$$\int_0^{2\pi} s\phi'\, d\phi' = \int_0^{2\pi} c\phi'\, d\phi' = 0 \quad \{\text{intégrale}\}$$

$$\vec{K} = \sigma\omega R \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ s\theta & 0 & c\theta \\ s\theta' c\phi' & s\theta' s\phi' & c\theta' \end{vmatrix}$$

$$d\vec{A} = \frac{\mu_0}{4\pi}\sigma\omega R^3 \frac{\int s\theta'\, d\theta'\, d\phi'}{\sqrt{r^2+R^2-2rRc\theta'}}\left[-\hat{x}\,\cancel{(c\theta s\theta' s\phi')} + \hat{y}\,(\cancel{c\theta s\theta' c\phi'} - s\theta c\theta') + \hat{z}\,\cancel{s\theta s\theta' s\phi'}\right]$$

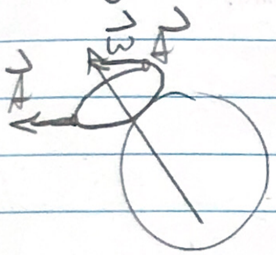
$0 \quad 0 \quad 0$

$$\vec{A}(\vec{r}) = -\hat{y}\frac{\mu_0}{2}\sigma\omega R^3 s\theta \underbrace{\int_0^{\pi} d\theta'\, s\theta' \frac{c\theta'}{\sqrt{r^2+R^2-2rRc\theta'}}}_{\int_{-1}^{1} du \frac{u}{\sqrt{r^2+R^2-2rRu}}} \qquad \{u = \cos\theta'\}$$

$$\left\{u = \frac{1}{-2rR}(r^2+R^2-2rRu) + \frac{r^2+R^2}{2rR}\right\}$$

$\Rightarrow$ 2 intégrales: $\int_{-1}^{1} du \sqrt{r^2+R^2-2rRu} \sim (\ )^{3/2}$ et $\int_{-1}^{1} \frac{du}{\sqrt{r^2+R^2-2rRu}} \sim (\ )^{1/2}$ /!\ avec $r<R$, $r>R$...

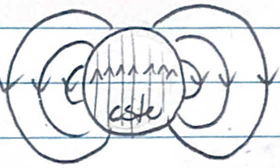
$$\vec{A} = -\hat{y}\frac{\mu_0}{2}\sigma\omega R^3 s\theta \times \frac{r^3+R^3-(r^2+rR+R^2)|r-R|}{3r^2R^2} = \begin{cases} -\hat{y}\dfrac{\mu_0\sigma\omega R}{3} r s\theta & r<R \\ -\hat{y}\dfrac{\mu_0\sigma\omega R^4}{3}\dfrac{s\theta}{r^2} & r>R \end{cases}$$



$\Rightarrow$

Dans le ref original, $\vec{A}(\vec{r}) = -\hat{\phi}\dfrac{\mu_0\sigma\omega R}{3}\begin{cases} r s\theta & r<R \\ \dfrac{R^3 s\theta}{r^2} & r>R \end{cases}$

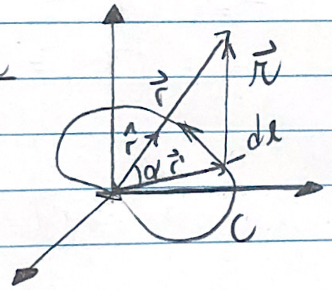
$$\vec{B} = \nabla\times\vec{A} = \begin{cases} \frac{2}{3}\mu_0\sigma\omega R\,\hat{z} & r<R \\ \frac{\mu_0\sigma\omega R^4}{3r^3}(2c\theta\,\hat{r} + s\theta\,\hat{\theta}) & r>R \end{cases}$$



identique à une sphère pol $\vec{P}$ = cste

Développement Multipolaire

Boucle de courant:



$$\frac{1}{\mathscr{r}} = (r^2+r'^2-2rr'c\alpha)^{-1/2} = \frac{1}{r}\underbrace{\left(1-2\frac{r'}{r}c\alpha + \left(\frac{r'}{r}\right)^2\right)^{-1/2}}$$

