# CS224n: NLP with Deep Learning

# Lecture 7: Vanishing Gradients and Fancy RNN

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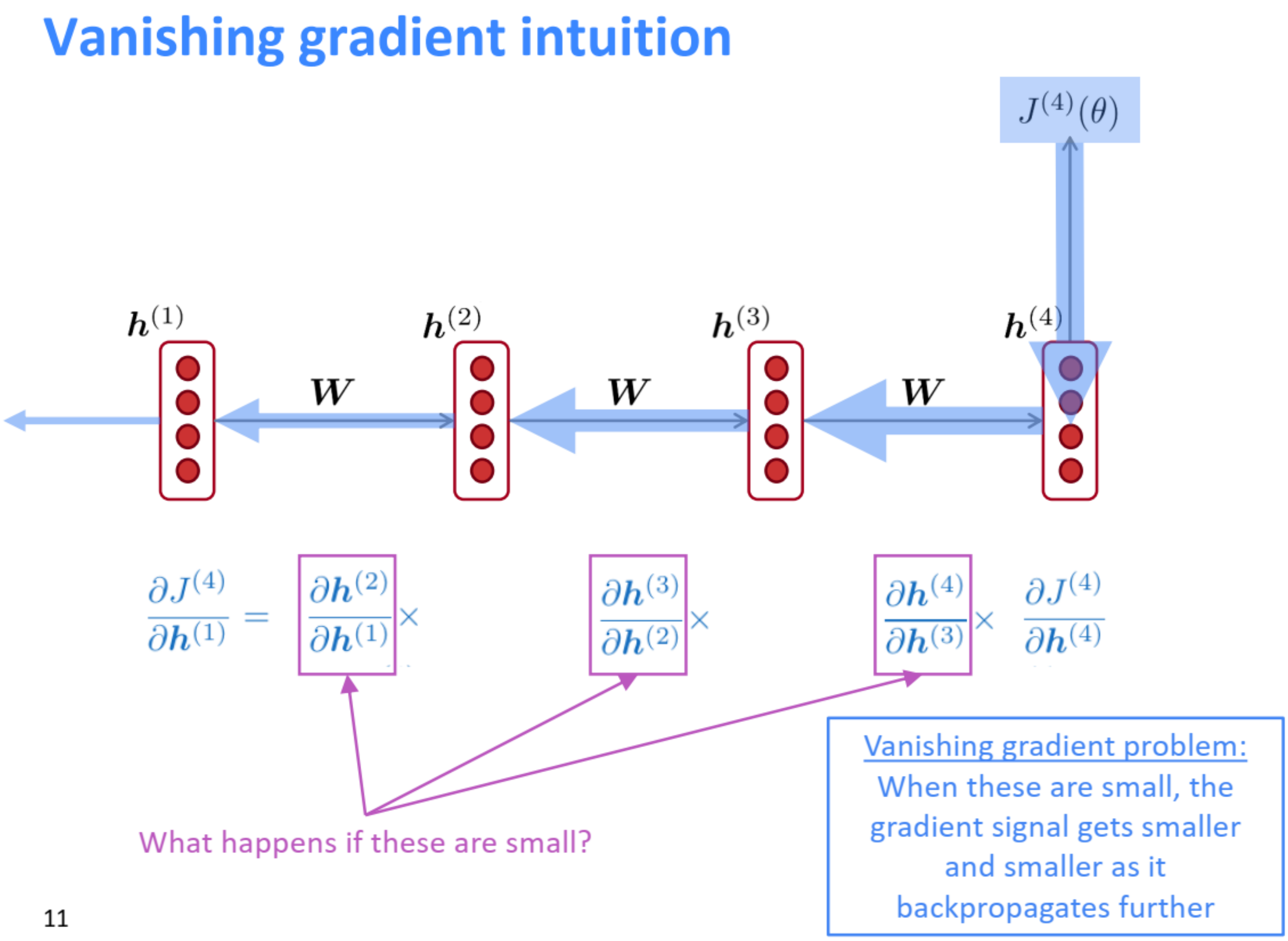
## Vanishing Gradient Problem

### Intuition

- When the gradients $\frac{\partial h^{(i)}}{\partial h^{(i-1)}}$ are small,
  the overall gradient is going to get smaller and smaller, as we go backwards




- If the largest eigenvalue of $W_h$ is < 1:

the gradient will shrink exponentially

→ Vanishing gradient !

### Proof Sketch

## Vanishing gradient proof sketch

- Recall: $$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h \boldsymbol{h}^{(t-1)} + \boldsymbol{W}_x \boldsymbol{x}^{(t)} + \boldsymbol{b}_1\right)$$

- Therefore: $$\frac{\partial \boldsymbol{h}^{(t)}}{\partial \boldsymbol{h}^{(t-1)}} = \mathrm{diag}\left(\sigma'\left(\boldsymbol{W}_h \boldsymbol{h}^{(t-1)} + \boldsymbol{W}_x \boldsymbol{x}^{(t)} + \boldsymbol{b}_1\right)\right)\boldsymbol{W}_h \qquad \text{(chain rule)}$$

- Consider the gradient of the loss $J^{(i)}(\theta)$ on step *i*, with respect to the hidden state $\boldsymbol{h}^{(j)}$ on some previous step *j*.

$$\frac{\partial J^{(i)}(\theta)}{\partial \boldsymbol{h}^{(j)}} = \frac{\partial J^{(i)}(\theta)}{\partial \boldsymbol{h}^{(i)}} \prod_{j<t\leq i} \frac{\partial \boldsymbol{h}^{(t)}}{\partial \boldsymbol{h}^{(t-1)}} \qquad \text{(chain rule)}$$

$$= \frac{\partial J^{(i)}(\theta)}{\partial \boldsymbol{h}^{(i)}} \boxed{\boldsymbol{W}_h^{(i-j)}} \prod_{j<t\leq i} \mathrm{diag}\left(\sigma'\left(\boldsymbol{W}_h \boldsymbol{h}^{(t-1)} + \boldsymbol{W}_x \boldsymbol{x}^{(t)} + \boldsymbol{b}_1\right)\right) \qquad \left(\text{value of } \frac{\partial \boldsymbol{h}^{(t)}}{\partial \boldsymbol{h}^{(t-1)}}\right)$$

If $W_h$ is small, then this term gets vanishingly small as *i* and *j* get further apart

