```

RungeKutta4.h

Description: Numerically approximates the solution to a system of Differential Equations

25c1ac, 12 lines

```
template<class F, class T>
T solveSystem(F f, T x, ld time, int iters) {
  double h = time / iters;
  for(int iter = 0; iter < iters; iter++) {
    T k1 = f(x);
    A k2 = f(x + 0.5 * h * k1); //6adf94
    A k3 = f(x + 0.5 * h * k2);
    A k4 = f(x + h * k3);
    x = x + h / 6.0 * (k1 + 2.0 * k2 + 2.0 * k3 + k4);
  } //004f46
  return x;
} //25c1ac
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage: `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`
Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

aa8530, 68 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair //20f308
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B; //a8b98c
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j]; //a00ca8
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; } //03bb56
      N[n] = -1; D[m+1][n] = 1;
    } //dcadf8

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv; //a86c76
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    } //df792b
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  } //193de8

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1; //8b65cd
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue; //f65882
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      } //170720
      if (r == -1) return false;
      pivot(r, s);
    } //d81c2f
  } //62b7d3

  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) { //dc34d7
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]); //db9144
        pivot(i, s);
      } //213eb8
    } //36d5c1
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  } //bc3870
}; //aa8530
```

6.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1; //c6c8fd
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k]; //979baa
    } //ebf330
  } //aa3042
  return res;
} //bd5cec
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i]; //155e04
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      } //3e9488
    } //7effce
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  } //666fb0
  return (ans + mod) % mod;
} //3313dc
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2 m)$

44c9ab, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m); //61ac86
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m) //9bbd0f
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break; //b9eea0
    } //e8dea5
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i]; //bc2598
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    } //34df26
    rank++;
```

```
  } //66cd8f

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i]; //9d7b80
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  } //55ec26
  return rank; // (multiple solutions if rank < m)
} //44c9ab
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" 08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i]; //46800e
fail:; } //08e495
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.

Time: $\mathcal{O}(n^2 m)$

fa2d7a, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0); //b3f2a0
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break; //4a27f9
    } //84b30e
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) { //31f207
      A[j].flip(i); A[j].flip(bc);
    } //bf5e08
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    } //0837c3
    rank++;
  } //4de1ff

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1; //c2244c
    rep(j,0,i) b[j] ^= A[j][i];
  } //fe12f5
  return rank; // (multiple solutions if rank < m)
} //fa2d7a
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.

Time: $\mathcal{O}(n^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) { //8ece41
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i; //baa3bb
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i]; //59c017
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k]; //293c3d
    } //4b5802
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  } //cd352a

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  } //fd4d51

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
} //ebfff6
```

MatrixInverse-mod.h

Description: Invert matrix A modulo a prime. Returns rank; result is stored in A unless singular (rank < n). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.

Time: $\mathcal{O}(n^3)$

"../number-theory/ModPow.h" a6f68f, 36 lines

```
int matInv(vector<vector<ll>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<ll>> tmp(n, vector<ll>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) { //4c70b5
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n) if (A[j][k]) {
      r = j; c = k; goto found;
    } //670a88
    return i;
found:
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c
        ]);
    swap(col[i], col[c]); //f483b9
    ll v = modpow(A[i][i], mod - 2);
    rep(j,i+1,n) {
      ll f = A[j][i] * v % mod;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod; //191
          b80
      rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
    } //3af408
    rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
    rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
    A[i][i] = 1;
  } //b5fe9f

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    ll v = A[j][i];
    rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
  } //597dbe

  rep(i,0,n) rep(j,0,n)
    A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod
        : 0);
  return n;
} //a6f68f
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

Time: $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i]; //5648ab
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i]; //13335c
    } //25f2e7
  } //7da0d1
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1]; //6bd4e6
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    } //94ec57
  } //4f78c5
  return b;
} //8f9fa8
```

JacobianMatrix.h

Description: Makes Jacobian Matrix using finite differences

75dc90, 15 lines

```
template<class F, class T>
vector<vector<T>> makeJacobian(F &f, vector<T> &x) {
  int n = sz(x);
  vector<vector<T>> J(n, vector<T>(n));
  vector<T> fX0 = f(x);
  rep(i, 0, n) { //6bdb0f
    x[i] += eps;
    vector<T> fX1 = f(x);
    rep(j, 0, n){
      J[j][i] = (fX1[j] - fX0[j]) / eps;
    } //8f9232
    x[i] -= eps;
  } //6c57a8
  return J;
} //75dc90
```

NewtonsMethod.h

Description: Solves a system on non-linear equations

jacobianMatrix.h 6af945, 10 lines

```
template<class F, class T>
void solveNonlinear(F f, vector<T> &x){
  int n = sz(x);
  rep(iter, 0, 100) {
    vector<vector<T>> J = makeJacobian(f, x);
    matInv(J); //0e4ed9
    vector<T> dx = J * f(x);
    x = x - dx;
  } //e79640
} //6af945
```

6.4 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.

Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if double)
  for (static int k = 2; k < n; k *= 2) { //beb684
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
  } //42ea68
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //9f2153
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    } //865e86
} //3b927f
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {}; //7ee20e
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  rep(i,0,sz(b)) in[i].imag(b[i]); //ea36b1
  fft(in);
  for (C& x : in) x *= x;
  rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n); //9893c9
  return res;
} //00ced6
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.

Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 22 lines

```
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {}; //ffecc4
  vl res(sz(a) + sz(b) - 1);
  int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
  vector<C> L(n), R(n), outs(n), outl(n);
  rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
  rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
  fft(L), fft(R); //f8a1f3
  rep(i,0,n) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
  } //455f55
  fft(outl), fft(outs);
  rep(i,0,sz(res)) {
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M; //0af53f
  } //26b37c
  return res;
} //b82773
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).

Time: $\mathcal{O}(N \log N)$

"../number-theory/ModPow.h" ced03d, 33 lines

```
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
  int n = sz(a), L = 31 - __builtin_clz(n); //c96375
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)}; //1759b1
    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
  } //5faa22
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //61bd17
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    } //35d5bf
} //29a029
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {}; //4001b0
  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s), n = 1
      << B;
  int inv = modpow(n, mod - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R); //6415db
  rep(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv %
      mod;
  ntt(out);
  return {out.begin(), out.begin() + s}; //70c6bc
} //ced03d
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.

Time: $\mathcal{O}(N \log N)$

464cf3, 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    } //398dab
  } //3431d0
  if (inv) for (int& x : a) x /= sz(a); // XOR only
} //57eeaf
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
} //464cf3
```

Minconv.h

Description: @param convex,arbitrary arrays where convex satisfies convex[i+1]-convex[i] <= convex[i+2]-convex[i+1] @returns array 'res' where 'res[k]' = the min of (a[i]+b[j]) for all pairs (i,j) where i+j==k

633806, 26 lines

```
vector<int> min_plus(const vector<int>& convex,
  const vector<int>& arbitrary) {
  int n = ssize(convex);
  int m = ssize(arbitrary);
  vector<int> res(n + m - 1, INT_MAX);
  auto dnc = [&](auto&& self, int res_le, int res_ri, //c890c6
          int arb_le, int arb_ri) -> void {
    if (res_le >= res_ri) return;
    int mid_res = (res_le + res_ri) / 2;
    int op_arb = arb_le;
    for (int i = arb_le; i < min(mid_res + 1, arb_ri); //00bcac
    i++) {
      int j = mid_res - i;
      if (j >= n) continue;
      if (res[mid_res] > convex[j] + arbitrary[i]) {
        res[mid_res] = convex[j] + arbitrary[i]; //c587b4
        op_arb = i;
      } //d9dac2
    } //12d663
    self(self, res_le, mid_res, arb_le,
    min(arb_ri, op_arb + 1));
    self(self, mid_res + 1, res_ri, op_arb, arb_ri);
  }; //133dea
  dnc(dnc, 0, n + m - 1, 0, m);
  return res;
  } //633806
```

gcdconv.h

Description: ssize(a)==ssize(b) gcdconv[k] = sum of (a[i]*b[j]) for all pairs (i,j) where gcd(i,j)==k

Time: $\mathcal{O}(N \log N)$

2dfb20, 16 lines

```
const int mod = 998'244'353;
vector<int> gcd_convolution(const vector<int>& a,
  const vector<int>& b) {
  int n = ssize(a);
  vector<int> c(n);
  for (int g = n - 1; g >= 1; g--) { //8423c4
    int64_t sum_a = 0, sum_b = 0;
    for (int i = g; i < n; i += g) {
      sum_a += a[i], sum_b += b[i];
      if ((c[g] -= c[i]) < 0) c[g] += mod;
    } //7021b5
    sum_a %= mod, sum_b %= mod;
    c[g] = (c[g] + sum_a * sum_b) % mod;
  } //22b2a9
  return c;
} //2dfb20
```

lcmconv.h

Description: ssize(a)==ssize(b) lcmconv[k] = sum of (a[i]*b[j]) for all pairs (i,j) where lcm(i,j)==k

ee1440, 16 lines

