# The Optimal Design of Climate Agreements

## Inequality, Trade, and Incentives for Carbon Policy

*Thomas Bourany*

THE UNIVERSITY OF CHICAGO

*EPIC lunch*

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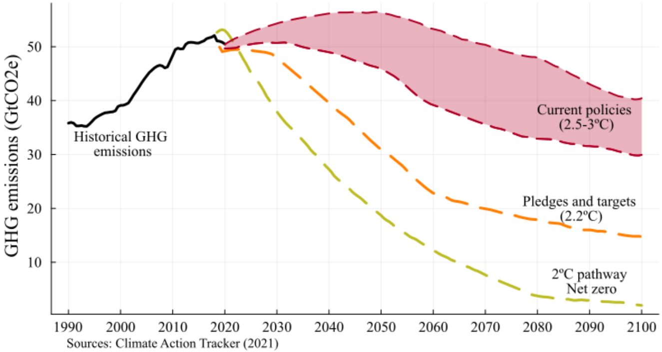
# Motivation

- Fighting climate change requires implementing ambitious carbon reduction policies
  - The “free-riding problem” causes climate inaction:
  - Climate policy redistributes across countries through:
    (i) change in climate (ii) energy markets, and (iii) reallocation of activity through trade



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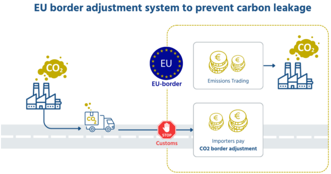
- Proposals to fight climate inaction and the free-riding problem:
  - Trade sanctions needed to give incentives to countries to reduce emissions meaningfully
  - "Climate club", Nordhaus (2015): trade sanctions on non-participations to sustain larger "clubs"
  - Carbon Border Adjustment mechanisms (CBAM), EU policy: carbon tariffs





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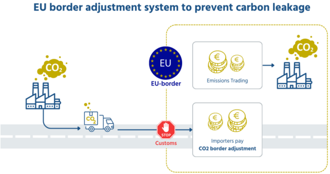


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▶ Preview of the results:

- Despite complete freedom of policy instruments, impossible to achieve the world's optimal policy with complete participation
- Beneficial to leave several fossil fuels producing countries outside of the climate agreement
- Welfare improvement with transfers, c.f. UN COP27's "loss and damage" fund

# Literature

- Theoretical model of climate agreements: cooperation
  - *Climate clubs and cooperation:* Nordhaus (2015), Barrett (1994), Harstad (2012), Maggi (2016), Barrett (2003, 2013, 2022), Iverson (2024), Hagen and Schneider (2021)
  - *Dynamics of coalition building:* Ray and Vohra (2015), Okada (2023), Nordhaus (2021), Harstad (2023), Maggi and Staiger (2022)

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  - *RA model:* Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
  - *HA model:* Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
  - *Spatial models:* Cruz, Rossi-Hansberg (2022, 2023) among others
  - *Non-cooperative or suboptimal taxation:* Chari, Kehoe (1990), Hassler, Krusell, Olovsson (2019)

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# Literature

- Nordhaus (2015)
  - Examine ”stable climate coalitions” (club imposing carbon tax) in a simple model
  - Abstract from General Equilibrium and distributional effects
  - Results: Penalty tariffs necessary to enforce a climate club
- Farrokhi, Lashkaripour (2024)
  - Study and characterize the optimal trade policy with climate externality
  - General static trade model. Results: unilateral tariffs not effective
  - Sequential search for one stable climate club if EU or US join.
- Main contribution:
  - Search for the *optimal* climate agreement
  - GE on good and energy market and redistribution effects are important
  - Cost of climate change is endogenous to policy: damages are non-linear
  - Analyze other distributional policies (transfers/taxes, *loss and damage funds*)
  - General framework for analyzing macrodynamics (c.f. Bourany (2024))

