Homework 4

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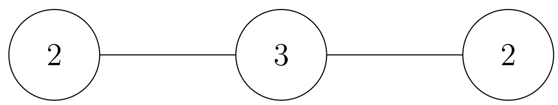
CS331 Algorithms and Complexity

Problem Q1. Given n samples and unambiguous ImgComp, design a divide and conquer algorithm which can report whether more than 50% of samples belong to a single class. Prove correctness and runtime.

Problem Q2.

Problem Q3(a).



In this example, the middle node will be chosen and the two side nodes discarded for a total weight of 3.
If the middle node was discarded and the two side nodes chosen, the total weight would have been $4 > 3$.

Problem Q3(b).

In this example, the maximum weight independent set includes nodes v_1 and v_4, but 1 is odd and 4 is even.

Problem Q3(c). Let the set of independent vertices $S_i \subset \{v_1, v_2, v_3, \ldots, v_i\}, i \leq n$ have some weight W_i
Let each vertex v_i have some weight w_i
Suppose S_i includes vertex v_i. S_i can therefore cannot include v_{i-1}, so $W_i = W_{i-2} + w_i$
Suppose S_i does not include v_i. $W_i = W_{i-1}$
Base case(s): $W_0 = 0, W_1 = w_1$
Algorithm: $W_i = max(W_{i-1}, W_{i-2} + w_i)$
return W_n
Memoization: W_j only has to be computed once for each unique j.
Proof of correctness:

Proof. Suppose W_n is not the maximum weight.
This means either W_{i-1} or W_{i-2} wasn't maximized.
This continues, with some W_{i-k} not being maximized.
However, W_0 and W_1 are always maximized by definition.
Thus, $W_{i-k}, k = i$ must be maximized.
This contradicts the original assumption, since each step is guaranteed to maximize the weight if the previous step is maximized.

□

Proof of runtime:

Proof. $W_0, W_1, \ldots, W_n$ will all be calculated exactly once due to memoization.
The time required to calculate W_i is $O(1)$, since it is a single comparison and a single addition.
Therefore, the total complexity is $O(n)$.

$\square$