Credit Cycles, Market Liquidity and Heterogeneous Firms

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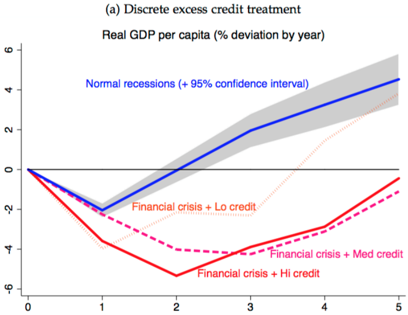
Introduction – Motivation



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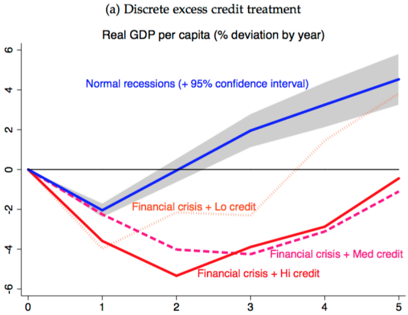

Importance of credit for recessions – from Jordà, Schularick, and Taylor (2013)

Cyclicality of top-1% and small firms – from Crouzet, Mehrotra, et al. (2020)

Research question

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  - Are recessions amplified by the collapse of large companies ...
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- Does the large firms reaction drive business cycles in this context ?
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$\Rightarrow$ *Aim of the paper :*
Examine the effects of firm heterogeneity on the economy's response to aggregate shocks in presence of (il)liquidity

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- What is the role of firm heterogeneity for aggregate fluctuations in presence of two dimensions of (il)liquidity ?
- Financial (il)liquidity : Two dimensions
  - *Market illiquidity :* Asset and capital lose value when sold (asset)
  - Funding illiquidity : Difficult to raise funds for investment (liability)
  - Consequence : Deleveraging, credit/asset-price feedback loop
- Firms heterogeneity :
  - Dispersion and *power-law* distribution
  - Hedging both idiosyncratic & aggregate risk in incomplete market
  - Credit and Collateral constraints

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$\Rightarrow$ *Aim :* Study of the three dimensions *together*

- Propose a theoretical framework with heterogeneous firms, collateral constraints and market illiquidity

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  - Macro-Finance : both Market and funding illiquidity
    - Financial frictions and credit cycles : BGG (99), Kiyotaki Moore (JPE 1997), Brunnermeier and Sannikov (2014) and many others ...
  - Heterogeneous agents :
    - Households : Aiyagari (94), Kaplan-Moll-Violante (2017),Benhabib, Bisin, and Zhu (2015), Achdou, Han, Lasry, Lions, and Moll (2017)
    - Firms : Moll (2014), Winberry (2016a), Mongey and Williams (2017), Khan and Thomas (2013) (JPE)
    - With aggregate shocks : Krusell and Smith (1998) (JPE), Winberry (2016b), Ahn, Kaplan, Moll, Winberry, and Wolf (2018)

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Framework : firms problem

- Heterogeneous entrepreneur :
  - Capital k_t subject to income shocks (CRS prod.)

$$dR_t = \bar{R}\, k_t\, dZ_t^i$$

    - Z_t productivity : Z_t idiosyncratic shock : $dZ_t^i = \mu^i(z_t)dt + \sigma_t dW_t^i$
    - z_t (and $\mu^i(z_t)$) : jump on productivity growth : n_z states process : here, 3-states : low state, median state, high state
    - Aggregate shock on average return $\bar{R}$ (more on this later) :
      $d\bar{R}_t = \eta(\overline{\mu}^R - \bar{R}_t)dt + \sigma_R^c\, \varepsilon_t^c dN_t^c$

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  - Two assets portfolio choice (à la Merton 73) :
    - Risk-free Bond b_t at rate r_t vs. Risky Capital k_t at rate R_t
  - Incomplete market : cannot self-insure against any risk
  - Collateral constraints and shocks on capital quality θ_t^c :

$$b_t \geq -\theta_t^c\, q_t\, k_t \qquad \Rightarrow \qquad k_t \leq \frac{1}{(1-\theta_t^c)q_t}\, a_t$$

