

**Task 1.****Definition of Neighbourhood**

$R_2$  is a neighbour of  $R_1$  when swapping the position of two adjacent values in  $R_2$  would transform  $R_2$  into  $R_1$ .

For instance, if  $R_1 = [1, 2, 3, 4, 5]$ , and  $R_2 = [2, 1, 3, 4, 5]$   $R_2$  would be a neighbour to  $R_1$  as the adjacent values 2, and 1 could have their positions swapped to make  $R_1$ .

$R_3 = [2, 1, 3, 5, 4]$  would be a neighbour of  $R_2$  but not  $R_1$ .

**Justification**

In this specific instance this neighbourhood works. If player A beat player B, the Kemeny Score would only include that value if player A was ranked lower than player B. Only their relative position (higher or lower) has an impact not the distance between them.

With that in mind, only the edge between the swapped values need to be considered, as any player with a higher rank would still be higher, and a lower still lower.

Therefore, finding the Change in Cost of the Kemeny Score is very simple.

Two values need to be taken from the matrix: the value of position row A, column B; and row B, column A.

Row A, Column B for a row major matrix refers to the instance of player A beating player B by some margin.

Row B, Column A for a row major matrix refers to the instance of player B beating player A by some margin.

These values are mutually exclusive. Either they are both 0 (draw), or only 1 is zero.

Let  $a$  be the value from position row A, column B.

Let  $b$  be the value from position row B, column A.

Let player A be the player ranked higher.

Let player B be the player ranked lower.

The change in cost will be:  $a - b$

If A had beaten B then  $a > 0$  and  $b = 0$ . Leading to an increase in the Kemeny Score as a better player has been "cheated" of their rank.

If B had beaten A then  $b > 0$  and  $a = 0$ . Leading to a decrease in the Kemeny Score as a better player is now ranked higher than a worse player.

If A and B drew, the change would be 0.

For all other players, the relative position of A and the relative position of B is the same. Hence, their values need not be considered.