12 **Source**: "On the difficulty of training recurrent neural networks", Pascanu et al, 2013. http://proceedings.mlr.press/v28/pascanu13.pdf

## Vanishing gradient proof sketch

- Consider matrix L2 norms:

$$\left\| \frac{\partial J^{(i)}(\theta)}{\partial \boldsymbol{h}^{(j)}} \right\| \leq \left\| \frac{\partial J^{(i)}(\theta)}{\partial \boldsymbol{h}^{(i)}} \right\| \|\boldsymbol{W}_h\|^{(i-j)} \prod_{j<t\leq i} \left\| \operatorname{diag}\left(\sigma'\left(\boldsymbol{W}_h \boldsymbol{h}^{(t-1)} + \boldsymbol{W}_x \boldsymbol{x}^{(t)} + \boldsymbol{b}_1\right)\right) \right\|$$

- Pascanu et al showed that that if the largest eigenvalue of $W_h$ is less than 1, then the gradient $\left\| \frac{\partial J^{(i)}(\theta)}{\partial \boldsymbol{h}^{(j)}} \right\|$ will shrink exponentially
  - Here the bound is 1 because we have sigmoid nonlinearity

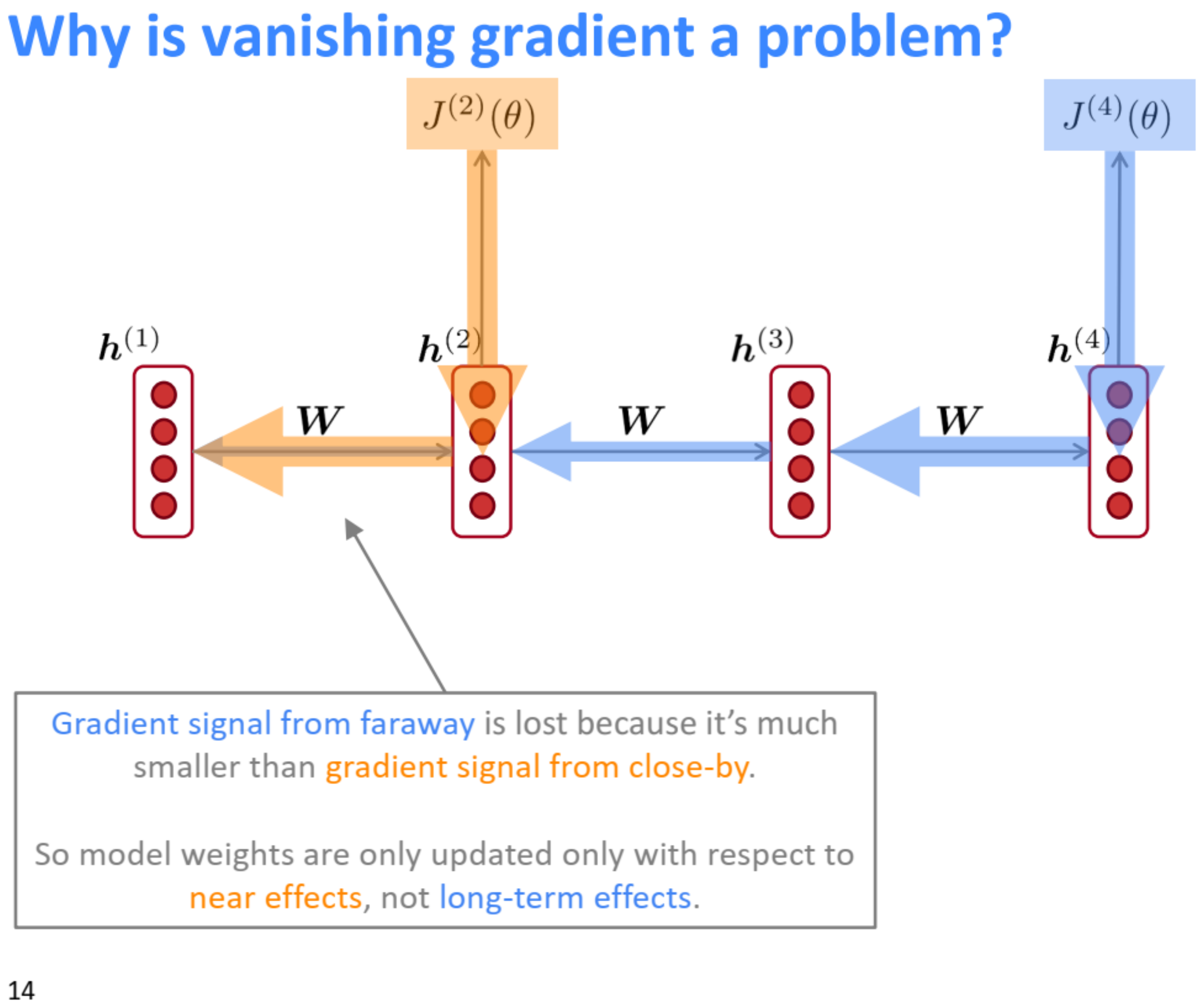
- There's a similar proof relating a largest eigenvalue >1 to exploding gradients

13 **Source**: "On the difficulty of training recurrent neural networks", Pascanu et al, 2013. http://proceedings.mlr.press/v28/pascanu13.pdf

**But why is this a problem?**

- The gradient from steps far away will be much smaller than gradients from close-by

→ The weights will only be updated with respect to close effects, not long-term !




