Conditional Probability

1. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent say that they are Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted.

   (a) What percent of voters participated in the local election?

   (b) A voter is chosen at random. Given that this person voted in the local election, what is the probability that they are an Independent?

2. The American Cancer Society estimates that about 1.7% of women have breast cancer. (`http://www.cancer.org/cancer/cancerbasics/cancer-prevalence`). Susan G. Komen For The Cure Foundation states that mammography correctly identifies about 78% of women who truly have breast cancer. (`http://ww5.komen.org/BreastCancer/AccuracyofMammograms.html`) An article published in 2003 suggests that up to 10% of all mammograms result in false positives for patients who do not have cancer. (`http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1360940`)

   When a patient goes through breast cancer screening there are two competing claims: patient had cancer and patient doesn't have cancer. Assuming these approximations are correct and if a mammogram yields a positive result, what is the probability that patient actually has cancer?

3. A common epidemiological model for the spread of diseases is the SIR model, where the population is partitioned into three groups: Susceptible, Infected, and Recovered. This is a reasonable model for diseases like chickenpox where a single infection usually provides immunity to subsequent infections. Sometimes these diseases can also be difficult to detect. Imagine a population in the midst of an epidemic where 60% of the population is considered susceptible, 10% is infected, and 30% is recovered. The only test for the disease is accurate 95% of the time for susceptible individuals, 99% for infected individuals, but 65% for recovered individuals. (Note: In this case accurate means returning a negative result for susceptible and recovered individuals and a positive result for infected individuals).

Draw a probability tree to reflect the information given above. If the individual has tested positive, what is the probability that they are actually infected?