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- Firms' control problem :

$$\max_{\{c_t,k_t\}_{t_0}^{\infty}} \mathbb{E}_{t_0} \int_{t_0}^{\infty} e^{-\rho_t t} u(c_t) dt$$

$$da_t = \Big[r_t\, a_t - c_t + q_t k_t \left(\bar{R}\, \mu^i(z_t) - r_t \right) \Big] dt + q_t k_t \bar{R}\, \sigma_t^i dW_t^i \qquad \text{(P)}$$

$$0 \leq k_t \leq a_t / (1 - \theta_t^c) q_t \qquad\qquad a_t \geq 0$$

  - Two states variables : assets $a_t = b_t + q_t k_t$ (size, or asset/networth) and productivity z_t
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- Problem analogous to :
  - Bewley-Huggett-Aiyagari, with collateral constraint :
    - generates high mass of firm $m(a, z)$ on the left tail of the distribution
  - Merton portfolio choice for large firms :
    - generates power law $m(a, z) \sim a^{-\zeta - 1}$ on right tail of distribution

Framework : firms problem

- Typical heterogenous agents problem (Mean Field Game)
  - Risk-free interest r_t determined by market clearing for bonds : $\int_{\mathcal{A}\times\mathcal{Z}} b_t(a,z)\, m(t,da,dz) = \underline{B} = 0$
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  - Capital price q_t (unique !) for capital market : $\int_{\mathcal{A}\times\mathcal{Z}} k_t(a,z)\, m(t,da,dz) = \underline{K}$
    - This creates a new element : Market liquidity

$$dq_t = q_t\, \mu_t^e\, dt + q_t\, \sigma_t^{c,e}\, \varepsilon_t^c\, dN_t^c$$

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- Extensions :
  - General equilibrium : Household consuming and pricing the asset
  - Government policy : stimulus and subsidy to demand
  - Entry and exit : deleveraging + default

Aggregate shocks

- Aggregate shocks : Jump-drift processes (jump dN^c) of size ε

$$dX_t = \eta(\overline{\mu}^X - X_t)dt + \sigma_X^c \, \varepsilon_t^c dN_t^c$$

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- Uncertainty shock, to entrepreneurs volatility σ_t^i
  - Examples : Bloom (2009) and more recently Mongey and Williams (2017) and induces "wait-and-see" effect and dispersion in firms

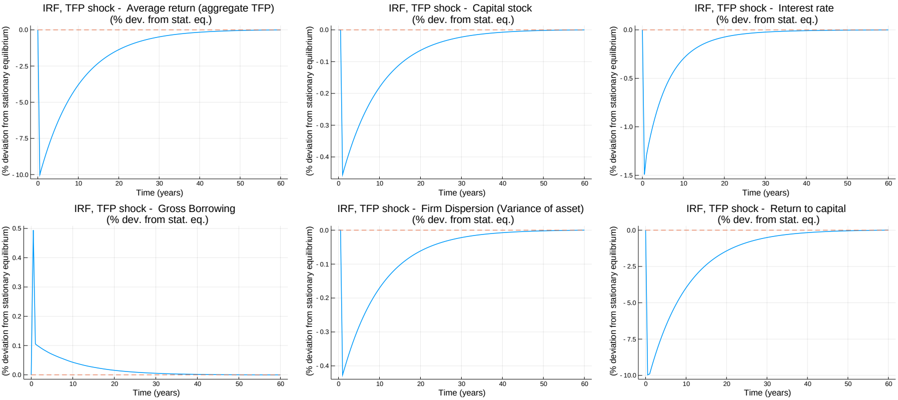
Aggregate shocks :

- Three experiments : Unanticipated aggregate shock :
    1. Standard recession : $\bar{R}_t$ drop of 10%
    2. Credit crunch : θ_t^c : drop of 15%
    3. Uncertainty shock : σ^2 : rise of 15%
- Revealed in the first year
- Generalized Ornstein Uhlenbeck process ($\approx$ AR(1) with $\rho = 0.9$)

$\Rightarrow$ For now : abstract from market illiquidity :

