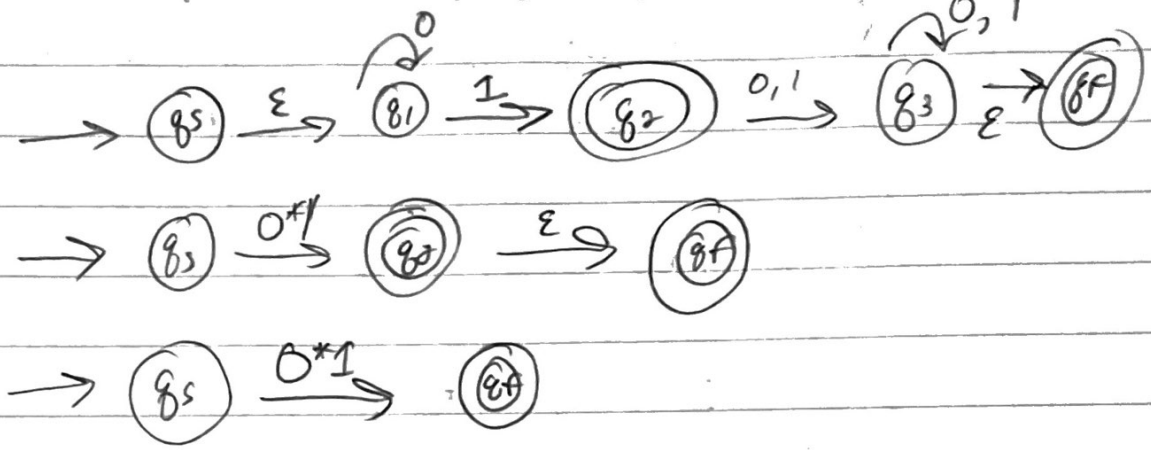
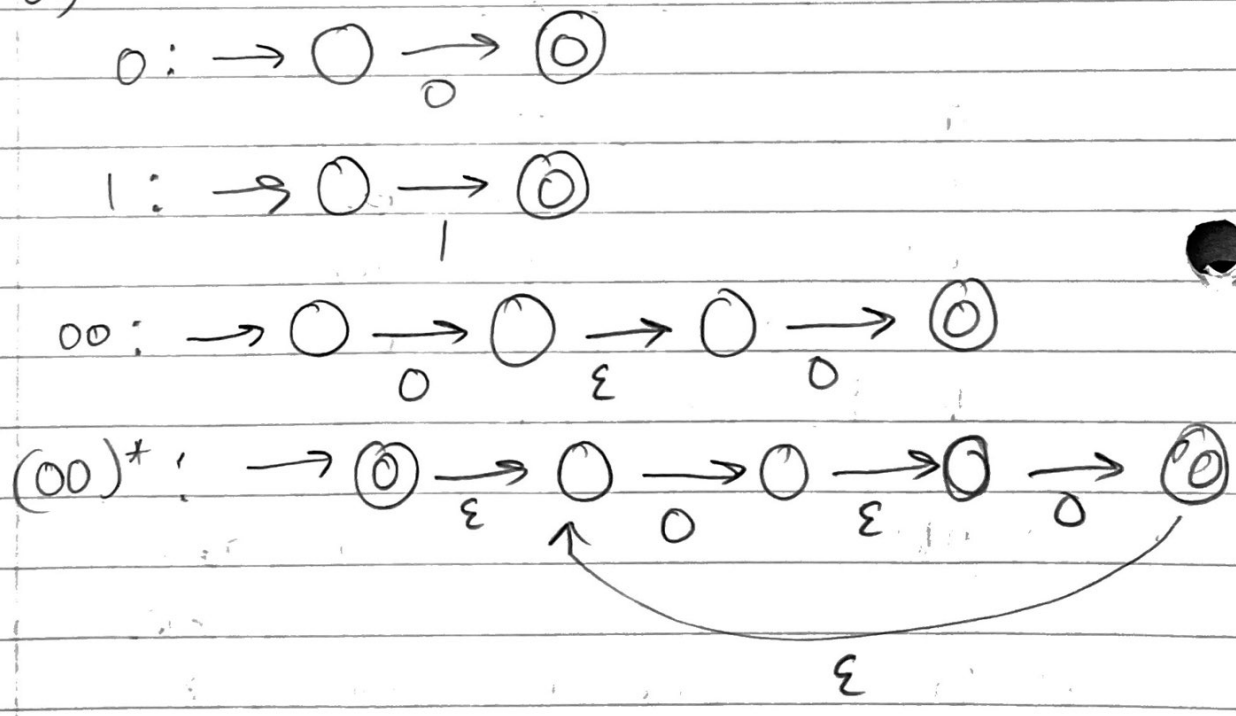
Thomas DeMasse - 7/13/23 - Foundations

