# The Optimal Design of Climate Agreements

## Inequality, Trade, and Incentives for Carbon Policy

WORK IN PROGRESS

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*Committee meeting*

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# Introduction

- Fighting climate change requires implementing ambitious carbon reduction policies
  - The "free-riding problem" causes climate inaction:
    the tax costs are local and the climate benefits are global
  - Moreover, such climate policy redistributes across countries through
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- Climate agreement boils down to a carbon price, a tariff rate and a choice of countries
- Build a Climate-Macro model with heterogeneous countries & trade and study the strategic implications of climate agreements and the optimal club design

- Preview of the result:
  - With enough policy instruments, the "coalitional Nash" climate agreement reproduces the world optimal policy: high carbon tax, high tariffs, participation of the entire world

- Literature:
  - Nordhaus (2015), Iverson (2024), Old theoretical literature on Climate Agreements
  - Trade Policy: Farrokhi, Lashkaripour (2021), Kortum, Weisbach (2022), Böhringer et al.
  - Public finance / Heterogeneous agents macro / spatial

# Model – Household & Firms

- Static and deterministic Neoclassical economy
  - countries $i \in \mathbb{I}$, heterogeneous in productivity $z_i$, temperature $T_i$, energy extraction cost $\mathcal{C}_i$
  - In each country, four agents:

1. Representative household problem $\mathcal{V}_i = \max_{c_{ij}} u(c_i)$

$$c_i = \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \qquad \sum_j c_{ij} \underbrace{(1+\mathrm{t}_{ij}^b)}_{\text{tariff}} \underbrace{\tau_{ij}}_{\text{iceberg cost}} \mathrm{p}_j = \underbrace{w_i \ell_i}_{\text{labor income}} + \underbrace{\pi_i^f}_{\text{fossil firm profit}} + \mathrm{t}_i^{ls}$$

2. Competitive final good firm:

$$\max_{\ell_i, e_i^f, e_i^r} \mathrm{p}_i \, \mathcal{D}(T_i) \, z_i f(\ell_i, e_i^f, e_i^r) - w_i \ell_i - (q^f + \mathrm{t}_i^f) e_i^f - q_i^r e_i^r$$

  - Externality: Damage function $\mathcal{D}(T_i)$, Inequality from $z_i$, Fossil energy tax: $\mathrm{t}_i^f$

3. Renewable energy firm: elastic supply $e_i^r$ at price $q_i^r = z^r \mathbb{P}_i$

# Model – Energy markets & Emissions

4. Competitive fossil fuels energy producer, extracting $e_i^x$

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x)\mathbb{P}_i$$

– Energy traded in international markets, at price $q^f$

$$E = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

- Climate system: effect on local temperature in $i$

$$T_i = \bar{T}_{i0} + \underbrace{\Delta_i}_{\text{pattern scaling}} \underbrace{\sum\nolimits_{\mathbb{I}} e_i^f}_{\text{GHG emission}}$$

- Market clearing for goods: (in expenditure)

$$\mathrm{p}_i \underbrace{y_i}_{\substack{\text{output} \\ = \mathcal{D}(T_i) z_i f(\cdot)}} = \sum_{k \in \mathbb{I}} \frac{s_{ki}}{1+\mathrm{t}_{ki}^b} \left(\mathrm{p}_k y_k + q^f(e_k^x - e_k^f) + \mathrm{t}_k^{ls}\right)$$

# Model – Equilibrium

- Given policies $\{\mathrm{t}^f_i, \mathrm{t}^b_{ij}, \mathrm{t}^{ls}_i\}_i$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e^f_i, e^x_i\}_{ij}$, states $\{T_i\}_i$ and prices $\{\mathrm{p}_i, w_i\}_i, q^f$ such that:
  - Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint
  - Firm choose inputs $\{e^f_i, e^r_i\}_i$ to max. profit
  - Fossil firms extract/produce $\{e^x_i\}_i$ to max. profit. + Elastic renewable supply $\{e^r_i\}$
  - Emissions $\mathcal{E}$ affects climate $\{T_i\}_i$.
  - Government budget clear $\sum_i \mathrm{t}^{ls}_i = \sum_i \mathrm{t}^f_i e^f_i + \sum_{i,j} \mathrm{t}^b_{ij} c_{ij} \tau_{ij} \mathrm{p}_j$
  - Prices $\{\mathrm{p}_i, w_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e^x_{it} = \sum_{\mathbb{I}} e^f_{it}$ and for each good

$$y_i := \mathcal{D}(T_i)\, z_i f(e^f_i) = \sum_{k\in\mathbb{I}} \tau_{ki} c_{ki} + \sum_{k\in\mathbb{I}} g_{ki}$$

with $g_{ki}$ net export of good $i$ to pay for costs of energy in $k$
In expenditure, with import shares $s_{ij} = \frac{c_{ij}\tau_{ij}\mathrm{p}_j}{c_i \mathbb{P}_i}$, it yields
$\mathrm{p}_i y_i = \sum_{k\in\mathbb{I}} \frac{s_{ki}}{1+\mathrm{t}^b_{ki}} \left(\mathrm{p}_k y_k + q^f (e^x_k - e^f_k) + \widetilde{\mathrm{t}}^{ls}_k\right)$

# Benchmarks

- Two different benchmarks:
  - World's planner maximizing world's welfare without participation constraints
    - Single carbon and absence of cross country transfers $\mathbb{J} = \mathbb{I}$
    - Optimal carbon tax $\mathrm{t}^f$ accounts for:
      (i) Redistribution motive, G.E. effects on (ii) energy markets and (iii) through trade
      + optimal tariffs for terms-of-trade manipulations
  - Local planner in country $i$ unilaterally choosing $\mathrm{t}_i^f$ and $\mathrm{t}_{ij}^b$
    - Optimal unilateral carbon tax (subsidy!) and tariffs for terms-of-trade manipulations
    - Nash equilibrium of $I$ agents choosing individually unilateral policies
  - Climate club $\mathbb{J} \subsetneq \mathbb{I}$

# Benchmark: Optimal world policy – Summary of results

- Consider a social planner maximizing the world's welfare:
  - Choose a single carbon tax $\mathrm{t}^f$ for the world $\mathbb{J} = \mathbb{I}$

