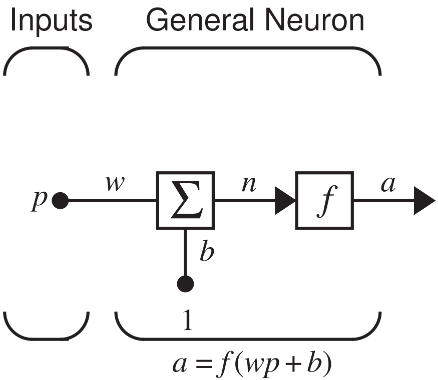
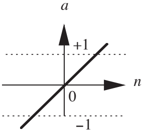
Multilayer Networks

Deep Learning

Basic network building block (neuron)

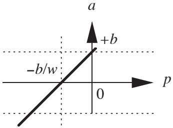


Transfer (activation) functions (1)



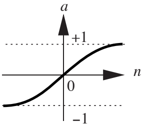
$a = purelin(n)$

Linear Function



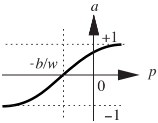
$a = purelin(wp+b)$

Single-Input *purelin* Neuron



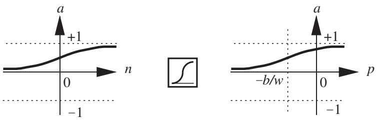
$a = tansig(n)$

Tan-Sigmoid Function



$a = tansig(wp + b)$

Single-Input *tansig* Neuron

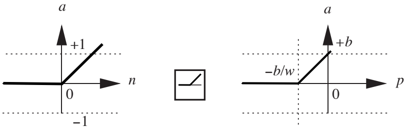


$a = logsig(n)$

Log-Sigmoid Transfer Function

$a = logsig(wp+b)$

Single-Input *logsig* Neuron



$a = poslin(n)$

Positive Linear Function

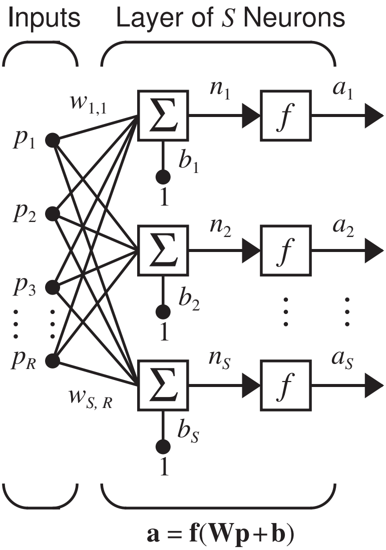
$a = poslin(wp + b)$

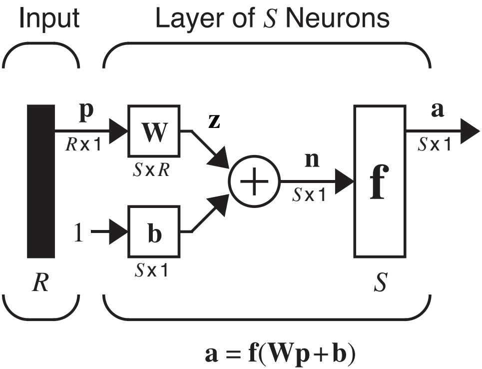
Single-Input *poslin* Neuron

Softmax

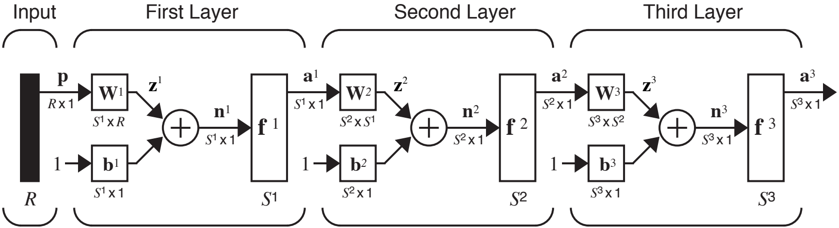
$$a_i = f_i(\mathbf{n}) = \frac{e^{n_i}}{\sum_{j=1}^{S} e^{n_j}}$$

Used at the output layer of a pattern recognition network with multiple output neurons.


Inputs
Layer of S Neurons
$w_{1,1}$
p_1
p_2
p_3
p_R
$w_{S,R}$
Σ
n_1
n_2
n_S
f
a_1
a_2
a_S
b_1
b_2
b_S
1
$\mathbf{a} = \mathbf{f}(\mathbf{Wp}+\mathbf{b})$

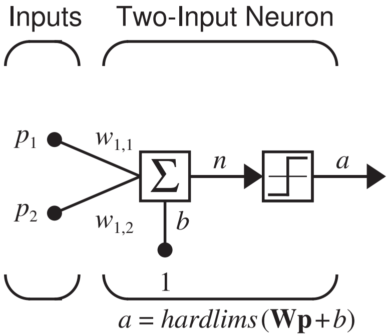

Input
Layer of S Neurons
p
R x 1
W
S x R
z
1
b
S x 1
n
S x 1
f
a
S x 1
R
S
a = f(Wp + b)

Multiple layer network



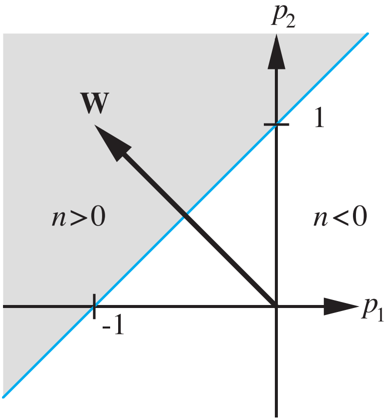
$\mathbf{a}^1 = \mathbf{f}^1(\mathbf{W}^1\mathbf{p}+\mathbf{b}^1)$ $\quad$ $\mathbf{a}^2 = \mathbf{f}^2(\mathbf{W}^2\mathbf{a}^1+\mathbf{b}^2)$ $\quad$ $\mathbf{a}^3 = \mathbf{f}^3(\mathbf{W}^3\mathbf{a}^2+\mathbf{b}^3)$

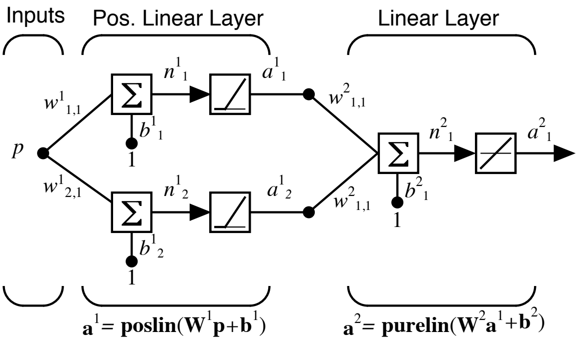
$$\mathbf{a}^3 = \mathbf{f}^3(\mathbf{W}^3\mathbf{f}^2(\mathbf{W}^2\mathbf{f}^1(\mathbf{W}^1\mathbf{p}+\mathbf{b}^1)+\mathbf{b}^2)+\mathbf{b}^3)$$



Decision boundary

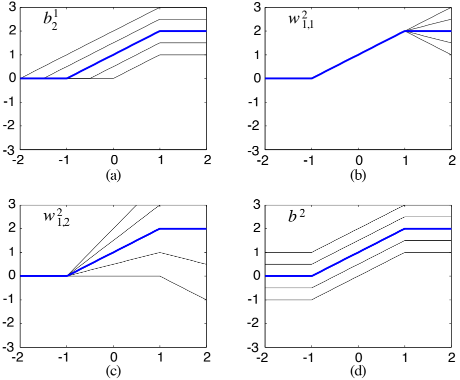
$$n = \mathbf{Wp} + b == 0$$

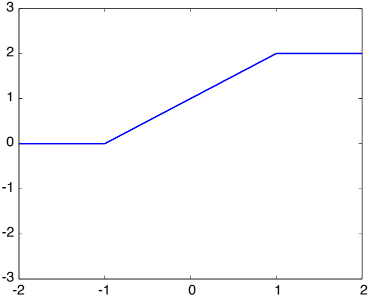


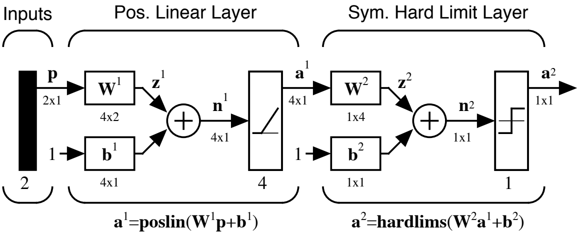

Inputs
Pos. Linear Layer
Linear Layer
p
w1 1,1
w1 2,1
n1 1
n1 2
a1 1
a1 2
b1 1
b1 2
1
1
w2 1,1
w2 1,1
n2 1
a2 1
b2 1
1
a1 = poslin(W1p+b1)
a2 = purelin(W2a1+b2)

$$\mathbf{W}^1 = \begin{bmatrix}1 & 1\end{bmatrix}^T, \mathbf{b}^1 = \begin{bmatrix}-1 & 1\end{bmatrix}^T$$

$$\mathbf{W}^2 = \begin{bmatrix}-1 & 1\end{bmatrix}, \mathbf{b}^2 = \begin{bmatrix}0\end{bmatrix}$$