**Remark:**

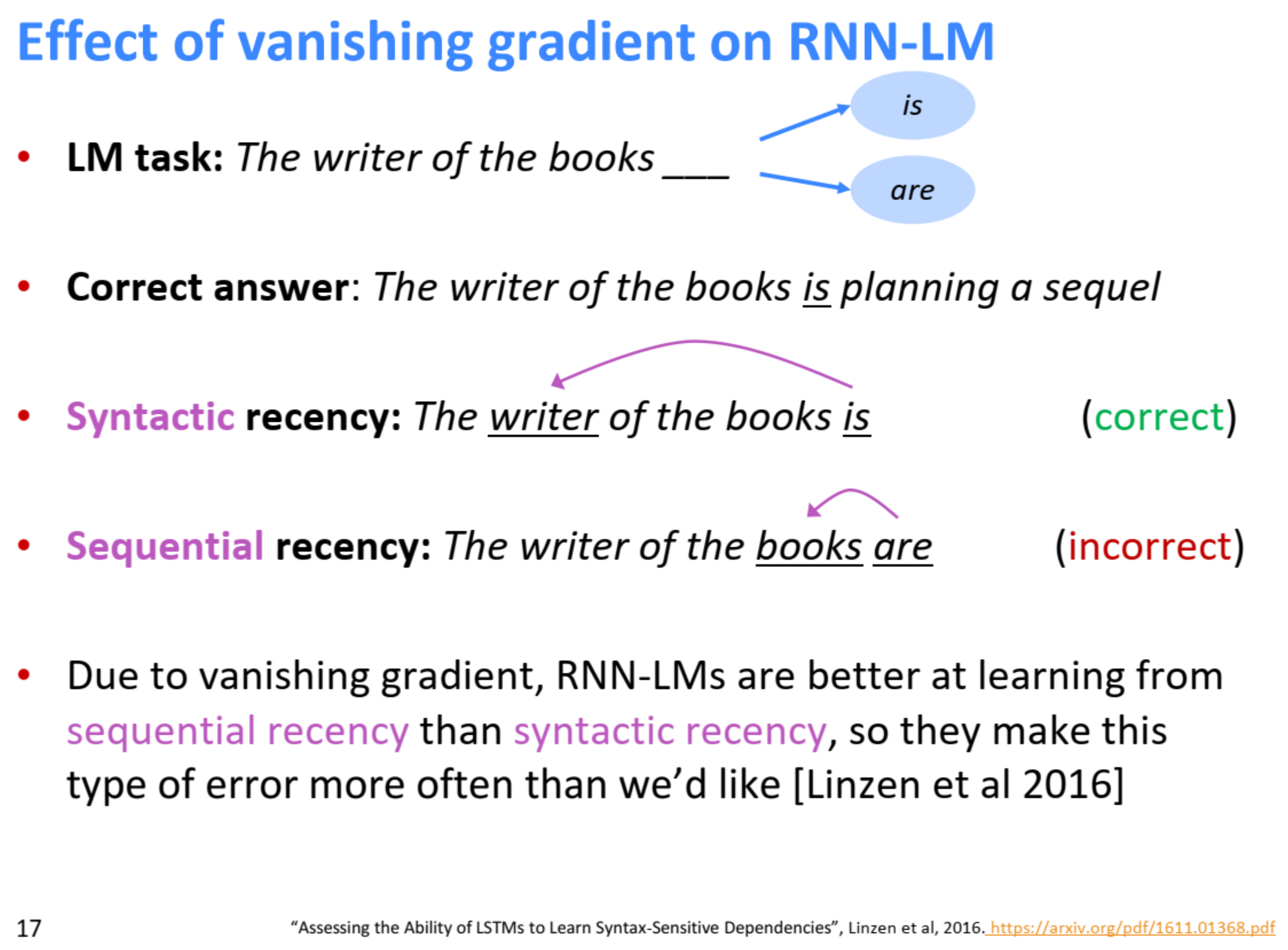
- We are studying $\frac{\partial J}{\partial h^{(i-1)}}$ because we have to calculate it to get $\frac{\partial J}{\partial W}$

**Effect on RNN-LM**

- Incapability of learning long-distance dependencies

- Syntactic recency (correct):
  Making dependencies to words close **in their syntax**
- Sequential recency (incorrect):
  Making dependencies to words close **spatially**

Unfortunately, RNN-LMs are better at learning sequential recency than syntactic recency:
(see example below)




### Exploding Gradient Problem

- If our gradient is too big, the update term for our parameter becomes too big

→ Take update steps too large, and get stuck in a local optimum

→ Get **Inf** or **Nan** values

**Solution**: Gradient clipping

- Scale down gradients that are higher than some thereshold

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## LSTM

- Reason to exist = To solve the vanishing gradient problem
- A RNN with ability to learn / preserve information from many timesteps ago

What about a RNN with separate memory?

### Structure

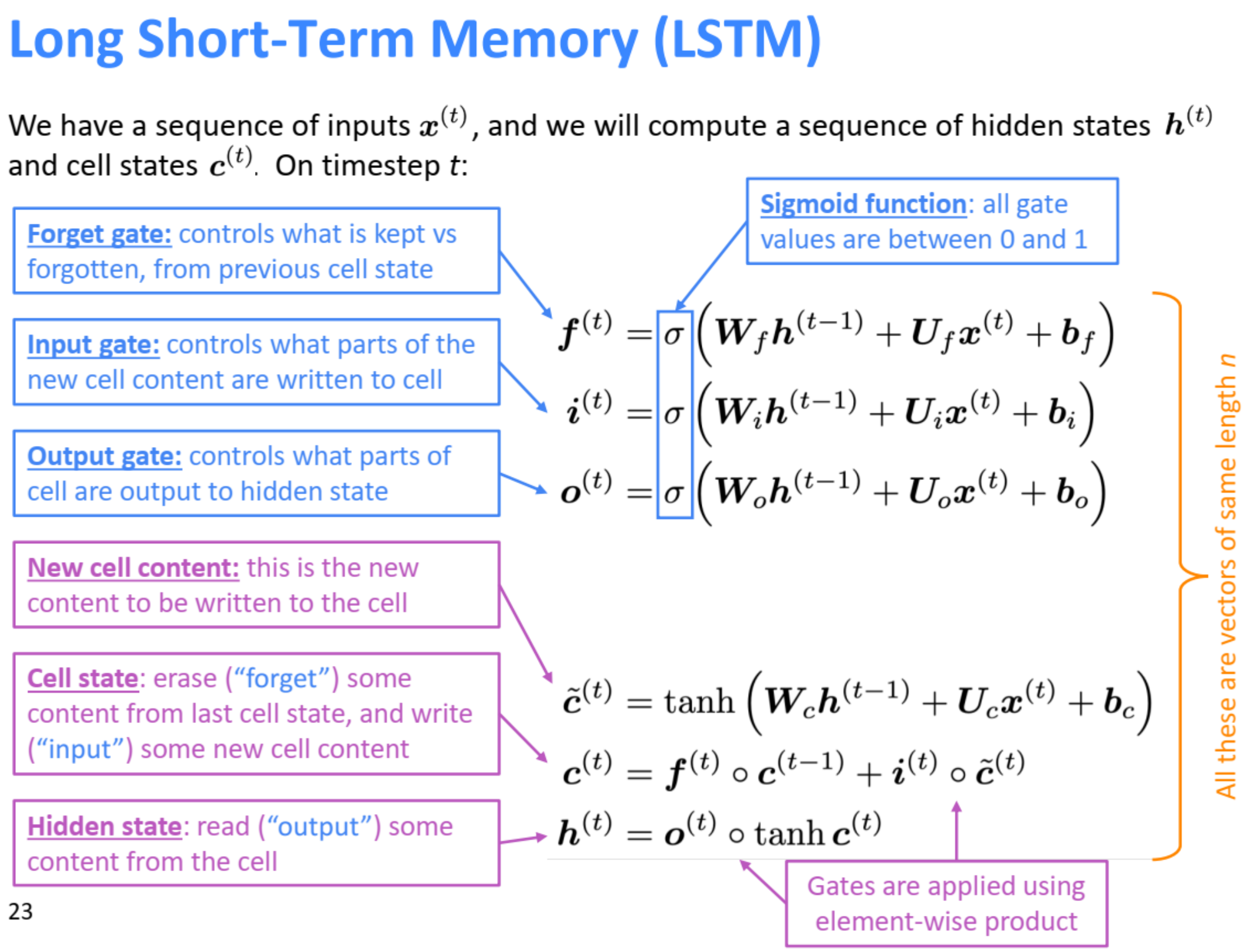
For each step $t$:

- a hidden state $h^{(t)}$: $(n, 1)$
- a cell state $c^{(t)}$: $(n, 1)$
  Stores long-term information

LSTM can erase, write and read information from cell !

This is done using gates of length $n$, whose values are in $[0, 1]$, thanks to our **sigmoid** friend:

- 1: open gate
- 0: closed gate




Cell state $c^{(t)}$ is the sum of:

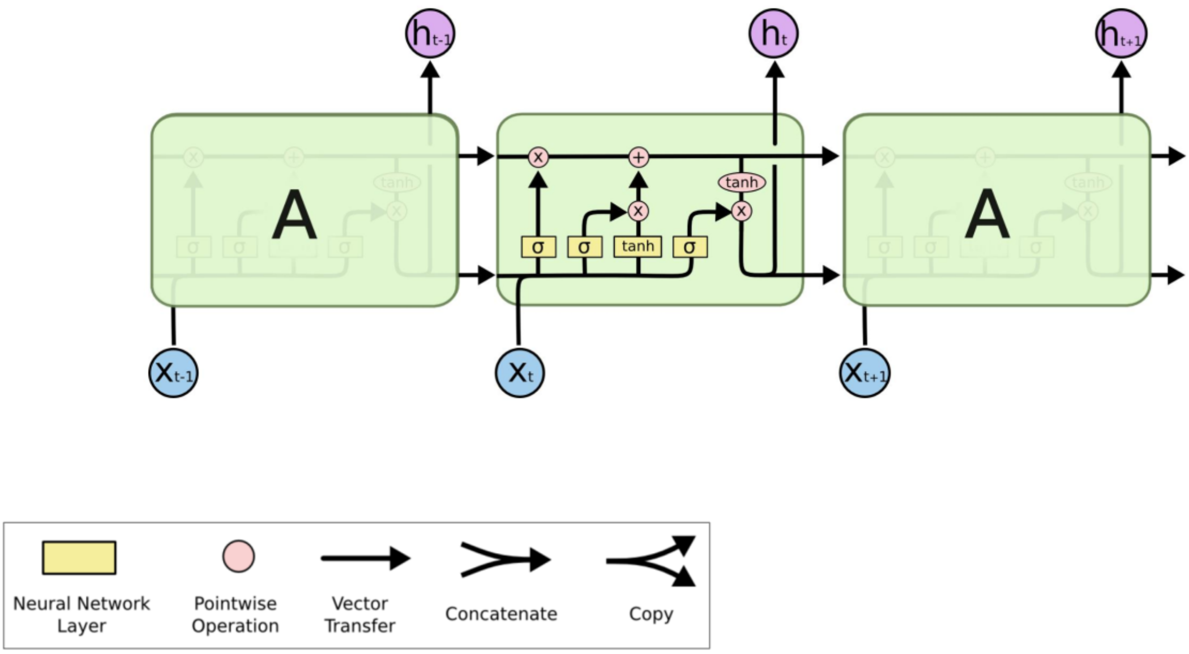
- previous cell state, masked by our forget gate
- our new cell, masked by our input gate

We can think of the hidden states as the ouput of the RNN

- These cell states ~ general memory, generally not accessible from the outside
- The hidden states will be passed on to the model

# Long Short-Term Memory (LSTM)

You can think of the LSTM equations visually like this:






**Source:** http://colah.github.io/posts/2015-08-Understanding-LSTMs/

# Long Short-Term Memory (LSTM)

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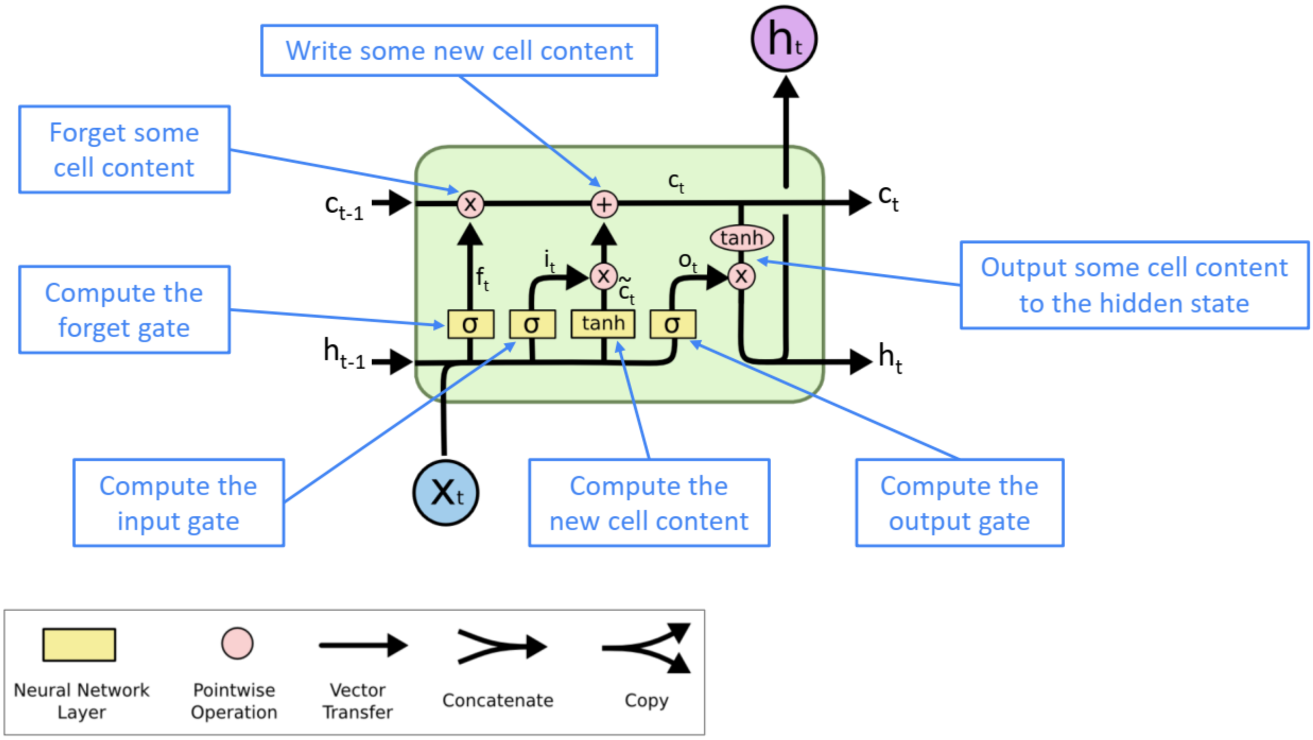