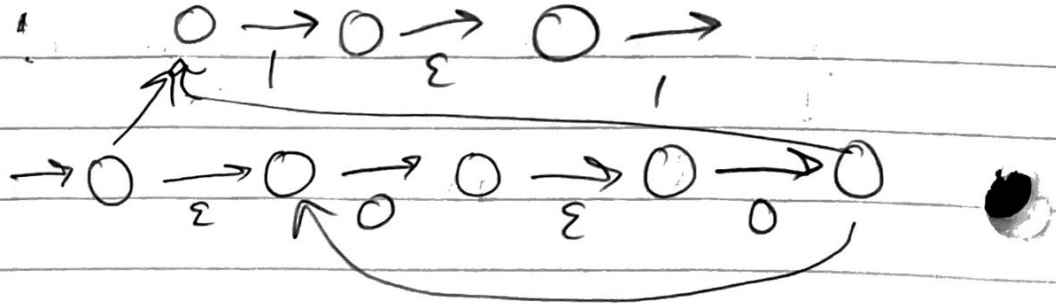
1.)



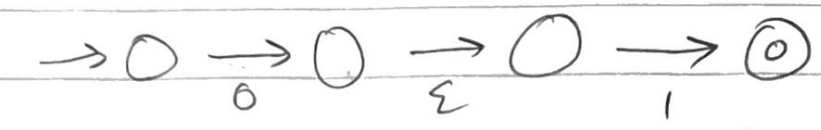
2.)



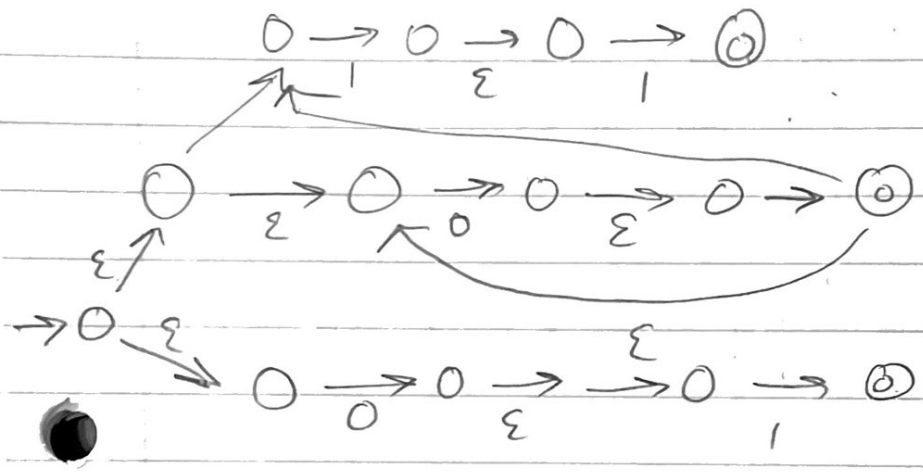
(00)*11:



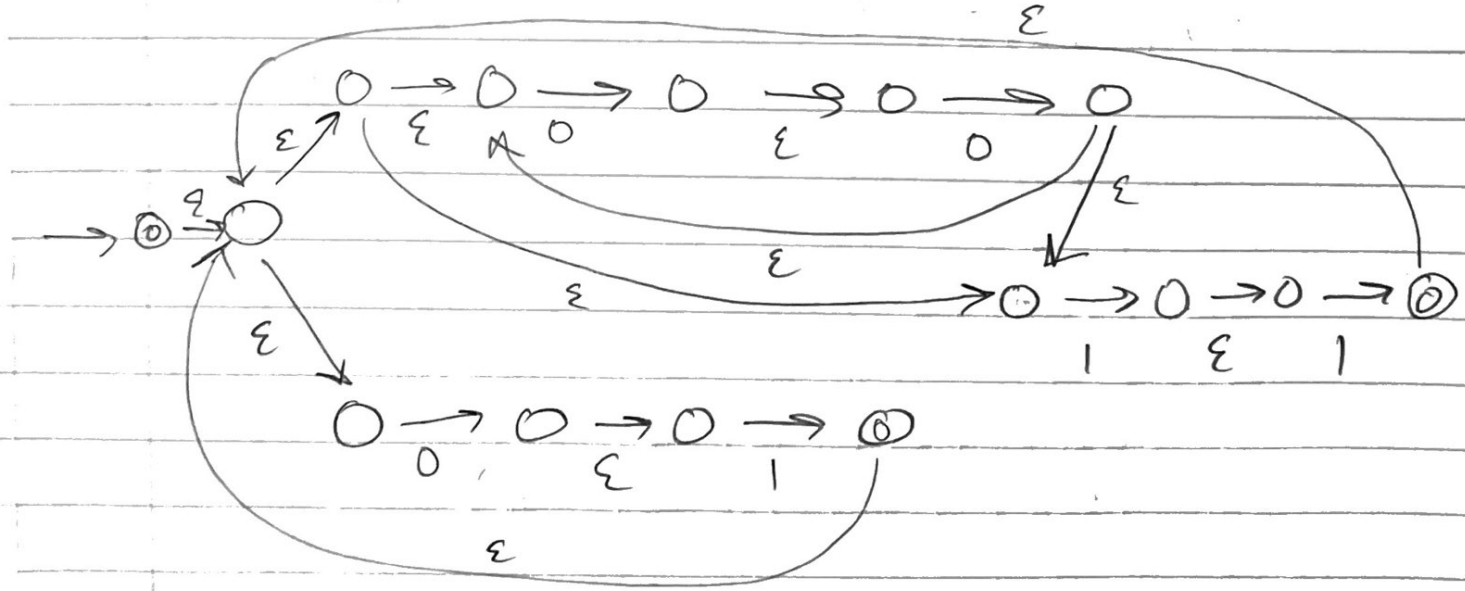
01 :



