APPENDIX 2

Geometry of smooth non-conforming surfaces in contact

The profile of each body close to the origin O can be expressed:

$$z_1 = (1/2R_1')x_1^2 + (1/2R_1'')y_1^2$$

and

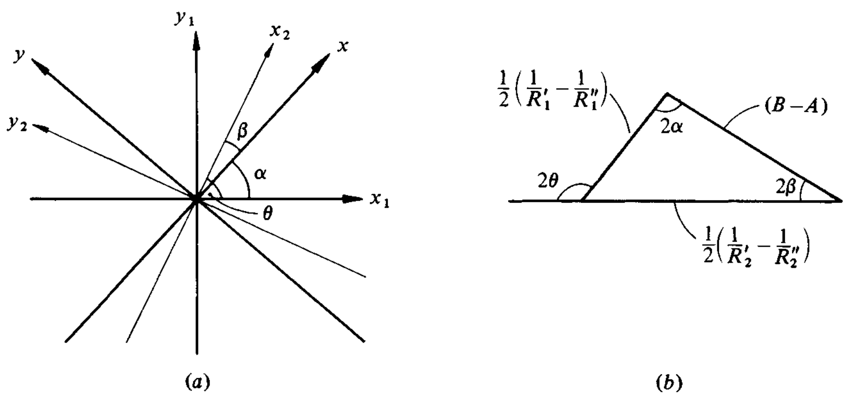
$$z_2 = -\{(1/2R_2')x_2^2 + (1/2R_2'')y_2^2\}$$

where the directions of the axes for each body are chosen to coincide with the principal curvatures of that body. In general the two sets of axes may be inclined to each other at an arbitrary angle θ, as shown in Fig. A2.1(a).

We now transform the coordinates to a common set of axes (x, y) inclined at α to x_1 and β to x_2 as shown. The gap between the surfaces can then be written

$$h = z_1 - z_2 = Ax^2 + By^2 + Cxy$$

Fig. A2.1



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where

$$C = \tfrac{1}{2}\left(\frac{1}{R_2'} - \frac{1}{R_2''}\right)\sin 2\beta - \tfrac{1}{2}\left(\frac{1}{R_1'} - \frac{1}{R_1''}\right)\sin 2\alpha$$

The condition that C should vanish, so that

$$h = Ax^2 + By^2$$

is satisfied by the triangle shown in Fig. A2.1(b), with the result:

$$A - B = \tfrac{1}{2}\left(\frac{1}{R_1'} - \frac{1}{R_1''}\right)\cos 2\alpha + \tfrac{1}{2}\left(\frac{1}{R_2'} - \frac{1}{R_2''}\right)\cos 2\beta$$

$$|A - B| = \tfrac{1}{2}\left\{\left(\frac{1}{R_1'} - \frac{1}{R_1''}\right)^2 + \left(\frac{1}{R_2'} - \frac{1}{R_2''}\right)^2 + 2\left(\frac{1}{R_1'} - \frac{1}{R_1''}\right)\left(\frac{1}{R_2'} - \frac{1}{R_2''}\right)\cos 2\theta\right\}^{1/2}$$

Finally

$$A + B = \tfrac{1}{2}\left(\frac{1}{R_1'} + \frac{1}{R_1''} + \frac{1}{R_2'} + \frac{1}{R_2''}\right)$$

from which the values of A $(= 1/2R')$ and B $(= 1/2R'')$ can be found. N.B. Concave curvatures are *negative.*