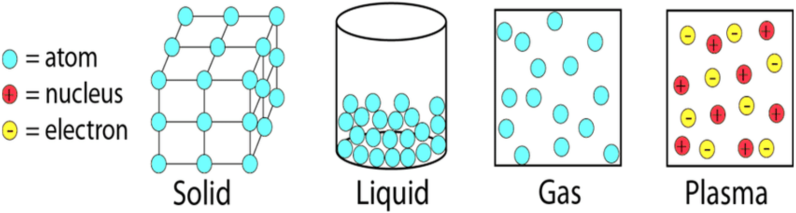
# Simulations in AC Tokamak Ramp Downs

Tom Malcolm

(The talk edition)

Nov. 2023

# What is plasma?



# What is fusion?

- Abundant process in nature - see stars
- Large amounts of energy released
- Occurs most easily in plasma

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \overset{\text{(fusion)}}{\rightarrow} {}^{4}\mathrm{He} + n + 17.59\ \mathrm{MeV}$$

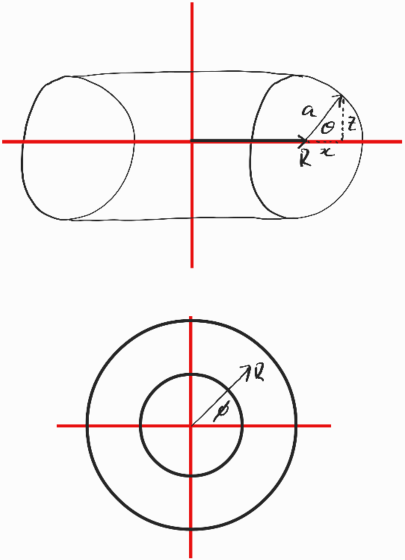
# What is a fusion reactor?

**What is it?**

- Controlled fusion for power generation
- Axisymmetric donut (spherical cow)

**Challenges**

- Confinement
- Runaway Electrons
- Sustaining the reaction



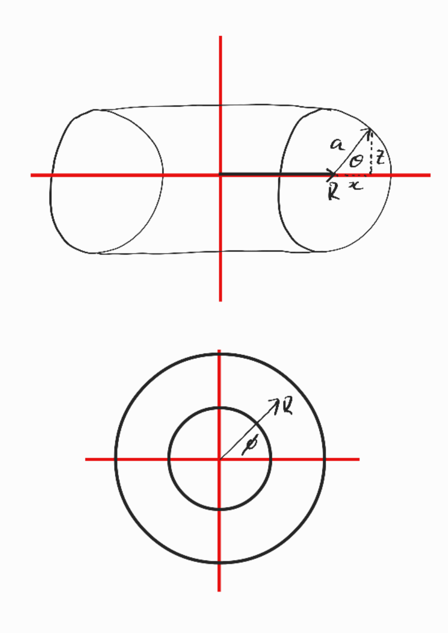
# What is a fusion reactor?

**What is it?**

- Controlled fusion for power generation
- Axisymmetric donut (spherical cow)

**Challenges**

- Confinement
- Runaway Electrons
- Sustaining the reaction



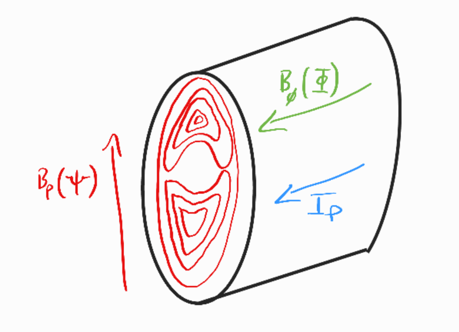
# What is a fusion reactor?

