

The Optimal design of Climate Agreements Inequality, Trade and Incentives for carbon policy

WORK IN PROGRESS

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Introduction

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 - ▶ Countries have differing incentives to join
e.g. cold, “closed” or fossil-rich countries are better off outside “climate clubs”
- ⇒ Designing a climate agreement entails determining *jointly* the level of carbon tax and the club of participating countries

Introduction – this project

- ▶ Trade-off between intensive margins and extensive margin :
 - Climate club with a small number of countries, higher tax and large emissions reductions
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 - Evaluate the welfare costs of global warming and solve optimal carbon policy
 - Analyze the strategic implications of joining climate agreements
 - Design the optimal size of the climate club

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 - Evaluate the welfare costs of global warming and solve optimal carbon policy
 - Analyze the strategic implications of joining climate agreements
 - Design the optimal size of the climate club
- ▶ Preview of the result :
 - Differing incentives to join club through exposure to GE effects on energy & good prices
 - Benefit : change in climate, cost through taxation, energy rent & GE effect on price
 - Trade matters as it creates interdependence across countries
 - Damages of climate change propagate across countries : SCC become correlated
 - Leakage effect of carbon taxation : reallocate activity to outside the club
 - Generate policy leverage for making the climate club sustainable

Literature

► Climate change & optimal carbon taxation

- RA model : Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
- HA model : Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
- Spatial models : Cruz, Rossi-Hansberg (2022, 2023)

⇒ *Optimal and constrained policy with heterogeneous countries & trade*

► Unilateral vs. climate club policies :

- Climate clubs : Nordhaus (2015), Non-cooperative taxation : Chari, Kehoe (1990), Suboptimal policy : Hassler, Krusell, Olovsson (2019)
- Trade policy : Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021), Hsiao (2022), Costinot, Donalson, Vogel, Werning (2015), Adao, Costinot (2022), Antras, Fort, Gutierrez, Tintelnot (2022)

⇒ *Climate cooperation and optimal design of climate club*

► Optimal policy in heterogeneous agents models

- Policy with limited instruments : Diamond (1973), Davila, Walther (2022)
- Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) ...

⇒ *Application to climate and carbon taxation policy*

Model – Household & Firms

- ▶ Deterministic Neoclassical economy, static (for today)
 - countries $i \in \mathbb{I}$, heterogeneous in productivity z_i , temperature τ_{it} , energy cost/reserves \mathcal{R}_{it}
 - In each country, 3 agents :
 - (i) HtM household, (ii) homogeneous good firm, (iii) (fossil) energy producer

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- ▶ Representative household problem in each country i :

$$\mathcal{V}_i = u(c_i) \quad \mathbb{P}_i c_i = w_i + \pi_i^f + t_i^L \quad c_i = \begin{cases} c_{ii} & \mathbb{P}_i = \mathbf{p}_i = 1 \\ \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} & \text{price} \end{cases} \quad (1+t_{ij}^b) d_{ij} p_j$$

- Labor income w_i from final good firm (labor norm. to 1), profit π_i^f from fossil firm

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$$\mathcal{V}_i = u(c_i) \quad \mathbb{P}_i c_i = w_i + \pi_i^f + t_i^{ls} \quad c_i = \begin{cases} c_{ii} & \mathbb{P}_i = \mathbf{p}_i = 1 \\ \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} & \text{price} \end{cases} \quad (1+t_{ij}^b) d_{ij} \mathbf{p}_j$$

- Labor income w_i from final good firm (labor norm. to 1), profit π_i^f from fossil firm
- ▶ Competitive homogeneous good producer in country i

$$\max_{e_i^f} \mathbf{p}_i \mathcal{D}(\tau_i) z_i f(e_i^f) - w_i - (q^f + t_i^e) e_i^f$$

- Fossil energy consumption e_i^f – emitting carbon – subject to price q^f and tax/subsidy t^f .
- Climate externality : effect of temperature on damage/TFP, $\mathcal{D}(\tau) \in (0, 1)$

Model – Energy markets & Emissions

► Competitive fossil fuels energy producer :

- Extraction of fossil energy e_{it}^x out of reserves \mathcal{R}_i

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}^f(e_i^x, \mathcal{R}_i)$$

- Energy traded in international markets, at price q^f

$$E = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

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► Climate system

- Fossil energy e^f releases GHGs in the atmosphere

$$\mathcal{E} = \sum_{\mathbb{I}} e_i^f$$

- Country's local temperature :

$$\tau_i = \bar{\tau}_{i0} + \Delta_i \mathcal{E}$$

- Linear model + Climate sensitivity/pattern scaling factor Δ_i

Model – Dynamics & extensions

1. Energy market

- Renewable energy firm in each country
- Price of clean energy trending down
- Fossil energy extraction/depleting reserves \Rightarrow Hotelling problem

2. Firm

- Use capital as well to produce
- Use an energy bundle of renewable and fossil energy

3. Households

- Consumption / saving in bonds / in capital \Rightarrow Keynes-Ramsey rule
- International markets to borrow bonds (in zero net supply)

4. Climate system with (short) inertia / closer to Integrated assessments models

5. Population growth dynamics for each country

6. (Exogenous) growth : TFP change and Energy-augmenting Directed TC.