- Partial equilibrium in q_t but general equilibrium in r_t

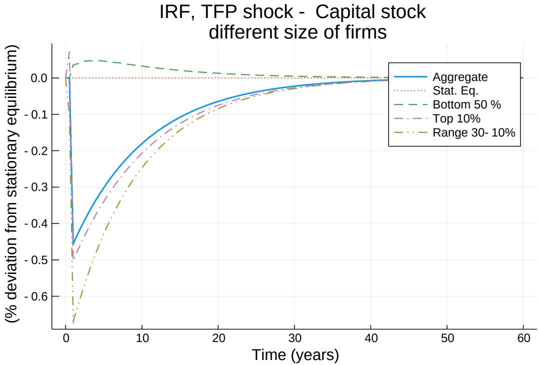
Supply side shock : ”Standard” recession



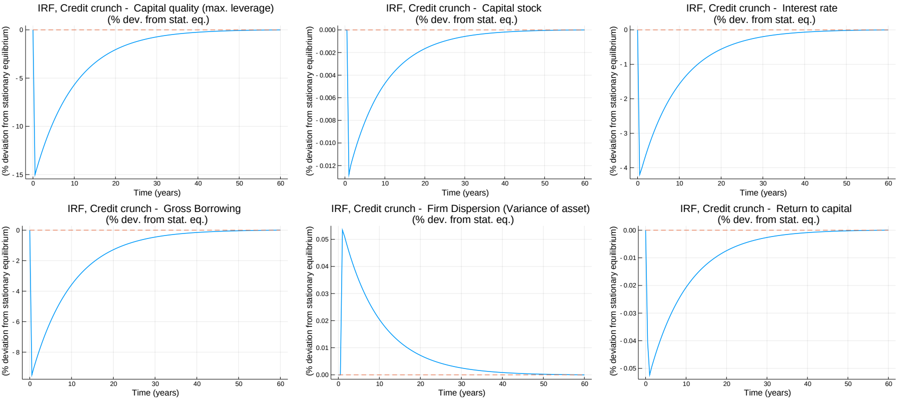
IRF – TFP shock to aggregate return $\bar{R}_t$

Supply side shock : "Standard" recession

- Decomposition by firms size :
  - Bottom firm behavior : constrained and smoother.
  - Top firms : change their exposure to capital risk : reduction in precautionary saving and income effect



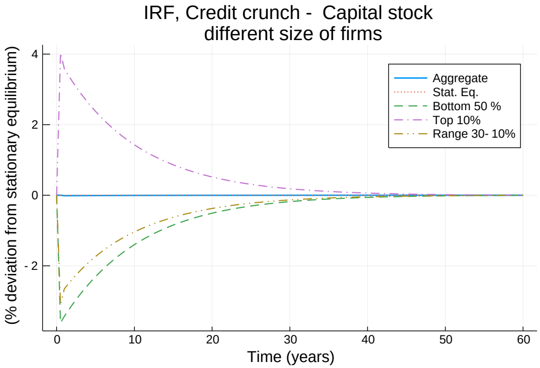
Capital quality shock : Credit crunch



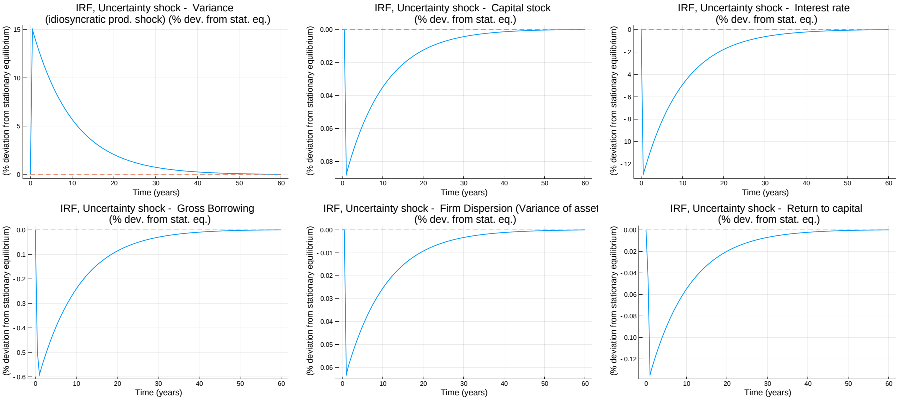
IRF – Credit crunch : shock θ_t^c

Capital quality shock : Credit crunch

- Decomposition by firms size :
  - Bottom firm behavior : deleveraging and large drop in capital
  - Top firms : benefit from the general equilibrium effect to invest