**What is it?**

- Controlled fusion for power generation
- Axisymmetric donut (spherical cow)

**Challenges**

- Confinement
- Runaway Electrons
- Sustaining the reaction



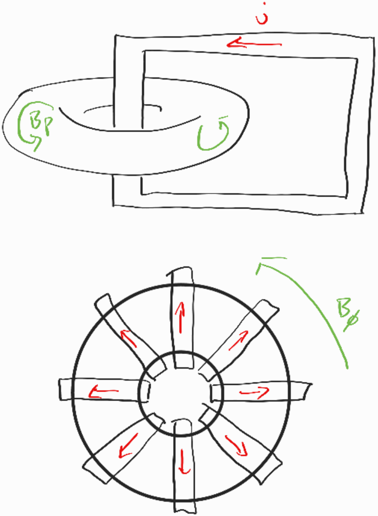
# What is an AC Tokamak?

**What is it?**

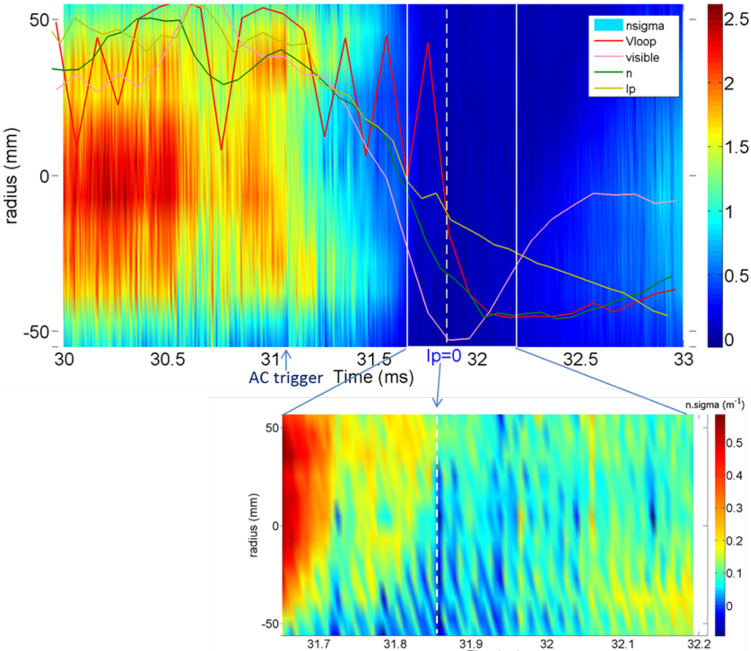
- Plasma current oscillates back and forth (AC)
- ISTTOK reactor (Artur in Portugal)

**Challenges?**

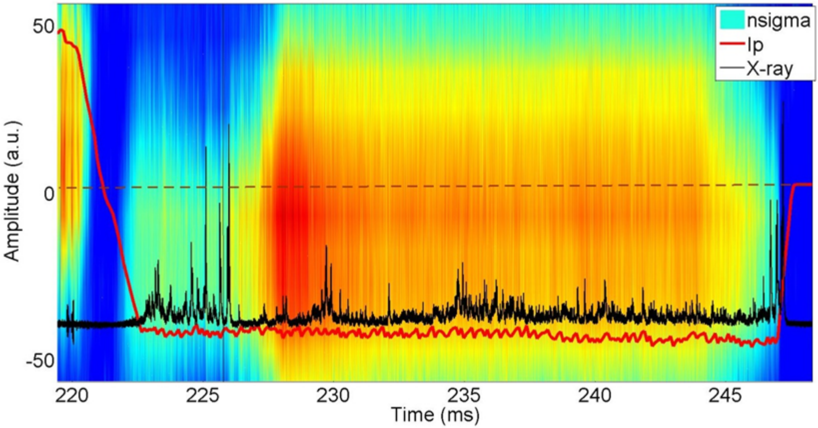
- Residual current density at $I_p = 0$
- Runaway electron population increase



# So what's the problem?



# So what's the problem?



# Modelling Plasma

**Fluid Dynamics**

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{u})$$

$$\rho \frac{\mathrm{D}}{\mathrm{D}t} \vec{u} = \vec{F}$$

**Maxwell's Equations**

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{E} = \frac{\rho_c}{\epsilon_0}$$

