

The Inequality of Climate Change and Optimal Energy Policy

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Introduction

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 - (i) energy markets, between importer and exporters
 - (ii) change in climate, benefit from warming vs. catastrophic condition
 - (iii) reallocation of activity through trade, the leakage effect
 - (+) higher income countries not exposed as much as developing economies.

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 - (iii) reallocation of activity through trade, the leakage effect
 - (+) higher income countries not exposed as much as developing economies.
- ▶ As a result, different countries are affected differently by carbon taxation,
 - ⇒ What is the optimal carbon policy in the presence of climate externality and inequality?
 - Optimal taxation design depends crucially on redistribution instruments i.e. lump-sum transfers across countries

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- ▶ What is the optimal carbon policy in the presence of climate externality and inequality?
- ▶ Study an Integrated Assessment Model (IAM) with heterogeneous countries to:
 - Evaluate the welfare costs of global warming (Social Cost of Carbon)
 - Solve for the optimal Ramsey policy for carbon taxation
 - Analyze the strategic implications of joining/designing climate agreements
 - Provide a numerical methodology for this Het. Agents model

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 - Analyze the strategic implications of joining/designing climate agreements
 - Provide a numerical methodology for this Het. Agents model
- ▶ Preview of the results:
 - Social Cost of Carbon need to be adjusted for inequality level
 - Taxation of energy also account for supply and demand elasticity
 - Country-specific taxes: poorer countries will pay relatively lower taxes

Literature

► Climate change & optimal carbon taxation

- RA model: Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
- HA model: Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
- Spatial models: Cruz, Rossi-Hansberg (2022, 2023) among others

⇒ *Optimal and constrained policy with heterogeneous countries & trade*

► Unilateral vs. climate club policies:

- Climate clubs: Nordhaus (2015), Non-cooperative taxation: Chari, Kehoe (1990), Suboptimal policy: Hassler, Krusell, Olovsson (2019)
- Trade policy: Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021), Hsiao (2022), Costinot, Donalson, Vogel, Werning (2015), Adao, Costinot (2022), Antras, Fort, Gutierrez, Tintelnot (2022)

⇒ *Climate cooperation and optimal design of climate club*

► Optimal policy in heterogeneous agents models

- Policy with limited instruments: Diamond (1973), Davila, Walther (2022)
- Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) ...

⇒ *Application to climate and carbon taxation policy*

Model – Household & Firms

- ▶ Static deterministic Neoclassical economy (for today)
 - countries $i \in \mathbb{I}$, heterogeneous in productivity z_i , temperature T_i , energy extraction cost C_i
 - In each country, 3 agents:
 - (i) HtM household, (ii) homogeneous good firm, (iii) (fossil) energy producer

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- ▶ Representative household problem in each country i (passive):

$$\mathcal{V}_i = u(c_i) \qquad c_i = w_i \ell_i + \pi_i^f + t_i^{ls}$$

- Labor income $w_i \ell_i$ from final good firm (labor supply fixed to $\bar{\ell}_i$), profit π_i^f from fossil firm

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- Labor income $w_i \ell_i$ from final good firm (labor supply fixed to $\bar{\ell}_i$), profit π_i^f from fossil firm
- ▶ Competitive homogeneous good producer in country i

$$\max_{e_i^f} \mathcal{D}(T_i) z_i f(e_i^f, \ell_i) - w_i \ell_i - (q^f + t_i^f) e_i^f$$

- Fossil energy demand e_i^f – emitting carbon – subject to price q^f and tax/subsidy t^f .
- Climate externality: effect of temperature on damage/TFP, $\mathcal{D}(T) \in (0, 1)$

Model – Energy markets & Emissions

► Competitive fossil fuels energy producer:

