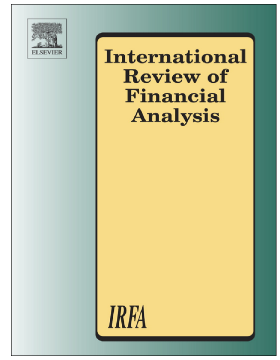

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Modeling Local Trends with Regime Shifting Models with Time-Varying Probabilities

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Modeling Local Trends with Regime Shifting Models with Time-Varying Probabilities

Abstract: In this paper we show that persistence and switching of trends are phenomena that appear in most long-lived stock return series. We model stock returns using a family of models based on hidden Markov models with duration-dependent transition probabilities. Trends are correlated so that aggregates such as indexes exhibit the same persistence and switching behavior as single stocks themselves. Hidden Markov models can thus explain medium-term momentum.

Keywords: regime shifting, hidden Markov models, duration-dependent Markov switching models, return models, momentum, reversals

JEL Classifications: C58, G10

1. INTRODUCTION

In this paper we investigate the persistence and switching of trends for most long-lived stock return series. We do so by modeling stock returns using a family of models based on hidden Markov models with duration-dependent transition probabilities. To do so we follow the empirical investigation proposed in Focardi, Fabozzi, and Mitov (2016). The key result of that paper is that a large universe of stock prices, such as the S&P500 universe, admits one integrated factor while all other factors are stationary processes. Their results were obtained by applying the methodology proposed by Nyblom and Harvey (2000) to estimate the number of integrated factors. Based on this finding, Focardi, Fabozzi, and Mitov (2016) describe a statistical arbitrage strategy that obtains a 27% annual return.

Moskowitz, Ooi, and Pedersen (2012) analyzed the persistence of returns in single time series. The authors make a distinction between two types of momentum. The first is the relative cross sectional momentum. In this type of momentum the difference of returns between the stocks that exhibited the highest returns and those that exhibited the lowest returns for periods of the order of 6-12 months will persist for several months. This property allows to construct long-short market neutral portfolios. The second type of momentum of consists in the permanence of returns time series of single stocks. This type of momentum allows to select stocks that will exhibit high returns.Analyzing stock prices over a long period, Hurst, Ooi, and Pedersen (2017) reported that persistence of stock returns would have allowed profitable strategies since 1880.

In this paper we continue the analysis of the drivers of stock prices considering the behavior of the same universe from the point of view of regimes. We empirically analyze the regime-shifting behavior of stock returns in a large universe of S&P500 stock return processes over the 36-year period 1980-2016. We find that stock returns are subject to shifts in mean and variance and that the regimes of different stocks are highly correlated.

In Fabozzi, Focardi, and Mitov (2016) analyzed the behavior of logprices and concluded that in large markets there is only one integrated stochastic trend. In this paper we analyze the behavior of logreturns and conclude that there is a fundamental similarity for local trends.

Any model of stock returns must take into account a number of stylized facts that have been revealed by empirical analysis. In particular (1) the autocorrelation of stock returns is very weak except possibly at very short time lags; (2) the square of returns and the absolute value of returns are persistent; (3) returns of large portfolios are autocorrelated, (4) stock returns exhibit momentum and reversal phenomena and (5) the distribution of returns is not normal but it has fat tails. (See Cont, 2001.)

Vector autoregressive (VAR) models do not easily explain the above stylized facts. In particular, momentum and reversals suggest explanations based on the persistence and reversals of trends. Hence, a possible model for stock returns should combine vector autoregressions with the switching of regimes. Markov switching VAR (MSVAR) models are vector autoregressions where the parameters of the autoregressions depend on the regime, with Markov chains driving the switching of regimes.

We look at trends as local trends, subject to switching between regimes of high expected returns (positive trends) and regimes of low expected returns (negative trends). In this paper, we propose a model of returns that offers a global framework for momentum and reversals.

Our approach employs regime-shifting models with time-varying probabilities to represent returns. For a large sample of stocks covering the January 1980-December 2016 period, we show that the permanence of expected returns is not a phenomenon limited to the highest and lowest returns but is diffused throughout the entire sample. In fact, the vast majority of stocks exhibit permanence of expected returns. Using duration-dependent Markov switching models, we show that there is a variety of switching regimes. In some stocks one regime prevails in the sense that it exhibits growing probability to stay in the same regime. In other stocks both regimes show increasing probabilities to stay in the same regime. In addition, trend switching is highly coordinated: stocks tend to remain in the same regime (e.g., high return or low return), resulting in the fact that the switching behavior is detected not only in individual stocks but also in indexes.

The paper is organized as follows. In Section 2 we provide an introduction to hidden Markov models with a fixed transition matrix and an introduction to duration-dependent Markov switching models,. In Section 3, we discuss empirical results obtained with both these families models. In Section 4 we discuss the application of these models. Our conclusions are summarized in Section 5.

2. THE MODEL

Initially proposed by Quandt (1958) and Goldfeld and Quandt (1973), regime-shifting models became a mainstream econometric tool through the work of Hamilton (1989, 1990). Hamilton proposed a Bayesian filter to compute loglikelihood so that estimation could be performed with usual maximum likelihood estimation (MLE) methods. Hamilton also proposed the expectation-maximization (EM) algorithm as an effective way to estimate these models. Diebold, Lee, and Weinbach (1994) extend the EM methodology to regime-shifting models with time-varying probabilities driven by exogenous variables. Kim, Piger, and Starz (2008) allow the regime switching to be driven by endogenous variables while Durland and McCurdy (1994) allow the transition probabilities to be driven by duration in the current state.

The simplest implementation of regime-shifting models are regressions whose parameters change in function of a two-state Markov chain. Samuelson (1991) calls regime-shifting models of this type a *momentum model* if the transition matrix exhibits a higher probability to stay in the current state than to switch to another state and a *rebound model* in the opposite case. Ryden et al. (1998) analyze how Markov switching models can reproduce stylized facts of asset returns. Ang and Timmermann (2012) and Guidolin (2011) provide comprehensive reviews of applications of Markov switching models in finance.

Among the papers most closely related to our work, Schaller and Van Norden (1997), Maheu and McCurdy (2000a, 2000b), and Alexander and Lazar (2009) investigate the switching behavior of stock returns. These papers conclude in favor of a switching behavior of stock returns at the level of indexes. Kanas (2003), Guidolin et al. (2009), and Pettenuzzo and Timmermann (2012) investigate the predictability of returns with Markov switching. There is agreement that predictability changes in function of regimes. The identification and forecast of bull and bear markets is discussed in McQueen and Thorley (1994), Lunde and Timmermann (2004), and Kole and van Dijk (2011).

