The Inequality of Climate Change
Heterogeneity and Optimal Energy policy

WORK IN PROGRESS

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Introduction – this project

- Marginal damages of climate & temperature varies across countries
  - Vary with the damage function : non-linearity matters a lot !
- What is the optimal taxation of energy in the presence of climate externality *and* heterogeneities ?
  - In contexts where fossil fuels taxation and climate policy redistributes across countries

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- Develop a simple and flexible model of climate economics
  - Standard NCG – IAM model with heterogeneous regions
  - Normative implications : Ramsey policy + possibility to study uncertainty

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- Develop a simple and flexible model of climate economics
  - Standard NCG – IAM model with heterogeneous regions
  - Normative implications : Ramsey policy + possibility to study uncertainty
- Evaluate the heterogeneous welfare costs of global warming
- Provide analytical formulas and a numerical methodology to compute the cost of carbon
  - Heterogeneity increases the welfare cost of carbon
- Solve world optimal carbon policy with heterogeneous regions
  - Does the optimal carbon tax coincide with the social cost of carbon ?
  - $\Rightarrow$ Maybe not, need to adjust for inequality level
  - What are the welfare gains of suboptimal policies ?

Model

- Neoclassical economy, in continuous time
  - countries/regions $i \in \mathbb{I}$: ex-ante heterogeneous : productivity z_i and more
  - ex-post heterogeneity in capital and temperature $\{k_i, \tau_i\}$
- Household problem in country i :

$$\mathcal{V}_{i,t_0} = \max_{\{c_{it}, e^f_{it}, e^r_{it}\}} \int_{t_0}^{\infty} e^{-\rho t}\, u(c_{it}, \tau_{it}) dt$$

- Dynamics of capital in every country i :

$$\dot{k}_{it} = \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - \bar{\delta} k_{it} + \theta_i \pi^f_t + \pi^r_{it} - (q^f_t + \mathbf{t}^f_{it}) e^f_{it} - (q^r_t + \mathbf{t}^r_{it}) e^r_{it} - c_{it} + \mathbf{t}^{ls}_{it}$$

  - Damage $\mathcal{D}^y(\tau_{it})$ affect country's production and consumption $u(\cdot, \tau_{it})$
  - Energy mix : $e_{it} = \mathscr{E}(e^f_{it}, e^r_{it} | \sigma_e)$ with fossil e^f_{it} – emitting carbon – vs. renewable e^r_t
  - Energy rents redistributed : share θ_i for fossils / fully for local renew. firm.
  - Prices, fossil q^f_t and non-carbon q^r_t (c.f. next slides)

Model – Energy markets

- Fossil fuels energy producer :
  - Extended-Hotelling problem (depleting reserves with stock effects and exploration)

$$\max_{\{E_t^f, \mathcal{I}_t\}_t} \int_0^\infty e^{-\rho t} \pi_t^f(q_t^f, E_t^f, \mathcal{R}_t) dt \qquad \text{with } \pi_t(E_t^f, \mathcal{R}_t) = q_t^{e,f} E_t^f - \mathcal{C}^f(E_t^f, \mathcal{R}_t) - \mathcal{C}^i(\mathcal{I}_t, \mathcal{R}_t)$$

$$s.t. \qquad E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta_R \mathcal{I}_t$$

  - Optimal pricing with finite-resources rents More details

$$q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \mathcal{C}_{\mathcal{I}}^i(\mathcal{I}_t, \mathcal{R}_t) = \delta_R \lambda_t^R$$

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$$q^{e,f}_t = \mathcal{C}^f_E(E^f_t, \mathcal{R}_t) + \lambda^R_t \qquad \mathcal{C}^i_{\mathcal{I}}(\mathcal{I}_t, \mathcal{R}_t) = \delta_R \lambda^R_t$$

- Renewable energy as a substitute technology *for each country* i (Static problem)

$$\pi^r_{it} = \max_{\{e^r_{it}\}} q^r_{it} e^r_{it} - \mathcal{C}^r(e^r_{it}) \qquad \Rightarrow \qquad q^r_{it} = \mathcal{C}^r_E(e^r_t)$$

  - Problem with prod. fct instead of marg. costs

Climate model :