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{c},\boldsymbol{e}\}_i} \sum_{i\in\mathbb{I}} \omega_i \, u(c_i) = \textstyle\sum_{\mathbb{I}} \mathcal{W}_i$$

  - Optimal climate policy depends on the availability of redistribution instruments (i.e. lump-sum transfers $\mathrm{t}_i^{ls}$ across countries)
  - Without redistribution motives, optimal Pigouvian carbon tax: $\mathrm{t}^f = SCC$

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  - Optimal climate policy depends on the availability of redistribution instruments (i.e. lump-sum transfers $\mathrm{t}_i^{ls}$ across countries)
  - Without redistribution motives, optimal Pigouvian carbon tax: $\mathrm{t}^f = SCC$
  - Otherwise, the optimal carbon tax should account for the distribution of (i) Local Damage $LCC_i$, (ii) energy supply terms-of-trade effects, (iii) energy demand distortions, (iv) all of them weighted by an index $\phi_i \propto \omega_i u'(c_i)$

$$\mathrm{t}^f = \underbrace{\sum\nolimits_i \phi_i \, LCC_i}_{= SCC} + \sum\nolimits_i \phi_i \, \text{Supply Distortion}_i + \sum\nolimits_i \phi_i \, \text{Demand Distortion}_i$$

- Details:

  ***Competitive equilibrium*** Details eq 0, ***First-Best***, with unlimited instruments Details eq 1, ***Second-best***, Ramsey policy with limited instruments Details eq 2

# Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.

- ***Definition*** A climate agreement is a set $\{\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b\}$, with $\mathbb{J} \subseteq \mathbb{I}$ countries and a C.E. $\{\boldsymbol{c}, \boldsymbol{e}, \boldsymbol{q}\}$ such that:
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax $\mathrm{t}^f$ on fossil energy
  - Countries can leave:
    If $j$ exits the agreement, club members $i \in \mathbb{J}$ pay uniform tariffs $\mathrm{t}^b_{ij} = \mathrm{t}^b$ on goods from $j$.
    They still trade with club members in energy at price $q^f$. *Extension 1:* The club $\mathbb{J}$ can also impose a tax $\mathrm{t}^{bf}$ on energy.

  - Exit decision:
    Subcoalition exit: only $\hat{\mathbb{J}}$ stay in the agreement, "Coalitional-Nash" / "Core"

- Participation constraints, indirect utility $U_i(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b) \equiv u(c_{i(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b)})$

$$U_i(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b) \geq U_i(\hat{\mathbb{J}}, \mathrm{t}^f, \mathrm{t}^b) \qquad \forall\, \hat{\mathbb{J}} \subseteq \mathbb{J} \backslash \{i\} \qquad \text{[Coalition-Nash equilibrium]}$$

# Design climate agreements 1 – Tax schedule + stability

- Consider a climate agreement $\{\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b\}$
  - Coalitional Nash eq. (or "core") $\mathbb{C}(\mathrm{t}^f, \mathrm{t}^b)$: robust to deviation of sub-coalitions:
    - No country $i$ would be better off than in the current agreement $\mathbb{J}$
    - note: the "core" $\mathbb{C}(\mathrm{t}^f, \mathrm{t}^b)$ can be empty
- Objective: search for the optimal climate agreement

$$\max_{\mathbb{J},\mathrm{t}^f,\mathrm{t}^b} \mathcal{W}(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b) = \max_{\mathrm{t}^f,\mathrm{t}^b} \max_{\mathbb{J}} \mathcal{W}(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b)$$

$$s.t. \qquad \mathbb{J} \in \mathbb{C}(\mathrm{t}^f, \mathrm{t}^b) = \Big\{ \mathcal{J} \mid U_i(\mathbb{J},\mathrm{t}^f,\mathrm{t}^b) \geq U_i(\hat{\mathbb{J}},\mathrm{t}^f,\mathrm{t}^b) \ \forall i \in \mathcal{J} \ \& \ \forall \ \hat{\mathbb{J}} \subseteq \mathcal{J} \backslash \{i\} \Big\}$$

  - Welfare, for coalition $\mathbb{J}$, weighting all countries $i \in \mathbb{I}$

$$\mathcal{W}(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b) = \sum_{i \in \mathbb{I}} \omega_i \, U_i(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b)$$

  - Current design: (i) choose taxes $\{\mathrm{t}^f, \mathrm{t}^b\}$,
    (ii) choose the coalition $\mathbb{J}$ s.t. participation constraints hold

# Design climate agreements 2 – Coalition-dependent taxes

- Search for an optimal climate agreement $\{\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b\}$

$$\max_{\mathbb{J},\mathfrak{t}^f,\mathfrak{t}^b} \mathcal{W}(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b) = \max_{\mathbb{J}} \max_{\mathfrak{t}^f(\mathbb{J}),\mathfrak{t}^b(\mathbb{J})} \mathcal{W}(\mathbb{J}, \mathfrak{t}^f(\mathbb{J}), \mathfrak{t}^b(\mathbb{J}))$$

$$s.t. \qquad \mathfrak{t}^f(\mathbb{J}), \mathfrak{t}^b(\mathbb{J}) \in \mathbb{C}(\mathbb{J}) = \left\{\mathfrak{t}^f, \mathfrak{t}^b \mid U_i(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b) \geq U_i(\mathbb{J}\backslash\{i\}, \mathfrak{t}^f(\mathbb{J}\backslash\{i\}), \mathfrak{t}^b(\mathbb{J}\backslash\{i\})) \;\; \forall i \in \mathcal{J} \right\}$$

  - Unilateral Nash eq. $\mathbb{C}$: robust to unilateral deviation
  - Welfare, for coalition $\mathbb{J}$, weighting all countries $i \in \mathbb{I}$

$$\mathcal{W}(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b) = \sum_{i\in\mathbb{I}} \omega_i \, U_i(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b)$$

  - Potential design: (i) choose the coalition $\mathbb{J}$
    (ii) choose the policies $\{\mathfrak{t}^f(\mathbb{J}), \mathfrak{t}^b(\mathbb{J})\}$ s.t. participation constraints hold
  - Differences:
    - Approach 1: current implementation (brute force), allow to study the coalition-Nash, computationally intensive
    - Approach 2: more flexible, but have to restrict to unilateral Nash