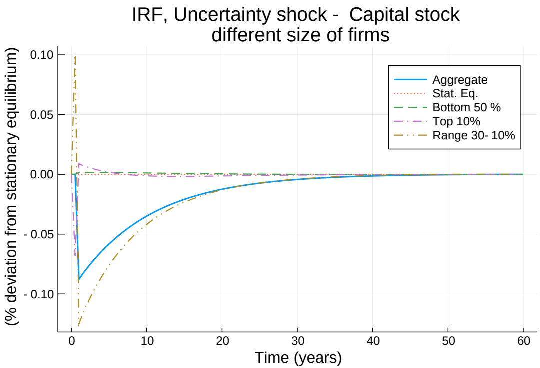
Uncertainty shock : Rise in volatility



IRF – Uncertainty shock : shock on σ_t

Uncertainty shock : Rise in volatility

- Decomposition by firms size :
  - Median/large firms reduce exposure while top firms benefit from equilibrium effect (interest) : opposite behaviors on impact !
  - Bottom firm behavior : constrained and barely affected.



Conclusion and Future research

- *Question :*
  What is the influence of firms heterogeneity on credit cycles ?
  - → Standard recessions driven by the median and larger firms
  - → Effects of credit crunch partly offset/smoothed by the hedging of largest firms (top 10%)
  - → Uncertainty shocks or TFP shock do not impact the bottom constrained firm
- Bridging macro-finance and heterogeneous agents literatures
  - Propose a theoretical framework with heterogeneous firms, collateral constraints, and market illiquidity, in presence of aggregate risk
  - Non-linearity of shocks + failure of approximate aggregation
  - Does market (il)liquidity matters ? *(next step on this project !)*

Future research – Empirical evidence

- Empirical relevance *(next step on this project !)*
  - Micro-level Firms heterogeneity
    - Cross-section data from Compustat
    - Largest firms : reacting to aggregate shocks + power law $\zeta \approx 1.0$

  $\Rightarrow$ Structural estimation (SMM)
    - What determine the firms distribution ?
    - Preference ? or structure/variance of risks ?
- Application to Boom and Bust dynamics
  - Great Recession : From credit crunch to economic downturn
- Aggregate risk : more elaborate treatment :
  - Exploiting MIT shocks (BKM, Auclert et al.),
    Tree structure of aggreg. shocks (Bourany-Achdou, cf. appendix)

Conclusion

- This paper examine the influence of firm heterogeneity in presence of market illiquidity and financial frictions
- It revisits the transmission and amplification mechanisms of different aggregate shocks that could explain deep recessions

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- THANK YOU FOR YOUR FEEDBACKS !

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Results : Stationary policy functions

- Characterization of value/policies functions $c^\star(a,z)$, $s^\star(a,z)$ & $k^\star(a,z)$
  - **Result 1 :** For each productivity $z \in \mathcal{Z}$, there is an interval over size (asset) such that all firms are constrained & have the same policy :

$$a \in [0, \gamma(\theta^c, q, z)] \qquad \Rightarrow \qquad k(a,z) = k(a) = \frac{a}{(1-\theta^c)q}$$

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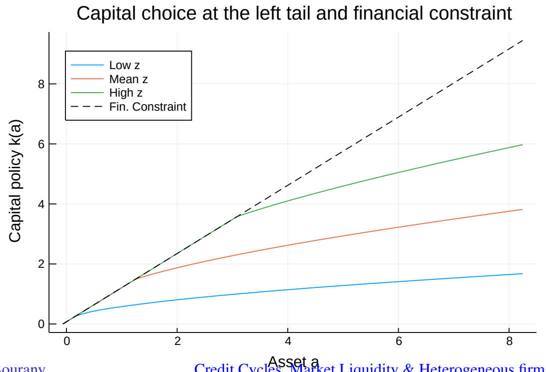