# Model – Household & Firms

- Deterministic Neoclassical economy
  - countries $i \in \mathbb{I}$, heterogeneous in productivity $z_i$, temperature $T_i$, energy extraction cost $\mathcal{C}_i$
  - In each country, five agents:

1. Representative household $\mathcal{V}_i = \max_{c_{ij}} u(c_i)$

$$c_i = \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \qquad \sum_j c_{ij} \underbrace{(1+\mathrm{t}_{ij}^b)}_{\text{tariff}} \underbrace{\tau_{ij}}_{\substack{\text{iceberg}\\\text{cost}}} \mathrm{p}_j = \underbrace{w_i \ell_i}_{\substack{\text{labor}\\\text{income}}} + \underbrace{\pi_i^f}_{\substack{\text{fossil firm}\\\text{profit}}} + \mathrm{t}_i^{ls}$$

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2. Competitive final good firm:

$$\max_{\ell_i, e_i^f, e_i^c, e_i^r} \mathrm{p}_i \, \mathcal{D}_i(\mathcal{E}) \, z_i \, f(\ell_i, e_i^f, e_i^c, e_i^r) - w_i \ell_i - (q^f + \mathrm{t}_i^{\varepsilon}) e_i^f - (q_i^c + \mathrm{t}_i^{\varepsilon}) e_i^c - q_i^r e_i^r$$

  - Externality: Damage function $\mathcal{D}_i(\mathcal{E})$, Income inequality from $z_i$, Carbon tax: $\mathrm{t}_i^{\varepsilon}$
  - Trade, *à la* Armington

# Model – Energy markets & Emissions

3. Competitive fossil fuels (oil-gas) producer, extracting $e_i^x$

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x)\mathbb{P}_i$$

   – Energy traded in international markets, at price $q^f$

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4. Coal energy firm: elastic supply $e_i^c$ at price $q_i^c = z_i^c \mathbb{P}_i$

5. Renewable energy firm: elastic supply $e_i^r$ at price $q_i^r = z_i^r \mathbb{P}_i$

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- Climate system: mapping from emission $\mathcal{E} = \sum_{\mathbb{I}} e_i^f + e_i^c$ to damage $\mathcal{D}_i(\mathcal{E})$

# Model – Equilibrium

- Given policies $\{\mathrm{t}_i^{\varepsilon}, \mathrm{t}_{ij}^{b}, \mathrm{t}_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e_i^f, e_i^c, e_i^r, e_i^x\}_{ij}$, emission $\{\mathcal{E}\}_i$ changing climate and prices $\{\mathrm{p}_i, w_i, q_i^c, q_i^r\}_i, q^f$ such that:
  - Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint
  - Firm choose inputs $\{e_i^f, e_i^c, e_i^r\}_i$ to max. profit
  - Oil-gas firms extract/produce $\{e_i^x\}_i$ to max. profit. + Elastic renewable, coal supplies $\{e_i^c, e_i^r\}$
  - Emissions $\mathcal{E}$ affects climate and damages $\mathcal{D}_i(\mathcal{E})$
  - Government budget clear $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^{\varepsilon}(e_i^f + e_i^c) + \sum_{i,j} \mathrm{t}_{ij}^b c_{ij} \tau_{ij} \mathrm{p}_j$
  - Prices $\{\mathrm{p}_i, w_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$ and for each good

$$y_i := \mathcal{D}_i(\mathcal{E})\, z_i f(\ell_i, e_i^f, e_i^r, e_i^r) = \sum_{k \in \mathbb{I}} \tau_{ki} c_{ki} + \sum_{k \in \mathbb{I}} \tau_{ki}(x_{ki}^f + x_{ki}^c + x_{ki}^r)$$

with $x_{ki}$ export of good $i$ as input in energy production in $k$
In expenditure, with import shares $s_{ij} = \frac{c_{ij}\tau_{ij}\mathrm{p}_j}{c_i \mathbb{P}_i}$, it yields

$$\mathrm{p}_i y_i = \sum_{k \in \mathbb{I}} \frac{s_{ki}}{1 + \mathrm{t}_{ki}^b} \left(\mathrm{p}_k y_k + q^f (e_k^x - e_k^f) + \tilde{\mathrm{t}}_k^{ls}\right)$$