dév en série de puissance

$$= \frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{r'}{r}\right)^l P_l(c\alpha)$$

$$\begin{cases} P_0(u) = 1 \\ P_1(u) = u \\ P_2(u) = \frac{3u^2-1}{2} \end{cases}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r}\oint d\vec{l}' \sum_l \left(\frac{r'}{r}\right)^l P_l(c\alpha) = A_0 + A_1 + A_2 + A_3 + \dots$$

$$\vec{A}_0 = \frac{\mu_0 I}{4\pi}\oint d\vec{l}' \quad \{f(b)-f(a)\} \Rightarrow A_0 = 0$$

$\{r' c\alpha = \vec{r}'\cdot\hat{r}\}$?

$$\{\vec{A}_{dip}(\vec{r})\} \Rightarrow \vec{A}_1 = \frac{\mu_0 I}{4\pi r^2}\oint d\vec{l}'\, r' c\alpha = \frac{\mu_0 I}{4\pi r^2}\oint d\vec{l}'\, \hat{r}\cdot\vec{r}'$$

C, S, $d\vec{a}$: $\vec{a} = \int d\vec{a}$ Aire vectorielle

Surface plane: $\vec{a} = \hat{n}\int da$ $\{\hat{n}\cdot \text{Aire}\}$

Aire vectorielle $\leq$ Aire usuelle

Lemme: Soit $\vec{c}$, un vecteur cste quelconque

$$\oint_C (\vec{c}\cdot\vec{r})\,d\vec{l} = \vec{a}\times\vec{c}$$

$\vec{a}$: Aire vectorielle d'une surface borné par C

Preuve: Théorème de Stokes

$$G \equiv \oint_C \vec{v}\cdot d\vec{l} = \int_S \nabla\times\vec{v}\cdot d\vec{a} \equiv D$$

Choix de $\vec{v}$: $\vec{v}(\vec{r}) = \vec{b}(\vec{c}\cdot\vec{r})$ (cste)

$$G = \oint \vec{b}(\vec{c}\cdot\vec{r})\cdot d\vec{l} = \vec{b}\cdot\oint_C(\vec{c}\cdot\vec{r})\,d\vec{l}$$

$$D = \int_S\left[\nabla\times(\vec{b}(\vec{c}\cdot\vec{r}))\right]\cdot d\vec{a}$$

$D=\sum_i \int_S [\vec c\times\vec b]_i\, da_i$

$=(\vec c\times\vec b)\cdot\int_S d\vec a$

$$\left\{[\nabla\times(\vec b(\vec c\cdot\vec r))]_i=\sum_{jk}\epsilon_{ijk}\partial_j\Big(b_k\sum_\ell c_\ell r_\ell\Big)\right.$$

p.r à r

$$=\sum_{jk}\epsilon_{ijk}b_k\underbrace{\sum_\ell c_\ell\delta_{j\ell}}_{c_j}$$

$$=\sum_{jk}\epsilon_{ijk}c_jb_k$$

$$=(\vec c\times\vec b)_i$$

$\left\{\begin{array}{l}0 \text{ si } j\neq\ell\\ 1 \text{ si } j=\ell\end{array}\right.$ $\partial_j r_\ell=\delta_{j\ell}$

parallélépipède, permutations de $\vec a,\vec b,\vec c$ possible

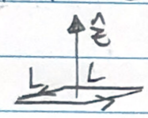
$=(\vec c\times\vec b)\cdot\vec a=(\vec a\times\vec c)\cdot\vec b$

$\vec b\cdot\oint_C(\vec c\cdot\vec r)\,d\vec l=\vec b\cdot(\vec a\times\vec c)$ $\{\forall\vec b\}$ $\Rightarrow$ $\boxed{\oint(\vec c\cdot\vec r)\,d\vec l=\vec a\times\vec c}$

