Math. Meth. Oper. Res. (2006) 64: 227–236
DOI 10.1007/s00186-006-0086-0

ORIGINAL ARTICLE

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Discrete–time delta hedging and the Black–Scholes model with transaction costs

Received: 6 April 2005 / Accepted: 3 March 2006 / Published online: 5 August 2006

Abstract The paper deals with the problem of discrete–time delta hedging and discrete-time option valuation by the Black–Scholes model. Since in the Black–Scholes model the hedging is continuous, hedging errors appear when applied to discrete trading. The hedging error is considered and a discrete-time adjusted Black–Scholes–Merton equation is derived. By anticipating the time sensitivity of delta in many cases the discrete-time delta hedging can be improved and more accurate delta values dependent on the length of the rebalancing intervals can be obtained. As an application the discrete-time trading with transaction costs is considered. Explicit solution of the option valuation problem is given and a closed form delta value for a European call option with transaction costs is obtained.

Keywords Delta hedging · Transaction costs

1 Introduction

The option valuation problem with transaction costs has been considered extensively in the literature. In many papers on option valuation with transaction costs the discrete-time trading is considered by the continuous-time framework of the Black–Scholes–Merton partial differential equation (BSM-pde); see e.g. [Leland 1985; Boyle and Vorst 1992; Hoggard et al. 1994; Avellaneda and Paras 1994; Toft 1996]. Since in continuous-time models the hedging is instantenuous, hedging errors appear when applied to discrete trading. In that case perfect replication is not possible, hence reducing the error is important for application.

The hedging error, defined as the difference between the return to the portfolio value and the return to the riskless asset over the rebalancing (rehedging)

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interval, depends on the length δt of the interval. When rebalancing intervals are relatively small and thus trading takes place very frequently, then the expected hedging error may be relatively small. For the analysis of the hedging error in the case where option values are described by the BSM-pde; see e.g. [Boyle and Emanuel 1980; Toft 1996; Mello and Neuhaus 1998]. However in presence of transaction costs more frequent trading may increase the accumulated costs considerably [Korn 2004; Kocinski 2004].

In continuous-time models the value of delta representing the number of shares held in the portfolio varies continuously with time. If $V(t, S)$ is the option value at time t and S the price of the underlying asset, then the delta value at time t is given by the partial derivative $V_S(t, S)$ of the current value of V; see e.g. [Black and Scholes 1973; Merton 1973].

In practice where the trading takes place discretely in time, the number of shares given by the delta is held constant over the rebalancing interval and thus the hedging error cannot be eliminated. This means that reducing the risk and the hedging error is of main practical interest. In the case when δt is sufficiently small, the current value of delta gives an acceptable value such that the hedging error is small. However, when the rebalancing is not that frequent, due to transaction costs for instance, then in general the hedging error will not be small. Moreover, if the delta is more sensitive with respect to time, the error will further increase.

In the papers of Leland and others, which consider the option pricing problem with transaction costs by the Black–Scholes model, the number of shares kept constant is given by the Black–Scholes delta at time t regardless of the time between successive rehedgings and the time sensitivity of delta..

In this paper the time between rehedgings and the time sensitivity of delta is considered. The objective is to incorporate the time change of delta over the rebalancing interval into the model implicitly so that the resulting equation can be directly transformed to the BSM pde and solved as the diffusion equation. In this way the number of shares kept constant between successive rehedgings can be derived explicitly by the BSM pde adjusted to discrete-time trading. A closed form discrete-time delta dependent on the interval length δt can be obtained. Short notes on further research and improvements of discrete hedging are given. As an application the case of option valuation with transaction costs is treated and explicitly solved.

The paper outline is as follows: in Sect. 2 the portfolio over the rebalancing interval is studied. The discrete-time adjusted Black–Scholes–Merton partial differential equation is derived and appropriate values for discrete-time hedging are given. In Sect. 3 the option valuation problem with transaction costs is considered and an appropriate equation describing the process is obtained. A specific example of a European call option is studied and a closed form representation of the discrete-time delta is given.