- Fossil energy input e^f_t causes climate externality

$$\mathcal{E}_t = \int_{\mathbb{I}} \xi_i \, e^f_{it} \, di$$

- World climate – cumulative GHG in atmosphere $\mathcal{S}_t$ leads to increase in temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- Impact of climate on country's local temperature :

$$\dot{\tau}_{i,t} = \zeta\Big(\Delta_i \chi \mathcal{S}_t - \big(\tau_{i,t} - \bar{\tau}_{i,t_0}\big)\Big)$$

  - Simple model : Climate sensitivity to carbon χ, Climate reaction/inertia ζ, Carbon content of fossils ξ, Country scaling factor Δ_i, Carbon exit for atmosphere δ_s

Model – Household Solution

- Household solves a consumption/saving/energy decision, as in the NCG
  - Using Pontryagin (PMP), we obtain a system of coupled ODEs More details

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  - Consumption/Saving Euler equation :

$$\dot{c}_{it} = c_{it}\frac{1}{\eta}(\tilde{f}_{k,it} - \rho)$$

  - Energy decisions :
    Static demand for the two sources of energy : fossil e^f_{it} and renewable $e^r_{i,t}$ for every i, taking prices $\{q^f, q^r\}$ as given

$$\begin{aligned} &\tilde{f}_{e,it} = \mathcal{Q}(q^f_{i,t} + \mathbf{t}^f_{it}, q^r_{i,t} + \mathbf{t}^r_{it}) \\ \Rightarrow \quad & e^f_{i,t} = \mathcal{Q}_{q^f}(q^f_{i,t} + \mathbf{t}^f_{it}, q^r_{i,t} + \mathbf{t}^r_{it})e_{it} \qquad e^r_{i,t} = \mathcal{Q}_{q^r}(q^f_{i,t} + \mathbf{t}^f_{it}, q^r_{i,t} + \mathbf{t}^r_{it})e_{i,t} \end{aligned}$$

  - with $\tilde{f}_{k,it} = \mathcal{D}(\tau_{it})z_{i,t}f_{k,it} - \bar{\delta}$ and $\tilde{f}_{e,it} = \mathcal{D}^y(\tau_{i,t})z_{i,t}f_e(k_{i,t}, e_{i,t})$, and $\mathcal{Q}(\cdot)$ are aggregators functions (e.g. CES) and $\mathcal{Q}_{q^f}(\cdot)$ demand for fossil.

Impact of increase in temperature

- Using Damage fct $\mathcal{D}^y(\tau_{i,t}) = e^{-\frac{1}{2}\gamma_i(\tau_{i,t}-\tau_i^\star)^2}$ and $u(c,\tau) = u(\mathcal{D}^u(\tau_{i,t})c)$, w/ $u(\hat{c}) = \frac{c^{1-\eta}}{1-\eta}$
- Marginal values of the climate variables : $\lambda^S_{i,t}$ and $\lambda^\tau_{i,t}$

$$\dot{\lambda}^\tau_{i,t} = \lambda^\tau_{i,t}(\rho+\zeta) + \overbrace{\gamma_i(\tau_{i,t}-\tau_i^\star)\mathcal{D}^y(\tau_{i,t})}^{-\partial_\tau \mathcal{D}^y(\tau_{i,t})} f(k_{i,t},e_{i,t})\lambda^k_{i,t} + \overbrace{\phi_i(\tau_{i,t}-\tau_i^\star)\mathcal{D}^u(\tau_{i,t})^{1-\eta}c_{i,t}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}^S_{i,t} = \lambda^S_{i,t}(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{i,t}$$

- Costate $\lambda^S_{i,t}$: marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{i,t}-\tau_i^\star$ & damage sensitivity of TFP γ_i and utility ϕ_i
  - Development level $f(k_{i,t},e_{i,t})$ and $c_{i,t}$
  - Climate params : χ climate sensitivity, Δ_i "catching up" of τ_i and ζ reaction speed

Local Social cost of carbon

- The marginal "externality damage" or "local social cost of carbon" (SCC) for region i :

$$LSCC_{i,t} := -\frac{\partial \mathcal{V}_{i,t}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{i,t}/\partial c_{i,t}} = -\frac{\lambda^S_{i,t}}{\lambda^k_{i,t}}$$

  - Ratio of marg. cost of carbon vs. the marg. value of consumption/capital
  - Theorem : ***Stationary LSCC*** :
    When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ, ϕ, temperature, and output, consumption.

