

## APPENDIX 1

### Cauchy Principal Values of some useful integrals

$$(1) \int_b^a \frac{ds}{x-s} = \ln \left( \frac{x-b}{a-x} \right), \quad b \leq x \leq a$$

$$(2) \int_{-1}^{+1} \frac{1-S^2}{X-S} dS = 2X + (1-X^2) \ln \left( \frac{1+X}{1-X} \right), \quad -1 \leq X \leq +1$$

$$(3) \int_{-1}^{+1} \frac{S(1-S^2)^{1/2}}{X-S} dS = \pi X^2 - \pi/2, \quad -1 \leq X \leq +1$$

$$(4) \int_{-1}^{+1} \frac{(1-S^2)^{1/2}}{X-S} dS = \pi X, \quad -1 \leq X \leq +1$$

$$(5) \int_{-1}^{+1} \frac{\text{sign}(S)(1-X^2)^{1/2} dS}{X-S} = 2(1-X^2)^{1/2} \ln \left\{ \frac{|X|}{1+(1-X^2)} \right\},$$
$$-1 \leq X \leq +1$$

$$(6) \int_{-1}^{+1} \frac{|S|(1-X^2)^{1/2} dS}{X-S} = 2X + 2X(1-X^2) \ln \left\{ \frac{|X|}{1+(1+X^2)^{1/2}} \right\},$$
$$-1 \leq X \leq +1$$