2 Discrete-time Black–Scholes–Merton partial differential equation

Let us denote by $V = V(t, S)$ the option value as a function of the underlying asset with price S and time t. Suppose that the price of the underlying at time t is given by

$$S(t) = S_0 \exp(\nu t + \sigma W(t)), \tag{2.1}$$

where *W(t)* is the Wiener process (Brownian motion), σ the annualized volatility and $\mu = \nu + (1/2)\sigma^2$ the expected annual rate of return. Then the change of $S = S(t)$ over the small noninfinitesimal interval of length δt is approximately equal to

$$\delta S = S(t + \delta t) - S(t) = \sigma S Z \sqrt{\delta t} + \mu S \delta t, \tag{2.2}$$

where Z is normally distributed variable with mean zero and variance one; in short $Z \sim N(0, 1)$ (Leland 1985; Hull 1997)

Let Π be the value of a portfolio at time t consisting of a long position in the option and a short position in *N(t)* units of shares with the price S

$$\Pi = V - N(t)S. \tag{2.3}$$

In the continuous-time model where the hedging is instantanuous and the replication is perfect, the number of shares N(t) is given by the current value of the derivative $V_S(t, S)$ (the delta), where *V(t, S)* is the solution of the Black–Scholes–Merton equation.

In practice where the trading takes place discretely in time, the number of shares given by the delta is held constant over the rebalancing interval $(t, t + \delta t)$. Thus when continuous-time models are applied to discrete-time trading, hedging errors appear. The hedging error, defined as the difference between the return to the portfolio value and the return to the riskless asset over the rehedging interval, depends on the length δt of the interval. This means when δt is sufficiently small, the current value of the derivative gives an acceptable approximation for the discrete-time hedging with small error. However, when the rebalancing is not frequent, especially in the case where the delta is sensitive with respect to time, higher errors may occur (cf. Remark 2.2). For the mean-variance analysis of the error, where V is given by the BSM equation (Boyle and Emanuel 1980; Toft 1996).

Remark 2.1 Suppose that after discrete hedging, for instance from a time point $t + \delta t$ to expiry, continuous hedging is assumed. Then the delta value at time $t + \delta t$ is determined by the partial derivative $V_S(t + \delta t, S + \delta S)$ where *V(t,S)* is the solution of the Black–Scholes–Merton equation. In that case for example the constant value of delta over the rehedging interval $(t, t + \delta t)$ can be approximated by $V_S(t + \delta t, S)$.

In the continuation we will consider the case, where hedging is in discrete time until expiry. We assume that hedging takes place at equidistant time points with rebalancing intervals of (equal) length δt. Throughout we assume that partial derivatives of *V(t, S)* are continuous functions on their respective domains. Moreover we let the trading be relatively frequent so that the interval length δt is relatively small. Then we have

Proposition 2.1 *Let $V = V(t, S)$ be the option value at time t and price S. Let σ be the annualized volatility and r the annual interest rate of a riskless asset continuously compounded. If the approximate number of shares $N(t)$ held short over the rebalancing interval of length δt is equal to:*

$$N(t) = V_S(t + \delta t, S), \tag{2.4}$$

where V(t, S) satisfies the Black–Scholes–Merton equation

$$V_t(t, S) + \frac{1}{2}\sigma_0^2 S^2 V_{SS}(t, S) + r_0 S V_S(t, S) - r_0 V(t, S) = 0 \tag{2.5}$$

with parameters

$$\sigma_0 = \sigma\Big/\sqrt{1 + r\delta t} \quad \textit{and} \quad r_0 = {}^{r}\!\big/_{1 + r\delta t}, \tag{2.6}$$

then the mean and the variance of the hedging error is of order $O(\delta t^2)$.

Proof Let us consider the return to the portfolio Π over the period $[t, t + \delta t]$, $t \in [0, T - \delta t]$. By assumption over the period of length δt the value of the portfolio changes by

$$\Delta\Pi = \Delta V - N(t)\delta S \tag{2.7}$$

as the number of shares *N(t)* is held fixed during the time step δt.

First we consider the change ΔV of the option value *V(t, S)* over the time interval $[t, t + \delta t]$ of length δt. We give the Taylor series expansion of the difference in the following way:

$$\begin{aligned} \Delta V &= V(t + \delta t, S + \delta S) - V(t, S) \\ &= (V(t + \delta t, S + \delta S) - V(t + \delta t, S)) \\ &\quad + (V(t + \delta t, S) - V(t, S)) = I + II \end{aligned} \tag{2.8}$$

where:

$$\begin{aligned} I &= (V(t + \delta t, S + \delta S) - V(t + \delta t, S)) \\ &= V_S(t + \delta t, S)(\delta S) + \frac{1}{2} V_{SS}(t + \delta t, S)(\delta S)^2 \\ &\quad + \frac{1}{6} V_{SSS}(t + \delta t, S)(\delta S)^3 + O(\delta t^2), \end{aligned} \tag{2.9}$$

and

$$II = (V(t + \delta t, S) - V(t, S)) = V_t(t + \delta t, S)(\delta t) + O(\delta t^2). \tag{2.10}$$

By (2.2) we get the approximation

$$\begin{aligned} \Delta V &= V_t(t + \delta t, S)(\delta t) + V_S(t + \delta t, S)(\delta S) \\ &\quad + \frac{1}{2} V_{SS}(t + \delta t, S)(\delta S)^2 + \frac{1}{6} V_{SSS}(t + \delta t, S)(\delta S)^3 + O(\delta t^2) \\ &= V_t(t + \delta t, S)(\delta t) + V_S(t + \delta t, S)(\delta S) \\ &\quad + \frac{1}{2} V_{SS}(t + \delta t, S)(\sigma S Z\sqrt{\delta t} + \mu S \delta t)^2 \\ &\quad + \frac{1}{6} V_{SSS}(t + \delta t, S)(\sigma S Z\sqrt{\delta t} + \mu S \delta t)^3 + O(\delta t^2) \end{aligned} \tag{2.11}$$

hence

$$\begin{aligned} \Delta V &= V_t(t + \delta t, S)(\delta t) + V_S(t + \delta t, S)(\delta S) \\ &\quad + \frac{1}{2} V_{SS}(t + \delta t, S)\left(\sigma^2 S^2 Z^2 \delta t + 2\sigma\mu S^2 Z \delta t^{\frac{3}{2}}\right) \\ &\quad + \frac{1}{6} V_{SSS}(t + \delta t, S)\sigma^3 S^3 Z^3 \delta t^{\frac{3}{2}} + O(\delta t^2). \end{aligned} \tag{2.12}$$

Thus by (2.7) it follows:

$$\begin{aligned}\Delta\Pi &= \Delta V - N(t)\delta S\\ &= V_t(t+\delta t, S)(\delta t) + [V_S(t+\delta t, S) - N(t)]\,(\delta S)\\ &\quad + \frac{1}{2}V_{SS}(t+\delta t, S)(\sigma^2 S^2 Z^2 \delta t + 2\sigma\mu\, S^2 Z\delta\, t^{\frac{3}{2}})\\ &\quad + \frac{1}{6}V_{SSS}(t+\delta t, S)\sigma^3 S^3 Z^3 \delta t^{\frac{3}{2}} + O(\delta t^2).\end{aligned} \tag{2.13}$$

By choosing the number of shares in the following way

$$N(t) = V_S(t+\delta t, S) \tag{2.14}$$

the δS term can be eliminated completely and the risk in (2.13) can be reduced. Hence we get

$$\begin{aligned}\Delta\Pi &= V_t(t+\delta t, S)(\delta t) + \frac{1}{2}V_{SS}(t+\delta t, S)(\sigma^2 S^2 Z^2 \delta t + 2\sigma\mu\, S^2 Z\delta\, t^{\frac{3}{2}})\\ &\quad + \frac{1}{6}V_{SSS}(t+\delta t, S)\sigma^3 S^3 Z^3 \delta t^{\frac{3}{2}} + O(\delta t^2).\end{aligned} \tag{2.15}$$

By assumption the amount Π can be invested in a riskless asset with an interest rate r compounded continuously. Then over the rehedging interval of length δt the return to the riskless investment is equal to

$$\Delta B = \Pi \exp(r\delta t) - \Pi = \Pi r\delta t \; + O(\delta t^2). \tag{2.16}$$

Let us define the hedging error ΔH as the difference between the return to the portfolio value $\Delta\Pi$ and the return to the riskless value ΔB over the rehedging interval of length δt. By (2.13) and (2.14) we obtain

$$\begin{aligned}\Delta H &= \Delta\Pi - \Delta B\\ &= V_t(t+\delta t, S)(\delta t) + \frac{1}{2}V_{SS}(t+\delta t, S)\left(\sigma^2 S^2 Z^2 \delta t + 2\sigma\mu\, S^2 Z\delta\, t^{\frac{3}{2}}\right)\\ &\quad + \frac{1}{6}V_{SSS}(t+\delta t, S)\sigma^3 S^3 Z^3 \delta t^{\frac{3}{2}} - \Pi r\delta t + O(\delta t^2)\\ &= V_t(t+\delta t, S)(\delta t) + \frac{1}{2}V_{SS}(t+\delta t, S)\left(\sigma^2 S^2 Z^2 \delta t + 2\sigma\mu\, S^2 Z\delta\, t^{\frac{3}{2}}\right)\\ &\quad + \frac{1}{6}V_{SSS}(t+\delta t, S)\sigma^3 S^3 Z^3 \delta t^{\frac{3}{2}} - (V(t,S) - SV_S(t+\delta t, S))r\delta t + O(\delta t^2).\end{aligned} \tag{2.17}$$

