

The Optimal Design of Climate Agreements

Inequality, Trade, and Incentives for carbon policy

WORK IN PROGRESS

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Introduction

- ▶ Fighting climate change requires implementing ambitious carbon reduction policies
 - However, one of the sources of international inaction is the “free-riding problem”
 - Implementing taxation of carbon and fossil fuels is costly for individual countries
 - Moreover, such climate policy redistributes across countries through
 - (i) energy markets (ii) change in climate, and (iii) reallocation of activity through trade
 - ▶ One proposal to solve climate inaction is the idea of “climate club” – Nordhaus (2015)
 - Climate coalitions taxing carbon are inherently “unstable”
 - Trade sanctions need to be imposed on non-participants to sustain a “club” and reduce emissions meaningfully
- ⇒ How can we design an optimal climate agreement that implements the optimal energy taxation in the presence of inequality and policy constraints?

Introduction – this project

- ▶ Trade-off between intensive margin and extensive margin:
 - Agreement with a small set of countries, high tax, large emissions reductions for members
 - Extensive agreements with a large number of countries but lower optimal tax to accommodate participation constraints
- Trade tariffs/sanctions for non-members are crucial for the stability of the agreement

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- ▶ Build an Integrated Assessment Model (IAM) with heterogeneous countries & trade to:
 - Evaluate the welfare costs of global warming and solve optimal carbon policy
 - Analyze the strategic implications of joining climate agreements
 - Design the optimal size of the climate club

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- ▶ Preview of the results:
 - For unilateral deviation – i.e. Nash-equilibrium climate agreement – the world optimal policy is sustainable: high carbon tax, high tariffs, participation of the entire world
 - Optimal agreements robust to “subcoalition deviation” depend on the trade-off between (i) gains from trade, (ii) climate damage and (iii) distortionary effects of the carbon tax

Literature

► Climate change & optimal carbon taxation

- RA model: Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
- HA model: Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
- Spatial models: Cruz, Rossi-Hansberg (2022, 2023) among others

⇒ *Optimal and constrained policy with heterogeneous countries & trade*

► Unilateral vs. climate club policies:

- Climate clubs: Nordhaus (2015), Non-cooperative taxation: Chari, Kehoe (1990), Suboptimal policy: Hassler, Krusell, Olovsson (2019)
- Trade policy: Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021), Hsiao (2022), Costinot, Donalson, Vogel, Werning (2015), Adao, Costinot (2022), Antras, Fort, Gutierrez, Tintelnot (2022)

⇒ *Climate cooperation and optimal design of climate club*

► Optimal policy in heterogeneous agents models

- Policy with limited instruments: Diamond (1973), Davila, Walther (2022)
- Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) ...

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Literature

► Nordhaus (2015)

- Examine "stable climate coalitions" (club imposing carbon tax) in a simple model
- Abstract from General Equilibrium and distributional effects
- Results: Penalty tariffs necessary to enforce a climate club

► Farrokhi, Lashkaripour (2021)

- Study and characterize the optimal trade policy with climate externality
- General static trade model. Results: unilateral tariffs not effective
- Sequential search for one stable climate club if EU or US join.

► Main contribution:

- Search for the *optimal* climate agreement
- GE on good and energy market and redistribution effects are first-order
- Cost of climate change is endogenous to policy (damages are non-linear)
- Possibility of analyzing other distributional policies (transfers, *loss and damage funds*)
- General framework for analyzing macrodynamics

Model – Household

- ▶ Deterministic Neoclassical economy, static (for today)
 - countries $i \in \mathbb{I}$, heterogeneous in productivity z_i , temperature T_i , energy extraction cost \mathcal{C}_i
 - In each country, 3 agents:
 - (i) Household, (ii) final good firm, (iii) (fossil) energy producer

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1. Representative household problem in each country i :

$$\mathcal{V}_i = \max_{c_{ij}} u(c_i) \quad \mathbb{P}_i c_i = w_i + \pi_i^f + t_i^{ls} \quad c_i = \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

$$c_{ij} \quad \text{at price} \quad \overbrace{(1+t_{ij}^b)}^{\text{tariff}} \overbrace{\tau_{ij}}^{\text{iceberg cost}} p_j$$

$$c_i \quad \text{at price} \quad \mathbb{P}_i = \left(\sum_j a_{ij} [(1+t_{ij}^b) \tau_{ij} p_j]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Labor income w_i from final good firm (labor normalized to 1), profit π_i^f from fossil firm

Model – Firms

2. Competitive final good producer in country i

$$\max_{e_i^f} p_i \mathcal{D}(T_i) z_i f(e_i^f) - w_i - (q^f + t_i^f) e_i^f$$

- Productivity / TFP residual z_i \Rightarrow creates inequalities across countries
- Fossil energy demand per unit of labor e_i^f , emitting carbon
Fossil energy price q^f and tax/subsidy t^f
- Climate externality on temperature T_i
 - Damage affect productivity: $\mathcal{D}(T) \in (0, 1]$

Model – Energy markets & Emissions

► Competitive fossil fuels energy producer:

- Supply fossil energy e_i^x by extraction at cost $\mathcal{C}_i^f(e_i^x)$

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x) \mathbb{P}_i$$

- Energy traded in international markets, at price q^f

$$E = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

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► Climate system

- Fossil energy e^f releases GHGs in the atmosphere

$$\mathcal{E} = \sum_{\mathbb{I}} e_i^f$$

- Country i 's local temperature:

$$T_i = \bar{T}_{i0} + \Delta_i \mathcal{E}$$

- Linear model + Climate sensitivity/pattern scaling factor Δ_i

Model – Equilibrium

- Given policies $\{t_i^f, t_{ij}^b, t_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e_i^f, e_i^x\}_{ij}$, states $\{T_i\}_i$ and prices $\{p_i\}_i, q^f$ such that:
 - Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint
 - Firm choose inputs $\{e_i^f\}_i$ to max. profit
 - Fossil firms extract/produce $\{e_i^x\}_i$ to max. profit.
 - Emissions \mathcal{E} affects climate $\{T_i\}_i$.
 - Government budget clear $\sum_i t_i^{ls} = \sum_i t_i^f e_i^f + \sum_{i,j} t_{ij}^b c_{ij} \tau_{ij} p_j$
 - Prices $\{p_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$ and for each good

$$y_i := \mathcal{D}(T_i) z_i f(e_i^f) = \sum_{k \in \mathbb{I}} \tau_{ki} c_{ki} + \sum_{k \in \mathbb{I}} g_{ki}$$

with g_{ki} net export of good i to pay for energy in k

In expenditure, with import shares $s_{ij} = \frac{c_{ij} \tau_{ij} p_j}{c_i p_i}$, it yields $p_i y_i = \sum_{k \in \mathbb{I}} \frac{s_{ki}}{1 + t_{ki}^b} (p_k y_k + \tilde{t}_k^{ls})$

Model – Extensions & Dynamics

- ▶ Quantitative model (today)
 - Use an energy bundle of fossil and renewable energy
 - Renewable energy input price constant (for now)
 - Use capital as well to produce at interest $r = \rho$ (BGP)
- ▶ Dynamics, extensions:
 1. Energy market:
 - Fossil energy extraction/exploration reserves \Rightarrow Hotelling problem
 2. Households
 - Consumption / saving in bonds and in capital \Rightarrow Euler equation, Keynes-Ramsey rule
 - International markets to borrow/lend bonds
 3. Climate system with inertia: standard IAMs
 4. Population dynamics
 5. (Exogenous) growth: TFP change and Energy-augmenting Directed TC.

Benchmark: Optimal world policy – Summary of results

- ▶ Consider a social planner maximizing world's welfare:

- Choose a single carbon tax τ^f for the world

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{c}, \mathbf{e}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- ▶ Summary:

- Optimal climate policy depends on the availability of redistribution instruments (i.e. lump-sum transfers t_i^{ls} across countries)
- Without redistribution motives, optimal Pigouvian carbon tax: $\tau^f = SCC$
- Otherwise, optimal carbon tax should account for
(i) inequality and local climate damage, (ii) energy supply elasticities, (iii) energy terms-of-trade redistribution effects, (iv) energy demand distortions

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- ▶ Details:

- **Competitive equilibrium** Details eq 0
 - **First-Best**, with unlimited instruments Details eq 1
 - **Second-best**, Ramsey policy with limited instruments Details eq 2

Ramsey Problem with participation constraints

- ▶ Consider that countries can “exit” climate agreement.
- ▶ **Definition** A climate agreement is a set $\{\mathbb{J}, t^f, t^b\}$, with $\mathbb{J} \subseteq \mathbb{I}$ countries and a C.E. $\{c, e, q\}$ such that:
 - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f on fossil energy
 - Countries can leave:
 If j exits the agreement, club members $i \in \mathbb{J}$ pay uniform tariffs $t_{ij}^b = t^b$ on goods from j .
 They still trade with club members in energy at price q^f . They obtain \hat{c}_j
 - Exit decision:
 Unilateral vs. subcoalition exit, s.t. only $\hat{\mathbb{J}}$ stay in the agreement: concept of “Core”

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Unilateral vs. subcoalition exit, s.t. only $\hat{\mathbb{J}}$ stay in the agreement: concept of “Core”
- ▶ Participation constraints:

$$u(c_i) \geq u(\hat{c}_i(\hat{\mathbb{J}})) \quad [\nu_i] \quad \forall \hat{\mathbb{J}} \subseteq \mathbb{J} \setminus \{i\}$$

Stable climate agreements

- ▶ Consider a climate agreement $\{\mathbb{J}, t^f, t^b\}$
 - It is a Nash equilibrium if it is stable to unilateral deviation, $\hat{\mathbb{J}} = \mathbb{J} \setminus \{i\}$
 - It belongs to “core” $\mathbb{C}(t^f, t^b)$ if it robust to deviation of sub-coalitions $\hat{\mathbb{J}} \subseteq \mathbb{J} \setminus \{i\}$
 - i.e. no subcoalition would be better off than in the current agreement \mathbb{J}
 - note: the “core” $\mathbb{C}(t^f, t^b)$ can be empty
- ▶ Objective: search for the optimal climate agreement such that:

$$\max_{\mathbb{J}, t^f, t^b} \mathcal{W}(\mathbb{J}, t^f, t^b) \quad s.t. \quad \mathbb{J} \in \mathbb{C}(t^f, t^b)$$

- ▶ Welfare, for coalition \mathbb{J} , weighting all countries $i \in \mathbb{I}$

$$\mathcal{W}(\mathbb{J}, t^f, t^b) = \sum_{i \in \mathbb{I}} \omega_i u(c_i) \quad s.t. \quad \{c, e, q\}_i \text{ is a C.E. for agreement } \{\mathbb{J}, t^f, t^b\}$$