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**Source:** http://colah.github.io/posts/2015-08-Understanding-LSTMs/

It is possible to set the forget gate so that the LSTM remembers everything from each time step

/!\ We could still have vanishing/exploding gradients, even when using LSTMs !

- With RNN, the hidden states were a bottleneck:
  all gradients have to pass through them !!
- With LSTM, we can view the cell as a shortcut connection
  THere is a potential route within the cell that doesn't make the gradient vanish !

## Applications

- Handwriting recognition
- Speech recognition
- Machine translation
- Parsing
- Image captioning

However, more recently, Transformers seem to have taken over ! !

# Gated Recurrent Units (GRU)

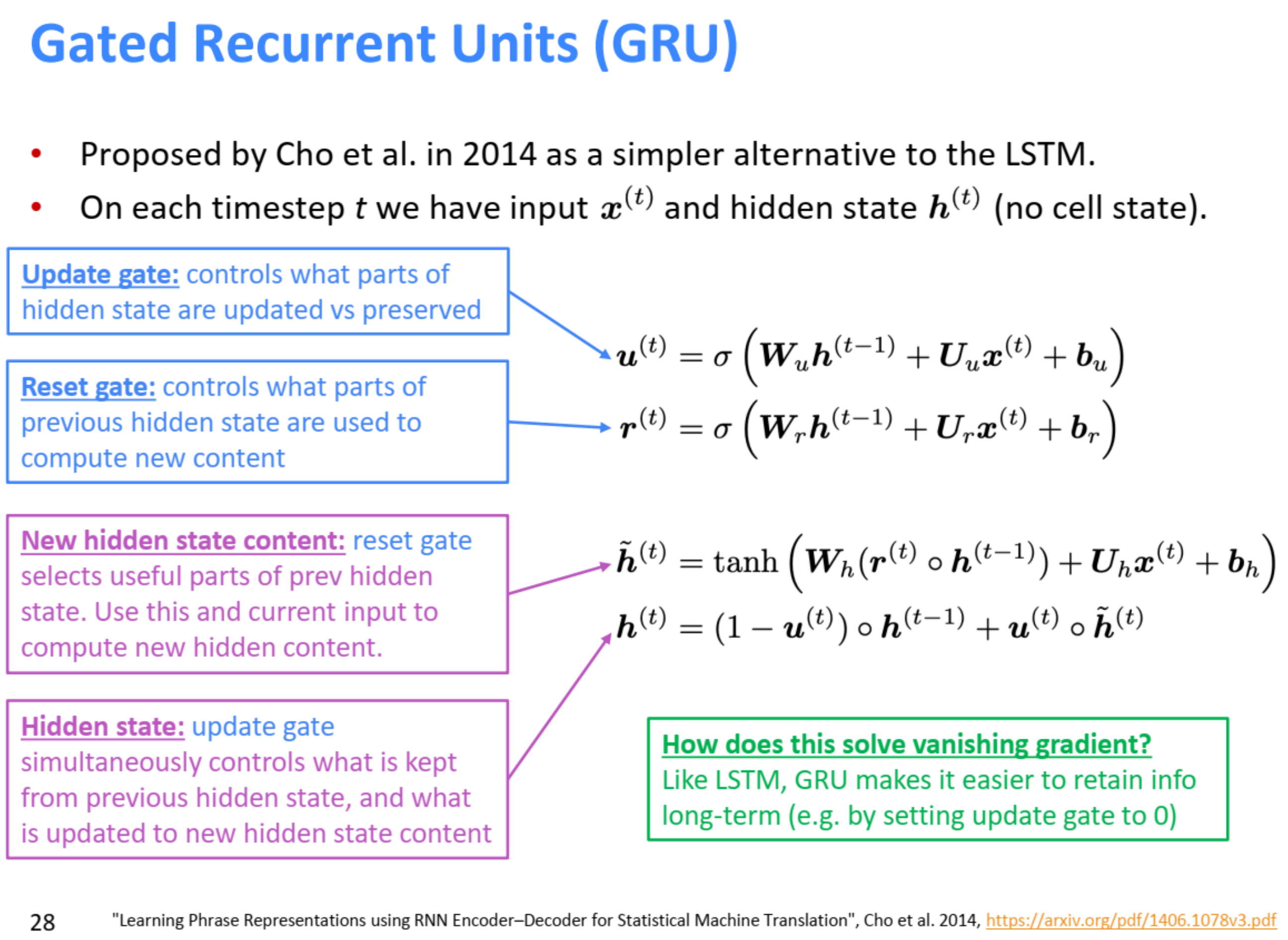
- Simpler alternative to LSTMs
- We have no cell state, only hidden states
- But we still have our cool gates!