Model – Equilibrium

► Equilibrium

- Given policies $\{t_i^e, t_{ij}^b, t_i^{ls}\}$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e_i^f, e_i^x\}_{ij}$, states $\{\tau_i\}_i$ and prices $\{p_i, q^f\}$ such that :
 - Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint
 - Firm choose policies $\{e_i^f\}_i$ to max. profit and Fossil firms extract/produce $\{e_i^x\}_i$ to max. profit.
 - Emissions \mathcal{E}_t affects climate $\{\tau_i\}_i$.
 - Prices $\{p_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$ and for each good $y_i := z_i \mathcal{D}(\tau_i) f(e_i^f) = \sum_k d_{ki} c_{ki} + \frac{q^e}{p_i} (e_i^f - e_i^x) + \mathcal{C}(e_i^x, \mathcal{R}_i)$

Model – Solution

- ▶ Step 0 : Competitive equilibrium / Business as usual : No policy $t_i = 0$
- ▶ Step 1 : Optimal (Ramsey) policy for the world
- ▶ Step 2 : Countries decide whether to join the climate club : participation constraints
- ▶ Step 3 : Optimal design of size \mathbb{J} and countries $j \in \mathbb{J}$ in the climate agreement

Step 0 : Competitive equilibrium

► Key objects :

- Marginal value of wealth $\lambda_i = \frac{u'(c_i)}{\mathbb{P}_i}$ with price index $\mathbb{P}_i = \left(\sum_j a_{ij} (d_{ij} p_j)^{1-\theta} \right)^{\frac{1}{1-\theta}}$
- Marginal value of carbon $\psi_i^{\mathcal{E}}$ for country i
- “Local social cost of carbon” (LCC) for region i :

$$LCC_i := \frac{\partial \mathcal{V}_i / \partial \mathcal{E}}{\partial \mathcal{V}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(\tau_i) z_{if}(e_i^f) p_i \quad (> 0 \text{ if heat causes losses})$$

Step 1 : World optimal Ramsey policy

- Maximizing welfare of the world Social Planner :

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_{it}) = \sum_{i \in \mathbb{I}} \mathcal{W}_i$$

- One single instrument : carbon tax τ^f on energy e_i^f , rebated lump-sum to HHs $t_i^{ls} = \tau^f e_i^f$

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- One single instrument : carbon tax \mathbf{t}^f on energy e_i^f , rebated lump-sum to HHs $\mathbf{t}_i^s = \mathbf{t}^f e_i^f$
- The planner takes into account (i) the **marginal value of wealth** λ_i as well as (ii) the **shadow value of good i** , from market clearing, μ_i :

$$\begin{array}{ll} \text{w/ trade :} & \omega_i u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij} (d_{ij} p_j)^{1-\theta} \left[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij} (1-s_{ij}) \right] \right)^{\frac{1}{1-\theta}} \\ \text{vs. w/o trade} & \omega_i u'(c_i) = \omega_i \lambda_i \end{array}$$

Step 1 : Optimal policy – Social Cost of Carbon

- Key objects : Local vs. Global Social Cost of Carbon :

$$scc := \frac{\partial \mathcal{W} / \partial \mathcal{E}}{\partial \mathcal{W} / \partial w} = \frac{\psi^{\mathcal{E}}}{\lambda} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

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- Inequality across regions :

$$\text{w/o trade : } \hat{\lambda}_i = \frac{\omega_i \lambda_i}{\lambda} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \leq 1$$

$$\text{w/ trade : } \hat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)} \leq 1 \quad \Rightarrow \quad \text{ceteris paribus, poorer countries have higher } \hat{\lambda}_i$$

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- Social Cost of Carbon integrates these inequalities :

$$SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i = \sum_{\mathbb{I}} LCC_i + \text{Cov}_i(\hat{\lambda}_i, LCC_i)$$

Step 1 : Optimal policy – Other motives

- ▶ Taxing fossil energy has additional redistributive effects :
 1. Through energy markets : distort supply, lowers eq. fossil price, benefit net importers
 2. Distort energy demand, of countries that need more or less energy
- ▶ New measure : Social Cost of Fossil (SCF)

$$SCF := \frac{\partial \mathcal{W} / \partial E_t^f}{\partial \mathcal{W}_t / \partial w_t} = c_{EE}^f \mathbb{Cov}_i \left(\hat{\lambda}_i, e_{it}^f - e_{it}^x \right) - \mathbb{Cov}_i \left(\hat{v}_i, \frac{q^e (1 - s_i^f)}{\sigma e_i^f} \right)$$

- with c_{EE}^f fossil energy supply elasticity, s_i^f energy cost share and σ energy demand elasticity

- ▶ Proposition 2 : Optimal fossil energy tax :

$$\Rightarrow \quad \mathfrak{t}^f = SCC + SCF$$

- Social cost of carbon : $SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i$

Step 2 : Ramsey Problem with participation constraints

- ▶ Consider that countries can “exit” climate agreement.
- ▶ For a climate “club” of $\mathbb{J} \subset \mathbb{I}$ countries :
 - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f
 - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $t^{b,r}$ on goods and get consumption \tilde{c}_i
 - Countries $i \notin \mathbb{J}$ can still trade in goods s.t. tariff/tax t^b with club members and countries outside the club

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 - Countries $i \notin \mathbb{J}$ can still trade in goods s.t. tariff/tax t^b with club members and countries outside the club
- ▶ Participation constraints :

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- ▶ Welfare :

$$\mathcal{W} = \max_{\{t, e, q\}_i} \sum_{\mathbb{J}} \omega_i u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i u(c_i)$$