```
const int mod = 998'244'353;
vector<int> lcm_convolution(const vector<int>& a,
  const vector<int>& b) {
  int n = ssize(a);
  vector<int64_t> sum_a(n), sum_b(n);
  vector<int> c(n); //f8bc27
  for (int i = 1; i < n; i++) {
    for (int j = i; j < n; j += i)
      sum_a[j] += a[i], sum_b[j] += b[i];
    sum_a[i] %= mod, sum_b[i] %= mod;
    c[i] = (c[i] + sum_a[i] * sum_b[i]) % mod; //2c8c40
    for (int j = i + i; j < n; j += i)
      if ((c[j] -= c[i]) < 0) c[j] += mod;
  } //2b66e9
  return c;
} //ee1440
```

Number theory (7)

7.1 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

6f684f, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.

Time: $\mathcal{O}(\sqrt{m})$

c040b8, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j; //2d9fb0
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
} //c040b8
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.

modsum(to, c, k, m) $= \sum_{i=0}^{\text{to}-1} (ki+c)\%m$. divsum is similar but for floored division.

Time: $\log(m)$, with a large constant.

5c5bc5, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); } //6bd037

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res; //d4b74d
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
} //4a574e

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
} //5c5bc5
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.

Time: $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow

bbbd8f, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
} //a9c350
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans; //6d3d5f
} //bbbd8f
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).

Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 19a793, 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2; //6aa127
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p); //94db39
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p; //2d5fcd
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p; //198af1
  } //ac3137
} //19a793
```

7.2 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.

Time: LIM=1e9 $\approx$ 1.5s

6b2912, 20 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM / 2;
  vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
  vector<pii> cp; //81984e
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
  } //e31824
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{}; //8834d0
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
    rep(i,0,min(S, R - L))
      if (!block[i]) pr.push_back((L + i) * 2 + 1);
  } //4de4a4
  for (int i : pr) isPrime[i] = 1;
  return pr;
} //6b2912
```

LinearSieve.h

Description: Finds smallest prime factor of each integer

Time: $\mathcal{O}(N)$

32eeca, 8 lines

```
const int LIM = 1000000;
vi lp(LIM+1), primes;

rep(i, 2, LIM + 1) {
  if (lp[i] == 0) primes.push_back(lp[i] = i);
  for (int j = 0; j < sz(primes) && i * primes[j] <= LIM &&
      primes[j] <= lp[i]; ++j)
    lp[i * primes[j]] = primes[j]; //91f1b5
} //32eeca
```

CountPrimes.h

Description: Count # primes <= N, can be modified to return sum of primes by setting f(p) = n, ps(n) = nth tri number.

Time: $\mathcal{O}\left(n^{3/4}\right)$

af82c0, 13 lines

```
ll countprimes(ll n) { //n>0
  vector<ll> divs,dp; ll sq = sqrtl(n);
  for (ll l = 1, r; l <= n && (r = n / (n / l)); l = r + 1)
    divs.push_back(r);
  auto idx = [&](ll x) -> int {
  return x <= sq ? x - 1 : (sz(divs) - n / x); }; //d740a2
  rep(i,0,sz(divs)) dp.push_back(divs[i]-1);
  for(ll p = 2; p*p <= n; ++p) // ^ ps(divs[i])-1
    if(dp[p-1]!=dp[p-2])
    for(int i = sz(divs)-1; divs[i]>=p*p && i>=0; i--)
      dp[i] -= (dp[idx(divs[i]/p)]-dp[p-2]);// *f(p);
  return dp.back(); //0b539f
} //af82c0
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

Time: 7 times the complexity of $a^b \bmod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s; //29e314
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  } //1fad05
  return 1;
} //60dcd1
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" d8d98d, 18 lines

```
ull pollard(ull n) {
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  auto f = [&](ull x) { return modmul(x, x, n) + i; }; //12dccb
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  } //0b4d32
  return __gcd(prd, n);
} //cd2ac3
vector<ull> factor(ull n) {
  if (n == 1) return {}; //6303f2
  if (isPrime(n)) return {n}; //74d420
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
} //d8d98d
```

GetFactors.h

Description: Gets all factors of a number N given the prime factorization of the number. as lists of primes and corresponding power

