Homework 7

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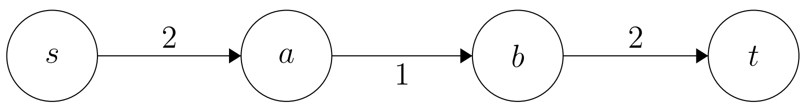
CS331 Algorithms and Complexity

Problem Q1(a). Prove or disprove
Claim: Consider an implementation of Ford-Fulkerson algorithm which does not create any backward edges in the residual graph. It is claimed that, there exists a constant $\alpha > 0$, such that for any flow network G, this modified implementation is guaranteed to find a flow of value at least α times the maximum-flow value in G

Problem Q1(b). Consider a network $G = (V, E)$ with a source s, a sink t, and a capacity c_e on every edge $e \in E$.
Claim: if f is a maximum $s - t$ flow in G, then f either saturates every edge out of s or saturates every edge into t.
False

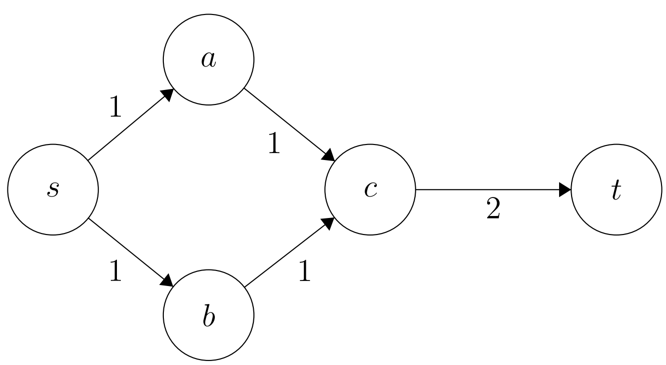


Proof.
The min cut is $A = \{s, a\}, B = \{b, t\}$. The value of this cut is 1, which implies the max flow $f = 1$. This does not saturate the edge leaving s or the edge entering t.

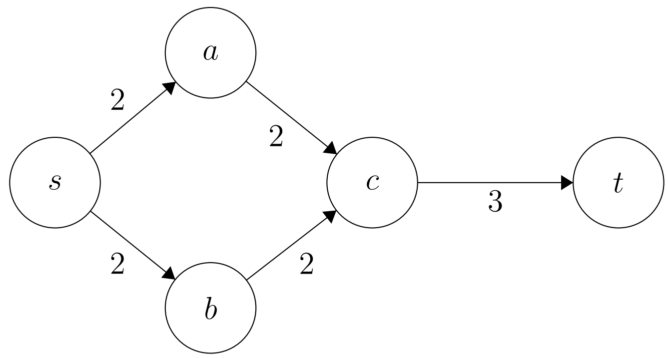
□

Problem Q1(c). Consider a network $G = (V, E)$ with a source s, a sink t, and a capacity c_e on every edge $e \in E$. Let A, B be a minimum $s - t$ cut with respect to the capacities c_e.
Claim: If we add 1 to every capacity, namely $\forall e \in E, c'_e = c_e + 1 \implies A, B$ is still a minimum $s - t$ cut with respect to c'_e
False

Proof. Let G be the graph as pictured below:



Note that a min cut in G is $A = \{s, a, b\}, B = \{c, t\}$ with a value of 2
Now consider G'



Note that the cut $A = \{s, a, b\}, B = \{c, t\}$ now has a value of 4, whereas the cut $A' = \{s, a, b, c\}, B' = \{t\}$ has a value of 3

$\square$

Problem Q2(a).

Problem Q2(b).