Credit Cycles, Market Liquidity and Heterogeneous Firms

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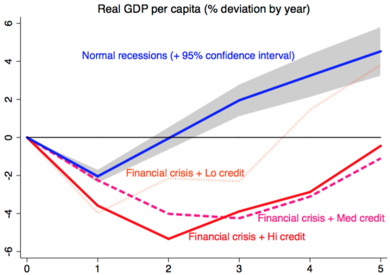
Introduction – Motivation

(a) Discrete excess credit treatment



Importance of credit for recessions – from Jordà, Schularick, and Taylor (2013)

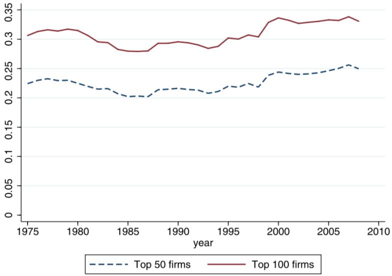


FIGURE 1.—Sum of the sales of the top 50 and 100 non-oil firms in Compustat, as a fraction of GDP. Hulten's theorem (Appendix B) motivates the use of sales rather than value added.

Granularity, top firms matter for aggregates – from Gabaix (2011)

Research question

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  - Are recessions amplified by the collapse of large companies ...
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- Does the large firms reaction drive business cycles in this context ?
  - *Transmission mechanisms* (does aggregation holds/distribution matters ?)

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$\Rightarrow$ *Aim of the paper :*
Examine the effects of firm heterogeneity on aggregate fluctuations in presence of different dimensions of (il)liquidity

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- What is the impact of firm heterogeneity on aggregate fluctuations in presence of two dimensions of (il)liquidity ?
- Financial (il)liquidity : Two dimensions
  - *Market illiquidity :* Asset and capital lose value when sold (asset)
  - Funding illiquidity : Difficult to raise funds for investment (liability)
  - Consequence : Deleveraging, credit/asset-price feedback loop
- Firms heterogeneity :
  - Dispersion and *power-law* distribution
  - Hedging both idiosyncratic & aggregate risk in incomplete market
  - Credit and Collateral constraints

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$\Rightarrow$ *Aim :* Study of the three dimensions *together*

- Propose a theoretical framework with heterogeneous firms, collateral constraints and market illiquidity

Research question – Theoretical literature

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  - Macro-Finance : both Market and funding illiquidity
    - Financial frictions and credit cycles : BGG (99), Kiyotaki Moore (JPE 1997), Brunnermeier and Sannikov (2014) and many others ...
  - Heterogeneous agents :
    - Households : Aiyagari (94), Kaplan-Moll-Violante (2017),Benhabib, Bisin, and Zhu (2015), Achdou, Han, Lasry, Lions, and Moll (2017)
    - Firms : Moll (2014), Winberry (2016a), Mongey and Williams (2017), Khan and Thomas (2013) (JPE)
    - With aggregate shocks : Krusell and Smith (1998) (JPE), Winberry (2016b), Ahn, Kaplan, Moll, Winberry, and Wolf (2018)

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Framework : firms problem

- Heterogeneous entrepreneur :
  - Capital k_t subject to income shocks (CRS prod.)

$$dR_t = \bar{R}\, k_t\, dZ_t^i$$

    - Z_t productivity : Z_t idiosyncratic shock : $dZ_t^i = \mu^i(z_t)dt + \sigma_t dW_t^i$
    - z_t (and $\mu^i(z_t)$) : jump on productivity growth : n_z states process : here, 3-states : low state, median state, high state
    - Aggregate shock on average return $\bar{R}$ (more on this later) :

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  - Two assets portfolio choice (à la Merton 73) :
    - Risk-free Bond b_t at rate r_t vs. Risky Capital k_t at rate R_t
  - Incomplete market : cannot self-insure against any risk
  - Collateral constraints and shocks on capital quality θ_t^c :

$$b_t \geq -\theta_t^c\, q_t\, k_t \qquad \Rightarrow \qquad k_t \leq \frac{1}{(1-\theta_t^c)q_t}\, a_t$$

Framework : firms problem

- Reduce to one state-variable (asset/networth) $a_t = b_t + q_t k_t$
- Optimization problem (without agg. risk)

$$\max_{\{c_t,k_t\}_{t_0}^{\infty}} \mathbb{E}_{t_0} \int_{t_0}^{\infty} e^{-\rho_t t} u(c_t) dt$$

$$da_t = \Big[r_t\, a_t - c_t + q_t k_t \left(\bar{R}\, \mu^i(z_t) - r_t \right) \Big] dt + q_t k_t \bar{R}\ \sigma_t^i dW_t^i \qquad \text{(P)}$$

$$0 \le k_t \le a_t / (1-\theta_t^c) q_t \qquad\qquad a_t \ge 0$$

  - Two states variables : assets a_t (size) and productivity z_t
  - Portfolio : capital k_t/bonds b_t determined endogenously
- Typical heterogenous agents problem (Mean Field Game)
  - (i) Optimal choices given by the Hamilton Jacobi Bellman (HJB)
  - (ii) Law of motion of the distribution by Kolmogorov Forward (KF)

Framework : firms problem

- Problem analogous to :
  - Bewley-Huggett-Aiyagari, with collateral constraint :
    - generates high mass of firm $m(a,z)$ on the left tail of the distribution
  - Merton portfolio choice for large firms :
    - generates power law $m(a,z) \sim a^{-\zeta-1}$ on right tail of distribution

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- Coupling : strategic complementarity/pecuniary externality
  - Risk-free interest r_t determined by market clearing

$$\int_{\mathcal{A}\times\mathcal{Z}} b_t(a,z)\, m(t,da,dz) = \underline{B} = 0 \quad \text{with} \quad b_t(a,z) = a - q_t k_t(a,z)$$

- New element : Market liquidity
  - Endogenous asset price q_t

Market liquidity : asset pricing

- Determination of asset price q_t : different solutions
  - Incomplete market : no hedging by firms
  - One price for capital that has different return for different firms :

$$\mathbb{E}^m[R_t] = \int_{\mathcal{A}\times\mathcal{Z}} \bar{R}\, k_t(a,z)\mu^i(z)\; m(t, da, dz)$$

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- Endogenous asset price q_t

$$dq_t = q_t\,\mu_t^e\,dt + q_t\,\sigma_t^{c,e}\,\varepsilon_t^c\,dN_t^c$$

  - Market liquidity mechanism :
    aggregate fluctuations in q_t exacerbate exogenous shocks

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  - Market liquidity mechanism :
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- Determination of q_t : No arbitrage :
  - Between risk-free rate r_t and a "market portfolio" of all firms :

$$q_t = \mathbb{E}\Big[\int_t^{\infty} e^{-r_\tau(\tau-t)}\mathbb{E}^m[R_\tau]d\tau\Big]$$

$$dq_t = \Big(q_t r_t - \mathbb{E}^m[R_\tau]\Big)dt \qquad\qquad \text{without aggr. risk yet}$$

Aggregate shocks – 1

- Aggregate shocks : Jump-drift processes (jump dN^c) of size ε

- Treated as unexpected (zero probability) events
- Stationary equilibrium without aggregate shocks :

$$\sigma_R^c = \sigma_\sigma^c = \sigma_\theta^c = \sigma_\rho^c = 0$$

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$$d\bar{R}_t = \eta(\overline{\mu}^R - \bar{R}_t)dt + \sigma_R^c \, \varepsilon_t^c dN_t^c$$

  - RBC-style exogenous supply shock
  - Here, affect the return of all entrepreneurs to capital

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- Change in patience/discount factor

$$d\rho_t = \eta(\mu^\rho - \rho_t)dt + \sigma_\rho^c \varepsilon_t^c dN_t^c$$

  - NK-style aggr. demand shock (e.g. Eggertsson and Krugman (2012))
  - Here, affect the patience of experts and saving motive

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Aggregate shocks – 2

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- Aggregate shocks : Jump-drift processes (jump dN^c) of size ε
- Capital quality shocks (experts)

$$d\theta_t^c = \eta(\mu^\theta - \theta_t^c)dt + \sigma_\theta^c \, \varepsilon_t^c dN_t^c$$

  - Driver of the Great Recession according to Khan and Thomas (2013) and Jermann and Quadrini (2012)
  - Implies deleveraging : large drop in investment by constrained firms

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$$d\sigma_t^i = \eta(\mu^\sigma - \sigma_t^i)dt + \sigma_\sigma^c \, \varepsilon_t^c dN_t^c$$

  - E.g. Bloom (2009) and more recently Mongey and Williams (2017)
  - Induces "wait-and-see" effect (and reduction investment) but more dispersion in firms : ambiguous effects on the distribution of firms
- Treated as unexpected (zero probability) events
- Stationary equilibrium without aggregate shocks :

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Stationary equilibrium

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  - Hamilton-Jacobi-Bellman equation : ⇒ pin down the value $v(t,a,z)$ and choices of firms

$$-\partial_t v(t,a,z) + \rho\, v(t,a,z) = \max_{\substack{c\geq 0\ ,\ a\geq 0 \\ 0\leq k_t\leq a/(1-\theta^c)a}} u(c) + \partial_a v(t,a,z)\left[ra - c + (\bar{R}\mu(z) - r)\,q\,k\right] \quad \textbf{(HJB)}$$

$$+\,\Delta_a v(t,a,z)(q_t k\bar{R}\ \sigma_t^i)^2/2 + \sum_{z'}^{n_z} \lambda_{z'}\left(v(t,a,z') - v(t,a,z)\right)$$

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  - Kolmogorov Forward : ⇒ pin down the measure of agents $m(t,a,z)$

$$\partial_t m(t,a,z) = -\partial_a\left[m(t,a,z)\left(r_t a - c_t^\star(a,z) + (\bar{R}\,\mu(z)\ - r)\,q_t k_t^\star(a,z)\right]\right] \quad \text{(KF)}$$

$$+ \partial_a^2\left[m(t,a,z)(q_t k_t^\star(a,z)\bar{R}\ \sigma^i)^2/2\right] - \lambda_z m(t,a,z) + \sum_{z'\neq z}^{n_z} \lambda_{z'} m(t,a,z')$$

$$\forall (t,a,z) \in [t_0,\infty]\times\mathcal{A}\times\mathcal{Z} = [t_0,\infty]\times[0,\infty)\times\{z_1\dots z_{n_z}\}$$

Stationary equilibrium

- A competitive industry stationary equilibrium (without aggregate shocks) is a set of prices $\{r_t, q_t\}_t$ such that :
  - The firms value and optimal policies c_t, k_t, b_t are given by the solution of HJB
  - The firms distribution $m_t(da, dz)$ is given by the solution of KF
  - Bond market clears $\int_{\mathcal{A}\times\mathcal{Z}} b_t(a, z)\, m_t(da, dz) = \underline{B}$
  - Asset market is priced by no arbitrage $r_t dt = \frac{dq_t + \mathbb{E}[R_t]}{q_t}$
- Extensions :
  - Alternative way of pricing the asset q_t
  - General equilibrium : Household consuming and pricing the asset
  - Government policy : stimulus and subsidy to demand
  - Entry and exit : deleveraging + default

Results : Stationary policy functions

- Characterization of value/policies functions $c^{\star}(a,z)$, $s^{\star}(a,z)$ & $k^{\star}(a,z)$
  - **Result 1 :** For each productivity $z \in \mathcal{Z}$, there is an interval over size (asset) such that all firms are constrained & have the same policy :

$$a \in [0, \gamma(\theta^c, q, z)] \qquad \Rightarrow \qquad k(a,z) = k(a) = \frac{a}{(1-\theta^c)q}$$

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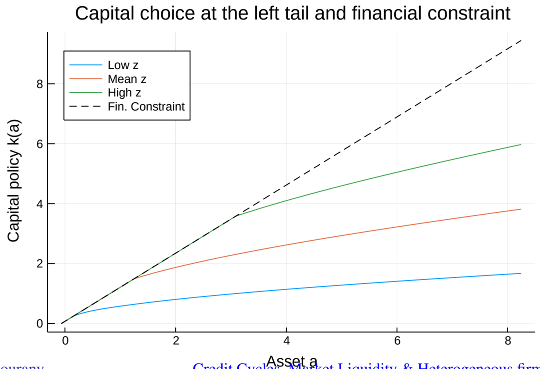


Results : Stationary policy functions

- Policy functions : $c(a,z), s(a,z)$ and $k(a,z)$ at the upper tail : linearity in state
  - **Prop. 1 :** In stat. eq., with CRRA = γ, and no jump in level of productivity growth, i.e. $\lambda_z = 0 \ \forall z$, firms consumption, saving and capital investment policies are asymptotically linear in wealth a, as wealth grows large : $a \to \infty$:

$$c(a,z) \sim \left(\frac{\rho - (1-\gamma) r}{\gamma} - \frac{1}{2} \frac{(\bar{R}\mu(z) - r)^2}{\bar{R}^2 \sigma^2} \frac{1-\gamma}{\gamma^2} \right) a$$

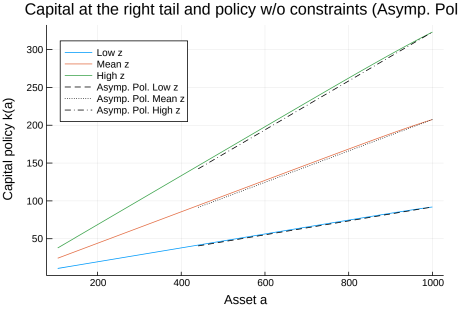
$$s(a,z) \sim \left(\frac{r - \rho}{\gamma} + \frac{1+\gamma}{2\gamma} \frac{(\bar{R}\mu(z) - r)^2}{\gamma \bar{R}^2 \sigma^2} \right) a$$

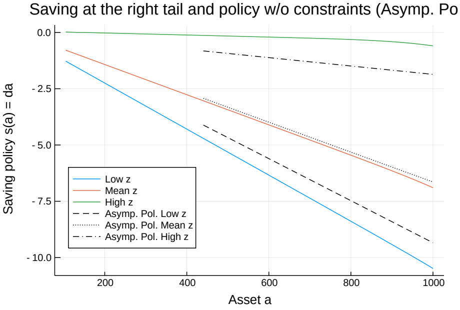
$$k(a,z) \sim \frac{\bar{R}\mu(z) - r}{\gamma \, q \, \bar{R}^2 \sigma^{i\,2}} \, a$$

with $f(x) \sim g(x)$ as $x \to \infty$ notation for $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$.

Results : Stationary policy functions

- Characterization of decisions $k(a,z)$ and $s(a,z)$ as wealth grows large : $a \to \infty$: linearity as a function of size :





Results : Stationary distribution of firms

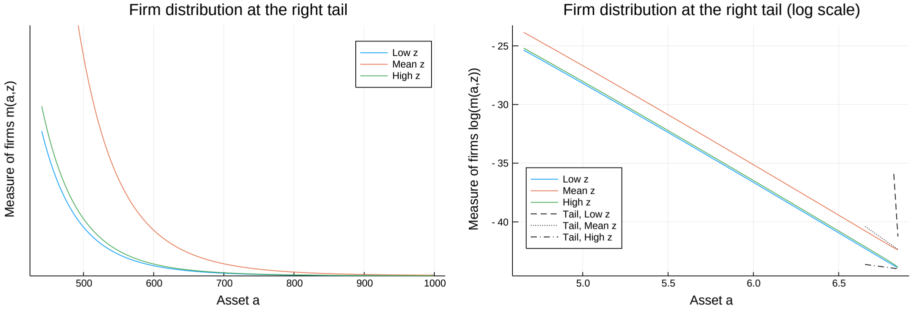
- Skewed firm distribution : mass highly concentrated on the left tail
  - Dual result of constrained capital policy $k^{\star}$
- Characterization of the right tail of the firm distribution :
- Pareto distribution : $1 - G(a) \sim \xi \, a^{-\zeta}$ with tail exponent $\zeta(z)$
  - Smoothing effect of precautionary saving. Case without jump z :

$$\zeta(z) = \gamma \left(\frac{2\bar{R}^2 \sigma^2 \, (\rho - r)}{(\bar{R} \, \mu(z) - r)^2} - 1 \right)$$

- Share of top firm $1/\zeta$ is
  - decreasing in volatility σ, risk aversion γ and time preference ρ
  - Increasing in interest rate r_t and excess return $\bar{R}\mu(z) - r_t$.

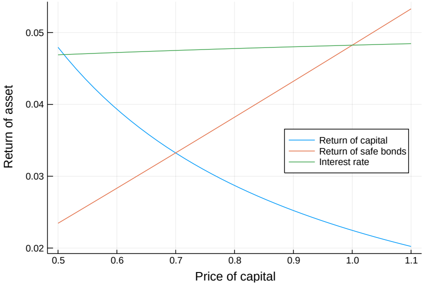
Result : Stationary equilibrium

- Firm distribution : Pareto : $1 - G(a) \sim \xi\, a^{-\zeta}$

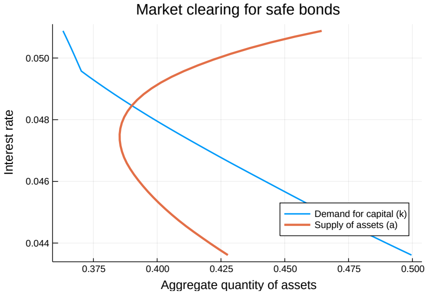


Equilibrium :

- Asset market (and market illiquidity) and bond market (and precautionary saving)



No arbitrage for asset (capital) : $\mathbb{E}[R_t]$ (blue) vs. $q_t r_t$ (red)



Market clearing for bonds b_t, due to interest r_t

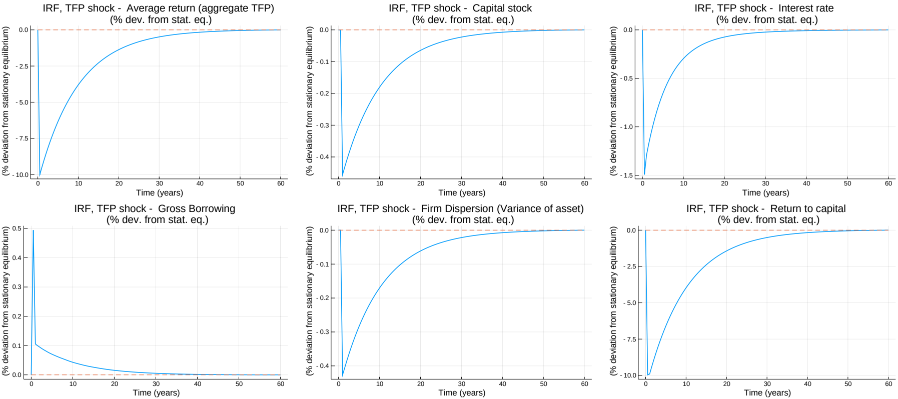
Aggregate shocks :

- Three experiments : Unanticipated aggregate shock :
  1. Standard recession : $\bar{R}_t$ drop of 10%
  2. Credit crunch : θ_t^c : drop of 15%
  3. Uncertainty shock : σ^2 : rise of 15%
- Revealed in the first year
- Generalized Ornstein Uhlenbeck process ($\approx$ AR(1) with $\rho = 0.9$)

$\Rightarrow$ For now : abstract from market illiquidity :

- Partial equilibrium in q_t but general equilibrium in r_t

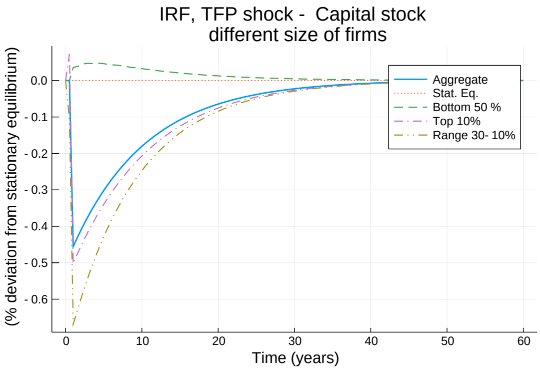
Supply side shock : "Standard" recession



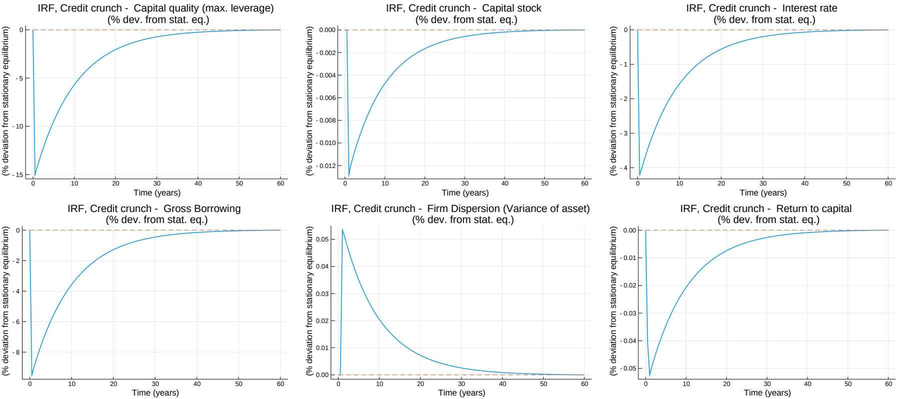
IRF – TFP shock to aggregate return $\bar{R}_t$

Supply side shock : "Standard" recession

- Decomposition by firms size :
  - Bottom firm behavior : constrained and smoother.
  - Top firms : change their exposure to capital risk : reduction in precautionary saving and income effect



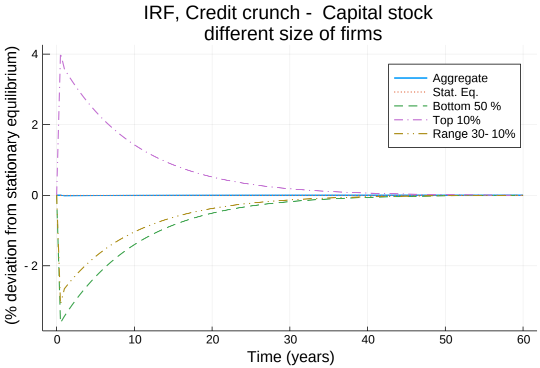
Capital quality shock : Credit crunch



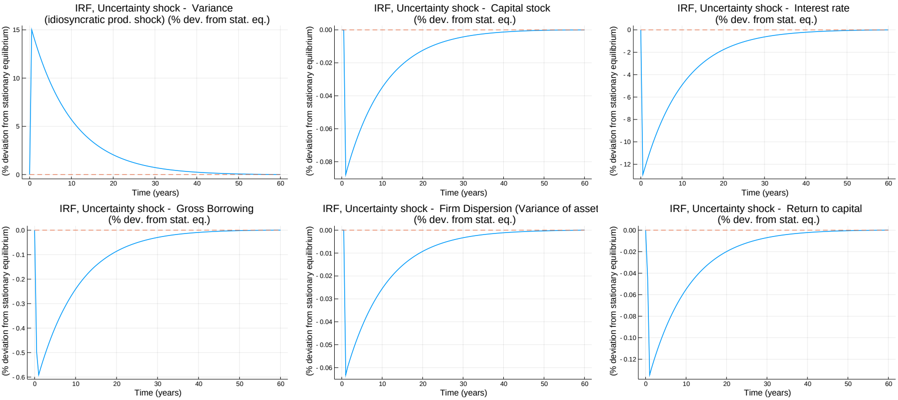
IRF – Credit crunch : shock θ_t^c

Capital quality shock : Credit crunch

- Decomposition by firms size :
  - Bottom firm behavior : deleveraging and large drop in capital
  - Top firms : benefit from the general equilibrium effect to invest



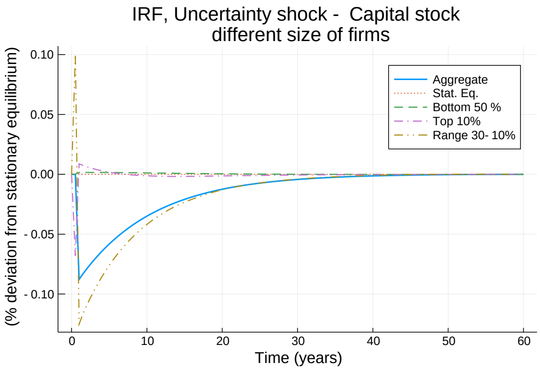
Uncertainty shock : Rise in volatility



IRF – Uncertainty shock : shock on σ_t

Uncertainty shock : Rise in volatility

- Decomposition by firms size :
  - Median/large firms reduce exposure while top firms benefit from equilibrium effect (interest) : opposite behaviors on impact !
  - Bottom firm behavior : constrained and barely affected.



Conclusion and Future research

- *Question :*
  What is the influence of firms heterogeneity on credit cycles ?
  - $\rightarrow$ Standard recessions driven by the median and larger firms
  - $\rightarrow$ Effects of credit crunch partly offset/smoothed by the hedging of largest firms (top 10%)
  - $\rightarrow$ Uncertainty shocks or TFP shock do not impact the bottom constrained firm
- Bridging macro-finance and heterogeneous agents literatures
  - Propose a theoretical framework with heterogeneous firms, collateral constraints, and market illiquidity, in presence of aggregate risk
  - Non-linearity of shocks + failure of approximate aggregation
  - Does market (il)liquidity matters ? *(next step on this project !)*

Future research – Empirical evidence

- Empirical relevance *(next step on this project !)*
  - Micro-level Firms heterogeneity
    - Cross-section data from Compustat
    - Largest firms : reacting to aggregate shocks + power law $\zeta \approx 1.0$

  $\Rightarrow$ Structural estimation (SMM)
    - What determine the firms distribution ?
    - Preference ? or structure/variance of risks ?
- Application to Boom and Bust dynamics
  - Great Recession : From credit crunch to economic downturn
- Aggregate risk : more elaborate treatment :
  - Exploiting MIT shocks (BKM, Auclert et al.),
    Tree structure of aggreg. shocks (Bourany-Achdou, cf. appendix)

Conclusion

- This paper examine the influence of firm heterogeneity in presence of market illiquidity and financial frictions
- It revisits the transmission and amplification mechanisms of different aggregate shocks that could explain deep recessions

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- This paper examine the influence of firm heterogeneity in presence of market illiquidity and financial frictions
- It revisits the transmission and amplification mechanisms of different aggregate shocks that could explain deep recessions
- THANK YOU FOR YOUR FEEDBACKS !

Achdou, Y., F. J. Buera, J.-M. Lasry, P.-L. Lions, and B. Moll (2014). Partial differential equation models in macroeconomics. *Philosophical Transactions of the Royal Society A : Mathematical, Physical and Engineering Sciences 372*(2028), 20130397.

Achdou, Y., F. Camilli, and I. Capuzzo-Dolcetta (2013). Mean field games : convergence of a finite difference method. *SIAM Journal on Numerical Analysis 51*(5), 2585–2612.

Achdou, Y. and I. Capuzzo-Dolcetta (2010). Mean field games : Numerical methods. *SIAM Journal on Numerical Analysis 48*(3), 1136–1162.

Achdou, Y., J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll (2017, November). Income and wealth distribution in macroeconomics : A continuous-time approach. *R & R, Review of Economic Studies* (NBER 23732).

Ahn, S., G. Kaplan, B. Moll, T. Winberry, and C. Wolf (2018). When inequality matters for macro and macro matters for inequality. *NBER Macroeconomics annual 32*(1), 1–75.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics 109*(3), 659–684.

Angeletos, G.-M. (2007). Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic Dynamics 10*(1), 1 – 30.

Auclert, A., B. Bardóczy, M. Rognlie, and L. Straub (2019). Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. Technical report, National Bureau of Economic Research.

Axtell, R. L. (2001). Zipf distribution of u.s. firm sizes. *Science 293*(5536), 1818–1820.

Becard, Y., D. Gauthier, and B. de France (2019). Collateral shocks.

Benhabib, J., A. Bisin, and S. Zhu (2011). The distribution of wealth and fiscal policy in economies with finitely lived agents. *Econometrica 79*(1), 123–157.

Benhabib, J., A. Bisin, and S. Zhu (2015). The wealth distribution in bewley economies with capital income risk. *Journal of Economic Theory 159*, 489–515.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. *The American Economic Review 79*(1), 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). Chapter 21 the financial accelerator in a quantitative business cycle framework. Volume 1 of *Handbook of Macroeconomics*, pp. 1341 – 1393. Elsevier.

Bewley, T. (1986). Stationary monetary equilibrium with a continuum of independently fluctuating consumers. *Contributions to mathematical economics in honor of Gérard Debreu 79*.

Bloom, N. (2009). The impact of uncertainty shocks. *econometrica 77*(3), 623–685.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018). Really uncertain business cycles. *Econometrica 86*(3), 1031–1065.

Boppart, T., P. Krusell, and K. Mitman (2018). Exploiting mit shocks in heterogeneous-agent economies : the impulse response as a numerical derivative. *Journal of Economic Dynamics and Control 89*, 68–92.

Bourany, T. (2018). Wealth distribution over the business cycle : A mean-field game with common noise. *Master Thesis, Sorbonne – Paris Diderot, Applied Mathematics Departments (LJLL)*.

Brunnermeier, M. K., T. M. Eisenbach, and Y. Sannikov (2012). Macroeconomics with financial frictions : A survey. Technical report, National Bureau of Economic Research.

Brunnermeier, M. K. and L. H. Pedersen (2008). Market liquidity and funding liquidity. *The review of financial studies 22*(6), 2201–2238.

Brunnermeier, M. K. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *American Economic Review 104*(2), 379–421.

Buera, F. J., R. N. F. Jaef, and Y. Shin (2015). Anatomy of a credit crunch : from capital to labor markets. *Review of Economic Dynamics 18*(1), 101–117.

Buera, F. J., J. P. Kaboski, and Y. Shin (2015). Entrepreneurship and financial frictions : A macrodevelopment perspective. *Annual Review of Economics 7*(1), 409–436.

Buera, F. J. and B. Moll (2015). Aggregate implications of a credit crunch : The importance of heterogeneity. *American Economic Journal : Macroeconomics 7*(3), 1–42.

Buera, F. J. and Y. Shin (2013). Financial frictions and the persistence of history : A quantitative exploration. *Journal of Political Economy 121*(2), 221–272.

Capuzzo-Dolcetta, I. and P.-L. Lions (1990). Hamilton-jacobi equations with state constraints. *Transactions of the American Mathematical Society 318*(2), 643–683.

Cardaliaguet, P. (2013/2018). Notes on mean field games. *Lecture notes from P.L. Lions' lectures at College de France and P. Cardaliaguet at Paris Dauphine*.

Carmona, R. and F. Delarue (2018). *Probabilistic Theory of Mean Field Games with Applications I-II*. Springer.

Carmona, R., F. Delarue, and D. Lacker (2016). Mean field games with common noise. *The Annals of Probability 44*(6), 3740–3803.

Christiano, L. J., R. Motto, and M. Rostagno (2014). Risk shocks. *American Economic Review 104*(1), 27–65.

Clementi, G. L., A. Khan, B. Palazzo, and J. K. Thomas (2014). Entry, exit and the shape of aggregate fluctuations in a general equilibrium model with capital heterogeneity. *Unpublished Working Paper*.

Clementi, G. L. and B. Palazzo (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal : Macroeconomics 8*(3), 1–41.

Crouzet, N., N. R. Mehrotra, et al. (2017). Small and large firms over the business cycle. Technical report.

Eggertsson, G. B. et al. (2003). Zero bound on interest rates and optimal monetary policy. *Brookings papers on economic activity 2003*(1), 139–233.

Eggertsson, G. B. and P. Krugman (2012). Debt, deleveraging, and the liquidity trap : A fisher-minsky-koo approach. *The Quarterly Journal of Economics 127*(3), 1469–1513.

Gabaix, X. (2009). Power laws in economics and finance. *Annu. Rev. Econ. 1*(1), 255–294.

Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica 79*(3), 733–772.

Gabaix, X., P. Gopikrishnan, V. Plerou, and H. E. Stanley (2003). A theory of power-law distributions in financial market fluctuations. *Nature 423*(6937), 267.

Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of monetary economics*, Volume 3, pp. 547–599. Elsevier.

Gilchrist, S., J. W. Sim, and E. Zakrajšek (2014). Uncertainty, financial frictions, and investment dynamics. Technical report, National Bureau of Economic Research.

Gopinath, G., Ş. Kalemli-Özcan, L. Karabarbounis, and C. Villegas-Sanchez (2017). Capital allocation and productivity in south europe. *The Quarterly Journal of Economics 132*(4), 1915–1967.

Guerrieri, V. and G. Lorenzoni (2017). Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics 132*(3), 1427–1467.

He, Z. and A. Krishnamurthy (2013). Intermediary asset pricing. *American Economic Review 103*(2), 732–70.

Hopenhayn, H. and R. Rogerson (1993). Job turnover and policy evaluation : A general equilibrium analysis. *Journal of political Economy 101*(5), 915–938.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica : Journal of the Econometric Society*, 1127–1150.

Jarrow, R. A., S. Lamichhane, et al. (2018). Asset price bubbles, market liquidity and systemic risk.

Jermann, U. and V. Quadrini (2012). Macroeconomic effects of financial shocks. *The American Economic Review 102*(1), 238–271.

Jordà, O., M. Schularick, and A. M. Taylor (2013). When credit bites back. *Journal of Money, Credit and Banking 45*(s2), 3–28.

Kaplan, G., B. Moll, and G. L. Violante (2018). Monetary policy according to hank. *American Economic Review 108*(3), 697–743.

Khan, A., T. Senga, and J. K. Thomas (2014). Default risk and aggregate fluctuations in an economy with production heterogeneity. *Unpublished Manuscript*.

Khan, A. and J. K. Thomas (2003). Nonconvex factor adjustments in equilibrium business cycle models : do nonlinearities matter ? *Journal of monetary economics 50*(2), 331–360.

Khan, A. and J. K. Thomas (2007). Inventories and the business cycle : An equilibrium analysis of (s, s) policies. *American Economic Review 97*(4), 1165–1188.

Khan, A. and J. K. Thomas (2008). Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. *Econometrica 76*(2), 395–436.

Khan, A. and J. K. Thomas (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy 121*(6), 1055–1107.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of political economy 105*(2), 211–248.

Kiyotaki, N. and J. Moore (2012). Liquidity, business cycles, and monetary policy.

Kocherlakota, N. et al. (2000). Creating business cycles through credit constraints. *Federal Reserve Bank of Minneapolis Quarterly Review 24*(3), 2–10.

Krusell, P. and A. A. Smith, Jr (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy 106*(5), 867–896.

Kumhof, M. and R. G. Rancière (2010). Inequality, leverage and crises.

Le Grand, F. and X. Ragot (2017). A class of tractable incomplete-market models to study asset returns and risk exposure.

Lucas Jr, R. E. and B. Moll (2014). Knowledge growth and the allocation of time. *Journal of Political Economy 122*(1), 1–51.

Moll, B. (2014). Productivity losses from financial frictions : Can self-financing undo capital misallocation ? *American Economic Review 104*(10), 3186–3221.

Mongey, S. and J. Williams (2017). Firm dispersion and business cycles : Estimating aggregate shocks using panel data. *Manuscript, New York University*.

Nuño, G. and C. Thomas (2017). Bank leverage cycles. *American Economic Journal : Macroeconomics 9*(2), 32–72.

Ottonello, P. and T. Winberry (2018). Financial heterogeneity and the investment channel of monetary policy. *Revise and resubmit, Econometrica*.

Punzi, M. T. and K. Rabitsch (2015). Investor borrowing heterogeneity in a kiyotaki–moore style macro model. *Economics Letters 130*, 75–79.

Quadrini, V. (2011). Financial frictions in macroeconomic fluctuations.

Reiter, M. (2009). Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control 33*(3), 649–665.

Reiter, M. (2010). Solving the incomplete markets model with aggregate uncertainty by backward induction. *Journal of Economic Dynamics and Control 34*(1), 28–35.

Reiter, M. (2018). Comments on" exploiting mit shocks in heterogeneous-agent economies : The impulse response as a numerical derivative" by t. boppart, p. krusell and k. mitman. *Journal of Economic Dynamics and Control 89*, 93–99.

Terry, S. J. (2017). Alternative methods for solving heterogeneous firm models. *Journal of Money, Credit and Banking 49*(6), 1081–1111.

Winberry, T. (2016a). Lumpy investment, business cycles, and stimulus policy. *Revise and resubmit, American Economic Review*.

Winberry, T. (2016b). A toolbox for solving and estimating heterogeneous agent macro models. *Forthcoming Quantitative Economics*.

Young, E. R. (2010). Solving the incomplete markets model with aggregate uncertainty using the krusell–smith algorithm and non-stochastic simulations. *Journal of Economic Dynamics and Control 34*(1), 36–41.

Appendix : Mathematical problems and Algorithms

- Heterogeneous agents (HA) :
  - → Usually no analytical solutions :
  - → Numerics : Value Fct. Iteration / finite difference for PDEs
  - For stationary equilibrium / MFG (mean field games)

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- HA models with aggregate shocks
  - $\rightarrow$ Infinite dimensional problem (Master equation) or Stochastic system of PDEs (SPDE)
  - Mean Field Games with common noise
  - Krusell and Smith (1998) (bounded rationality), Reiter (2010) (projection/perturbation methods) or exploiting MIT shocks (BKM)

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- HA models with aggregate shocks, and non-trivial distribution and non-linear dynamics
  → ... ?
  - Achdou-Bourany (2018)
  - Ongoing work by other researchers

Appendix : Tree structure for aggregate shocks, Achdou-Bourany

- Achdou-Bourany (2018)
  - Master thesis under supervision of Y. Achdou

Appendix : Tree structure for aggregate shocks, Achdou-Bourany

- Achdou-Bourany (2018)
  - Master thesis under supervision of Y. Achdou
- Main idea : approximate the process for the agg. shock Z_t by a finite number of "simple" shocks :
  - Every ΔT (deterministic times), Z_t jumps stochastically to one of the K outcomes
  - Repeat this : a finite M number of "waves" of uncertainty
  - This way, you can build a tree of K^M paths of Z_t with deterministic branches separated by stochastic shocks
  - Taking $\Delta T \to 0$, you can approximate any process (e.g. Donsker's theorem for Brownian motion)
  - Need to link the branches together in an appropriate way

Appendix : Tree structure for aggregate shocks : Achdou-Bourany

- Grafting branches :
  - On each branch (between each shock), compute the evolution of the system : HJB and KF : $v(a,z_j,\tilde{Z})$ and $g(a,z_j,\tilde{Z})$
- To account for future and past shocks ?

  $\Rightarrow$ use boundary conditions of the PDEs !

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$\Rightarrow$ use boundary conditions of the PDEs !

- t_m^- time before revelation of the shock ($Z_{t_m^-} = Z_m$)
- t_m^+ : time when shocks hits ($Z_{t_m^+} = Z_{m+1}$ take K values)

$$v(a,z_j,Z_m) = \sum_{k|Z_{m+1}=Z_k} \mathbb{P}(Z_{m+1}|Z_m)\, v(a,z_j,Z_{m+1})$$

$$g(a,z_j,Z_m) = g(a,z_j,Z_{m+1})$$

- Agents are forward looking, form expectations over the different future branches (paths of Z_t)
- Continuity of $g(\cdot)$ in time t

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$$v(a,z_j,Z_m) = \sum_{k|Z_{m+1}=Z_k} \mathbb{P}(Z_{m+1}|Z_m)\, v(a,z_j,Z_{m+1})$$

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  - Agents are forward looking, form expectations over the different future branches (paths of Z_t)
  - Continuity of $g(\cdot)$ in time t
- Loop to find eq. fixed point on the entire tree (all branches !)
  - Problem : computationally heavy/slow !

Appendix : Tree structure for aggregate shocks : Achdou-Bourany

- This type of algorithm :
  - Global : no linearization, simulate the entire set of shocks histories
  - Non-linear : shock of different variance/sign with different effects : break the certainty equivalence !
  - Keep track of the movement of the entire distribution
  - $>$ ... But stark assumption of the structure of uncertainty (finite sequence of deterministic time of revelation of shocks)
  - $>$ ... and slow (exponentially growing complexity)
- Simulation of the model above. Need to compare with :
  - Krusell Smith : distribution of agents matters beyond first moment
  - Reiter/Perturbation methods : credit crunch and market illiquidity feedback loops create non-linear dynamics
  - BKM and Auclert et al : anticipation/hedging/pricing of aggregate risk changes transmission of shocks !
- This comparison project will be implemented very soon...