Quiz 1 - Thomas DeMasse

1.) (a) $2^2 = 4$ $= \{4, 81\}$
$9^2 = 81$

(b) $S = \{x \mid x \in N \text{ and } 5 \leq x \leq 25\}$

2.) (a) $A = \{1,2\}$ and $B = \{3,4\}$

$A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$
$B \times A = \{(3,1),(3,2),(4,1),(4,2)\}$
this shows the Cartesian Product is not equal

(b) $P(\{2,3,5\}) = \{\{\},\{2\},\{3\},\{5\},\{2,3\},\{2,5\},$
$\{3,5\},\{2,3,5\}\}$

3.) (a) False because the number of strings that can be formed from the alphabet is infinite.

(b) True because a string is a sequence of characters in the alphabet $\{a,b,c\}$ that is a Finite set. Therefore, each individual string must be a finite # of characters.

(c) False $(ba)^3 = bababa$

(d) True, "abc" occurs consectively in the String a"abc"bc

(e) true because {c, bac, cbd} are a subset of Σ^*

4.) for $xRy \iff x-y$ is divisible by 5 to be an equivalence relation, we have to check if it's reflexive, symmetric, and transitive.

Reflexive: If $x \in \mathbb{Z}$ then $x-x$ is divisible by 5 since $x-x=0$.

Symmetric $x, y \in \mathbb{Z}$ and xRy holds.
$x-y$ is divisible by 5 which means $y-x$ is divisible by 5

Transitive: $x, y, z \in \mathbb{Z}$ and xRy and yRz hold.
Since $x-y$ and $y-z$ are divisible by 5 that means their sum is divisible by 5.

Since the relation is reflexive, symmetric, and transitive then it is an equivalence relation

3.) $\sqrt{3} = a/b$, a rational # can be expressed as a fraction of two integers, with a non-zero denominator

$b \neq 0$ and there are no common factors.

$$\sqrt{3} = \frac{a}{b}$$

$$3 = \left(\frac{a}{b}\right)^2$$

$$3 = \frac{a^2}{b^2}$$

$3b^2 = a^2$, this means a^2 can be divisible by 3. So a is divisible by 3.

$a = 3k$, k is an integer

$3b^2 = (3k)^2$

$\frac{3b^2}{3} = \frac{9k^2}{3}$ → This shows b^2 is divisible by 3

$b^2 = 3k^2$ and b divisible by 3

If $\sqrt{3}$ is a rational number, both a and b must be divisible by 3. But if we assume that $\frac{a}{b}$ is in its simplist form, this is a contradiction.

So $\sqrt{3}$ is not a rational number.