$\vec A_1(\vec r)=\frac{\mu_0 I}{4\pi r^2}\oint_C d\vec l'\,\overbrace{\hat r}^{\{\vec c\}}\cdot\overbrace{\vec r'}^{\{\vec r\}}$ $\Rightarrow$ $\vec A_1(\vec r)=\frac{\mu_0 I}{4\pi r^2}\,\vec a\times\hat r=\frac{\mu_0}{4\pi r^2}(I\vec a)\times\hat r$ → propriété de la boucle

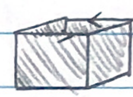
Moment dip. mag. $\vec m\equiv I\vec a=I\int_S d\vec a$ $\boxed{\vec A_1(\vec r)=\frac{\mu_0}{4\pi}\frac{\vec m\times\hat r}{r^2}}=\frac{\mu_0}{4\pi}\frac{\vec m\times\vec r}{r^3}$

↳ $\int_{S_1}d\vec a=\int_{S_2}d\vec a\iff\oint_S d\vec a=0$ {aire vect. ind de S}



<u>Calcul de $\vec m$</u>

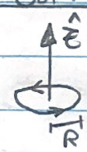
$\vec m=I\int d\vec a=I\int\hat z\,da=I\hat z L^2$ <u>Autre surface?</u>



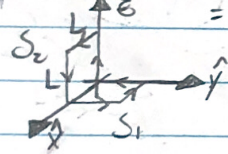
la contribution des faces opposées s'annulent

<u>Boucle planaire:</u> $\vec m=I\hat nA$

$=\hat z I\pi R^2$



<u>Machin:</u>



$\vec m=I\int_S d\vec a=I\int_{S_1}d\vec a+I\int_{S_2}d\vec a=I(\hat zL^2+\hat yL^2)$

$=IL^2(\hat y+\hat z)=\underbrace{IL^2\sqrt2}_{m}\,\underbrace{\frac{\hat y+\hat z}{\sqrt2}}_{\hat n}$

{$\vec A=L^2\sqrt2\,\hat n$ est l'air maximal qu'un observateur peut voir} vue à 45°

<u>Dipole électrique</u>

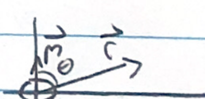
$d\,{}^{q}_{-q}$ $\vec p=q\vec d$ $V\approx V_{dip}$ {lim $q\to\infty$ et qd fixe} $V=V_{dip}$

<u>en magnétique:</u> $\vec a$ $\vec m=I\vec a$, {lim $I\to\infty$, Ia fixe}: dip. mag. pur

$\vec A\approx\vec A_{dip}$ $\qquad\vec A=\vec A_{dip}$

<u>Champ magnétique dipolaire</u>

$\vec B_{dip}=\nabla\times\vec A_{dip}$, où $\vec A_{dip}=\frac{\mu_0}{4\pi}\frac{\vec m\times\hat r}{r^2}=\frac{\mu_0}{4\pi}\frac{s_\theta\hat\phi}{r^2}$

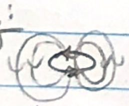


$\Rightarrow\vec B_{dip}=\frac{\mu_0 m}{4\pi}\nabla\times\left(\frac{s_\theta\hat\phi}{r^2}\right)$ $\left\{\nabla\times(V_\phi\hat\phi)=\frac{1}{rs_\theta}\partial_\theta(s_\theta V_\phi)\hat r-\frac1r\partial_r(rV_\phi)\hat\theta\right\}$

$=\frac{\mu_0 m}{4\pi}\left[\frac{1}{rs_\theta}\partial_\theta\left(\frac{s_\theta^2}{r^2}\right)\hat r-\frac1r\partial_r\left(\frac{s_\theta}{r}\right)\hat\theta\right]$

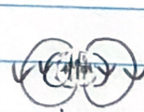
$\boxed{\vec B_{dip}=\frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat r+\sin\theta\,\hat\theta\right]}$ $\left\{\vec E_{dip}=\frac{p}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat r+s_\theta\hat\theta\right]\right\}$

<u>Mag:</u>

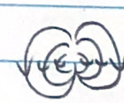


<u>phys</u> identique à élec <u>pur</u>

<u>Élec:</u>



<u>phys</u>



<u>pur</u>