$$LSCC_{i,t} \equiv \frac{\chi\,\Delta_i}{\widetilde{\rho} + \delta^s}\left(\tau_{i,\infty} - \tau^\star_i\right)\left[\gamma_i\, y_{i,\infty} + \phi_i\, c_{i,\infty}\right]$$

    – More general formula : Here, Proof : Here + What determine temperatures ? Details Temperature

Model – Equilibrium

- Two types of interactions
  - On climate (externality) + heterogeneous effects of temperatures
  - On energy market + redistribution effects of energy rent
  - No bilateral flows (eq. doesn't exist with continuum and trade or migration)

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  - On climate (externality) + heterogeneous effects of temperatures
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  - No bilateral flows (eq. doesn't exist with continuum and trade or migration)
- Equilibrium
  - Given, initial conditions $\{k_0, \tau_0\}$ and country-specific policies $\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, \mathbf{t}^{ls}_{it}\}$, a competitive equilibrium is a continuum of sequences of states $\{k_{it}, \tau_{it}\}_{i,t}$ and $\{\mathcal{S}_t, \mathcal{T}_t, \mathcal{R}_t\}_t$ and policies $\{c_{i,t}, e^f_{i,t}, e^r_{i,t}\}_{i,t}$ and $\{E^f_t, \mathcal{E}_t, \mathcal{I}_t\}_t$, and price sequences $\{q^f_t, q^r_t\}$ such that :

  - Households choose policies $\{c_{i,t}, e^f_{i,t}, e^r_{i,t}\}_{i,t}$ to max utility s.t. budget constraint, giving $\dot{k}_{it}$
  - Renewable energy firm produce $\{e^r_{it}\}$ to max static profit
  - Fossil fuel firm extract and explore $\{E^f_t, \mathcal{I}_t\}$ to max profit, yielding $\dot{\mathcal{R}}_t$
  - Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{i,t}\}_{i,t}$.
  - Prices $\{q^f_t, q^r_t\}$ adjust to clear the markets : $E^f_t = \int_{\mathbb{I}} e^f_{it} di$ and $e^r_{it} = e^r_{it}$

Optimal policy

- Social planner, First best with a full set of instruments :
  - Solves world's inequality, using lump-sum transfers such that $\lambda_t^k = u'(c_{it}) = u'(c_{jt}), \forall i,j \in \mathbb{I}$
  - Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^S}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)

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- Second best / Ramsey planner :
  - Doesn't have access to lump-sum transfers
  - Can only use region-i-specific distortive energy taxes : $\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r\}$
  - Redistribute lump sum the tax revenues : $\mathbf{t}_{it}^{ls} = \mathbf{t}_{it}^f e_{it}^f + \mathbf{t}_{it}^r e_{it}^r$

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- Questions :
  - Is the level of energy tax regions specific ?
    - $\Rightarrow$ No under Negishi Pareto weights, yes under other weights (e.g. Uniform)
  - What is the level of the Pigouvian tax ?
    - $\Rightarrow$ $\propto$ Welfare cost/climate damage : "social costate" for carbon $\mathcal{S}$, i.e. ψ^S
    - $\Rightarrow$ Inequality/Heterogeneity in damage change the *level* of this tax

The Ramsey Problem

- Consider a Social Planner that care about aggregate welfare :

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t}^f_{it},\mathbf{t}^r_{it},c_{it},e^f_{it},e^r_{it},k_{it},\lambda^k_{it},\tau_{it},\mathcal{S}_t,\mathcal{R}_t,\mathcal{I}_t,\lambda^{\mathcal{R}}_t\}_{i,t}} \int_{t_0}^{\infty}\int_{\mathbb{I}} e^{-\bar{\rho}t}\,\omega_i\, u(c_{i,t},\tau_{i,t})\, di\, dt$$

subject to

  - Optimality conditions of households, for c_i, e^f_i, e^r_i and k_i
  - Optimality conditions of the Fossil firm, for E^f, $\mathcal{I}$ and $\mathcal{R}$
  - Optimality condition of the renewable firm, for e^r_i
  - Climate and temperature dynamics τ_i and $\mathcal{S}$
  - Given Pareto weights ω_i

$\Rightarrow$ Large scale system of ODE More details - Hamiltonian

  - A Ramsey plan is a set $\{\mathbf{t}^f_{it},\mathbf{t}^r_{it},\mathbf{t}^{ls}_{it}\}_{it}$ s.t. the competitive equilibrium is maximizing welfare