Step 2 : Ramsey Problem with participation constraints

► Participation constraints

$$u(c_{it}) \geq u(\tilde{c}_{it}) \quad [\nu_{it}]$$

► Proposition 3.1 : Second-Best social valuation with participation constraints

- Participation incentives change our measure of inequality

$$\text{w/ trade :} \quad \omega_i(1+\nu_i)u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij}(d_{ij}p_j)^{1-\theta} \left[\omega_i \tilde{\lambda}_i + \omega_j \tilde{\mu}_j + \tilde{\eta}_{ij}(1-s_{ij}) \right] \right)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(\tilde{\lambda}_i + \tilde{\mu}_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(\tilde{\lambda}_i + \tilde{\mu}_i)} \neq \hat{\lambda}_i$$

$$\text{vs. w/o trade} \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(1+\nu_j)u'(c_j)} \neq \hat{\lambda}_i$$

- Similarly, the “effective Pareto weights” are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

Step 2 : Participation constraints & Optimal policy

► Proposition 3.2 : Second-Best taxes :

- Taxation with imperfect instruments :
 - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
 - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu \sigma}{q^e(1-s^f)}$
with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
- Optimal fossil energy tax $t^f(\mathbb{J})$:

$$\Rightarrow t^f(\mathbb{J}) = \text{SCC} + \text{SCF}$$

$$= \frac{1}{1 - \vartheta_{\mathbb{J}^c}} \sum_{i \in \mathbb{I}} \tilde{\lambda}_i \text{LCC}_i + \frac{1}{1 - \vartheta_{\mathbb{J}^c}} C_{EE}^f \sum_{i \in \mathbb{I}} \tilde{\lambda}_i (e_i^f - e_i^x) - \sum_{i \in \mathbb{J}} \tilde{v}_i \frac{q^f(1-s_i^f)}{\sigma e_i^f}$$

- Optimal tariffs/export taxes $t^{b,r}(\mathbb{J})$ and $t^b(\mathbb{J})$: In search for a closed-form expression
As of now, only opaque system of equations (fixed point w/ demand/multipliers)

Step 3 : Optimal Design of a Climate Agreement – Naive approach

- ▶ Tradeoff extensive/intensive margin
- ▶ Reduction in emissions $\mathcal{E} = \sum_{i \in \mathbb{I}} e_i^f$ depends both on :
 - The level of tax t^f , since high $t^f \Leftrightarrow$ large change in emissions $\Delta \mathcal{E}(\mathbb{J})$
 - The *number* of countries \mathbb{J} in a stable coalition

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 - The *number* of countries \mathbb{J} in a stable coalition
- ▶ Naive approach :
 - Combinatorial problem : $\mathcal{P}(\mathbb{I})$ with $2^{|\mathbb{I}|}$ choices

$$\max_{\mathbb{J} \in \mathcal{P}(\mathbb{I})} \mathcal{W}(\mathbb{J})$$

- Choice of countries \mathbb{J} yields optimal taxes $\{t^f(\mathbb{J}), t^{b,r}(\mathbb{J}), t^b(\mathbb{J})\}$
- Search for complementarity

$$\Delta \mathcal{W}(\mathbb{J}', j) := \mathcal{W}(\mathbb{J}' \cup j) - \mathcal{W}(\mathbb{J}') > \Delta \mathcal{W}(\mathbb{J}, j) \quad \text{when } \mathbb{J}' \supset \mathbb{J} \quad \text{for all } j \in \mathbb{I}$$

Step 3 : Optimal Design – Alternative approach

► Alternative approach : choosing policy first

- From a level of the tax t^f and t^b imposed on club \mathbb{J} , we can deduce the number of countries $\tilde{\mathbb{J}}$ with binding participation constraints

$$\tilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\tilde{c}_i) \quad \forall i \in \tilde{\mathbb{J}}$$

- Search for the couple $\{t^f, t^b\}$ such that $\mathbb{J} = \tilde{\mathbb{J}}$

► What determines the choice of a country to join the climate agreement ?

- Benefit : lower temperature path τ_i , reduce in energy price q^e , increase in domestic good price p_i , etc.
- Costs : carbon tax, tariffs on countries outside the club, decrease in fossil rent

Step 3 : Countries' incentives – Model w/o trade in goods

- ▶ Experiment : Model with trade in energy but not in “goods”
 - Start from the equilibrium where carbon tax $\tau^f(\mathbb{J}) = 0$,
⇒ country i is indifferent to join the club \mathbb{J} or not
 - Linear approximation around that point for small changes in the carbon tax $d\tau^f$

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- Linear approximation around that point for small changes in the carbon tax dt^f
- Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})} = & -e_i dt^f - \gamma_i(\tau_i - \tau_{i0})y_i \Delta_i (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^e \nu}{E_{\mathbb{I}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \pi_i \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \end{aligned}$$

- Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}} = - \left(E_{\mathbb{J}} - E_{\mathbb{J} \setminus \{i\}} \right) \frac{\sigma dt^f}{q^e (1 - s^f)} \frac{1}{1 + \frac{\nu \sigma}{(1 - s^f)}}$$

- Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, Climate damage γ_i

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- Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, Climate damage γ_i

Step 3 : Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$
 - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})c_i} = & -e_i dt^f - \gamma_i(\tau_i - \tau_{i0})\eta_i^y \Delta_i(dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^e \nu}{E_{\mathbb{I}}}(dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E}(dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & + \eta_i^y \left(\frac{dp_i}{p_i} \Big|_{i \in \mathbb{J}} - \frac{dp_i}{p_i} \Big|_{i \notin \mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j \in \mathbb{I}} s_{ij} \left(\frac{dp_j}{p_j} \Big|_{i \in \mathbb{J}} - \frac{dp_j}{p_j} \Big|_{i \notin \mathbb{J}} \right) \end{aligned}$$

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► Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$

- Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})c_i} = & -e_i dt^f - \gamma_i(\tau_i - \tau_{i0})\eta_i^y \Delta_i(dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^e \nu}{E_{\mathbb{J}}}(dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E}(dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & + \eta_i^y \left(\frac{dp_i}{p_i} \Big|_{i \in \mathbb{J}} - \frac{dp_i}{p_i} \Big|_{i \notin \mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j \in \mathbb{I}} s_{ij} \left(\frac{dp_j}{p_j} \Big|_{i \in \mathbb{J}} - \frac{dp_j}{p_j} \Big|_{i \notin \mathbb{J}} \right) \end{aligned}$$

- GE effect on energy markets, same as before
- GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k \in \mathbb{I}} \mathcal{P}_k \alpha_{ki} \left(\frac{dv_k}{v_k} - \theta \frac{dt_{ki}^b}{1+t_{ki}^b} \right) \quad \alpha_{ki} = \frac{c_{ki} d_{ki} p_i}{\sum_{\ell} c_{k\ell} d_{k\ell} p_{\ell}} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+t_{ki}^b)v_i}$$

- Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{v_i p_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Step 3 : Countries' incentives – Armington Model with trade in goods

► Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$

- Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})c_i} = & -e_i dt^f - \gamma_i(\tau_i - \tau_{i0})\eta_i^y \Delta_i(dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^e \nu}{E_{\mathbb{J}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & + \eta_i^y \left(\frac{dp_i}{p_i} \Big|_{i \in \mathbb{J}} - \frac{dp_i}{p_i} \Big|_{i \notin \mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j \in \mathbb{I}} s_{ij} \left(\frac{dp_j}{p_j} \Big|_{i \in \mathbb{J}} - \frac{dp_j}{p_j} \Big|_{i \notin \mathbb{J}} \right) \end{aligned}$$

- GE effect on energy markets, same as before
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$$\frac{dv_i}{v_i} = \sum_{k \in \mathbb{I}} \mathcal{P}_k \alpha_{ki} \left(\frac{dv_k}{v_k} - \theta \frac{dt_{ki}^b}{1+t_{ki}^b} \right) \quad \alpha_{ki} = \frac{c_{ki} d_{ki} p_i}{\sum_{\ell} c_{k\ell} d_{k\ell} p_{\ell}} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+t_{ki}^b) v_i}$$

- Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i p_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Step 3 : Numerical approach

► Algorithm : sequential approach

1. Start from the second-best optimal policy $\{t^f, t^b\}$, on the world $\mathbb{J} = \mathbb{I}$
2. From tax levels imposed on club \mathbb{J} , deduce the number of countries $\tilde{\mathbb{J}}$ with binding participation constraints

$$\tilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\tilde{c}_i) \quad \forall i \in \tilde{\mathbb{J}}$$

- Search for t^f that yield $\mathbb{J} = f(\mathbb{J}, t^f, t^b)$
3. If $Im(f(\mathbb{J}, t^f, t^b)) \subsetneq \mathbb{J}$, remove countries one-by-one
 4. Repeat (2-3) until convergence – fixed point of $\tilde{\mathbb{J}} = f(\mathbb{J}, t^f, t^b)$ – or unraveling

Quantification and numerical method

► Quantification [More details](#)

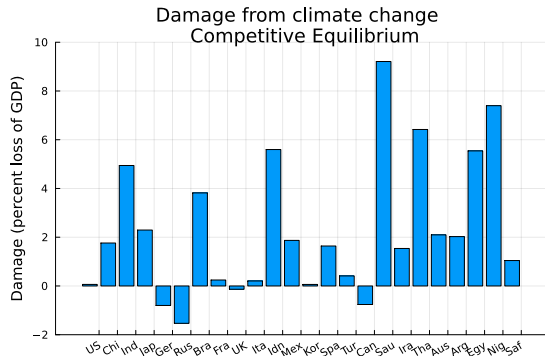
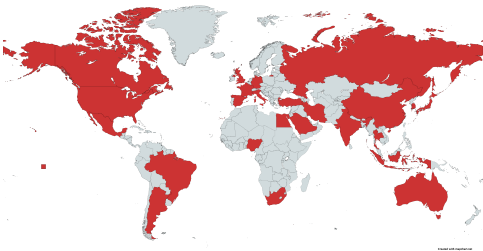
- Production $\bar{y} = zf(k, e^f, e^r)$ with Nested CES capital/energy $\sigma_y < 0$ and fossil/renewable $\sigma_e > 1$. Calibrate parameters to match GDP / energy shares data.
- Quadratic damage as in Nordhaus DICE $y = \mathcal{D}_i(\tau)\bar{y}$ with $\mathcal{D}_i(\tau) = e^{-\gamma(\tau-\tau_i)^2}$
- Energy parameters to match production/reserves

► Numerical method [More details](#)

- Sequential approach : rely on Pontryagin Maximum Principle
- Can simulate models with arbitrary numbers of dimensions of heterogeneity

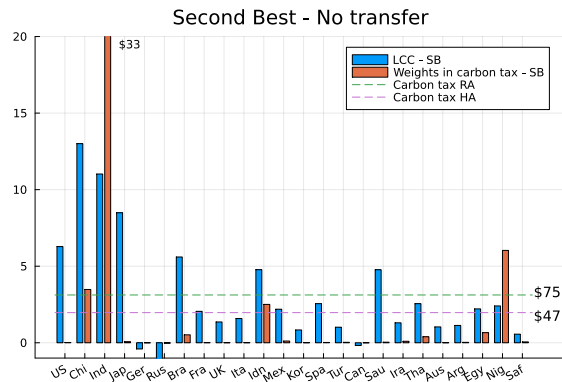
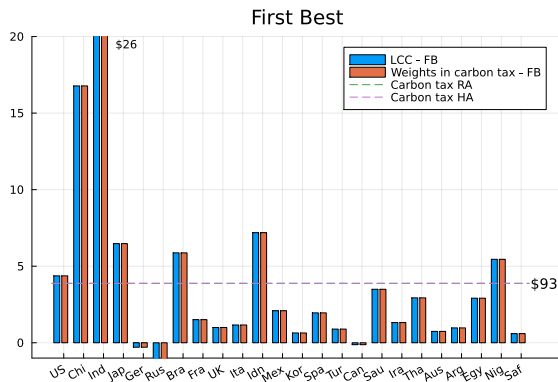
Numerical Application – Competitive equilibrium