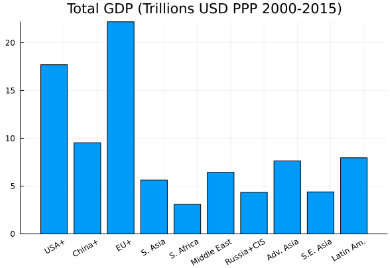
# Quantification

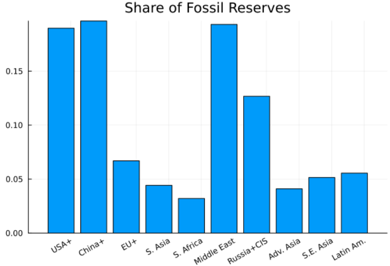
- Quadratic damage as in Nordhaus DICE $y = \mathcal{D}_i(T)\bar{y}$ with $\mathcal{D}_i(T) = e^{-\gamma(T_i - T_{i0})^2}$
- Energy parameters to match production/reserves,
  - Isoelastic cost function $\mathcal{C}_i(e_i^x) = \bar{\nu}_i \left(e_i^x/\mathcal{R}_i\right)^{1+\nu_i}\mathcal{R}_i$
  - Use $\bar{\nu}_i, \nu_i$ to match $e_i^x$ and $\pi_i^f$
- Armington model,
  - Iceberg cost $\tau_{ij}$ projected on distance and preferences $a_{ij}$ to match import shares $s_{ij} = \frac{c_{ij}\tau_{ij}p_j}{c_i\mathbb{P}_i}$
- Production $\bar{y} = zf(\ell_i, k_i, e_i^f, e_i^r)$
  - Nested CES energy vs. labor-capital Cobb-Douglas bundle $k_i^\alpha \ell_i^{1-\alpha}$ (elasticity $\sigma_y < 1$), and fossil/renewable $\sigma_e > 1$, $CES(e_i^f, e_i^r)$
  - TFP, and DTC, $z_i, z_i^e$, calibrated to match GDP / energy shares data.
- Pareto weights $\omega_i$:
  - Imply no redistribution motive, $\bar{c}_i$ consumption in initial equilibrium $t = 2000$

$$\omega_i = \frac{1}{u'(\bar{c}_i)}$$

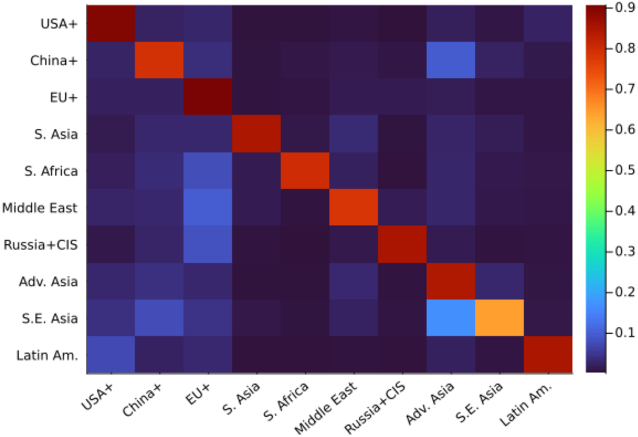
- Details More details

# Numerical Application - Sample of "10 regions"




- Data on trade shares $s_{ij} = \frac{c_{ij}\tau_{ij}\mathrm{p}_j}{c_i\mathbb{P}_i}$, 10 regions, 2015



# Theoretical investigation: decomposing the welfare effects

- Experiment:
  - Start from the equilibrium where carbon tax $\mathrm{t}_j^f = 0, \mathrm{t}_{jk}^b = 0, \forall j,$
  - Change in welfare: Linear approximation around that point $\Rightarrow$ small changes in carbon tax $d\mathrm{t}_j^f, \forall j$ and tariffs $d\mathrm{t}_{j,k}^b, \forall j, k$

$$\frac{d\mathcal{V}_i}{u'(c_i)} = \eta_i^c \frac{d\mathrm{p}_i}{\mathrm{p}_i} + \left[\eta_i^c \gamma_i \frac{1}{\bar{\nu}} - \eta_i^c s_i^e + \eta_i^\pi \left(1 + \frac{1}{\bar{\nu}}\right)\right] \frac{dq^f}{q^f} + \dots$$

  - Difference in the GE effect on energy markets $\frac{dq^f}{q^f} \approx \bar{\nu} \frac{dE^f}{E^f} + \dots$, due to taxation

$$\frac{dq^f}{q^f} = -\sum_j \nu_j^f \frac{d\mathrm{t}_j^f}{\mathrm{t}_j^f} + \sum_i \nu_j^{p,R} \frac{d\mathrm{p}_j}{\mathrm{p}_j} + \sum_{j,k} \nu_j^{R,f,z,qr} s_{j,k} \frac{d\mathrm{t}_{jk}^b}{\mathrm{t}_{jk}^b}$$

    - Trade and leakage effect: GE impact of $\mathrm{t}_j^f$ and $\mathrm{t}_{j,}^b$ on $y_i$ and $\mathrm{p}_i$
    - Simplifying assumption: no renewable

◦ Params: $\sigma$ energy demand elast$^y$, $s^e$ energy cost share, $\bar{\nu}$ energy supply inverse elas$^y$, Climate damage $\gamma_i$

# Decomposing the welfare effects: gains from trade

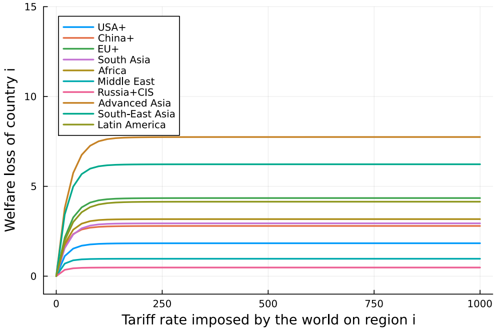
- Start from the equilibrium where carbon tax $t_j^f = 0, t_{jk}^b = 0, \forall j$,
- Change in welfare: Linear approximation around that point $\Rightarrow$ small changes in carbon tax $dt_j^f, \forall j$ and tariffs $dt_{j,k}^b, \forall j, k$

$$\frac{d\mathrm{p}}{\mathrm{p}} = \left[\mathbf{I} - \mathbf{T} - (\theta - 1)\left[\mathbf{T} \odot \mathbf{S} - (\mathbf{T} \odot \mathbf{I})'\right]\right]^{-1} \left((\mathbf{T} - \mathbf{I})\frac{dy}{y} + \left(\mathbf{T}[(\theta-1)\mathbf{I} - \theta\mathbf{S}] \odot \frac{dt^b}{t^b}\right)\mathbb{1}\right)$$

$$\frac{dy_i}{y_i} = \alpha_i^{p,qr}\frac{d\mathrm{p}_i}{\mathrm{p}_i} - \alpha_i^{qf}\frac{dt_i^f}{t_i^f} + \dots$$