The Ramsey Problem – Measure of inequality

- Shadow value of capital / wealth

$$[c_{it}] \qquad \psi^k_{it} = \underbrace{\omega_i u_c(c_i, \tau_{it})}_{=\text{direct effect}} + \underbrace{\phi^c_{it} u_{cc}(c_i, \tau_{it})}_{=\text{effect on savings}}$$

  - In the background, Household FOCs pinning down c_{it} and dynamics for ψ^k_{it}
  - Depends on Pareto weights ω_i
- Measure of inequality and redistribution motive :

$$\widehat{\psi}^k_{it} = \frac{\psi^k_{it}}{\overline{\psi}^k_t} \lessgtr 1$$

$$\text{low } z_i, k_i \qquad \Rightarrow \qquad \text{low } \; c_i, \; \text{high } \psi^k_{i,t} \approx \omega_i u'(c_i) p_i \; > \overline{\psi}^k_t = \int_{\mathbb{I}} \psi^k_{i,t} di$$

  - With ω_i the Negishi weights, we have $\widehat{\psi}^k_{it} = 1$

The Ramsey Problem – Measure for Social Cost of Carbon

- Shadow value of carbon $\psi_t^{\mathcal{S}}$ and of temperature $\psi_{jt}^{\mathcal{T}}$

$$\dot{\psi}_t^{\mathcal{S}} = \left(\tilde{\rho} + \delta^s\right)\psi_t^{\mathcal{S}} - \int_{\mathbb{I}} \Delta_j \zeta \chi \psi_{jt}^{\mathcal{T}} dj$$

  - Social planner summarizes climate damages in one variable $\psi_t^{\mathcal{S}} = \frac{\partial \mathcal{W}_t}{\partial \mathcal{S}}$
  - Integrate all welfare costs of temperatures $\psi_{jt}^{\mathcal{T}}$, i.e. local costs of carbon/climate damages :

$$LSCC_{it}^{sp} = \frac{\psi_{it}^{\mathcal{S}}}{\psi_{it}^{k}}$$

- Can reexpress the welfare cost of carbon WCC_t

$$WCC_t = -\frac{\psi_t^{\mathcal{S}}}{\overline{\psi}_{it}^{k}} = \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}_{it}^{k}, LSCC_{j,t}^{sp}\right) + \mathbb{E}_j[LSCC_{j,t}^{sp}] > \mathbb{E}_j[LSCC_{j,t}^{sp}] = SCC_t$$

The Ramsey Problem – Measure for Fossil price redistribution

- Shadow value of fossil price ϕ_t^{Ef}

$$SVF_t = \frac{\phi_t^{Ef}}{\overline{\psi}_{it}^{k}} = \int_{\mathbb{I}} \widehat{\psi}_{jt}^{k} e_{jt}^{f} dj - \partial_{q^f} \pi^f(\cdot) \int_{\mathbb{I}} \widehat{\psi}_{jt}^{k} \theta_j dj$$
$$= \mathbb{C}\text{ov}_j\left(\widehat{\psi}_{jt}^{k}, e_{jt}^{f}\right) - E_t^f \mathbb{C}\text{ov}_j\left(\widehat{\psi}_{jt}^{k}, \theta_{jt}^{f}\right)$$

- SVF is the shadow value of changing (endogenously !) the fossil price q_t^f
  - Low price q_t^f benefit fossil consumers, especially if they're poor / high $\widehat{\psi}_{it}^{k}$
  - Low price q_t^f hurts the fossil firm owners θ_{jt}^{f} especially if they have high $\widehat{\psi}_{it}^{k}$
  - In practice, $SVF_t > 0$

Optimal Fossil taxation

- Optimal policy for fossil energy, FOC of Ramsey planner :

$$\Big(\frac{\mathcal{Q}^2_{q^f}}{f_{ee,it}} + \mathcal{Q}_{q^f q^f}\Big)\Big[-\xi_i \frac{\psi^S_t}{\overline{\psi}^k_t} + \frac{\phi^{Ef}_t}{\overline{\psi}^k_t} \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it}\mathbf{t}^f_{it}\Big] + \cdots = 0$$

$$\Rightarrow \qquad \widehat{\psi}^k_{it}\, \mathbf{t}^f_{it} = \xi_i WCC_t + SVF_t \mathcal{C}^f_{EE} \qquad \& \qquad \mathbf{t}^r_{it} = 0$$