- Data : 24 countries, (G20+4 large countries)



Local Cost of Carbon & Carbon Tax – First and Second Best

► Difference $LCC_i = \frac{\psi_i^\mathcal{E}}{\lambda_i^w}$ vs. $\hat{\lambda}_i^w LCC_i = \frac{\psi_i^\mathcal{E}}{\lambda_i^w}$ since $SCC = \sum_{\mathbb{I}} \hat{\lambda}_i^w LCC_i$



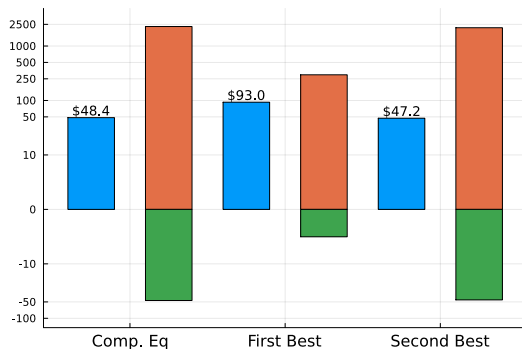
Comparison - Value of wealth vs. Social Cost of Carbon

- Social Cost of Carbon can be decomposed :

$$SCC := -\frac{\partial \mathcal{W} / \partial \mathcal{E}}{\partial \mathcal{W} / \partial c} = \frac{\psi^{\mathcal{E}}}{\lambda} = \frac{\sum_{\mathbb{I}} \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \lambda_i}$$

- Here plot that decomposition :

$$\log(SCC) = \log(\psi^{\mathcal{E}}) - \log(\lambda)$$



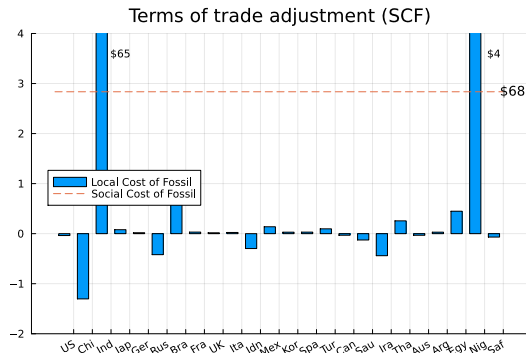
Local Cost of Fossil and Terms of Trade Adjustment

- Social Cost of Fossil Energy :

$$SCF = \mathcal{C}_{EE} \sum_{\mathbb{I}} \hat{\lambda}_i (e_i^f - e_i^x) \quad \mathcal{C}_{EE}^{f-1} = \sum_{\mathbb{I}} \mathcal{C}_{i,e^x}^{f-1}$$

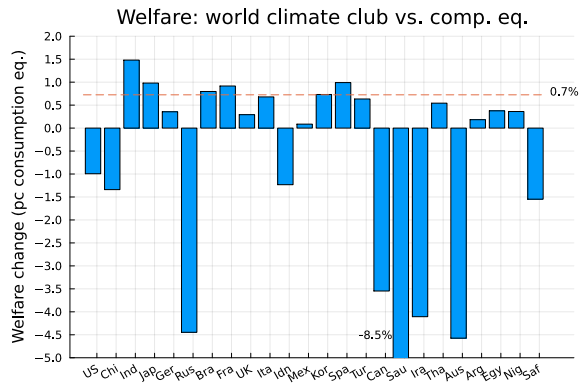
- Here plotting local cost of fossil :

$$LCF_i = \hat{\lambda}_i (e_i^f - e_i^x)$$



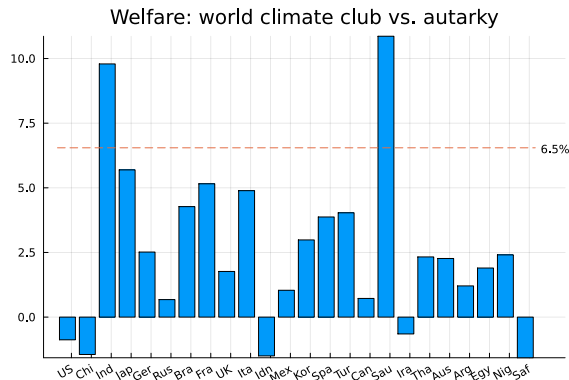
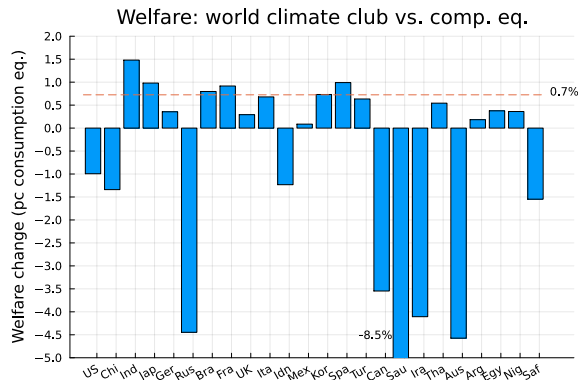
Winner and losers – Second Best vs. Competitive equilibrium

- ▶ Difference $\mathcal{W}_i(\text{II})$ (second-best climate club) vs. \mathcal{V}_i (no climate club)
- ▶ .



