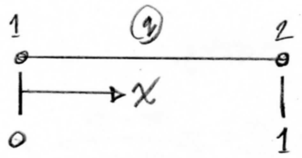
MECH-462 Assignment # 8 Thomas Bement 24099822

Q3) 

$\Rightarrow$ Linear element so linear mapping

$$\xi = \frac{x - x_i}{x_j - x_i} \quad \begin{cases} N_1(\xi) = 1 - \xi \\ N_2(\xi) = \xi \end{cases}$$

$$\underline{B}(x) = \frac{d\underline{N}(x)}{dx} = \begin{pmatrix} \frac{x_j - 1}{x_j - x_i} \\ \frac{1 - x_i}{x_j - x_i} \end{pmatrix}$$

$$\underline{N}_q(\xi) = \begin{pmatrix} N_1(\xi) \\ N_2(\xi) \end{pmatrix} = \begin{pmatrix} 1 - \xi \\ \xi \end{pmatrix}$$

$$\underline{N}_q(x) = \begin{pmatrix} N_1(x) \\ N_2(x) \end{pmatrix} = \begin{pmatrix} 1 - \frac{x - x_i}{x_j - x_i} \\ \frac{x - x_i}{x_j - x_i} \end{pmatrix}$$

$$\frac{d\underline{N}(x)}{dx} = \begin{pmatrix} 1 - \frac{1 - x_i}{x_j - x_i} \\ \frac{1 - x_i}{x_j - x_i} \end{pmatrix} = \begin{pmatrix} \frac{x_j - \cancel{x_i} - 1 + \cancel{x_i}}{x_j - x_i} \\ \frac{1 - x_i}{x_j - x_i} \end{pmatrix} = \begin{pmatrix} \frac{x_j - 1}{x_j - x_i} \\ \frac{1 - x_i}{x_j - x_i} \end{pmatrix}$$

$$\underset{\sim}{k} = \int_{x_i}^{x_j} EA\, \underline{B}(x)\, \underline{B}^T(x)\, dx$$

$EA = ¢$, $x_i = 0$ & $x_j = 1$

$$\underline{B}(x)\,\underline{B}^T(x) = \begin{pmatrix} \frac{x_j - 1}{x_j - x_i} \\ \frac{1 - x_i}{x_j - x_i} \end{pmatrix} \begin{pmatrix} \frac{x_j - 1}{x_j - x_i}, & \frac{1 - x_i}{x_j - x_i} \end{pmatrix}$$

$$\underline{B}(x)\,\underline{B}^T(x) = \begin{pmatrix} \frac{1-1}{1-0} \\ \frac{1-0}{1-0} \end{pmatrix} \begin{pmatrix} \frac{1-1}{1-0}, & \frac{1-0}{1-0} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0, & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = ¢$$

$$\underset{\sim}{k} = EA \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \int_0^1 dx = EA \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x \Big|_0^1 = EA \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} (1 - 0) = E\cancel{A}^{1} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = E \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\underset{\sim}{k}_{11} = 0 \Rightarrow \boxed{\frac{\underset{\sim}{k}_{11}}{E} = \frac{E(0)}{\cancel{E}} = \frac{0}{1} = 0}$$

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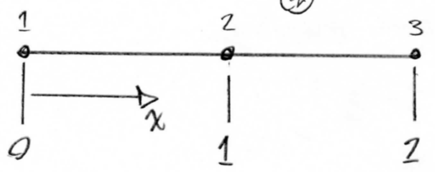
a

Q4)