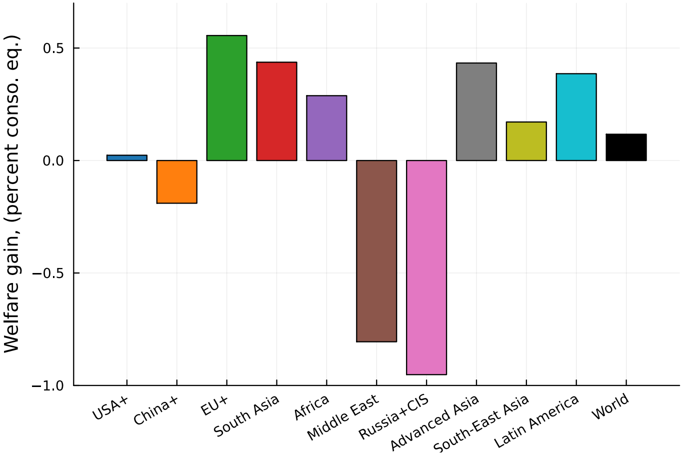
∘ Params: **S** Trade share matrix, **T** income flow matrix, $\theta$, Armington CES

– Loss from trade from large tariffs / autarky:



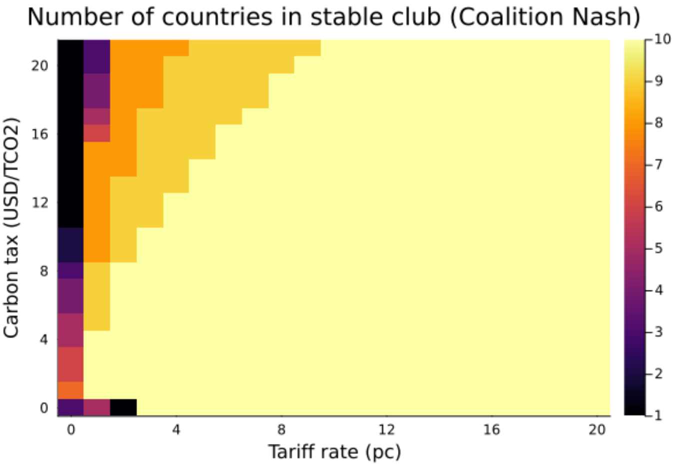
# Gains from cooperation – Second Best

- Optimal carbon tax, Second Best: $\sim \$19/tCO_2$ $(\sim \$64/tC)$
- Reduce fossil fuels / $CO_2$ emissions by 6% compared to Business as Usual (BAU)
- Welfare difference between World Second-Best Policy and BAU (Comp. Eq.)

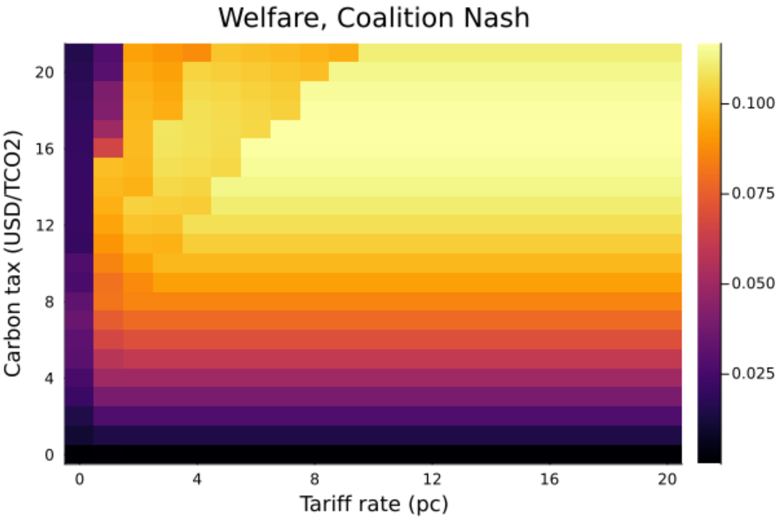


# Optimal coalition

- With this set of $(t^f, t^b) \Rightarrow$ can sustain any coalition
- Result sensitive to Pareto weights



# Taxes combination can recover any climate coalition



# Two extensions: climate agreements, retaliation and lack of commitment

- Consider a climate agreement $\{\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b\}$
  - Coalitional Nash eq. (or "core") $\mathbb{C}(\mathrm{t}^f, \mathrm{t}^b)$: robust to deviation of sub-coalitions

1. Countries outside the club decide on a retaliation trade policy $\mathrm{t}^r$
   – General approach: search for optimal agreement in $\mathbb{I}-\mathbb{J}+1$ players continuous Nash games

$$\max_{\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b} \mathcal{W}(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b, \mathrm{t}^r) \qquad s.t. \qquad \mathbb{J} \in \mathbb{C}(\mathrm{t}^f, \mathrm{t}^b, \mathrm{t}^r)$$

$$\max_{\mathrm{t}^r} \mathcal{V}_i(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b, \mathrm{t}^r) \qquad \forall\, i \in \mathbb{I}\backslash\mathbb{J}$$