Winner and losers – Second Best vs. Outside options

- ▶ Difference $\mathcal{W}_i(\mathbb{I})$ (second-best climate club) vs. \mathcal{V}_i (no climate club)
- ▶ Difference $\mathcal{W}_i(\mathbb{I})$ (second-best climate club) vs. $\mathcal{W}_i(\mathbb{I} \setminus \{i\})$ (outside options)



Conclusion

- ▶ In this project, I solve for the optimal climate policy
 - Accounting for inequality as it depends on the availability of transfer mechanisms
 - Redistributing through general equilibrium on energy and good markets \Rightarrow terms-of-trade effects
- ▶ Climate agreement design jointly solves for :
 - The optimal choice of countries participating
 - The carbon tax level, both for correcting externality & respecting participation constraints
- ▶ Differing incentive to join
 - Benefit : change in climate due to participation, cost through taxation, loss in energy rent, GE effect on price
 - Complementarity : the larger the group, the higher the effect on (1) climate, (2) energy price, (3) price of outside countries $i \notin \mathbb{J}$

Appendices

More details – Capital market

- In each country, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} &= \mathcal{D}_i^y(\tau_{it})z_{if}(k_{it}, e_{it}) - (\delta + n + \bar{g})k_{it} + \iota_{it} \\ \dot{b}_{it} &= r^*b_{it} + \theta_i\pi_t^f + \pi_{it}^r - (q_t^f + \mathfrak{t}_{it}^f)e_{it}^f - (q_t^r + \mathfrak{t}_{it}^r)e_{it}^r - \iota_{it} - c_{it} + \mathfrak{t}_{it}^{ls} \\ b_{it} &\geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^*w_{it} + \mathcal{D}_i^y(\tau_{it})z_{if}(k_{it}, e_{it}) - (\bar{\delta} + r_t^*)k_{it} + \theta_i\pi_t^f + \pi_{it}^r - (q_t^f + \mathfrak{t}_{it}^f)e_{it}^f - (q_t^r + \mathfrak{t}_{it}^r)e_{it}^r - c_{it} + \mathfrak{t}_{it}^{ls}$$

$$k_{it} \leq \frac{1}{1 - \vartheta} w_{it}$$

- Two polar cases :
- $\vartheta \rightarrow 0$, full autarky (no trade), and $w_{it} = k_{it}$
 - $\vartheta \rightarrow 1$, full financial integration :

$$k_{it} \quad s.t. \quad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it})z_{if}(k_{it}, e_{it}) - (\delta + n + \bar{g})k_{it} = r_t^*$$

Impact of increase in temperature

- Marginal values of the climate variables : λ_{it}^s and λ_{it}^τ

$$\dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho + \zeta) + \overbrace{\gamma_i(\tau_{it} - \tau_i^*) \mathcal{D}^y(\tau_{it}) f(k_{it}, e_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} \lambda_{it}^k + \overbrace{\phi_i(\tau_{it} - \tau_i^*) \mathcal{D}^u(\tau_{it})^{1-\eta} c_{it}^{1-\eta}}^{\partial_\tau u(c, \tau)}$$

$$\dot{\lambda}_{it}^s = \lambda_{it}^s(\rho + \delta^s) - \zeta \chi \Delta_i \lambda_{it}^\tau$$

- Costate λ_{it}^s : marg. cost of 1Mt carbon in atmosphere, for country i . Increases with :
- Temperature gaps $\tau_{it} - \tau_i^*$ & damage sensitivity of TFP γ_i^y and utility γ_i^u
 - Development level $f(k_{it}, e_{it})$ and c_{it}
 - Climate params : χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
 - [back](#)

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature \Leftrightarrow solving ODE

$$\dot{\lambda}_{it}^{\tau} = \lambda_t^{\tau}(\tilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^*)\mathcal{D}^y(\tau)f(k, e)\lambda_t^k + \phi(\tau - \tau^*)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}_t^S = \lambda_t^S(\tilde{\rho} + \delta^S) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda_{it}^{\tau}$$

- Solving for λ_t^{τ} and λ_t^S , in stationary equilibrium $\dot{\lambda}_t^S = \dot{\lambda}_t^{\tau} = 0$

$$\lambda_{it}^{\tau} = - \int_t^{\infty} e^{-(\tilde{\rho} + \zeta)u} (\tau_u - \tau^*) \left(\gamma \mathcal{D}^y(\tau_u) y_{\tau} \lambda_u^k + \phi \mathcal{D}^u(\tau_u) u(c_u) \right) du$$

$$\lambda_{it}^{\tau} = - \frac{1}{\tilde{\rho} + \Delta\zeta} (\tau_{\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{\infty}) y_{\infty} \lambda_{\infty}^k + \phi \mathcal{D}^u(\tau_{\infty}) u(c_{\infty}) \right)$$

$$\lambda_t^S = - \int_t^{\infty} e^{-(\tilde{\rho} + \delta^S)u} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,u}^{\tau} dj du$$

$$= \frac{1}{\tilde{\rho} + \delta^S} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,\infty}^{\tau}$$

$$= - \frac{\chi}{\tilde{\rho} + \delta^S} \frac{\zeta}{\tilde{\rho} + \zeta} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{j,\infty}) y_{\infty} \lambda_{j,\infty}^k + \phi \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

$$\lambda_t^S \xrightarrow{\zeta \rightarrow \infty} - \frac{\chi}{\tilde{\rho} + \delta^S} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{j,\infty}) y_{j,\infty} \lambda_{j,\infty}^k + \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