$\Rightarrow$ Quadratic element $\therefore$ quadradic mapping

$$\xi = 2\,\frac{x - x_i}{x_k - x_i} \qquad x_i = 0,\quad x_k = 2 \quad \Rightarrow \xi = 2\,\frac{x-0}{2-0} = x$$

$$N_1(\xi) = N_1(x) = \frac{1}{2}(1-x)(2-x)$$

$$N_2(\xi) = N_2(x) = x\,(2-x)$$

$$N_3(\xi) = N_3(x) = \frac{1}{2}x\,(x-1)$$

$$\underset{\sim}{N}_q(x) = \begin{pmatrix} \frac{1}{2}(1-x)(2-x) \\ x(2-x) \\ \frac{1}{2}x(x-1) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(2-3x+x^2) \\ (2x-x^2) \\ \frac{1}{2}(x^2-x) \end{pmatrix}$$

$$\frac{d\underset{\sim}{N}_q(x)}{dx} = \begin{pmatrix} \frac{1}{2}(2x-3) \\ (2-2x) \\ \frac{1}{2}(2x-1) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(2x-3) \\ 2(1-x) \\ \frac{1}{2}(2x-1) \end{pmatrix}$$

$$\underset{\sim}{B}(x) = \frac{dN_q(x)}{dx} = \begin{pmatrix} \frac{1}{2}(2x-3) \\ 2(1-x) \\ \frac{1}{2}(2x-1) \end{pmatrix}$$

$A = 1 \therefore EA = E$

$$\underset{\sim}{B}(x)\,\underset{\sim}{B}^T(x) = \begin{pmatrix} \frac{1}{2}(2x-3) \\ 2(1-x) \\ \frac{1}{2}(2x-1) \end{pmatrix}\left(\frac{1}{2}(2x-3),\ 2(1-x),\ \frac{1}{2}(2x-1)\right)$$

$$\underset{\sim}{B}(x)\,\underset{\sim}{B}^T(x) = \begin{pmatrix} \frac{1}{4}(4x^2-12x+9) & 1(2x-2x^2-3+3x) & \frac{1}{4}(4x^2-8x+3) \\ 1(-2x^2+5x-3) & 4(1-2x+x^2) & 1(2x-1-2x^2+x) \\ \frac{1}{4}(4x^2-8x+3) & 1(-2x^2+3x-1) & \frac{1}{4}(4x^2-4x+1) \end{pmatrix}$$

$$\underset{\sim}{k} = \int_0^2 EA\,\underset{\sim}{B}(x)\,\underset{\sim}{B}^T(x)\,dx = E\int_0^2 \begin{pmatrix} (x^2-3x+\frac{9}{4}) & (-2x^2+5x-3) & (x^2-2x+\frac{3}{4}) \\ (-2x^2+5x-3) & (\frac{x^2}{4}-\frac{x}{2}+\frac{1}{4}) & (-2x^2+3x-1) \\ (x^2-2x+\frac{3}{4}) & (-2x^2+3x-1) & (x^2-x+\frac{1}{4}) \end{pmatrix} dx$$

$$\underset{\sim}{k} = E\left.\begin{pmatrix} (\frac{x^3}{3}-\frac{3x^2}{2}+\frac{9x}{4}) & (\frac{-2x^3}{3}+\frac{5x^2}{2}-3x) & (\frac{x^3}{3}-x^2+\frac{3x}{4}) \\ (\frac{-2x^3}{3}+\frac{5x^2}{2}-3x) & (\frac{x^3}{12}-\frac{x^2}{4}+\frac{x}{4}) & (\frac{-2x^3}{3}+\frac{3x^2}{2}-x) \\ (\frac{x^3}{3}-x^2+\frac{3x}{4}) & (\frac{-2x^3}{3}+\frac{3x^2}{2}-x) & (\frac{x^3}{3}-\frac{x^2}{2}+\frac{x}{4}) \end{pmatrix}\right|_0^2 = E\begin{pmatrix} \frac{7}{6} & -\frac{4}{3} & \frac{1}{6} \\ -\frac{4}{3} & \frac{1}{6} & -\frac{4}{3} \\ \frac{1}{6} & -\frac{4}{3} & \frac{7}{6} \end{pmatrix}$$

$$\underset{\sim}{k}_{22} = \frac{E}{6} \Rightarrow \boxed{\frac{\underset{\sim}{k}_{22}}{E} = \frac{\cancel{E}(1)}{\cancel{E}(6)} = \frac{1}{6} \cong 0.17}$$