ECEN 4843 HW 4 Thomas Kidd

Question 1:

The news media has shown the astronauts placing laser retroreflectors on the moon. Use the expansion law for Gaussian beams to predict the diameter of a laser beam when it hits the moon. In the textbook we are also given that $\lambda_0 = 6943\AA$ Consider the two cases:

(a) A laser rod of 2cm diameter

The smaller the starting diameter of the beam, the larger the spread of the beam will be as we increase the distance the path has traveled. We also know that $\omega_0 = \frac{1}{2}\text{diameter} = 1cm$. Based on NASA: https://spaceplace.nasa.gov/moon-distance/en/ the moon is **384,400 km** away from Earth on average. We can then do some calculations:

First lets find z_0 where z_0 is the distance where the beam radius increases by $\sqrt{2}$

$$z_0 = \frac{\pi n \omega_0^2}{\lambda_0} = \frac{\pi * 1 * 1cm^2}{0.00006943cm} = 45248.345cm$$

To find the actual beam width at the distance of the moon z we can use the following equation:

$$\omega^2(z) = \omega_0^2 \left[1 + \left(\frac{z}{z_0}\right)^2\right]$$

This results in (for a large z like this we can also ignore the $+1$):

$$\omega^2(z) = 1cm^2 \left[1 + \left(\frac{3.844 * 10^{10}cm}{45248.345cm}\right)^2\right] = 7.217 * 10^{11}cm^2$$

$$\omega(3.844 * 10^{10}cm) = \sqrt{7.217 * 10^{11}cm^2} = 8.495 * 10^5cm$$

(b) This same laser sent through a telescope backward such that the beam starts with a diameter of 2 meters

This beam has a much larger diameter so we should see a smaller beam width at the moon than that of the previous laser. We also know that $\omega_0 = \frac{1}{2}\text{diameter} = 100cm$. Based on NASA: https://spaceplace.nasa.gov/moon-distance/en/ the moon is **384,400 km** away from Earth on average. We can then do some calculations:

First lets find z_0 where z_0 is the distance where the beam radius increases by $\sqrt{2}$

$$z_0 = \frac{\pi n \omega_0^2}{\lambda_0} = \frac{\pi * 1 * (100cm)^2}{0.00006943cm} = 4.525 * 10^8cm$$

To find the actual beam width at the distance of the moon z we can use the following equation:

$$\omega^2(z) = \omega_0^2 \left[1 + \left(\frac{z}{z_0}\right)^2\right]$$

This results in (for a large z like this we can also ignore the $+1$):

$$\omega^2(z) = (100cm)^2 \left[1 + \left(\frac{3.844 * 10^{10}cm}{4.525 * 10^8cm}\right)^2\right] = 7.217 * 10^7cm^2$$

$$\omega(3.844 * 10^{10}cm) = \sqrt{7.217 * 10^7cm^2} = 8495cm$$

(c) Eye damage intensities are in the range of 10 $\mu W / cm2$. If the laser on earth produced a pulse power of 10MW, was there danger to the astronauts from the optical radiation.

Now we can calculate the intensity of the beam at t this new distance. In order to find the intensity of the wave we must find the power of the wave. Because we are assuming that this is a Gaussian beam we can use the following formula for calculating the intensity

$$I_p = \frac{P}{\pi \frac{\omega^2}{2}}$$

For the laser with a radius of 1cm and a power of 10MW we get the following intensity

$$I_p = \frac{10MW}{\pi \frac{7.217 * 10^{11} cm^2}{2}} = 8.821 * 10^{-6} \frac{W}{cm^2}$$

This shows us that the intensity from problem (a) is $8.821 \frac{\mu W}{cm^2} < 10 \frac{\mu W}{cm^2}$ is safe.

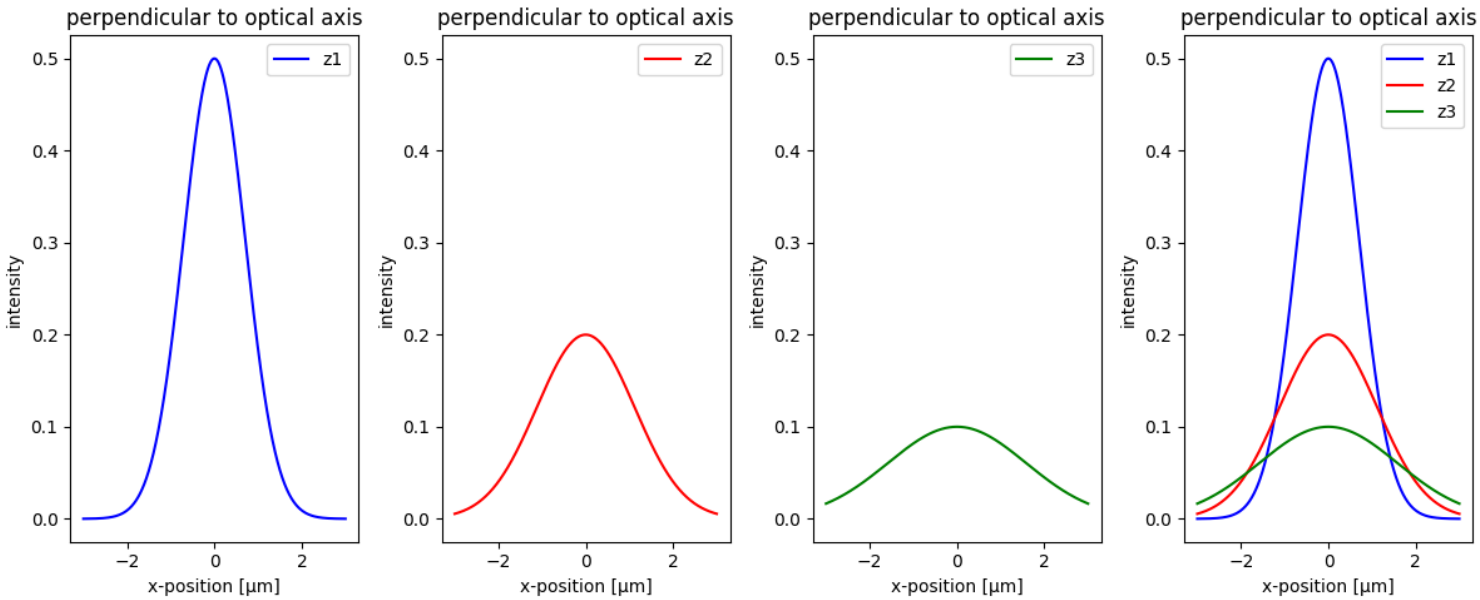
For the laser with a radius of 100cm and a power of 10MW we get the following intensity