Cost of carbon / Marginal value of temperature

► *Proposition (Stationary LSCC) :*

When $t \rightarrow \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \rightarrow \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ , **marg. damage** γ_i^y , γ_i^u , **temperature**, and **output, consumption**.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^*) \left(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \right)$$

- Stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
- Fast temperature adjustment $\zeta \rightarrow \infty$
- [Back](#)

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta \chi \xi \omega \int_{t_0}^T e^{(n+\bar{g})t - \delta_s(T-t)} q_t^f - \sigma_e \int_{j \in \mathbb{I}} (z_j z_{j,t}^e \mathcal{D}(\tau_{j,t}))^{\sigma-1} y_{j,t} q_{j,t}^{\sigma_e - \sigma} dj dt$$

- Geographical factors determining warming Δ_i
- Climate sensitivity χ & carbon exit from atmosphere δ_s
- Growth of population n , aggregate productivity \bar{g}
- Deviation of output from trend y_i & relative TFP z_j
- Directed technical change z_t^e , elasticity of energy in output σ
- Fossil energy price q^{ef} and Hotelling rent $g^{qf} \approx \dot{\lambda}_t^R / \lambda_t^R = \rho$
- Change in energy mix, renewable share ω , price q_t^r & elasticity of source σ_e

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity [More details](#)

$$\frac{\dot{\tau}_T}{\tau_T} \propto n + \bar{g}^y - (1 - \sigma)(g^{z^e} - \tilde{\gamma}) + (\sigma_e - \sigma)(1 - \omega)g^{q^r} - (\sigma_e(1 - \omega) + \sigma\omega)g^{q^f}$$

Equilibrium – Mean Field Games

- Uniqueness : Lasry-Lions monotonicity on the function :

$$\int_{\mathbb{I}} [f(t, x, \mu', \alpha) - f(t, x, \mu, \alpha)] d(\mu' - \mu)(x) \geq 0 \quad \mu, \mu' \in \mathcal{P}(\mathbb{X})$$

- Work in progress : checking such conditions along the transition

$$\sum_{i \in \mathbb{I}} (u(c^*(w, \tau, p')) - u(c^*(w, \tau, p))) [p'(w, \tau) - p(w, \tau)] \geq 0$$

with $p'(w, \tau)$ empirical distribution $p'(w, \tau) = \frac{1}{\mathbb{I}} \sum_{i \in \mathbb{I}} \delta_{\{x_i = (w, \tau)\}} \equiv$ population distribution !

- Mean Field approximation & Carmona Delarue (2013)

- Mean-Field is an ε -equilibrium of the N -player game when $N \rightarrow \infty$
- Require symmetry and invariance under permutation
- [Back](#)

Sequential solution method

► Summary of the model :

- ODEs for states $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^\tau, \lambda_{it}^s\}_{it}$
- Non-linear equations (FOCs) for household controls $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness [More details](#)

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- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness [More details](#)

► Global Numerical solution :

- Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
- Express as a large vector $\mathbf{y} = \{\mathbf{x}, \boldsymbol{\lambda}, \mathbf{c}, \mathbf{q}\}$ in a large non-linear function

$$F(\mathbf{y}) = \mathbf{0}$$

- Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

► Why use a sequential approach ?

- *Global approach* : Only need to follow the trajectories for i agents :
 - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
Productivity z_i *Population* p_i , *Temperature scaling* Δ_i , *Fossil energy cost* $\bar{\nu}_i$, *Energy mix* $\epsilon_i, \omega_i, z_i^r$,
Local damage $\gamma_i^y, \gamma_i^u, \tau_i^*$, *Directed Technical Change* z_i^e
 - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
For now : *Wealth* w_{it} , *temperature* τ_{it} , *reserves* \mathcal{R}_{it} , *Carbon* S_t
Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
 - Newton method & Non-linear solvers very efficient

► Why not :

- Numerical constraint to solve a large system of ODEs and non-linear equations :
 ⇒ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
- Relying on numerical solvers/structure of the problem can be opaque

[back](#)

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \varepsilon(e^f, e^r)) = \left[(1 - \epsilon_i)^{\frac{1}{\sigma_y}} k^{\alpha \frac{\sigma_y - 1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} (z_i^e \varepsilon_i(e^f, e^r))^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega_i^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1 - \omega_i)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

- Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
- Today : $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
- Future : $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

- Damage functions in production function y :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm, y} (\tau - \tau_i^*)^2}$$

- Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+, y} \mathbb{1}_{\{\tau > \tau_i^*\}} + \gamma^{-, y} \mathbb{1}_{\{\tau < \tau_i^*\}}$
- Today $\gamma_i^{\pm, y} = \bar{\gamma}^{\pm, y}$ & $\tau_i^* = \bar{\alpha} \tau_{it0} + (1 - \bar{\alpha}) \tau^*$

Quantification – Energy markets

► Fossil production e_{it}^x and reserve \mathcal{R}_{it}

- Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i} \left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i} \mathcal{R}$
- Now : $\bar{\nu}_i$ to match extraction data e_i^x , \mathcal{R}_{it} calibrated to *proven reserves* data from BP.
 $\nu_i = \nu = 1$ quadratic extraction cost.
- Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost \mathcal{C}_e & extraction data e_i^x (BP, IEA)

Quantification – Energy markets

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 $\nu_i = \nu = 1$ quadratic extraction cost.
- Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost \mathcal{C}_e & extraction data e_i^x (BP, IEA)

► Renewable : Production \bar{e}_{it}^r and price q_{it}^r

- Now : $q_{it}^r = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
- Future : Choose z_i^r to match the energy mix (e_i^f, e_i^r)

[back](#)

Quantification – Future Extensions :

► Damage parameters :

- $\gamma_i^{\pm,y}$ depends on daily temperature distribution $\tau \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
- Use Climate Lab's (Greenstone et al) estimates for damage γ_i ?