## Gates

- Update gate:
  controls what is updated or preserved
  ~ input & forget gates for LSTM

- Reset gate:
  selects which parts of the previous state are useful

- News hidden state = combination of previous hidden state, and computed new content




GRU compared to LSTM:

- Quicker
- Less parameters
- Similar performances

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# Other fixes

## Gradient clipping

(See above)

## Skip connections

- Create direct connections from lower layers to newer layers
  → Allowing the gradients to flow more easily

Example: ResNet

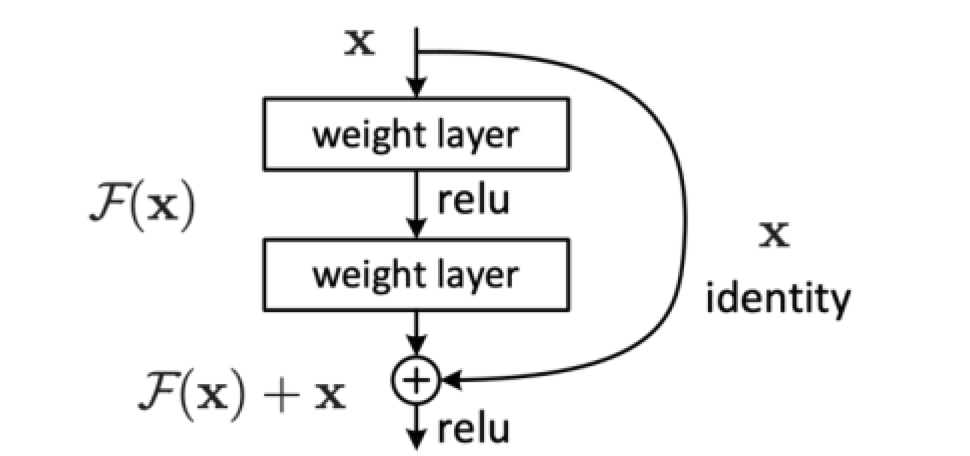


Figure 2. Residual learning: a building block.

- DenseNet:
  Going even further: connect every layer to each other!

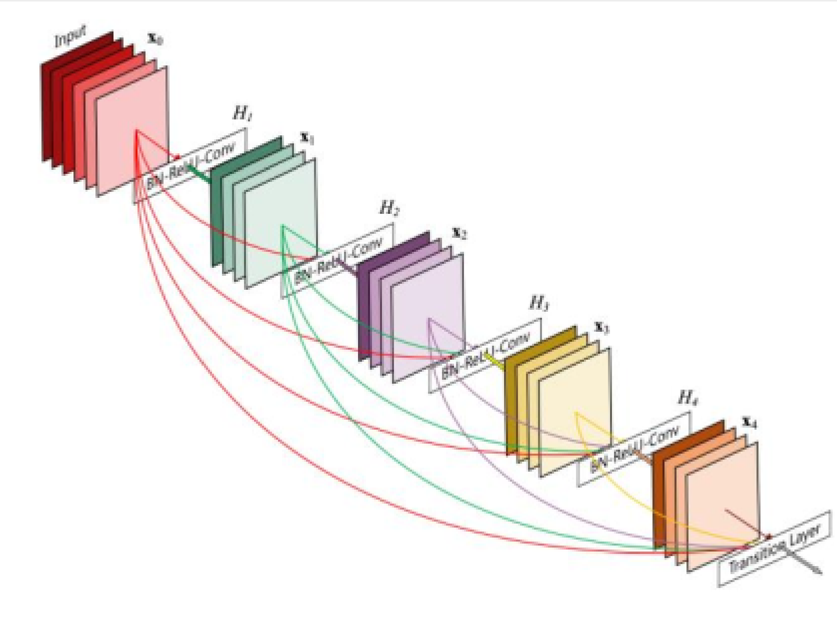


**Figure 1:** A 5-layer dense block with a growth rate of $k = 4$. Each layer takes all preceding feature-maps as input.

# Recap 1

## Recap

- Today we've learnt:
  - Vanishing gradient problem: what it is, why it happens, and why it's bad for RNNs
  - LSTMs and GRUs: more complicated RNNs that use gates to control information flow; they are more resilient to vanishing gradients

