

Thomas DeMasse - HW3

1.) State Stack

$q_1 \rightarrow q_2$ []

$q_2 \rightarrow q_2$ [#]

$q_3 \rightarrow q_3$ [#]

$q_3 \rightarrow q_3$ [#]

$q_3 \rightarrow q_4$ []

q_4

either 2x |

3x |

Rem

The language recognized by this PDA is as follows: In order for the language to work, it must have either twice or triple the amount of 1s as it does 0s.
For ex) 001111 would work
Where 00111 would not

2.) Separate A into 2 Separate languages $\Rightarrow S_1, S_2$
 $A_1 = \{a^i b^j c^k \mid i, j, k \geq 0, i = j\}$

\Rightarrow in this language we must have the same number of a's and b's given the condition $i = j$ for $a^i b^j$

$S_1 \rightarrow S_1 c \mid A \mid \epsilon$
 $A \rightarrow a A b \mid \epsilon$

$A_2 = \{a^i b^j c^k \mid i, j, k \geq 0, j = k\}$

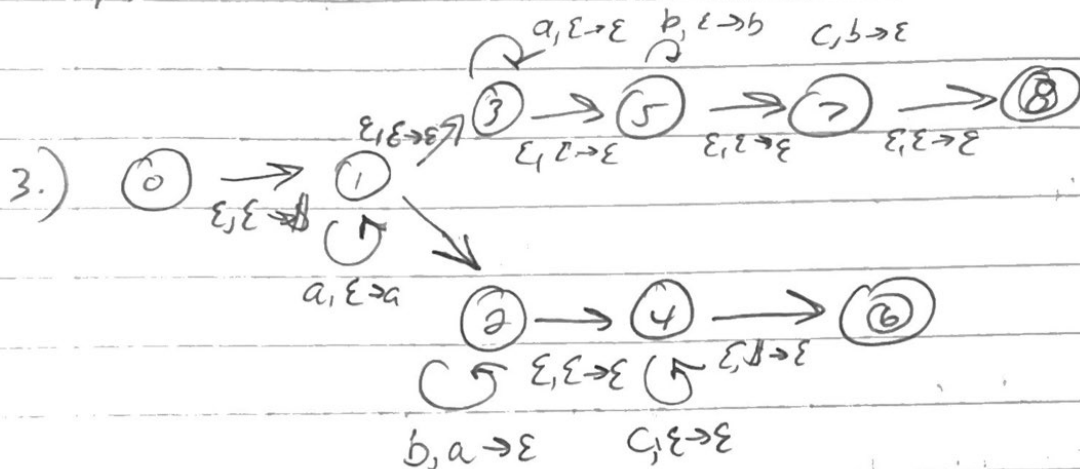
\Rightarrow In this language we must have the same number of b's as c's given the condition

$b^j c^k$ where $j = k$

$S_2 \rightarrow a S_2 \mid B \mid \epsilon$
 $B \rightarrow b B c \mid \epsilon$

The Grammar is ambiguous because each subsequent language derived from either S_1 or S_2 we get the "same" result

$S \rightarrow S_1$ $S \rightarrow S_2$
 $S_1 \rightarrow A$ results in S $S_2 \rightarrow B$ results in S
 $A \rightarrow \epsilon$ $B \rightarrow \epsilon$



The PDA for the language works by reading the a 's first.
 As it reads, when it splits off into 2 branches
 if either branch finds an accepting state, this causes
 the whole machine to accept.

- 4.) Both language A and B are context free languages performing an intersection on the results in a new language which we hope is also context free: this being $L(A \cap B) = L$. Given the fact that L_A has b 's and c 's equal and L_B has a 's and b 's equal, the constraint on the new language must be that all a, b, c are equal.

Using 2.36 from our book, it is proven that Language L is not regular. The intersection of A and B is not closed under intersection. The pumping lemma with pumping constant P can be used.

$$\text{let } S = a^p b^p c^p \Rightarrow |S| \geq p$$

Possible $\forall XY, |S|+1 \leq P$

all either $a's, b's, c's$ and since $c's$ and $b's$ are same $b's$ and $c's$

Condition 1 states that: for each

$$i \geq 0 \cup v^i x y^i z \in A$$

however when $i=0$ $|v^0 x y^0 z| < |S|$

because both v and y cannot be empty, the string will always have a character less than that of our pumping constant P which violates Condition 3 where $|vxy| \leq P$, it's not closed under intersection

5.) We can use the pumping lemma for CFL from the book for simplicity sake I will use 3 as pumping constant P .

$$\begin{aligned} \text{Case 1: } \{b^3\} &= \{b b b b b b\} \text{ let } i = 0 \\ \{v^i x y^i z\} &\rightarrow \{b b^0 b b^0 b b\} \\ &= \{b b b\} \notin L \end{aligned}$$

$$\begin{aligned} \text{Case 2: } \{b^3\} &= \{b b b b b b\} \text{ } i=0 \\ &\quad \underbrace{\quad \quad \quad}_{vxy} \quad \underbrace{\quad \quad \quad}_z \\ \{(b b b)^0 b b b\} &\subseteq \{b b b\} \notin L \end{aligned}$$

Case 3: $\{b^{3^i}\} = \{bbbbbb\}$ $i=0$
 $\{b^0 bbbbb\} = \{bbbbbb\} \notin L$

In any case such that $P=3$ and $i=\emptyset$, there will not exist a case that will generate a string that is an element of L because it is a factorial expression such that $k \geq 0: 1, 1, 2, 6, 24, 120 \dots k^i$

There is no possible way to arrange $uvxy^2$ to produce a string of $P+1$. The cases prove this where b^P will always be less than $b^{(P+1)}$