

The Inequality of Climate Change & Optimal Energy policy

WORK IN PROGRESS

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Introduction

- ▶ What is the optimal taxation of energy in the presence of climate externality *and* inequality?
 - In a context where fossil fuels taxation and climate policy redistribute across countries

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 - In a context where fossil fuels taxation and climate policy redistribute across countries
- ▶ Develop a simple and flexible model of climate economics
 - Standard IAM model with heterogeneous regions
 - Normative implications : Optimal Ramsey policy for carbon taxation
 - Provide a numerical methodology and a quantitative model

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 - In a context where fossil fuels taxation and climate policy redistribute across countries
- ▶ Develop a simple and flexible model of climate economics
 - Standard IAM model with heterogeneous regions
 - Normative implications : Optimal Ramsey policy for carbon taxation
 - Provide a numerical methodology and a quantitative model
- Evaluate the heterogeneous welfare costs of global warming
 - Climate damages & temperature varies across countries
 - ⇒ Inequality increases the welfare cost of carbon
- Solve world optimal carbon policy with heterogeneous regions
 - Does the optimal carbon tax coincide with the social cost of carbon ?
 - ⇒ Depends on transfer policy : need to adjust the tax for inequality level

Preview of the findings

- ▶ In a large class of IAM models, optimal carbon policy goes hand in hand with the availability of redistribution instruments
- ▶ Case 1 : (First Best)
 - Energy tax common for all countries i and proportional to the Social Cost of Carbon
 - Lump-sum taxes and transfers redistribute across countries
- ▶ Case 2 :
 - Without such instruments, energy taxes are country i -specific & account for redistribution
 - Tax scales with Pareto weights ω_i and marg. utility of consumption $U'(c_i)$
 \Rightarrow lower energy tax for poorer countries
 - Also accounts for redistribution through energy markets, due to changes in terms-of-trade
- ▶ Case 3 :
 - If countries can exit climate agreements, one needs to account for participation constraints
 - Tax may be lower for countries with better outside options

Model – Household

- ▶ Deterministic Neoclassical economy, in continuous time
 - heterogeneous countries $i \in \mathbb{I}$
 - In each country, 4 agents : (i) representative household, (ii) homogeneous good firm, (iii) fossil and (iv) renewable energy producers.
- ▶ Representative household problem in each country i :

$$\mathcal{V}_{it_0} = \max_{\{c_{it}, k_{it}, b_{it}\}} \int_{t_0}^{\infty} e^{-\rho t} u(c_{it}) dt$$

- ▶ Dynamics of wealth of country i , $w_{it} = b_{it} + k_{it}$ [More details](#)

$$\dot{w}_{it} := \dot{k}_{it} + \dot{b}_{it} = y_{it} + \pi_{it}^f + \pi_{it}^r + r_t^* b_{it} + (r_t^* - \bar{\delta}) k_{it} - c_{it} + t_{it}^{ls}$$

- Labor income y_{it} from homogeneous good firm.
- All the lower-case variables are expressed per unit of efficient labor $y_{it} = Y_{it}/(L_{it}A_{it})$

Model – Representative Firm

- Competitive homogeneous good producer in country i

$$\max_{k_{it}, e_{it}^f, e_{it}^r} \mathcal{D}^y(\tau_{it}) z_i f(k_{it}, e_{it}^f, e_{it}^r) - r_t^* k_{it} - (q_t^f + t_{it}^f) e_{it}^f - (q_t^r + t_{it}^r) e_{it}^r - y_{it}$$

- Energy mix with fossil e_{it}^f – emitting carbon – subject to price q_t^f and tax/subsidy t_{it}^f . Similarly “clean” renewable e_{it}^r , at price q_t^r and tax t_{it}^r .
- No international trade in goods and Labor is immobile

Model – Energy markets

► Competitive fossil fuels energy producer :

- Static problem (for now) extract energy e_{it}^x depleting reserves \mathcal{R}_{it}

$$\pi_{it}^f = \max_{e_{it}^x} q_t^f e_{it}^x - \mathcal{C}_i^f(e_{it}^x, \mathcal{R}_{it})$$

$$\dot{\mathcal{R}}_{it} = -e_{it}^x \quad \mathcal{R}_{it_0} = \mathcal{R}_{i0} \quad \mathcal{R}_{it} \geq 0$$

- Fossil energy traded in international markets :

$$\int_{\mathbb{I}} e_{it}^f di = \int_{\mathbb{I}} e_{it}^x di$$

- Optimal extraction

$$q_t^f = \mathcal{C}_e^f(e_{it}^x, \mathcal{R}_{it})$$

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- Fossil energy traded in international markets :

$$\int_{\mathbb{I}} \bar{e}_{it}^f di = \int_{\mathbb{I}} e_{it}^x di$$

- Optimal extraction

$$q_t^f = \mathcal{C}_e^f(e_{it}^x, \mathcal{R}_{it})$$

► Renewable energy as a substitute technology in each country i (Static problem for now)

$$\pi_{it}^r = \max_{\{\bar{e}_{it}^r\}} q_{it}^r \bar{e}_{it}^r - \mathcal{C}_i^r(\bar{e}_{it}^r) \quad \Rightarrow \quad q_{it}^r = \mathcal{C}_e^r(\bar{e}_{it}^r) = z_{it}^r$$

Climate system

- ▶ Fossil energy input e_t^f causes climate externality

$$\mathcal{E}_t = \int_{\mathbb{I}} e_{it}^f di$$

- ▶ Cumulative GHG in atmosphere \mathcal{S}_t increases temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- ▶ Country's local temperature :

$$\dot{\tau}_{it} = \zeta \left(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \bar{\tau}_{it_0}) \right)$$

- Linear model : Climate sensitivity to carbon χ , Climate reaction/inertia ζ , Country i linear pattern scaling factor Δ_i , Carbon exit from atmosphere δ_s

Model – Solution

► Case 0 : Competitive equilibrium

- Absence of Policies : Taxes $t_{it}^f = t_{it}^r = t_{it}^{ls} = 0$

Model – Solution

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► Case 1 : First Best

- Planner maximize aggregate welfare

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r, \mathbf{t}_{it}^{ls}, \dots\}_{it}} \int_{\mathbb{I}} \int_{t_0}^{\infty} e^{-\bar{\rho}t} \omega_i u(c_{it}) dt di$$

- All instruments available $\mathbf{t} = \{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r, \mathbf{t}_{it}^{ls}\}_{it}$, including transfers across countries.

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► Case 2 : Limited transfers

- Ramsey policy, where lump-sum transfers across countries are prohibited
- Country-specific energy taxes $\mathbf{t}_{it}^f, \mathbf{t}_{it}^r$ and lump-sum (local) rebate $\mathbf{t}_{it}^{ls} = \mathbf{t}_{it}^f e_{it}^f + \mathbf{t}_{it}^r e_{it}^r$

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► Case 3 (Work in progress)

- Prohibited lump-sum transfers & countries can exit climate agreements
- Participation constraint, with \bar{c}_{it} consumption in autarky

$$u(c_{it}) \geq u(\bar{c}_{it}) \quad \forall t \geq 0$$

Model – Equilibrium

► Equilibrium

- Given, initial conditions $\{w_{i0}, \tau_{i0}, \mathcal{R}_{i0}\}$ and country-specific policies $\{t_{it}^f, t_{it}^r, t_{it}^{ls}\}$, a **competitive equilibrium** is a continuum of sequences of states $\{w_{it}, \tau_{it}, \mathcal{R}_{it}\}_{it}$ and $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, policies $\{c_{it}, b_{it}, k_{it}, e_{it}^f, e_{it}^r, e_{it}^x\}_{it}$ and price sequences $\{r_t^*, q_t^f, q_t^r\}$ such that :
 - Households choose policies $\{c_{it}, b_{it}\}_{it}$ to max utility s.t. budget constraint, giving \dot{w}_{it}
 - Firm choose policies $\{k_{it}, e_{it}^f, e_{it}^r\}_{it}$ to max profit
 - Fossil and renewables firms extract/produce $\{e_{it}^x, \bar{e}_{it}^r\}_{it}$ to max static profit, yielding $\dot{\mathcal{R}}_t$
 - Emissions \mathcal{E}_t affects climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{it}\}_{it}$.
 - Prices $\{r_t^*, q_t^f, q_t^r\}$ adjust to clear the markets : $\int_{\mathbb{I}} e_{it}^x di = \int_{\mathbb{I}} e_{it}^f di$ and $e_{it}^r = \bar{e}_{it}^r$, and $\int_{\mathbb{I}} b_{it} di = 0$, with bonds $b_{it} = w_{it} - k_{it}$
- Existence and Uniqueness : Mean Field Game result under Lasry-Lions Monotonicity condition (work in progress)

Case 0 : Competitive equilibrium

► Household consumption/saving problem

- Using Pontryagin Max. Principle : states $\{x\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}\}$, controls $\{c\} = \{c_{it}, b_{it}, k_{it}\}$ and costates $\{\lambda\} = \{\lambda_{it}^w, \lambda_{it}^\tau, \lambda_{it}^s\} \Rightarrow$ system of coupled ODEs.

$$\mathcal{H}^{hh}(\{x\}, \{c\}, \{\lambda\}) = u(c_i) + \lambda_{it}^w \dot{w}_{it} + \lambda_{it}^\tau \dot{\tau}_{it} + \lambda_{it}^s \dot{S}_t$$

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- ODE for the costate for wealth $\lambda_{it}^w = u'(c_{it}) \Rightarrow$ Euler equation
- The “local social cost of carbon” (LCC) for region i :

$$LCC_{it} := -\frac{\partial \mathcal{V}_{it} / \partial \mathcal{S}_t}{\partial \mathcal{V}_{it} / \partial w_{it}} = -\frac{\lambda_{it}^s}{\lambda_{it}^w}$$

- ODEs for Costates : temperature λ_{it}^τ and carbon λ_{it}^s , [More details](#)
- Stationary equilibrium closed-form formula, analogous to GHKT (2014) [Here](#)

Case 1 : First-Best, Optimal policy with transfers

- First-Best, Maximizing welfare of the Social Planner :

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t}, \mathbf{x}, \mathbf{c}, \mathbf{q}\}_{it}} \int_{\mathbb{I}} \int_{t_0}^{\infty} e^{-\bar{\rho}t} \omega_i u(c_{it}) dt di = \int_{\mathbb{I}} \mathcal{W}_{it_0} di$$

- Full set of instruments $\mathbf{t} = \{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r, \mathbf{t}_{it}^{ls}\}$, including transfers *across countries*

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- Full set of instruments $\mathbf{t} = \{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r, \mathbf{t}_{it}^{ls}\}$, including transfers *across countries*

- Social Planner Hamiltonian

States $\{x\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}\}_{it}$, controls $\{c\} = \{c_{it}, b_{it}, k_{it}, e_{it}^f, e_{it}^r, e_{it}^x\}_{it}$ and costates $\{\psi\} = \{\psi_{it}^w, \psi_{it}^\tau, \psi_{it}^s\}_{it} \Rightarrow$ system of coupled ODEs.

$$\mathcal{H}^{sp}(\{x\}, \{c\}, \{\psi\}) = \int_{\mathbb{I}} \omega_i u(c_i) di + \int_{\mathbb{I}} \left(\psi_{it}^w \dot{w}_{it} + \psi_{it}^\tau \dot{\tau}_{it} + \psi_{it}^s \dot{\mathcal{S}}_{it} \right) di$$

Social Cost of Carbon :

► Key Objects : Social Cost of Carbon

► Local :

$$LCC_{it} := -\frac{\partial \mathcal{W}_{it} / \partial \mathcal{S}_t}{\partial \mathcal{W}_{it} / \partial w_{it}} = -\frac{\psi_{it}^S}{\psi_{it}^w}$$

- Intuitives ODEs for costates ψ_{it}^S and ψ_{it}^w [More details](#)

► Global :

$$SCC_t := -\frac{\partial \mathcal{W}_t / \partial \mathcal{S}_t}{\partial \mathcal{W}_t / \partial w_t} = -\frac{\psi_t^S}{\psi_t^w} = -\frac{\int_{\mathbb{I}} \psi_{it}^S di}{\int_{i \in \mathbb{I}} \psi_{it}^w di}$$

Case 1 : First-Best, Optimal policy with transfers

- Proposition 1 : Optimal carbon tax :

$$t_t^S = -\frac{\psi_t^S}{\psi_t^W} = SCC_t$$

- Same as in Representative Agent economy, c.f. GHKT (2014)

$$SCC_t = -\frac{\psi_t^S}{\psi_t^W} = -\int_{\mathbb{I}} \frac{\psi_{it}^S}{\psi_t^W} di = \mathbb{E}_i[LCC_{it}]$$

- Lump-sum transfers redistribute across countries, s.t.

$$\omega_i u'(c_{it}) = \psi_{it}^W = \psi_t^W = \psi_{jt}^W = \omega_j u'(c_{jt}) \quad \forall i, j \in \mathbb{I}$$

- Imply cross-countries lump-sum transfers $\exists i$ s.t. $T_i \geq 0$ or $\exists j$ s.t. $T_j \leq 0$
- There exist Pareto weights $\{\omega_i\}$ shutting down redistribution $T_i = 0$, e.g. $\omega_i = 1/u'(c_{it})$

Case 2 : Ramsey policy with limited transfers

► Second best without access to lump-sum transfers

- Only region- i -specific distortive energy taxes : $\{t_{it}^f, t_{it}^r\}$.
 \Rightarrow Tax receipts redistributed lump-sum : $t_{it}^{ls} = t_{it}^f e_{it}^f + t_{it}^r e_{it}^r$
- Implies inequality across regions :

$$\hat{\psi}_{it}^w = \frac{\psi_{it}^w}{\psi_t^w} = \frac{\omega_i u'(c_{it})}{\int_{\mathbb{I}} \omega_j u'(c_{jt}) dj} \leq 1$$

\Rightarrow ceteris paribus, poorer countries have higher $\hat{\psi}_{it}^w$

- Social Cost of Carbon integrates these inequalities :

$$\begin{aligned} SCC_t &= - \int_{\mathbb{I}} \hat{\psi}_{it}^w \underbrace{\frac{\psi_{it}^S}{\psi_{it}^w}}_{=-LCC_{it}} di \\ SCC_t &= \text{Cov}_i \left(\hat{\psi}_{it}^w, LCC_{it} \right) + \mathbb{E}_i [LCC_{it}] \end{aligned}$$

Case 2 : Ramsey Problem – Optimal Carbon & Energy Policy

- Optimal Pigouvian carbon tax :

$$t_{it}^S = \frac{1}{\widehat{\psi}_{it}^w} SCC_t$$

- Integrate redistribution motives, both :
 - for the distribution of tax : countries with higher $\widehat{\psi}_{it}^w \propto \omega_i u'(c_{it})$ have lower tax t_{it}^S
 - for the level : $SCC_t = \text{Cov}_i(\widehat{\psi}_{it}^w, LCC_{it}) + \mathbb{E}_i[LCC_{it}]$
- Implementation : taxing carbon amounts to taxing fossil fuels/energy

Case 2 : Ramsey Problem – Optimal Carbon & Energy Policy

- Taxing fossil energy has additional redistributive effects :
 - Lowering the equilibrium price of fossil fuels benefit importers and hurt exporters
 - New measure of this effect : Social Cost of Fossil (SCF)

$$SCF_t := \frac{\partial \mathcal{W}_t / \partial E_t^f}{\partial \mathcal{W}_t / \partial w_t} = \mathcal{C}_{EE}^f \text{Cov}_i \left(\widehat{\psi}_{it}^w, e_{it}^f - e_{it}^x \right) \quad \mathcal{C}_{EE}^f = \left(\int_{i \in \mathbb{I}} \frac{1}{\mathcal{C}_{i,e^x e^x}^f} dj \right)^{-1}$$

- with \mathcal{C}_{EE}^f and $\mathcal{C}_{i,e^x e^x}^f \propto$ fossil energy supply elasticity

Case 2 : Ramsey Problem – Optimal Carbon & Energy Policy

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 - Lowering the equilibrium price of fossil fuels benefit importers and hurt exporters
 - New measure of this effect : Social Cost of Fossil (SCF)

$$\textcolor{red}{SCF}_t := \frac{\partial \mathcal{W}_t / \partial E_t^f}{\partial \mathcal{W}_t / \partial w_t} = \mathcal{C}_{EE}^f \text{Cov}_i \left(\widehat{\psi}_{it}^w, \textcolor{red}{e}_{it}^f - \textcolor{red}{e}_{it}^x \right) \quad \mathcal{C}_{EE}^f = \left(\int_{i \in \mathbb{I}} \frac{1}{\mathcal{C}_{i, e^x e^x}^f} dj \right)^{-1}$$

– with \mathcal{C}_{EE}^f and $\mathcal{C}_{i, e^x e^x}^f \propto$ fossil energy supply elasticity

- ▶ Proposition 2 : Optimal fossil energy tax :

$$\Rightarrow \quad \mathfrak{t}_{it}^f = \frac{1}{\widehat{\psi}_{it}^w} [\textcolor{green}{SCC}_t + \textcolor{red}{SCF}_t]$$

- ▶ What about renewable energy e_t^r ?
 - Not traded, with constant return to scale, and not carbon intensive, hence :

$$\mathfrak{t}_{it}^r = 0$$

Case 3 : Ramsey Problem with participation constraints

- ▶ Assume that lump-sum transfers are prohibited & countries can exit climate agreements
 - Participation constraint, with \bar{c}_i autarky consumption (no trade in energy/assets)

$$u(c_{it}) \geq u(\bar{c}_{it}) \quad [\nu_{it}]$$

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- ▶ Proposition 3 : Second-Best without transfers & participation constraints
 - Participation incentive change our measure of inequality

$$\tilde{\psi}_{it}^w \propto \omega_i u'(c_{it}) + \nu_{it} u'(c_{it}) \neq \hat{\psi}_{it}^w$$

- Optimal fossil energy tax :

$$\Rightarrow \quad \mathbf{t}_{it}^f = \frac{1}{\tilde{\psi}_{it}^w} [\textcolor{green}{SCC}_t + \textcolor{red}{SCF}_t]$$

– With levels changing $\textcolor{green}{SCC}_t = \text{Cov}_j(\tilde{\psi}_{it}^w, \textcolor{green}{LSCC}_{jt}) + \mathbb{E}_j[\textcolor{green}{LSCC}_{jt}]$

$$\textcolor{red}{SCF}_t = \mathcal{C}_{EE}^f \text{Cov}_j(\tilde{\psi}_{it}^w, \textcolor{red}{e}_{jt}^f - \textcolor{red}{e}_{jt}^x)$$

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \varepsilon(e^f, e^r)) = \left[(1 - \epsilon) \frac{1}{\sigma_y} k^{\alpha \frac{\sigma_y - 1}{\sigma_y}} + \epsilon \frac{1}{\sigma_y} (z_i^e \varepsilon(e^f, e^r))^{\frac{\sigma_y - 1}{\sigma_y}} \right] \frac{\sigma_y}{\sigma_y - 1}$$

$$\varepsilon(e^f, e^r) = \left[\omega \frac{1}{\sigma_e} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1 - \omega) \frac{1}{\sigma_e} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}} \right] \frac{\sigma_e}{\sigma_e - 1}$$

- Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
- Today : $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
- Future : $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

- Damage functions in production function y :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm, y} (\tau - \tau_i^*)^2}$$

- Asymmetry in damage to match empirical evidence, with
 $\gamma^y = \gamma^{+, y} \mathbb{1}_{\{\tau > \tau_i^*\}} + \gamma^{-, y} \mathbb{1}_{\{\tau < \tau_i^*\}}$
- Today $\gamma_i^{\pm, y} = \bar{\gamma}^{\pm, y}$ & $\tau_i^* = \bar{\alpha} \tau_{it_0} + (1 - \bar{\alpha}) \tau^*$

Quantification – Energy markets

► Fossil production e_{it}^x and reserve \mathcal{R}_{it}

- Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i} \left(\frac{e^x}{\mathcal{R}} \right)^{1+\nu_i} \mathcal{R}$
- Now : $\bar{\nu}_i = \bar{\nu}$ and $\nu_i = \nu$ and \mathcal{R}_{it} calibrated to *proven reserves* data from BP.
- Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost \mathcal{C}_e & extraction level data e_i^x (BP, IEA)

Quantification – Energy markets

- ▶ Fossil production e_{it}^x and reserve \mathcal{R}_{it}
 - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i} \left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i} \mathcal{R}$
 - Now : $\bar{\nu}_i = \bar{\nu}$ and $\nu_i = \nu$ and \mathcal{R}_{it} calibrated to *proven reserves* data from BP.
 - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost \mathcal{C}_e & extraction level data e_i^x (BP, IEA)

- ▶ Renewable : Production \bar{e}_{it}^r and price q_{it}^r
 - Now : $q_{it}^r = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
 - Future : Choose z_i^r to match the energy mix (e_i^f, e_i^r)

Quantification – Future Extensions :

- ▶ Damage parameters :
 - $\gamma_i^{\pm,y}$ depends on daily temperature distribution $\tau \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
 - Use Climate Lab's (Greenstone et al) estimates for damage γ_i
- ▶ Fossil Energy markets :
 - Divide fossils e_{it}^f / e_{it}^x into oil/gas/coal
 - Match the production/cost/reserves data across countries
 - Use a dynamic model : extraction/exploration a la Hotelling
- ▶ Renewables Energy markets :
 - Make the problem dynamic with investment in capacity C_{it}^r
- ▶ Population dynamics
 - Match UN forecast for growth rate / fertility

Calibration

TABLE – Baseline calibration (★ = subject to future changes)

<i>Technology & Energy markets</i>			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
\bar{g}	0.01★	Long run TFP growth	Conservative estimate for growth
g_e	0.01★	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	-0.01★	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	2★	Extraction elasticity of fossil energy	Cubic extraction cost
<i>Preferences & Time horizon</i>			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	0.01★	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