- Pigouvian tax :
  - Integrate several redistribution motives :
  - Changes in carbon/climate impacts in WCC_t
  - Changes in fossil fuel rent redistribution SVF along supply curve $\mathcal{C}^f_{EE}$ of fossil firm
  - Depends on the country's income level $\widehat{\psi}^k_{it}$: set a lower tax on poorer, high $\widehat{\psi}^k_{it}$ countries

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- Welfare costs of suboptimal taxes :
  - Proportional to $\big(\frac{\mathcal{Q}^2_{q^f}}{f_{ee,it}} + \mathcal{Q}_{q^f q^f}\big)$, i.e. depends on the curvature of production function f_{ee}, elasticity $\mathcal{Q}_{q^f}$ and curvature $\mathcal{Q}_{q^f q^f}$ of fossil demand

Numerical Applications

- Coming soon

Conclusion

- Climate change has redistributive effects & heterogeneous impacts
- Redistributive effects of policy
  - Pigouvian tax that covers aggregate marginal damages
  - Can account for inequality both for welfare cost climates and redistribution effects of
- Study suboptimal policies and
  -
- Future plans
  - Dynamics on the capacity of renewable ?
  - Endogenous growth in TFP/energy saving technology
    Learning-by-doing : positive externality ?
  - Uncertainty

Appendices

More details – Energy market

- Fossil fuel producer : price the Hotelling rent with the maximum principle :

$$\mathcal{H}^m(\mathcal{R}_t, \lambda_t^R, E_t, \mathcal{I}_t^e) = \pi_t(E_t^f, \mathcal{I}_t^f, \mathcal{R}_t) + \lambda_t^R(\delta^R \mathcal{I}_t^e - E_t)$$

- Rent λ_t^R grows with interest ρ and with the marginal gain of increasing reserves

$$\begin{aligned} \dot{\lambda}_t^R &= \rho \lambda_t^R - \partial_R \mathcal{C}(E_t^f, \mathcal{I}_t^f, \mathcal{R}_t) \\ &= \rho \lambda_t^R + \frac{\bar{\nu}\nu}{1+\nu}\left(\frac{E_t^\star}{R_t}\right)^{1+\nu} + \frac{\bar{\mu}\mu}{1+\mu}\left(\frac{I_t^\star}{R_t}\right)^{1+\mu} \\ \dot{\lambda}_t^R &= \rho \lambda_t^R + \frac{\bar{\nu}^{-1/\nu}\nu}{1+\nu}\left(q^{e,f} - \lambda_t^R\right)^{1+1/\nu} + \frac{\bar{\mu}^{-1/\mu}\mu}{1+\mu}\left(\delta^R \lambda_t^R\right)^{1+1/\mu} \end{aligned}$$

- Because of decreasing return to scale and Hotelling rents : profits are > 0

$$\pi_t(E_t^f, \mathcal{R}_t, \lambda_t^R) = \frac{1+\nu-\bar{\nu}}{1+\nu}\left(\frac{E_t^f}{\mathcal{R}_t}\right)^{1+\nu}\mathcal{R}_t + \lambda_t^R E_t^f - \frac{\bar{\mu}^{-1/\mu}}{1+\mu}\left(\delta^r \lambda_t^R\right)^{1+\frac{1}{\mu}}$$

back

More details – PMP – Competitive equilibrium

- Household problem : State variables $s_{i,t} = (k_i, \tau_i, z_i, p_i, \Delta_i)$ Back
- Pontryagin Maximum Principle

$$\mathcal{H}^{hh}(s, \{c\}, \{e^f\}, \{e^r\}, \{\lambda\}) = u(c_i, \tau_i) + \lambda^k_{i,t}\Big(\mathcal{D}(\tau_{it})f(k_t, e_t) - (n + \bar{g} + \delta)k_t - q^f_t e^f_{it} - q^r_{it} e^r_{it} - c_t\Big)$$
$$+ \lambda^\tau_{i,t}\zeta\Big(\Delta_i\,\chi\,\mathcal{S}_t - (\tau_{it} - \tau_{i0})\Big) + \lambda^S_{i,t}\Big(\mathcal{E}_t - \delta^s\mathcal{S}_t\Big)$$

$$[c_t] \qquad u'(c_{it}) = \lambda^k_{i,t}$$

$$[e^f_t] \qquad MPe^f_{it} = \mathcal{D}(\tau_{i,t})z\,\partial_e f(k_{i,t}, e_{i,t})\left(\frac{e^f_{i,t}}{\omega e_{i,t}}\right)^{-\frac{1}{\sigma_e}} = q^f_t$$