Time: $\mathcal{O}\left(\sqrt[3]{N}\right)$

493617, 5 lines

```
void getFactors(auto &primes, auto &pws, auto &divs, int i = 0,
    ll n = 1) {
  if(i == pws.size()) return void(divs.push_back(n));
  for(ll j = 0, pow = 1; j <= pws[i]; j++, pow *= primes[i])
    getFactors(primes, pws, divs, i+1, n * pow);
} //493617
```

mobiusFunction.h

Description: Computes mobius function, example code for counting coprime pairs

1783cc, 13 lines

```
//Mobius function
vector<int> mu(maxv); mu[1] = 1;
for(int i = 1; i < mu.size(); i++)
    for(int j = 2*i; j < mu.size(); j+=i)
        mu[j]-=mu[i];

//Count coprime pairs
ll ans = 0; //b800ad
for(int d = 1; d<maxv; d++){
    ll sum = 0;
    for(int j = 0; j < maxv; j+=d) sum+=freq[j];
    ans+=(mu[d]*choose2(sum));
} //1783cc
```

7.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
} //33ba8f
```

CRT.h

Description: Chinese Remainder Theorem.

crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x; //000521
} //04d93a
```

7.3.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.

Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p$ $\forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i) //9fb18b
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
} //cf7d6d
```

7.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \ge 0$, finds the closest rational approximation p/q with $p, q \le N$. It will obey $|p/q - x| \le 1/qN$.

For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.

Time: $\mathcal{O}(\log N)$

dd6c5e, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
       a = (ll)floor(y), b = min(a, lim), //82cd25
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives us a
      // better approximation; if b = a/2, we *may* have one.
      // Return {P, Q} here for a more canonical approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q); //3c2b26
    } //451a2f
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ}; //32957f
    } //ec2d82
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  } //a15756
} //dd6c5e
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \le N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}

Time: $\mathcal{O}(\log(N))$

27ab3e, 25 lines

```
struct Frac { ll p, q; }; //feaca1

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo; //7f70d6
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step; //3067db
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q}; //306933
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      } //a40ec9
    } //d35347
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv; //fc82fe
  } //2c9a8f
  return dir ? hi : lo;
} //27ab3e
```

Fraction.h

Description: Safer struct for representing fractions/rationals. Comparison is 100% overflow safe; other ops are safer but can still overflow. All ops are $O(\log N)$.

8ff7f8, 32 lines

```
template<class T> struct QO {
  T a, b;
  QO(T p, T q = 1) {
    T g = gcd(p, q);
    a = p / g;
    b = q / g; //6d7843
    if (b < 0) a = -a, b = -b; } //fe71bc
  T gcd(T x, T y) const { return __gcd(x, y); } //044c49
  QO operator+(const QO& o) const {
```

```
    T g = gcd(b, o.b), bb = b / g, obb = o.b / g;
    return {a * obb + o.a * bb, b * obb}; } //b90212
  QO operator-(const QO& o) const {
    return *this + QO(-o.a, o.b); } //970b3b
  QO operator*(const QO& o) const {
    T g1 = gcd(a, o.b), g2 = gcd(o.a, b);
    return {(a / g1) * (o.a / g2), (b / g2) * (o.b / g1)}; }
  QO operator/(const QO& o) const {
    return *this * QO(o.b, o.a); } //961352
  QO recip() const { return {b, a}; } //527d54
  int signum() const { return (a > 0) - (a < 0); } //b6aa22
  static bool lessThan(T a, T b, T x, T y) {
    if (a / b != x / y) return a / b < x / y;
    if (x % y == 0) return false;
    if (a % b == 0) return true;
    return lessThan(y, x % y, b, a % b); } //cab1f0
  bool operator<(const QO& o) const {
    if (this->signum() != o.signum() || a == 0)
      return a < o.a;
    if (a < 0) return lessThan(abs(o.a), o.b, abs(a), b);
    else return lessThan(a, b, o.a, o.b); } //6ce8a3
  friend ostream& operator<<(ostream& cout, const QO& o) {
    return cout << o.a << "/" << o.b; } }; //8ff7f8
```

7.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k\cdot(m^2-n^2),\ \ b = k\cdot(2mn),\ \ c = k\cdot(m^2+n^2),$$

with $m > n > 0$, $k > 0$, $m\perp n$, and either m or n even.

7.6 Primes

$p = 962592769$ is such that $2^{21} \mid p-1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

7.7 Estimates

$\sum_{d|n} d = O(n \log\log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

7.8 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1\le m\le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1\le m\le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (8)

8.1 Permutations

8.1.1 Factorial

n	1 2 3 4	5	6	7	8	9	10
$n!$	1 2 6 24	120	720	5040	40320	362880	3628800

n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14

n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.

Time: $\mathcal{O}(n)$

044568, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
    use |= 1 << x;                      // (note: minus, not ~!)
  return r;
} //044568
```

8.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

8.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor\frac{n!}{e}\right\rceil$$

8.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

8.2 Partitions and subsets

8.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ \ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0 1 2 3 4 5	6	7	8	9	20	50	100
$p(n)$	1 1 2 3 5 7	11	15	22	30	627	~2e5	~2e8

8.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.

