Problem Set 4, 1 October 2018 Name: _______________ , Section: ___

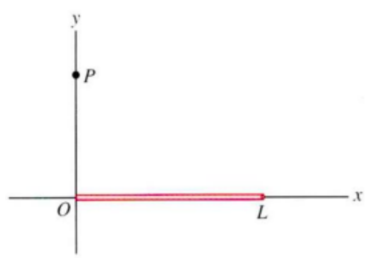
Problem – (P28.10)* A total amount of positive charge Q is spread onto a nonconducting, flat, circular annulus of inner radius a and outer radius b. The charge is distributed so that the charge density (charge per unit area) is given by $\sigma = k/r^3$, where r is the distance from the center of the annulus to any point on it. Show that (with $V = 0$ at infinity) the potential at the center of the annulus is given by

$$V = \frac{Q}{8\pi\epsilon_0}\left(\frac{a+b}{ab}\right).$$

Solution:

Problem – (P28.13) On a thin rod of length L lying along the x axis with one end at the origin ($x = 0$), as in the figure below, there is a charge per unit length given by $\lambda = kr$, where k is a constant and r is the distance from the origin.

a) Taking the electrostatic potential at infinity to be zero, find V at the point P on the y axis.
b) Determine the vertical component, E_y, of the electric field at P from the result of part (a) and also by direct calculation.
c) Why cannot E_x, the horizontal component of the electric field at P, be found using the result of part (a)?
d) At what distance from the rod along the y axis is the potential equal to one-half the value at the left end of the rod?



Solution:

Problem – ¶3. In lecture, we calculated the amount of electrostatic energy contained in the interior electric field of a uniform sphere of radius R and charge density ρ. The result was,

$$U = \frac{2\pi\rho^2}{45\epsilon_0} R^5.$$

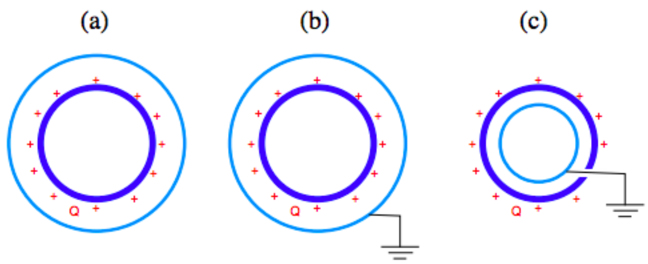
For this problem, complete the calculation by calculating the energy stored in the *exterior* electric field, and verify that the *total* stored energy matches the work done to assemble the sphere.

Solution:

Problem – 4. Consider an infinitely long line of charge with linear charge density λ . The line of charge is parallel to and a distance d above an infinite grounded conducting plane. Sketch the resulting electric field lines in the half-space above the plane, including any surface charges. Then using the concepts discussed in lecture, determine the electric field $E(x)$ at the surface of the plane, as a function of the horizontal distance x from the perpendicular between the plane and line.

Solution:

Problem – 5. Consider a hollow conducting sphere that carries a net positive charge Q. Next, a second initially uncharged concentric conducting sphere is brought into proximity with the first (without touching it). In scenario (a), the second sphere is placed outside the first and left *ungrounded*. In scenario (b), the second sphere is outside the first and *grounded*. In scenario (c), the second sphere is *inside* the first and also grounded.



For all three scenarios, describe the final arrangement of charge on the second sphere and the electric field (if any) outside the second sphere.

Solution:

Problem – 6. A cylindrical capacitor is made from two long thin concentric metal cylinders of length L and radii a and b. ($L \gg a$ and $a > b$)

a) Using the definition of capacitance $C = QV$, calculate the capacitance per unit length C/L of this configuration.

b) Repeat the capacitance calculation, this time using the stored energy $U = \frac{1}{2}\frac{Q^2}{C}$.

Solution: