The Optimal Design of Climate Agreements

Inequality, Trade, and Incentives for Carbon Policy

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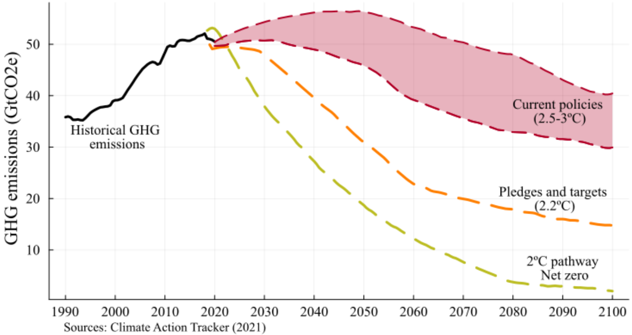
Motivation

- Fighting climate change requires implementing ambitious carbon reduction policies



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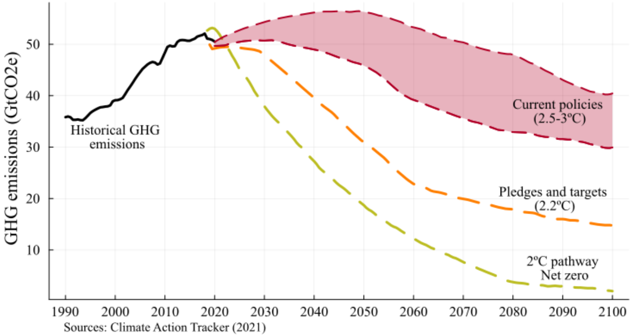

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  - The "free-riding problem" causes climate inaction
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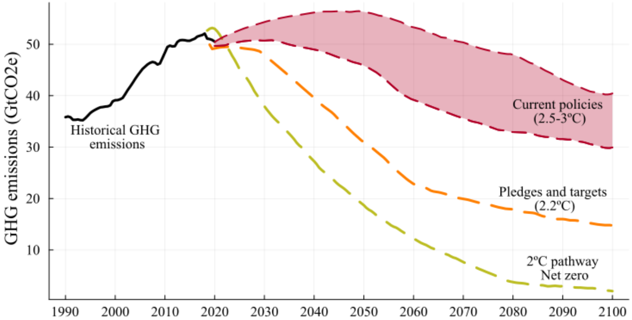
  - Climate policy redistributes across countries through:
    (i) change in climate (ii) energy markets, and (iii) reallocation of activity through trade



Sources: Climate Action Tracker (2021)

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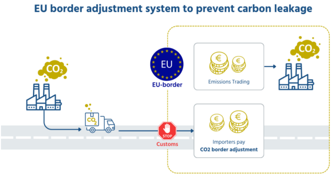
  - Carbon Border Adjustment mechanisms (CBAM), EU policy: carbon tariffs



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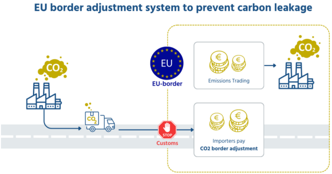

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▶ Preview of the results:

- Despite complete freedom of policy instruments, impossible to achieve the world's optimal policy with complete participation
- Beneficial to leave several fossil fuels producing countries outside of the climate agreement
- Welfare improvement with transfers, c.f. UN COP27's "loss and damage" fund

Literature

- Theoretical model of climate agreements: cooperation
  - *Climate clubs and cooperation:* Nordhaus (2015), Barrett (1994), Harstad (2012), Maggi (2016), Barrett (2003, 2013, 2022), Iverson (2024), Hagen and Schneider (2021)
  - *Dynamics of coalition building:* Ray and Vohra (2015), Okada (2023), Nordhaus (2021), Harstad (2023), Maggi and Staiger (2022)

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Literature

- Nordhaus (2015)
  - Examine ”stable climate coalitions” (club imposing carbon tax) in a simple model
  - Abstract from General Equilibrium and distributional effects
  - Results: Penalty tariffs necessary to enforce a climate club

- Farrokhi, Lashkaripour (2024)
  - Study and characterize the optimal trade policy with climate externality
  - General static trade model. Results: unilateral tariffs not effective
  - Sequential search for one stable climate club if EU or US join.

- Main contribution:
  - Search for the *optimal* climate agreement
  - GE on good and energy market and redistribution effects are important
  - Cost of climate change is endogenous to policy: damages are non-linear
  - Analyze other distributional policies (transfers/taxes, *loss and damage funds*)
  - General framework for analyzing macrodynamics (c.f. Bourany (2024))

Model – Household & Firms

- Deterministic Neoclassical economy
  - countries $i \in \mathbb{I}$, heterogeneous in productivity z_i, temperature T_i, energy extraction cost $\mathcal{C}_i$
  - In each country, five agents:

1. Representative household $\mathcal{U}_i = \max_{c_{ij}} u(c_i)$

$$c_i = \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \qquad \sum_j c_{ij} \underbrace{(1+\mathrm{t}_{ij}^b)}_{\text{tariff}} \underbrace{\tau_{ij}}_{\text{iceberg cost}} \mathrm{p}_j = \underbrace{w_i \ell_i}_{\text{labor income}} + \underbrace{\pi_i^f}_{\text{fossil firm profit}} + \mathrm{t}_i^{ls}$$

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2. Competitive final good firm:

$$\max_{\ell_i, e_i^f, e_i^c, e_i^r} \mathrm{p}_i \, \mathcal{D}_i(\mathcal{E}) \, z_i \, F(\ell_i, e_i^f, e_i^c, e_i^r) - w_i \ell_i - (q^f + \mathrm{t}_i^{\varepsilon}) e_i^f - (q_i^c + \mathrm{t}_i^{\varepsilon}) e_i^c - q_i^r e_i^r$$

  - Externality: Damage function $\mathcal{D}_i(\mathcal{E})$, Income inequality from z_i, Carbon tax: $\mathrm{t}_i^{\varepsilon}$
  - Trade, *à la* Armington

Model – Energy markets & Emissions

3. Competitive fossil fuels (oil-gas) producer, extracting e_i^x

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x)_{\mathbb{P}_i}$$

– Energy traded in international markets, at price q^f

$$E^f = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

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4. Coal energy firm: elastic supply e_i^c at price $q_i^c = z_i^c \mathbb{P}_i$

5. Renewable energy firm: elastic supply e_i^r at price $q_i^r = z_i^r \mathbb{P}_i$

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- Climate system: mapping from emission $\mathcal{E} = \sum_{\mathbb{I}} e_i^f + e_i^c$ to damage $\mathcal{D}_i(\mathcal{E})$

Model – Equilibrium

- Given policies $\{\mathrm{t}_i^{\varepsilon}, \mathrm{t}_{ij}^{b}, \mathrm{t}_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e_i^f, e_i^c, e_i^r, e_i^x\}_{ij}$, emission $\{\mathcal{E}\}_i$ changing climate and prices $\{\mathrm{p}_i, w_i, q_i^c, q_i^r\}_i, q^f$ such that:
  - Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint
  - Firm choose inputs $\{e_i^f, e_i^c, e_i^r\}_i$ to max. profit
  - Oil-gas firms extract/produce $\{e_i^x\}_i$ to max. profit. + Elastic renewable, coal supplies $\{e_i^c, e_i^r\}$
  - Emissions $\mathcal{E}$ affects climate and damages $\mathcal{D}_i(\mathcal{E})$
  - Government budget clear $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^{\varepsilon}(e_i^f + e_i^c) + \sum_{i,j} \mathrm{t}_{ij}^{b} c_{ij} \tau_{ij} \mathrm{p}_j$
  - Prices $\{\mathrm{p}_i, w_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$ and for each good

$$y_i := \mathcal{D}_i(\mathcal{E})\, z_i\, F(\ell_i, e_i^f, e_i^r, e_i^r) = \sum_{k\in\mathbb{I}} \tau_{ki} c_{ki} + \sum_{k\in\mathbb{I}} \tau_{ki}(x_{ki}^f + x_{ki}^c + x_{ki}^r)$$

with x_{ki} export of good i as input in energy production in k
In expenditure, with import shares $s_{ij} = \frac{c_{ij}\tau_{ij}\mathrm{p}_j}{c_i \mathbb{P}_i}$, it yields

$$\mathrm{p}_i y_i = \sum_{k\in\mathbb{I}} \frac{s_{ki}}{1+\mathrm{t}_{ki}^b} \left(\mathrm{p}_k y_k + q^f(e_k^x - e_k^f) + \tilde{\mathrm{t}}_k^{ls}\right)$$

Ramsey Problem with endogenous participation

- ***Definition:*** A climate agreement is a set $\{\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}\}$ of $\mathbb{J} \subseteq \mathbb{I}$ countries and a C.E. s.t.:
  - Countries $i \in \mathbb{J}$ pay carbon tax t^{ε}
  - If j exits agreement, club members $i \in \mathbb{J}$ impose uniform tariffs $\mathrm{t}^{b}_{ij} = \mathrm{t}^{b}$ on goods from j
    They still trade with club members in oil-gas at price q^f
  - Exit: unilateral deviation $\mathbb{J}\backslash\{j\}$, $\Rightarrow$ ***Nash equilibrium***
- Participation constraints, given indirect utility $\mathcal{U}_i(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) \equiv u\big(c_{i(\mathbb{J},\mathrm{t}^{\varepsilon},\mathrm{t}^{b})}\big)$

$$\mathcal{U}_i(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) \geq \mathcal{U}_i(\mathbb{J}\backslash\{i\}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) \qquad \text{[Nash equilibrium]}$$

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- Objective: search for the optimal *and stable* climate agreement

$$\max_{\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}} \mathcal{W}(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) = \max_{\mathrm{t}^{\varepsilon}, \mathrm{t}^{b}} \max_{\mathbb{J}} \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{U}_i(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b})$$

$$s.t. \qquad \mathbb{J} \in \mathbb{S}(\mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) = \left\{\mathcal{J} \mid \mathcal{U}_i(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) \geq \mathcal{U}_i(\mathbb{J}\backslash\{i\}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) \ \forall i \in \mathcal{J}\right\}$$

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  - Alternative: ***Coalitional Nash-equilibrium*** $\mathbb{C}(\mathrm{t}^{f}, \mathrm{t}^{b})$: robust of sub-coalitions deviations:

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$$\max_{\mathbb{J}, t^{\varepsilon}, t^{b}} \mathcal{W}(\mathbb{J}, t^{\varepsilon}, t^{b}) = \max_{t^{\varepsilon}, t^{b}} \max_{\mathbb{J}} \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{U}_i(\mathbb{J}, t^{\varepsilon}, t^{b})$$

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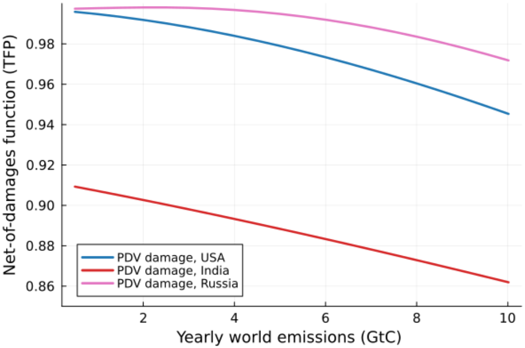
    - No country i and subcoalition $\hat{\mathbb{J}}$ would be better off than in the current agreement $\mathbb{J}$

  - Current design: (i) choose taxes $\{t^{f}, t^{b}\}$,
    (ii) choose the coalition $\mathbb{J}$ s.t. participation constraints hold

  - Solution method (Nash equilibrium):
    - relies on the complementarity of the combinatorial discrete choice problem and use a "squeezing procedure", c.f. Jia (2008), Arkolakis, Eckert, Shi (2023), to handle the problem

Quantification – Climate system and damage

- Static economic model:
  decisions $e_i^f + e_i^c$ taken "once and for all", $\mathcal{E} = \sum_i e_i^f + e_i^c$
  - Climate system:
  $$\dot{\mathcal{S}}_t = \mathcal{E} - \delta_s \mathcal{S}_t$$
  $$T_{it} = \bar{T}_{i0} + \Delta_i \mathcal{S}_t$$
  - Path of period damages heterogeneous across countr
  Quadratic (c.f. Nordhaus-DICE)
  $$\mathcal{D}(T_{it} - T_i^\star) = e^{-\gamma(T_{it} - T_i^\star)^2}$$
  - Economic feedback in Present discounted value
  $$\mathcal{D}_i(\mathcal{E}) = \bar{\rho}_i \int_0^\infty e^{-(\rho - n_i + \eta \bar{g}_i)t} \mathcal{D}\left(T_{it} - T_i^\star\right) dt$$
  - Similarly for $LCC_i, SCC_i \ldots$



Quantification – Welfare and production, trade, energy

- Pareto weights ω_i: Imply no redistribution motive
  $\bar{c}_i$ conso in initial equilbrium $t = 2020$ w/o climate change

$$\omega_i = \frac{1}{u'(\bar{c}_i)} \qquad \Leftrightarrow \qquad C.E.(\bar{c}_i) \in \underset{\bar{c}_i}{\operatorname{argmax}} \sum_i \omega_i u(\bar{c}_i)$$

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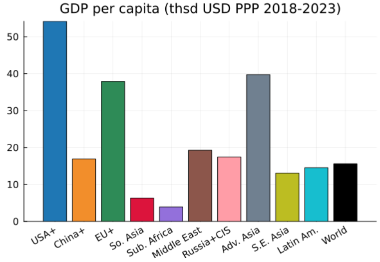
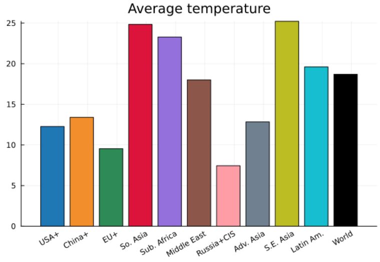
- Energy parameters to match production/reserves,
  - Isoelastic cost function $\mathcal{C}_i(e_i^x) = \bar{\nu}_i \big(e_i^x / \mathcal{R}_i\big)^{1+\nu_i} \mathcal{R}_i$
  - Use $\bar{\nu}_i, \nu_i$ to match e_i^x and π_i^f ,
- Armington model,
  - Iceberg cost τ_{ij} projected on distance and preferences a_{ij} to match import shares $s_{ij} = \frac{c_{ij}\tau_{ij}p_j}{c_i \mathbb{P}_i}$
- Production $\bar{y} = zf(\ell_i, k_i, e_i^f, e_i^c, e_i^r)$
  - Nested CES energy vs. labor-capital Cobb-Douglas bundle $k_i^{\alpha} \ell_i^{1-\alpha}$ (elasticity $\sigma_y < 1$), and fossil/renewable $\sigma_e > 1$, $CES(e_i^f, e_i^r)$
  - TFP, and DTC, z_i, z_i^e, z_i^c, z_i^r, calibrated to match GDP / energy shares / energy mix data.