8.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1!k_2!...k_n!}$.

a0a312, 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
} //a0a312
```

8.3 General purpose numbers

8.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

8.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ \ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

8.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

8.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

8.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

8.3.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

8.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Strings (9)

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

d4375c, 16 lines

```
vi pi(const string& s) {
  vi p(sz(s));
  rep(i,1,sz(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]); //21a657
  } //6c1f11
  return p;
} //9cb7fc

vi match(const string& s, const string& pat) {
  vi p = pi(pat + '\0' + s), res;
  rep(i,sz(p)-sz(s),sz(p))
    if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat)); //
        68390b
  return res;
} //d4375c
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

3ae526, 12 lines

```
vi Z(string S) {
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]]) //fe9318
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  } //1fcbd4
  return z;
} //3ae526
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)}; //daf4bc
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1]) //508df3
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  } //21a1fb
  return p;
} //e7ad79
```

Eertree.h

Description: Generates an eertree on str. *cur* is accurate at the end of the main loop before the final assignment to *t*.
Time: $\mathcal{O}(|S|)$

288121, 35 lines

```
struct eertree{
    static constexpr int ALPHA = 26;
    struct node{ //sInd is starting index of an occurrence
        array<int,ALPHA> down;
        int slink, ln, sInd, freq = 0;
        node(int slink, int ln, int eInd = -1): //5dff69
            slink(slink), ln(ln), sInd(eInd-ln+1) {
                fill(begin(down),begin(down)+ALPHA,-1);
            } //6a8cb3
    }; //aa06f7
    vector<node> t = {node(0,-1),node(0,0)}; //b4be49
    eertree(string &s){
        int cur = 0, k = 0;
        for(int i = 0; i < sz(s); i++){
            char c = s[i]; int cID = c-'a'; //first chracter
            while(k<=0 || s[k-1] != c) //e85b7f
                k = i - t[cur = t[cur].slink].ln;
            #define TCD t[cur].down[cID]
            if(TCD == -1){
                TCD = sz(t);
                t.emplace_back(-1,t[cur].ln+2,i); //8f1444
                if(t.back().ln > 1){
                    do k = i - t[cur = t[cur].slink].ln;
                    while(k<=0 || s[k-1] != c);
                    t[sz(t)-1].slink = TCD;
                } else t[sz(t)-1].slink = 1; //519576
                cur = sz(t)-1;
            } else cur = TCD;
            t[cur].freq++;
            k = i - t[cur].ln+1;
        } //f67fbd
        for(int i = sz(t)-1; i > 1; i--) //update frequencies
            t[t[i].slink].freq += t[i].freq;
    } //6acbda
}; //288121
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$

d07a42, 8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; } //20f912
  } //b2e25e
  return a;
} //d07a42
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$

38db9f, 23 lines

```
struct SuffixArray {
  vi sa, lcp;
  SuffixArray(string& s, int lim=256) { // or basic_string<int>
    int n = sz(s) + 1, k = 0, a, b;
    vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
    sa = lcp = y, iota(all(sa), 0); //74da6a
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
      p = j, iota(all(y), n - j);
      rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i,0,n) ws[x[i]]++; //499169
      rep(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
```

```
    rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
      (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
  } //f30252
  rep(i,1,n) rank[sa[i]] = i;
  for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
    for (k && k--, j = sa[rank[i] - 1];
        s[i + k] == s[j + k]; k++);
  } //22a139
}; //38db9f
```

SuffixArrayQuery.h

Description: Various helper queries for suffix array problems inputs are 0 based input/output is inc-ex

Time: lenlcp: $\mathcal{O}(1)$, cmpsub: $\mathcal{O}(1)$, findstr: $\mathcal{O}(log(n))$

92e674, 27 lines