- Remainder of this lecture:
  - Bidirectional RNNs
  - Multi-layer RNNs

Both of these are pretty simple

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# More fancy RNN variants

## Bidirectional RNN

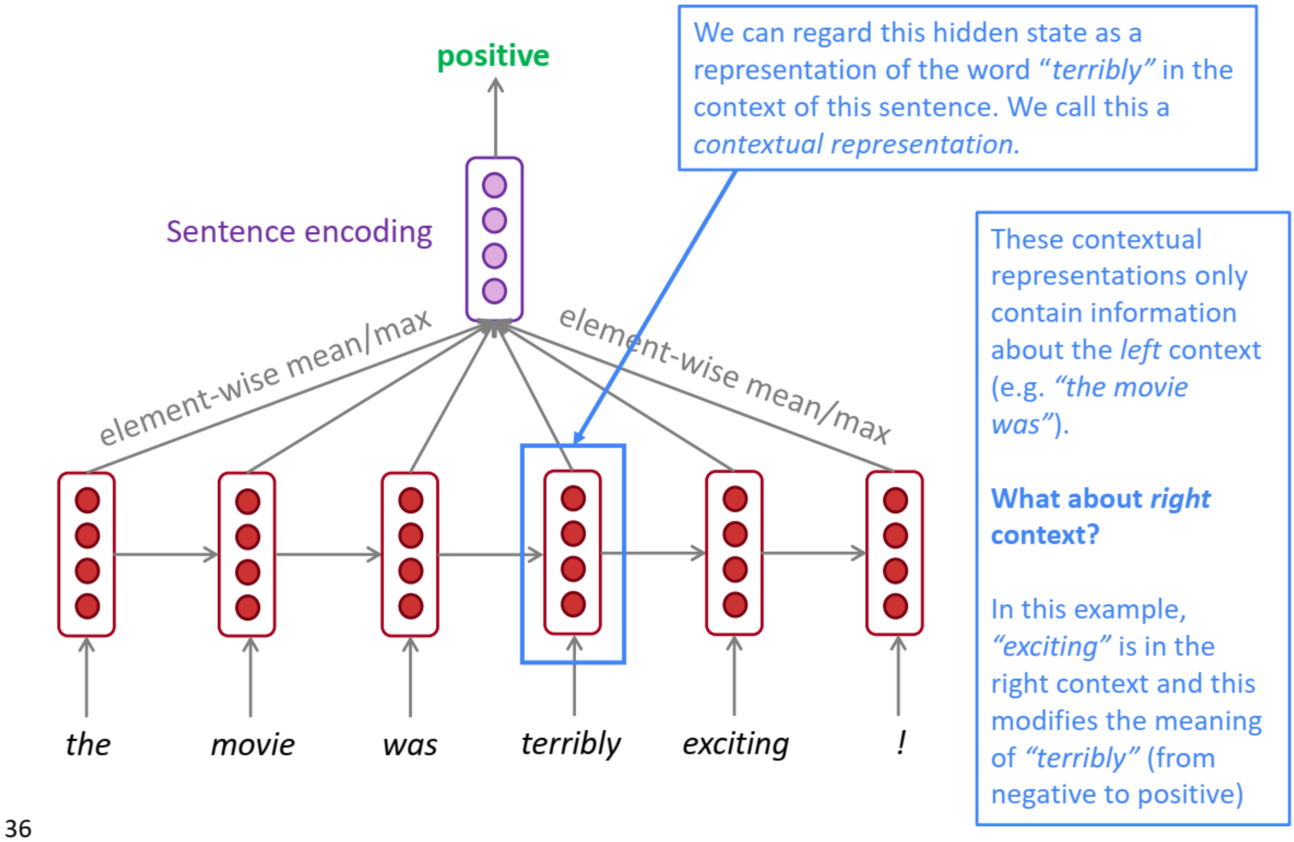
### Motivation

- Our contextual representation only gets information from the left context !

We might want to get context coming from the right direction as well !

# Bidirectional RNNs: motivation

Task: Sentiment Classification

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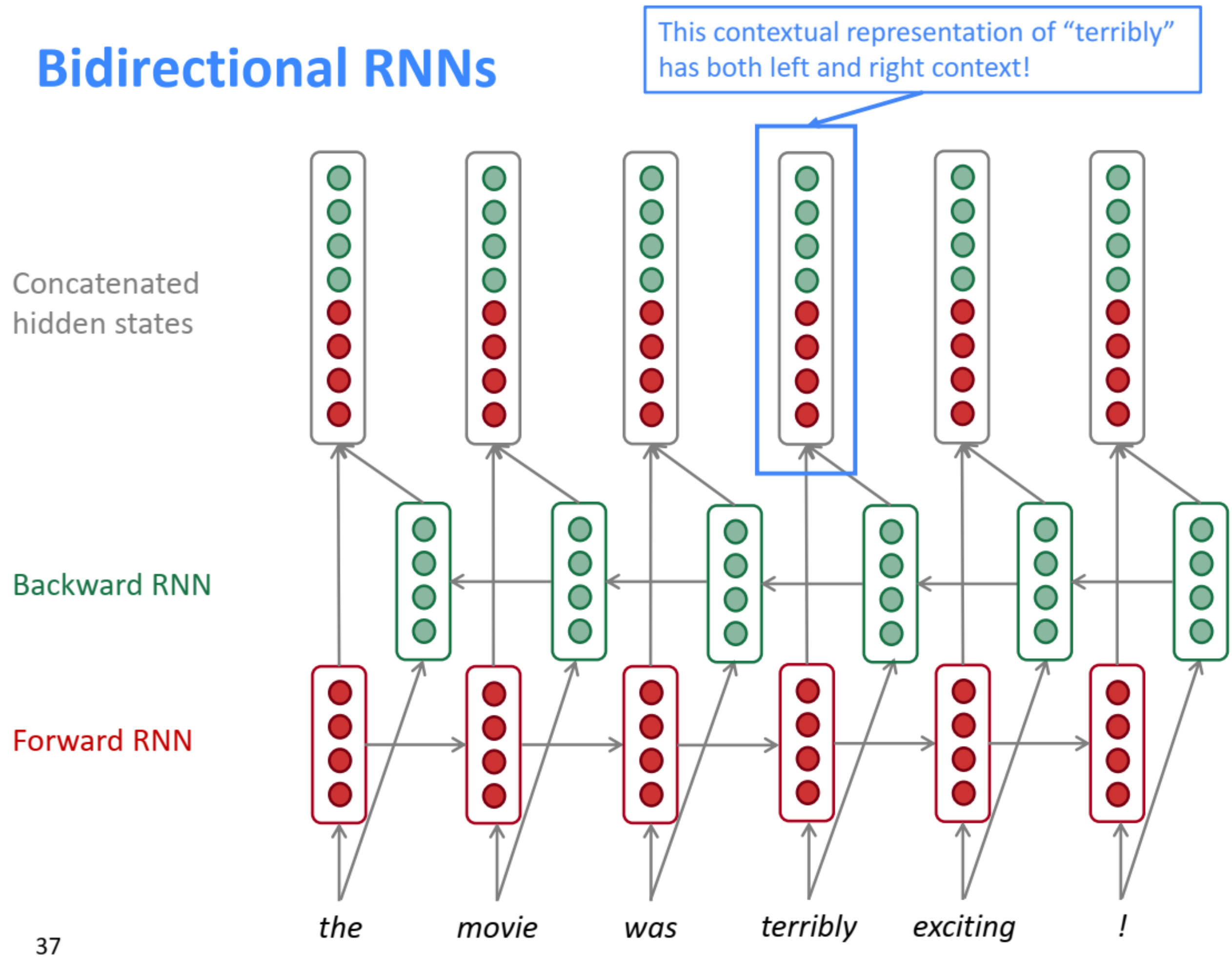
- We have 2 parallel RNN: one for each direction
- Hidden states = [Hidden state from left, Hidden state from right]



Bidirectional RNNs
This contextual representation of "terribly" has both left and right context!
Concatenated hidden states
Backward RNN
Forward RNN
the
movie
was
terribly
exciting
!
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**Notations**

## Bidirectional RNNs

On timestep *t*:

This is a general notation to mean "compute one forward step of the RNN" – it could be a vanilla, LSTM or GRU computation.

Forward RNN $\overrightarrow{\boldsymbol{h}}^{(t)} = \boxed{\mathrm{RNN}_{\mathrm{FW}}}(\overrightarrow{\boldsymbol{h}}^{(t-1)}, \boldsymbol{x}^{(t)})$

Backward RNN $\overleftarrow{\boldsymbol{h}}^{(t)} = \mathrm{RNN}_{\mathrm{BW}}(\overleftarrow{\boldsymbol{h}}^{(t+1)}, \boldsymbol{x}^{(t)})$

Generally, these two RNNs have separate weights

Concatenated hidden states $\boxed{\boldsymbol{h}^{(t)}} = [\overrightarrow{\boldsymbol{h}}^{(t)}; \overleftarrow{\boldsymbol{h}}^{(t)}]$

We regard this as "the hidden state" of a bidirectional RNN. This is what we pass on to the next parts of the network.

