

$$\begin{aligned}
&T_p\\
&(x,U)\\
&p\big|\\
&x_i\big|_p\\
&i=\\
&1,\dots,m\\
&i\big|_p(f):=\\
&\partial_i(f\circ\\
&x^{-1})\big|_{x(p)}\\
&\partial_i\\
&i\\
&(x_1\big|_p,\dots,x_m\big|_p)\\
&p\\
&\sum_{i=1}^mv(x_i)x_i\big|_p=\\
&\sum_{i=1}^m\xi x_i\big|_p.\\
&??\\
&_1\big|_p(x^j)=\\
&\delta_{ij}\\
&(x_1\big|_p,\dots,x_m\big|_p)\\
&p\\
&f: \\
&U'\subset\rightarrow\\
&U'\subset\\
&U\\
&p\in\\
&U'\\
&f_i: \\
&U'\rightarrow\\
&\sum_{i=1}^m(x'-\\
&x'(p))f.\\
&f_i(p)=\\
&x_i\big|_p(f)\\
&tangentialvectors.pdf\\
&\psi(U)-\\
&\psi(U_0)=\\
&\int_0^1t\psi(tU+\\
&(1-\\
&t)U_0)t\\
&U=\\
&x(q)\\
&q\in\\
&U_0=\\
&x(p)\\
&\psi(U)-\\
&\psi(U_0)=\\
&\sum_i(U^i-\\
&U_0^i)\int_0^1\underbrace{\psi U'(tU+(1-t)U_0)t}_{:=\psi_i(U)}\\
&f_i=\\
&\psi_i\circ\\
&f_i: \\
&U'\subset\rightarrow\\
&f_i\\
&\psi(U)-\\
&\psi(U_0)=\\
&\psi(x(q))-\\
&\psi(x(p))=\\
&f(q)-\\
&f(p)\\
&U^i=\\
&x^i(q)\\
&U_0^i=\\
&x^i(p)\\
&\psi_i(U)=\\
&\psi_i(x(1))=\\
&f_i(q)\\
&\sum_{i=1}^n(x_i(q)-\\
&x_i(p))f_i(q)\\
&i\big|_p(f)=\\
&\partial_i\underbrace{(f\circ x^{-1})}_{\psi}\big|_{x(p)}\\
&=\\
&\partial_i\psi\big|_{x(p)}\\
&=\\
&\psi_i(x(p))=\\
&f_i(p)
\end{aligned}$$