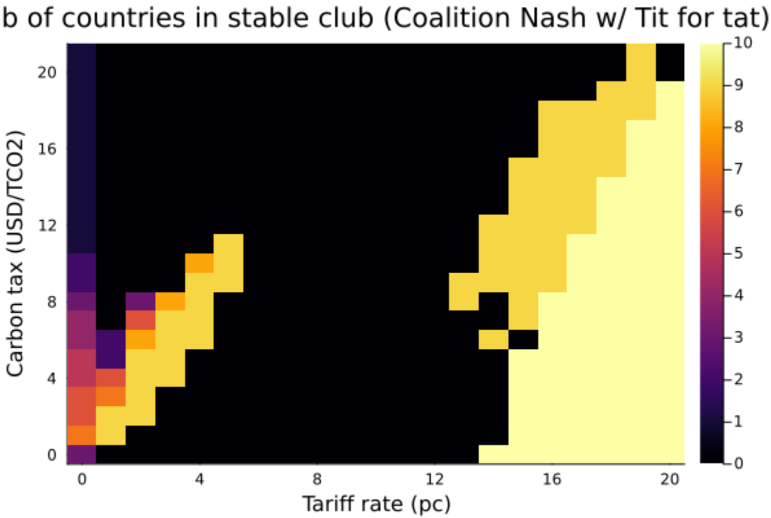
   – Simple experiment: tit-for-tat: $\mathrm{t}^r = \mathrm{t}^b$ equal retaliation

2. Countries within the club deviate from applying a retaliation trade policy $\mathrm{t}^b$
   – Individual / unilateral policy $\tilde{\mathrm{t}}^f_i, \tilde{\mathrm{t}}^b_{ij}$

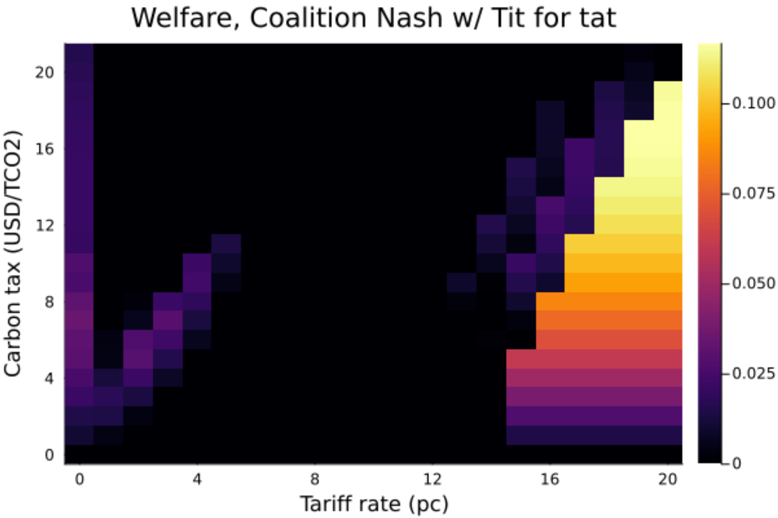
$$\max_{\tilde{\mathrm{t}}^f_i, \tilde{\mathrm{t}}^b_{ij}} \mathcal{V}_i(\tilde{\mathrm{t}}^f_i, \tilde{\mathrm{t}}^b_{ij}, \mathbb{J}, \mathrm{t}^f, \mathrm{t}^b) \qquad \forall\, i \in \mathbb{J}$$

   – Additional participation constraint for the climate agreement: $U_i(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b) \geq \mathcal{V}_i(\tilde{\mathrm{t}}^f_i, \tilde{\mathrm{t}}^b_{ij}, \mathbb{J}, \mathrm{t}^f, \mathrm{t}^b)$

# Retaliation break climate coalition



# Retaliation break climate coalition



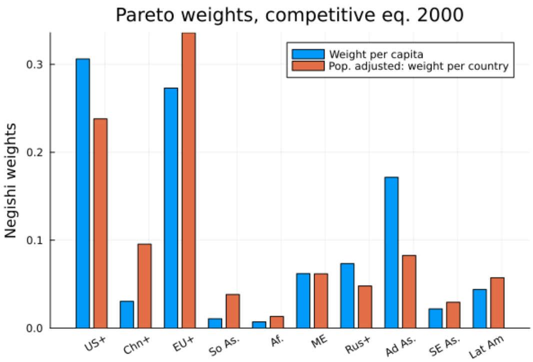
# Conclusion

- In this project, I solve for the optimal climate policy
  - Accounting for inequality as it depends on the availability of transfer mechanisms
  - Redistributing through GE effects on energy and good markets $\Rightarrow$ terms-of-trade effects
- Climate agreement design jointly solves for:
  - The optimal choice of countries participating
  - The carbon tax level, both for correcting externality & respecting participation constraints
- Optimal coalition: strong result, with enough freedom of instruments, can replicate any coalition
  - Positive G.E effet on energy market and large(r) welfare cost of tariffs compared to cost of carbon taxation
- Extensions:
  - More intricate game-theoretical considerations
  - Extend this to dynamic settings: intertemporal tradeoffs

# Appendices

# Welfare and Pareto weights

- Welfare:
$$\mathcal{W}(\mathbb{J}) = \sum_{i \in \mathbb{I}} \omega_i \, u(c_i)$$
- Pareto weights $\omega_i$:
$$\omega_i = \frac{1}{u'(\bar{c}_i)}$$
for $\bar{c}_i$ consumption in initial equilibrium "without climate change", i.e. year $= 2000$
- Imply no redistribution motive in $t = 2000$
$$\omega_i u'(\bar{c}_i) = \omega_j u'(\bar{c}_j) \qquad \forall i, j \ \in \mathbb{I}$$
- Climate change, taxation, and climate agreement (tax + tariffs) have redistributive effects $\Rightarrow$ change distribution of $c_i$.



# Step 0: Competitive equilibrium & Trade

- Each household in country $i$ maximize utility and firms maximize profit
- Standard trade model results:
  - Consumption and trade:

$$s_{ij} = \frac{c_{ij} p_{ij}}{c_i \mathbb{P}_i} = a_{ij} \frac{(\tau_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}(\tau_{ik}(1+\mathrm{t}^b_{ik})\mathrm{p}_k)^{1-\theta}} \qquad \& \qquad \mathbb{P}_i = \Big(\sum_j a_{ij}(\tau_{ij}\mathrm{p}_j)^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

  - Energy consumption doesn't internalize climate damage:

$$\mathrm{p}_i MPe_i = q^e$$

  - Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

  - "Local Social Cost of Carbon", for region $i$

$$LCC_i = \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e^f_i) \frac{\mathrm{p}_i}{\mathbb{P}_i} \qquad (> 0 \text{ if heat causes losses})$$

back

# Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - Full array of instruments: cross-countries lump-sum transfers $\mathrm{t}_i^{ls}$, individual carbon taxes $\mathrm{t}_i^f$ on energy $e_i^f$, bilateral tariffs $\mathrm{t}_{ij}^b$
  - Budget constraint: $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^f e_i^f + \sum_{i,j} \mathrm{t}_{ij}^b c_{ij} \tau_{ij} \mathrm{p}_j$

