Homework 3

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CS331 Algorithms and Complexity

Problem Q1. Prove each of the following heuristics to be ideal or not ideal for scheduling jobs on a machine while minimizing:

$$\sum_{i=1}^{n} w_i C_i$$

1. Smallest time first.
2. Most important first.
3. Maximum $\frac{w(i)}{t(i)}$.

Problem Q2(a). Briefly explain which steps of Dijkstra's algorithm fail when a graph can have negative weight edges.

Problem Q2(b). Given a set of 4-tuples (s_i, d_i, p_i, q_i), where each entry corresponds to source, destination, departure time, arrival time respectively, calculate the plan that arrives at d_i as early as possible, while allowing for at least 1 hour for each connecting flight.

Problem Q3(a). Give an algorithm to find the minimum-product spanning tree.

Let $G(V, E)$ be the graph in question
for $\forall e \in E$ **do**
    $weight(e) = ln(weight(e))$
end for
Run Prim's algorithm on the new graph G'

Problem Q3(b). Prove that if the weights of an undirected graph are unique, there exists a unique minimum spanning tree.

Problem Q3(c). Let $\mathcal{T}_G$ be the set of minimum spanning trees for some graph G. Let T be a spanning tree for G. Prove that:

$$\forall e \in T, e \in T' \in \mathcal{T}_G \implies T \in \mathcal{T}_G$$