- Supply fossil energy e_i^x by extraction at cost \mathcal{C}_i^f

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x)$$

- Energy traded in international markets, at price q^f

$$E = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

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► Climate system

- Fossil energy e^f releases GHGs in the atmosphere

$$\mathcal{E} = \sum_{\mathbb{I}} e_i^f$$

- Country's local temperature:

$$T_i = \bar{T}_{i0} + \Delta_i \mathcal{E}$$

– Linear model + Climate sensitivity/pattern scaling factor Δ_i

Model – Equilibrium

► Equilibrium

- Given policies $\{t_i^f, t_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_i, e_i^f, e_i^x\}_i$, states $\{T_i\}_i$ and prices $\{q^f, w_i\}_i$ such that:
 - Households choose $\{c_i\}_i$ to max. utility s.t. budget constraint
 - Firm choose policies $\{e_i^f\}_i$ to max. profit
 - Fossil firms extract/produce $\{e_i^x\}_i$ to max. profit.
 - Emissions \mathcal{E}_t affects climate $\{T_i\}_i$.
 - Government budget clear $\sum_i t_i^{ls} = \sum_i t_i^f e_i^f$
 - Prices q^f adjust to clear the markets for energy $\sum_{\mathbb{I}} e_i^x = \sum_{\mathbb{I}} e_i^f$ The good market clearing holds by Walras law

Model – Dynamics & extensions

1. Firm

- Use capital as well to produce
- Use an energy bundle of renewable and fossil energy

2. Energy market

- Renewable energy firm in each country
- Price of clean energy trending down
- Fossil energy extraction/depleting reserves \Rightarrow Hotelling problem

3. Households

- Consumption / saving in bonds / in capital \Rightarrow Keynes-Ramsey rule
- International markets to borrow bonds (in zero net supply)

4. Climate system with inertia / closer to standard IAMs

5. Population growth dynamics (for each country)

6. (Exogenous) growth: TFP change and Energy-augmenting Directed TC.

Optimal world policy – Summary of results

- ▶ **Equilibrium 0:** Competitive equilibrium Details eq 0
 - Passive policies $t^f = 0$, and large cost of climate change
- ▶ **Equilibrium 1:** First-Best, with unlimited instruments Details eq 1
 - Welfare: $\mathcal{W} = \max_{\{t, c, e\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$
 - Social Planner redistribute across countries with lump-sum transfers t_i^{ls}
 - Set the optimal Pigouvian carbon tax to $t^f = SCC$
- ▶ **Equilibrium 2:** Second-best Ramsey policy, with limited instruments Details eq 2
 - Optimal carbon tax accounts for (i) inequality and local climate damage, (ii) energy supply elasticities, (iv) energy demand distortions
- ▶ **Equilibrium 3:** Countries can exit climate agreements Details eq 3
 - All formulas corrected for participation constraints (multipliers affect distribution weights)
 - Optimal design of climate agreement \Rightarrow JMP

Quantification

► Quantification and calibration [More details](#)

- Quadratic damage as in Nordhaus DICE

$$y_i = \mathcal{D}_i(T)\bar{y} \quad \text{with} \quad \mathcal{D}_i(T) = e^{-\gamma_i(T-T_0)^2}$$

- Energy parameters to match production/reserves, Isoelastic cost function

$$\mathcal{C}_i(e_i^x) = \bar{\nu}_i (e_i^x / \mathcal{R}_i)^{1+\nu} \mathcal{R}_i$$

- Cost $\bar{\nu}_i$ and Reserves \mathcal{R}_i to match BP data for production and reserve
- Production $\bar{y} = z^f(\ell_i, k_i, e_i^f, e_i^r)$, labor, capital, fossil, renewable
 - Nested CES energy vs. labor-capital Cobb-Douglas bundle (elasticity $\sigma_y < 1$), and fossil/renewable $\sigma_e > 1$.
 - TFP, and DTC, z_i, z_i^e , calibrated to match GDP / energy shares data.
- Population, from WDI data

Competitive equilibrium

► Key objects:

- Marginal value of wealth $\lambda_i^w = u'(c_i)$
- Marginal value of carbon ψ_i^S for country i
- “Local social cost of carbon” (LCC) for region i :

$$LCC_i := -\frac{\partial \mathcal{V}_i / \partial \mathcal{S}}{\partial \mathcal{V}_i / \partial c_i} = \frac{\psi_i^S}{\lambda_i^w} = -\Delta_i \mathcal{D}'(T_i) z_i^f(e_i^f) > 0$$

- Stationary equilibrium closed-form formula, analogous to GHKT (2014) [Closed Form Solution Here](#)

First-Best, Optimal policy with transfers

- First-Best, Maximizing welfare of the Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{x}, \mathbf{c}, \mathbf{q}\}_i} \sum_{\mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- Full set of instruments $\mathbf{t} = \{\mathbf{t}_i^f, \mathbf{t}_i^{ls}\}$, including transfers *across countries*