**? MHD Equations ?**

# Modelling Plasma

## MHD Equations

$$\frac{\mathrm{D}}{\mathrm{D}t}\rho = \rho\nabla\cdot\vec{u}$$

$$\rho\frac{\mathrm{D}}{\mathrm{D}\vec{u}} = -\nabla p + \vec{j}\times\vec{B} + \nabla\cdot\vec{\Pi}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}p = -\gamma p\nabla\cdot\vec{u} + (\gamma - 1)\left[-\nabla\cdot\vec{q} + \vec{\Pi} : \nabla\vec{u} + \eta j^2\right]$$

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\vec{E}$$

$$\mu_0\vec{j} = \nabla\times\vec{B}$$

$$\vec{E} + \vec{u}\times\vec{B} = \eta\vec{j}$$

# Modelling Plasma

**Too cumbersome... Ideal MHD!**

- Resistivity is negligible
- No external source of heat / energy
- No extra source of pressure (e.g. gravitational)

$$\frac{\mathrm{D}}{\mathrm{D}t}\rho = \rho \nabla \cdot \vec{u}$$
$$\rho \frac{\mathrm{D}}{\mathrm{D}\vec{u}} = -\nabla p + \vec{j} \times \vec{B}$$
$$\frac{\mathrm{D}}{\mathrm{D}t} p = -\gamma p \nabla \cdot \vec{u}$$
$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$
$$\mu_0 \vec{j} = \nabla \times \vec{B}$$
$$\vec{E} + \vec{u} \times \vec{B} = 0$$

# Grad-Shafranov Equation

**Assumptions**

- Ideal MHD
- Axisymmetry
- Magnetic field topology fixed with respect to fluid (frozen-in flux condition)

$$x\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial \Psi}{\partial x}\right)+\frac{\partial^2 \Psi}{\partial z^2}=-\mu_0 x^2\frac{\partial p}{\partial \Psi}-\mu_0^2 f(\Psi)\frac{\partial f}{\partial \Psi}$$

$f(\Psi)$ is poloidal current density flux function; $p(\Psi)$ is pressure density flux function; and $\Psi(x,z)$ is the poloidal magnetic flux function.

# Grad-Shafranov-Helmholtz Equation

**Notes**

- Introduced by Wang
- Allows for current reversals
- Introduces $(a_1, a_2, \alpha)$

$$\left(x\frac{\partial}{\partial x}\frac{1}{x}\frac{\partial}{\partial x}+\frac{\partial^2}{\partial z^2}\right)\psi = -\frac{1}{2}x^2\frac{\partial\beta}{\partial\psi}-\frac{1}{2}\frac{\partial g^2}{\partial\psi} = -xj_\phi$$

$$-\frac{1}{2}\frac{\partial\beta}{\partial\psi} = a_1$$

$$-\frac{1}{2}\frac{\partial g^2}{\partial\psi} = -a_2-\alpha^2\psi(x,z)$$

Oh also, has analytic solution! Boils down from an eigenvalue problem.

# Grad-Shafranov-Helmholtz Equation

**Solution**

$$\psi(x,z) = x \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} \frac{(-1)^l 2a_n^u}{k v_l a_n^d (\alpha^2 - \lambda_{n,l}^2)} \left[c_n J_1(\mu_n x) + N_1(\mu_n x)\right] \cos(v_l z)$$