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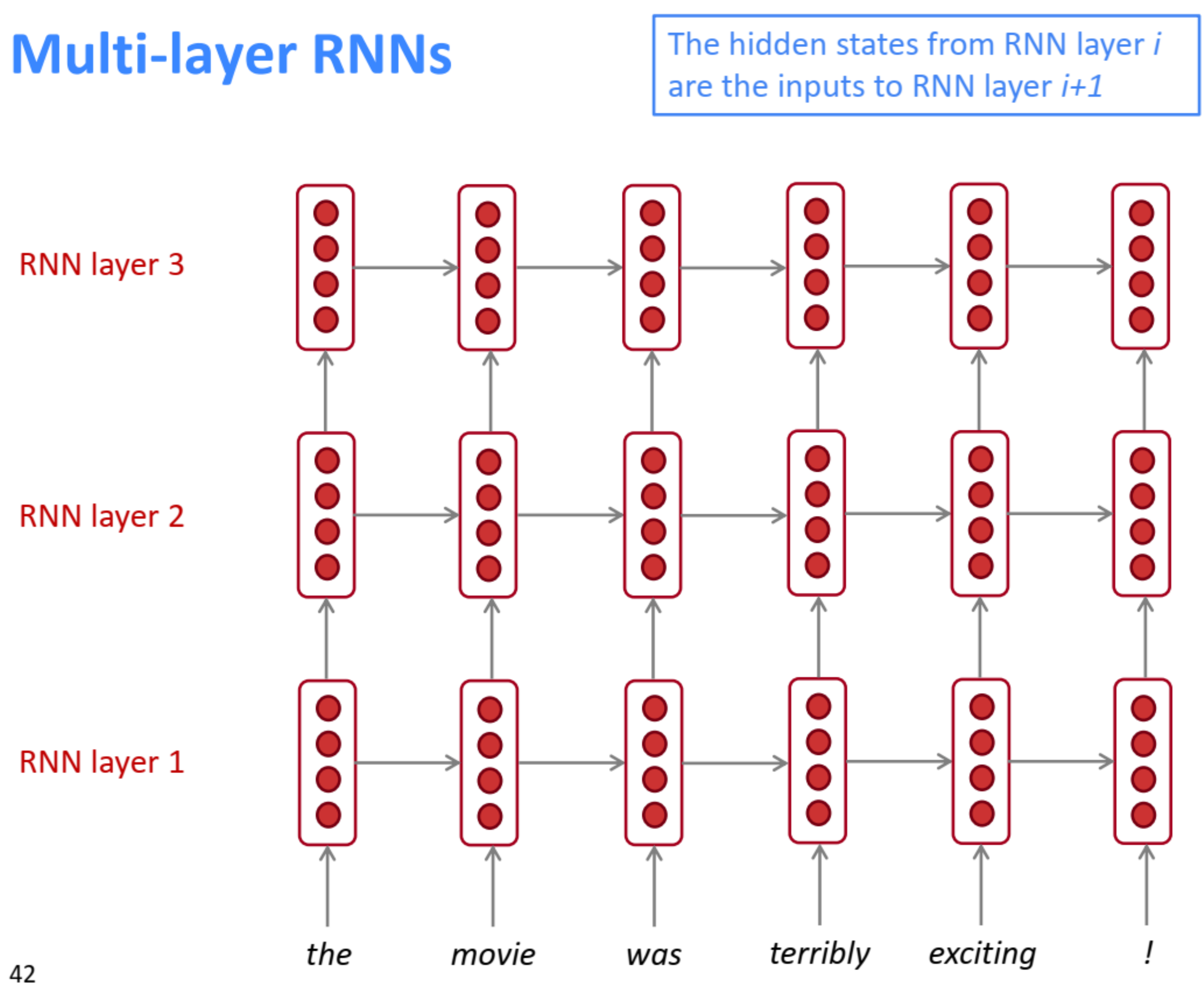
- The 2 RNNs are trained together, not separately

- Not useful for Language Modelling (as the Right context is missing, by definition! )

### Multi-layer RNN

- Using multiple RNNs
- Allows to compute more complex representations

- Hidden states of layer $i$ are the inputs to layer $i + 1$

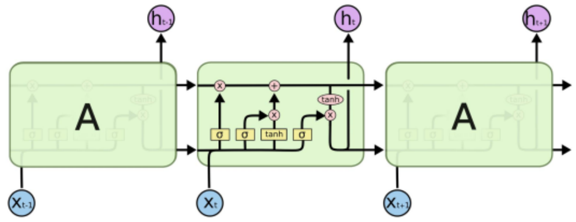



- RNNs have to be computed sequentially
  → Expensive to compute, can't go too deep

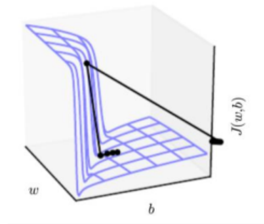
## Recap 2: Practical Takeaways

# In summary

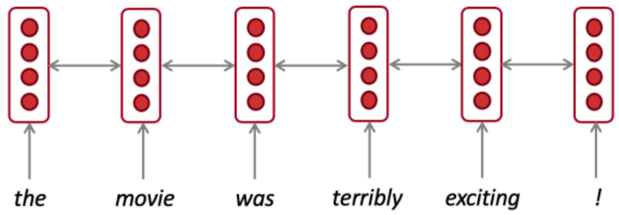
Lots of new information today! What are the practical takeaways?



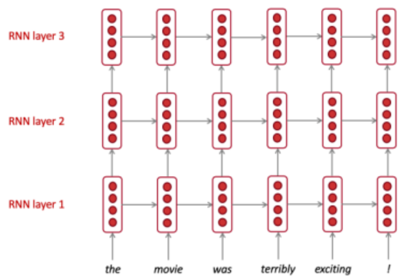

1. LSTMs are powerful but GRUs are faster




2. Clip your gradients




3. Use bidirectionality when possible




4. Multi-layer RNNs are powerful, but you might need skip/dense-connections if it's deep