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- Full set of instruments $\mathbf{t} = \{\mathbf{t}_i^f, \mathbf{t}_i^{ls}\}$, including transfers *across countries*
- Key objects: Local vs. Global Social Cost of Carbon,

$$SCC^{fb} := -\frac{\partial \mathcal{W} / \partial \mathcal{S}}{\partial \mathcal{W} / \partial \bar{c}} = \frac{\psi_t^S}{\lambda_t^w} = \frac{\sum_{\mathbb{I}} \psi_i^S}{\frac{1}{I} \sum_{\mathbb{I}} \lambda_i^w}$$

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{S}}{\partial \mathcal{W}_i / \partial c_i} = \frac{\psi_i^S}{\lambda_i^w}$$

First-Best, Optimal policy with transfers

- Proposition 1: Optimal carbon tax:

$$\mathfrak{t}^f = SCC^{fb}$$

- Result as in Representative Agent economy, c.f. Nordhaus DICE (1996), GHKT (2014)

$$SCC^{fb} = \frac{\psi^S}{\lambda^w} = - \sum_{\mathbb{I}} \frac{\psi_i^S}{\lambda_i^w} = \sum_{\mathbb{I}} LCC_i$$

- Lump-sum transfers redistribute across countries, s.t.

$$\omega_i u'(c_i) = \lambda_i^w = \bar{\lambda}^w = \lambda_j^w = \omega_j u'(c_j) \quad \forall i, j \in \mathbb{I}$$

- Imply cross-countries lump-sum transfers $\exists i \text{ s.t. } \mathfrak{t}_i^{ls} \geq 0$ or $\exists j \text{ s.t. } \mathfrak{t}_j^{ls} \leq 0$

Ramsey policy with limited transfers

- Second best without access to lump-sum transfers: choice of a carbon tax $\{t^f, t^r\}$

- Tax receipts redistributed lump-sum: $t_i^{ls} = t^f e_i^f$
- Inequality across regions:

$$\hat{\lambda}_i^w = \frac{\omega_i \lambda_i^w}{\bar{\lambda}^w} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \leq 1$$

⇒ ceteris paribus, poorer countries have higher $\hat{\lambda}_i^w$

- Social Cost of Carbon integrates these inequalities:

$$SCC^{sb} = \sum_{\mathbb{I}} \hat{\lambda}_i^w LCC_i$$

$$SCC^{sb} = \sum_{\mathbb{I}} LCC_i + \text{Cov}_i(\hat{\lambda}_i^w, LCC_i)$$

Ramsey Problem – Optimal Carbon and Energy Policy

- Taxing fossil energy has additional redistributive effects:
 1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
 2. Distort energy demand, of countries that need more or less energy

$$\text{Supply Distortion}^{sb} + \text{Demand Distortion}^{sb} = \mathcal{C}_{EE}^f \text{Cov}_i \left(\hat{\lambda}_i, e_i^f - e_i^x \right) - \text{Cov}_i \left(\hat{v}_i, \frac{q^f (1-s_i^f)}{\sigma e_i} \right)$$

- Params: \mathcal{C}_{EE}^f agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

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$$\text{Supply Distortion}^{sb} + \text{Demand Distortion}^{sb} = \underbrace{\mathcal{C}_{EE}^f}_{\text{agg. supply distortion}} \underbrace{\text{Cov}_i(\hat{\lambda}_i, e_i^f - e_i^x)}_{\text{terms-of-trade redistribution}} - \underbrace{\text{Cov}_i\left(\hat{v}_i, \frac{q^f(1-s_i^f)}{\sigma_{e_i}}\right)}_{\text{demand distortion}}$$

◦ Params: \mathcal{C}_{EE}^f agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

► Proposition 2: Optimal fossil energy tax:

$$\Rightarrow \quad \mathfrak{t}^f = \text{SCC}^{sb} + \text{Supply Distortion}^{sb} + \text{Demand Distortion}^{sb}$$

– Social cost of carbon: $\text{SCC}^{sb} = \sum_{\mathbb{I}} \hat{\lambda}_i^w \text{LCC}_i$