Approach: joint solution of policy and coalition

- Choosing policy $\{t^f, t^b\}$ at the same time as the set of countries \mathbb{J}
 - From one agreement $\{\mathbb{J}, t^f, t^b\}$, one can deduce the set $\tilde{\mathbb{J}}$ of countries with binding participation constraints

$$\tilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\hat{c}_i) \quad \forall i \in \tilde{\mathbb{J}} \quad \& \quad \forall \hat{\mathbb{J}} \subseteq \mathbb{J} \setminus \{i\}$$

- Search for the couple $\{t^f, t^b\}$ such that $\mathbb{J} = \tilde{\mathbb{J}}$
 - Difficult (but feasible)
 - Can be extended to dynamic settings:
choose a path of $\{t_t^f, t_t^b\}$ instead of a path of combinations of clubs
- Heuristics for an algorithm:
 - Start from the world agreement $\{\mathbb{I}, t^{f*}, t^{b*}\}$
 - For $\{\mathbb{J}, t^f, t^b\}$, deduce the country with binding participation constraints $\tilde{\mathbb{J}} = f(\mathbb{J}, t^f, t^b)$
 - Search for a fixed point $\mathbb{J} = f(\mathbb{J}, t^f, t^b)$
- Today: Brute force solution with a small number of countries

Leverage the combinatorial discrete choice literature

- ▶ Trade Literature on sourcing decisions:
 - Combinatorial discrete choice , c.f. Antras, Fort, Tintelnot (2017), Jia (2008), **Arkolakis, Eckert, Shi (2023)**, Alfaro-Ureña, Castro-Vincenzi, Fanelli, Morales (2024)

- ▶ Search for complementarity, given **one** policy $\{t^f, t^b\}$

- Key concept: Single crossing difference in choice - from below (SCD-C):

$$\Delta \mathcal{W}(\mathbb{J}', j) := \mathcal{W}(\mathbb{J}' \cup j) - \mathcal{W}(\mathbb{J}') > \Delta \mathcal{W}(\mathbb{J}, j) \quad \text{when } \mathbb{J}' \supseteq \mathbb{J} \quad \text{for all } j \in \mathbb{I}$$

- Adding an extra member j is increasingly profitable with the size of the club \mathbb{J}
 - Can use a “squeezing procedure” to bypass the combinatorial problem

- ▶ Search optimal combination for **all** policies $\{t^f, t^b\}$?

- Single crossing difference in type (SCD-T):

$$\text{Define } \Lambda_i(\mathbb{J}) = \{t^f, t^b \in \mathbb{R}_+^2 \mid \Delta \mathcal{W}(\mathbb{J}, i) > 0\}$$

- Welfare function has SCD-T if $\Lambda_i(\mathbb{J})$ is a connected set

Quantification

- ▶ Quadratic damage as in Nordhaus DICE $y = \mathcal{D}_i(T)\bar{y}$ with $\mathcal{D}_i(T) = e^{-\gamma_i(T-T_{i0})^2}$
- ▶ Energy parameters to match production/reserves,
Isoelastic cost function $\mathcal{C}_i(e_i^x) = \bar{\nu}_i(e_i^x/\mathcal{R}_i)^{1+\nu}\mathcal{R}_i$
- ▶ Armington model, distance τ_{ij} and preferences a_{ij} to match import shares $s_{ij} = \frac{c_{ij}\tau_{ij}p_j}{c_i\mathbb{P}_i}$
- ▶ Production $\bar{y} = zf(\ell_i, k_i, e_i^f, e_i^r)$
 - Nested CES energy vs. labor-capital Cobb-Douglas bundle $k_i^\alpha \ell_i^{1-\alpha}$ (elasticity $\sigma_y < 1$), and fossil/renewable $\sigma_e > 1$, $CES(e_i^f, e_i^r)$
 - TFP, and DTC, z_i, z_i^e , calibrated to match GDP / energy shares data.
- ▶ Details [More details](#)

Welfare and Pareto weights

- Recall welfare:

$$\mathcal{W}(\mathbb{J}) = \sum_{i \in \mathbb{I}} \omega_i u(c_i)$$

- Choose Pareto weights ω_i per country such that:

$$\omega_i = \frac{1}{u'(c_i)}$$

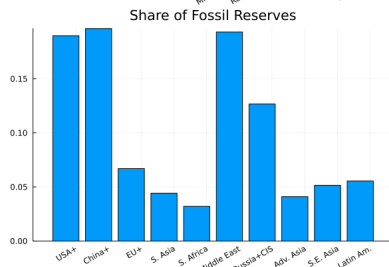
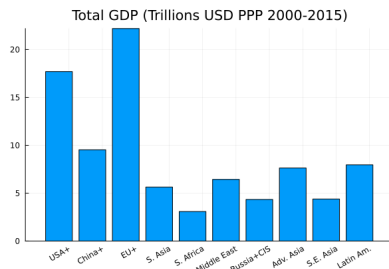
for c_i consumption in initial equilibrium “without climate change“, i.e. year = 2000

- Imply that there is no redistribution motive (in $t = 2000$)

$$\omega_i u'(c_i) = \omega_j u'(c_j) \quad \forall i, j \in \mathbb{I}$$

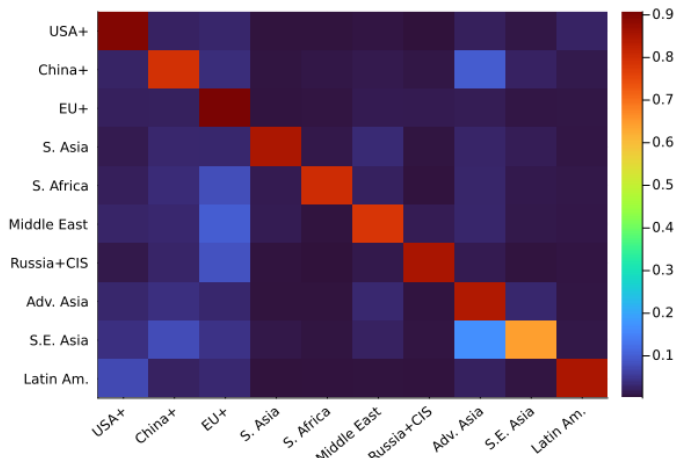
- Climate change, carbon taxation and climate agreement (tax and tariffs) have redistributive effects \Rightarrow will change distribution of c_i .

Numerical Application - Sample of “10 regions”



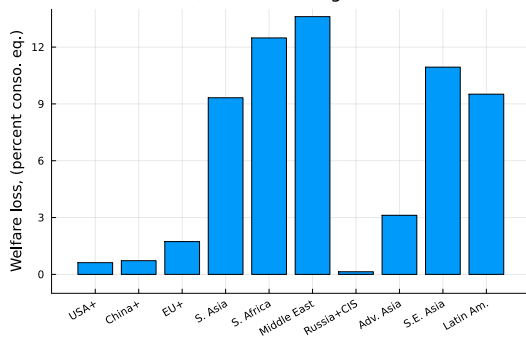
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- Data on trade shares $s_{ij} = \frac{c_{ij}T_{ij}P_j}{c_iP_i}$, 10 regions, Average 2000-2015

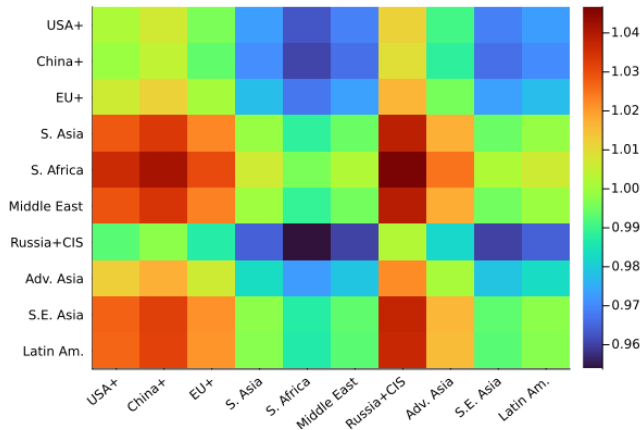


Cost of Climate Change

Welfare loss, Climate change, 2050 vs. 2000



► Trade reallocation, change in shares s_{ij} due to climate change



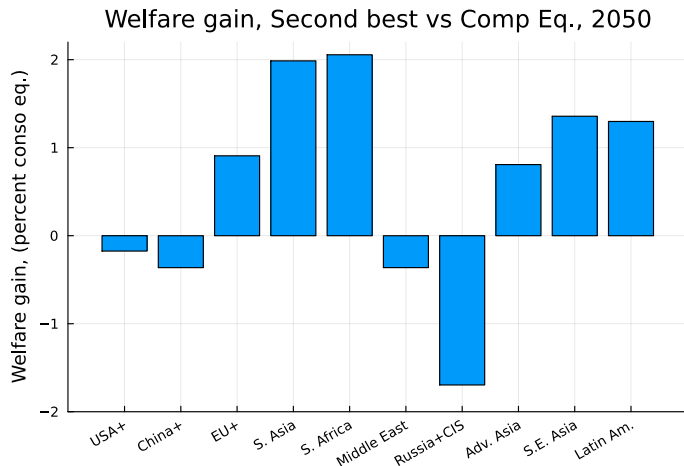
Gains from trade

- ▶ Static problem: Calibration subject to changes
- ▶ Losses from increasing trade tariffs:
- ▶ Losses from autarky

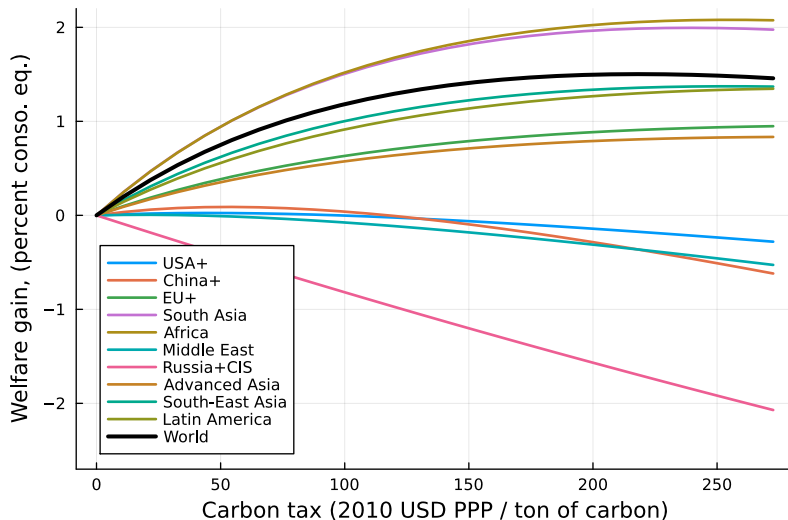
Picture of gains from trade per country here

Gains from cooperation – Second Best

- ▶ Static problem: Calibration subject to changes
- ▶ Optimal carbon tax, Second Best: $\sim \$215/tC$ ($\sim \$880/tCO_2$)
- ▶ Reduce fossil fuels / CO_2 emissions by 24% compared to Business as Usual (BAU)
- ▶ Welfare difference between World Second-Best Policy and BAU (Comp. Eq.)

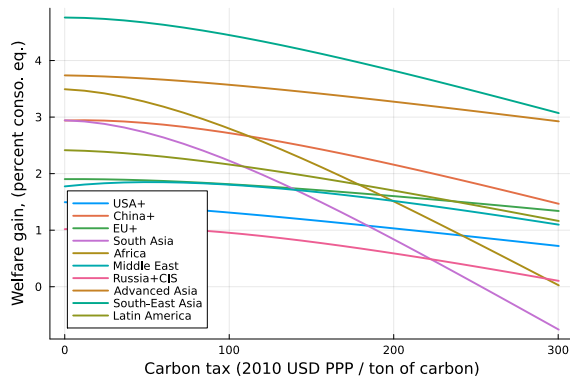
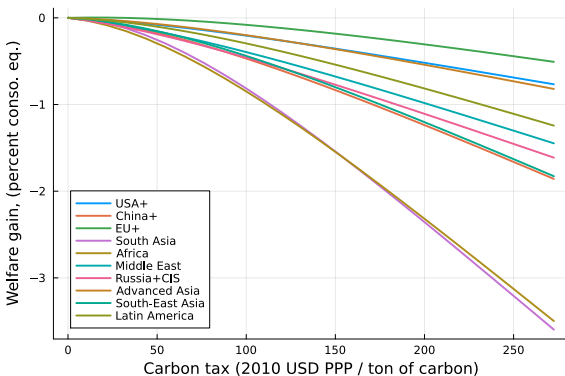


Gains from cooperation – Second Best – Tax variation



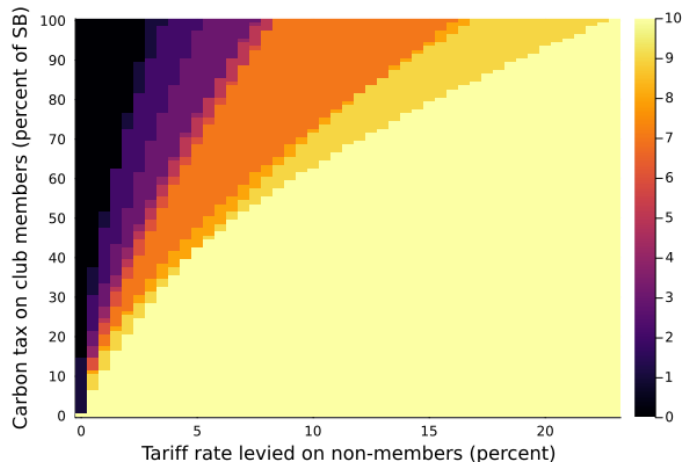
Free riding problem ... solved with trade tariffs

- Welfare difference between World Second-Best Policy and Unilateral deviation
 - Recover Nordhaus (2015) result



Taxes combination can recover any climate coalition

- ▶ Choice of any couples $(t^f, t^b) \in \mathbb{R}_+^2$ allow to enforce any coalitions (any number of countries)
 - ▶ Trade penalties change the country's outside options, ruling out unilateral deviations
- ⇒ One can reproduce the second-best: full-cooperation, high-tax and maximum welfare



Nash equilibrium: conclusion – Optimal agreement is the Second-Best

- ▶ With flexible t^f, t^b , the "agreement designer" can reproduce the world's optimal policy
 - Carbon taxation corrects externality, accounting for terms-of-trade / redistribution effects
 - Trade tariffs serve as penalties to enforce the stability of the club

Nash equilibrium: conclusion – Optimal agreement is the Second-Best

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 - Carbon taxation corrects externality, accounting for terms-of-trade / redistribution effects
 - Trade tariffs serve as penalties to enforce the stability of the club
- ▶ Same mechanisms with conditional transfers (loss and damages funds – COP27)
 - Ensure stability (change one side of the participation constraint)
 - How to decide/rationalize who "deserves" the funds? the poorest? the most vulnerable? or the countries with the highest outside options?
 - Coase type of arguments: harder to bargain on I -instruments (c.f. Weitzman 2014)
- ▶ Obvious conclusion?
 - With enough instruments, easy to reach full coordination
 - In practice, coordination failure to implement binding agreements

Conclusion

- ▶ In this project, I solve for the optimal climate policy
 - Accounting for inequality as it depends on the availability of transfer mechanisms
 - Redistributing through GE effects on energy and good markets \Rightarrow terms-of-trade effects
- ▶ Climate agreement design jointly solves for:
 - The optimal choice of countries participating
 - The carbon tax level, both for correcting externality & respecting participation constraints
- ▶ Can reproduce any coalition with arbitrary trade tariffs or conditional transfers
 - Can achieve the second-best, world climate agreement and largest emission reductions
 - Objective to extend this to dynamic settings where the tradeoffs are less obvious/more realistic.

Appendices

Step 0: Competitive equilibrium & Trade

- ▶ Each household in country i maximize utility and firms maximize profit
- ▶ Standard trade model results:
 - Consumption and trade:

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i \mathbb{P}_i} = a_{ij} \frac{(\tau_{ij}(1+t_{ij}^b)p_j)^{1-\theta}}{\sum_k a_{ik}(\tau_{ik}(1+t_{ik}^b)p_k)^{1-\theta}} \quad \& \quad \mathbb{P}_i = \left(\sum_j a_{ij}(\tau_{ij}p_j)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Energy consumption doesn't internalize climate damage:

$$p_i MPe_i = q^e$$

- Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

- “Local Social Cost of Carbon”, for region i

$$LCC_i = \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_{if}(e_i^f) \frac{p_i}{\mathbb{P}_i} \quad (> 0 \text{ if heat causes losses})$$

Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- Full array of instruments: cross-countries lump-sum transfers \mathbf{t}_i^{ls} , individual carbon taxes \mathbf{t}_i^f on energy \mathbf{e}_i^f , bilateral tariffs \mathbf{t}_{ij}^b
 - Budget constraint: $\sum_i \mathbf{t}_i^{ls} = \sum_i \mathbf{t}_i^f \mathbf{e}_i^f + \sum_{i,j} \mathbf{t}_{ij}^b c_{ij} \tau_{ij} \mathbf{p}_j$
- Maximize welfare subject to
- Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1: World First-best policy

► Social planner results:

- Consumption:

$$\omega_i u'(c_i) = \left[\sum_j a_{ij} (\tau_{ij} \omega_j \mu_j)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

- Energy use:

$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

- Social cost of carbon:

$$SCC = - \frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}'_j(T_j) \bar{y}_j}{\frac{1}{I} \sum_j \omega_j \mu_j}$$

back

Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- One single instrument: uniform carbon tax \mathfrak{t}^f on energy e_i^f
 - Rebate tax lump-sum to HHs $\mathfrak{t}_i^{ls} = \mathfrak{t}^f e_i^f$
- Ramsey policy: Primal approach, maximize welfare subject to
- Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
 - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
 - Trade-off faced by the planner:
 - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

back

Step 2: World optimal Ramsey policy

- The planner takes into account

- (i) the marginal value of wealth λ_i
- (ii) the shadow value of good i , from market clearing, μ_i :

$$\text{w/o trade} \quad \omega_i u'(c_i) = \omega_i \lambda_i$$

$$\text{vs. w/ trade in goods:} \quad \omega_i u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij} (\tau_{ij} p_j)^{1-\theta} \left[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij} (1 - s_{ij}) \right] \right)^{\frac{1}{1-\theta}}$$

- Relative welfare weights, representing inequality

$$\text{w/o trade:} \quad \hat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{j \in \mathbb{I}} \omega_j u'(c_j)} \leq 1 \quad \Rightarrow \quad \text{ceteris paribus, poorer countries have higher } \hat{\lambda}_i$$

$$\text{vs. w/ trade:} \quad \hat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{j \in \mathbb{I}} \omega_j (\lambda_j + \mu_j)} \leq 1$$

Step 2: Optimal policy – Social Cost of Carbon

► Key objects: Local vs. Global Social Cost of Carbon:

- Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i
- “Local social cost of carbon” (LCC) for region i :

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_{if}(e_i^f) p_i \quad (> 0 \text{ if heat causes losses})$$

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- Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W} / \partial \mathcal{E}}{\partial \mathcal{W} / \partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

- Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{Cov}_i(\hat{\lambda}_i, LCC_i)$$

Step 2: Optimal policy – Other motives

- ▶ Taxing fossil energy has additional redistributive effects:
 1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
 2. Distort energy demand, of countries that need more or less energy
- ▶ New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W} / \partial E}{\partial \mathcal{W} / \partial w} = \mathcal{C}_{EE}^f \mathbb{Cov}_i \left(\hat{\lambda}_i, e_i^f - e_i^x \right) - \mathbb{Cov}_i \left(\hat{\lambda}_i, \frac{q^f (1 - s_i^f)}{\sigma} \right)$$

- Params: \mathcal{C}_{EE}^f agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

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$$SVF := \frac{\partial \mathcal{W} / \partial E}{\partial \mathcal{W} / \partial w} = \underbrace{C_{EE}^f}_{\text{agg. supply distortion}} \underbrace{\text{Cov}_i(\hat{\lambda}_i, e_i^f - e_i^x)}_{\text{terms-of-trade redistribution}} - \underbrace{\text{Cov}_i\left(\hat{\lambda}_i, \frac{q^f(1-s_i^f)}{\sigma}\right)}_{\text{demand distortion}}$$

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$$\textcolor{red}{SVF} := \frac{\partial \mathcal{W} / \partial E}{\partial \mathcal{W} / \partial w} = \mathcal{C}_{EE}^f \mathbb{Cov}_i \left(\hat{\lambda}_i, e_i^f - e_i^x \right) - \mathbb{Cov}_i \left(\hat{\lambda}_i, \frac{q^f (1 - s_i^f)}{\sigma} \right)$$

- Params: \mathcal{C}_{EE}^f agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity
- ▶ Proposition 2: Optimal fossil energy tax:

$$\Rightarrow \quad t^f = \textcolor{green}{SCC} + \textcolor{red}{SVF}$$

- Social cost of carbon: $\textcolor{green}{SCC} = \sum_{\mathbb{I}} \hat{\lambda}_i \textcolor{blue}{LCC}_i$

Step 3: Ramsey Problem with participation constraints

- ▶ Consider that countries can “exit” climate agreement.
- ▶ For a climate “club” of $\mathbb{J} \subset \mathbb{I}$ countries:
 - Countries $i \in \mathbb{J}$ are subject to a carbon tax τ^f
 - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $\tau^{b,r}$ on goods and get consumption \tilde{c}_i
 - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff τ^b with club members and countries outside the club. They still trade with the club members in energy at price q^f

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 - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f
- ▶ Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- ▶ Welfare:

$$\mathcal{W} = \max_{\{t, e, q\}_i} \sum_{\mathbb{J}} \omega_i u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i u(c_i)$$

Step 3: Ramsey Problem with participation constraints

► Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

► Proposition 3.1: Second-Best social valuation with participation constraints

- Participation incentives change our measure of inequality

$$\text{w/ trade:} \quad \omega_i(1+\nu_i)u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij}(\tau_{ij}p_j)^{1-\theta} \left[\omega_i \tilde{\lambda}_i + \omega_j \tilde{\mu}_j + \tilde{\eta}_{ij}(1-s_{ij}) \right] \right)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(\tilde{\lambda}_i + \tilde{\mu}_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(\tilde{\lambda}_i + \tilde{\mu}_i)} \neq \hat{\lambda}_i$$

$$\text{vs. w/o trade} \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(1+\nu_j)u'(c_j)} \neq \hat{\lambda}_i$$

- Similarly, the “effective Pareto weights” are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

Step 3: Participation constraints & Optimal policy

► Proposition 3.2: Second-Best taxes:

- Taxation with imperfect instruments:
 - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
 - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu \sigma}{q^f (1-s^f)}$
with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
- Optimal fossil energy tax $t^f(\mathbb{J})$:

$$\begin{aligned} \Rightarrow \quad t^f(\mathbb{J}) &= \text{SCC} + \text{SVF} \\ &= \frac{1}{1 - \vartheta_{\mathbb{J}^c}} \sum_{i \in \mathbb{I}} \tilde{\lambda}_i \text{LCC}_i + \frac{1}{1 - \vartheta_{\mathbb{J}^c}} C_{EE}^f \sum_{i \in \mathbb{I}} \tilde{\lambda}_i (e_i^f - e_i^x) - \sum_{i \in \mathbb{J}} \tilde{\lambda}_i \frac{q^f (1-s_i^f)}{\sigma} \end{aligned}$$

- Optimal tariffs/export taxes $t^{b,r}(\mathbb{J})$ and $t^b(\mathbb{J})$: In search for a closed-form expression
As of now, only opaque system of equations (fixed point w/ demand/multipliers)

Countries' incentives – Model w/o trade in goods

- ▶ Experiment: Model with trade in energy but not in “goods”
 - Start from the equilibrium where carbon tax $\tau^f(\mathbb{J}) = 0$,
 \Rightarrow country i is indifferent to join the club \mathbb{J} or not
 - Linear approximation around that point \Rightarrow small changes in carbon tax $d\tau^f$

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 - Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})} = & -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta y_i \Delta_i (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \pi_i \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \end{aligned}$$

- Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}} = - \left(E_{\mathbb{J}} - E_{\mathbb{J} \setminus \{i\}} \right) \frac{\sigma dt^f}{q^f (1 - s^f)} \frac{1}{1 + \frac{\nu \sigma}{(1 - s^f)}}$$

- Params: σ energy demand elast^y, s^f energy cost share, ν energy supply elas^y, Climate damage γ_i and curv. δ

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Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$
 - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})c_i} = & -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & + \eta_i^y \left(\frac{dp_i}{p_i} \Big|_{i \in \mathbb{J}} - \frac{dp_i}{p_i} \Big|_{i \notin \mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j \in \mathbb{I}} s_{ij} \left(\frac{dp_j}{p_j} \Big|_{i \in \mathbb{J}} - \frac{dp_j}{p_j} \Big|_{i \notin \mathbb{J}} \right) \end{aligned}$$

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- GE effect on energy markets, same as before
- GE effect on goods markets, equilibrium on expenditure $v_i = p_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k \in \mathbb{I}} \mathcal{P}_k \alpha_{ki} \left(\frac{dv_k}{v_k} - \theta \frac{dt_{ki}^b}{1+t_{ki}^b} \right) \quad \alpha_{ki} = \frac{c_{ki} \tau_{ki} p_i}{\sum_{\ell} c_{k\ell} \tau_{k\ell} p_{\ell}} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+t_{ki}^b) v_i}$$

- Params: σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i p_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

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Complementarity in coalition formation – Model w/o trade in goods

- Is marginal gain $\Delta\mathcal{W}(\mathbb{J}, j) := \mathcal{W}(\mathbb{J} \cup j) - \mathcal{W}(\mathbb{J})$ “growing” in \mathbb{J} ?
- Linear approximation for small $\{t^f, t^b\}$

$$\begin{aligned} \Delta\mathcal{W}(\mathbb{J}, j) = & -\omega_j u'(c_j) e_j dt^f + \left[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) \Delta_i \gamma_i (T_i - T_{i0})^\delta y_i \right] \frac{\sigma e_j dt^f}{q^f (1 - s^f + \nu \sigma)} \\ & + \left[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) e_i \right] \frac{1}{1 + \frac{1-s^f}{\nu \sigma}} \frac{e_j dt^f}{E_{\mathbb{I}}} - \left[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) \pi_i \right] \frac{(1+\nu)}{E_{\mathbb{I}}} \frac{\sigma e_j dt^f}{q^f (1 - s^f + \nu \sigma)} \end{aligned}$$

- Free-riding problem: $\Delta\mathcal{W}(\mathbb{J}, j)$ could be negative
- If $\Delta\mathcal{W}(\mathbb{J}, j) > 0$, what effects does \mathbb{J} have on marginal gain?
 - Marginal climate benefit decreases in \mathbb{J} , since temperature T_i declines!
 - G.E. effect on energy price: $E_{\mathbb{I}}$, q and π^f decreases with \mathbb{J} , effect on demand ambiguous
 - Similar formula for the case with trade tariffs: Work in progress.