```
struct SAQuery{
  SuffixArray sa; RMQ<int> lcp;
  string s; vector<int> sainv;
  SAQuery(SuffixArray sa,string s):sa(sa), lcp(sa.lcp), s(s){
    sainv.resize(sz(s)+1);
    rep(i,0,sz(sa.sa)) sainv[sa.sa[i]] = i; //700e88
  } //859793
  int len_lcp(int u, int v){
    if(u==v) return sz(s)-u;
    auto[l,r] = minmax(sainv[u],sainv[v]);
    return lcp.query(l+1,r+1);
  } //given [) substr of s: s1<s2->-1,s1==s2->0,s1>s2->1
  int cmp_sub(int l1, int r1, int l2, int r2){ //787424
    auto sgn = [](int x){ return (x>0)-(x<0); }; //7a8522
    int len1 = r1-l1+1, len2 = r2-l2+1;
    return len_lcp(l1,l2) < min(len1,len2)
      ? sgn(sainv[l1]-sainv[l2]): sgn(len1-len2);
  } //73a96e
  pair<int, int> find_str(int s_l, int s_r) {
    auto cmp = [&](int i, bool flip) -> bool {
      return flip ^ (len_lcp(i, s_l) < s_r - s_l); }; //67dcf3
    auto it = begin(sa.sa) + sainv[s_l];
    int l=lower_bound(begin(sa.sa),it,0,cmp)-begin(sa.sa);
    int r=lower_bound(it+1,end(sa.sa),1,cmp)-begin(sa.sa);
    return {l, r}; //# -> r-l
  } //102c76
}; //92e674
```

findSubstr.h

Description: returns inc-exclusive range of occurences of needle string inside suffix array, assumes global sa structure and global s (haystack)

Time: $\mathcal{O}(|ndl|log(|s|))$

7d1493, 9 lines

```
pair<int,int> find_str(const string &ndl){
  auto le = lower_bound(begin(sa.sa)+1,end(sa.sa), 0,
  [&](int i, int) -> bool { return lexicographical_compare
    (begin(s) + i, end(s), all(ndl)); });
  auto ri = lower_bound(le, end(sa.sa), 0,
  [&](int i, int) -> bool { return mismatch(begin(s) + i,
    end(s), all(ndl)).second == end(ndl);}); //022ea5
  return {le-begin(sa.sa),ri-begin(sa.sa)}; //[)
} //7d1493
```

SuffixAutomaton.h

Description: Creates a partial DFA (DAG) that accepts all suffixes, with suffix links. One-to-one map between a path from the root and a substring. len is the longest-length substring ending here. pos is the first index in the string matching here. term is whether this node is a terminal (aka a suffix)

Time: construction takes $\mathcal{O}(N \log K)$, where $K =$ Alphabet Size

1914a9, 22 lines

```
struct st { int len, pos, term; st *link; map<char, st*> next;
    };
st *suffixAutomaton(string &str) {
  st *last = new st(), *root = last;
  for(auto c : str) {
    st *p = last, *cur = last = new st{last->len + 1, last->len
        };
    while(p && !p->next.count(c)) //4cd1a8
      p->next[c] = cur, p = p->link;
    if (!p) cur->link = root;
    else {
      st *q = p->next[c];
      if (p->len + 1 == q->len) cur->link = q; //1cc2d6
      else {
        st *clone = new st{p->len+1, q->pos, 0, q->link, q->
            next};
        for (; p && p->next[c] == q; p = p->link)
          p->next[c] = clone;
        q->link = cur->link = clone; //08d876
      } //b49887
    } //31bf7e
  } //76ccab
  while(last) last->term = 1, last = last->link;
  return root;
} //1914a9
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

Time: $\mathcal{O}(26N)$

aae0b8, 50 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; } //e2aa04
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) { //a822f9
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; } //810ece
      v=t[v][c]; q=l[v];
    } //6b58ee
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m]; //d6dde8
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    } //451104
  } //0b7995

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t); //ab059b
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
  } //e6a350

  // example: find longest common substring (uses ALPHA = 28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1; //dc2e91
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3) //f72e9f
      best = max(best, {len, r[node] - len});
    return mask;
  } //526a4c
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  } //9dc48b
}; //aae0b8
```

Hashing.h

Description: Self-explanatory methods for string hashing.

4b8fa1, 19 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
typedef uint64_t ull; //41d24d
struct H {
  ull x; H(ull x=0) : x(x) {} //80cf70
  H operator+(H o) { return x + o.x + (x + o.x < x); } //1f9d48
  H operator-(H o) { return *this + ~o.x; } //98ccfa
  H operator*(H o) { auto m = (__uint128_t)x * o.x;
    return H((ull)m) + (ull)(m >> 64); } //4eff44
  ull get() const { return x + !~x; } //f17b1d
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); } //442
      de3
}; //40d284
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

HashInterval.h

Description: Various self-explanatory methods for string hashing.

"Hashing.h" 122649, 12 lines