Moreover we have

$$(V(t,S) = V(t+\delta t, S)) - V_t(t+\delta t, S)(\delta t) \; + \; O(\delta t^2) \tag{2.18}$$

hence it follows:

$$\begin{aligned}\Delta H = \Delta\Pi - \Delta B = (1 + r\delta t)V_t(t+\delta t, S)(\delta t)\\ +\frac{1}{2}V_{SS}(t+\delta t, S)\left(\sigma^2 S^2 Z^2 \delta t + 2\sigma\mu S^2 Z \delta t^{\frac{3}{2}}\right)\\ +\frac{1}{6}V_{SSS}(t+\delta t, S)\sigma^3 S^3 Z^3 \delta t^{\frac{3}{2}} - (V(t+\delta t, S)\\ - SV_S(t+\delta t, S))r\delta t + O(\delta t^2).\end{aligned} \tag{2.19}$$

Let us assume for a moment that there exists a solution *V(t, S)* of the following partial differential equation

$$\begin{aligned}(1 + r\delta t)V_t(t+\delta t, S) \ + \frac{1}{2}\sigma^2 S^2\ V_{SS}(t+\delta t, S)\\ -(V(t+\delta t, S) - SV_S(t+\delta\, t, S))r = 0\end{aligned} \tag{2.20}$$

for $t \in [0, T - \delta t]$. Then from (2.20) we obtain

$$\begin{aligned}\Delta H = \frac{1}{2}V_{SS}(t+\delta t, S)\left(\sigma^2 S^2 (Z^2 - 1)\delta t + 2\sigma\mu\, S^2 Z\delta\, t^{\frac{3}{2}}\right)\\ +\frac{1}{6}V_{SSS}(t+\delta t, S)\sigma^3 S^3 Z^3 \delta t^{\frac{3}{2}} + O(\delta t^2).\end{aligned} \tag{2.21}$$

Taking expectations and noting that $Z \sim N(0,1)$ we get the mean

$$E(\Delta H) = O(\delta t^2) \tag{2.22}$$

and the variance

$$V(\Delta H) = \left(\frac{1}{2}V_{SS}(t+\delta t, S)\sigma^2 S^2\right)^2 2\delta t^2\ + O(\delta t^3) \tag{2.23}$$

Therefore when the option value *V(t, S)* satisfies the following partial differential equation:

$$\begin{aligned}V_t(t+\delta t, S) + \frac{1}{2}\sigma_0^2 S^2\ V_{SS}(t+\delta t, S)\\ + r_0 S V_S(t+\delta\, t, S)) - r_0 V(t+\delta t, S) = 0,\end{aligned} \tag{2.24}$$

where σ_0 and $r_0$are given by (2.6), then the hedging error has mean zero of order $O(\delta t^2)$ and variance of order $O(\delta t^2)$. By a shift $V_1(t) = V(t + \delta\mathrm{t})$ from (2.24) the Black–Scholes–Merton equation for $V_1(t)$ with expiry at $t = T - \delta t$ is obtained. Hence the BSM Eq. (2.5) with adjusted volatility σ_0 and interest rate r_0 readily follows. □

Remark 2.2 We note that in ΔV appearing in (2.8)–(2.12) the time derivative of V_S is implicitly considered by the relation:

$$V_S(t+\delta\, t, S) = V_S(t, S) + V_{St}(t, S)\delta t + O(\delta t^2) \tag{2.25}$$

Hence in (2.13) the term $V_{St}\delta t\delta S$ can be avoided and the obtained equation can be transformed to the diffusion equation.

Remark 2.3 By Expansion (2.11) it readily follows that higher order derivatives V_{SS} and V_{SSS} can be included into *N(t)* and hedging further improved. Since in Proposition 2.1 the form of the Black–Scholes–Merton pde is preserved, the model can be generalized to models with dividends and models with time dependent volatility and interest rates.