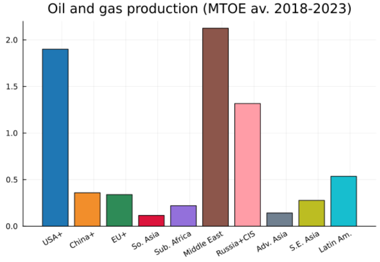
▶ Details More details Details Pareto weights

Quantitative application – Sample of 10 “regions”

- Sample of 10 “regions”: (i) US+Canada, (ii) China+HK, (iii) EU+UK+Schengen, (iv) South Asia, (v) Sub-saharian Africa, (vi) Middle-East+Maghreb, (vii) Russia+CIS, (viii) Japan+Korean+Australia+Asian Dragons, (ix) South-East Asia (Asean), (x) Latin America WIP: 25 countries + 5 regions
- Data (Avg. 2018-2023) on macro variables, energy markets, trade shares $s_{ij} = \frac{c_{ij}\tau_{ij}\mathrm{p}_j}{c_i\mathbb{P}_i}$, etc.







Details Trade shares – details

Optimal policy : benchmarks

- Three policy benchmarks, c.f. Bourany (2024), without endogenous participation
  - ***First-Best***, Social planner maximizing global welfare with unlimited instruments
    - Pigouvian result: Carbon tax = Social Cost of Carbon
    - Relies heavily on cross-country transfers to offset redistributive effects

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  - ***Second-Best:*** Social planner, single carbon tax without transfers
    - Optimal carbon tax t^{ε} correct climate externality, but also accounts for:
      (i) Redistribution motives, G.E. effects on (ii) energy markets and (iii) trade leakage

$$\mathrm{t}^{\varepsilon} = \underbrace{\sum_i \phi_i \, LCC_i}_{=SCC} + \sum_i \phi_i \, \text{Supply Redistrib}_i^{\circ} + \sum_i \phi_i \, \text{Demand Distort}_i^{\circ} - \sum_i \text{Trade Redistrib}_i^{\circ} \qquad \phi_i \propto \omega_i u'(c_i)$$

- Details: *Competitive equilibrium* Details eq 0, *First-Best*, with unlimited instruments Details eq 1, *Second-best*, Ramsey policy with limited instruments Details eq 2

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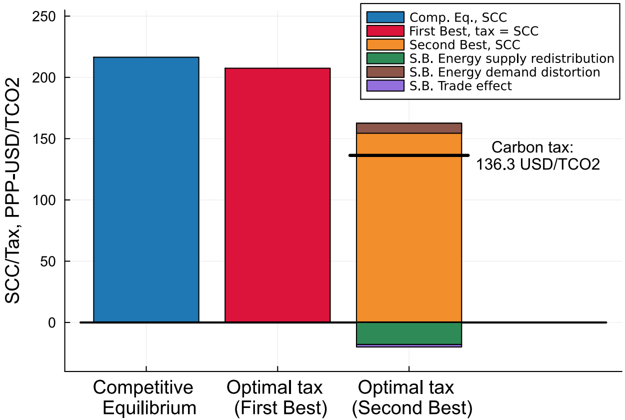
    - Details: *Competitive equilibrium* Details eq 0, *First-Best*, with unlimited instruments Details eq 1, *Second-best*, Ramsey policy with limited instruments Details eq 2

  - ***Unilateral policy:*** Local planner in country i unilaterally choosing $\mathrm{t}_i^{\varepsilon}$ and t_{ij}^{b}
    - Optimal unilateral carbon tax (subsidy!) and tariffs for terms-of-trade manipulations
    - Nash equilibrium of $\mathbb{I}$ countries choosing individually unilateral policies

Second-Best climate policy

- Accounting for redistribution implies to set a tax lower than the Social Cost of Carbon



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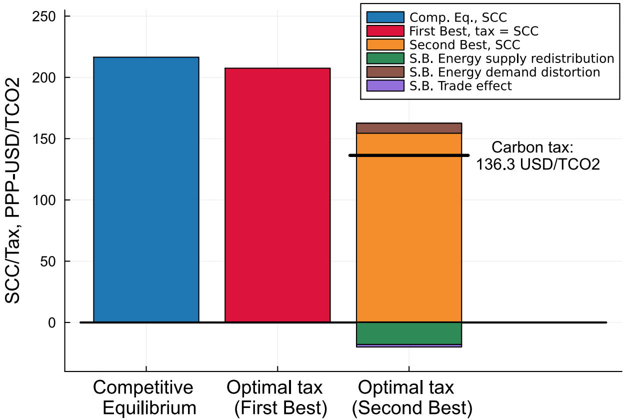
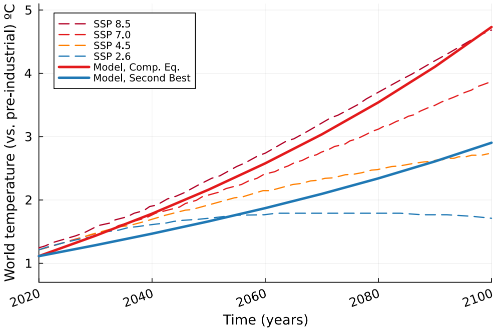
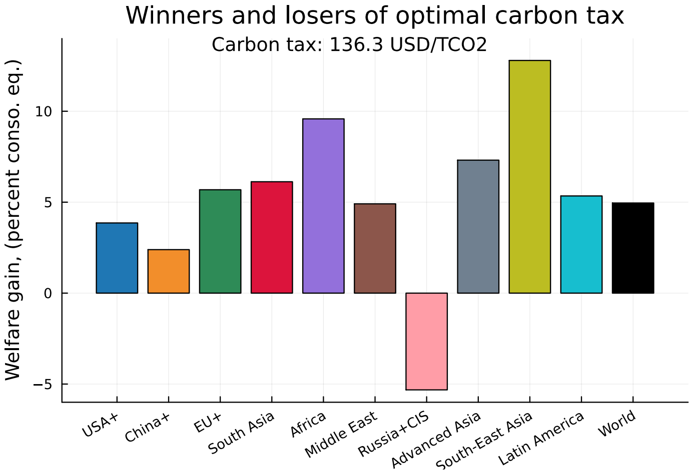




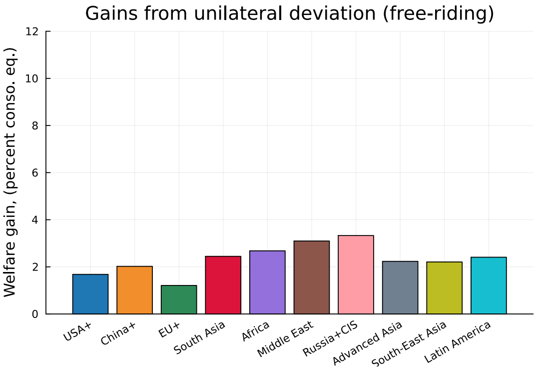
Gains from cooperation – World Optimal policy

- Optimal carbon tax (Second Best): $\sim \$136/tCO_2$
- Reduce fossil fuels / CO_2 emissions by 40% compared to Business as Usual (BAU)
- Welfare difference btw world optimal policy w/o participation constraints vs BAU (Comp. Eq.)



Trade-off – Cost of Carbon Taxation vs. Gains from trade

Gains from unilateral exit from agreement vs. Gains from trade, i.e. loss from tariffs/autarky



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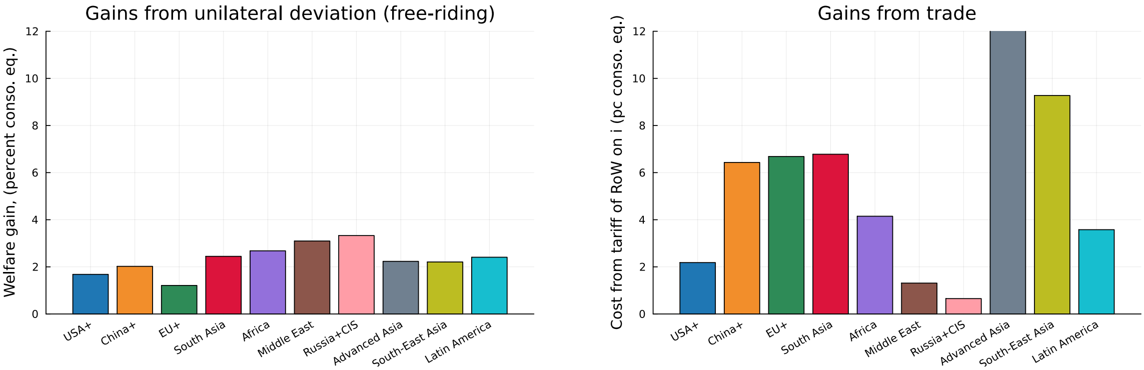


Theoretical investigation: decomposing the welfare effects

- Experiment:
  - Start from the equilibrium where carbon tax $\mathrm{t}_j^{\varepsilon} = 0, \mathrm{t}_{jk}^{b} = 0, \forall j,$
  - Change in welfare: Linear approximation around that point $\Rightarrow$ small changes in carbon tax $d\mathrm{t}_j^{\varepsilon}, \forall j$ and tariffs $d\mathrm{t}_{j,k}^{b}, \forall j, k$ for a club J_i

$$\frac{d\mathcal{V}_i}{u'(c_i)} = \eta_i^c d\ln \mathrm{p}_i + \Big[-\eta_i^c \bar{\gamma}_i \frac{1}{\bar{\nu}} - \eta_i^c s_i^e s_i^f + \eta_i^\pi (1+\tfrac{1}{\bar{\nu}}) \Big] d\ln q^f - \Big[\eta_i^c s_i^e (s_i^c + s_i^r) + \eta_i^\pi \frac{1}{\nu} + 1 \Big] d\ln \mathbb{P}_i$$

  - GE effect on energy markets $d\ln q^f \approx \bar{\nu} d\ln E^f + \ldots$, due to taxation

$$d\ln q^f = -\frac{\bar{\nu}}{1+\bar{\gamma}+\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i) + \bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \widetilde{\lambda}_i^f \mathrm{J}_i d\mathrm{t}^{\varepsilon} + \sum_i \beta_i d\ln \mathrm{p}_i$$

    - Climate damage $\bar{\gamma}_i = \gamma(T_i - T_i^{\star}) T_i\, s^{E/S}$
    - Trade and leakage effect: GE impact of $\mathrm{t}_j^{\varepsilon}$ and $\mathrm{t}_{j,}^{b}$ on y_i and p_i

○ Params: σ energy demand elast^y, s^e energy cost share, $\bar{\nu}$ energy supply inverse elas^y

Decomposing the welfare effects: gains from trade

- Start from the equilibrium where carbon tax $\mathrm{t}^f_j = 0, \mathrm{t}^b_{jk} = 0, \forall j$,
- Change in welfare: Linear approximation around that point $\Rightarrow$ small changes in carbon tax $d\mathrm{t}^f_j, \forall j$ and tariffs $d\mathrm{t}^b_{j,k}, \forall j, k$

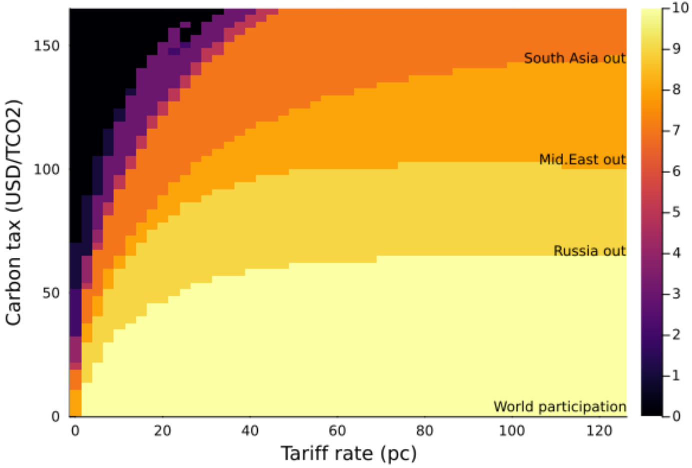
$$d \ln \mathrm{p} = \mathbf{A}^{-1}\Big[-(\mathbf{I}-\mathbf{T}\odot v^y)\alpha^{y,qf} + \mathbf{T}\big(v^{e^x}\odot\tfrac{1}{\nu} + v^{e^f}\tfrac{\sigma^y}{1-s^e} + v^{ne}\big) - \Big((\mathbf{I}-\mathbf{T}\odot v^y)\alpha^{y,z} - \tfrac{\sigma^y}{1-s^e}\Big)\bar{\gamma}\tfrac{1}{\bar{\nu}}\Big] d\ln q^f$$
$$+ \Big[-(\mathbf{I}-\mathbf{T}\odot v^y)\alpha^{y,qf} + \mathbf{T}\big(v^{e^f}\odot\tfrac{\sigma^y}{1-s^e}\big)\Big]\odot \mathrm{J} d\ln \mathrm{t}^{\varepsilon} + \theta\Big(\mathbf{TS}\odot\mathbf{J}\odot d\ln \mathrm{t}^b - \mathbf{T}(\mathbb{1}+\mathbf{S}')\odot(\mathbf{J}\odot d\ln \mathrm{t}^b)'\Big)$$

- ○ Params: **S** Trade share matrix, **T** income flow matrix, θ, Armington CES
- ○ General equilibrium (and leakage) effects summarized in a complicated matrix **A**: price affect energy demand, oil-gas extraction, energy trade balance, output, etc.

Details Market Clearing for good

Climate Agreements: Intensive vs. Extensive Margin

- **Intensive margin:**
  higher tax, emissions ↓, welfare ↑
- **Extensive margin:**
  higher tax, participation ↓,
  free-riding and emissions ↑

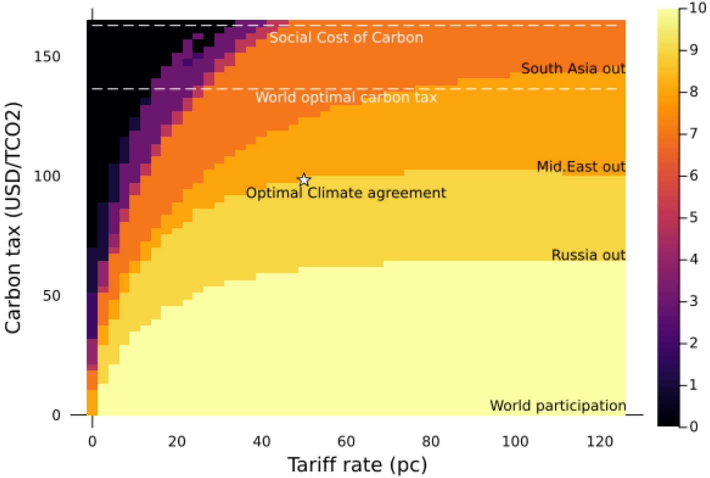


Optimal Climate Agreement

- Despite full freedom of instruments (t^{ε}, t^{b})

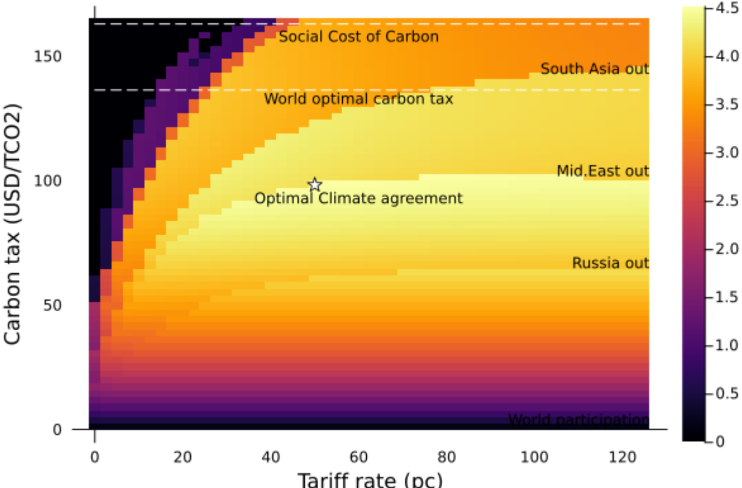
  $\Rightarrow$ can not sustain an agreement with Russia & Middle East

  $\Rightarrow$ need to reduce carbon tax from \$136 to \$98
- Intuition: relatively cold and closed economy, and fossil-fuel producers



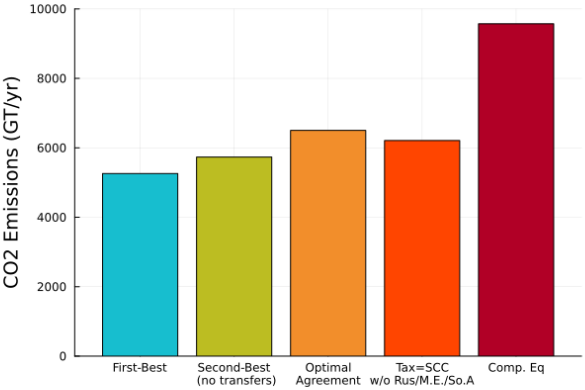
Climate agreement and welfare

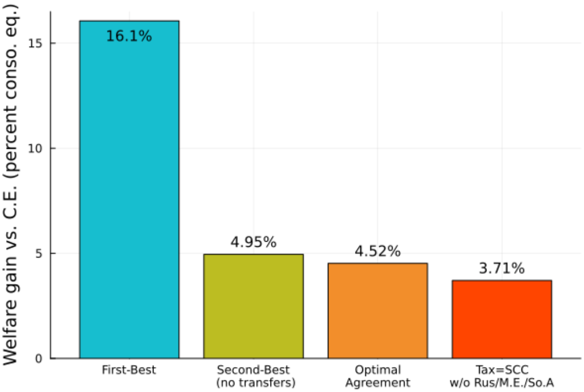
Recover 91% of welfare gains, i.e. 4.5% out of 5% conso equivalent.



Welfare and emission reduction: Different metrics!

- Agreements with tariffs recover 91% of welfare gains from the Second-Best – optimal carbon tax without transfers – at a cost of increasing emissions by 13%
- First-best allocation relies heavily on transfers to be able to impose a higher carbon tax



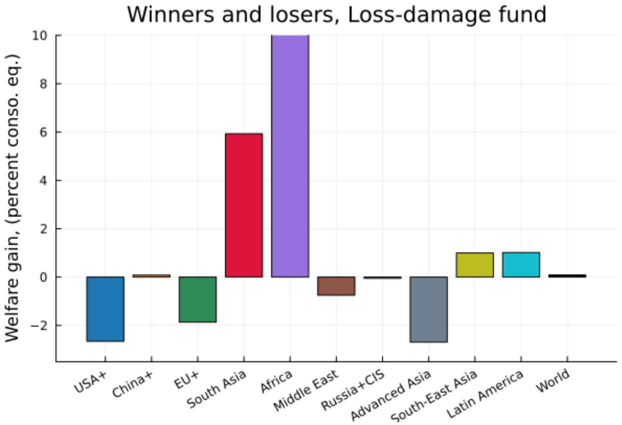


Transfers – Loss and damage funds

- COP28 Major policy proposal: *Loss and damage funds* for countries vulnerable to the effects of climate change
- Simple implementation in our context: lump-sum receipts of carbon tax revenues:

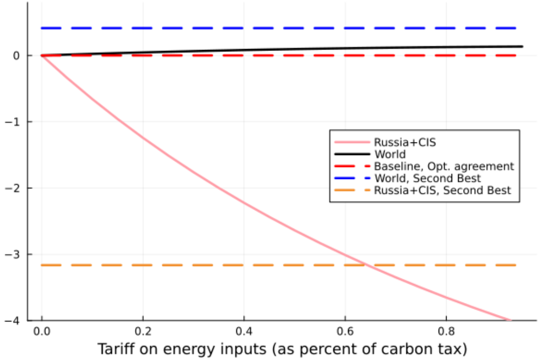
$$t_i^{ls} = (1-\alpha)t^{\varepsilon}\varepsilon_i + \alpha\frac{1}{\mathcal{P}}\sum_j t^{\varepsilon}\varepsilon_j$$

- In practice: transfers from large emitters to low emitters



Taxation of fossil fuels energy inputs

- Current climate club:
  only imposes penalty tariffs on final goods, not on energy imports
  - Empirically relevant, c.f. Shapiro (2021): inputs are more emission-intensives but trade policy is biased against final goods output
- Alternative: tax energy import from non-participants $t_{ij}^{bf} = \beta t^b \mathbb{1}\{i \in \mathbb{J}, j \notin \mathbb{J}\}$



Dynamic coalition formation

- Current "equilibrium": $\mathrm{t}_i^{\varepsilon} = 0$, $\mathrm{t}_{ij}^{b} = 0$
- Optimal club equilibrium $\mathrm{t}_i^{\varepsilon} = \mathrm{t}^{\varepsilon\star}$, $\mathrm{t}_{ij}^{b} = \mathrm{t}^{b\star}\mathbb{1}\{i \in \mathbb{J}, j \notin \mathbb{J}\}$
- Optimal agreement follows the planner taxes and participation decision:

$$\mathbb{J}^\star = \mathbb{J}\left(\mathrm{t}^{\varepsilon\star}, \mathrm{t}^{b\star}\right)$$

▶ What is driving the coordination failure?

- Possible explanation: coalition building and *bargaining* may never reach such equilibrium:

$$\bar{\mathbb{J}}_{t_0}(0,0) = \mathbb{I} \quad \xrightarrow[t\to T]{?} \quad \bar{\mathbb{J}}_T\left(\mathrm{t}^{\varepsilon\star}, \mathrm{t}^{b\star}\right) = \mathbb{J}^\star$$

- Can we find a sequence $\mathbb{J}_t, \mathrm{t}_t^f, \mathrm{t}_t^b$ such that

$$\{\mathbb{J}_{t_0}, \mathrm{t}_{t_0}^f, \mathrm{t}_{t_0}^b\} = \{\bar{\mathbb{J}}_{t_0}, 0, 0\} \qquad\qquad \{\mathbb{J}_T, \mathrm{t}_T^{\varepsilon}, \mathrm{t}_T^b\} = \{\bar{\mathbb{J}}_T, \mathrm{t}^{f\star}, \mathrm{t}^{b\star}\}$$

- Instruments used by leader countries (e.g. E.U., U.S. or China?) to reach such agreement?

Conclusion

- In this project, I solve for the optimal design of climate agreements
  - Correcting for inequality, redistribution effects through energy markets and trade leakage, as well as free-riding incentives
- Climate agreement design jointly solves for:
  - The optimal choice of countries participating
  - The carbon tax and tariff levels, accounting for both the climate externality, redistributive effects and the participation constraints
- Optimal coalition depends on the trade-off between
  - the gain from cooperation and free riding incentives
  - the gain from trade, i.e. the cost of retaliatory tariffs

  $\Rightarrow$ Need a large coalition and a carbon at 70% of the world optimum
- Extensions:
  - Extend this to dynamic settings: coalition building
  - Explore additional policy proposal to improve the optimal agreement

Conclusion

Thank you!

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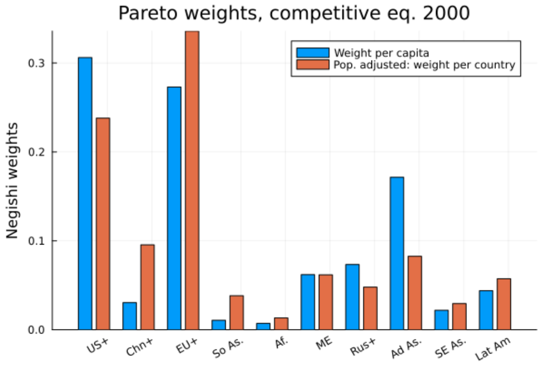
Appendices

Welfare and Pareto weights

- Welfare:
$$\mathcal{W}(\mathbb{J}) = \sum_{i\in\mathbb{I}} \omega_i \, u(c_i)$$
- Pareto weights ω_i:
$$\omega_i = \frac{1}{u'(\bar{c}_i)}$$
  for $\bar{c}_i$ consumption in initial equilibrium “without climate change“, i.e. year $= 2020$
- Imply no redistribution motive in $t = 2020$
$$\omega_i u'(\bar{c}_i) = \omega_j u'(\bar{c}_j) \qquad \forall i,j \ \in \mathbb{I}$$
- Climate change, taxation, and climate agreement (tax + tariffs) have redistributive effects
  $\Rightarrow$ change distribution of c_i



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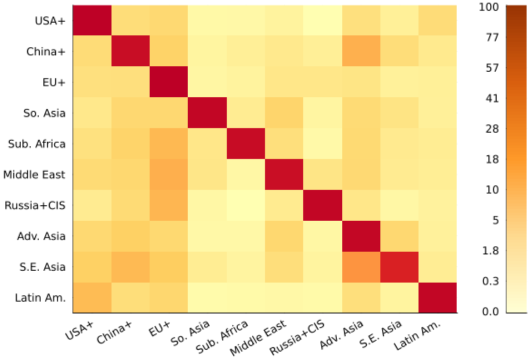
Quantification – Trade model

- Armington Trade model:

$$s_{ij} \equiv \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_i} = a_{ij}\frac{((1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k)^{1-\theta}}$$

- CES $\theta = 5.63$ estimated from a gravity regression
- Iceberg cost τ_{ij} as projection of distance
  $\log \tau_{ij} = \beta \log d_{ij}$
- Preference parameters a_{ij} identified as remaining variation in the trade share s_{ij}
  $\Rightarrow$ policy invariant



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Step 0: Competitive equilibrium & Trade

- Each household in country i maximize utility and firms maximize profit
- Standard trade model results:
  - Consumption and trade:

$$s_{ij} = \frac{c_{ij} p_{ij}}{c_i \mathbb{P}_i} = a_{ij} \frac{(\tau_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}(\tau_{ik}(1+\mathrm{t}^b_{ik})\mathrm{p}_k)^{1-\theta}} \qquad \& \qquad \mathbb{P}_i = \left(\sum_j a_{ij}(\tau_{ij}\mathrm{p}_j)^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

  - Energy consumption doesn't internalize climate damage:

$$\mathrm{p}_i MPe_i = q^e$$

  - Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

  - "Local Social Cost of Carbon", for region i

$$LCC_i = \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial c_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = \Delta_i \gamma (T_i - T_i^\star)\mathrm{p}_i y_i \qquad (> 0 \text{ for warm countries})$$

back

Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - Full array of instruments: cross-countries lump-sum transfers t_i^{ls}, individual carbon taxes t_i^f on energy e_i^f, unrestricted bilateral tariffs t_{ij}^b
  - Budget constraint: $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^f e_i^f + \sum_{i,j} \mathrm{t}_{ij}^b c_{ij} \tau_{ij} \mathrm{p}_j$

- Maximize welfare subject to
  - Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1: World First-best policy

- Social planner results:
  - Consumption:

$$\omega_i u'(c_i) = \Big[\sum_j a_{ij}(\tau_{ij}\omega_j\mu_j)^{1-\theta}\Big]^{\frac{1}{1-\theta}} = \mathbb{P}_i \qquad\qquad \omega_i \frac{u'(c_i)}{\mathbb{P}_i} = \bar{\lambda}$$

  - Energy use:

$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

  - Social cost of carbon:

$$SCC = \sum_j \omega_j \Delta_j \gamma (T_i - T_i^\star) y_j \mu_j$$

  - Decentralization:
    large transfers to equalize marg. utility + carbon tax = SCC

$$\mathrm{t}^\varepsilon = SCC \qquad\qquad \mathrm{t}_i^{lb} = c_i^\star \mathbb{P}_i - w_i \ell_i + \pi_i^f \qquad s.t. \quad u'(c_i^\star) = \bar{\lambda}\mathbb{P}_i/\omega_i$$

back

Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - One single instrument: uniform carbon tax t^f on energy e_i^f
  - Rebate tax lump-sum to HHs $\mathrm{t}_i^{ls} = \mathrm{t}^\varepsilon e_i^f + \mathrm{t}^\varepsilon e_i^c$

- Ramsey policy: Primal approach, maximize welfare subject to
  - Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
  - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand $[\upsilon_i]$ & supply $[\theta_i]$, etc.
  - Trade-off faced by the planner:
    - (i) Correcting climate externality, (ii) Redistributive effects,
      (iii) Distort energy demand and supply (iv) Distort good demand

back

Step 2: World optimal Ramsey policy

- The planner takes into account
  - (i) the marginal value of wealth λ_i
  - (ii) the shadow value of good i, from market clearing, μ_i:
  - (iii) the shadow value of bilateral trade ij, from household FOC, η_{ij}:

$$\text{w/ free trade} \qquad u'(c_i) = \lambda_i$$

$$\text{vs. w/ Armington trade} \qquad u'(c_i) = \lambda_i \left(\sum_{j \in \mathbb{I}} a_{ij} (\tau_{ij} \mathrm{p}_j)^{1-\theta} \left[1 + \frac{\omega_j}{\omega_i} \frac{\mu_j}{\lambda_i} - \frac{\eta_{ij}}{\theta \lambda_i} (1 - s_{ij}) \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Relative welfare weights, representing inequality

$$\widehat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \lesseqgtr 1 \qquad \Rightarrow \qquad \text{ceteris paribus, poorer countries have higher } \widehat{\lambda}_i$$

Step 2: Optimal policy – Social Cost of Carbon

- Key objects: Local vs. Global Social Cost of Carbon:
  - Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i
  - "Local social cost of carbon" (LCC) for region i:

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = \Delta_i \gamma (T_i - T_i^{\star}) y_i \mathrm{p}_i$$

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  - Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W} / \partial \mathcal{E}}{\partial \mathcal{W} / \partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i \lambda_i}$$

  - Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i \big(\widehat{\lambda}_i, LCC_i \big)$$

Step 2: Optimal policy – Other motives

- Taking fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy
  3. Reallocate goods production, which is then supplied internationally

$$\text{Supply Redistrib}^{\circ sb} + \text{Demand Distort}^{\circ sb} - \text{Trade effect}^{sb} = \underbrace{\mathcal{C}^f_{EE}}_{\substack{\text{agg. supply} \\ \text{inv. elast}^y}} \underbrace{\mathbb{C}\text{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right)}_{\substack{\text{energy T-o-T} \\ \text{redistrib}^\circ}} - \underbrace{\mathbb{C}\text{ov}_i\left(\widehat{\upsilon}_i, \frac{q^f(1-s^e_i)}{\sigma_i e_i}\right)}_{\substack{\text{demand} \\ \text{distortion}}} - q^f \underbrace{\mathbb{E}_j[\widehat{\mu}_j]}_{\substack{\text{good T-o-T} \\ \text{redistrib}^\circ}}$$

∘ Params: $\mathcal{C}^f_{EE}$ agg. fossil inv. elasticity, s^e_i energy cost share and σ_i energy demand elasticity

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  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
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$$\text{Supply Redistrib}^{\circ sb} + \text{Demand Distort}^{\circ sb} - \text{Trade effect}^{sb} = \underbrace{\mathcal{C}^f_{EE}}_{\substack{\text{agg. supply}\\ \text{inv. elast}^y}} \underbrace{\mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}_i, e^f_i - e^x_i\Big)}_{\substack{\text{energy T-o-T}\\ \text{redistrib}^\circ}} - \underbrace{\mathbb{C}\mathrm{ov}_i\Big(\widehat{v}_i, \frac{q^f(1-s^e_i)}{\sigma_i e_i}\Big)}_{\substack{\text{demand}\\ \text{distortion}}} - q^f \underbrace{\mathbb{E}_j[\widehat{\mu}_j]}_{\substack{\text{good T-o-T}\\ \text{redistrib}^\circ}}$$

◦ Params: $\mathcal{C}^f_{EE}$ agg. fossil inv. elasticity, s^e_i energy cost share and σ_i energy demand elasticity

- *Proposition 2:* Optimal fossil energy tax:

$$\Rightarrow \quad t^f = SCC^{sb} + \text{Supply Redistribution}^{sb} + \text{Demand Distortion}^{sb} - \text{Trade effect}^{sb}$$

– Reexpressing demand terms:

$$t^\varepsilon = \left(1 + \mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}^w_i, \frac{\widehat{\sigma_i e_i}}{1-s^e_i}\Big)\right)^{-1}\left[\sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}^w_i, LCC_i\Big) + \mathcal{C}^f_{EE}\,\mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}^w_i, e^f_i - e^x_i\Big) - q^f\,\mathbb{E}_j[\widehat{\mu}_j]\right]$$

back

Step 3: Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.
- For a climate "club" of $\mathbb{J} \subset \mathbb{I}$ countries:
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $\mathrm{t}^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f

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  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f
- Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

- Welfare:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{\mathbb{J}} \omega_i \, u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i \, u(c_i)$$

Step 3: Ramsey Problem with participation constraints

- Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- *Proposition 3.1:* Second-Best social valuation with participation constraints
  - Participation incentives change our measure of inequality

$$\text{w/ trade:} \qquad \omega_i(1+\nu_i)u'(c_i) = \Big(\sum_{j\in\mathbb{I}} a_{ij}(\tau_{ij}\mathrm{p}_j)^{1-\theta}\Big[\omega_i\widetilde{\lambda}_i + \omega_j\widetilde{\mu}_j + \widetilde{\eta}_{ij}(1-s_{ij})\Big]^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)} \neq \widehat{\lambda}_i$$

$$\text{vs. w/o trade} \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_j(1+\nu_j)u'(c_j)} \neq \widehat{\lambda}_i$$

  - Similarly, the "effective Pareto weights" are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

Step 3: Participation constraints & Optimal policy

- *Proposition 3.2:* Second-Best taxes:
  - Taxation with imperfect instruments:
    - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
    - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu\sigma}{q^f(1-s^f)}$
      with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
  - Optimal fossil energy tax $\mathrm{t}^f(\mathbb{J})$:

$$\Rightarrow \qquad \mathrm{t}^f(\mathbb{J}) = SCC + SVF$$

$$= \frac{1}{1-\vartheta_{\mathbb{J}^c}} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i LCC_i + \frac{1}{1-\vartheta_{\mathbb{J}^c}} C^f_{EE} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i (e^f_i - e^x_i) - \sum_{i\in\mathbb{J}} \widetilde{\lambda}_i \frac{q^f(1-s^f_i)}{\sigma}$$

  - Optimal tariffs/export taxes $\mathrm{t}^{b,r}(\mathbb{J})$ and $\mathrm{t}^b(\mathbb{J})$: In search for a closed-form expression
    As of now, only opaque system of equations (fixed point w/ demand/multipliers)

back

Welfare decomposition

- Armington model of trade with energy:
  - Linearized market clearing

$$\Big(\frac{d\mathrm{p}_i}{d\mathrm{p}_i} + \frac{dy_i}{y_i}\Big) = \sum_k \boldsymbol{t}_{ik}\Big[\Big(\frac{\mathrm{p}_k y_k}{v_k}\Big)(d\ln \mathrm{p}_k + d\ln y_k) + \frac{q^f e^x_k}{v_k} d\ln e^x_k - \frac{q^f e^f_k}{v_k} d\ln e^f_k + \frac{q^f (e^x_k - e^f_k)}{v_k} d\ln q^f$$

$$+ \theta \sum_h \big(s_{kh} d\ln \mathrm{t}_{kh} - (1{+}s_{ki}) d\ln \mathrm{t}_{ki}\big) + (\theta{-}1) \sum_h \big(s_{kh} d\ln \mathrm{p}_h - d\ln \mathrm{p}_i\big)\Big]$$

  - Fixed point for price level $d\ln p_i$

$$\Big[(\mathbf{I}{-}\mathbf{T}{\odot}v^y)[\mathbf{I}{-}\alpha^{y,p}{\odot}\mathbf{I}] + \mathbf{T}(v^{e^x}{\odot}\tfrac{1}{\nu}) + \mathbf{T}v^{e^f}\tfrac{\sigma^y}{1-s^e} - (\theta{-}1)(\mathbf{TS} - \mathbf{T}') - \Big((\mathbf{I}{-}\mathbf{T}{\odot}v^y)\alpha^{y,z} {-} \tfrac{\sigma^y}{1-s^e}\Big){\odot}\bar{\gamma}\mathbf{I}{\odot}\big(\tfrac{\lambda^x}{\nu}\big)'\Big] d\ln \mathrm{p}$$

$$\Big[- (\mathbf{I} - \mathbf{T}{\odot}v^y)\alpha^{y,qf} + \mathbf{T}\big(v^{e^x}{\odot}\tfrac{1}{\nu} {+} v^{e^f}\tfrac{\sigma^y}{1-s^e} + v^{ne}\big) - \Big((\mathbf{I} - \mathbf{T}{\odot}v^y)\alpha^{y,z} - \tfrac{\sigma^y}{1-s^e}\Big)\bar{\gamma}\tfrac{1}{\bar{\nu}}\Big] d\ln q^f$$

$$+ \Big[- (\mathbf{I} - \mathbf{T}{\odot}v^y)\alpha^{y,qf} + \mathbf{T}\big(v^{e^f}{\odot}\tfrac{\sigma^y}{1-s^e}\big)\Big]{\odot}\mathbf{J} d\ln \mathrm{t}^{\varepsilon} + \theta\Big(\mathbf{TS}{\odot}\mathbf{J}{\odot}d\ln \mathrm{t}^b - \mathbf{T}(\mathbb{1}{+}\mathbf{S}'){\odot}(\mathbf{J}{\odot}d\ln \mathrm{t}^b)'\Big)$$

back

Countries' incentives – Model w/o trade in goods

- Experiment: Model with trade in energy but not in "goods"
  - Start from the equilibrium where carbon tax $t^f(\mathbb{J}) = 0$,
    $\Rightarrow$ country i is indifferent to join the club $\mathbb{J}$ or not
  - Linear approximation around that point $\Rightarrow$ small changes in carbon tax dt^f

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  - Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})} = -e_i d\mathfrak{t}^f - \gamma_i (T_i - T_{i0})^{\delta} y_i \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \pi_i \frac{(1+\nu)}{E} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

  - Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}} = -\Big(E_{\mathbb{J}} - E_{\mathbb{J}\setminus\{i\}}\Big) \frac{\sigma\, d\mathfrak{t}^f}{q^f(1-s^f)} \frac{1}{1 + \frac{\nu\sigma}{(1-s^f)}}$$

○ Params: σ energy demand elast^y, s^f energy cost share, ν energy supply elas^y, Climate damage γ_i and curv. δ

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Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned}\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = & -e_i d\mathrm{t}^f - \gamma_i (T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \\ & - e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \\ & + \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}} \Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij} \Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}} \Big)\end{aligned}$$

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  - GE effect on energy markets, same as before
  - GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{p}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki} \left(\frac{dv_k}{v_k} - \theta \frac{d\mathrm{t}_{ki}^b}{1+\mathrm{t}_{ki}^b}\right) \qquad \alpha_{ki} = \frac{c_{ki}\tau_{ki}\mathrm{p}_i}{\sum_\ell c_{k\ell}\tau_{k\ell}\mathrm{p}_\ell}\frac{v_k}{v_i} = s_{ki}\frac{v_k}{(1+\mathrm{t}_{ki}^b)v_i}$$

    – Params: σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

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    - Params: σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Complementarity in coalition formation – Model w/o trade in goods

- Is marginal gain $\Delta\mathcal{W}(\mathbb{J},j) := \mathcal{W}(\mathbb{J}\cup j) - \mathcal{W}(\mathbb{J})$ "growing" in $\mathbb{J}$?
  - Linear approximation for small $\{\mathfrak{t}^f, \mathfrak{t}^b\}$

$$\Delta\mathcal{W}(\mathbb{J},j) = -\omega_j u'(c_j) e_j d\mathfrak{t}^f + \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i)\Delta_i\gamma_i(T_i - T_{i0})^\delta y_i\Big] \frac{\sigma\, e_j d\mathfrak{t}^f}{q^f(1-s^f+\nu\sigma)}$$
$$+ \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i) e_i\Big] \frac{1}{1+\frac{1-s^f}{\nu\sigma}} \frac{e_j d\mathfrak{t}^f}{E_{\mathbb{I}}} - \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i)\pi_i\Big] \frac{(1+\nu)}{E_{\mathbb{I}}} \frac{\sigma\, e_j d\mathfrak{t}^f}{q^f(1-s^f+\nu\sigma)}$$

  - Free-riding problem: $\Delta\mathcal{W}(\mathbb{J},j)$ could be negative
  - If $\Delta\mathcal{W}(\mathbb{J},j) > 0$, what effects does $\mathbb{J}$ have on marginal gain?
    - Marginal climate benefit decreases in $\mathbb{J}$, since temperature T_i declines!
    - G.E. effect on energy price: $E_{\mathbb{I}}$, q and π^f decreases with $\mathbb{J}$, effect on demand ambiguous
    - Similar formula for the case with trade tariffs: Work in progress.

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(T_i) z_i F(k, \varepsilon(e^f, e^r))$

$$F_i(\varepsilon(e^f,e^c,e^r), \ell) = \left[(1-\epsilon)^{\frac{1}{\sigma_y}} \left(\bar{k}^{\alpha} \ell^{1-\alpha}\right)^{\frac{\sigma_y - 1}{\sigma_y}} + \epsilon^{\frac{1}{\sigma_y}} \left(z_i^e \; \varepsilon_i(e^f,e^c,e^r)\right)^{\frac{\sigma_y-1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y-1}}$$

$$\varepsilon_i(e^f, e^c, e^r) = \left[(\omega^f)^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e-1}{\sigma_e}} + (\omega^c)^{\frac{1}{\sigma_e}} (e^c)^{\frac{\sigma_e-1}{\sigma_e}} + (\omega^r)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e-1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e-1}}$$

  - Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2019-23 (avg. PPP).
  - Technology: $\omega^f = 56\%, \omega^c = 27\%, \omega^f = 17\%$, $\epsilon = 12\%$ for all i
  - Calibrate (z_i^e) to match Energy/GDP $q^e e_i / \mathrm{p}_i y_i$

- Damage functions in production function y:

$$\mathcal{D}_i^y(T) = e^{-\gamma_i^{\pm,y}(T - T_i^\star)^2}$$

  - Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y} \mathbb{1}_{\{T > T_i^\star\}} + \gamma^{-,y} \mathbb{1}_{\{T < T_i^\star\}}$
  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $T_i^\star = \bar{\alpha} T_{it_0} + (1-\bar{\alpha}) T^\star$

Quantification – Energy markets

- Fossil production e^x_{it} and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now: $\bar{\nu}_i$ to match extraction data e^x_i, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. ν_i extraction cost curvature to match profit $\pi^f_i = \frac{\bar{\nu}_i \nu_i}{1+\nu_i}(\frac{e^x_i}{\mathcal{R}_i})^{\nu_i}\mathcal{R}_i\mathbb{P}_i$
  - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e^x_i (BP, IEA)
- Coal and Renewable: Production $\bar{e}^r_i, \bar{e}^x_i$ and price q^c_i, q^r_i
  - Calibrate $q^c_i = z^c\mathbb{P}_i$, $q^r_{it} = z^r\mathbb{P}_i$
    Choose z^c_i, z^r_i to match the energy mix (e^f_i, e^c_i, e^r_i)
- Population dynamics
  - Match UN forecast for growth rate / fertility

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Calibration

Table: Baseline calibration ($\star$ = subject to future changes)

Technology & Energy markets			
α	0.35	Capital share in $F(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $F(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2022)
ω^f	0.56	Fossil energy share in $e(\cdot)$	Oil-gas/Energy ratio
ω^c	0.27	Coal energy share in $e(\cdot)$	Coal/Energy ratio
ω^r	0.17	Non-carbon energy share in $e(\cdot)$	Non-carbon/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
Preferences & Time horizon			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	1.5	Risk aversion	
n	$0.01^\star$	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
ω_i	$1/u'(c_i)$	Pareto weights	Negishi / Status-quo Social Planner
T	400	Time horizon	Dynamic st

Calibration

Table: Baseline calibration ($\star$ = subject to future changes)

Climate parameters			
ξ	0.81	Emission factor	Conversion 1 $MTOE \Rightarrow 1\ MT\ CO_2$
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature $\sim 11-15$ years
χ	$2.3/1e6$	Climate sensitivity	Pulse experiment: $100\ GtC \equiv 0.23^\circ C$ medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment: $100\ GtC \equiv 0.15^\circ C$ long-term warming
$\gamma^{\oplus}$	$0.003406^\star$	Damage sensitivity	Nordhaus' DICE
$\gamma^{\ominus}$	$0.25 \times \gamma^{\oplus\ \star}$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^T	0.5	Weight historical climate for optimal temp.	Marginal damage correlated with initial temp.
$T^\star$	14.5	Optimal yearly temperature	Average spring temperature / Developed economies
Parameters calibrated to match data			
p_i		Population	Data – World Bank
z_i		TFP	To match GDP Data (WDI)
T_i		Local Temperature	Match population-weighted temperature
$\mathcal{R}_i$		Local Fossil reserves	Data, Energy Institute Energy review
ν_i		Extraction elasticity of fossil energy	Match Data, energy rent, WDI
z_i^e		Directed Technical Change (energy)	Match Data, energy intensity, Energy Institute

Sequential solution method

- Summary of the model:
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, T_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^T_{it}, \lambda^{\mathcal{S}}_t, \lambda^{\mathcal{R}}_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

Sequential solution method

- Summary of the model:
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, T_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^T_{it}, \lambda^{\mathcal{S}}_t, \lambda^{\mathcal{R}}_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)
- Global Numerical solution:
  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method: Pros and Cons

- Why use a sequential approach?
  - *Global approach*: *Only* need to follow the trajectories for i agents:
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity:
    Productivity z_i Population p_i, Temperature scaling Δ_i, Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$, Local damage $\gamma_i^y, \gamma_i^u, T_i^\star$, Directed Technical Change z_i^e
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables:
    For now: Wealth w_{it}, temperature T_{it}, reserves $\mathcal{R}_{it}$, Carbon $\mathcal{S}_t$
    Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
  - Newton method & Non-linear solvers very efficient
- Why not:
  - Numerical constraint to solve a large system of ODEs and non-linear equations:

  $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

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