- Maximize welfare subject to
  - Market clearing for good $[\mu_i]$, market clearing for energy $\mu^e$

back

# Step 1: World First-best policy

- Social planner results:
  - Consumption:

$$\omega_i u'(c_i) = \Big[\sum_j a_{ij}(\tau_{ij}\omega_j\mu_j)^{1-\theta}\Big]^{\frac{1}{1-\theta}}$$

  - Energy use:

$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

  - Social cost of carbon:

$$SCC = -\frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}_j'(T_j)\bar{y}_j}{\frac{1}{I}\sum_j \omega_j \mu_j}$$

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# Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - One single instrument: uniform carbon tax $\mathrm{t}^f$ on energy $e_i^f$
  - Rebate tax lump-sum to HHs $\mathrm{t}_i^{ls} = \mathrm{t}^f e_i^f$

- Ramsey policy: Primal approach, maximize welfare subject to
  - Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
  - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
  - Trade-off faced by the planner:
    - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

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# Step 2: World optimal Ramsey policy

- The planner takes into account
  (i) the marginal value of wealth $\lambda_i$
  (ii) the shadow value of good $i$, from market clearing, $\mu_i$:

$$\text{w/o trade} \qquad \omega_i u'(c_i) = \omega_i \lambda_i$$

$$\text{vs. w/ trade in goods:} \qquad \omega_i u'(c_i) = \Big( \sum_{j \in \mathbb{I}} a_{ij} (\tau_{ij} \mathrm{p}_j)^{1-\theta} \Big[ \omega_i \lambda_i + \omega_j \mu_j + \eta_{ij}(1 - s_{ij}) \Big]^{1-\theta} \Big)^{\frac{1}{1-\theta}}$$

- Relative welfare weights, representing inequality

$$\text{w/o trade:} \qquad \widehat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \lessgtr 1 \qquad \Rightarrow$$

ceteris paribus, poorer countries have higher $\widehat{\lambda}_i$

$$\text{vs. w/ trade:} \qquad \widehat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)} \lessgtr 1$$

# Step 2: Optimal policy – Social Cost of Carbon

- Key objects: Local vs. Global Social Cost of Carbon:
  - Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country $i$
  - "Local social cost of carbon" (LCC) for region $i$:

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e_i^f) \mathrm{p}_i \qquad (> 0 \text{ if heat causes losses})$$

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  - Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W} / \partial \mathcal{E}}{\partial \mathcal{W} / \partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

  - Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i = \sum\nolimits_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, LCC_i\big)$$

## Step 2: Optimal policy – Other motives

- Taxing fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy
- New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \mathcal{C}^f_{EE}\, \mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}_i, e^f_i - e^x_i\Big) \; - \; \mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\Big)$$

∘ Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, $s^f_i$ energy cost share and $\sigma$ energy demand elasticity

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$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \underbrace{\mathcal{C}^f_{EE}}_{\substack{\text{agg. supply}\\ \text{distortion}}} \underbrace{\mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right)}_{\substack{\text{terms-of-trade}\\ \text{redistribution}}} - \underbrace{\mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\right)}_{\substack{\text{demand}\\ \text{distortion}}}$$

◦ Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, $s^f_i$ energy cost share and $\sigma$ energy demand elasticity

# Step 2: Optimal policy – Other motives

- Taking fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
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- New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \mathcal{C}^f_{EE}\, \mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}_i, e^f_i - e^x_i\Big) \;-\; \mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\Big)$$

∘ Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, $s^f_i$ energy cost share and $\sigma$ energy demand elasticity

- *Proposition 2:* Optimal fossil energy tax:

$$\Rightarrow \qquad t^f = SCC + SVF$$

  – Social cost of carbon: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i\, LCC_i$

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# Step 3: Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.
- For a climate "club" of $\mathbb{J} \subset \mathbb{I}$ countries:
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax $\mathrm{t}^f$
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $\mathrm{t}^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff $\mathrm{t}^b$ with club members and countries outside the club. They still trade with the club members in energy at price $q^f$

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  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff $\mathrm{t}^b$ with club members and countries outside the club. They still trade with the club members in energy at price $q^f$
- Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

- Welfare:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{\mathbb{J}} \omega_i\, u(c_i) + \sum_{\mathbb{J}^c} \alpha\omega_i\, u(c_i)$$

# Step 3: Ramsey Problem with participation constraints

- Participation constraints
$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$
- *Proposition 3.1:* Second-Best social valuation with participation constraints
  - Participation incentives change our measure of inequality

$$\text{w/ trade:} \qquad \omega_i(1+\nu_i)u'(c_i) = \Big(\sum_{j\in\mathbb{I}} a_{ij}(\tau_{ij}\mathrm{p}_j)^{1-\theta}\Big[\omega_i\widetilde{\lambda}_i + \omega_j\widetilde{\mu}_j + \widetilde{\eta}_{ij}(1-s_{ij})\Big]^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)} \neq \widehat{\lambda}_i$$

$$\text{vs. w/o trade} \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_j(1+\nu_j)u'(c_j)} \neq \widehat{\lambda}_i$$

  - Similarly, the "effective Pareto weights" are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

# Step 3: Participation constraints & Optimal policy

- *Proposition 3.2:* Second-Best taxes:
  - Taxation with imperfect instruments:
    - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
    - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu\sigma}{q^f(1-s^f)}$
    with $\nu$ fossil supply elasticity, $\sigma$ energy demand elasticity and $s^f$ energy cost share.
  - Optimal fossil energy tax $\mathrm{t}^f(\mathbb{J})$:

$$\Rightarrow \quad \mathrm{t}^f(\mathbb{J}) = SCC + SVF$$

$$= \frac{1}{1-\vartheta_{\mathbb{J}^c}} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i \, LCC_i + \frac{1}{1-\vartheta_{\mathbb{J}^c}} \mathcal{C}^f_{EE} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i (e^f_i - e^x_i) - \sum_{i\in\mathbb{J}} \widetilde{\lambda}_i \frac{q^f(1-s^f_i)}{\sigma}$$