Step 2: Ramsey Problem – Country-specific energy tax

- ▶ Suppose the planner has access to a *distribution* of carbon price.
- ▶ Proposition 3: Optimal country-specific fossil energy tax:

$$\Rightarrow \quad \mathfrak{t}^f = \frac{1}{\widehat{\lambda}_i^w} [\textcolor{green}{SCC}^{sb} + \textcolor{brown}{Supply Distortion}^{sb}]$$

– Social cost of carbon: $\textcolor{green}{SCC}^{sb} = \sum_{\mathbb{I}} \widehat{\lambda}_i^w LCC_i$

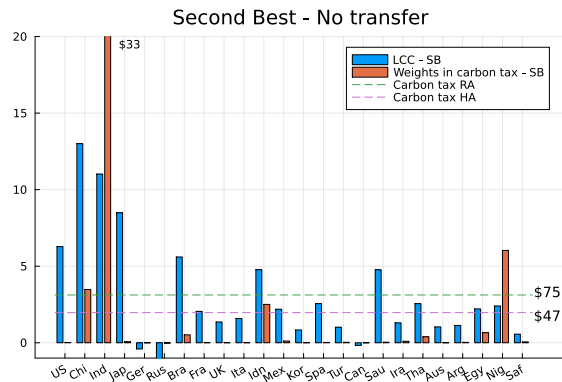
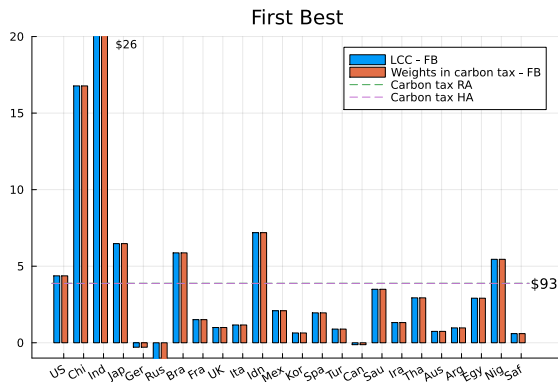
⇒ Reduce the tax burden for poorer/more “valuable” countries

Step 2: Ramsey Problem – Extensions

- ▶ Trade block à la Armington Details eq 2
 - Additional trade-off/distortion on goods important for the trade network
- ▶ Dynamic consideration (in the paper)
 - Valuation of reserves (Hotelling rent), carbon tax serves as an instrument for intertemporal substitution of fossil production
 - ▶ Heal, Schlenker (2019), Cruz, Rossi-Hansberg (2022)
 - Curb capital demand and distort consumption/saving decision, c.f. H.A. models

Local Cost of Carbon & Carbon Tax – First and Second Best

► Difference $LCC_i = \frac{\psi_i^S}{\lambda_i^w}$ vs. $\hat{\lambda}_i^w LCC_i = \frac{\psi_i^S}{\lambda^w}$ since $SCC = \sum_{\mathbb{I}} \hat{\lambda}_i^w LCC_i$



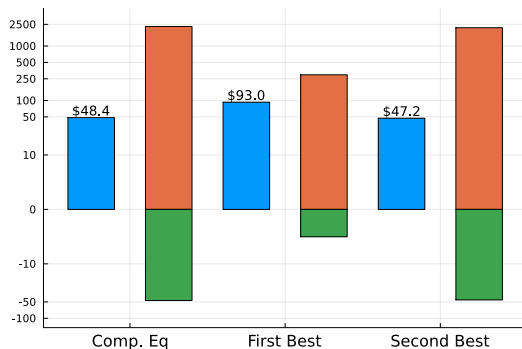
Comparison - Value of wealth vs. Social Cost of Carbon

- Social Cost of Carbon can be decomposed:

$$SCC := -\frac{\partial \mathcal{W} / \partial \mathcal{S}}{\partial \mathcal{W} / \partial c} = \frac{\psi^S}{\bar{\lambda}^w} = \frac{\sum_{\mathbb{I}} \psi_i^S}{\frac{1}{I} \sum_{\mathbb{I}} \lambda_i^w}$$