```
struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i], //c3c119
      pw[i+1] = pw[i] * C;
  } //67307e
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  } //39481a
}; //122649
```

LyndonFactorization.h

Description: A string is called simple (or a Lyndon word), if it is strictly smaller than any of its own nontrivial suffixes. Examples of simple strings are: a, b, ab, aab, abb, $ababb$, $abcd$. It can be shown that a string is simple, if and only if it is strictly smaller than all its nontrivial cyclic shifts. Next, let there be a given string s. The Lyndon factorization of the string s is a factorization $s = w_1 w_2 \dots w_k$, where all strings w_i are simple, and they are in non-increasing order $w_1 \geq w_2 \geq \dots \geq w_k$. It can be shown, that for any string such a factorization exists and that it is unique.

Time: $\mathcal{O}(N)$

0e6ce6, 20 lines

```
vector<string> duval(string const& s) {
    int n = s.size();
    int i = 0;
    vector<string> factorization;
    while (i < n) {
```

```
        int j = i + 1, k = i; //d0372e
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                k++; //8d1eaa
            j++;
        } //cf42b4
        while (i <= k) {
            factorization.push_back(s.substr(i, j - k));
            i += j - k;
        } //46a6db
    } //14171a
    return factorization;
} //0e6ce6
```

Wildcard.h

Description: string matching with wildcards, returns boolean vector of size s-p+1 representing if a match occurs at this start position, wild cards are repsented by 0 and can be in s,p or both.
Time: $\mathcal{O}((n+m)log(n+m))$ b0e86b, 24 lines

```
vector<vl> make_powers(const vl& v) {
    int n = sz(v);
    vector<vl> pws(3, vl(n)); pws[0] = v;
    rep(k,1,3) rep(i,0,n) //mod?
        pws[k][i] = pws[k-1][i]*v[i];
    return pws; //a00fe1
} //10e306

vector<bool> wildcard_pattern_matching(const vl& s,
    const vl& p) {
    int n = sz(s), m = sz(p);
    auto s_pws = make_powers(s), p_pws = make_powers(p);
    for (auto& p_pw : p_pws) reverse(all(p_pw)); //cd7088
    vector<vl> res(3);
    rep(pw_hay,0,3) //ntt
        res[pw_hay] = conv(s_pws[pw_hay], p_pws[2 - pw_hay]);
    vector<bool> mtch(n - m + 1);
    rep(i,0,sz(mtch)){ //890a02
        int id = i + m - 1;
        auto num = res[0][id] - 2 * res[1][id] + res[2][id];
        mtch[i] = !num; //num == 0
    } //934360
    return mtch;
} //b0e86b
```

AhoCorasick-Tyler.h

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$. 647ca9, 47 lines

```
const int ABSIZE = 26;

struct node {
  int nxt[ABSIZE];
  vi ids = {}; //d04adb
  int prv = -1, link = -1;
  char c;
  int linkMemo[ABSIZE];

  node(int prv = -1, char c = '$'): prv(prv), c(c) { //ec9f1e
    fill(all(nxt), -1);
    fill(all(linkMemo), -1);
  } //16055b
}; //432cad

vector<node> trie(1);

void addWord(string &s, int id) {
  int cur = 0; //aa1bc0
  for(char c: s) {
    int idx = c - 'a';
    if(trie[cur].nxt[idx] == -1) {
      trie[cur].nxt[idx] = sz(trie);
      trie.emplace_back(cur, c); //23b9d2
    } //ba2978
    cur = trie[cur].nxt[idx];
  } //35f152
  trie[cur].ids.push_back(id);
} //1dfc37

int getLink(int cur);

int calc(int cur, char c) {
  int idx = c - 'a'; //e9a88a
  auto &ret = trie[cur].linkMemo[idx];
  if(ret != -1) return ret;
  if(trie[cur].nxt[idx] != -1)
    return ret = trie[cur].nxt[idx];
  return ret = cur == 0 ? 0 : calc(getLink(cur), c); //1a4276
} //c61f02

int getLink(int cur) {
  auto &ret = trie[cur].link;
  if(ret != -1) return ret;
  if(cur == 0 || trie[cur].prv == 0) return ret = 0; //be881f
  return ret = calc(getLink(trie[cur].prv), trie[cur].c);
} //647ca9
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$ edce47, 23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it); //a98b04
  } //381108
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  } //5783d8
  return is.insert(before, {L,R});
} //d57d47