Results : Stationary policy functions

- Policy functions : $c(a,z), s(a,z)$ and $k(a,z)$ at the upper tail : linearity in state
  - **Prop. 1 :** In stat. eq., with CRRA = γ, and no jump in level of productivity growth, i.e. $\lambda_z = 0 \; \forall z$, firms consumption, saving and capital investment policies are asymptotically linear in wealth a, as wealth grows large : $a \to \infty$:

$$c(a,z) \sim \left(\frac{\rho - (1-\gamma) r}{\gamma} - \frac{1}{2} \frac{(\bar{R}\mu(z) - r)^2}{\bar{R}^2 \sigma^2} \frac{1-\gamma}{\gamma^2} \right) a$$

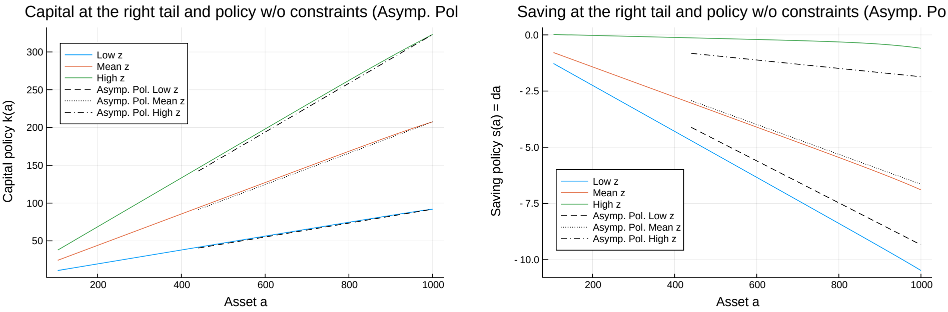
$$s(a,z) \sim \left(\frac{r - \rho}{\gamma} + \frac{1+\gamma}{2\gamma} \frac{(\bar{R}\mu(z) - r)^2}{\gamma \, \bar{R}^2 \sigma^2} \right) a$$

$$k(a,z) \sim \frac{\bar{R}\mu(z) - r}{\gamma \, q \, \bar{R}^2 \sigma^{i\,2}} \, a$$

with $f(x) \sim g(x)$ as $x \to \infty$ notation for $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$.

Results : Stationary policy functions

- Characterization of decisions $k(a,z)$ and $s(a,z)$ as wealth grows large : $a \to \infty$: linearity as a function of size :



Results : Stationary distribution of firms

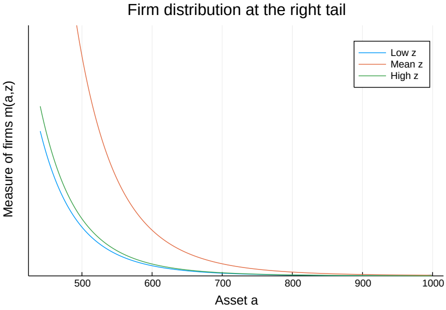
- Skewed firm distribution : mass highly concentrated on the left tail
  - Dual result of constrained capital policy $k^\star$
- Characterization of the right tail of the firm distribution :
- Pareto distribution : $1 - G(a) \sim \xi\, a^{-\zeta}$ with tail exponent $\zeta(z)$
  - Smoothing effect of precautionary saving. Case without jump z :

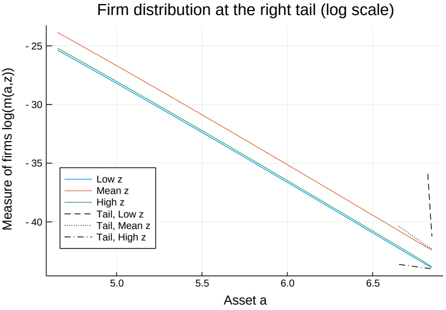
$$\zeta(z) = \gamma \left(\frac{2\bar{R}^2 \sigma^2 \, (\rho - r)}{(\bar{R}\, \mu(z) - r)^2} - 1 \right)$$

- Share of top firm $1/\zeta$ is
  - decreasing in volatility σ, risk aversion γ and time preference ρ
  - Increasing in interest rate r_t and excess return $\bar{R}\mu(z) - r_t$.