  - Optimal tariffs/export taxes $\mathrm{t}^{b,r}(\mathbb{J})$ and $\mathrm{t}^b(\mathbb{J})$: In search for a closed-form expression
  As of now, only opaque system of equations (fixed point w/ demand/multipliers)

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# Countries' incentives – Model w/o trade in goods

- Experiment: Model with trade in energy but not in "goods"
  - Start from the equilibrium where carbon tax $t^f(\mathbb{J}) = 0$,
    $\Rightarrow$ country $i$ is indifferent to join the club $\mathbb{J}$ or not
  - Linear approximation around that point $\Rightarrow$ small changes in carbon tax $dt^f$

# Countries' incentives – Model w/o trade in goods

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  - Start from the equilibrium where carbon tax $\mathfrak{t}^f(\mathbb{J}) = 0$,
    $\Rightarrow$ country $i$ is indifferent to join the club $\mathbb{J}$ or not
  - Linear approximation around that point $\Rightarrow$ small changes in carbon tax $d\mathfrak{t}^f$
  - Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})} = -e_i d\mathfrak{t}^f - \gamma_i (T_i - T_{i0})^\delta y_i \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \pi_i \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

  - Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}} = -\left(E_{\mathbb{J}} - E_{\mathbb{J}\setminus\{i\}}\right) \frac{\sigma\, d\mathfrak{t}^f}{q^f(1-s^f)} \frac{1}{1 + \frac{\nu\sigma}{(1-s^f)}}$$

○ Params: $\sigma$ energy demand elast$^y$, $s^f$ energy cost share, $\nu$ energy supply elas$^y$, Climate damage $\gamma_i$ and curv. $\delta$

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# Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i d\mathrm{t}^f - \gamma_i(T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$+ \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}}\Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij}\Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}}\Big)$$

# Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i d\mathrm{t}^f - \gamma_i(T_i - T_{i0})^\delta \eta_i^y \Delta_i(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
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  - GE effect on energy markets, same as before
  - GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki}\Big(\frac{dv_k}{v_k} - \theta\frac{d\mathrm{t}_{ki}^b}{1+\mathrm{t}_{ki}^b}\Big) \qquad \alpha_{ki} = \frac{c_{ki}\tau_{ki}\mathrm{p}_i}{\sum_\ell c_{k\ell}\tau_{k\ell}\mathrm{p}_\ell}\frac{v_k}{v_i} = s_{ki}\frac{v_k}{(1+\mathrm{t}_{ki}^b)v_i}$$

    – Params: $\sigma$ energy demand elasticity, $s^f$ energy cost share, $\nu$ energy supply elasticity, share of output $y$ in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

# Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta \, \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \; + \; \eta_i^f \frac{(1+\nu)}{E} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$+ \eta_i^y \Big( \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}} \Big) - s_{i\mathbb{J}^c} dt^b - \sum_{j\in\mathbb{I}} s_{ij} \Big( \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}} \Big)$$

  - GE effect on energy markets, same as before
  - GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki} \Big( \frac{dv_k}{v_k} - \theta \frac{dt_{ki}^b}{1+t_{ki}^b} \Big) \qquad \alpha_{ki} = \frac{c_{ki}\tau_{ki}\mathrm{p}_i}{\sum_\ell c_{k\ell}\tau_{k\ell}\mathrm{p}_\ell} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+t_{ki}^b)v_i}$$

    – Params: $\sigma$ energy demand elasticity, $s^f$ energy cost share, $\nu$ energy supply elasticity, share of output $y$ in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

# Complementarity in coalition formation – Model w/o trade in goods

- Is marginal gain $\Delta\mathcal{W}(\mathbb{J},j) := \mathcal{W}(\mathbb{J}\cup j) - \mathcal{W}(\mathbb{J})$ "growing" in $\mathbb{J}$ ?
  - Linear approximation for small $\{\mathrm{t}^f, \mathrm{t}^b\}$

$$\Delta\mathcal{W}(\mathbb{J},j) = -\omega_j u'(c_j) e_j d\mathrm{t}^f + \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i)\Delta_i \gamma_i (T_i - T_{i0})^\delta y_i\Big] \frac{\sigma\, e_j d\mathrm{t}^f}{q^f(1-s^f+\nu\sigma)}$$
$$+ \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i) e_i\Big] \frac{1}{1+\frac{1-s^f}{\nu\sigma}} \frac{e_j d\mathrm{t}^f}{E_{\mathbb{I}}} - \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i)\pi_i\Big] \frac{(1+\nu)}{E_{\mathbb{I}}} \frac{\sigma\, e_j d\mathrm{t}^f}{q^f(1-s^f+\nu\sigma)}$$

  - Free-riding problem: $\Delta\mathcal{W}(\mathbb{J},j)$ could be negative
  - If $\Delta\mathcal{W}(\mathbb{J},j) > 0$, what effects does $\mathbb{J}$ have on marginal gain?
    - Marginal climate benefit decreases in $\mathbb{J}$, since temperature $T_i$ declines!
    - G.E. effect on energy price: $E_{\mathbb{I}}$, $q$ and $\pi^f$ decreases with $\mathbb{J}$, effect on demand ambiguous
    - Similar formula for the case with trade tariffs: Work in progress.

# Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(T_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \ell, \varepsilon(e^f, e^r)) = \Big[(1-\epsilon_i)^{\frac{1}{\sigma_y}} \left(k^{\alpha}\ell^{1-\alpha}\right)^{\frac{\sigma_y - 1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} \left(z_i^e\ \varepsilon_i(e^f, e^r)\right)^{\frac{\sigma_y - 1}{\sigma_y}}\Big]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \Big[\omega_i^{\frac{1}{\sigma_e}} \left(e^f\right)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\omega_i)^{\frac{1}{\sigma_e}} \left(e^r\right)^{\frac{\sigma_e - 1}{\sigma_e}}\Big]^{\frac{\sigma_e}{\sigma_e - 1}}$$

  - Calibrate TFP $z_i$ to match $y_i = GDP_i$ per capita in 2011 (PPP).
  - Today: $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all $i$
  - Future: $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix $(e_i^f, e_i^r)$

- Damage functions in production function $y$:

$$\mathcal{D}_i^y(T) = e^{-\gamma_i^{\pm,y}(T - T_i^\star)^2}$$

  - Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y}\mathbb{1}_{\{T > T_i^\star\}} + \gamma^{-,y}\mathbb{1}_{\{T < T_i^\star\}}$
  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $T_i^\star = \bar{\alpha} T_{it_0} + (1-\bar{\alpha}) T^\star$