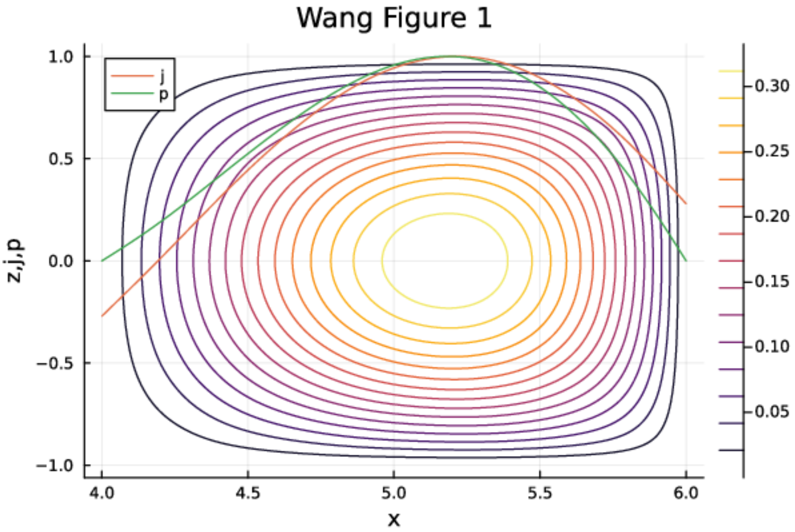
to go alongside

$$\beta(x,z) = \beta_0 - 2a_1 \psi(x,z)$$
$$j_\phi(x,z) = -a_1 x + \frac{1}{x} a_2 + \frac{\alpha^2}{x} \psi(x,z)$$

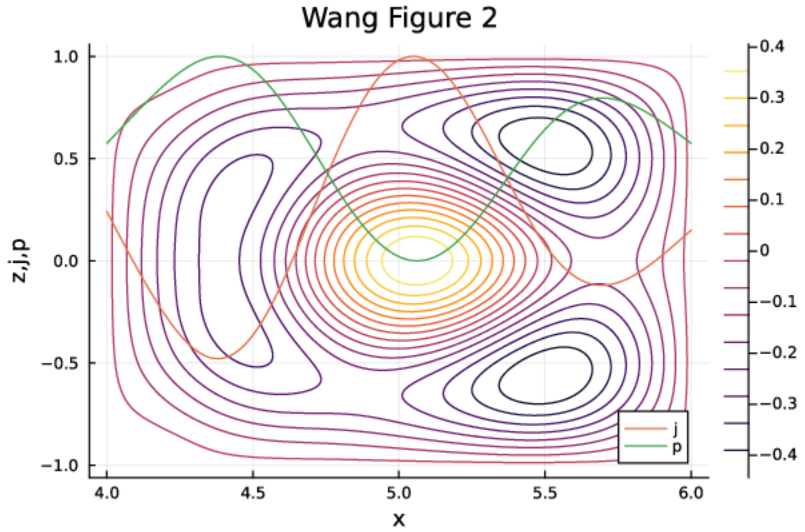
Now have:

- Poloidal magnetic flux
- Current density
- Pressure density

# Wang Reproduction



# Wang Reproduction



# Awesome

Achievement get: reproduce figures

Achievement not (yet) getted: animate figures

# Awesome

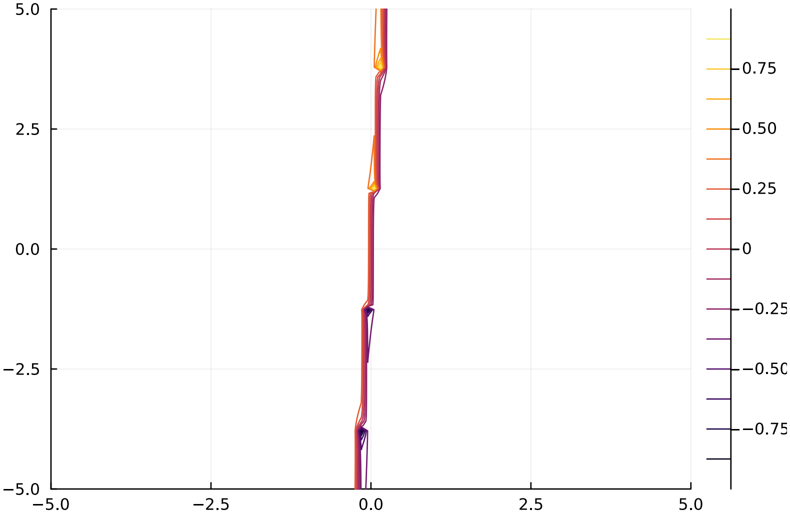
Achievement get: reproduce figures

Achievement not (yet) getted: animate figures

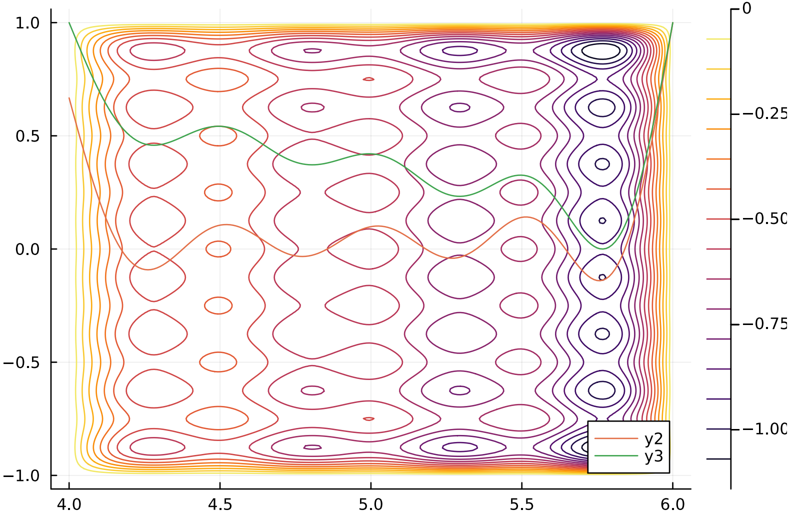
**Data Fitting**

- Currently just specify $(a_1, a_2, \alpha)$
- Want to input data and get those values
- Least squares data fitting time

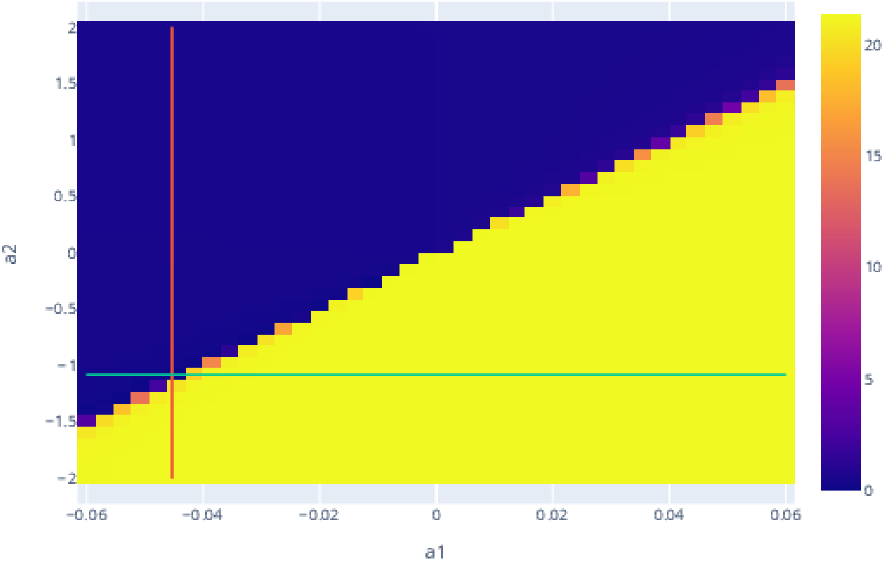
# Uh oh



# Uh oh



# What's going on?



# How fix?

- Include more data (using current density... could add pressure?)
- Use a different model?
- Use a different optimisation method?
- Modify our optimisation approach?