$$[e^r_t] \qquad MPe^r_{it} = \mathcal{D}(\tau_{i,t})z\,\partial_e f(k_{i,t}, e_{i,t})\left(\frac{e^r_{i,t}}{(1-\omega)e_{i,t}}\right)^{-\frac{1}{\sigma_e}} = q^r_{it}$$

$$[k_t] \qquad \dot{\lambda}^k_t = -\lambda^k_t\big(\mathcal{D}(\tau_{i,t})\partial_k f(k_{i,t}, e_{i,t}) - \delta - \bar{g} - n - \rho\big)$$

- Fossil Energy Monopoly :

$$\mathcal{H}^m(\mathcal{R}_t, \lambda^R_t, E^f_t, \mathcal{I}_t) = \pi_t(E^f_t, \mathcal{I}_t, \mathcal{R}_t) + \lambda^R_t(\delta^R\mathcal{I}_t - E^f_t)$$

$$[\mathcal{R}_t] \qquad \dot{\lambda}^R_t = \rho\lambda^R_t + \frac{\bar{\nu}\nu}{1+\nu}\left(\frac{E^\star_t}{R_t}\right)^{1+\nu} + \frac{\bar{\mu}\mu}{1+\mu}\left(\frac{I^\star_t}{R_t}\right)^{1+\mu}$$

$$[E^f_t] \qquad q^{e,f}_t = \nu_E(E, \mathcal{R}) + \lambda^R_t = \bar{\nu}\left(\frac{E_t}{\mathcal{R}_t}\right)^\nu + \lambda^R_t$$

$$[\mathcal{I}_t] \qquad \lambda^R_t\delta^R = \mu_I(I_t, R_t) = \bar{\mu}\left(\frac{\mathcal{I}_t}{\mathcal{R}_t}\right)^\mu \qquad \mathcal{I}_t = R_t\left(\frac{\lambda^R_t\delta}{\bar{\mu}}\right)^{1/\mu}$$

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}^{\tau}_{i,t} = \lambda^{\tau}_t(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\lambda^k_t + \phi(\tau - \tau^\star)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}^S_t = \lambda^S{}_t(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda^{\tau}_{i,t}$$

- Solving for λ^{τ}_t and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^S_t = \dot{\lambda}^{\tau}_t = 0$

$$\lambda^{\tau}_{i,t} = -\int_t^\infty e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_u)y_\tau\lambda^k_u + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda^{\tau}_{i,t} = -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_\infty)y_\infty\lambda^k_\infty + \phi\mathcal{D}^u(\tau_\infty)u(c_\infty)\Big)$$

$$\begin{aligned}
\lambda^S_t &= -\int_t^\infty e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\tau}_{j,u}dj\,du \\
&= \frac{1}{\widetilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\tau}_{j,\infty} \\
&= -\frac{\chi}{\widetilde{\rho}+\delta^s}\frac{\zeta}{\widetilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_\infty\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

$$\lambda^S_t \xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj$$

Cost of carbon / Marginal value of temperature

- Closed form solution for CC :
  - In stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - no internalization of externality (business as usual)

$$LSCC_{i,t} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s}(\tau_\infty - \tau^\star)\Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty\Big)$$

- Heterogeneity + uncertainty about models Back

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega \int_{t_0}^{T} e^{(n+\bar{g})t-\delta_s(T-t)} q_t^{f\ -\sigma_e} \int_{j\in\mathbb{I}} \left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e-\sigma}\, dj\, dt$$

  - Geographical factors determining warming Δ_i
  - Climate sensitivity χ & carbon exit from atmosphere δ_s
  - Growth of population n, aggregate productivity $\bar{g}$
  - Deviation of output from trend y_i & relative TFP z_j
  - Directed technical change z_t^e, elasticity of energy in output σ
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}_t^R/\lambda_t^R = \rho$
  - Change in energy mix, renewable share ω, price q_t^r & elasticity of source σ_e

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto\ n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \widetilde{\gamma}\right) + (\sigma_e - \sigma)(1-\omega)g^{q^r} - (\sigma_e(1-\omega)+\sigma\omega)g^{q^f}$$