The approach has been also applied in more recent studies, like to the pricing of options as in Han (2018) or to the modeling and analysis of stock return movements, exchange rates and interest rates as in Kim (2017). Some other recent works proposed an enhanced version of regime-switching model, as in Chang (2017), where the proposed approach uses model regime switching through an autoregressive latent factor able to forecast the regime-switching according to a threshold level of the factor, or a modified Time-Varying Transition Probability version of model regime-switching, as in Bazzi (2016), where the classical TVTP approach has been slightly modified by considering transition probabilities evolving over time by means of an observation driven model (time-varying probability is generated as a score of a predictive likelihood function).

We propose a simplified model of (log)prices where logreturns are represented as a hidden Markov model (HMM). In this model, logprices are conditionally normal. Unconditional distributions, however, exhibit fat tails due to the mixture of normal variables.

Qualifications are needed. As we will see in the next section, most stocks do not comprise a stock index for very long periods of time. In order to apply HMM models long series of returns are needed. Therefore, HMMs are applicable only to a subset of stocks. Our model analyzes the behavior of stocks that exist for long periods of time, i.e., 36 years. We will first discuss a model with fixed transition probabilities and then introduce a variable probability model.

Note that we are interested in modelling trends over long time periods, in the range of weeks or months. High frequency data might reveal micro regimes perhaps intra-day. In this work we are not interested in this type of behavior.

2.1. *Hidden Markov Models with Fixed Probabilities*

Let's first consider HMM with fixed transition probabilities. In HMM we observe a sequence of emissions from states that are not observed. States are assumed to evolve as a first-order Markov chain with N states: s_i, i = 1, 2,… N.. The time between state transition is fixed. Let's assume that the transition between states is governed by a probability transition matrix **T**:

$$\mathbf{T} = \begin{bmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & \ddots & \vdots \\ p_{N1} & \cdots & p_{NN} \end{bmatrix}, \quad \sum_{j=1}^{N} p_{ij} = 1 \qquad (1)$$

where p_{ij} is the probability of transition from state s_i at time $t-1$ to state s_j at time t: $p_{ij} = P\left(s_j \middle| s_i\right)$. In each state s_i the system emits the observable variable y following a normal distribution $y \approx N(\mu(s_i), \sigma(s_i))$.

We will first suppose that there are only two states and that the transition matrix relative to stock i is the following fixed and time-invariant matrix:

$$T_i = \begin{bmatrix} p_{i,11} & p_{i,12} \\ p_{i,21} & p_{i,22} \end{bmatrix}, \; p_{i,11} = p_i, p_{i,12} = 1 - p_i, p_{i,21} = 1 - q_i, p_{i,22} = q_i \qquad (2)$$

We write our model of logreturns for each stock as follows:

$$r_{i,t} = p_{i,t} - p_{i,t} = \delta_i(s) + \sigma_i(s)\varepsilon_{i,t} \tag{3}$$

where both coefficients $\delta_i(s)$ and $\sigma_i(s)$ depend on the state *s*. The average sojourn time of stock *i* in state *j*, that is, the time stock *i* spends in state *j*, is $AT_{ij} = 1/(1 - p_{i.jj})$.

We assume only two states and do not assume any dynamics within each state essentially to limit the number of parameters to estimate. Since we want to analyze duration-dependent Markov switching models, using more states and/or adding a dynamic to each state would increase an already large number of parameters to estimate. Therefore, our choice has to be considered a trade-off between analyzing duration dependence versus capturing short-term dynamics.

We assume a transition matrix for each stock. The regime-shifting model captures the state transitions of each stock return process. The market-state transitions such as the transition between bull and bear markets will emerge through the correlation between trend switching of all stocks.

2.2 *Hidden Markov Models with Time-Varying Transition Probabilities*

We now relax the assumption that the probabilities of the transition matrix are constant. *Time-varying transition probabilities* (TVTP) have been described in the literature. Several different ways of representing and estimating TVTP have been proposed. We adopt the model proposed by Durland and McCurdy (1994) called *Duration-Dependent Markov Switching* (DDMS) where the transition probabilities depend on the duration of the system in each state. The objective is to understand if the probability of staying in a state increases or decreases with the time the stock has already spent in that state.

In particular, in the case of a two-state system, the probability to move from state *i* at time *t*–1 to state *j* at time *t* depends on the time the system has already spent in state *i*.

$$p_{ij,d} = P\left(S_t = s_j \middle| S_{t-1} = s_i, D_{t-1} = d\right),\ i = 1,2 \tag{4}$$

where *d* is the number of periods the system has spent in state *i*. For example, if

$S_{t-1} = 1, S_{t-2} = 1, S_{t-3} = 2$, then duration is equal 2. Durland and McCurdy assume that the system has a finite memory τ; that is, the transition probability is conditioned at most by the past τ steps and is independent from step $\tau + 1$.

A Markov chain with finite memory τ is called an τ-order Markov chain. It is well known that a 2-state τ-order Markov chain is equivalent to a first-order 2τ-state Markov chain. To illustrate this point, consider a 2-state Markov chain with states $s_1 = 1,\ s_2 = 2$. Suppose the chain has a memory $\tau = 3$ Consider a new set of four expanded states formed as ordered pairs of states and durations $ES_t = (s_i, d)$ and labeled from e_1 to e_6: $e_1 \equiv (s_1,1), e_2 \equiv (s_1,2), e_3 \equiv (s_1,3), e_4 \equiv (s_2,1), e_5 \equiv (s_2,2), e_6 \equiv (s_2,3)$. In each pair, the first number is the state and the second number is the duration.

We can write down explicitly the sequences that characterize expanded states as follows:

$$(S_{t-1} = 2, S_t = 1) \to ES_t = e_1 \equiv (1,1) \qquad (S_{t-1} = 1, S_t = 2) \to ES_t = e_4 \equiv (2,1)$$
$$(S_{t-2} = 2, S_{t-1} = 1, S_t = 1) \to ES_t = e_2 \equiv (1,2) \qquad (S_{t-2} = 1, S_{t-1} = 2, S_t = 2) \to ES_t = e_5 \equiv (2,2)$$
$$(S_{t-2} = 1, S_{t-1} = 1, S_t = 1) \to ES_t = e_3 \equiv (1,3) \qquad (S_{t-2} = 2, S_{t-1} = 2, S_t = 2) \to ES_t = e_6 \equiv (2,3)$$

The following table represents a possible sequence of seven time steps. The second row represents states *s* while the third row represents expanded states *e*. At time *t*-3 the expanded state ES_{t-3} is not determined from the sequence. At time *t*-2 $ES_{t-2} = e_4$ because $S_{t-2} = s_2$ and its duration is 1 as $S_{t-3} = s_1$ and so on for the following steps.

time	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$
states	s_1	s_2	s_1	s_1	s_1	s_2	s_2
expanded states	$\cdots$	e_4	e_1	e_2	e_3	e_1	e_2

We can now compute the probabilities of transition from extended states in function of the probabilities $p_{ij,d}$. When in extended state e_i, the system can be followed by either s_1 or s_2 and we can write the following table:

$t-1$	ES_{t-1}	S_t	t	ES_t	P_{ij}
$(\cdots,s_2,s_1)$	$e_1 \equiv (s_1,1)$	s_1	$(\cdots,s_2,s_1,s_1)$	$e_2 \equiv (s_1,2)$	$P_{12} = p_{11,1}$
$(\cdots,s_2,s_1)$	$e_1 \equiv (s_1,1)$	s_2	$(\cdots,s_2,s_1,s_2)$	$e_4 \equiv (s_2,1)$	$P_{14} = p_{12,1}$
$(\cdots,s_2,s_1,s_1)$	$e_2 \equiv (s_1,2)$	s_1	$(\cdots,s_2,s_1,s_1,s_1)$	$e_3 \equiv (s_1,3)$	$P_{23} = p_{11,2}$
$(\cdots,s_2,s_1,s_1)$	$e_2 \equiv (s_1,2)$	s_2	$(\cdots,s_2,s_1,s_1,s_2)$	$e_4 \equiv (s_2,1)$	$P_{24} = p_{12,2}$
$(\cdots,s_1,s_1,s_1)$	$e_3 \equiv (s_1,3)$	s_1	$(\cdots,s_1,s_1,s_1,s_1)$	$e_3 \equiv (s_1,3)$	$P_{33} = p_{11,3}$
$(\cdots,s_1,s_1,s_1)$	$e_3 \equiv (s_1,3)$	s_2	$(\cdots,s_1,s_1,s_1,s_2)$	$e_4 \equiv (s_2,1)$	$P_{34} = p_{12,3}$
$(\cdots,s_1,s_2)$	$e_4 \equiv (s_2,1)$	s_1	$(\cdots,s_1,s_2,s_1)$	$e_1 \equiv (s_1,1)$	$P_{41} = p_{21,1}$
$(\cdots,s_1,s_2)$	$e_4 \equiv (s_2,1)$	s_2	$(\cdots,s_1,s_2,s_2)$	$e_5 \equiv (s_2,2)$	$P_{45} = p_{22,1}$
$(\cdots,s_1,s_2,s_2)$	$e_5 \equiv (s_2,2)$	s_1	$(\cdots,s_1,s_2,s_2,s_1)$	$e_1 \equiv (s_1,1)$	$P_{51} = p_{21,2}$
$(\cdots,s_1,s_2,s_2)$	$e_5 \equiv (s_2,2)$	s_2	$(\cdots,s_1,s_2,s_2,s_2)$	$e_6 \equiv (s_2,3)$	$P_{56} = p_{22,2}$
$(\cdots,s_2,s_2,s_2)$	$e_6 \equiv (s_2,3)$	s_1	$(\cdots,s_2,s_2,s_2,s_1)$	$e_1 \equiv (s_1,1)$	$P_{61} = p_{21,3}$
$(\cdots,s_2,s_2,s_2)$	$e_6 \equiv (s_2,3)$	s_2	$(\cdots,s_2,s_2,s_2,s_2)$	$e_6 \equiv (s_2,3)$	$P_{66} = p_{22,3}$

For example, P_{23} is the probability that the sequence of states $(S_{t-3}=s_2, S_{t-2}=s_1, S_{t-1}=s_1)$ will continue into the sequence $(S_{t-3}=s_2, S_{t-2}=s_1, S_{t-1}=s_1, S_{t-1}=s_1)$. P_{23} is equal to the probability that the state s_1 with duration 2 will remain unchanged.

Note that there are six extended states e_i and therefore, in principle, 36 transition probabilities of which only 12 are different from zero because many transitions are not possible. For example, the transition from e_6 to e_4 is impossible. Hence the following sparse 6×6 transition matrix applies:

$$\{P_{ij}\} = \begin{bmatrix} 0 & P_{12} & 0 & P_{14} & 0 & 0 \\ 0 & 0 & P_{23} & P_{24} & 0 & 0 \\ 0 & 0 & P_{33} & P_{34} & 0 & 0 \\ P_{41} & 0 & 0 & 0 & P_{45} & 0 \\ P_{51} & 0 & 0 & 0 & 0 & P_{56} \\ P_{61} & 0 & 0 & 0 & 0 & P_{66} \end{bmatrix} \tag{5}$$

In this way, the original 2-state, 3-order Markov chain is transformed into an equivalent 6-state first order chain. Note that each row has only two non-zero entries because the chain can only move to one of two states. We can generalize this example and write the following rules:

$$
\begin{aligned}
& j = 1, \tau + 1 \le i \le 2\tau \Rightarrow P_{ij} = p_{21,i-\tau} \\
& j = \tau + 1, 1 \le i \le \tau \Rightarrow P_{ij} = p_{12,i} \\
& 1 < j < \tau, i = j - 1 \Rightarrow P_{ij} = p_{11,i} \\
& \tau + 1 < j < 2\tau, i = j - 1 \Rightarrow P_{ij} = p_{22,i-\tau} \\
& P_{\tau\tau} = p_{11,\tau} \\
& P_{2\tau 2\tau} = p_{22,\tau}
\end{aligned}
\tag{6}
$$

In order to capture a reasonable memory depth, the number of parameters to estimate in full generality rapidly becomes excessive. For example, with a memory depth of six periods, we need 12 states and would therefore need to estimate 24 parameters.

In order to solve this problem, Durland and McCurdy (1994) assume that the duration-dependent transition probabilities are parameterized as logistic functions of the duration as follows:

$$
\begin{aligned}
& p(d) \equiv p_{11,d} = \frac{\exp(\beta_1 + \beta_2 d)}{1 + \exp(\beta_1 + \beta_2 d)} \\
& p_{12,d} = 1 - p_{11,d} = \frac{1}{1 + \exp(\beta_1 + \beta_2 d)} \\
& q(d) \equiv p_{22,d} = \frac{\exp(\beta_3 + \beta_4 d)}{1 + \exp(\beta_3 + \beta_4 d)} \\
& p_{21,d} = 1 - p_{22,d} = \frac{1}{1 + \exp(\beta_3 + \beta_4 d)}
\end{aligned}
\tag{7}
$$

We can rewrite the transition matrix of the previous example as follows:

$$
\{P_{ij}\} = \begin{bmatrix}
0 & p(1) & 0 & 1 - p(1) & 0 & 0 \\
0 & 0 & p(2) & 1 - p(2) & 0 & 0 \\
0 & 0 & p(3) & 1 - p(3) & 0 & 0 \\
1 - q(1) & 0 & 0 & 0 & q(1) & 0 \\
1 - q(2) & 0 & 0 & 0 & 0 & q(2) \\
1 - q(3) & 0 & 0 & 0 & 0 & q(3)
\end{bmatrix}
\tag{8}
$$

The choice of the logistic function is suggested essentially by mathematical convenience. The logistic function ensures that probabilities always remain confined in the (0.1) interval. In addition, it is flexible enough to represent either a sharp transition between two regimes, a steady growth, or a steady decay. If $\beta_2 > 0$, then the probability $p(d)$ grows with *d* and tends to 1 when

d tends to infinity. If $\beta_2 < 0$ then $p(d)$ decreases and tends to 0 when d tends to infinity. The same considerations apply for β_4 and the probability $q(d)$.

With this logistic function parameterization, we cannot pretend to faithfully capture the dependence of the transition matrix on duration. However, we can reasonably expect to understand if the probability of switching increases or not with duration.

2.2. *The Estimation of HMM*

Two main methods have been proposed for estimating HMMs: Gibbs sampling and MLE method. We have chosen the MLE method which maximizes the likelihood of the model on the sample data. Direct maximization of likelihood on all possible paths of the hidden states is not computationally feasible. Hamilton (1989) introduced a recursive filter to estimate the loglikelihood conditional on r initial data:

$$L_y(\vartheta) = \log p(y_T, \ldots, y_1 | y_0, \ldots, y_{-r+1}, \vartheta) \tag{9}$$

where ϑ is the set of parameters of the model.

The loglikelihood $L_y(\vartheta)$ is a highly nonlinear function of the parameters ϑ It can be maximized using an optimization software. To estimate our models we developed our own software. We implemented the estimation procedures in Matlab, using function *fminsearch* for the direct optimization of the loglikelihood $L_y(\vartheta)$.

To estimate the DDMS, we implemented the algorithm in Durland and McCurdy (1994). First we define the transition matrix (8) for the equivalent fixed transition HMM. Using this transition matrix, we compute the loglikelihood function and maximize the loglikelihood using the Matlab function *fminsearch.* Initial values are considered parameters to be maximized. In the Appendix the Matlab code used to estimate the loglihood is provided.

As a check, we also estimated our models using the software MS_Regress made available on-line by Professor Marcelo Perlin of the Federal University of Rio Grande do Sul, Porto Alegre, Brazil. Using MS-Regress we obtained the same results as with our own software.[1]

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[1] MS_Regress is available at https://sites.google.com/site/marceloperlin/matlab-code/ms_regress---a-package-for-markov-regime-switching-models-in-matlab. AlthoughMS_Regress works well, we developed our own program because we needed to estimate duration-dependent models.

2.3. *The Empirical Setting*

Before analyzing our empirical results, we have to discuss our empirical settings and its limitations. HMM are long-run models of stock behavior. As we are looking for switches in regimes (i.e., switches of local trends with an expected duration of two or three years), we need to study samples formed by long time series that include several regime switches. However, as already mentioned, individual stocks do not generally have long lives.

In fact, consider our sample which is formed of all stocks that have been in the S&P500 universe for the period January 1980 to December 2016. We consider returns over consecutive periods of 20 days. This granularity is approximately equivalent to monthly returns but it simplifies computations. There are 466 20-day periods in our sample and a total of 1,284 stocks. However, due to various corporate actions, many S&P500 stocks begin or cease to exist during the considered period. Only 207 stocks exist on the period considered in its entirety; the return process of these stocks form the sample of interest for our analysis.

This creates serious challenges for analyzing long-run stock behavior. Simply put, we cannot analyze the long-run behavior of stocks that do not exist for long periods. Our analysis is limited to those stocks that exist for the entire period under examination.

We can analyze stocks that survive for a long time, in particular we can analyze their switching behavior. What we cannot do is draw conclusions that involve short-lived stocks; Nevertheless we can still learn a lot about the switching behavior of long-lived stocks.

The literature thus far has focused on analyzing trend-shifting behavior in aggregated indexes and not individual stocks. In this way, the problem of how to deal with short-lived stocks is circumvented. However, the start up or closing of firms and corporate actions such as through mergers and acquisitions might induce regime shifts.

Our objective is to understand (1) if individual stocks are subject to switching behavior, (2) how the switching behavior of individual stocks results in the switching behavior of indices. The 203 long lived stocks represent in each moment only two fifth of the constituents of S&P 500 index. A priori there is no reason to believe that the switching behavior of the index be correlated with the switching behavior of its long-lived constituents. For example, the switching behavior of an index could be driven only by a few stocks while the others could produce only noise.

Hence, we cannot limit our analysis to indexes but must analyze the behavior of both indexes and individual stocks. In so doing, we have to accept the intrinsic limitations due to available samples of stocks: most stocks are relatively short-lived and only a subset survives for long periods.

2.4. *Empirical Results*

Let us now analyze the first set of empirical results obtained estimating HMMs with fixed transition probability matrices. We begin by analyzing if stock return processes follow local trends. For each of the 207 stocks in our analysis, consider the logprices: *pi(t)*, *t*=1,...,466, *i*=1,...,207 and the relative logreturns obtained as differences of logprices: *ri(t) = pi(t) - pi(t-1)*, *t*=2,...466, *i*=2,...,207.

Consider first an index obtained taking the average of logprices:

$$Index(t) = \frac{1}{N}\sum_{i=1}^{N} p_i(t)$$

$$\Delta Index(t) = \frac{1}{N}\sum_{i=1}^{N} r_i(t)$$

The return of this index is only approximately equal to the logarithm of the return of an equally weighted portfolio formed with 207 stocks. Table 1 summarizes the mean and variance for the Index and the equally weighted index Index EW and compare them with the average mean and variance for all stocks. For the Index, we estimate a 2-state HMM, obtaining the following results:

Probability transition matrix = [0.97283 0.25824; 0.02717 0.74176]

Mean returns = [0.0050518; -0.028038]

Expected sojourn times in periods of 20 days= [36.805; 3.8724]

<Place Table 1 about here>

The expected sojourn times are computed using the formula $\text{Expected sojourn times} = \frac{1}{1 - TM(i,i)}$. Upward trends with an average positive logreturn of 0.0050518 have an expected duration of approximately three years (36.805 20-day periods) while downward trends have a slightly negative expected return of -0.028038 and a duration of approximately less than three months (3.8724 20-day periods). The spread between upward and downward trends of logprices is 0.033090.

To test the significance of these results, we apply the likelihood ratio test against the null of only one state. By estimating a simple random walk, we obtain a mean return of 0.0017055.

It is well known that the likelihood ratio test between two states and only one state (i.e., switching behavior against the null of non-switching behavior) has a non-standard distribution. Hansen (1992) provides a strategy to compute likelihood tests and Garcia (1998) provides critical values for the loglikelihood test. The critical value for 99% confidence level is 17.67.