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<i>Climate parameters</i>			
ξ	0.81	Emission factor	Conversion 1 <i>MTOE</i> \Rightarrow 1 <i>MT CO₂</i>
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature \sim 11–15 years
χ	2.1/1e6	Climate sensitivity	Pulse experiment : 100 <i>GtC</i> \equiv 0.21°C medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment : 100 <i>GtC</i> \equiv 0.16°C long-term warming
γ^{\oplus}	0.00234★	Damage sensitivity	Nordhaus' DICE
γ^{\ominus}	$0.2 \times \gamma^{\oplus}$ ★	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^{τ}	0.2★	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
τ^{\star}	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
<i>Parameters calibrated to match data</i>			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
τ_i		Local Temperature	To match temperature of largest city
\mathcal{R}_i		Local Fossil reserves	To match data from BP Energy review

Sequential solution method

► Summary of the model :

- ODEs for states $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^\tau, \lambda_{it}^s\}_{it}$
- Non-linear equations (FOCs) for household controls $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$

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► Global Numerical solution :

- Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
- Express as a large vector $\mathbf{y} = \{\mathbf{x}, \boldsymbol{\lambda}, \mathbf{c}, \mathbf{q}\}$ in a large non-linear function

$$F(\mathbf{y}) = \mathbf{0}$$

- Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

► Why use a sequential approach ?

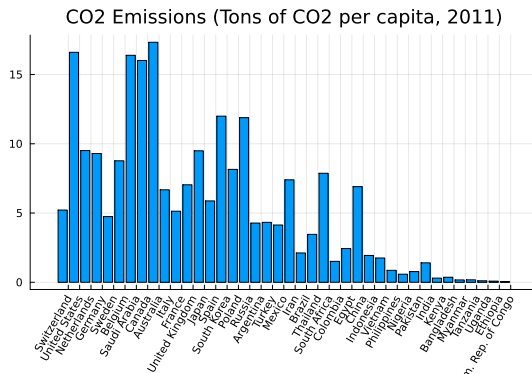
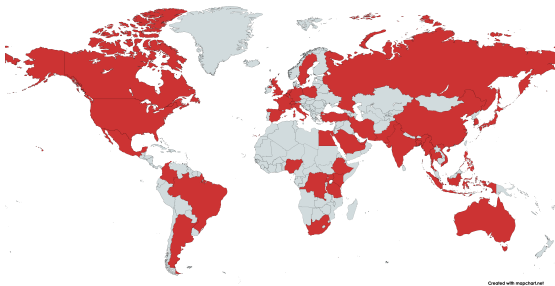
- *Global approach* : Only need to follow the trajectories for i agents :
 - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
*Productivity z_i Population p_i , Temperature scaling Δ_i , Fossil energy cost \bar{v}_i , Energy mix $\epsilon_i, \omega_i, z_i^r$,
 Local damage $\gamma_i^y, \gamma_i^u, \tau_i^*$, Directed Technical Change z_i^e*
 - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
*For now : Wealth w_{it} , temperature τ_{it} , reserves \mathcal{R}_{it} , Carbon S_t
 Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)*
 - Newton method & Non-linear solvers very efficient

► Why not :

- Numerical constraint to solve a large system of ODEs and non-linear equations :
 ⇒ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
- Relying on numerical solvers/structure of the problem can be opaque

Numerical Application – Competitive equilibrium

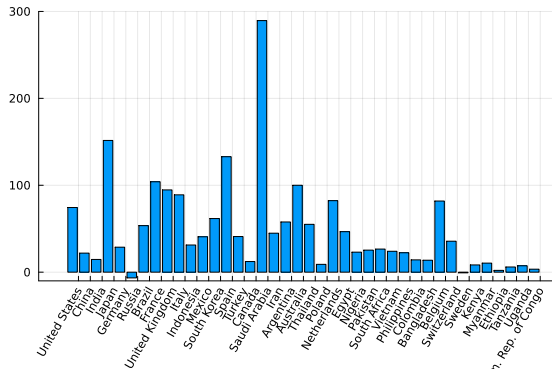
- ▶ Data : 40 countries, 25 largest countries either both GDP and population
- ▶ Work in progress (quantification/algorithm) subject to changes