By Proposition 2.1 it follows directly that in the discrete-time case the number of shares held fixed over the rebalancing interval depends on the length δt of the interval. In addition by Remark 2.2, if δt is not small then the values of delta at time t and $t + \delta t$ may considerably differ, especially when the delta is more sensitive to the change in time. If in (2.21) $|Z|$ is near one, the gamma term is near zero and so other terms of the hedging error prevail. In that case by (2.4) the hedging error can be substantially reduced. Moreover, when options vega and rho are high, then by (2.6) the adjustment of parameters with respect to δt can be considered.

3 Transaction costs

It is known that transaction costs can be included into the Black–Scholes–Merton equation by considering appropriately adjusted volatilility (Leland 1985; Avellaneda and Paras 1994; Toft 1996).

We denote here transaction costs by c_{tr}. These costs depend on the volume of transactions at rehedging and a constant percentage k.

Let us consider again the portfolio at time t consisting of one option long and a short position in $N(t)$ units of stock S with the value $\Pi = V - N(t)S$.

Let k represent the round trip transaction costs, measured as a fraction of the volume of transactions. Then the transactions costs of rehedging over the rehedging interval of length δt are equal to

$$c_{tr} = \frac{k}{2} S \left| N(t + \delta t) - N(t) \right|, \tag{3.1}$$

for the details (Leland 1985; Avellaneda and Paras 1994). If the discrete time trading is considered, then by (2.4) we have $N(t) = V_S(t + \delta t, S)$and the volume can be approximated in the following way:

$$\begin{aligned} N(t + \delta t) - N(t) &= V_S(t + 2\delta t, S + \delta S) - V_S(t + \delta t, S) \\ &= V_{SS}(t + \delta t, S)\delta S \;+\; O(\delta t) \end{aligned}$$

Let us assume

$$k < \sigma \sqrt{\frac{\pi \delta t}{2}} \tag{3.2}$$

Then the transaction costs are equal to

$$\begin{aligned} c_{tr} &= \frac{k}{2} S \left| V_{SS}(t + \delta t, S) \right| |\delta S| + O\left(\delta t^{\frac{3}{2}}\right) \\ &= \frac{k}{2} \sigma S^2 \left| V_{SS}(t + \delta t, S) \right| |Z| \sqrt{\delta t} + O\left(\delta t^{\frac{3}{2}}\right). \end{aligned} \tag{3.3}$$

In some cases it is possible to assume that $S^2 V_{SS}$ is of order $O(\delta t^{1/2})$; for instance see Leland (1985) for a European call option.

In the same way as in Sect. 2 [relations (2.13)–(2.15)] the change in value of the portfolio can be studied. By (2.14) then it follows:

$$\begin{aligned}\Delta\Pi &= \Delta V - N(t)\delta S - c_{tr} = V_t(t+\delta t, S)(\delta t)\\ &\quad + \frac{1}{2}V_{SS}(t+\delta t, S)(\sigma^2 S^2 Z^2 \delta t - \frac{k}{2}\sigma S^2\,|V_{SS}(t+\delta t, S)|\,|Z|\sqrt{\delta t} + O\left(\delta t^{\frac{3}{2}}\right)\end{aligned} \tag{3.4}$$

By assumption that the amount Π can be invested in a riskless asset, it follows that the return to the riskless investment is equal to

$$\begin{aligned}\Delta B &= \Pi r\delta t + O(\delta t^2) = (V(t, S) - V_S(t+\delta t, S)S)r\delta t + O(\delta t^2)\\ &= (V(t+\delta t, S) - V_t(t+\delta t, S)\delta t - V_S(t+\delta t, S)S)r\delta t + O(\delta t^2)\end{aligned} \tag{3.5}$$

In the same way as in (2.19) the hedging error can be obtained

$$\begin{aligned}\Delta H &= \Delta\Pi - \Delta B\\ &= (1 + r\delta t)V_t(t+\delta t, S)(\delta t) + \frac{1}{2}V_{SS}(t+\delta t, S)\sigma^2 S^2 Z^2 \delta t\\ &\quad -(V(t+\delta t, S) - SV_S(t+\delta t, S))r\,\delta t\\ &\quad -\frac{k}{2}\sigma S^2\,|V_{SS}(t+\delta t, S)|\,|Z|\sqrt{\delta t} + O\left(\delta t^{\frac{3}{2}}\right)\end{aligned} \tag{3.6}$$