We compute the loglikelihood test statistic as the difference of the likelihoods of models with one (random walk) and two states and we obtain the value 65.294. Applying the Garcia (1998) critical value, it is clear that we can reject the assumption of no switching.

These results seem to confirm that the index is not a random walk but that it is formed by a sequence of local trends. Similar results have already been reported in the literature. Schaller and Van Norden (1997) found strong evidence of switching behavior in the CRSP value weighted index for the period 1927-1989. They consider various specifications for the switching behavior: switching in mean, switching in variance, and switching in both mean and variance. Using the Garcia (1998) test, they find evidence of switching with all specifications.

The conclusion of Schaller and Van Norden (1997) is also in agreement with the conclusion of Maheu and McCurdy (2000) who identify two states in an index computed on a much broader sample over almost 200 years (1802-1995). They use a model with transition matrices that are duration-dependent. We will discuss these models in the next section.

As a next step, we perform the same analysis on each of the 207 long-lived stocks in our sample. All the 207 stocks exhibit a loglikelihood ratio in excess of 17.67 and pass the loglikelihood ratio test against the null of no switching[2]. We can therefore conclude that the totality of the stocks in our sample exhibit switching behavior, oscillating between two trends.

Thus our first principal conclusion is: stock price processes follow local trends and oscillate between extended periods of higher and lower expected returns. Let's now look at the parameters of the Markov switching models of each stock.

Table 2 summarizes the mean and variance of returns in each state for the Index and the average for all 207 stocks.

<Place Table 2 about here>

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[2] A list of the companies in our sample and tables with the loglikelihood ratio and other test results for each company are available from the authors upon request.

Panel (a) of Figure 1 plots, for each stock, the values of the mean return in state S1 and state S2, while Panel (b) plots the sorted values of the spread of returns (i.e., the difference between expected returns in state S1 and state S2). The expected logreturns in state S1 range from -0.0071982 to 0.020829 with an average of 0.010340; a similar result was obtained for state S2 with a range from -0.88961 to 0.017725 and an average of -0.15545. The corresponding values for the index are -0.028038 and 0.0050518. The spread ranges from a minimum of -0.019110 to a maximum of 0.89849, with an average of 0.16579. The spread of the index is 0.033090.

<Place Figure 1 about here>

Let's now analyze the transition matrices and the expected sojourn time in each state. Figure 2 represents, for each stock, the expected sojourn time in state S1 and state S2. Sojourn times are cut to a maximum of 153.83 days (winsorized). The average sojourn time is 36.601 periods in state S1 and 2.9849 periods in state S2. These sojourn times correspond approximately to average local trends of 720 days of up-trends and down-trends of average length 60 days. Table 3 compares the transition matrix of the Index with the average of transition matrices of the 207 stocks.

<Place Figure 2 about here>

<Place Table 3 about here>

The correlation coefficient between the probabilities p and q of the different stocks is C($p.q$) = 0.059196 and the correlation coefficient between the durations (winsorized) is C = 0.15015. There is a negative correlation C = -0.23574 between the loglikelihood ratio and the spread.

The previous analysis makes it clear that stock price processes, at least in the universe under consideration, move following local trends. If we aggregate stock price processes forming an index, we still find that the index moves following local trends.

2.5. *The Joint Behavior of Local Trends*

Results obtained thus far show that in the sample we consider: (1) the logreturns of both stocks and indexes follow a Markov switching behavior and (2) the logreturns of stocks and the differences between stock logreturns and index logreturns follow the same Markov switching behavior. The Markov switching behavior of stock returns implies that logprices follow local trends. If the starting points and the average lengths of local trends of different stocks were mutually independent, the switching behavior of individual stocks would be lost — or at least significantly reduced — in aggregate. The fact that both stocks and indexes present a robust switching behavior and that both logreturns and the residuals after subtracting the index present robust switching behavior indicates that trends move together.

In order to quantify the co-movements between local trends, let's first describe the trending behavior of each stock. In order to do so, we have to determine the sequence of states for each process. However states are not observable and can only be inferred. Of course inference about states is not deterministic — we can infer only the most probable sequence of states.

From the estimation procedure, we already have the smoothed probabilities for each state in each moment. We can therefore easily derive the most probable sequence of states, choosing at each moment the state with the highest probability. Panel (a) of Figure 3 plots the probability that the index be in each of the two states S1 and S2 for each period, while Panel (b) plots the average over the 207 stocks of the probabilities that each stock be in each of the two states S1 and S2 for each period.

<Place Figure 3 about here>

Looking at the two panels in Figure 3, one clearly sees that the probability of being in the S1 (high return) or S2 (low return) follows the same pattern in most stocks. If regimes switches were independent, then the average of probabilities would be almost flat. Instead, the two panels show a pattern quite similar to the pattern of the probabilities of states S1 and S2 for the index.

We compute the correlations between each pair of smoothed probabilities. There are 207×206/2 = 42,642 distinct pairs of stocks. The average of all correlations is 0.10931 while the average of the absolute values of all correlations is 0.11987. To gain a better understanding of the correlation structure of the smoothed probabilities of different stock return processes, we perform a hierarchical clustering of the smoothed probabilities processes. We use two distance measures: Euclidean distances and correlation distances.

Consider two series of smoothed probabilities relative to stocks *i.j* $\mathrm{SmPr}(t,i), \mathrm{SmPr}(t,j), t=1,\ldots,T$. The Euclidean distance between the two series is defined as follows:

$$\mathrm{ED}_{\mathrm{i,j}} = \sqrt{\sum_{\mathrm{t=1}}^{\mathrm{T}} \left(\mathrm{SmPr}(t,i) - \mathrm{SmPr}(t,j)\right)^2} \qquad (10)$$

while the correlation distance is defined as one minus the correlation coefficient.

We perform hierarchical clustering using the largest distance algorithm, which uses as distance between two clusters the largest distance between objects in the clusters. Figure 4 represents dendrograms that illustrate the structure of clusters. Consider, for example, Panel (b) of Figure 4 which is the dendrogram of clustering obtained with correlation distance. The top of the figure represents the maximum correlation distance. At each step clusters split in two.

<Place Figure 4 about here>

Given that we use the maximum distance algorithm, at any split the distance between the newly formed clusters is equal to the maximum distance in the cluster above. In general, the height of each U-shaped subgraph in the dendrogram (which appears on the left side of the diagram) is the maximum distance in the cluster above. As we increase the number of clusters, they become increasingly correlated and capture specific market sectors.

From the analysis discussed in this section, we can conclude that in the sample under consideration, 100% of the 207 stock return processes follow local trends. Trends are highly coordinated. Aggregation in an equally weighted index does not eliminate the trending behavior. The index exhibits a regime-switching behavior with expected sojourn time shorter than the individual processes.

Using monthly returns, we first estimate a DDMS model with a four-period memory. We estimate this model for the index and for each stock. This model captures how transition probability evolves with duration, going from from one month to four months.