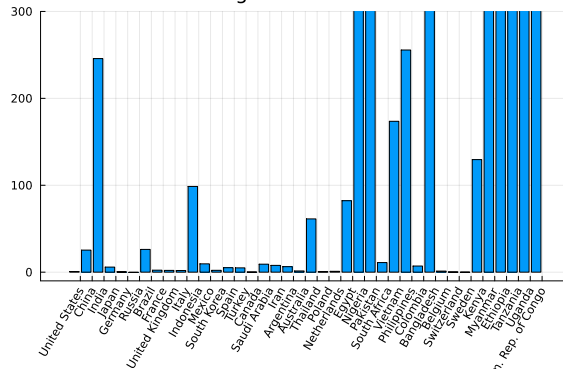
Local Cost of Carbon

► Difference $LCC_i = \frac{\lambda_{it}^S}{\lambda_{it}^w}$ and $LWCC_{it} = \hat{\lambda}_{it}^w LCC_{it} = \frac{\lambda_{it}^S}{\lambda_t^w}$

LSCC in 2100



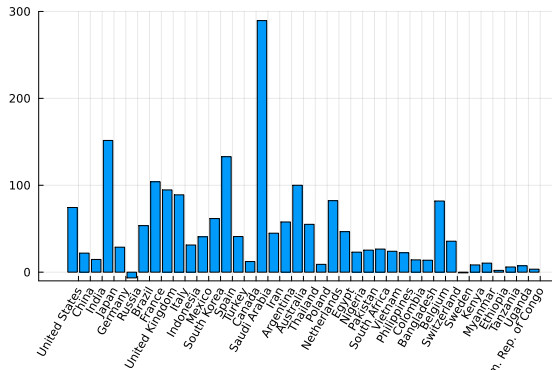
Reweighted LSCC in 2100



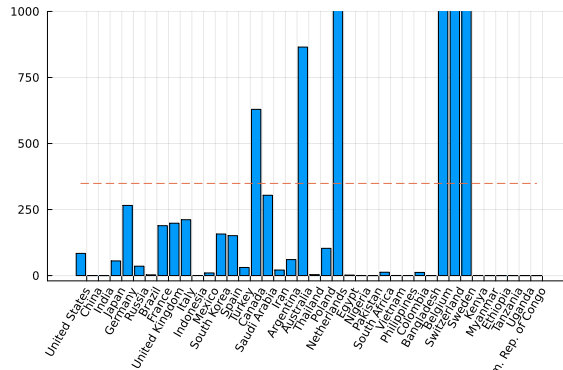
Social Cost of Carbon and Carbon Tax

► Difference $LCC_i = \frac{\lambda_{it}^S}{\lambda_{it}^w}$ and $\mathbf{t}_{it}^S = (1/\hat{\lambda}_{it}^w)SCC$

LSCC in 2100



Carbon tax in 2100



Conclusion

- ▶ Climate change has redistributive effects & heterogeneous impacts
- ▶ Optimal carbon policy take into account inequality and redistribution
 - Depends on the availability of transfer mechanisms
 - Level of Pigouvian tax & Social Cost of Carbon exacerbated by inequality
 - Distribution of carbon & energy taxes inversely related to distribution of consumption
 - Energy tax also depends on redistribution through changes in terms-of-trade
- ▶ Future improvement in the calibration / quantification & numerical method

Appendices

More details – Capital market

- In each country, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} &= \mathcal{D}_i^y(\tau_{it})z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g})k_{it} + \iota_{it} \\ \dot{b}_{it} &= r^*b_{it} + \theta_i\pi_t^f + \pi_{it}^r - (q_t^f + \mathfrak{t}_{it}^f)e_{it}^f - (q_t^r + \mathfrak{t}_{it}^r)e_{it}^r - \iota_{it} - c_{it} + \mathfrak{t}_{it}^{ls} \\ b_{it} &\geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^*w_{it} + \mathcal{D}_i^y(\tau_{it})z_i f(k_{it}, e_{it}) - (\bar{\delta} + r_t^*)k_{it} + \theta_i\pi_t^f + \pi_{it}^r - (q_t^f + \mathfrak{t}_{it}^f)e_{it}^f - (q_t^r + \mathfrak{t}_{it}^r)e_{it}^r - c_{it} + \mathfrak{t}_{it}^{ls}$$

$$k_{it} \leq \frac{1}{1 - \vartheta} w_{it}$$

- Two polar cases :
- $\vartheta \rightarrow 0$, full autarky (no trade), and $w_{it} = k_{it}$
 - $\vartheta \rightarrow 1$, full financial integration :

$$k_{it} \quad s.t. \quad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it})z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r_t^*$$

