APPENDIX 3

Summary of Hertz elastic contact stress formulae

$$E^* \equiv \left(\frac{1-\nu_1{}^2}{E_1} + \frac{1-\nu_2{}^2}{E_2}\right)^{-1}$$

$$R \equiv (1/R_1 + 1/R_2)^{-1}$$

(*a*) *Line contacts* (load P per unit length)

Semi-contact-width:

$$a = \left(\frac{4PR}{\pi E^*}\right)^{1/2}$$

Max. contact pressure:

$$p_0 = \frac{2}{\pi}\frac{P}{a} = \left(\frac{PE^*}{\pi R}\right)^{1/2}$$

Max. shear stress:

$$\tau_1 = 0.30 p_0 \text{ at } x = 0, \quad z = 0.78a$$

(*b*) *Circular point contacts* (load P)

Radius of contact circle:

$$a = \left(\frac{3PR}{4E^*}\right)^{1/3}$$

Max. contact pressure:

$$p_0 = \left(\frac{3P}{2\pi a^2}\right) = \left(\frac{6PE^{*2}}{\pi^3 R^2}\right)^{1/3}$$

Approach of distant points:

$$\delta = \frac{a^2}{R} = \left(\frac{9}{16}\frac{P^2}{RE^{*2}}\right)^{1/3}$$

Max. shear stress:

$\tau_1 = 0.31 p_0$ at $r = 0, \quad z = 0.48a$

Max. tensile stress:

$\sigma_r = \frac{1}{3}(1 - 2\nu)p_0$ at $r = a, \quad z = 0$

(*c*) *Elliptical point contacts* (load P)

a = major semi-axis; b = minor semi-axis; $c = (ab)^{1/2}$; R' and R'' are major and minor *relative* radii of curvature (see Appendix 2); equivalent radius of curvature $R_e = (R'R'')^{1/2}$

$a/b \approx (R'/R'')^{2/3}$

$$c = (ab)^{1/2} = \left(\frac{3PR_e}{4E^*}\right)^{1/3} F_1(R'/R'')$$

Max. contact pressure:

$$p_0 = \frac{3P}{2\pi ab} = \left(\frac{6PE^{*2}}{\pi^3 R_e^2}\right)^{1/3} [F_1(R'/R'')]^{-2/3}$$

Approach of distant points:

$$\delta = \left(\frac{9P^2}{16R_e E^{*2}}\right)^{1/3} F_2(R'/R'')$$

The functions $F_1(R'/R'')$ and $F_2(R'/R'')$ are plotted in Fig. 4.4 (p. 97). To a first approximation they may be taken to be unity.
For values of maximum shear stress τ_1, see Table 4.1 (p. 99).