Back

More details – PMP – Ramsey Optimal Allocation

- Hamiltonian :

$$
\begin{aligned}
\mathcal{H}^{sp}(s,\{c\},\{e^f\},\{e^r\},\{\lambda\},\{\psi\}) = & \int_{\mathbb{I}} \omega_i u(c_i,\tau_i) p_i di \\
& + \psi^k_{i,t}\Big(\mathcal{D}(\tau_{it}) f(k_{it},e_{it}) - (n+\bar{g}+\delta)k_t + \theta_i \pi(E^f_t,\mathcal{I}_t,\mathcal{R}_t) + \pi^r_{it}(e^r_{it}) - (q^f_t + \mathbf{t}^f_{it})e^f_{it} - (q^r_{it}+\mathbf{t}^r_{it})e^r_{it} - c_t + \mathbf{t}^{ls}_t\Big) \\
& + \psi^S_t\Big(\mathcal{E}_t - \delta^s \mathcal{S}_t\Big) + \psi^\tau_{it}\zeta\Big(\Delta_i \chi \mathcal{S}_t - (\tau_{it}-\tau_{i0})\Big) + \psi^{\mathcal{R}}_{it}\Big(-E^f_t + \delta^R \mathcal{I}_t\Big) \\
& + \psi^{\lambda k}_{i,t}\Big(\lambda^k_t(\rho - r_t)\Big) + \psi^{\lambda R}_t\Big(\rho\lambda^R_t + \mathcal{C}^f_{\mathcal{R}}(E^f_t,\mathcal{I}_t,\mathcal{R}_t)\Big) + \phi^c_{it}\big(u_c(c_i,\tau_{it}) - \lambda^k_{it}\big) \\
& + \phi^{ef}_{it}\Big(e^f_{it} - \mathcal{Q}_{q^f}(q^f_t + \mathbf{t}^f_{it}, q^r_t + \mathbf{t}^r_{it})e_{it}\Big) + \phi^{er}_{it}\Big(e^r_{it} - \mathcal{Q}_{q^r}(q^f_t + \mathbf{t}^f_{it}, q^r_t + \mathbf{t}^r_{it})e_{it}\Big) \\
& + \phi^e_{it}\Big(f_e(k_{it},e_{it}) - \mathcal{Q}(q^f_t + \mathbf{t}^f_{it}, q^r_t + \mathbf{t}^r_{it})\Big) + \phi^{Ef}_t\Big(q^f_t - \mathcal{C}^f_E(\cdot) - \lambda^{\mathcal{R}}_t\Big) + \phi^{Er}_{it}\Big(q^r_{it} - \mathcal{C}^r_e(\cdot)\Big) + \phi^{\mathcal{I}f}_t\big(\delta\lambda^{\mathcal{R}}_t - \mathcal{C}^f_{\mathcal{I}}(\cdot)\big)
\end{aligned}
$$

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Ramsey Optimal Allocation - FOCs

- FOCs w.r.t. $\{c_{it}, e_{it}, e^f_{it}, e^r_{it}, \mathcal{I}_t\}$, prices $\{q^f_t, q^r_{it}\}$ and taxes, denoting $\tilde{q}_{it} = q_t + \mathbf{t}_{it}$

$$[c_{it}] \qquad \psi^k_{it} = \underbrace{\omega_i u_c(c_i, \tau_{it}) p_i}_{=\text{direct effect}} + \underbrace{\phi^c_{it} u_{cc}(c_i, \tau_{it})}_{=\text{effect on savings}}$$

$$[e_{it}] \qquad \psi^k_{it} f_{e,it} + \phi^e_{it} f_{ee,it} - \phi^{ef}_{it} \mathcal{Q}_{q^f} - \phi^{er}_{it} \mathcal{Q}_{q^r} = 0 \qquad \Rightarrow \qquad \phi^e_{it} = \frac{1}{f_{ee,it}} \left(\phi^{ef}_{it} \mathcal{Q}_{q^f} + \phi^{er}_{it} \mathcal{Q}_{q^r} - \psi^k_{it} f_{e,it} \right)$$

$$[e^f_{it}] \qquad \phi^{ef}_{it} = \psi^k_{it} \tilde{q}^f_t - \psi^k_{it} \mathbf{t}^f_{it} - \xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) \qquad [e^r_{it}] \qquad \phi^{er}_{it} = \psi^k_{it} \tilde{q}^r_t - \psi^k_{it} \mathbf{t}^r_{it} + \phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot)$$