- Here plot that decomposition:

$$\log(SCC_t) = \log(\psi^S) - \log(\lambda^w)$$



Local Cost of Fossil and Terms of Trade Adjustment

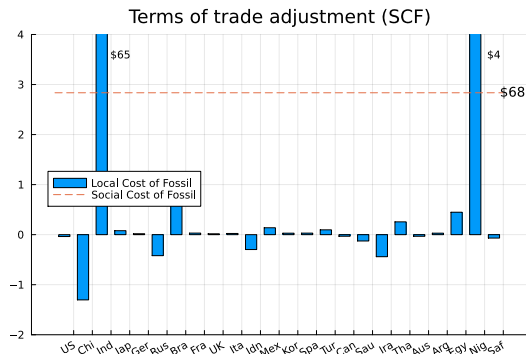
- Social Cost of Fossil Energy:

$$SCF_t = c_{EE}^f \sum_{\mathbb{I}} \hat{\lambda}_{it}^w (e_{it}^f - e_{it}^x)$$

$$c_{EE}^{f-1} = \sum_{\mathbb{I}} c_{i,e^x}^{f-1} = \nu \frac{q^f}{E}$$

- Here plotting local cost of fossil:

$$LCF_{it} = \hat{\lambda}_{it}^w (e_{it}^f - e_{it}^x)$$



Conclusion

- ▶ In this project, I solve for the optimal climate policy
 - Accounting for inequality as it depends on the availability of transfer mechanisms
 - Redistributing through GE effects on energy and good markets \Rightarrow terms-of-trade effects
 - Additional trade-related and dynamic motives co-funded in energy taxation

 - ▶ Incentives and implementability
 - What if some countries deviate from apply the appropriate energy tax?
 - Game theoretical consideration due to participation constraints
 - Implementation of a “climate club”: penalty tariffs for non-participants crucial for enforcing carbon policy
- \Rightarrow Job Market Paper: “The Optimal Design of Climate Agreements”

Appendices

Step 0: Competitive equilibrium & Trade

- ▶ Each household in country i maximize utility and firms maximize profit
- ▶ Standard trade model results:
 - Consumption and trade:

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i p_i} = a_{ij} \frac{(d_{ij}(1+t_{ij}^b)p_j)^{1-\theta}}{\sum_k a_{ik}(d_{ik}(1+t_{ik}^b)p_k)^{1-\theta}} \quad \& \quad p_i = \left(\sum_j a_{ij}(d_{ij}p_j)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Energy consumption doesn't internalize climate damage:

$$p_i M P e_i = q^e$$

- Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

- “Local Social Cost of Carbon”, for region i

$$LCC_i = \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_{if}(e_i^f) \frac{p_i}{p_i} \quad (> 0 \text{ if heat causes losses})$$

Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- Full array of instruments: cross-countries lump-sum transfers \mathbf{t}_i^{ls} , individual carbon taxes \mathbf{t}_i^f on energy \mathbf{e}_i^f , bilateral tariffs \mathbf{t}_{ij}^b
 - Budget constraint: $\sum_i \mathbf{t}_i^{ls} = \sum_i \mathbf{t}_i^f \mathbf{e}_i^f + \sum_{i,j} \mathbf{t}_{ij}^b c_{ij} d_{ij} \mathbf{p}_j$
- Maximize welfare subject to
- Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1: World First-best policy

► Social planner results:

- Consumption:

$$\omega_i u'(c_i) = \left[\sum_j a_{ij} (d_{ij} \omega_j \mu_j)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

- Energy use:

$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

- Social cost of carbon:

$$SCC = - \frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}'_j(T_j) \bar{y}_j}{\frac{1}{I} \sum_j \omega_j \mu_j}$$

back

Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- One single instrument: uniform carbon tax \mathfrak{t}^f on energy e_i^f
 - Rebate tax lump-sum to HHs $\mathfrak{t}_i^{ls} = \mathfrak{t}^f e_i^f$
- Ramsey policy: Primal approach, maximize welfare subject to
- Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
 - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
 - Trade-off faced by the planner:
 - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

back

Step 2: World optimal Ramsey policy

- The planner takes into account

- (i) the marginal value of wealth λ_i
- (ii) the shadow value of good i , from market clearing, μ_i :

w/o trade $\omega_i u'(c_i) = \omega_i \lambda_i$

vs. w/ trade in goods: $\omega_i u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij} (d_{ij} p_j)^{1-\theta} \left[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij} (1 - s_{ij}) \right] \right)^{\frac{1}{1-\theta}}$