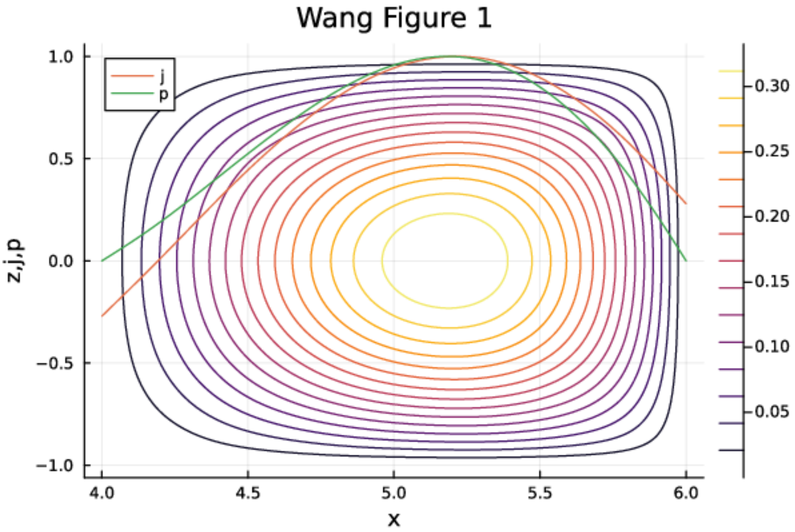
# How fix?

- Include more data (using current density... could add pressure?)
- Use a different model?
- Use a different optimisation method?
- Modify our optimisation approach?

**Now:**

- Provide initial guess which is "close to" actual $(a_1, a_2, \alpha)$
- Works! (we've just replicated the Wang figure again, but with data instead)

# Simulated



Wang Figure 1
j
p
z,j,p
x
1.0
0.5
0.0
−0.5
−1.0
4.0
4.5
5.0
5.5
6.0
0.30
0.25
0.20
0.15
0.10
0.05

# Current Inversion

**Goal:**

- Simulate quiescent phase of current reversal
- Observe magnetic field topology changes
- Pressure density effects
- Do this using data (foreshadowing ISTTOK)

**How:**

- Solve for $(a_1, a_2, \alpha)$ in successive equilibrium time slices
- Use solution of one to feed in as initial guess for next
- Question: what do we use as a guess for first time slice?

## Simulated Reversal

If you're seeing this, something has gone wrong... General Kenobi.
DONT FORGET TO PUT ANIMATION HERE

# Simulated Reversal

If you're seeing this, something has gone wrong... General Kenobi.
DONT FORGET TO PUT ANIMATION HERE

# Simulated Quiescent Reversal

If you're seeing this, something has gone wrong... General Kenobi.
DONT FORGET TO PUT ANIMATION HERE

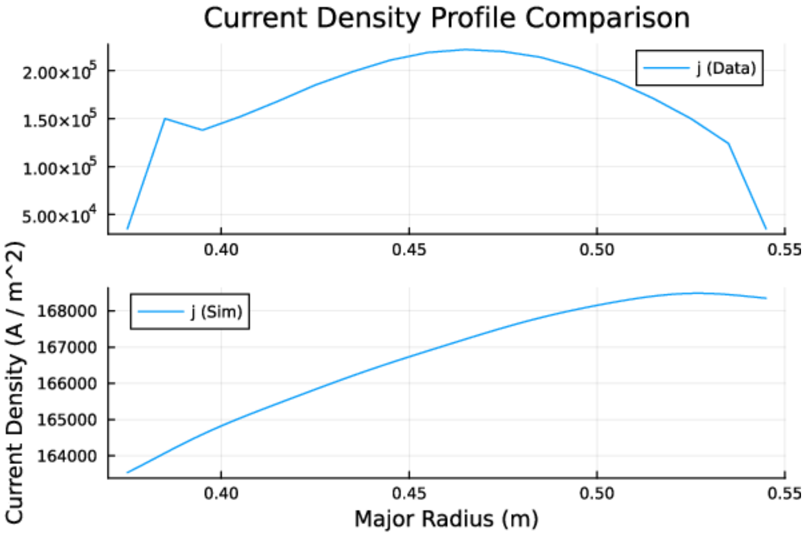
# Simulated Quiescent Reversal

If you're seeing this, something has gone wrong... General Kenobi.
DONT FORGET TO PUT ANIMATION HERE