(((00)*(11)∪(01))*:



Final NFA (((00)*(11)∪(01))*



3.) $L = \{ababa^2, a^2ba^2ba^3, a^3ba^3ba^4 \ldots\}$

Divide the string $ababa^2$ into 3 parts.

$x = a, y = ba, z = ba^2$

$|y| > 0 \Rightarrow |z| > 0$ (true)

$|xy| \leq P \Rightarrow |3| \leq 6$ (true)

$xy^i z \Rightarrow a(ba)^0 ba^2 \Rightarrow alba^2 \notin L$

$(i=0) \rightarrow$ (False)

$alba^2$ is not in L

4.) $|A| L_A = \{01\}$

$S_A \rightarrow 01$

$V = \{S_A\}$

$\Sigma = \{0, 1\}$

$R = \{S_A \rightarrow 01\}$

$S = S_A$

The CFG of L_A is:

$(\{S_A\}, \{0, 1\}, \{S_A \rightarrow 01\}, S_0)$

$L_B = \{0^n 1^n \mid n \geq 0\}$

$S_B \rightarrow 0^n 1^n \rightarrow 0S_1 1 \mid \varepsilon$

$V = \{S_B\}$

$\Sigma = \{0, 1\}$

$R = \{S_B \rightarrow 0S_1 1 \mid \varepsilon$

$S = S_B$

The CFG of L_B is:

$(\{S_B\}, \{0, 1\}, \{S_1 \rightarrow 0S_1 1 \mid \varepsilon\}, S_1)$

Concantenation: $V = \{S_A, S_B\}$

$\Sigma = \{0, 1\}$

$R = \{S_1 \rightarrow S_A S_B\}$

$S = S_1$

four-Tuple: $(\{S_A, S_B\}, \{0, 1\}, \{S_1 \rightarrow S_A S_B\}, S_1)$

(b) Step 1: New Start Symbol

$N \rightarrow S$

$S \rightarrow S_A S_B$

$S_A \rightarrow 01$

$S_B \rightarrow 0S_A 1 \mid \varepsilon$

Step 2: Removal of the ε rules

$N \rightarrow S$

$S \rightarrow S_A S_B \mid S_A$

$S_A \rightarrow 01$

$S_B \rightarrow 0S_A 1$

Step3: Removal of unit rules

$N \rightarrow S_A S_B \mid 01$

$S \rightarrow S_A S_B \mid 01$

$S_A \rightarrow 01$

$S_B \rightarrow 0 S_A 1$

Step 4: Splitting rules into Parts.

$N \rightarrow S_A S_B \mid 01$

$S \rightarrow S_A S_B \mid 01$

$S_A \rightarrow 01$

$S_B \rightarrow 0 S_C$

$S_C \rightarrow S_A 1$

Step 5: Replacing terminals with variables

$N \rightarrow S_a S_b 1$

$S \rightarrow S_a S_B 1$

$S_A \rightarrow XY$

$S_B \rightarrow XS_C$

$S_C \rightarrow A_y$

$X \rightarrow 0$

$y \rightarrow 1$

5.) $X = 01 | 01Y$

$Y = 01X01X | \varepsilon$

The four-tuple for this CFG would be

$V = \{X, Y\}$

$\Sigma = \{0, 1\}$

$R = \{X \to 01 | 01Y$

$Y \to 01X0 | X | \varepsilon\}$

$S = X$

$SO \to (\{X, Y\}, \{0,1\}, \{X \to 01 | 01Y, Y \to 01X0 | X | \varepsilon\},$
$X\}$

6.) $V = \{R_1, R_2, R_3\}$

$\Sigma = \{0, 1, \varepsilon\}$

$R = \{R_1 \to 0R_1, R_1 \to 1R_2, R_2 \to 0R_3, R_2 \to 1R_3,$
$R_2 \to \varepsilon, R_3 \to 0R_3, R_3 \to 1R_3\}$

$S = S_1$

The Four tuple is:

$(\{R_1, R_2, R_3\}, \{\}, \{0, 1, \varepsilon\}, \{R_1 \to 0R_1, R_1 \to 1R_2, R_2 \to 0R_3,$
$R_2 \to 1R_3, R_2 \to \varepsilon, R_3 \to 0R_3, R_3 \to 1R_3\}, S_1)$