MSIM 441/541 & ECE 406/506
Computer Graphics & Visualization

Homework Seven

Assigned November 5, Due 12:00 PM November 12
Thomas Laverghetta

Overview

This homework covers the second half of the lecture on Chapter 4: Geometric Objects and Transformations. Please only submit one single file that includes solutions to the tasks specified below.

Tasks

1. Show that translations are commutative. That is, if matrix $\mathbf{T}(d_1)$ represents a translation by d_1 and matrix $\mathbf{T}(d_2)$ represents a translation by d_2, then $\mathbf{T}(d_1)\mathbf{T}(d_2) = \mathbf{T}(d_2)\mathbf{T}(d_1)$.

Since, $T(d_i)$ will be same data structure no matter d_i selected and given dot-product operational rules. Then $T(d_1)T(d_2) = T(d_1 + d_2)$. Therefore, order of operations does not matter.

$$T(d_i) = \begin{bmatrix} 1 & 0 & \dots & 0 & a_1 \\ 0 & 1 & & 0 & a_2 \\ \vdots & & \ddots & & \vdots \\ & & & 1 & a_n \\ 0 & & \dots & 0 & 1 \end{bmatrix}$$

$$\therefore d_1 = [b_1 \quad b_2 \quad \dots \quad b_n], d_2 = [c_1 \quad c_2 \quad \dots \quad c_n]$$

$$\rightarrow T(d_1)T(d_2) = T(d_2)T(d_1) = T(d_1 + d_2) = \begin{bmatrix} 1 & 0 & \dots & 0 & b_1 + c_1 \\ 0 & 1 & & 0 & b_2 + c_2 \\ \vdots & & \ddots & & \vdots \\ & & & 1 & b_n + c_n \\ 0 & & \dots & 0 & 1 \end{bmatrix}$$

2. Show that scaling transformations are commutative. That is, if matrix $\mathbf{S}(\alpha_1,\alpha_2,\alpha_3)$ presents a scaling transformation and matrix $\mathbf{S}(\beta_1,\beta_2,\beta_3)$ represents another scaling transformation, then $\mathbf{S}(\alpha_1,\alpha_2,\alpha_3)\mathbf{S}(\beta_1,\beta_2,\beta_3)=\mathbf{S}(\beta_1,\beta_2,\beta_3)\mathbf{S}(\alpha_1,\alpha_2,\alpha_3)$.

Since, $S(v_1,v_2,v_3)$ will be same data structure no matter (v_1,v_2,v_3) selected and given dot-product operational rules. Then $S(\alpha_1,\alpha_2,\alpha_3)S(\beta_1,\beta_2,\beta_3) = S(\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3)$. Therefore, order of operations does not matter.

$$S(v_1,v_2,v_3) = \begin{bmatrix} v_1 & 0 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & 0 & v_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore S(\alpha_1,\alpha_2,\alpha_3)S(\beta_1,\beta_2,\beta_3) = S(\beta_1,\beta_2,\beta_3)S(\alpha_1,\alpha_2,\alpha_3) = \begin{bmatrix} \alpha_1\beta_1 & 0 & 0 & 0 \\ 0 & \alpha_2\beta_2 & 0 & 0 \\ 0 & 0 & \alpha_3\beta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Calculate the 4×4 matrices corresponding to the following OpenGL commands.
   1) glTranslate3f(5, 10, 5);
      i. $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
   2) glRotatef(45, 1, 0, 0);
      i. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(45) & -\sin(45) & 0 \\ 0 & \sin(45) & \cos(45) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
4. Calculate the 4×4 matrices corresponding to the following transformations (one matrix for each sub-task).
   1) A translation of (2, 0, 2) followed by a rotation of 90° about y axis.
      i. $\vec{p}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
   2) A rotation of 90° about y axis followed by a translation of (3, 0, 3).
      i. $\vec{p}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 0 & 3 & 1 \end{bmatrix}$

3) What conclusion can you draw from the results in 1) and 2)?
    i. It is easy to stack operations in the order you want to preform them to get desired end rotations or translations.

5. Reading materials: [Angel08] Chapter 4 and OpenGL red book Chapter 3 except Advanced sections.