Energy shocks and aggregate fluctuations
Is decoupling possible?

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Abstract

How important is energy for economic fluctuations? The energy sector – such as oil and electricity – is a complementary factor in industrial production processes. A decline of energy sources – due to declining supply of fossil-fuels (peak oil) – or political decisions to shrink greenhouse gas emissions – could have important implication for the macroeconomy. We measure the contribution of energy sector shocks to business cycles using a simple RBC model that features high degree of complementarity and non-linearity in production. The first preliminary result is that the expansion in energy supply was an important driver of output growth in the first half of the post WWII period. The vanishing of this source of growth can explain part of the decline of output since the first oil shock.

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To produce any given motion, to spin a certain weight of cotton, or weave any quantity of linen, there is required steam; to produce the steam, fuel; and thus the price of fuel regulates effectively the cost of mechanical power. Abundance and cheapness of fuel are hence main ingredients in industrial success. It is for this reason that in England the active manufacturing districts mark, almost with geological accuracy, the limits of the coal fields.

Robert John Kane –*The Industrial Resources of Ireland (1844)*

1 Introduction

How important is energy for business cycles? Access to energy resources has always been a crucial factor in economic development and growth. All the sectors of our economies – manufacturing, transportation, and services – rely on energy and electricity supply, with various degrees of complementarity in production. Hence, supply shocks to this underlying factor have effects on economic fluctuations. For example, the recessions of 1973, 1990, and 2008 were all preceded by stagnation in energy supply or by oil price shocks[1]. Furthermore, there has been widespread evidence of a decline in energy supplies, following "Hubbert peak" theory, for most energy supplies, mainly oil, gas and coal.

Second, there have been major debates on the role of macroeconomic fiscal and monetary policies in the fight against global warming and a transition away from fossil fuel energies[2]. However, fossil fuels represent still $\approx 80\%$ of energy sources in the World and the United States, and the energy consumption is the primary source[3] of greenhouse gas emissions (GHG). Macroeconomic policies would aim at keeping world temperatures below $1.5°/2°C$ by decreasing world GHG to zero by 2050. This climate action would have an enormous impact on energy demand in the medium run and should be implemented with the insight that this factor is essential to production.

This paper aims at providing a framework for such insight. What are the spillovers of energy supply shocks on the rest of the economy? We examine the different reallocation channels away from energy toward other factors, e.g., labor, or toward a reduction in aggregate activity. Are amplification effects large when we take into account strong complementarity and non-linearities in an otherwise standard model? Moreover, can energy shocks be measured as a significant driver of business cycle fluctuations?

We provide a theoretical model to think about these questions. It recast some conventional issues in energy and environmental economics into a broader macroeconomic approach. First, we provide a straightforward Real Business Cycle model, where energy is an additional factor to the traditional labor and capital. Moreover, we consider a nested production structure that features complementarity between these three factors: it starts from the intuition that capital can not

[1]For example, Hamilton (2009) and Baumeister and Kilian (2016) propose different views on the causes and the consequences of oil price shocks on economic activity.

[2]For two recent examples: Schnabel (2021) and Villeroy de Galhau (2021)

[3]Energy consumption, i.e., supplied to different sectors, represents 75% of greenhouse gas emission (GHG) emissions at the world level, far beyond the share from of agriculture (18%). Out of the total, 24% of GHG is emitted through energy use in industry, 16% by transportation, and 17% by energy use in buildings.

physically produce without energy inputs. This production structure is consistent with results in the energy economics literature, c.f., the estimations on microdata by Henningsen et al. (2019) or the macro approach of Atkeson and Kehoe (1999), finding such elasticity being significantly lower than 1.

Supply shocks to the energy sector imply strong and persistent effects on firms' production. In our context, even if the energy sector represents a small share of total output, around $\approx 10\%$, the impact of a supply shock is $65 - 75\%$ larger. Moreover, this amplification mechanism crucially depends on the reallocation channels. The fact that capital is a slow-moving variable or is subject to adjustment costs is key[4]. It results in reallocation effects toward labor, consistent with empirical evidence: for example, Kehrig and Ziebarth (2017) in the case of oil prices, or Allcott, Collard-Wexler, and O'Connell (2016) in the case of electricity supply distribution.

Moreover, we consider a model solution and estimation that measure second-order effects – beyond usual linearization techniques. Aggregation of multi-sector economies – as in our simple case with an energy sector – leads to significant second-order effects amplifying granular shocks. Baqaee and Farhi (2019) extend Hulten's theorem and show the sales share are no longer sufficient statistics: a negative shock to a sector with 10% sales share can be beyond 10% with complementarities in production and lack of reallocation channels toward the troubled sector. We show an example of this theoretical point in our context: the energy sector may have small sale shares[5] but causes a more significant decline in total output. Moreover, in the presence of additional market frictions, like nominal rigidities or mark-up, the role of the production structure becomes first-order, as shown in Baqaee and Farhi (2020).

I propose a business cycle accounting in the spirit of Chari, Kehoe, and McGrattan (2007) where energy supply shocks and energy augmenting technological shocks contribute to the change in GDP. In addition to these two energy sector shocks, we add TFP shock, Labor wedge shock, Market clearing shock (government consumption and net export) and match the post WWII data on output, investment, labor, government spending and export and energy consumed in physical units. I provide a simulation and Bayesian estimation methods to first filter the process. For each variable in this sample, we compute the contribution of each shock to the observable variables.

Our main result show that energy supply shocks and the expansion of this input in the first half of the post WWII period have explained around 30% of the fluctuation in real GDP forecast around the trend: without this augmentation of energy as underlying factor, the GDP level would have been 30% lower. The decline of this growth in the last part of the sample 1990-2020 could in part due to the vanishing of this source of growth. This is not compensated by a rise in TFP or energy-augmenting technology. The second more surprising result is the lack of contribution of this energy shock to the recessions of 1973, 1980 and 2008: the energy shock have kept an almost-

[4]In the first version of this model, adjustment costs are absent but will be added in the next version.

[5]An example given by Baqaee and Farhi (2019): in the U.S., the entire electricity market have the same sales share as Walmart. A negative 50% shock to the entire electricity sector would have a much more adverse impact than one on Walmart.

constant share in the output decomposition, while TFP and labor wedge played a large role. Our last result relates to the fall in labor hours: as seen in the toy model, when energy supply expand, the firm reallocation and substitute away from labor toward machine. As a result, in first part of the sample between 1949 and 1990, between 20 and 30% of the decline in labor could be attributed to an expansion of labor supply. Most of the variation are still explained by the labor wedge and more comprehensive analysis are required

Our objective, in a latter phase of this project, is threefold. First, these results require checking alternative specification of the energy production function and household preferences. We present two avenues of improvement of this model in the last section. Second, our particle filtering algorithm is ready to use, but to a lack of time I did not include the result in this draft yet. The next objective would be to embed the likelihood function obtained by Sequential Monte Carlo into a Monte Carlo Markov Chain for estimation purpose.

Third, if time permits, the final objective would be to extend this model to a medium-scale DSGE model, as Smets and Wouters (2007). Adding nominal rigidities makes the energy supply spillover over inflation and price dispersion: it distorts firms marginal costs, affect firms that can not change their price in the short-run and affect the price index faced by consumer.

This paper relates to (too many) different strands of the macroeconomic literature. First, I am not the first to draw link between energy and oil price shocks and the economy. Starting from empirical studies looking for correlation between oil price and GDP growth, Hamilton (1983) and Mork (1989), Hamilton (1996) , many studies have used macroeconometric methods to investigate the causes and consequences of such shocks, for example with Kilian (2009), Mork, Olsen, and Mysen (1994), Hamilton (2009) or Kilian and Murphy (2014).

Many studies have incorporated energy and oil as a factor in the RBC model, with the early work by Kim and Loungani (1992) and a surprisingly large – and surprisingly not so famous – literature using the New Keynesian DSGE framework, as, Dhawan and Jeske (2008), Milani (2009), Dhawan et al. (2010), Engemann et al. (2011). Some have focused on small open economy importing oil, such as the UK, in Millard (2011) or Aminu (2018). Many of these articles have focused on the impact of these shocks, without response of efficiency of production, what I call energy saving/energy augmenting productivity shifter, with the notable exception Acurio Vasconez, Giraud, Mc Isaac, and Pham (2015).

In comparison to this articles, I build a general equilibrium framework where energy is one source of aggregate fluctuations among others standard macroeconomic shocks – technological progress, time-varying friction on the good and labor market. Moreover, all these papers relate to the particular case of oil price shocks. Oil is an essential input in production processes, in particular for transportation or petrochemical products. However, it only represents an average of $35-45\%$ of total energy supplied in the U.S. Our focus is on total energy consumption, which also includes natural gas (averaging $20-30\%$ between 1990-2020), coal ($20-15\%$), nuclear ($7-8\%$) and renewable energy (hydroelectric, biomass, solar and wind, for $8-10\%$). Despite high correlation between energy prices (mainly gas and oil products), focusing on total energy may hide certain reallocation channels within the energy sector.

Second, the present work relates to another field of the literature of energy economics, with a more careful modeling of the oil producers and the impact of the price dynamics using microdata. This includes Kellogg (2014), Asker et al. (2019), Bornstein et al. (2021), Allcott, Collard-Wexler, and O'Connell (2016) or Kehrig and Ziebarth (2017). In a future version of this project, we follow this cautious way of microfounding the energy production and estimate the parameters with macroeconomic data.

Moreover, there has been numerous call for alternative modeling of the economy to include the interactions between economic fluctuations, fossil fuels consumption, GHG emissions and climate change. These works, usually outside the standard literature in economics, include Tverberg (2012), Bardi (2011), Grandjean and Giraud (2017) the work by Jancovici (2019) and some articles from the scientific literature, c.f. Hall et al. (2014), King and Van Den Bergh (2018), Capellan-Perez, de Castro, and Gonzalez (2019). Many of them call for a merging of integrated assessment models a la Meadows et al. (1972) – c.f. a recently updated version in Branderhorst (2020) – with standard DSGE models used in economics. One of the future objective of this project – likely in a different paper – could be to include some of these variables that could interact with economic fluctuations, such as GHG emissions, population growth and the impact on different sectors.

Some articles in the economic literature have tried to include climate change, starting with Nordhaus and Boyer (2000) and technological progress on the transition path toward a carbon-neutral economy, c.f. Hassler et al. (2019), Golosov et al. (2014) or Acemoglu et al. (2016) and the chapter Hassler et al. (2016) that summarizes the recent work by these authors. . However, in this paper, I try to provide a realistic modeling of the energy sector and the interaction with business cycle fluctuation: a decline in energy sector production has an impact on growth through the complementarity in production. Therefore, transition to a carbon-neutral economy with empirically sensible technological progress can be accompanied with a large decline in growth.

A last literature the present paper relates to is the field of production network. After Hulten, the first studies to incorporate multiple industries in the RBC model were in Long and Plosser (1983) and later extended by Horvath (1998), and recently, a new literature has emerged, with for example Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). I follow the theoretical intuition of Baqaee and Farhi (2019), and Baqaee and Farhi (2020) showing that complementarities in production imply larger amplification of shocks. If our structural approach incorporate different sector, we would borrow from the structural approach of Foerster, Sarte, and Watson (2011) and Atalay (2017) and the recent work by Winberry and vom Lehn (2020). Moreover, note that both Rubbo (2020) and Baqaee and Farhi (2020) explicitly mention theoretical exercise where the energy sector supply oil or energy to other intermediate goods producers, with an impact on aggregate production and inflation.

This paper – very preliminary as it is – could go further in the direction of one of these different literature. My objective would be to decide which of the previously mentioned orientation is the most promising. In the first section, we mention a toy model aimed a proving intuitions for the impact of an energy shock. In the second section we build the baseline model with an adhoc production function for energy. In the third section we briefly describe the empirical strategy, while in the four we present the preliminary result of the linear analysis of the model. The last section propose the different paths that this project will eventually take.

2 A toy model: complementarity implies amplification

Why are complementarities important for the transmission of energy shocks? We consider the simple model, where final output is produced with energy and labor:

$$Y = F(L, E)$$

In the following we consider two simple examples: Cobb Doublas or Leontieff:

$$Y_1 = E^\eta L^{1-\eta} \qquad\qquad Y_2 = \min\{E, L\}$$

The rest of the static model is standard: the household sector is trivial with CRRA utility function with parameter σ and labor supply disutility, with Inverse Frisch elasticity φ :

$$\max_{C,L} u(C) - v(L), \quad s.t. \qquad C = WL \qquad \Rightarrow \frac{u'(C)}{v'(L)} = L^\varphi C^\sigma = W$$

With a Cobb-Douglas function, production is approximately log-linear.

$$d \log Y = \eta d \log E + (1 - \eta) d \log K$$

In implies that income effect and substitution effect in production cancel out, keeping the share of energy constant. Using the labor market optimality condition, we obtain that labor supply also reacts:

$$\begin{aligned} (1-\eta) L^\eta E^\eta = W &= L^\varphi C^\sigma = L^\varphi (WL)^\sigma \\ \Rightarrow \qquad d \log L &= \tfrac{(1-\sigma)\eta}{\varphi + (1-\sigma)\eta + \sigma} d \log E \\ \Rightarrow \qquad d \log Y &= \tfrac{(1+\varphi)\eta}{\varphi + (1-\sigma)\eta + \sigma} d \log E \end{aligned}$$

if $\sigma \neq 1$. In the log case $\sigma = 1$ – and more generally in the case of Balanced-growth preferences – there is no wealth effect on labor supply, s.t. $d \log L = 0$ and output reacts: $d \log Y \approx \eta \alpha d \log E$ With reasonable parameters values it gives: $\frac{(1-\sigma)\eta}{\varphi + (1-\sigma)\eta + \sigma} \approx -0.04$, a (weak) smoothing effect!

In the case of a Leontieff production function

$$Y = \min\{E, L\} \Rightarrow \qquad d \log Y = d \log L = d \log E$$

Here the amplification mechanism $\approx 1 \gg \frac{(1+\varphi)\eta}{\varphi + (1-\sigma)\eta + \sigma}$ given the Domar weight being small $\eta \ll 1$. Moreover, concerning the fluctuation in the price of energy.

In the Cobb Douglas case, price change isn't very large: $p_E \approx \frac{\partial Y}{\partial E}$

$$d \log p^E \approx \underbrace{-\tfrac{(\sigma+\varphi)(1-\eta)}{\varphi + (1-\sigma)\eta + \sigma}}_{\approx -0.84} d \log E$$

However in the case of Leontieff, where we see a much larger amplification effect on prices!

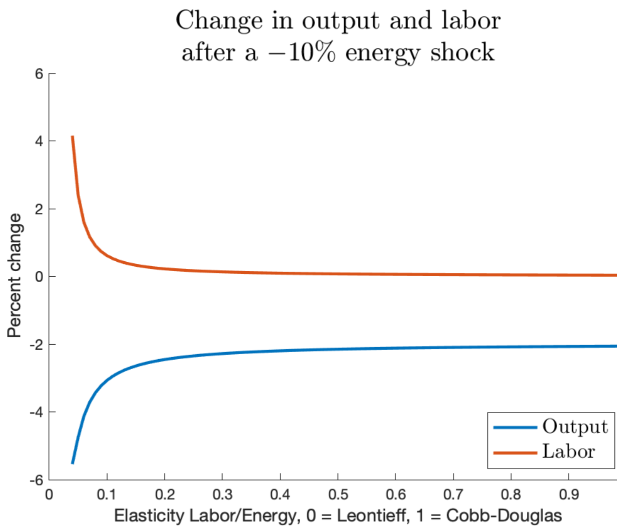
$$d\log p^E = -d\log W = \underbrace{-\tfrac{\varphi+\sigma}{1-\sigma}}_{\approx -22.5} d\log E$$

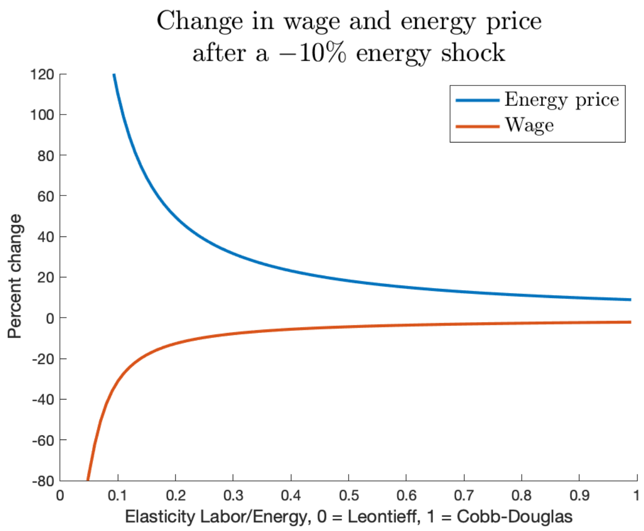
We know consider a CES aggregator production function as an intermediary case between the Leontieff and Cobb Douglas functions:

$$Y = \left[\eta E^{\frac{\varepsilon_e - 1}{\varepsilon_e}} + (1-\eta) L^{\frac{\varepsilon_e - 1}{\varepsilon_e}}\right]^{\frac{\varepsilon_e}{\varepsilon_e - 1}}$$

We consider a drop of 10% of energy supply compared to steady-state and plot in the following graph the resulting change – in percentage point deviation from steady-state – of the main variables. We see that labor supply reacts positively:

In our simple calibration, the elasticity of consumption is quite small ($\sigma \approx 1.1$) to see how the amplification works in a model with relatively small wealth effects of the labor supply. In the following figure, we vary the degree of complementarity ε_e between ≈ 0.05 (very close to Leontieff) up to $\varepsilon_e \approx 1$ (very close to Cobb-Douglas). We see that the complementary effect increase the energy share to high levels (more than twofold when $\varepsilon_e < 0.1$).





3 A RBC Model with energy

We present a model analogous to the most standard real business cycle model à la Kydland-Prescott. The main differences are : (i) a non-standard production function for output and (ii) the presence of an energy sector. We present these two features first before describing the rest of the model.

3.1 Firm

The representative firm produces a final good Y_t using three inputs: labor, i.e. hours L_t, capital K_{t-1} and energy E_t. These factors of productions are assembled in a Nested-CES structure: In an outer nest, machines M_t and labor L_t are combined together to produce output Y_t:

$$Y_t = \mathcal{F}_t(M_t, L_t) = Z_t \Big[\alpha^{\frac{1}{\varepsilon_y}} M_t^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1-\alpha)^{\frac{1}{\varepsilon_y}} L_t^{\frac{\varepsilon_y - 1}{\varepsilon_y}} \Big]^{\frac{\varepsilon_y}{\varepsilon_y - 1}} \tag{1}$$

the constant elasticity of substitution (CES) between machines and labor is $\varepsilon_y \in (0, \infty)$[6] while the labor share is the constant $1 - \alpha \in (0, 1)$. This production technology is subject to fluctuations in total factor productivity (TFP) Z_t, that follows an stationary AR(1) process in log:

$$\log Z_t = \rho_z \log Z_{t-1} + \sigma_z \omega_{zt} \tag{2}$$

In an inner nest, Energy E_t and capital K_{t-1} as combined together to create machines M_t:

$$M_t = \mathcal{M}_t(E_t, K_{t-1}) = \Big[\eta^{\frac{1}{\varepsilon_e}} (\zeta_t^s E_t)^{\frac{\varepsilon_e - 1}{\varepsilon_e}} + (1-\eta)^{\frac{1}{\varepsilon_e}} K_{t-1}^{\frac{\varepsilon_e - 1}{\varepsilon_e}} \Big]^{\frac{\varepsilon_e}{\varepsilon_e - 1}} \tag{3}$$

this constant-elasticity of substitution between capital and energy is $\varepsilon_e \in (0, \infty)$ and an energy share in production $\eta \in (0, 1)$. Moreover, this production function is also subject to changes in an "energy-saving" or "energy augmenting" technology ζ_t^s, that follows a AR(1) process in logs:

$$\log \zeta_t^s = \rho_s \log \zeta_{t-1}^s + \sigma_s \omega_t^s \tag{4}$$

As shown in the toy model above, this CES formulation nests two special cases:

- If $\varepsilon_e \to 0$, $\mathcal{M} \sim$ Leontieff, with $\mathcal{M}(E, K) = \min\{E/\eta, K/(1-\eta)\}$
- If $\varepsilon_e \to 1$, $\mathcal{M} \sim$ Cobb-Douglas, with $\mathcal{M}(E, K) = (E/\eta)^\eta (K/(1-\eta))^{1-\eta}$

This production defines an implicit demand function for energy inputs. Indeed, a positive productivity shock Z_t or a negative shock to the "energy-enhancing" factor Z_t^s makes the firm increases its input in energy. As a result, the price of energy, as defined by the marginal product

[6] Here the elasticity is general, even if, in practice, we will assume a certain degree of complementarity $\varepsilon_y \in (0, 1]$.

of energy *MPE*[7] increases. The price/*MPE* writes as:

$$Q_t^E = \frac{\partial \mathcal{F}_t(M_t, L_t)}{\partial M} \frac{\partial \mathcal{M}_t(E_t, K_{t-1})}{\partial E} = \alpha^{1/\varepsilon_y} Y_t^{1/\varepsilon_y} M_t^{(1/\varepsilon_e)-(1/\varepsilon_y)} \eta E^{-1/\varepsilon_e}$$

The rest of the firm sector is standard. Labor L_t is supplied by the household sector, and rewarded with a wage $W_t = MPL_t = \frac{\partial \mathcal{F}_t(M_t, L_t)}{\partial L} = (1-\alpha)^{1/\varepsilon_y} Y_t^{1/\varepsilon_y} L_t^{-1/\varepsilon_y}$, while the capital is accumulated following the law of motion, with investment I_t.

$$K_{t-1} = (1-\delta)K_t + I_t$$

The capital is rented at the rate : $R_t^k = 1 + r_t^k - \delta$ with the net interest r_t^k written as :

$$r_t^k = \frac{\partial \mathcal{F}_t(M_t, L_t)}{\partial K} \frac{\partial \mathcal{M}_t(E_t, K_{t-1})}{\partial E} = \alpha^{1/\varepsilon_y} Y_t^{1/\varepsilon_y} M_t^{(1/\varepsilon_e)-(1/\varepsilon_y)} K_{t-1}^{-1/\varepsilon_e} \tag{5}$$

3.2 An energy production function

We now specify the supply function for this energy factor. We consider the following ad-hoc formulation :

$$E_t = \xi_t^e E_{t-1}^{\nu} (Q_{t-1}^E)^{\mu} \qquad \nu \in (0, 1] \quad \& \quad \mu > 0 \tag{6}$$

The energy supply depends on three elements: an exogenous supply shifter, past energy and past prices of energy. First, this supply curve shifts when exogenous fluctuations increase the energy generation, for example with the stochastic discovery and exploitation of new coal mines or oil fields, or the opening of a new trade route (with the Middle East, Europe or the rest of North America). This is modeled as usual with a AR(1) process ξ_t in logs, following:

$$\log \xi_t^e = \rho_e \log \xi_{t-1}^e + \sigma_e \omega_t^e \tag{7}$$

Second, it reacts to past energy level E_{t-1}, with elasticity ν: this is justified for several reasons: (i) Fossil fuels extraction – which average 80-90% of energy source in our sample – and energy generation in general are highly energy-intensive activities. Using energy as a factor of production can be rationalized in models with micro-funded production functions of the type $E_t = \mathcal{E}(E_{t-1}, K_{t-1}, L_t)$. Moreover, (ii) there exists a high degree of inertia in energy production. In particular, most sources of fossil fuels are extracted at maximum capacity of the oil- and gas-fields or hydroelectric power plant. The exploitation typically features a characteristic decline of production over time, in the spirit of the Hoteling finite resource extraction framework. We choose a formulation with an elasticity $\nu < 1$, such that $\log E_t = \nu \log E_{t-1}$ drifts back to zero in the case of an absence of external supply shock.

[7] Using the same terminology than gross return of capital begin the marginal product of capital (MPK) and same for wage and marginal product of labor (MPL), as it is assumed here, in the standard RBC fashion.

Relatedly, these justifications related to the concept of *Energy Return of Investment*, or EROI.

$$\text{EROI} = \frac{E \text{ produced}}{E \text{ Invested}}$$

There have been important studies, e.g. Hall, Lambert, and Balogh (2014) or Brockway, Owen, Brand Correa, and Hardt (2019) analyzing the decline in the EROI for fossil fuels, from the traditional $\sim 25-30{:}1$ to 15:1 in 2010, and the lower EROI for electricity generation, around $\sim 6-12{:}1$, c.f. the studies by Raugei et al. (2012), Bhandari, Collier, Ellingson, and Apul (2015) and King and Van Den Bergh (2018)[8]. This decline is outside the scope of this business-cycle model, but this justifies a positive prior on ν – and hence strong autocorrelation of energy supply.

The third element determining energy supply is of course the price of energy Q^E. We assume this price-elasticity to be positive $\mu > 0$, consistent with upward sloping supply curve. However, we consider the past price as a driver of current energy supply (Q^E and not Q^E_t or $\mathbb{E}_t[Q^E_{t+1}]$. The reason for this choice relates to the general idea that energy supply takes time to be produced. Indeed, energy supplier often produce at maximum physical capacity (e.g. at the oil/gas field level for example), and production does not react to current prices. However, drilling reacts to current prices, leading to an increases in energy supply in the future. Price needs to exceed a threshold break-even price to initiate production, and marginal costs are usually small in comparison to fixed costs. Empirical evidence of these ideas can be found in Anderson, Kellogg, and Salant (2018) or in Heal and Schlenker (2019), and in the appendix in figure fig. 11 and appendix C. Note that this setting is conservative in the sense that evaluation and development of oil/gas fields or coal mines typically takes 10 to 15 years, and not one quarter as it is the case in this model. Moreover, empirical evidence that the supply curve is steep (at least in the short-term) justify our prior for low parameters $\nu < 0.5$.

These three points create a large degree of inertia in energy supply, which is consistent with empirical observations. The next step would be the micro-fund appropriately such formulation[9].

[8] Traditionally, EROI for fossil fuels (conventional oil and natural gas) averaged 25:1. However, non conventional extraction – like Tar sands and Shale Oil that requires energy-intensive fracking – have shown to have an EROI lower than 10. Despite an average low level, solar panel and photovoltaic electricity production have shown a return on energy increasing, from less than 2:1 in 1980 to around 6:1 in 2010, c.f. Hall, Lambert, and Balogh (2014). Even with reestimation of the photovoltaic to around $\sim 6-12{:}1$ for electricity generation, as shown in Raugei et al. (2012), the level stay significantly smaller than fossil fuels. Moreover, several Green Growth scenarios – achieving 100 % renewable electric system globally by 2060, as unlikely as it might be – would also reduce the efficiency of the energy sector from current 12:1 to 5:1, according Capellan-Perez et al. (2019). Moreover, in this context, the net energy available to the society is expressed as $\textit{Net Energy} = \textit{Energy Produced} \times (1 - \frac{1}{EROI})$. This function is highly non-linear when $EROI$ decreases below 5:1, causing an abrupt reduction in energy available for other sectors (this is called the "energy cliff"). A more rigorous analysis can be found in Hall, Balogh, and Murphy (2009) and Brockway, Owen, Brand Correa, and Hardt (2019).

[9] A first attempt of such micro foundation is the following:
Suppose a monopolistic energy supply sector, located outside the rest of the economy – for simplicity and consistent with the relative absence of positive spillovers from the fossil-fuel extractions into the rest of the real economy (think of the Dutch disease being a negative confirmation). The profit, as function of energy output next period $E_t = \mathcal{E}(E_{t-1}, F_{t-1})$. but paid a price Q^E_{t-1} for reasons explained above) is the following:

$$\Pi_{t-1} = \max_{F_t > 0} Q^E_{t-1}\mathcal{E}(E_{t-1}, F_t) - \bar{C}N_t - E_{t-1}Q_{t-1}$$

3.3 Household sector

This economy is composed of a representative household that maximizes its intertemporal utility, consume the final good C_t and supply labor L_t. The standard maximization problem is the following:

$$\max_{C_t,L_t} \mathbb{E}_{t_0}\Big[\sum_{t=t_0}^{\infty} U(C_t, L_t)\Big] \qquad\qquad U(C_t, L_t) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}$$

$$C_t + A_{t+1} = A_t R_t^k + \chi_t^\ell W_t L_t$$

We consider the very standard functional form for preferences, where the Constant Relative Risk Aversion coefficient is σ and the inverse of Frisch elasticity of labor supply is φ. The household budget constraint is standard, at the exception of a labor wedge χ_t. This exogenous variable that represents a generic distortion between the labor demand from employers and labor supplied by the household. This follows the approach of business cycle accounting by Chari, Kehoe, and McGrattan (2007). This shock evolves following an AR(1) process:

$$\log \chi_t^\ell = \rho_\ell \log \chi_{t-1}^\ell + \sigma_\ell \omega_t^\ell \tag{8}$$

This leads to the Euler equation and Labor-Consumption tradeoff :

$$\begin{aligned} C_t^{-\sigma} &= \mathbb{E}\big[\beta C_{t+1}^{-\sigma} R_{t+1}^k\big] = \mathbb{E}\big[\beta C_{t+1}^{-\sigma}(1 + \partial_K \mathcal{F}(\mathcal{M}(E_t, K_{t-1}), L_t) - \delta)\big] \\ C_t^\sigma L_t^\varphi &= \chi_t^\ell W_t = \chi_t^\ell \partial_L \mathcal{F}(\mathcal{M}(E_t, K_{t-1}), L_t) \end{aligned} \tag{9}$$

3.4 Resource constraint and equilibrium

To close the model we consider the resource constraint on the final goods:

$$C_t + I_t + G_t + Q_t^E E_t = Y_t \tag{10}$$

We add an additional shock G_t representing the market clearing wedge, i.e. government consumption and net export that could crowds out the market for final good.

$$\log G_t = \rho_g \log G_{t-1} + \sigma_g \omega_t^g \tag{11}$$

We assume that the energy producer sells energy to a wholesale trader or a retailler at the secured price Q_{t-1}^E (for reasons explained above) before making the investment in the factor F_{t-1}. As a result, the FOC writes: $Q_{t-1}^E \partial_F \mathcal{E}(E_{t-1}, F_t) = \bar{C}$ and $F_t = \partial_F \mathcal{E}^{-1}(\bar{C}/Q_{t-1}^E | E_{t-1})$, where this function is increasing since $\mathcal{E}(\cdot)$ has diminishing marginal return in F_t. With a simple production $\mathcal{E}(E_{t-1}, F_t) = E_{t-1}^\gamma \frac{F_t^{1-\lambda}}{1-\lambda}$ when decreasing return with $\lambda > 0$, we obtain $F_t = \bar{C}^{1/\lambda}(Q_{t-1}^E E_{t-1}^\gamma)^{-1/\lambda}$ and hence :

$$E_t = \frac{\bar{C}^{1/\lambda}}{1-\lambda} E_{t-1}^{\gamma\frac{\lambda-1}{\lambda}} (Q_{t-1}^E)^{\frac{\lambda-1}{\lambda}}$$

In our original formulation, we normalize $\bar{C}$ such that the factor is one, and $\mu = \gamma\frac{\lambda-1}{\lambda}$ and $\nu = \frac{\lambda-1}{\lambda}$. In our estimation, we find $\nu \approx 0.7246$, and $\mu \approx 0.3041$, which is consistent with $\lambda \approx 1.44$ and $\gamma \approx 2.38$

Note that the price paid for energy $Q_t^E E_t$, i.e. the return of this energy sector is *not* re-distributed to the Household. This imposes an additional frictions that appears in this market clearing. This could be interpreted as the consumption of the entrepreneur that owns the energy sector $C_t^E = Q_t^E E_t$.

Equilibrium The definition of general equilibrium is standard: it consists of a sequence of exogenous shocks $\{Z_t, \zeta_t^s, \xi_t^e, \chi^\ell, G_t\}_{t>0}$ and prices $\{R_t^k, W_t, Q_t^E\}$ such that : (i) households maximizes utility following the optimality condition eq. (9), (ii) firms use inputs K_{t-1}, E_t and choose investment I_t and labor L_t to maximize output Y_t (iii) the energy sector produces E_t given in eq. (6) and (iv) the good market eq. (10) clears.

4 Energy supply shock: Simple model analytics

Before exploring the estimation and the filtering of this model, we explore some simple impulse response of the main shock we consider: a one time reduction is energy shock, of around 3%, slightly less than the magnitude found in the data.

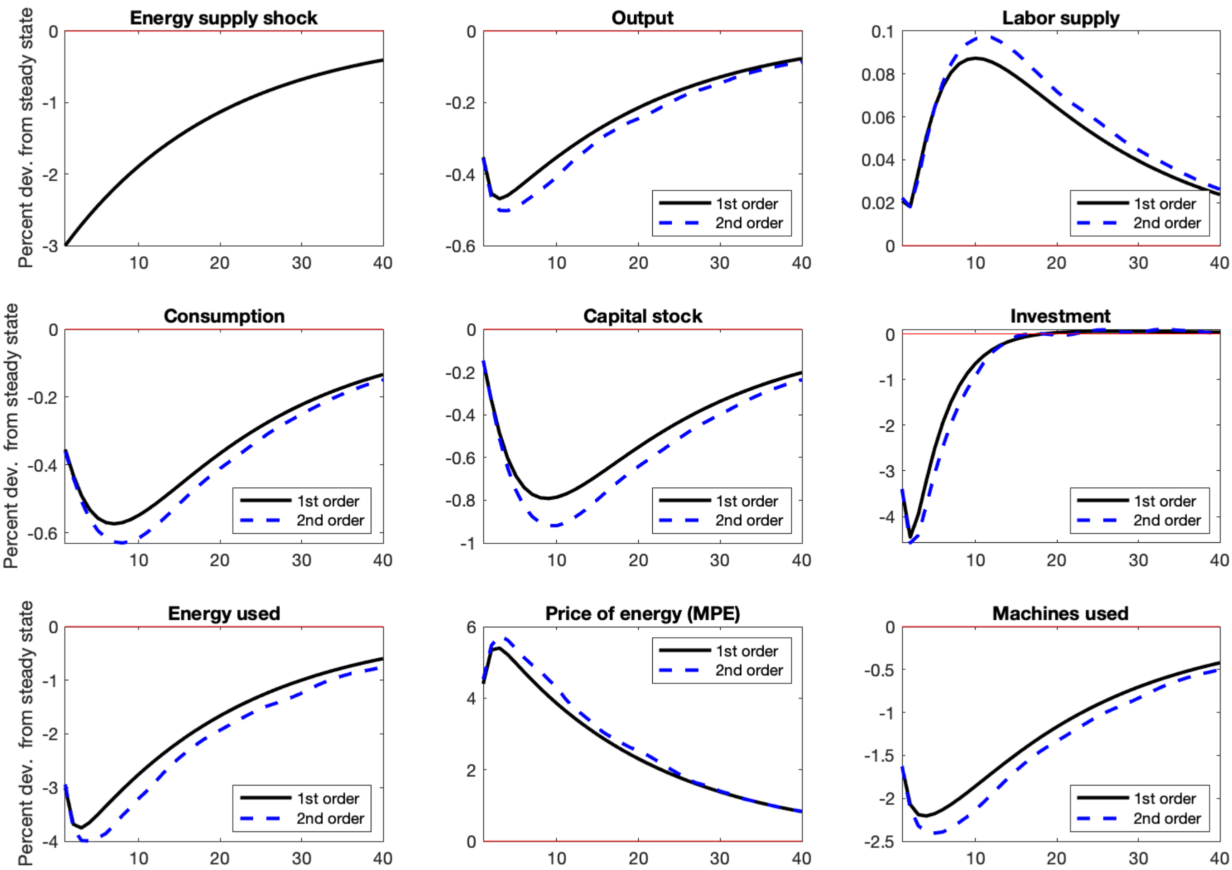


Figure 1: Impulse response to a 3% decline in ω_e, ξ_t and hence E_t

The calibration and estimation of the model parameters will be detailed in the next section, but we briefly give a first glimpse of the parameters values

Parameters	α	η	ε_y	ε_e
Value	0.4	0.3	0.7	0.5

The other parameters are standard and can be found in the estimation section below. We just need to mention that with this parametrization the energy share $\lambda^e = Q_t^E E_t / Y_t \approx \alpha\eta$ approximates 12% in our model, consistent with U.S. data. Following the network approach of the Hulten's theorem[10], the impact of a drop in energy supply of 3% should be proportional to this Domar weight λ^e, reducing GDP by 0.36%. We see that in our model, this drop is significantly lower due the complementarity of energy in the CES production structure. This is particularly true when we simulate the model using a second order Taylor approximation, in the blue dashed line in the graph.

Indeed, the average impact of a shock ω_t^e can be written as a kind of "energy supply multiplier" – dynamic analog of the experiment of the "toy model" section:

$$m^e(\omega^e) = \frac{1}{N}\sum_{t=0}^{N} \frac{Y_t(\omega^e) - \bar{Y}}{\lambda^e \omega_t^e} = \begin{cases} 1.606 & \text{at the 1st order only} \\ 1.817 & \text{w/ 2nd order effects} \end{cases}$$

for a shock of 3% on impact. The second order effects are stronger, the larger the size of the shock.

To see why complementarities are important we compare – in the same spirit as the toy model section – the model response to a 1% supply shock – for ease of comparison – with two values of the parameter ε_e, the elasticity of substitution between capital and energy. The result is displayed in **??**

With value $\varepsilon_e = 0.5$ the production displays a high degree of complementarity and capital stock and investment drops by almost 1%. Machines production fall and firms substitute toward labor, following the same mechanism as in the toy model. Marginal product of Labor rises boosting labor supply. The marginal product of energy, pinning down the price spikes at 5%. Overall, output declines by 0.15% with a persistence two stronger than the one of the underlying shock.

This picture is very different in an economy closer to Cobb Douglas, where $\varepsilon \approx 0.8$. In this context, the spillover on output is not distinguishable from the steady state Domar weight $\lambda^e = Q^E E/Y$ and there is no amplification mechanism on the rest of the economy.

[10]The effects of a supply shocks to sector/producer i is derived by Hulten's theorem – a first order approximation, exact for Cobb Douglas production function – or the second order extension by Baqaee and Farhi (2019).

$$d\log Y = \overbrace{\lambda' d\log A}^{\text{first order effects}} + \underbrace{d\log A' \frac{d\lambda}{d\log A} d\log A}_{\text{second order effects}}$$

In our case, these second order effects are significant due to complementarity in the CES production structure.

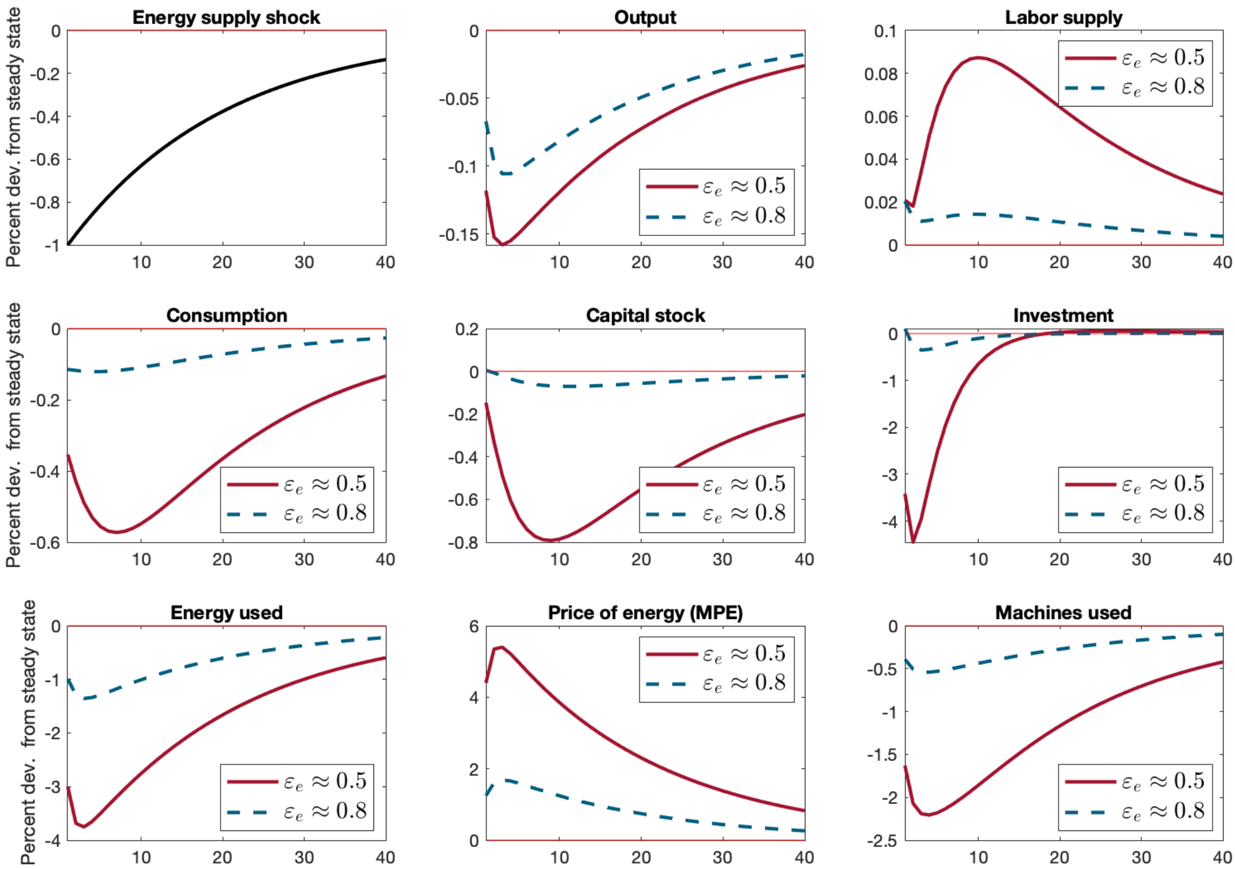


Figure 2: Impulse response to a 1% energy shock, comparison with different elasticities capital/energy ε_e

5 Estimation and shock filtering

5.1 Data

In the empirical section, I use U.S. Data, on GDP, investment, government consumption and net export, labor in hours and energy consumption in physical unit[11]. These time series are deflated with the GDP deflator, and divided by adult population. They span the last 72 years, from the first quarter of 1949 to the first quarter of 2020 – the last 3 quarters of 2020 being omitted due the ongoing pandemic. Since our model does not have any unit-root in the exogenous shock processes, in the estimation I use the series in log deviation from the linear trend of the period 1949-2007. Overall, I follow the methodology of Guerron-Quintana (2010) and Pfeifer (2014). More details on the data and the log-deviation data can be found in the appendix.

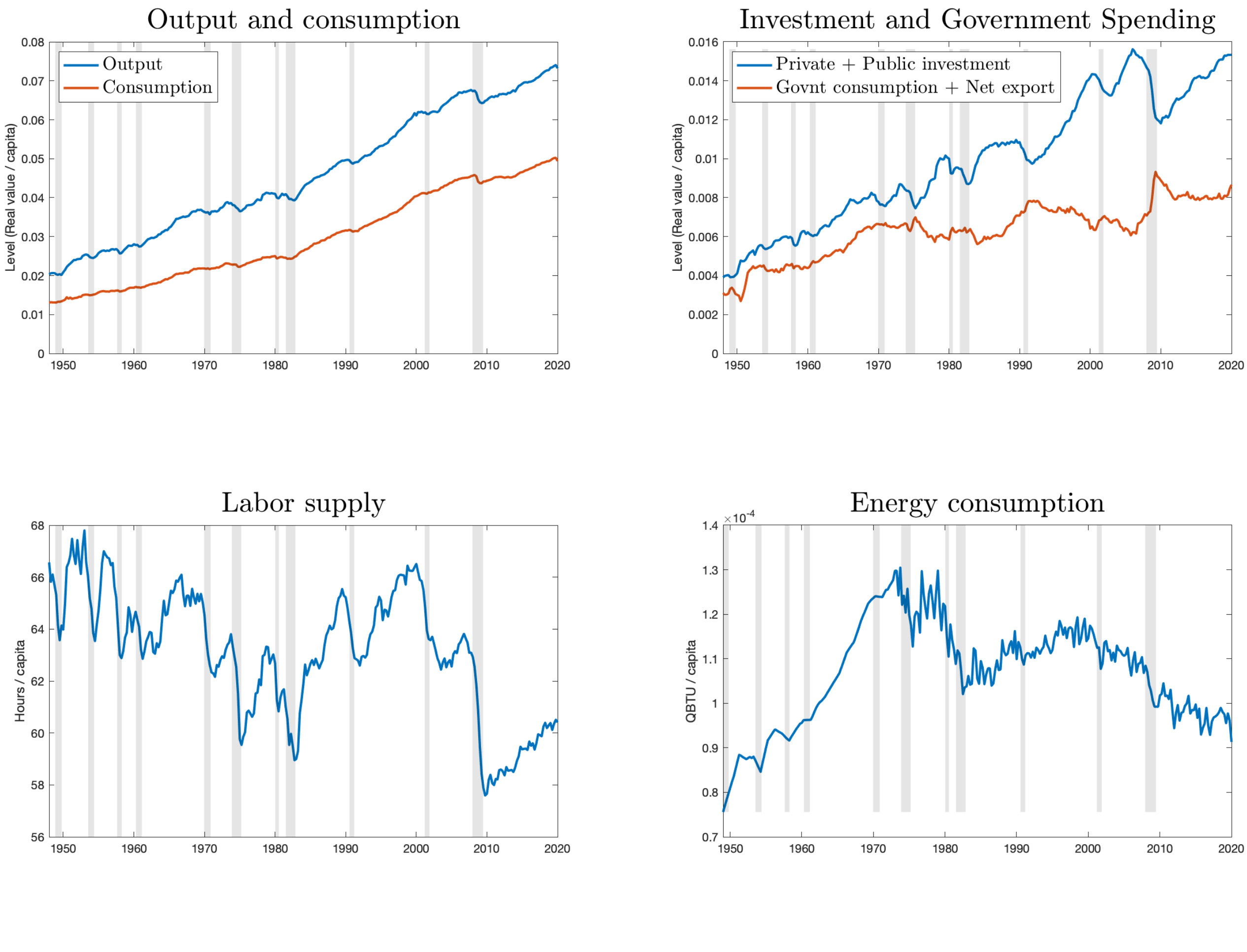


Figure 3: Sample U.S. data 1949 Q1 - 2020 Q1, in level per capita, deflated

[11]Here I use British Thermal Unit (QBTU, with Q for quadrillion) unit used by the U.S. Energy Information Administration. Alternative energy measures are: $1BBTU \approx 3GW{\cdot}h \approx 2.5Toe$ (resp. Billions BTU, Giga Watt Hours and Tons of Oil Equivalent). Hence the order of magnitude of this graph is around 90−120 Million BTU ≈ 30−35 MWh per adult for one quarter. Note that only 10% of this amount is consumed under the form of electricity. Note also that the world average is around 6 MWh per capita per quarter, and the European average 12.5 MWh (so respectively 5 and 3 times less).

5.2 Model solution and estimation methods

In the next (preliminary) section, I compute the model in a linear fashion. The model is linearized using dynare routines, c.f. Adjemian et al. (2011), taking the first order finite difference approximation of the model solution around the steady-state. As mentioned above, our steady-state is the linear trend of 1949-2007.

For a generic model, we have n endogenous states x_t with steady-state $\bar{x}$, m-exogenous shocks ω_{t+1} and k-observable y_t, with measurement errors η_{t+1}. In this case, we compute the state-transition equation and the measurement equation:

$$x_{t+1} = F(x_t - \bar{x}) + G\omega_{t+1}$$
$$y_t = Hx_t + K\eta_{t+1}$$

where F, G, H, K are respectively $n\times n$, $n\times m$, $n\times k$ matrices of the different parameters θ.

In a second step, we compute the model non-linearly to be able to address the complementarity in the production structure of this economy. A second order approximation writes:

$$x_{t+t} = F_1(x_t - \bar{x}) + (x_t - \bar{x})' F_2(x_t - \bar{x}) + G_1\omega_{t+1} + \omega'_{t+1}G_2\omega_{t+1} + \omega'_{t+1}G_3(x_t - \bar{x})$$

where F_2, G_2, and G_3 are resp. $n\times n\times n$, $n\times m\times m$, $n\times m\times n$ tensors. Note that in the variance of the shocks $\Sigma(\omega_t)$ is too large and the non-linearities are too strong, the model solution becomes explosive and diverges. One would need to use pruning. I refrain from such solution for now and consider small variance of shocks.

More generally, a fully-non-linear approach would solve:

$$x_{t+t} = \mathcal{F}(x_t, \omega_{t+1})$$

In this context, the matrices F_1, F_2, G_1, G_2 and G_3 are related to the Jacobian matrix and Hessian (in this case a tensor) of the function $\mathcal{F}(\cdot)$.

5.2.1 The filtering problem

First, given parameters θ and a sample (in level) $\mathscr{Y}_T = \{\mathcal{Y}_1, \mathcal{Y}_2, \dots \mathcal{Y}_T\}$, I use Kalman filtering techniques to extract the path of shocks that maximize the likelihood $\mathcal{L}(\theta; \mathcal{Y}_T)$

$$\mathcal{L}(\theta; \mathcal{Y}_T) = p(y_0|\theta) \prod_{t=1}^{T} p(\mathcal{Y}_t | \mathscr{Y}_{t-1}, \theta)$$

The distribution of the initial condition $f(y_0|\theta)$ is computed by the stationary equilibrium – which is Normal for a linear Gaussian model. To evaluate the predictive density of the latent variable x_t, we use the identity :

$$p(\mathcal{Y}_t | \mathscr{Y}_{t-1}, \theta) = \int_{\mathbb{Y}} g(\mathcal{Y}_t | y_t, \theta) f(y_t | \mathscr{Y}_{t-1}, \theta) dy_t$$

The density $g(\mathcal{Y}_t|y_t)$ results from the measurement equation $y_t = Hx_t + K\eta_{t+1}$. Without measurement errors, i.e. a degenerate distribution for η_t, we simply get the Dirac mass $g(\cdot) = \delta_{\mathcal{Y}_t}(\cdot)$ and $p(\mathcal{Y}_t|\mathscr{Y}_{t-1}, \theta) = f(\mathcal{Y}_t|\mathscr{Y}_{t-1}, \theta)$. That's our situation in the linear case. The density $f(y_t|\mathscr{Y}_{t-1}, \theta)$ results from both the state-transition equation and the measurement equation. Bayesian recursive estimation allows for this inference, since the endogenous variables x_t are unobserved: First, we forecast y_t given past data $\mathscr{Y}_{t-1}$ and then we filter, given the new observation $\mathcal{Y}_t$

$$
\begin{aligned}
f(y_t|\mathscr{Y}_{t-1}, \theta) &= \int_{\mathbb{Y}} h(y_t|y_{t-1}, \theta)\ell(y_{t-1}|\mathscr{Y}_{t-1})dy_{t-1} \\
\ell(y_t|\mathscr{Y}_t, \theta) &\propto g(\mathcal{Y}_t|y_t, \theta) f(y_t|\mathscr{Y}_{t-1}, \theta)
\end{aligned}
$$

where $f(y_t|\mathscr{Y}_{t-1}, \theta)$ is a prior given by the transition equation and the measurement equation.

In general, for generic n-dimensional problems these computations can be notoriously hard. In the linear-Gaussian case, when transition dynamics and measurement equations are all linear, and shocks process all Normally distributed, Kalman filtering allows to compute a closed form for the likelihood function $\mathcal{L}(\theta; \mathcal{Y}_T)$.

In the non-linear case, sequential Monte Carlo simulation allows to approximate the likelihood function. Without going into to ample details, I follow the most standard Bootstrap particle filter.

0. Initialize the state $s_0 \sim p(s_0)$ according to some prior
1. ***Forecasting*** Given x_{t-1}, Simulate by Monte Carlo N particles $\widetilde{x}_t^j \sim p(x_t|x_{t-1})$
   Note: one could use importance sampling $q(x_t|x_{t-1})$ if $p(x_t|x_{t-1})$ is too complicated to simulate
2. ***Prediction*** Given $\widetilde{x}_t^j$ predict the observable $\widetilde{y}_t^j$ and evaluate its likelihood $p(\mathcal{Y}_t|\widetilde{x}_t^j)$, i.e. probability of forecast error $\widetilde{y}_t^j - y_t$ compared to the observed $\mathcal{Y}_t$. Associate each particle with an incremental weight $\widetilde{w}_t^j = p(\mathcal{Y}_t|\widetilde{x}_t^j)$
3. ***Update/Filtering*** Changes the weights $\widetilde{W}_t^j = \frac{\widetilde{w}_t^j W_{t-1}^j}{\sum_k \widetilde{w}_t^k W_{t-1}^k}$ associated with for each x_t^j
4. ***Resampling*** Use these weights to resample the particles: Draw N new particles with these probabilities $\widetilde{W}_t^j$ with new weights W_t^j. Usually do this step only when the variance of weights is high, i.e. if $ESS_t = N/\sum_k \widetilde{W}_t^k$

An important limitation of this filtering method is to need to draw many "unlikely" particles – in the sense that they have very little chance of mapping to observed $\mathcal{Y}_t$. A forthcoming extension would be to use some conditionally optimal particle filter, where the sample distribution $q(x_t|x_{t-1}) = p(x_t|x_{t-1}, \mathcal{Y}_t) \propto p(\mathcal{Y}_t|x_t)p(x_t|x_{t-1})$. In this case, the weights need to be updated with $\widetilde{w}_t^j = \int_{\mathbb{X}} p(\mathcal{Y}_t|x_t)p(x_t|\widetilde{x}_{t-1}^j)dx_{t-1} = p(\mathcal{Y}_t|\widetilde{x}_{t-1}^j)$ instead of $p(\mathcal{Y}_t|\widetilde{x}_t^j)$ in the Bootstrap particle filter.

In any case, the likelihood function can be computed as a result of this filtering procedure, as well any posterior moment

$$
\mathbb{E}[h(x_t)]\widetilde{W}_t^j = \frac{1}{M}\sum_{j=1}^{M} h\left(s_t^j\right) W_t^j \overset{\text{SLLN/CLT}}{\approx} \mathbb{E}\left[h\left(s_t\right) \mid \mathcal{Y}_{1:t}, \theta\right]
$$

Moreover, in the Efficient Importance Sampling (EIS) of DeJong et al. (2013), the likelihood $p(y_t|Y_{t-1})$ is evaluated and the weights $\widetilde{w}_t^j \& \widetilde{W}_t^j$ are updated using a continuous & parametric kernel $\varphi_t(s_{t-1}, s_t|psi)$, and choose the parameters to minimize some variance. This facilitate the likelihood derivation, in situation where the Monte Carlo approximation can be irregular for some parameters range worsening the Bayesian inference problem.

5.2.2 The estimation procedure

The second step of this approach is the Bayesian estimation of parameters θ. We evaluate the likelihood for many parameter values for θ (Monte Carlo) and accept or reject the path of this process for θ to fit the observations (Markov Chain). This family of Markov Chains Monte Carlo algorithms are embedded in dynare routines and I rely on this for the first step of the estimation.

The description of the Metropolis Hasting algorithm with particle filter is forthcoming.

5.3 Calibration and estimation: Preliminary results in the Linear analysis

For some parameters I calibrate the model with standard values. I use conventional parametrization for time preference and depreciation [12]. The share of labor and energy match empirical facts. For the energy, given that I used energy as a physical unit, and not a market share, I match the sales share of the four sectors: Oil, Mining, Petroleum and Utilities. Combined, they average $12 - 16\%$ of GDP. I took the conservative lower bound of this interval to show how a small energy sector can still generate significant contribution to output fluctuation.

Parameter		Value	Justification
β	Discount factor	0.98	Standard for 4 % yearly interest rate
α	Share of machines in production	0.4	Standard: Match a Labor share ≈ 0.6
η	Share of energy in production	0.3	Match energy share $\approx 12\%$
δ	Depreciation of capital	0.04	Standard

The rest of the parameters are estimated. In a first step of this analysis, I use the Dynare MCMC routines to obtain first estimates of the parameters. In a later version of this analysis, I use the likelihood obtained by the particle filtering to estimate these parameters using MCMC algorithms.

Taking conventional prior on these parameters, the main novelly concerns the elasticity of substitution in the production function. Starting from a wide range for the elasticity between labor and machines ε_y from 0.4 to 1.1, i.e. including the Cobb Douglas case, the estimation shows a strong degree of complementarity ≈ 0.68. Similarly, the complementarity between energy and capital is also very strong with ≈ 0.53, which is higher than the estimate found in the microdata – which can go as low as 0.05 c.f. Henningsen et al. (2019). The Frisch elasticity of labor supply is

[12] However, I will change them due to a miscalculation in the yearly/quarterly interest rate

Parameter		Post. mean	Prior mean	Prior Std dev	Prior shape
ε_y	Elasticity Machine/Labor	0.6890	0.75	0.18	Beta
ε_e	Elasticity Energy/Capital	0.5384	0.45	0.18	Beta
σ	CRRA	1.7464	2.0	0.3	Normal
φ	Inverse Frisch elasticity	3.6377	2.5	0.5	Normal
μ	Price elasticity of energy output	0.3074	0.2	2.5	Inverse Gamma
ν	Past energy elasticity of energy	0.7210	0.75	0.1	Beta

also pretty low, at around $1/\varphi \approx 0.27$ which is significantly lower than usual estimates for DSGE models, c.f. Guerron-Quintana (2010) where the range goes from 0.5 to 1.5. This underlies the lack of reallocation effects increase the importance of energy shock as contributor to output fluctuation.

In addition, I also estimated the standard deviations $\sigma_z, \sigma_\ell, \sigma_s, \sigma_e, \sigma_g$ (but don't display them in the table[13]) and filter the shock processes using the Kalman filtering.

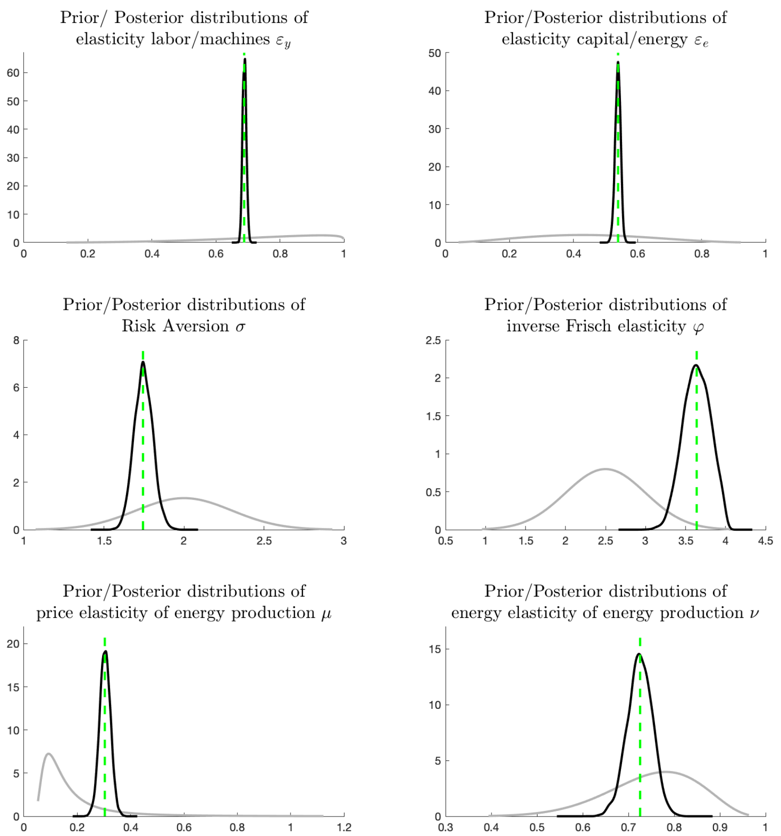


Figure 4: Posterior distribution of 6 parameters values for the production structure, preferences and the energy supply function

[13]The result for this estimation: $\sigma_z, \sigma_\ell, \sigma_s, \sigma_e, \sigma_g = [0.0125, 0.0256, 0.0342, 0.0375, 0.0453]$, starting from an inverse Gamma distribution, centered on 0.1 (two to ten times larger!) with a std. dev. of 3. The posterior is very concentrated and such low values are robust to different parametrization of the other model parameters

6 Preliminary results: Linear analysis

In this section, we present the first set of results on business cycle accounting. I compute the historical shock decomposition of the five observables – output, investment, labor, government spending and export and energy supply. For each variable in this sample, I compute the estimation of the shock process – TFP shock, Labor wedge shock, Market clearing shock and energy saving and supply shocks – via Kalman filter and smoother. We can decompose the historical deviations of the endogenous variables, e.g. real GDP and labor, from their respective steady state values into the contribution coming from the various shocks.

The next figure show such decomposition for output. Consistent with the conclusions of the Real Business Cycle literature, the TFP shock Z_t is driving most of the upward movement in growth. Moreover, this efficiency wedge, combined with the labor wedge are the main driver of large economic downturn, like the Great recession or the crisis of 1982, as shown in Brinca, Chari, Kehoe, and McGrattan (2016).

Moreover, the contribution of the energy shock deserves to be noted: it contributed to a significant share – around 30-40%, to the growth of output away from the linear trend in the first half of the sample. However, as noted in the data above, the energy consumption per capita has dropped substantially since the first oil/energy crisis of 1973. As a result, in the second half of the sample, the lower deviation of output is explained in part by this decline in energy supply *growth*. Energy is still a contributor, but less and less, until its contribution become close to zero after the Great Recession.

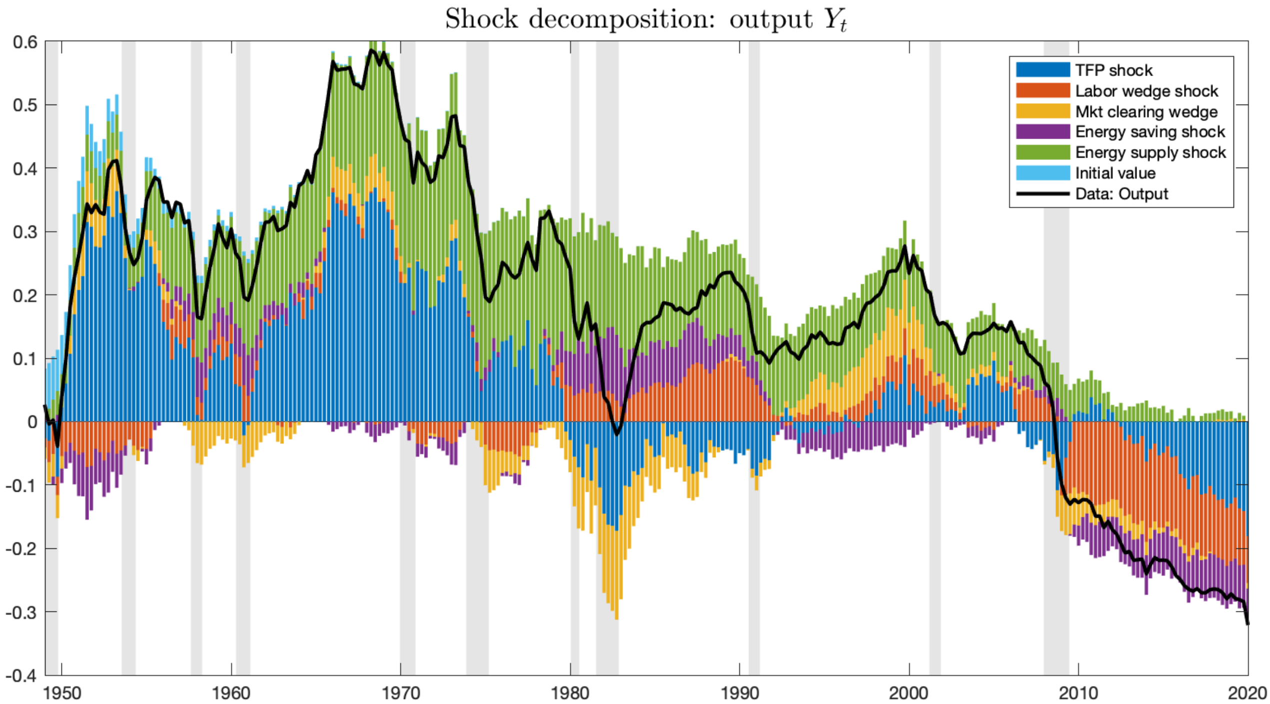


Figure 5: Shock decomposition for output Y_t, U.S. data 1949 - 2020

Moreover, this decline in energy consumption is not followed by positive energy augmenting technology shock: if anything this energy saving technological progress is negative in the period 1990-2020. This implies that productive activities have been becoming more energy intensive, which contributed around $10 - 15\%$ to the decline in output below trend in the last ten years.

A somewhat surprising result is the lack of contribution of this energy supply shock to the 1973 and 2008 crises, where oil price shocks contributed to the deepening of the recessions to some extent. We do not observe this pattern in the accounting of the different recessions, as recorded by the NBER (grey areas). The reason for this absence is twofold:

(i) First our focus is on total energy, broadly defined, not only oil, which represent roughly 30% of the total physical amount, the data matched by the model. As a consequence, we do not observe the sales share in *values* of the oil industry: it is likely that the oil price shock increased the share spent on this particular source of energy, causing reallocation *within* the energy sector, in addition to a drop in energy supply. There are diverse consequences of such reallocation, the main spillover being a drop in TFP during the adaption process during the oil shock, as explained in Dhawan, Jeske, and Silos (2010). Such fall in productivity would not be measured as an energy supply shock in our accounting procedure, explaining the weak role during crises[14].

(ii) Our model doesn't not feature prices and underlying sources of inflation, including price setting with nominal rigidities or fiscal theory of the price level – where a realistic s-shaped fiscal rule for the government need to be added. I believe the contribution to the energy shocks changes the price of the underlying factors and firms' marginal costs, leading to increase in price dispersion. This spillover would be captured as a labor wedge in this model, as explained by Chari, Kehoe, and McGrattan (2007).

As a result, we do not see an important contribution of energy supply shock and this paper can not contribute directly to the debate of the effects in oil price shocks raised by Hamilton (2009) and Baumeister and Kilian (2016).

[14] A more precise analysis of these reallocations and transmission channels is the one of the next step in this project.

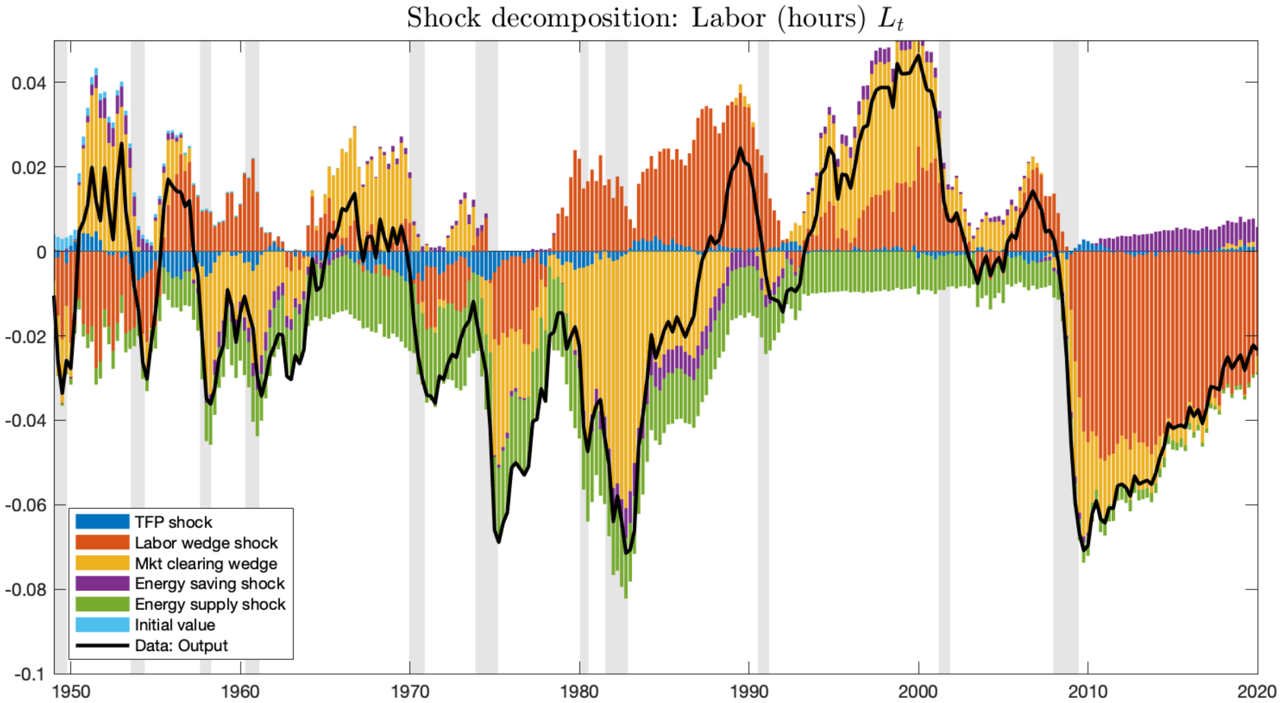


Figure 6: Shock decomposition for labor hours L_t, U.S. data 1949 - 2020

In this second plot, we show the contribution of energy to the labor share, and the decline in hours between 1950 and 2010. Energy, through machines and capital, is a substitute to labor – with some degree of complementarities as measured by ε_y. As a result, a rise in energy supply between 1949 and 1980 have forced a decline in the number of hours, with a contribution between 15 and 30%, a share that is more important in the recovery of recessions, for example in the years 1980. Moreover, as explained in Chari, Kehoe, and McGrattan (2007) the large decline in labor during the Great Recession can be mainly explained by an increase in the labor wedge.

If these two sets of results are surprising, we propose different modeling strategies to account for a role for energy in recessions.

7 Alternative model specification

I propose several directions for future extensions of the model. The first one relates to the specification of the energy production. The second one develop the model in two directions: (i) adding more quantitative in this real business cycles, and (ii) developing a medium scale DSGE model a la Smets and Wouters (2007).

7.1 A productive sector for energy

We describe a more detailed version of the energy sector model. We take the model of Bornstein, Krusell, and Rebelo (2021) off the shelf, since it is the most complete, yet parsimonious model of oil production. Our model address energy production in general rather than conventional oil, but the features present in this framework still hold for most energy sources: gas, coal, nuclear power and hydroelectric production. The main reason is the present of lags in investment and slow moving exploration capital that reduce the volatility of investment in the energy sector and imply an Hotelling rule for prices.

We assume the competitive firm running the energy sector is maximizing the discounted flow of profit:

$$V_0^E = \max_{I_t^E, E_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_t \Big[Q_t^E E_t - I_t^E - \bar{C}\Big(\xi_t^e \frac{E_t}{\mathcal{C}_t^E}\Big)^{\nu} \mathcal{C}_t^E \Big] \tag{12}$$

It chooses its optimal level of production E_{t+1} at time t, i.e. one period in advance, given the prices Q_t^E, the production capacities (or reserves) $\mathcal{C}^E$ and an exogenous ξ_t^e supply/cost shock. The term $\bar{C}\big(\xi_t^e \frac{E_t}{\mathcal{C}_t^E}\big)^{\nu} \mathcal{C}_t^E$ represents the operating costs of producing E_t, which is linear in capacities to ensure constant return to scale. Moreover, the supply shock ξ_t^e follows an AR(1):

$$\log \xi_t^e = \rho_e \log \xi_{t-1}^e + \sigma_e \omega_t^e \tag{13}$$

since the energy supply E_t is optimally chosen by the producer, the supply shock is not a shock on quantities but a shock on costs.

To model lags in investment, Bornstein et al. (2021) introduce a parsimonious way of describing the exploration, or increase in capacity of production, with a variable X_t^E called "development capital" (or exploration capital in Bornstein et al. (2021)) and represents the exploration and development of production infrastructures. It follows a law of motion:

$$X_{t+1}^E = (1-\lambda) X_t^E + \Theta \big(I_t^E\big)^{\theta} \tag{14}$$

Investment I_t^E in production of energy add to this development capital, but only a fraction λ of this capital materializes into capacity of production in every period. Moreover, this investment has diminishing return, with curvature $\theta < 1$: without this feature, investment would be very volatile, moving sharply with energy prices. This share λ could represent the probability of finding exploitable energy reserves, like oil, gas, or a new source of energy that is marketable.

The production capacity (or oil reserves in Bornstein et al. (2021)) evolves with development of infrastructures and investment, through X_t^E and declines with production E_t:

$$\mathcal{C}_{t+1}^E = \mathcal{C}_t^E - E_t + \lambda X_t^E \tag{15}$$

This formulation and the timing[15] ensure the possibility of "time to build" between investment, development of capacities and the production of energy.

The First Order conditions imply four expectation equations for the four variables $E_t, I_t^E, X_t^E, \mathcal{C}_t^E$, that will be added to the rest of the model[16], and the parameters for the lag of energy development λ, for the investment elasticity of development θ and the elasticity of the cost ν, can be estimated from the energy data.

7.2 Additional features for the real business cycle model

In addition of this energy sector microfoundation, Here is an enumeration of the potential features that should (and will) be added to this model in the next version of this project.

1. Preferences : Consumption-Labor tradeoff

First, we need to consider more general preferences, that allow both for wealth effects of energy shocks on the labor supply, and are consistent with balanced-growth path. Indeed, changes in real wages due to a change in energy supply yield income and substitution effects: these two channels should cancel out to be consistent with balance growth.

King-Plosser-Rebelo King, Plosser, and Rebelo (1988) preferences are generally written as follow, where the third formulation follows Trabandt and Uhlig (2011) and Shimer (2009):

$$U(C,L) = \log(C) - \psi \frac{L^{1+\varphi}}{1+\varphi} \qquad \text{when} \qquad \sigma = 1$$

$$U(C,L) = \frac{1}{1-\sigma} C_t^{1-\sigma} v(L) \qquad \text{when} \qquad \sigma \neq 1$$

$$U(C,L) = \frac{1}{1-\sigma}\Big(C_t^{1-\sigma}\Big(1 - \psi(1-\sigma)\frac{L^{1+\varphi}}{1+\varphi}\Big)^{\sigma} - 1\Big)$$

[15] The timing is as follow: the period starts with the shocks on the macroeconomic shocks $Z_t, \chi_t^e, \zeta_t^s, \xi_t^e$, and the supply shock ξ_t. A fraction λ of the exploration capacity materializes into new reserves capacity, and production occurs according to the predetermined extraction rate. At the end of the period, the firm chooses its investment and its extraction rate for the next period.

[16] Defining the fraction $s_t^E = \frac{E_t}{\mathcal{C}_t^E}$ the supply of energy as share of capacities, and $\mu_t^{\mathcal{C}}$ and μ_t^X the respective Lagrange multipliers of the law of motions for the production capacity $\mathcal{C}_t^E$ and development capital X_t^E, the FOC write:

$$\mathbb{E}_t\Big[\Lambda_{t+1}\big(Q_{t+1}^E - \nu\bar{C}(\xi_t^e s_{t+1}^E)^{\nu-1} - \mu_{t+1}^{\mathcal{C}}\big)\Big] = 0$$

$$\mu_t^{\mathcal{C}} = \mathbb{E}_t\Big[\Lambda_{t+1}\big(Q_{t+1}^E s_{t+1}^E + (1-s_t^E)\mu_{t+1}^{\mathcal{C}} - \bar{C}(s_t^E)^{\nu}\big)\Big]$$

$$\mu_t^X = \lambda\mu_t^R + (1-\lambda)\mathbb{E}_t\big(\Lambda_{t+1}\mu_{t+1}^X\big) \qquad \mu_t^X = \frac{I_t^{1-\theta}}{\theta\Theta}$$

In such case, the Labor optimality condition and the Stochastic discount factor are the following:

$$\frac{U_L(C,L)}{U_C(C,L)} = \frac{C}{1-\sigma}\frac{v'(L)}{v(L)} = -\sigma\psi\frac{CL^{\varphi}}{v(L)^{1/\sigma}} = \chi_t w_t$$

$$\Lambda_{t+1} = \beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\sigma}\frac{v(L_{t+1})}{v(L_t)} \qquad \text{with} \qquad v(L) := \Big(1-\psi(1-\sigma)\frac{L^{1+\varphi}}{1+\varphi}\Big)^{\sigma}$$

where the marginal rate of substitution of consumption for leisure is increasing in L, and more importantly the marginal utility of consumption $\propto C^{-\sigma}v(L)$ increases when household work more. It has implication at business cycle frequency, where for example a decline in output below trend – because of negative TFP or energy supply shock – decreases the ratio consumption-output ratio, decreasing the labor supply – as if the household where facing a higher labor taxes (and labor wedge χ_t^{ℓ}).

For the the other polar case, for GHH preferences the wealth effect on labor supply is absent, which can shut down one channel of reallocation. These preference are the following:

$$U(C,L) = \frac{1}{1-\sigma}\Big(C_t - \psi\frac{L^{1+\varphi}}{1+\varphi}\Big)^{1-\sigma}$$

The Labor optimality condition and the Stochastic discount factor are the following:

$$L_t^{\varphi} = \frac{\chi_t w_t}{\psi} \qquad \Lambda_{t+1} = \beta\frac{M_{t+1}}{M_t} \qquad \text{with} \qquad M_t := \Big(C_t - \psi\frac{L^{1+\varphi}}{1+\varphi}\Big)^{-\sigma}$$

2. Preferences : Habit formation

In addition, to match the slow-moving hump-shaped dynamics of consumption, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) add external habit formation. This implies that the utility level depends on deviation from previous consumption expenditure:

$$U(C_t,L_t) = \begin{cases} u(C_t - hC_{t-1}) - v(L) & = \log(C_t - hC_{t_1}) - \frac{L^{1+\varphi}}{1+\varphi} \\ u(C_t - hC_{t_1})v(L_t) & = \frac{1}{1-\sigma}(C_t - hC_{t_1})^{1-\sigma}v(L_t) \end{cases}$$

respectively in the separable vs. non separable BGP preferences. The parameter h is usually estimated to be between 0.6 and 0.7. This allows to reconcile the dynamics of investment and hump-shape consumption. The stochastic discount factor becomes:

$$\Lambda_{t+1} = \beta\Big(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}\Big)^{-\sigma}\frac{v(L_{t+1})}{v(L_t)} \qquad \text{with} \qquad v(L) := \Big(1-\psi(1-\sigma)\frac{L^{1+\varphi}}{1+\varphi}\Big)^{\sigma}$$

Moreover, if we include consumption in the set of observable, c.f. Guerron-Quintana (2010), we would probably need to add an additional shock preference $\widetilde{\rho}$ (to consumption or to time discount rate), where

$$\log\widetilde{\rho}_t = \rho_c\widetilde{\rho}_{t-1} + \sigma_c\omega_t^c$$

3. Production: capital adjustment cost and utilization

To induce more lags in investment, the RBC and DSGE literature usually adds second order adjustment cost of capital $\Gamma(I_t, I_{t-1})$ that discounts the rate of the investment where added to the law of motion of capital :

$$K_{t+1} = (1-\delta)K_t + I_t\big[1 - \Gamma\big(\frac{I_t}{I_{t-1}}\big)\big]$$

with $\Gamma(I_t, I_{t-1})$ satisfying the condition: $\Gamma(1) = \Gamma'(1) = 0$ and $\Gamma''(1) = \kappa > 0$.

This features of the model induces the agents to be forward looking and induces a slower reaction of investment to the price of capital. It has two effects in the case of energy shock: it slows down the reallocation of firm resources toward capital in cases where energy supply decline, but it also reduce the pace of disinvestment when machines can produce because of a reduction in supply.

Moreover, one could (but I will probably not) include an additional shock κ_t that change the price of investment relative to consumption good: $\widetilde{I_t} = \kappa_t I_t$

$$\log \kappa_t = \rho_i \kappa_{t-1} + \sigma_i \omega_t^i$$

The literature on medium-scale DSGE also adds (but I will probably not do so) variable utilization of the capital stock, as a changing the effective capital the household rents to the firm:

$$K_t^s = u_t K_{t-1}$$

the return on capital becomes $R_t^k u_t$ while the cost of capital is $\upsilon(u_t)K_{t-1}$, where $\upsilon(\cdot)$ is a convex function. It could add interesting dynamics when interplaying with change in energy supply and cost of adjustment of capital. However, given the conclusion of Smets and Wouters (2007) where the absence of variable utilization does not increase the standard error of TFP in DSGE models, I will refrain to add another layer of complexity.

7.3 Medium scale New Keynesian Model

In this section, I follow the traditional New Keynesian literature on DSGE, following Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) by adding nominal rigidities and wage rigidities. From these important contributions, I also follow the large literature that have reconsidered the different features of these models and the underlying empirical strategy, Del Negro et al. (2007), Justiniano and Primiceri (2008), Fernández-Villaverde and Guerrón-Quintana (2020).

A brief description of our future baseline model could go as follow – it borrows from Faria-e Castro (2018) which uses Rotemberg pricing for non linear estimation.

A firm sector consists of final goods retailers and intermediate goods producers. Final goods retailers are perfectly competitive and employ a continuum of intermediate goods varieties indexed by $k \in [0, 1]$ to produce the final good using a Dixit-Stiglitz aggregator with constant elasticity of substitution ε

$$Y_t = \left[\int_0^1 Y_t(k)^{\frac{\varepsilon}{\varepsilon-1}} dk\right]^{\frac{\varepsilon-1}{\varepsilon}}$$

There is a continuum of intermediate goods producers, each producing a different variety k, all with the production function as in eq. (1):

$$Y_t(k) = \mathcal{F}_t(\mathcal{M}_t(E_t(k), K_{t-1}(k)), L_t)$$

Given constant elasticity of substitution between varieties, each firm faces a demand schedule of the type

$$Y_t(k) = \left[\frac{P_t(k)}{P_t}\right]^{-\varepsilon} Y_t$$

I assume that firms are subject to menu costs as in Rotemberg (1982), with a standard quadratic functional form of the type

$$\mathcal{C}^P\left(P_t(k), P_{t-1}(k)\right) = \frac{\lambda}{2} Y_t \left[\frac{P_t(k)}{P_{t-1}(k)}\Pi^{-1} - 1\right]^2$$

where Π is the inflation target set by the central bank (so that it is free to adjust to keep up with trend inflation) and η is the menu cost parameter. The first-order condition for an individual price-setting firm k combined with the assumption of a symmetric equilibrium yields a standard (nonlinear) Phillips curve that relates inflation to aggregate output:

$$\lambda \frac{\Pi_t}{\Pi}\left(\frac{\Pi_t}{\Pi} - 1\right) + \varepsilon\left(\frac{\varepsilon - 1}{\varepsilon} - MC_t\right) = \lambda \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}\frac{\Pi_{t+1}}{\Pi}\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\right] \tag{16}$$

The main difference between our model and the rest of the New Keynesian literature is the marginal cost faced by firms. Due to the nested CES production structure, it writes as follow:

$$\begin{aligned} MC_t &= \frac{1}{Z_t}\left[\alpha (MC_t^M)^{1-\varepsilon_y} + (1-\alpha) W_t^{1-\varepsilon_y}\right]^{\frac{1}{1-\varepsilon_y}} \\ MC_t^M &= \left[\eta (Q_t^E)^{1-\varepsilon_e} + (1-\eta)(Q_t^k)^{1-\varepsilon_e}\right]^{\frac{1}{1-\varepsilon_e}} \end{aligned} \tag{17}$$

where W_t is the wage faced by firms – that can be sticky with a Philipps curves coming from the Union problem, and MC_t^M is the marginal cost of machines, Q_t^E the price of machine (the MPE) and Q_t^k the price of capital (Tobin's Q) that relates to the MPK and depends on the adjustment cost of capital in a dynamic fashion:

$$\begin{aligned} Q_t^k &= \mathbb{E}_t\Big[\Lambda_{t,t+1}\big((1 + r_t^k - \delta) + (1-\delta)Q_{t+1}^k\big)\Big] \\ 1 &= Q_t^k\Big(1 - \Gamma(\frac{I_t}{I_{t-1}}) - \Gamma'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}}\Big) + \mathbb{E}_t\Big[\Lambda_{t,t+1}\, Q_t^k\, \Gamma'(\frac{I_t}{I_{t-1}})(\frac{I_t}{I_{t-1}})^2\Big] \end{aligned} \tag{18}$$

8 Conclusion

Forthcoming

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A Additional motivation: Energy supply and GDP growth are correlated

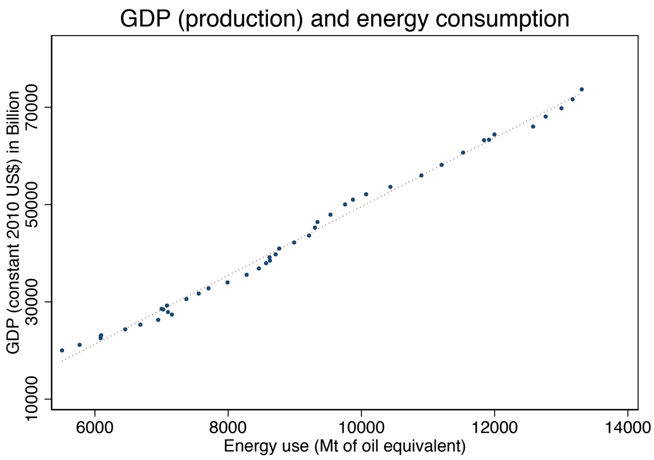

Figure 7: Linear relation at the world level

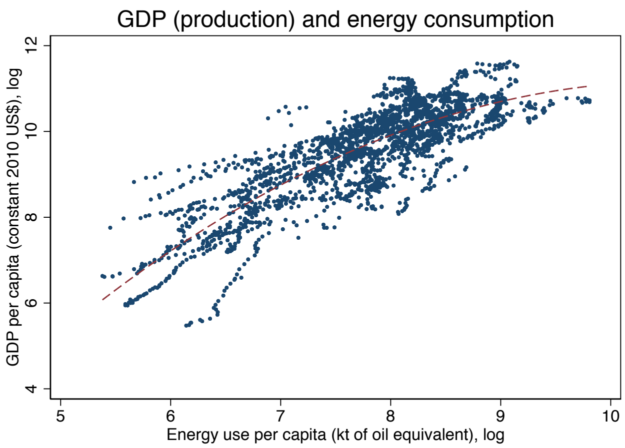

Figure 8: Cross country evidence, 60 countries 1960-2017

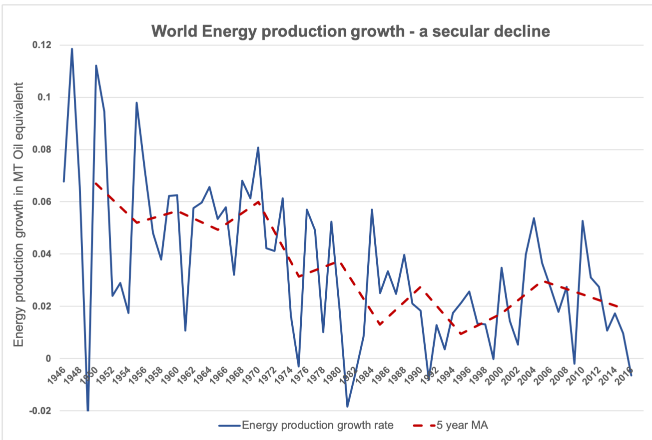

Figure 9: Decline in world energy production

B Detrended data: source and details

The time period of this sample span from 1949 to 2020 and included 289 quarters/observations. The source of the data used in the paper is the Federal Bank of St Louis (FRED):

- Output Y_t: Nominal Gross Domestic Product: Series GDP
- Consumption C_t: Personal consumption expenditure: Series PCEC
- Investment I_t: sum of two terms
  - Fixed private investment, Series FPI
  - Gross Government investment, Series A782RC1Q027SBEA
- Market clearing wedge G_t: sum of two terms:
  - Government consumption expenditures, Series A955RC1Q027SBEA
  - Net export, Series NETEXP
- Inflation: Π_t: GDP Implicit Price Deflator: Series GDPDEF
- Labor L_t: Nonfarm Business Sector: Average Weekly Hours, Series PRS85006023

All the variables in values (and real in the model) are deflated with the GDP deflator and divided by the population (adult above 16, series CNP16OV).

The notable exception is the energy consumption: this is issued by the U.S. Energy Information Administration. The data is yearly on the years 1949-1972 and monthly on the years 1973-2020. I compute a linear interpolation to convert the data at the quarterly level on the first subsample. This series is not seasonally adjusted, and I regress this series on months fixed effects to remove the seasonal effects (a forthcoming plan will be to use the X13-ARIMA package[17] since this is the common approach used by the Census, the BLS and the Federal Reserve System. The data on energy is measured in the physical unit : British Thermal Unit (QBTU, with Q for quadrillion) Some equivalent in alternative energy measures could be: $1BTU \approx 1kJ$ (kilo Joule), or ≈ 0.25 kcal, or $\approx 0.3kW{\cdot}h$. Moreover $1BBTU \approx 3GW{\cdot} \approx 2.5Toe$ (Tons of Oil Equivalent).

In the next figure, we display the time series used in the estimation. As the data in level display an important trend (especially in the data in value: GDP, consumption, investment government spending, net export), I choose not to take a stand on its source (technological progress or ... energy supply !) and do not include any unit-root in the exogenous shock processes $\{Z_t, \zeta_t, \xi_t, \chi_t\}$. Instead I remove the linear trend of the period 1949-2007 of these series taken in logs (one linear trend for each log-series). Indeed, the Great recession have caused a regime shift in many of these series (related to diverse hypotheses: hysteresis of the Great Recession, secular stagnation, ZLB, market power, change in the regulatory environment, etc.) and I choose not to included as part of the overall trend from 1949 to 2020. The series log-detrended are displayed in the following graph.

[17]When I'll manage to make it work on standard software.

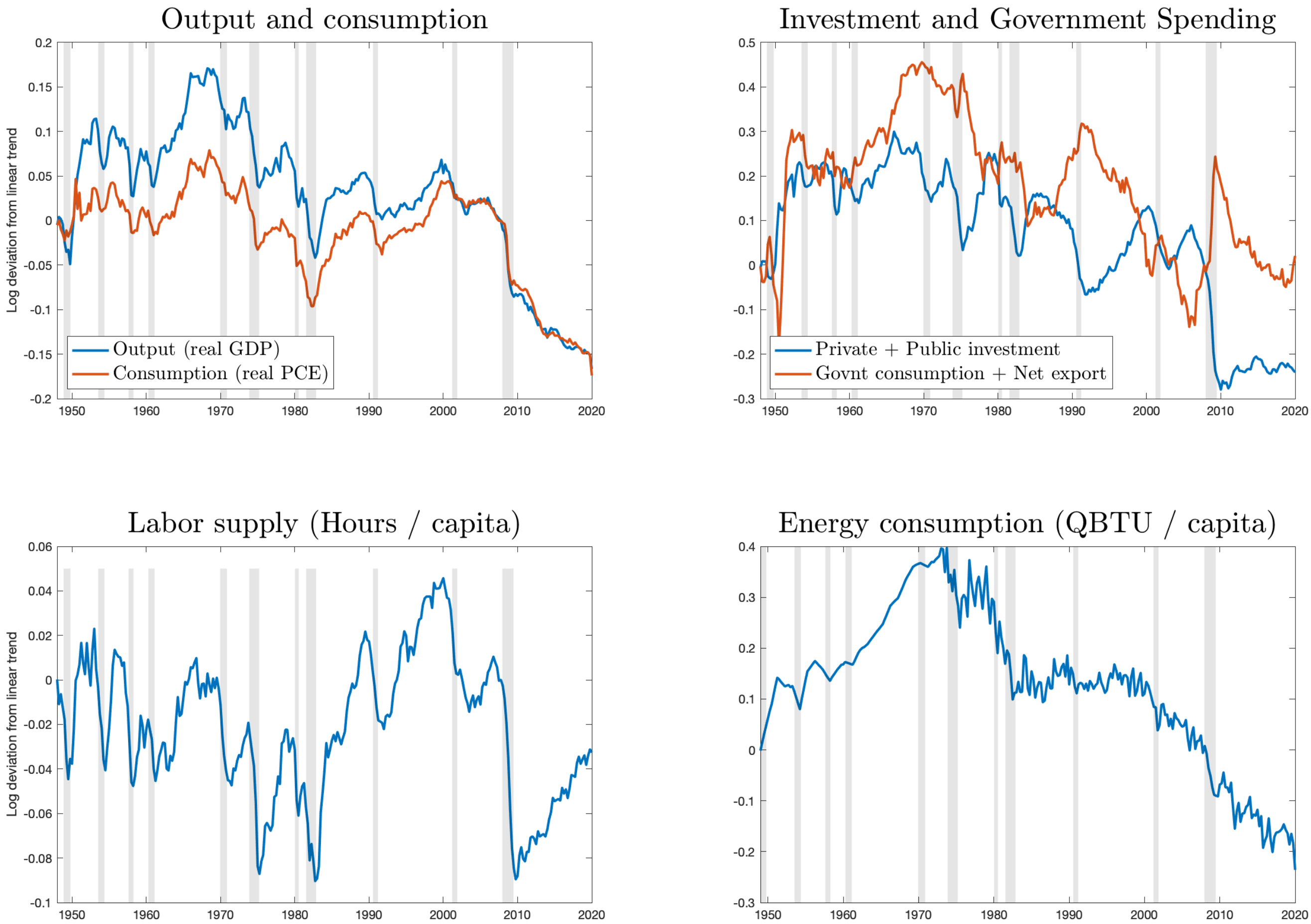

Figure 10: Sample: U.S. data 1949 Q1 - 2020 Q1, log deviation from 1949-2007 trend

C A production function for energy: two additional motivations

First, as shown in Anderson, Kellogg, and Salant (2018), (i) Production from Existing Wells does not respond to prices, Drilling and rig Activity does respond to price incentives, (iii) the industry cost structure explains these price responses: rate of production is physically constrained, marginal costs are very small compared to prices, fixed-costs are nonzero and the drilling of new oil wells and the rental price of drilling rigs react to oil price shocks.

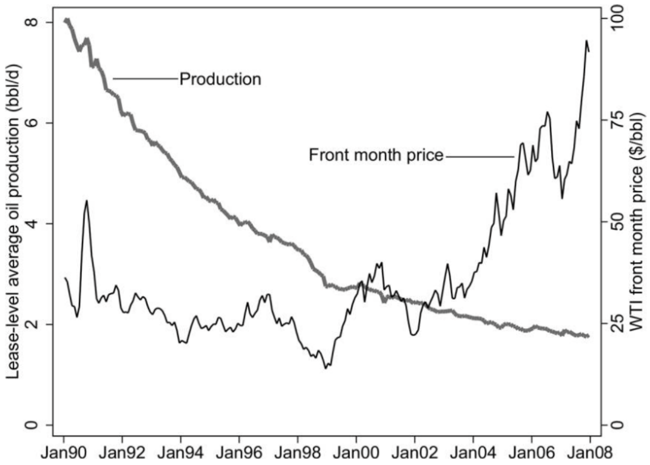

FIG. 1.—Crude oil prices and production from existing wells in Texas. This figure presents crude oil front-month ("spot") prices and daily average lease-level production from leases on which there was no rig activity (so that all production comes from preexisting wells). All prices are real 2007 dollars. See the text for details.

Figure 11: Anderson, Kellogg, and Salant (2018)

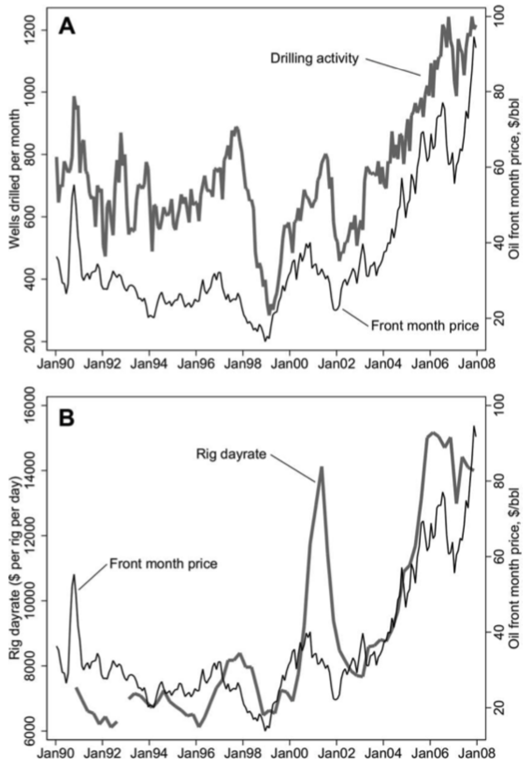

FIG. 2.—Texas rig activity versus crude oil spot prices. Panel A shows the total number of new wells drilled across all leases in our data set. Panel B shows dayrates for the Gulf Coast/South Texas region, for rigs with depth ratings between 6,000 and 9,999 feet. The dayrate data are quarterly rather than monthly. Data are available beginning in Q4 1990, and data for Q4 1992 are missing. Oil prices are real 2007 dollars. See the text for details.

Figure 12: Anderson, Kellogg, and Salant (2018)

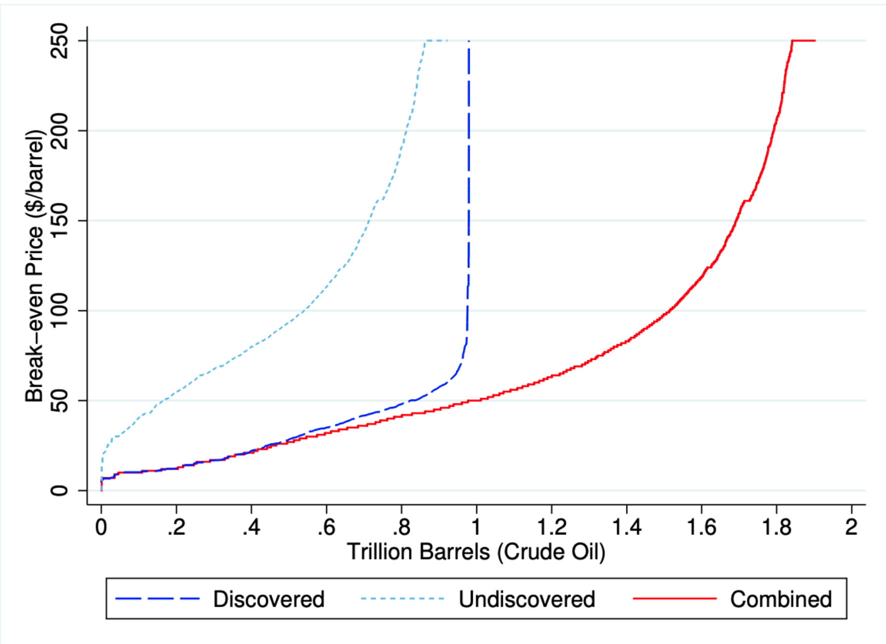

Figure 13: Supply curve for Oil producers (world level), Rystad energy data. Undiscovered reserves need to be explored and developed (i.e. drilled) to start producing oil

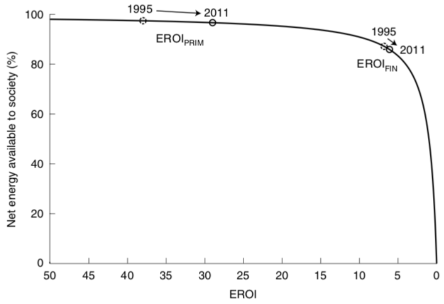

Figure 14: Net energy cliff: with EROI (Energy Return on Investment) on the x-axis and the Net Energy $= 1 - \frac{1}{EROI}$ on the y-axis, from Brockway et al. (2019)

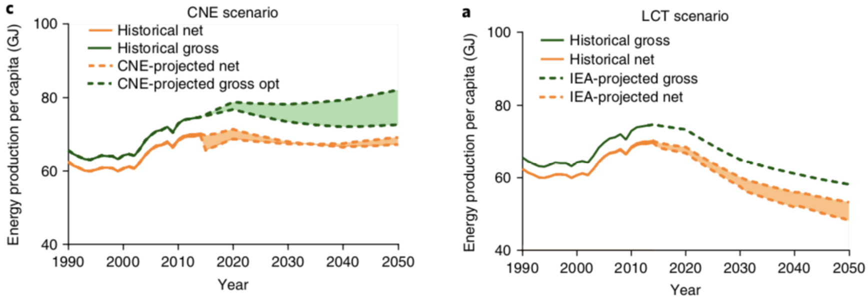

Figure 15: Stagnation or Decline in world energy per capita, depending on the Scenario: either a Low-Carbon transition (LCT, aimed at satisfying the $2°C$ limit), or a Constant Net Energy (CNE, aimed at maintaining the energy consumption per capita), from King and Van Den Bergh (2018)

D Additional IRFs for the RBC model with energy

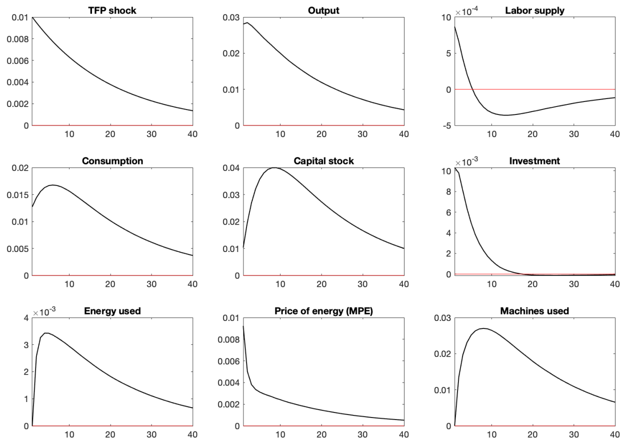

Figure 16: IRF to a TFP shock ω_z on Z^s_t

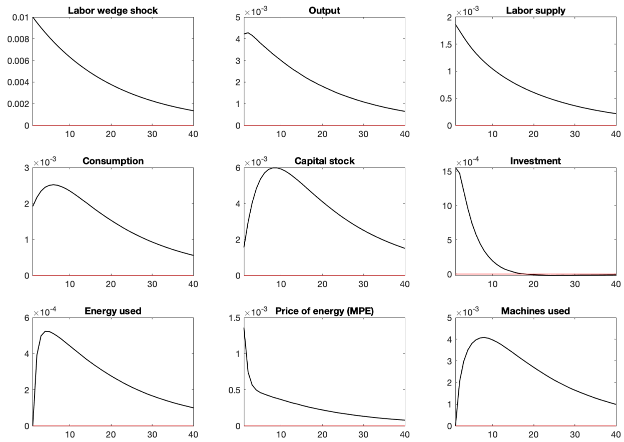

Figure 17: IRF to a Labor wedge shock ω_ℓ on ζ_t

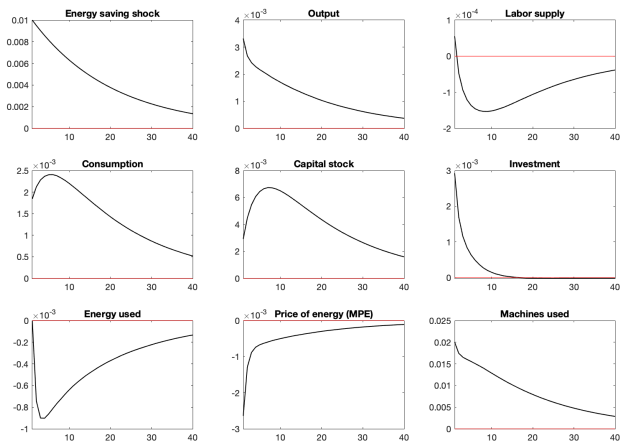

Figure 18: IRF to an Energy augmenting shock ω_s on ζ^s_t

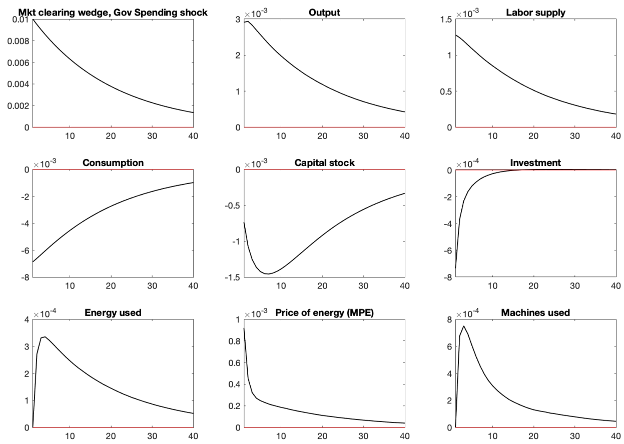

Figure 19: IRF to government spending/market clearing wedge shock ω_g on g_t