$$I_p = \frac{10MW}{\pi \frac{7.217 * 10^{7} cm^2}{2}} = 0.0882 \frac{W}{cm^2}$$

This shows us that the intensity from problem (a) is $0.0882 \frac{W}{cm^2} > 10 \frac{\mu W}{cm^2}$ is NOT safe.

Question 2

We were able to calculate that $z_0 = 6.2831$ and thus with some help of python we were able to plot the following graphs:



```
import numpy as np

import matplotlib.pyplot as plt

from time import sleep,time

import matplotlib as mpl

import matplotlib.cm as cm


# declaring variables


n = 1

lamda_0 = 500*10**(-9)

omega_0 = 1*10**(-3)


wavelength=lamda_0

k0=2*np.pi/wavelength

c=299792458

omega0=k0*c


z_0 = np.pi*n*(np.power(omega_0,2))/lamda_0


print(z_0)


# at each z_0 we drop the radius or beam width by a square root of 2


z_1 = 1*z_0

z_2 = 2*z_0

z_3 = 3*z_0


# calculating intensity


def gaussian_beam(k,omega, x,y,z,zr,w0,t):

    rho2=x**2+y**2

    w=w0*np.sqrt(1+(z/zr)**2)

    R=z*(1+(zr/z)**2)
```

```
    psi=np.arctan(z/zr)

    E=w0*np.exp(-rho2/w**2)*np.exp(1j*(k*z+k*rho2/(2*R)-psi))*np.exp(-1j*omega*t)/w

    return(E)


x=np.linspace(-3e-6,3e-6,1000)

z=np.linspace(-10e-6,10e-6,1000)

y=np.linspace(0,0,1000)



# field_1=gaussian_beam(k0,omega0,x,y,z,z_1,1e-6,0)

# field_2=gaussian_beam(k0,omega0,x,y,z,z_2,1e-6,0)

# field_3=gaussian_beam(k0,omega0,x,y,z,z_3,1e-6,0)


field_1=gaussian_beam(k0,omega0,x,y,z_1,z_0,1e-6,0)

field_2=gaussian_beam(k0,omega0,x,y,z_2,z_0,1e-6,0)

field_3=gaussian_beam(k0,omega0,x,y,z_3,z_0,1e-6,0)


extent = np.min(z)*1e6, np.max(z)*1e6,np.min(x)*1e6, np.max(x)*1e6



plt.figure(figsize=(12,5))


ax1 = plt.subplot(141)

plt.plot(x*1e6,np.abs(field_1[:1000])**2, color = "Blue", label = "z1")

plt.xlabel('x-position [μm]')

plt.ylabel('intensity')

plt.title('perpendicular to optical axis')

plt.tight_layout()

plt.legend()


ax2 = plt.subplot(142, sharey=ax1, sharex=ax1)

plt.plot(x*1e6,np.abs(field_2[:1000])**2, color = "Red", label = "z2")

plt.xlabel('x-position [μm]')

plt.ylabel('intensity')
```

```
plt.title('perpendicular to optical axis')

plt.tight_layout()

plt.legend()


ax3 = plt.subplot(143, sharey=ax1, sharex=ax1)

plt.plot(x*1e6,np.abs(field_3[:1000])**2, color = "Green", label = "z3")

plt.xlabel('x-position [µm]')

plt.ylabel('intensity')

plt.title('perpendicular to optical axis')

plt.tight_layout()

plt.legend()


ax4 = plt.subplot(144, sharey=ax1, sharex=ax1)

plt.plot(x*1e6,np.abs(field_1[:1000])**2, color = "Blue", label = "z1")

plt.plot(x*1e6,np.abs(field_2[:1000])**2, color = "Red", label = "z2")

plt.plot(x*1e6,np.abs(field_3[:1000])**2, color = "Green", label = "z3")

plt.xlabel('x-position [µm]')

plt.ylabel('intensity')

plt.title('perpendicular to optical axis')


plt.legend()

plt.show()
```