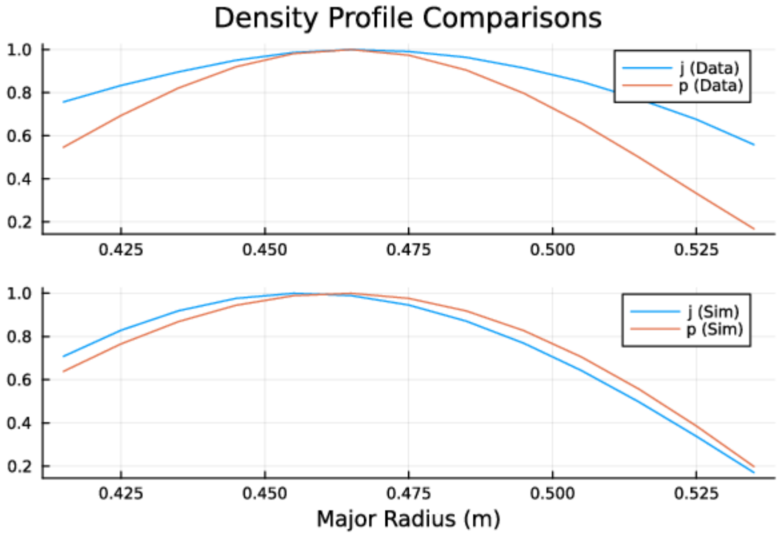
# ISTTOK Data

- Current density profile, pressure density profile, $v_{\text{loop}}$
- Use current density to get $(a_1, a_2, \alpha)$
- Compare simulated density profiles to data for accuracy of model
- ...
- Profit?

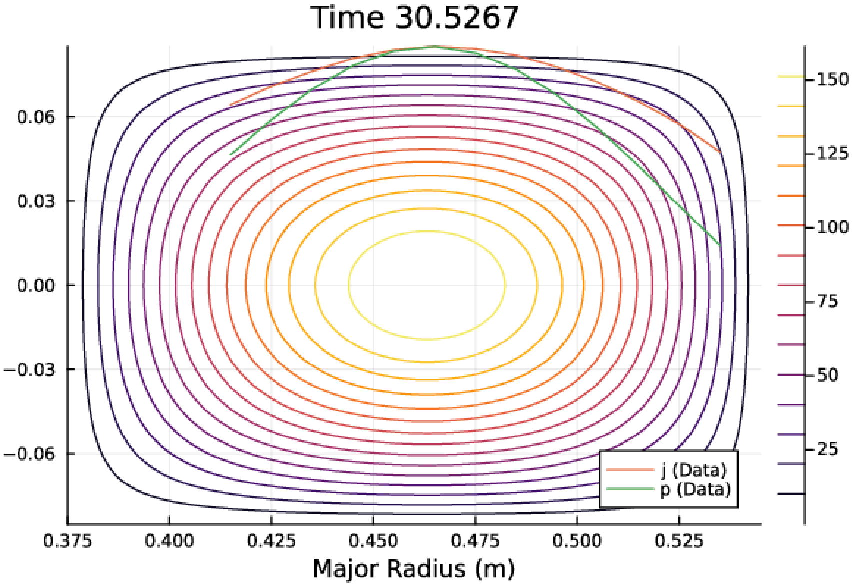
# Initial ISTTOK Simulation



# Scientific Creative Liberty Simulations



# ISTTOK Magnetic Field Topology



# The Other Time Slices?

- Model struggles to converge - highly parameter sensitive
- Data quite imprecise - manual manipulation to achieve above
- Still question of first time slice's parameter guess
- Combine what we can model with other simulations (work with Artur)

**Further Work**

- Data interpolation methods (inter- and intra- time slice)
- Electric field information (helpful for REs)
- Inclusion of pressure density profile in fitting (compare $v_{\text{loop}}$ instead)

# (BONUS) Cool Ion Diagnostics