# Ramsey Problem with endogenous participation

- ***Definition:*** A climate agreement is a set $\{\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}\}$ of $\mathbb{J} \subseteq \mathbb{I}$ countries and a C.E. s.t.:
  - Countries $i \in \mathbb{J}$ pay carbon tax $\mathrm{t}^{\varepsilon}$
  - If $j$ exits agreement, club members $i \in \mathbb{J}$ impose uniform tariffs $\mathrm{t}^{b}_{ij} = \mathrm{t}^{b}$ on goods from $j$
    They still trade with club members in oil-gas at price $q^{f}$
  - Exit: unilateral deviation $\mathbb{J}\backslash\{j\}$, $\Rightarrow$ ***Nash equilibrium***
- Participation constraints, given indirect utility $U_i(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) \equiv u(c_{i(\mathbb{J},\mathrm{t}^{\varepsilon},\mathrm{t}^{b})})$

$$U_i(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) \geq U_i(\mathbb{J}\backslash\{i\}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) \qquad \text{[Nash equilibrium]}$$

- Objective: search for the optimal *and stable* climate agreement

$$\max_{\mathbb{J},\mathrm{t}^{\varepsilon},\mathrm{t}^{b}} \mathcal{W}(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) = \max_{\mathrm{t}^{\varepsilon},\mathrm{t}^{b}} \max_{\mathbb{J}} \sum_{i\in\mathbb{I}} \omega_i \, U_i(\mathbb{J}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b})$$

$$s.t. \qquad \mathbb{J} \in \mathbb{S}(\mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) = \left\{\mathcal{J} \mid U_i(\mathbb{J},\mathrm{t}^{\varepsilon},\mathrm{t}^{b}) \geq U_i(\mathbb{J}\backslash\{i\},\mathrm{t}^{\varepsilon},\mathrm{t}^{b}) \ \forall i \in \mathcal{J}\right\}$$

# Ramsey Problem with endogenous participation

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  - Alternative: ***Coalitional Nash-equilibrium*** $\mathbb{C}(\mathrm{t}^{f}, \mathrm{t}^{b})$: robust of sub-coalitions deviations:

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# Ramsey Problem with endogenous participation

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    - No country $i$ and subcoalition $\hat{\mathbb{J}}$ would be better off than in the current agreement $\mathbb{J}$

  - Current design: (i) choose taxes $\{\mathrm{t}^{f}, \mathrm{t}^{b}\}$,
    (ii) choose the coalition $\mathbb{J}$ s.t. participation constraints hold

  - Solution method (Nash equilibrium):
    - relies on the complementarity of the combinatorial discrete choice problem and use a "squeezing procedure", c.f. Jia (2008), Arkolakis, Eckert, Shi (2023), to handle the problem

# Quantification

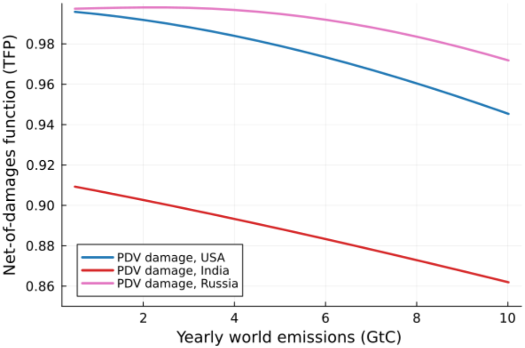
- Energy parameters to match production/reserves,
  - Isoelastic cost function $\mathcal{C}_i(e_i^x) = \bar{\nu}_i \left(e_i^x / \mathcal{R}_i\right)^{1+\nu_i} \mathcal{R}_i$
  - Use $\bar{\nu}_i, \nu_i$ to match $e_i^x$ and $\pi_i^f$ ,
- Armington model,
  - Iceberg cost $\tau_{ij}$ projected on distance and preferences $a_{ij}$ to match import shares $s_{ij} = \frac{c_{ij}\tau_{ij}p_j}{c_i\mathbb{P}_i}$
- Production $\bar{y} = zf\left(\ell_i, k_i, e_i^f, e_i^r\right)$
  - Nested CES energy vs. labor-capital Cobb-Douglas bundle $k_i^\alpha \ell_i^{1-\alpha}$ (elasticity $\sigma_y < 1$), and fossil/renewable $\sigma_e > 1$, $CES(e_i^f, e_i^r)$
  - TFP, and DTC, $z_i, z_i^e$, calibrated to match GDP / energy shares data.
- Pareto weights $\omega_i$:
  - Imply no redistribution motive, $\bar{c}_i$ consumption in initial equilibrium $t = 2020$

$$\omega_i = \frac{1}{u'(\bar{c}_i)}$$

- Details More details Details Pareto weights

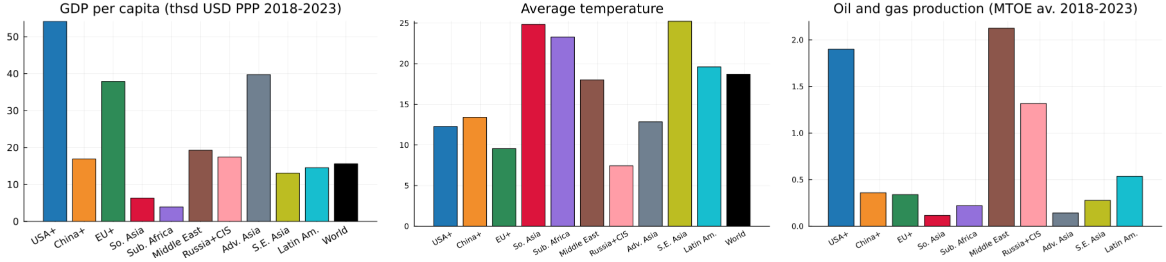
# Quantification – Climate system and damage

- Static economic model:
  decisions $e_i^f + e_i^c$ taken "once and for all", $\mathcal{E} = \sum_i e_i^f + e_i^c$
  - Climate system:
  $$\dot{\mathcal{S}}_t = \mathcal{E} - \delta_s \mathcal{S}_t$$
  $$T_{it} = \bar{T}_{i0} + \Delta_i \mathcal{S}_t$$
  - Path of period damages heterogeneous across countr
    Quadratic (c.f. Nordhaus-DICE)
  $$\mathcal{D}_i(T_{it}) = e^{-\gamma (T_{it} - T_i^\star)^2}$$
  - Economic feedback in Present discounted value
  $$\mathcal{D}_i(\mathcal{E}) = \int_0^\infty e^{-\rho t} \mathcal{D}(T_{it}) dt$$
  - Similarly for $LCC_i, SCC_i \dots$



# Quantitative application – Sample of 10 "regions"

- Sample of 10 "regions": (i) US+Canada, (ii) China+HK, (iii) EU+UK+Schengen, (iv) South Asia, (v) Sub-saharian Africa, (vi) Middle-East+Maghreb, (vii) Russia+CIS, (viii) Japan+Korean+Australia+Asian Dragons, (ix) South-East Asia (Asean), (x) Latin America WIP: 25 countries + 5 regions
- Data (Avg. 2018-2023) on macro variables, energy markets, trade shares $s_{ij} = \frac{c_{ij}\tau_{ij}\mathrm{p}_j}{c_i\mathbb{P}_i}$, etc.