We obtain the following estimate for the beta coefficients related to the index: β_1 = 3.6813, β_2 = 1.2042, β_3 = 20.094, and β_4 = -2.3730. The positive value of β_2 suggests that up-trends are self-reinforcing while the negative value of β_4 suggests that down-trends will disappear quickly.

Coefficients β_2 and β_4 are one positive and one negative and therefore the probabilities of staying in the same state increase with duration for p while decreses for q. Next we compute the coefficients $\beta_1, \beta_2, \beta_3, \beta_4$ for each stock. The average of these coefficients for all stocks is the following: β_1 = 3.4493, β_2 = -11.7691, β_3 = -14.9014, and β_4 = 21.8231.

Let's look at the distribution of the coefficients β_2 and β_4. Figure 5 shows the sorted coefficients β_2 and β_4. The number of coefficients $\beta_2 \geq 0$ is 112, of these, 46 stocks have $\beta_4 < 0$ (while the remaining 66 obviously have $\beta_4 \geq 0$,). The number of coefficients $\beta_2 < 0$ is 95; of these 55 con $\beta_4 \geq 0$, (the remaining 40 have $\beta_4 < 0$) as illustrated in Figure 5.

<Place Figure 5 about here>

Table 4 summarizes the mean and variance of returns in each state for the Index and compares them with the average for all 207 stocks in the case of time-varying transition probabilities.

<Place Table 4 about here>

4. APPLICATION OF THE PREVIOUS ANALYSIS

The previous empirical analysis has shown that a large fraction of the long-lived stocks in the S&P500 universe (in this case 207) exhibit switching behavior and follow local trends. The average time that each stock spends in each trend can be computed from the estimated transition matrices. Local trends are correlated and tend to move together; that is, stocks tend to move together to high or low states.

Our analysis shows that we can generalize the notion of momentum. The econometric fact behind momentum strategies is the persistence of high/low returns at both the stock and portfolio levels. That is, stocks and portfolios that exhibit high returns in the past continue to show high returns and vice versa. Alternatively, we can say that for those stocks that exhibited highest or lowest returns in some past time window, past average returns estimate expected returns, and therefore, the past trend will continue in the future.

However, our analysis shows that the persistence of returns is generalized to all long-lived stocks. This empirical fact shows that momentum behavior is common to almost all long-lived stocks. Our analysis shows that persistence of returns is not reserved to the highest or lowest returns but it is common to all returns of long lived stocks.

The previous analysis shows that long-lived stock return processes exhibit regime shifting behavior. Using time-varying transition matrices, we show that transition matrices are duration-dependent. In ourv sample, 29 stocks exhibit positive β_2 and negative β_4. These stocks clearly present persistence of up-trends. Seventy-nine stocks exhibit both positive β_2 and $\beta_{4.}$

These stocks might exhibit switching from positive to negative trends. 100 stocks exhibit negative β_2 Of these 58 exhibit positive β_4 and therefore exhibit persistence of negative trends while the remaining 42 stocks exhibit switching behavior.

Persistence of returns has important consequences from the application point of view. Changes of trends, especially change towards negative trends, is clearly very important for risk management. However, our analysis shows that it is difficult to diversify trends using long lived stocks as they are highly correlated. The risk of trend reversal cannot be reduced by diversification in the universe of long-lived stocks.

In a dynamic portfolio management approach, evaluation of the probability of persistence of trends can be an important component of the portfolio formation strategy. It is well known that estimation of expected returns is very difficult. HMMs offer a better understanding of the dynamics of expected returns at least for long-lived stocks.

5. CONCLUSIONS

In this paper, we analyzed the Markov switching behavior of stocks in the S&P500 universe. Considering only those 207 stocks that appeared at least once in the S&P500 and that survived through the entire sampling period from January 1, 1980 to December 31, 2016, we found that 100% of the stocks exhibit statistically significant trend-switching behavior. Trends are correlated insofar as they tend to be all in the up or down trend. This fact justifies why not only individual stocks but also an index formed with the same stocks exhibit switching behavior.

Using duration dependent Markov switching models, we also establish that the probability of staying in each state negatively depends on duration, that is, if a stocks remains for many periods in the same state, its probability of remaining in the same state tends to decrease or to increase.

The permanence of high-low return states offers a framework for momentum. Momentum is not an isolated phenomenon of those stocks that exhibit particularly high or low returns. Actually most returns, not only the highest and lowest, exhibit permanence and therefore some form of momentum.

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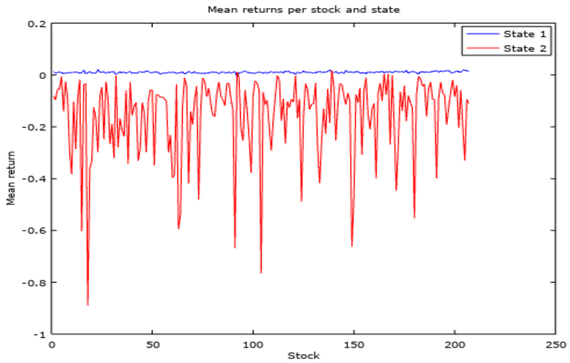
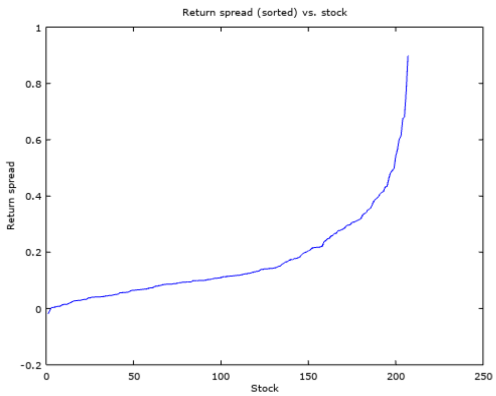
APPENDIX Matlab code for estimating the loglikelihood of our model:

```
function f = CLLS(W,y,Parameters)

T=length(y);

p=exp(W(1))/(1+exp(W(1)));
q=exp(W(2))/(1+exp(W(2)));
mu(1)=W(3);
mu(2)=W(4);
sigma(1)=W(5);
sigma(2)=W(6);

if Parameters.OIP==0
    rho=Parameters.rho;
elseif Parameters.OIP==1
    rho=exp(W(7))/(1+exp(W(7)));
end

for t=1:T
    for j=1:2
        fs(j,t)= normpdf(y(t),mu(j),sigma(j)) ;
    end
end

[tmv]=[p 1-p;1-q q];

Ps_F(1,1)=rho;
Ps_F(2,1)=1-rho;
Ps_Y(:,1)=Ps_F(:,1);

for t=2:T
     Ps_F(:,t)=tmv'*Ps_Y(:,t-1);
    Ps_Y(:,t)=Ps_F(:,t).*fs(:,t)/([1 1]*(Ps_F(:,t).*fs(:,t)));

end
```

```
for ts=1:T
    Z(ts)=log(Ps_F(1,ts)*fs(1,ts)+Ps_F(2,ts)*fs(2,ts));
end

f=-(sum(Z));
```



Panel (a): Mean returns per stock and state



Panel (b): Return spread (sorted) vs. stock

Figure 1: Returns in each state and their sorted spread

Panel (a) represents the expected returns of each stock in States 1 and 2. The expected logreturns in State S1 range from -0.0063 to 0.0211 with an average of 0.0104; in State S2 they range from -0.7718 to 0.1642 with an average of -0.1177. The corresponding values for the index are 0.0055 and -0.0164. Panel (b) represents the spread between returns in the two different states. The spread ranges from a minimum of -0.7811 to a maximum of 0.1553, with an average of -0.1281. The spread of the index is 0.0219. Panel (b) of Figure 1 plots the spread between the two states'.