- Relative welfare weights, representing inequality

w/o trade: $\hat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \leq 1 \quad \Rightarrow \quad \text{ceteris paribus, poorer countries have higher } \hat{\lambda}_i$

vs. w/ trade: $\hat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)} \leq 1$

Step 2: Optimal policy – Social Cost of Carbon

► Key objects: Local vs. Global Social Cost of Carbon:

- Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i
- “Local social cost of carbon” (LCC) for region i :

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_{if}(e_i^f) p_i \quad (> 0 \text{ if heat causes losses})$$

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- Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W} / \partial \mathcal{E}}{\partial \mathcal{W} / \partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

- Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{Cov}_i(\hat{\lambda}_i, LCC_i)$$

Step 2: Optimal policy – Other motives

- ▶ Taxing fossil energy has additional redistributive effects:
 1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
 2. Distort energy demand, of countries that need more or less energy
- ▶ New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W} / \partial E}{\partial \mathcal{W} / \partial w} = \mathcal{C}_{EE}^f \mathbb{Cov}_i \left(\hat{\lambda}_i, e_i^f - e_i^x \right) - \mathbb{Cov}_i \left(\hat{\lambda}_i, \frac{q^f (1 - s_i^f)}{\sigma} \right)$$

- Params: \mathcal{C}_{EE}^f agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

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 2. Distort energy demand, of countries that need more or less energy
- ▶ New measure: Social Value of Fossil (SVF)

$$\textcolor{red}{SVF} := \frac{\partial \mathcal{W} / \partial E}{\partial \mathcal{W} / \partial w} = \mathcal{C}_{EE}^f \mathbb{Cov}_i \left(\hat{\lambda}_i, e_i^f - e_i^x \right) - \mathbb{Cov}_i \left(\hat{\lambda}_i, \frac{q^f (1 - s_i^f)}{\sigma} \right)$$

- Params: \mathcal{C}_{EE}^f agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity
- ▶ Proposition 2: Optimal fossil energy tax:

$$\Rightarrow \quad t^f = \textcolor{green}{SCC} + \textcolor{red}{SVF}$$

- Social cost of carbon: $\textcolor{green}{SCC} = \sum_{\mathbb{I}} \hat{\lambda}_i \textcolor{blue}{LCC}_i$

Step 3: Ramsey Problem with participation constraints

- ▶ Consider that countries can “exit” climate agreement.
- ▶ For a climate “club” of $\mathbb{J} \subset \mathbb{I}$ countries:
 - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f
 - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $t^{b,r}$ on goods and get consumption \tilde{c}_i
 - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f

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- ▶ Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- ▶ Welfare:

$$\mathcal{W} = \max_{\{t, e, q\}_i} \sum_{\mathbb{J}} \omega_i u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i u(c_i)$$

Step 3: Ramsey Problem with participation constraints

► Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

► Proposition 3.1: Second-Best social valuation with participation constraints

- Participation incentives change our measure of inequality

$$\text{w/ trade:} \quad \omega_i(1+\nu_i)u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij}(d_{ij}p_j)^{1-\theta} \left[\omega_i \tilde{\lambda}_i + \omega_j \tilde{\mu}_j + \tilde{\eta}_{ij}(1-s_{ij}) \right] \right)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(\tilde{\lambda}_i + \tilde{\mu}_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(\tilde{\lambda}_i + \tilde{\mu}_i)} \neq \hat{\lambda}_i$$

$$\text{vs. w/o trade} \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(1+\nu_j)u'(c_j)} \neq \hat{\lambda}_i$$

- Similarly, the “effective Pareto weights” are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

Step 3: Participation constraints & Optimal policy

► Proposition 3.2: Second-Best taxes:

- Taxation with imperfect instruments:
 - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
 - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu \sigma}{q^f(1-s^f)}$
with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
- Optimal fossil energy tax $t^f(\mathbb{J})$:

$$\begin{aligned} \Rightarrow \quad t^f(\mathbb{J}) &= \text{SCC} + \text{SVF} \\ &= \frac{1}{1 - \vartheta_{\mathbb{J}^c}} \sum_{i \in \mathbb{I}} \tilde{\lambda}_i \text{LCC}_i + \frac{1}{1 - \vartheta_{\mathbb{J}^c}} C_{EE}^f \sum_{i \in \mathbb{I}} \tilde{\lambda}_i (e_i^f - e_i^x) - \sum_{i \in \mathbb{J}} \tilde{\lambda}_i \frac{q^f(1-s_i^f)}{\sigma} \end{aligned}$$