Details Trade shares – details

# Optimal policy : benchmarks

- Three policy benchmarks, c.f. Bourany (2024), without endogenous participation
  - ***First-Best***, Social planner maximizing global welfare with unlimited instruments
    - Pigouvian result: Carbon tax = Social Cost of Carbon
    - Relies heavily on cross-country transfers to offset redistributive effects

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  - ***Second-Best:*** Social planner, single carbon tax without transfers
    - Optimal carbon tax $\mathrm{t}^{\varepsilon}$ correct climate externality, but also accounts for:
      (i) Redistribution motives, G.E. effects on (ii) energy markets and (iii) trade leakage

$$\mathrm{t}^{\varepsilon} = \underbrace{\sum_i \phi_i \, LCC_i}_{=SCC} + \sum_i \phi_i \,\text{Supply Redistrib}_i^{\circ} - \sum_i \phi_i \,\text{Demand Distort}_i^{\circ} - \sum_i \phi_i \,\text{Trade Redistrib}_i^{\circ} \qquad \phi_i \propto \omega_i u'(c_i)$$

    - Details: *Competitive equilibrium* Details eq 0, *First-Best*, with unlimited instruments Details eq 1, *Second-best*, Ramsey policy with limited instruments Details eq 2

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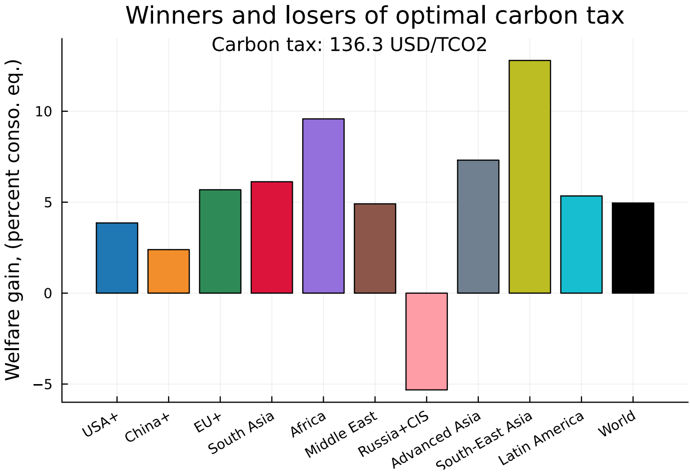
$$\mathrm{t}^{\varepsilon} = \underbrace{\sum_i \phi_i \, LCC_i}_{=SCC} + \sum_i \phi_i \,\text{Supply Redistrib}_i^{\circ} - \sum_i \phi_i \,\text{Demand Distort}_i^{\circ} - \sum_i \phi_i \,\text{Trade Redistrib}_i^{\circ} \qquad \phi_i \propto \omega_i u'(c_i)$$

    - Details: *Competitive equilibrium* Details eq 0, *First-Best*, with unlimited instruments Details eq 1, *Second-best*, Ramsey policy with limited instruments Details eq 2

  - ***Unilateral policy:*** Local planner in country $i$ unilaterally choosing $\mathrm{t}_i^{\varepsilon}$ and $\mathrm{t}_{ij}^{b}$
    - Optimal unilateral carbon tax (subsidy!) and tariffs for terms-of-trade manipulations
    - Nash equilibrium of $\mathbb{I}$ countries choosing individually unilateral policies

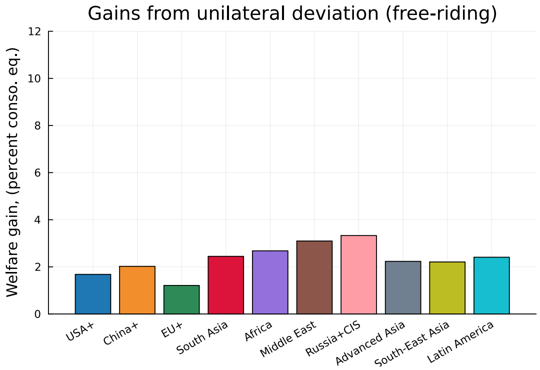
# Gains from cooperation – World Optimal policy

- Optimal carbon tax (Second Best): $\sim \$136/tCO_2$
- Reduce fossil fuels / $CO_2$ emissions by 40% compared to Business as Usual (BAU)
- Welfare difference btw world optimal policy w/o participation constraints vs BAU (Comp. Eq.)