Result : Stationary equilibrium

- Firm distribution : Pareto : $1 - G(a) \sim \xi\, a^{-\zeta}$





Appendix : Mathematical problems and Algorithms

- Heterogeneous agents (HA) :
  - $\rightarrow$ Usually no analytical solutions :
  - $\rightarrow$ Numerics : Value Fct. Iteration / finite difference for PDEs
  - For stationary equilibrium / MFG (mean field games)

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  - → Infinite dimensional problem (Master equation) or Stochastic system of PDEs (SPDE)
  - Mean Field Games with common noise
  - Krusell and Smith (1998) (bounded rationality), Reiter (2010) (projection/perturbation methods) or exploiting MIT shocks (BKM)

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- HA models with aggregate shocks, and non-trivial distribution and non-linear dynamics
  - → ... ?
  - Achdou-Bourany (2018)
  - Ongoing work by other researchers

Appendix : Tree structure for aggregate shocks, Achdou-Bourany

- Achdou-Bourany (2018)
  - Master thesis under supervision of Y. Achdou

Appendix : Tree structure for aggregate shocks, Achdou-Bourany

- Achdou-Bourany (2018)
  - Master thesis under supervision of Y. Achdou
- Main idea : approximate the process for the agg. shock Z_t by a finite number of "simple" shocks :
  - Every ΔT (deterministic times), Z_t jumps stochastically to one of the K outcomes
  - Repeat this : a finite M number of "waves" of uncertainty
  - This way, you can build a tree of K^M paths of Z_t with deterministic branches separated by stochastic shocks
  - Taking $\Delta T \to 0$, you can approximate any process (e.g. Donsker's theorem for Brownian motion)
  - Need to link the branches together in an appropriate way

Appendix : Tree structure for aggregate shocks : Achdou-Bourany

- Grafting branches :
  - On each branch (between each shock), compute the evolution of the system : HJB and KF : $v(a, z_j, \tilde{Z})$ and $g(a, z_j, \tilde{Z})$
- To account for future and past shocks ?
  - $\Rightarrow$ use boundary conditions of the PDEs !

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$\Rightarrow$ use boundary conditions of the PDEs !

  - t_m^- time before revelation of the shock ($Z_{t_m^-} = Z_m$)
  - t_m^+ : time when shocks hits ($Z_{t_m^+} = Z_{m+1}$ take K values)

$$v(a, z_j, Z_m) = \sum_{k | Z_{m+1} = Z_k} \mathbb{P}(Z_{m+1} | Z_m) \, v(a, z_j, Z_{m+1})$$

$$g(a, z_j, Z_m) = g(a, z_j, Z_{m+1})$$

  - Agents are forward looking, form expectations over the different future branches (paths of Z_t)
  - Continuity of $g(\cdot)$ in time t

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    - Agents are forward looking, form expectations over the different future branches (paths of Z_t)
    - Continuity of $g(\cdot)$ in time t
- Loop to find eq. fixed point on the entire tree (all branches !)
  - Problem : computationally heavy/slow !

Appendix : Tree structure for aggregate shocks : Achdou-Bourany

- This type of algorithm :
  - Global : no linearization, simulate the entire set of shocks histories
  - Non-linear : shock of different variance/sign with different effects : break the certainty equivalence !
  - Keep track of the movement of the entire distribution
  - > ... But stark assumption of the structure of uncertainty (finite sequence of deterministic time of revelation of shocks)
  - > ... and slow (exponentially growing complexity)
- Simulation of the model above. Need to compare with :
  - Krusell Smith : distribution of agents matters beyond first moment
  - Reiter/Perturbation methods : credit crunch and market illiquidity feedback loops create non-linear dynamics
  - BKM and Auclert et al : anticipation/hedging/pricing of aggregate risk changes transmission of shocks !
- This comparison project will be implemented very soon...