We note that $E(|Z|) = \sqrt{2/\pi}$ for $Z \sim N(0,1)$. Therefore by assuming that *V(t, S)* satisfies the following partial differential equation:

$$\begin{aligned}&(1 + r\delta t)V_t(t+\delta t, S) + \frac{1}{2}\sigma^2 S^2 V_{SS}(t+\delta t, S)\\ &-\frac{k}{2}\sigma S^2\sqrt{\frac{2}{\pi\delta t}}\,|V_{SS}(t+\delta t, S)|\\ &-(V(t+\delta t, S) - SV_S(t+\delta t, S))r = 0\end{aligned} \tag{3.7}$$

it follows that the hedging error can be written in the following way:

$$\begin{aligned}\Delta H &= \frac{1}{2}\sigma^2 S^2 V_{SS}(t+\delta t, S)(Z^2 - 1)\delta t\\ &\quad -\frac{k}{2}\sigma S^2\,|.V_{SS}(t+\delta t, S)|\left(|Z| - \sqrt{\frac{2}{\pi}}\right)\sqrt{\delta t} + O\left(\delta t^{\frac{3}{2}}\right)\end{aligned} \tag{3.8}$$

Hence by assumption (3.2) we have

$$\begin{aligned}&E(\Delta H) = O\left(\delta t^{\frac{3}{2}}\right) \qquad \text{and}\\ &V(\Delta H) = \left(\frac{k}{2}\sigma S^2 V_{SS}(t+\delta t, S)\right)^2\left(1 - \frac{2}{\pi}\right)\delta t\; + O(\delta t^2)\end{aligned} \tag{3.9}$$

By (2.6) and a transformation $t + \delta t \rightarrow t$ from Eq. (3.7) we get

$$V_t(t, S) + \frac{1}{2}\sigma_0^2 S^2 V_{SS}(t, S) - \frac{k}{2}\sigma_0 S^2 \sqrt{\frac{2}{\pi \delta t}}\, |V_{SS}(t, S)|$$
$$-(V(t, S) - SV_S(t, S))r_0 = 0 \tag{3.10}$$

Eq. (3.10) is nonlinear. In the case where $V_{SS}(t, S)$ is of the same sign for all values t and S the equation can be reduced to a linear equation. For instance a European call or put option satisfy the requirement. Therefore the following result can be obtained.

Proposition 3.1 *Let $C(t, S)$ be the value of the European call option, σ be the annualized volatility, r the annual interest rate of a riskless asset and let the trading take place discretely with rebalancing intervals of length δt. If k satisfies* (3.2) *and if the approximate number of shares $N(t)$ held short over the rebalancing interval is equal to: $N(t) = C_S(t + \delta\, t, S)$,where $C(t, S)$ satisfies the Black–Scholes–Merton equation*

$$C_t(t, S) \; + \frac{1}{2}\sigma_1^2 S^2\, C_{SS}(t, S) \; + r_1 SC_S(t, S)) - r_1 C(t, S) = 0, \tag{3.11}$$

With parameters

$$\sigma_1^2 = \frac{\sigma^2}{1 + r\delta t}\left(1 - \frac{k}{\sigma}\sqrt{\frac{2}{\pi \delta t}}\right) \quad \text{and} \quad r_1 = \frac{r}{1 + r\delta t}, \tag{3.12}$$

then the mean and the variance of the hedging error are of order $O(\delta t^{3/2})$and $O(\delta t^2)$ respectively.

Proof By relations (3.4)–(3.10) the proof can be given in the same way as that of Proposition 2.1. We only note that the second derivative C_{SS} of the solution of the BSM equation is given by the Gaussian or normal density function and hence it is a positive function. Thus the Eq. (3.11) is a direct consequence of Eq. (3.10).

For a short position in a call option and a long position in the shares we have $\Pi = -V + N(t)S$. Thus in (3.4) and in subsequent equations all the signs with the exception of the transaction costs term have to be changed. Therefore we conclude: □

Corolarry 3.1 *Let the portfolio consist of a short position in the European call option and a long position in N(t) shares. If $N(t) = C_S(t + \delta\, t, S)$, where $C(t, S)$ satisfies the BSM equation (3.11) with parameter σ_1 given by : $\sigma_1^2 = (\sigma^2/1 + r\delta t)$ $\left(1 + (k/\sigma)\sqrt{2/\pi \delta t}\right)$, then the mean and the variance of the hedging error are of order $O(\delta t^{3/2})$and $O(\delta t^2)$respectively.*

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