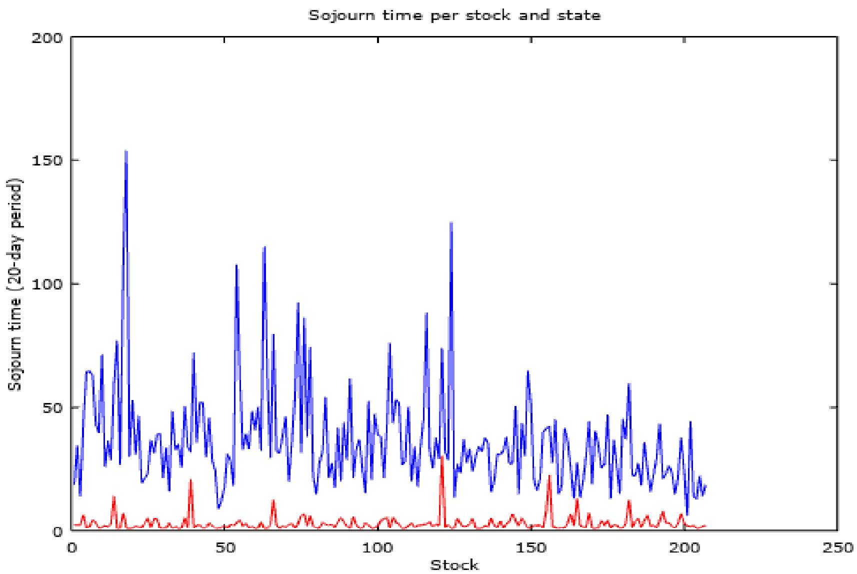
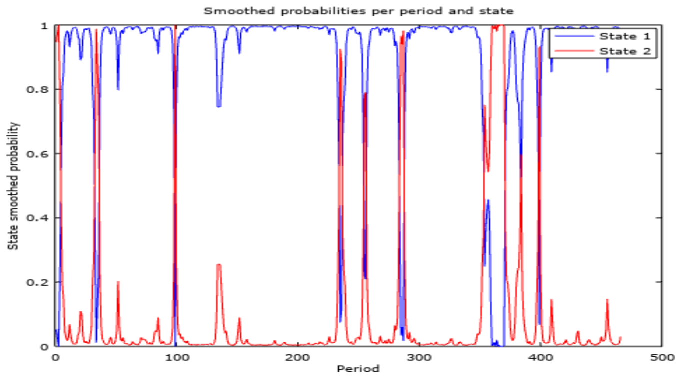
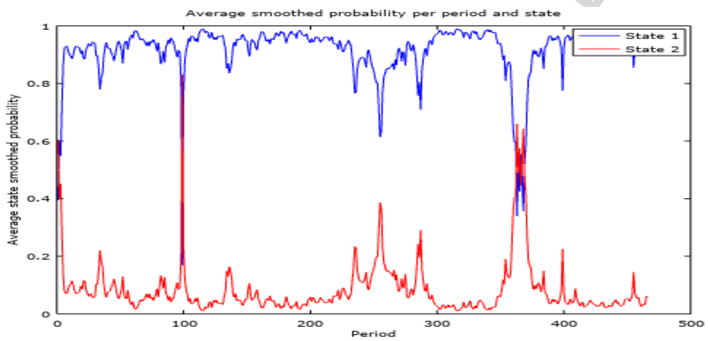


Figure 2: Expected sojourn time per stock and state

Sojourn times are cut to a maximum of 99.5 days (winsorized). The average sojourn time is 29.80 periods in State S1 and 3.45 periods in State S2. These sojourn times correspond approximately to average local trends of 600 days of up-trends and down-trends of average length 70 days.



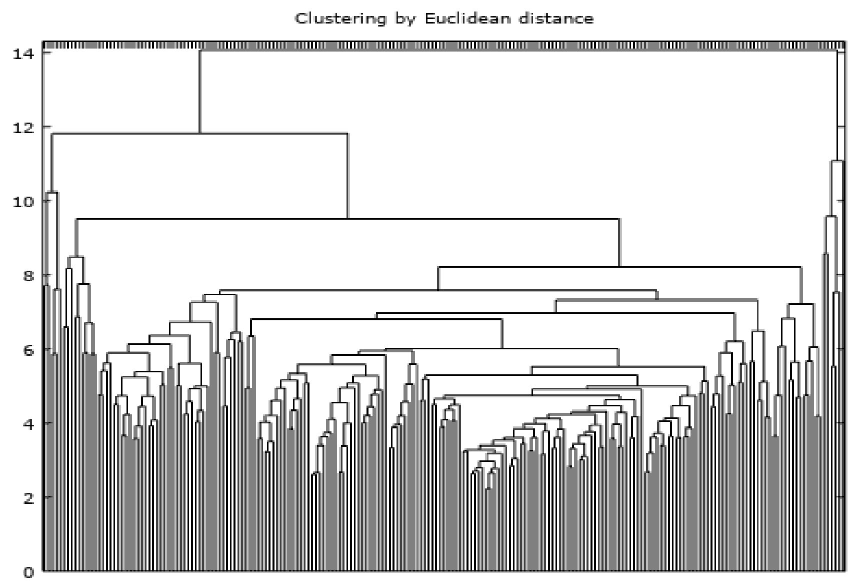
Panel (a): Smoothed probabilities per period and state