- Optimal tariffs/export taxes $t^{b,r}(\mathbb{J})$ and $t^b(\mathbb{J})$: In search for a closed-form expression
As of now, only opaque system of equations (fixed point w/ demand/multipliers)

Sequential solution method

► Summary of the dynamic model:

- ODEs for states $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^\tau, \lambda_t^S, \lambda_{it}^{\mathcal{R}}\}_{it}$
- Non-linear equations (FOCs) for household controls $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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► Global Numerical solution:

- Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
- Express as a large vector $\mathbf{y} = \{\mathbf{x}, \boldsymbol{\lambda}, \mathbf{c}, \mathbf{q}\}$ in a large non-linear function

$$F(\mathbf{y}) = \mathbf{0}$$

- Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Impact of increase in temperature

- Marginal values of the climate variables: λ_{it}^s and λ_{it}^τ

$$\dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho + \zeta) + \overbrace{\gamma_i(\tau_{it} - \tau_i^*) \mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it}, e_{it}) \lambda_{it}^k + \overbrace{\phi_i(\tau_{it} - \tau_i^*) \mathcal{D}^u(\tau_{it})}^{\partial_\tau u(c, \tau)} c_{it}^{1-\eta}$$

$$\dot{\lambda}_{it}^s = \lambda_{it}^s(\rho + \delta^s) - \zeta \chi \Delta_i \lambda_{it}^\tau$$

- Costate λ_{it}^s : marg. cost of 1Mt carbon in atmosphere, for country i . Increases with:
- Temperature gaps $\tau_{it} - \tau_i^*$ & damage sensitivity of TFP γ_i^y and utility γ_i^u
 - Development level $f(k_{it}, e_{it})$ and c_{it}
 - Climate params: χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
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Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature \Leftrightarrow solving ODE

$$\dot{\lambda}_{it}^{\tau} = \lambda_t^{\tau}(\tilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^*)\mathcal{D}^y(\tau)f(k, e)\lambda_t^k + \phi(\tau - \tau^*)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}_t^S = \lambda_t^S(\tilde{\rho} + \delta^S) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda_{it}^{\tau}$$

- Solving for λ_t^{τ} and λ_t^S , in stationary equilibrium $\dot{\lambda}_t^S = \dot{\lambda}_t^{\tau} = 0$

$$\lambda_{it}^{\tau} = - \int_t^{\infty} e^{-(\tilde{\rho} + \zeta)u} (\tau_u - \tau^*) \left(\gamma \mathcal{D}^y(\tau_u) y_{\tau} \lambda_u^k + \phi \mathcal{D}^u(\tau_u) u(c_u) \right) du$$

$$\lambda_{it}^{\tau} = - \frac{1}{\tilde{\rho} + \Delta\zeta} (\tau_{\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{\infty}) y_{\infty} \lambda_{\infty}^k + \phi \mathcal{D}^u(\tau_{\infty}) u(c_{\infty}) \right)$$

$$\lambda_t^S = - \int_t^{\infty} e^{-(\tilde{\rho} + \delta^S)u} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,u}^{\tau} dj du$$

$$= \frac{1}{\tilde{\rho} + \delta^S} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,\infty}^{\tau}$$

$$= - \frac{\chi}{\tilde{\rho} + \delta^S} \frac{\zeta}{\tilde{\rho} + \zeta} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{j,\infty}) y_{\infty} \lambda_{j,\infty}^k + \phi \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

$$\lambda_t^S \xrightarrow{\zeta \rightarrow \infty} - \frac{\chi}{\tilde{\rho} + \delta^S} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{j,\infty}) y_{j,\infty} \lambda_{j,\infty}^k + \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

Cost of carbon / Marginal value of temperature

► *Proposition (Stationary LSCC):*

When $t \rightarrow \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \rightarrow \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ , **marg. damage** γ_i^y , γ_i^u , **temperature**, and **output, consumption**.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^*) \left(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \right)$$

- Stationary equilibrium: $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
- Fast temperature adjustment $\zeta \rightarrow \infty$
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