void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second; //51cff5
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
} //edce47
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$ 9e9d8d, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first; //a166e4
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at])); //201b40
      at++;
    } //470978
    if (mx.second == -1) return {}; //f1e40b
    cur = mx.first;
    R.push_back(mx.second);
  } //cd0c49
  return R;
} //9e9d8d
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$ 753a4c, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q; //a2e0d8
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  } //5b694f
} //69b73b
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q); //587254
  g(i, to, q);
} //753a4c
```

10.2 Misc. algorithms

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$ 2932a0, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
  if (S.empty()) return {}; //be1376
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
```

```
  rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0}); //f6ef94
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i}; //26a0a3
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  } //f2ee22
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
} //2932a0
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
Time: $\mathcal{O}(N \max(w_i))$ b20ccc, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1); //11fd10
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x]) //51a6b1
      v[x-w[j]] = max(v[x-w[j]], j);
  } //d2bd39
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
} //b20ccc
```

maskloop.h

3e4515, 6 lines

```
//iterate submask
for (int submask = mask; submask;
  submask = (submask - 1) & mask)
//iterate supermask
for (int supermask = mask; supermask < (1 << n);
  supermask = (supermask + 1) | mask) //3e4515
```

10.3 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^2)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i)\le k<hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}((N + (hi - lo)) \log N)$ d38d2b, 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; } //ce585d
  int hi(int ind) { return ind; } //f742b2
  ll f(int ind, int k) { return dp[ind][k]; } //29ea0c
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1; //13ddb0
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1); //4993b6
    rec(mid+1, R, best.second, HI);
  } //116ea5
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
}; //d38d2b
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

Time.h

Description: Measure time elapsed fe7d8c, 3 lines

```
using namespace std::chrono;
auto t1 = steady_clock::now();
duration_cast<microseconds>(t2 - t1).count()/1e6
```

10.5 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to a (mod b) in the range $[0, 2b)$. 751a02, 8 lines

```
typedef unsigned long long ull;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {} //551bab
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;
  } //03d237
}; //751a02
```

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.
Usage: `./a.out < input.txt`
Time: About 5x as fast as cin/scanf. 7b3c70, 17 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin); //bba013
  } //e9a035
  return buf[bc++]; // returns 0 on EOF
} //0261eb

int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt(); //bc51ee
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
} //7b3c70
```

BumpAllocator.h

Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation. 745db2, 8 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s]; //e69924
} //0c4c77
void operator delete(void*) {} //745db2
```

SmallPtr.h

Description: A 32-bit pointer that points into BumpAllocator memory.
"BumpAllocator.h" 2dd6c9, 10 lines

```
template<class T> struct ptr {
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
    assert(ind < sizeof buf);
  } //bda3ee
  T& operator*() const { return *(T*)(buf + ind); } //36a0d6
  T* operator->() const { return &**this; } //c82e36
  T& operator[](int a) const { return (&**this)[a]; } //dd2aa9
  explicit operator bool() const { return ind; } //881391
}; //2dd6c9
```

BumpAllocatorSTL.h

Description: BumpAllocator for STL containers.
Usage: `vector<vector<int, small<int>>> ed(N);` bb66d4, 14 lines

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;
```

```
template<class T> struct small {
  typedef T value_type;
  small() {} //beaa7e
  template<class U> small(const U&) {} //a4e63a
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);
  } //16a7ac
  void deallocate(T*, size_t) {} //92a617
}; //bb66d4
```

SIMD.h

Description: Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern "_mm(256)?_name_(si(128|256)|epi(8|16|32|64)|pd|ps)". Not all are described here; grep for _mm_ in /usr/lib/gcc/*/4.9/include/ for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and #define __SSE__ and __MMX__ before including it. For aligned memory use _mm_malloc(size, 32) or int buf[N] alignas(32), but prefer loadu/storeu.

551b82, 43 lines

```
#pragma GCC target ("avx2") // or sse4.1
#include "immintrin.h"

typedef __m256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))

// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256, _mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts of x
// sad_epu8: sum of absolute differences of u8, outputs 4xi64
// maddubs_epi16: dot product of unsigned i7's, outputs 16xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g. _epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|hi)

int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m; //597c94
  int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; } //22ddd0
mi zero() { return _mm256_setzero_si256(); } //4823c6
mi one() { return _mm256_set1_epi32(-1); } //3889b7
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }

ll example_filteredDotProduct(int n, short* a, short* b) {
  int i = 0; ll r = 0; //a0b618
  mi zero = _mm256_setzero_si256(), acc = zero;
  while (i + 16 <= n) {
    mi va = L(a[i]), vb = L(b[i]); i += 16;
    va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
    mi vp = _mm256_madd_epi16(va, vb); //9738d4
    acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
      _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)));
  } //8de72e
  union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[i];
  for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; // <- equiv
  return r;
} //551b82
```