

# Homework 2

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CS331 Algorithms and Complexity

**Problem Q1(a).** What is the number of topological orderings in this directed graph?

**Solution:**  $2^{n+1}$

**Problem Q1(b).** Let  $T$  be a tree such that every node in  $T$  has either 2 children or 0 children. If  $T$  has  $n \geq 1$  leaves, prove that the total number of nodes in  $T$  is  $2n - 1$ .

**Problem Q2.** Let  $s$  be the vertex of a connected undirected graph  $G$ . Let  $T_{G,s}^B$  and  $T_{G,s}^D$  respectively be the trees obtained by running BFS and DFS on graph  $G$  starting at node  $s$ . Prove

$$T_{G,s}^B \equiv T_{G,s}^D \implies G \text{ is acyclic}$$

**Problem Q3.** Given  $n$  images and  $m$  unambiguous matches, design an algorithm that runs in  $O(m+n)$  time and uniquely labels  $n$  images as either A or B, such that two images reported to be the same by ImgComp get the same label, and two images reported to be different by ImgComp get different labels.

**Problem Q4.**