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(T_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \ell, \varepsilon(e^f, e^r)) = \left[(1 - \epsilon_i)^{\frac{1}{\sigma_y}} (k^\alpha \ell^{1-\alpha})^{\frac{\sigma_y-1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} (z_i^e \varepsilon_i(e^f, e^r))^{\frac{\sigma_y-1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y-1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega_i^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e-1}{\sigma_e}} + (1 - \omega_i)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e-1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e-1}}$$

- Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
- Today: $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
- Future: $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

- Damage functions in production function y :

$$\mathcal{D}_i^y(T) = e^{-\gamma_i^{\pm,y}(T-T_i^*)^2}$$

- Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y} \mathbb{1}_{\{T > T_i^*\}} + \gamma^{-,y} \mathbb{1}_{\{T < T_i^*\}}$
- Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $T_i^* = \bar{\alpha} T_{it0} + (1 - \bar{\alpha}) T^*$

Quantification – Energy markets

- ▶ Fossil production e_{it}^x and reserve \mathcal{R}_{it}
 - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i} \left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i} \mathcal{R}$
 - Now: $\bar{\nu}_i$ to match extraction data e_i^x , \mathcal{R}_{it} calibrated to *proven reserves* data from BP.
 $\nu_i = \nu = 1$ quadratic extraction cost.
 - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost \mathcal{C}_e & extraction data e_i^x (BP, IEA)

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- ▶ Renewable: Production \bar{e}_{it}^r and price q_{it}^r
 - Now: $q_{it}^r = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
 - Future: Choose z_i^r to match the energy mix (e_i^f, e_i^r)

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Quantification – Future Extensions:

► Damage parameters:

- $\gamma_i^{\pm,y}$ depends on daily temperature distribution $T \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
- Use Climate Lab's (Greenstone et al) estimates for damage γ_i ?

► Fossil Energy markets:

- Divide fossils e_{it}^f / e_{it}^x into oil/gas/coal
- Match the production/cost/reserves data across countries
- Use a dynamic model: extraction/exploration a la Hotelling

► Renewables Energy markets:

- Make the problem dynamic with investment in capacity C_{it}^r

► Population dynamics

- Match UN forecast for growth rate / fertility

Calibration

Table: Baseline calibration (★ = subject to future changes)

<i>Technology & Energy markets</i>			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
\bar{g}	0.01★	Long run TFP growth	Conservative estimate for growth
g_e	0.01★	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	-0.01★	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	2★	Extraction elasticity of fossil energy	Cubic extraction cost
<i>Preferences & Time horizon</i>			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	0.01★	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

Table: Baseline calibration (★ = subject to future changes)

<i>Climate parameters</i>			
ξ	0.81	Emission factor	Conversion 1 <i>MTOE</i> \Rightarrow 1 <i>MT CO₂</i>
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature \sim 11–15 years
χ	2.1/1e6	Climate sensitivity	Pulse experiment: 100 <i>GtC</i> \equiv 0.21°C medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment: 100 <i>GtC</i> \equiv 0.16°C long-term warming
γ^{\oplus}	0.00234★	Damage sensitivity	Nordhaus' DICE
γ^{\ominus}	$0.2 \times \gamma^{\oplus}$ ★	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^T	0.2★	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
T^{\star}	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
<i>Parameters calibrated to match data</i>			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
T_i		Local Temperature	To match temperature of largest city
\mathcal{R}_i		Local Fossil reserves	To match data from BP Energy review

Sequential solution method

► Summary of the model:

- ODEs for states $\{\mathbf{x}\} = \{w_{it}, T_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^T, \lambda_t^S, \lambda_{it}^{\mathcal{R}}\}_{it}$
- Non-linear equations (FOCs) for household controls $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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► Global Numerical solution:

- Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
- Express as a large vector $\mathbf{y} = \{\mathbf{x}, \boldsymbol{\lambda}, \mathbf{c}, \mathbf{q}\}$ in a large non-linear function

$$F(\mathbf{y}) = \mathbf{0}$$

- Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method: Pros and Cons

► Why use a sequential approach?

- *Global approach: Only need to follow the trajectories for i agents:*
 - Arbitrary (!) number of dimension of *ex-ante* heterogeneity:
*Productivity z_i Population p_i , Temperature scaling Δ_i , Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$,
 Local damage $\gamma_i^y, \gamma_i^u, T_i^*$, Directed Technical Change z_i^e*
 - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables:
For now: Wealth w_{it} , temperature T_{it} , reserves \mathcal{R}_{it} , Carbon S_t
Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
 - Newton method & Non-linear solvers very efficient

► Why not:

- Numerical constraint to solve a large system of ODEs and non-linear equations:
 ⇒ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
- Relying on numerical solvers/structure of the problem can be opaque

back