$$[\tilde{q}^f_{it}] \qquad \phi^e_{it} \mathcal{Q}_{q^f} + \phi^{ef}_{it} \mathcal{Q}_{q^f q^f} + \phi^{er}_{it} \mathcal{Q}_{q^r q^f} = 0$$

$$\Rightarrow \qquad \left(\frac{\mathcal{Q}^2_{q^f}}{f_{ee,it}} + \mathcal{Q}_{q^f q^f} \right) \left[-\xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it} \mathbf{t}^f_{it} \right] + \left(\frac{\mathcal{Q}_{q^f} \mathcal{Q}_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^r q^f} \right) \left[\phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot) - \psi^k_{it} \mathbf{t}^f_{it} \right]$$

$$[\tilde{q}^r_{it}] \qquad \phi^e_{it} \mathcal{Q}_{q^r} + \phi^{ef}_{it} \mathcal{Q}_{q^f q^r} + \phi^{er}_{it} \mathcal{Q}_{q^r q^r} = 0$$

$$\Rightarrow \qquad \left(\frac{\mathcal{Q}_{q^f} \mathcal{Q}_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^f q^r} \right) \left[-\xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it} \mathbf{t}^f_{it} \right] + \left(\frac{\mathcal{Q}^2_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^r q^r} \right) \left[\phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot) - \psi^k_{it} \mathbf{t}^f_{it} \right] = 0$$

$$[q^f_{it}] \qquad \phi^{Ef}_t = \int_{\mathbb{I}} \psi^k_{jt} e^f_{jt} dj - \partial_{q^f} \pi^f(\cdot) \int_{\mathbb{I}} \theta_j \psi^k_{jt} dj \qquad [q^r_{it}] \qquad \phi^{Er}_{it} = \psi^k_{it} e^r_{it} - \psi^k_{it} \partial^r_q \pi^r_{it} = 0$$

$$[\mathcal{I}_t] \qquad \delta\, \psi^{\mathcal{R}}_t + \partial^2_{\mathcal{R}\mathcal{I}} \mathcal{C}(\cdot)\, \psi^{\lambda,\mathcal{R}}_t - \phi^{\mathcal{I}}_t \partial^2_{\mathcal{I}\mathcal{I}} \mathcal{C}(\cdot) = 0$$

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Ramsey Optimal Allocation - FOCs

- Backward equations for planner's costates

$$[k_i] \qquad \dot{\psi}^k_{it} = \psi^k_{it}(\tilde{\rho} - r_{it}) + \psi^{\lambda k}_{it}\lambda^k_{it}\partial_k MPk_i + \frac{f_{ek,it}}{f_{ee,it}}\Big[- \xi\psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it}\mathbf{t}^f_{it}\Big]$$

$$[\mathcal{S}_i] \qquad \dot{\psi}^{\mathcal{S}}_t = (\tilde{\rho} + \delta^s)\psi^{\mathcal{S}}_t - \int_{\mathbb{I}} \Delta_j \zeta\chi\psi^\tau_{jt} dj$$

$$[\tau_i] \qquad \dot{\psi}^\tau_t = (\tilde{\rho} + \zeta)\psi^\tau_t - \Big(\omega_i u_\tau(c_{it}, \tau_{it}) + \psi^k_{it}\mathcal{D}'(\tau_{it})f(k_{it}, e_{it}) + \phi^c_{it}u_{c,\tau}(c_{it}, \tau_{it}) + \mathcal{D}'(\tau_{it})f_e\phi^e_{it}\Big)$$

$$[\mathcal{R}] \qquad \dot{\psi}^{\mathcal{R}}_t = \psi^{\mathcal{R}}_t\Big(\tilde{\rho} - \partial^2_{\mathcal{R}\mathcal{R}}\mathcal{C}(\cdot)\Big) - \phi^{Ef}_t \partial^2_{\mathcal{R}E}\mathcal{C}(\cdot)$$

$$[\lambda^k_i] \qquad \dot{\psi}^{\lambda,k}_t = \psi^{\lambda,k}_t\big[\tilde{\rho} - (\rho - r_{i,t})\big] + \phi^c_{i,t}$$

$$[\lambda^{\mathcal{R}}_i] \qquad \dot{\psi}^{\lambda,\mathcal{R}}_t = \psi^{\lambda,\mathcal{R}}_t(\tilde{\rho} - \rho) + \phi^{Ef}_t - \delta\phi^{\mathcal{I}f}_t$$

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