# Quantification – Energy markets

- Fossil production $e^x_{it}$ and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now: $\bar{\nu}_i$ to match extraction data $e^x_i$, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. $\nu_i$ extraction cost curvature to match profit $\pi^f_i = \frac{\bar{\nu}_i \nu_i}{1+\nu_i}(\frac{e^x_i}{\mathcal{R}_i})^{\nu_i}\mathcal{R}_i\mathbb{P}_i$
  - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data $e^x_i$ (BP, IEA)

# Quantification – Energy markets

- Fossil production $e^x_{it}$ and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
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  - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data $e^x_i$ (BP, IEA)

- Renewable: Production $\bar{e}^r_{it}$ and price $q^r_{it}$
  - Now: $q^r_{it} = z^r e^{-g_r t}$, with $g_r$ growth rate in renewable energy price decreases.
  - Future: Choose $z^r_i$ to match the energy mix $(e^f_i, e^r_i)$

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# Quantification – Future Extensions:

- Damage parameters:
  - $\gamma_i^{\pm,y}$ depends on daily temperature distribution $T \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
  - Use Climate Lab's (Greenstone et al) estimates for damage $\gamma_i$ ?
- Fossil Energy markets:
  - Divide fossils $e^f_{it}/e^x_{it}$ into oil/gas/coal
  - Match the production/cost/reserves data across countries
  - Use a dynamic model: extraction/exploration a la Hotelling
- Renewables Energy markets:
  - Make the problem dynamic with investment in capacity $C^r_{it}$
- Population dynamics
  - Match UN forecast for growth rate / fertility

# Calibration

Table: Baseline calibration ($\star$ = subject to future changes)

| | | | |
|---|---|---|---|
| *Technology & Energy markets* | | | |
| $\alpha$ | 0.35 | Capital share in $f(\cdot)$ | Capital/Output ratio |
| $\epsilon$ | 0.12 | Energy share in $f(\cdot)$ | Energy cost share (8.5%) |
| $\sigma$ | 0.3 | Elasticity capital-labor vs. energy | Complementarity in production (c.f. Bourany 2020) |
| $\omega$ | 0.8 | Fossil energy share in $e(\cdot)$ | Fossil/Energy ratio |
| $\sigma_e$ | 2.0 | Elasticity fossil-renewable | Slight substitutability & Study by Stern |
| $\delta$ | 0.06 | Depreciation rate | Investment/Output ratio |
| $\bar{g}$ | $0.01^\star$ | Long run TFP growth | Conservative estimate for growth |
| $g_e$ | $0.01^\star$ | Long run energy directed technical change | Conservative / Acemoglu et al (2012) |
| $g_r$ | $-0.01^\star$ | Long run renewable price decrease | Conservative / Match price fall in R.E. |
| $\nu$ | $2^\star$ | Extraction elasticity of fossil energy | Cubic extraction cost |
| *Preferences & Time horizon* | | | |
| $\rho$ | 0.03 | HH Discount factor | Long term interest rate & usual calib. in IAMs |
| $\eta$ | 2.5 | Risk aversion | |
| $n$ | $0.01^\star$ | Long run population growth | Conservative estimate for growth |
| $\omega_i$ | 1 | Pareto weights | Uniforms / Utilitarian Social Planner |
| $T$ | 90 | Time horizon | Horizon 2100 years since 2010 |

# Calibration

Table: Baseline calibration ($\star$ = subject to future changes)

| | | | |
|---|---|---|---|
| *Climate parameters* | | | |
| $\xi$ | 0.81 | Emission factor | Conversion 1 $MTOE \Rightarrow 1\ MT\ CO_2$ |
| $\zeta$ | 0.3 | Inverse climate persistence / inertia | Sluggishness of temperature $\sim 11-15$ years |
| $\chi$ | $2.1/1e6$ | Climate sensitivity | Pulse experiment: $100\, GtC \equiv 0.21^{\circ}C$ medium-term warming |
| $\delta_s$ | 0.0014 | Carbon exit from atmosphere | Pulse experiment: $100\, GtC \equiv 0.16^{\circ}C$ long-term warming |
| $\gamma^{\oplus}$ | $0.00234^{\star}$ | Damage sensitivity | Nordhaus' DICE |
| $\gamma^{\ominus}$ | $0.2 \times \gamma^{\oplus\ \star}$ | Damage sensitivity | Nordhaus' DICE & Rudik et al (2022) |
| $\alpha^{T}$ | $0.2^{\star}$ | Weight historical climate for optimal temp. | Marginal damage decorrelated with initial temp. |
| $T^{\star}$ | 15.5 | Optimal yearly temperature | Average spring temperature / Developed economies |
| *Parameters calibrated to match data* | | | |
| $p_i$ | | Population | Data – World Bank 2011 |
| $z_i$ | | TFP | To match GDP Data – World Bank 2011 |
| $T_i$ | | Local Temperature | To match temperature of largest city |
| $\mathcal{R}_i$ | | Local Fossil reserves | To match data from BP Energy review |

# Sequential solution method

- Summary of the model:
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, T_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^T_{it}, \lambda^{\mathcal{S}}_t, \lambda^{\mathcal{R}}_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q_t^f, r_t^\star\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)
- Global Numerical solution:
  - Discretize agents (countries) space $i \in \mathbb{I}$ with $M$ and time-space $t \in [t_0, t_T]$ with $T$ periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and $N$ equations with gradient-descent – Newton-Raphson methods.

# Sequential method: Pros and Cons

- Why use a sequential approach?
  - *Global approach*: *Only* need to follow the trajectories for $i$ agents:
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity:
    *Productivity $z_i$ Population $p_i$, Temperature scaling $\Delta_i$, Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$, Local damage $\gamma_i^y, \gamma_i^u, T_i^\star$, Directed Technical Change $z_i^e$*
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables:
    *For now: Wealth $w_{it}$, temperature $T_{it}$, reserves $\mathcal{R}_{it}$, Carbon $\mathcal{S}_t$*
    *Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)*
  - Newton method & Non-linear solvers very efficient

- Why not:
  - Numerical constraint to solve a large system of ODEs and non-linear equations:

  $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either $M$ or $T$ can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

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