► Fossil Energy markets :

- Divide fossils e_{it}^f / e_{it}^x into oil/gas/coal
- Match the production/cost/reserves data across countries
- Use a dynamic model : extraction/exploration a la Hotelling

► Renewables Energy markets :

- Make the problem dynamic with investment in capacity C_{it}^r

► Population dynamics

- Match UN forecast for growth rate / fertility

Calibration

TABLE – Baseline calibration (★ = subject to future changes)

<i>Technology & Energy markets</i>			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
\bar{g}	0.01★	Long run TFP growth	Conservative estimate for growth
g_e	0.01★	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	-0.01★	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	2★	Extraction elasticity of fossil energy	Cubic extraction cost
<i>Preferences & Time horizon</i>			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	0.01★	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

TABLE – Baseline calibration (★ = subject to future changes)

<i>Climate parameters</i>			
ξ	0.81	Emission factor	Conversion 1 <i>MTOE</i> \Rightarrow 1 <i>MT CO₂</i>
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature \sim 11–15 years
χ	2.1/1e6	Climate sensitivity	Pulse experiment : 100 <i>GtC</i> \equiv 0.21°C medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment : 100 <i>GtC</i> \equiv 0.16°C long-term warming
γ^{\oplus}	0.00234★	Damage sensitivity	Nordhaus' DICE
γ^{\ominus}	$0.2 \times \gamma^{\oplus}$ ★	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^{τ}	0.2★	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
τ^{\star}	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
<i>Parameters calibrated to match data</i>			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
τ_i		Local Temperature	To match temperature of largest city
\mathcal{R}_i		Local Fossil reserves	To match data from BP Energy review

Step 4 : the Design of a Climate agreement

- Welfare effect : 1st order :

$$\begin{aligned} \delta(\mathbb{J}, j) &= \mathcal{W}_{t_0}(\mathbb{J} \cup \{j\}) - \mathcal{W}_{t_0}(\mathbb{J}) = \omega_j u(c_{jt}) + \sum_{i \in \mathbb{J}} \Delta \mathcal{W}_i \\ \Delta \mathcal{W}_i &\approx d\mathcal{W}_i = \lambda_i^w (1 - \theta_i) \underbrace{(\epsilon_i^e + (1 - \epsilon_i^e) \alpha \sigma_i^{k/e})}_{\text{production } f(k,e)} \underbrace{(-\omega_i^f \sigma_i^f + (1 - \omega_i^f) \sigma_i^{r/f})}_{\text{energy use } \varepsilon(e^f, e^r)} \left(\underbrace{\frac{t^f}{q^f + t^f} \frac{dt^f}{t^f}}_{\text{tax change}} + \underbrace{\frac{q^f}{q^f + t^f} \frac{dq^f}{q^f}}_{\text{GE effect}} \right) \\ &\quad + \lambda_i^w \underbrace{\theta_i (1 + \frac{1}{\nu_i})}_{\text{fossil rent/supply}} \underbrace{\frac{q^f}{q^f + t^f} \frac{dq^f}{q^f}}_{\text{GE effect}} + \underbrace{\psi_i^S}_{\tau_i \text{ damage}} \left[\underbrace{\chi \sum_{j \in \mathbb{I}} \varepsilon_j \sigma_j^f}_{\text{climate sens}^{ty}} \right] \left(\underbrace{\frac{t^f}{q^f + t^f} \frac{dt^f}{t^f}}_{\text{tax change}} + \underbrace{\frac{q^f}{q^f + t^f} \frac{dq^f}{q^f}}_{\text{GE effect}} \right) \end{aligned}$$

- **Direct effect** on energy use on production and substitutability with renewable
 cost-share ϵ_e , fossil-share ω_i , elasticity σ_j^f & capital-energy cross elast^{ty}. $\sigma_{k,e}$, fossil-renewable cross elast^{ty}. $\sigma_i^{r/f}$
- **Indirect effect** through energy market fossil rent θ_i , supply elasticity ν_i
- **Indirect climate effect** of a reduction in world emissions

Sequential solution method

► Summary of the model :

- ODEs for states $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^\tau, \lambda_t^S, \lambda_{it}^{\mathcal{R}}\}_{it}$
- Non-linear equations (FOCs) for household controls $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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► Global Numerical solution :

- Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
- Express as a large vector $\mathbf{y} = \{\mathbf{x}, \boldsymbol{\lambda}, \mathbf{c}, \mathbf{q}\}$ in a large non-linear function

$$F(\mathbf{y}) = \mathbf{0}$$

- Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

► Why use a sequential approach ?

- *Global approach* : Only need to follow the trajectories for i agents :
 - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
Productivity z_i *Population* p_i , *Temperature scaling* Δ_i , *Fossil energy cost* $\bar{\nu}_i$, *Energy mix* $\epsilon_i, \omega_i, z_i^r$,
Local damage $\gamma_i^y, \gamma_i^u, \tau_i^*$, *Directed Technical Change* z_i^e
 - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
For now : *Wealth* w_{it} , *temperature* τ_{it} , *reserves* \mathcal{R}_{it} , *Carbon* S_t
Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
 - Newton method & Non-linear solvers very efficient

► Why not :

- Numerical constraint to solve a large system of ODEs and non-linear equations :
⇒ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
- Relying on numerical solvers/structure of the problem can be opaque

back