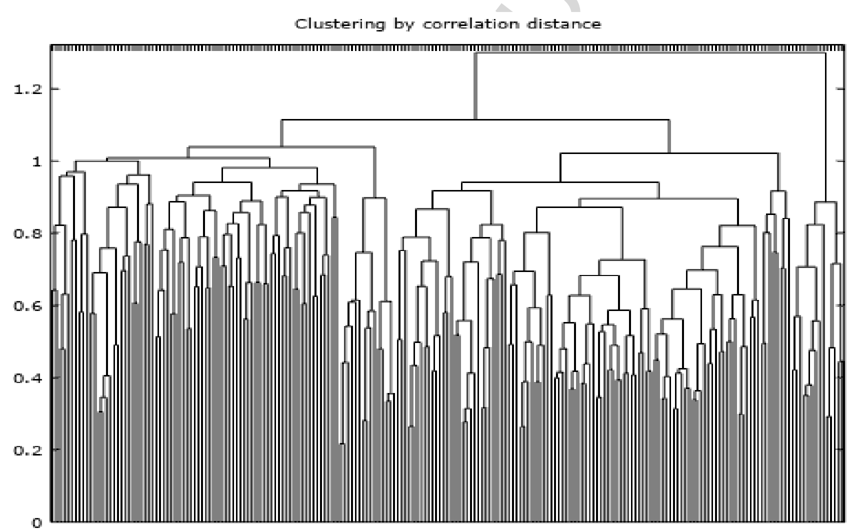
Panel (b) Average smoothed probability per period and state

Figure 3: Smoothed probabilities for the index and their average for all stocks

Panel (a) plots the probability that the index be in each of the two states S1 and S2 for each period. Panel (b) plots the average over the 203 stocks of the probabilities that each stock be in each of the two states S1 and S2 for each period. Looking at the two panels, one clearly sees that the probability of being in the S1 (high return) or S2 (low return) follows the same pattern in most stocks. If regimes switches were independent, then the average of probabilities would be almost flat. Instead, the two panels show a pattern quite similar to the pattern of the probabilities of states S1 and S2 for the index.



Panel (a): Dendrograms of stocks according to Euclidean distance



Panel (b): Dendrogram according to correlation distances

Figure 4: Dendrograms of hierarchical clustering of stocks

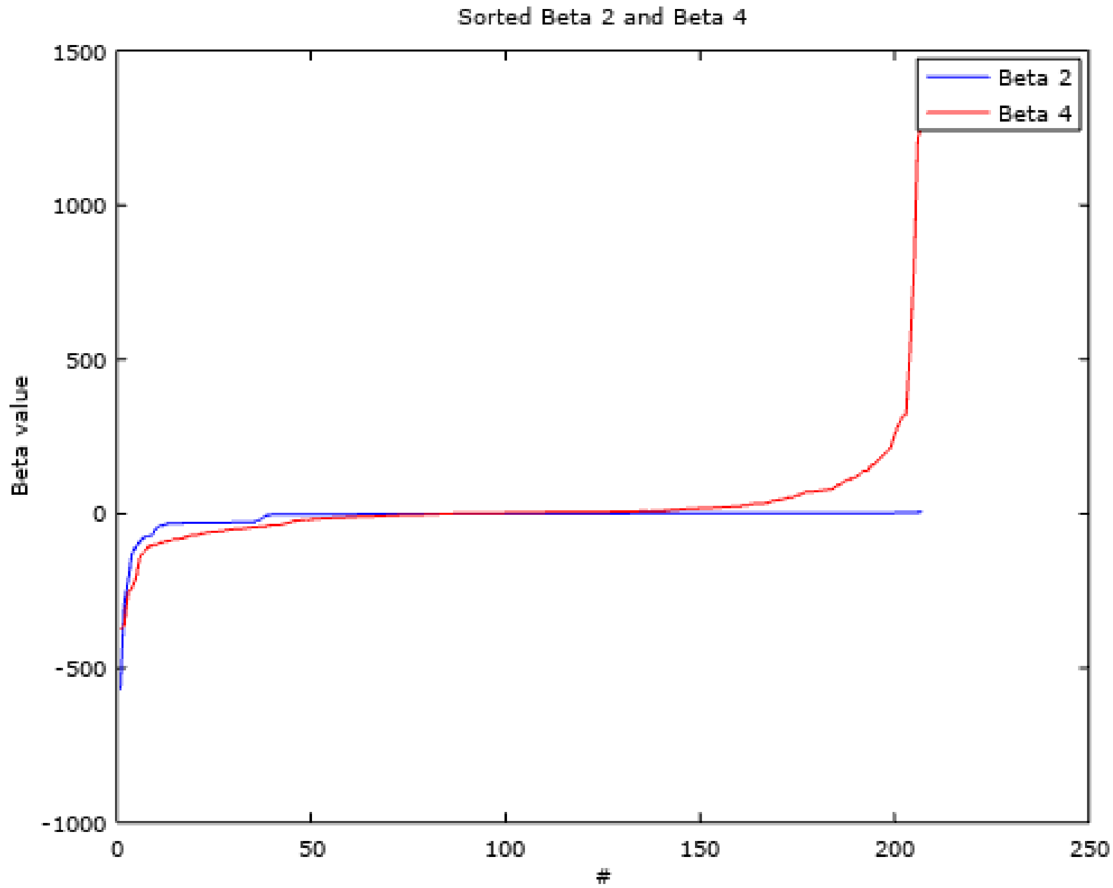


Figure 5. Sorted beta coefficients of time-varying transition matrices

This figure shows the sorted coefficients β_2 and β_4. The number of coefficients $\beta_2 \geq 0$ is 103, of these , 29 stocks have $\beta_4 < 0$ (while the remaining 74 obviously have $\beta_4 \geq 0$,). The number of coefficients $\beta_2 < 0$ is 100; of these 58 con $\beta_4 \geq 0$, (the remaining 42 have $\beta_4 < 0$).

Table 1: Mean and variance of the index and the equally weighted index EW

This table summarizes the mean and variance of the Index and the equally weighted Index EW and compares them with the average of mean and variance of returns for all 207 stocks.

	Mean	Variance
Index EW	0.0017055	0.0022931
Index	0.0023072	0.0023193
Average of all 207 stocks	0.0017055	0.012032

Table 2: Constant transition probabilities for Index and average of all stocks

This table compares the mean and variance of returns in each state for the Index with the average for all 207 stocks

Constant transition probabilities				
	State 1		State 2	
	Mean return	Variance	Mean return	Variance
Index	0.0050518	0.001187085	-0.0280384	0.011091376
Average of all 207stocks	0.010340	0.0045130	-0.155450	0.0802956

Table 3: Transition matrix of the index and average of stocks' transition matrices

This table compares the transition matrix of the Index with the average of transition matrices of the 207 stocks

	Transition matrix (index)		Transition matrix (all stocks)	
	State 1	State 2	State 1	State 2
State 1	0.97283	0.25824	0.96466	0.55100
State 2	0.02717	0.74176	0.03534	0.44090

Highlights

Persistence and switching of trends appear in most long-lived stock returns
Hidden Markov models with duration-dependent transition probabilities use
Aggregates and individual stocks exhibit same persistence and switching behavior
Hidden Markov models can explain medium-term momentum