Impact of increase in temperature

- Marginal values of the climate variables : λ_{it}^s and λ_{it}^τ

$$\dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho + \zeta) + \overbrace{\gamma_i(\tau_{it} - \tau_i^*) \mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it}, e_{it}) \lambda_{it}^k + \overbrace{\phi_i(\tau_{it} - \tau_i^*) \mathcal{D}^u(\tau_{it})}^{\partial_\tau u(c, \tau)} c_{it}^{1-\eta}$$

$$\dot{\lambda}_{it}^s = \lambda_{it}^s(\rho + \delta^s) - \zeta \chi \Delta_i \lambda_{it}^\tau$$

- Costate λ_{it}^s : marg. cost of 1Mt carbon in atmosphere, for country i . Increases with :
- Temperature gaps $\tau_{it} - \tau_i^*$ & damage sensitivity of TFP γ_i^y and utility γ_i^u
 - Development level $f(k_{it}, e_{it})$ and c_{it}
 - Climate params : χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
 - [back](#)

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature \Leftrightarrow solving ODE

$$\dot{\lambda}_{it}^{\tau} = \lambda_t^{\tau}(\tilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^*)\mathcal{D}^y(\tau)f(k, e)\lambda_t^k + \phi(\tau - \tau^*)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}_t^S = \lambda_t^S(\tilde{\rho} + \delta^S) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda_{it}^{\tau}$$

- Solving for λ_t^{τ} and λ_t^S , in stationary equilibrium $\dot{\lambda}_t^S = \dot{\lambda}_t^{\tau} = 0$

$$\lambda_{it}^{\tau} = - \int_t^{\infty} e^{-(\tilde{\rho} + \zeta)u} (\tau_u - \tau^*) \left(\gamma \mathcal{D}^y(\tau_u) y_{\tau} \lambda_u^k + \phi \mathcal{D}^u(\tau_u) u(c_u) \right) du$$

$$\lambda_{it}^{\tau} = - \frac{1}{\tilde{\rho} + \Delta\zeta} (\tau_{\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{\infty}) y_{\infty} \lambda_{\infty}^k + \phi \mathcal{D}^u(\tau_{\infty}) u(c_{\infty}) \right)$$

$$\lambda_t^S = - \int_t^{\infty} e^{-(\tilde{\rho} + \delta^S)u} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,u}^{\tau} dj du$$

$$= \frac{1}{\tilde{\rho} + \delta^S} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,\infty}^{\tau}$$

$$= - \frac{\chi}{\tilde{\rho} + \delta^S} \frac{\zeta}{\tilde{\rho} + \zeta} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{j,\infty}) y_{\infty} \lambda_{j,\infty}^k + \phi \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

$$\lambda_t^S \xrightarrow{\zeta \rightarrow \infty} - \frac{\chi}{\tilde{\rho} + \delta^S} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{j,\infty}) y_{j,\infty} \lambda_{j,\infty}^k + \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

Cost of carbon / Marginal value of temperature

► *Proposition (Stationary LSCC) :*

When $t \rightarrow \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \rightarrow \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ , **marg. damage** γ_i^y , γ_i^u , **temperature**, and **output, consumption**.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^*) \left(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \right)$$

- Stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
- Fast temperature adjustment $\zeta \rightarrow \infty$
- [Back](#)

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta \chi \xi \omega \int_{t_0}^T e^{(n+\bar{g})t - \delta_s(T-t)} q_t^f - \sigma_e \int_{j \in \mathbb{I}} (z_j z_{j,t}^e \mathcal{D}(\tau_{j,t}))^{\sigma-1} y_{j,t} q_{j,t}^{\sigma_e - \sigma} dj dt$$

- Geographical factors determining warming Δ_i
- Climate sensitivity χ & carbon exit from atmosphere δ_s
- Growth of population n , aggregate productivity \bar{g}
- Deviation of output from trend y_i & relative TFP z_j
- Directed technical change z_t^e , elasticity of energy in output σ
- Fossil energy price q^{ef} and Hotelling rent $g^{qf} \approx \dot{\lambda}_t^R / \lambda_t^R = \rho$
- Change in energy mix, renewable share ω , price q_t^r & elasticity of source σ_e

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity [More details](#)

$$\frac{\dot{\tau}_T}{\tau_T} \propto n + \bar{g}^y - (1 - \sigma)(g^{z^e} - \tilde{\gamma}) + (\sigma_e - \sigma)(1 - \omega)g^{q^r} - (\sigma_e(1 - \omega) + \sigma\omega)g^{q^f}$$