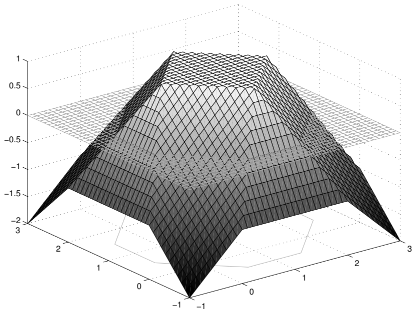


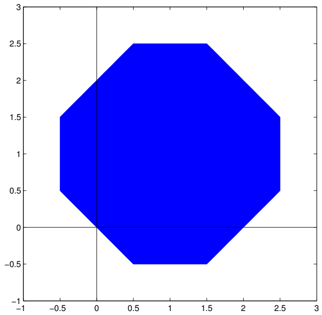


$$\mathbf{W}^1 = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}^T, \mathbf{b}^1 = \begin{bmatrix} -1 & 3 & 1 & 1 \end{bmatrix}^T$$

$$\mathbf{W}^2 = \begin{bmatrix} -1 & -1 & -1 & -1 \end{bmatrix}, \mathbf{b}^2 = \begin{bmatrix} 5 \end{bmatrix}$$

2D Poslin network surface and decision boundary





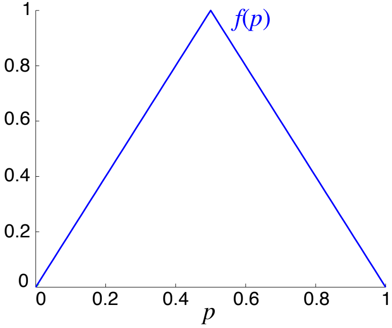
Why do we need deep networks?

- Theoretically, two-layer networks (with sufficient hidden neurons) can approximate any practical function or decision region.
- In practice, deep networks have been able to solve problems that two-layer networks were not previously able to.
- The theoretical explanation of this is not complete.
- We will describe a couple of initial explanations.
- The idea is that deeper networks can produce more efficient realizations of some complex functions than two layer networks can.

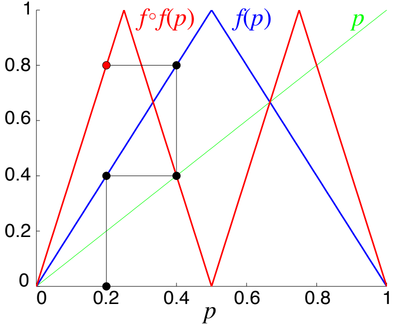
- Consider again the earlier two-layer network, but change the weights.

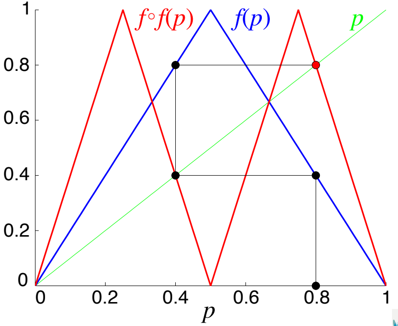
$$\mathbf{W}^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{b}^1 = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}, \mathbf{W}^2 = \begin{bmatrix} 2 & -4 \end{bmatrix}, \mathbf{b}^2 = \begin{bmatrix} 0 \end{bmatrix}$$

- We get the following function.



- Now cascade the function with itself: $f \circ f(p) = f(f(p))$.
- We get the following function (in red).
- By adding a single layer we have doubled the complexity.

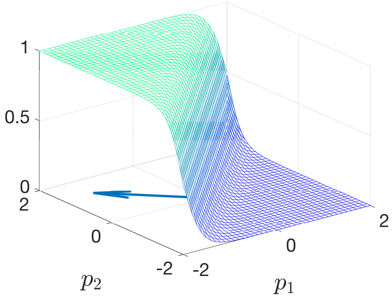




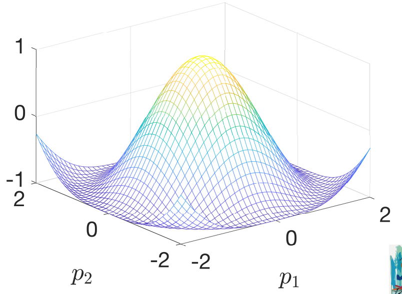
Cascading layers (3)

- Each time we add a layer (a linear increase in the number of neurons), we double the number of neurons that would be required for a two layer network to implement the same function.
- Two-layer networks can create any function that can be created with a deep network.
- The number of neurons required in the two-layer network increase geometrically.
- The number of neurons increase linearly in the equivalent deep network.

- A sigmoid neuron response does not change orthogonal to the weight direction, which creates a ridge function.
- It is difficult to approximate a radial function with a sum of ridges (two-layer network).
- With the addition of a third layer, a 2-4-2-1 network can easily make an accurate approximation.



Sigmoid Neuron Response



Radial Function