# Trade-off – Cost of Carbon Taxation vs. Gains from trade

Gains from unilateral exit from agreement vs. Gains from trade, i.e. loss from tariffs/autarky



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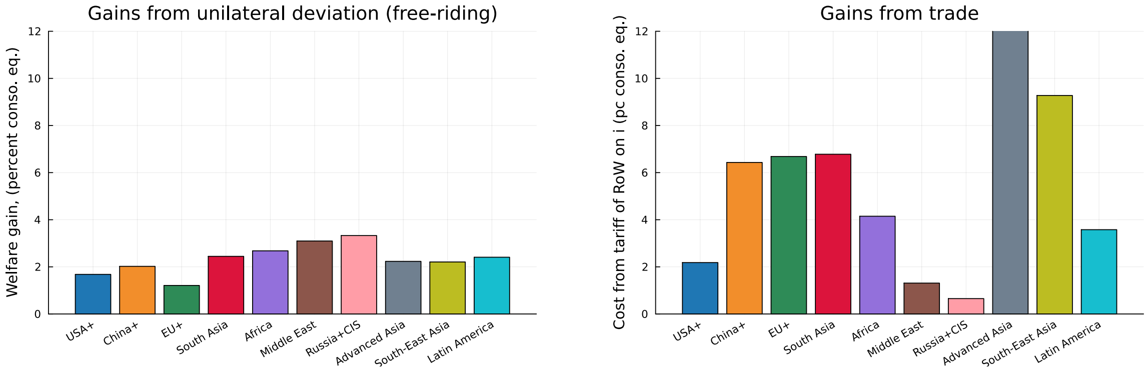


# Theoretical investigation: decomposing the welfare effects

- Experiment:
  - Start from the equilibrium where carbon tax $\mathrm{t}_j^f = 0, \mathrm{t}_{jk}^b = 0, \forall j,$
  - Change in welfare: Linear approximation around that point $\Rightarrow$ small changes in carbon tax $d\mathrm{t}_j^f, \forall j$ and tariffs $d\mathrm{t}_{j,k}^b, \forall j, k$

$$\frac{d\mathcal{V}_i}{u'(c_i)} = \eta_i^c \frac{d\mathrm{p}_i}{\mathrm{p}_i} + \left[\eta_i^c \gamma_i \frac{1}{\bar{\nu}} - \eta_i^c s_i^e + \eta_i^\pi \left(1+\frac{1}{\bar{\nu}}\right)\right] \frac{dq^f}{q^f} + \dots$$

  - Difference in the GE effect on energy markets $\frac{dq^f}{q^f} \approx \bar{\nu} \frac{dE^f}{E^f} + \dots$, due to taxation

$$\frac{dq^f}{q^f} = -\sum_j \nu_j^f \frac{d\mathrm{t}_j^f}{\mathrm{t}_j^f} + \sum_i \nu_j^{p,R} \frac{d\mathrm{p}_j}{\mathrm{p}_j} + \sum_{j,k} \nu_j^{R,f,z,qr} s_{j,k} \frac{d\mathrm{t}_{jk}^b}{\mathrm{t}_{jk}^b}$$

    - Trade and leakage effect: GE impact of $\mathrm{t}_j^f$ and $\mathrm{t}_{j,}^b$ on $y_i$ and $\mathrm{p}_i$
    - Simplifying assumption: no renewable

∘ Params: $\sigma$ energy demand elast$^y$, $s^e$ energy cost share, $\bar{\nu}$ energy supply inverse elas$^y$, Climate damage $\gamma_i$

# Decomposing the welfare effects: gains from trade

- Start from the equilibrium where carbon tax $\mathrm{t}_j^f = 0, \mathrm{t}_{jk}^b = 0, \forall j$,
- Change in welfare: Linear approximation around that point $\Rightarrow$ small changes in carbon tax $d\mathrm{t}_j^f, \forall j$ and tariffs $d\mathrm{t}_{j,k}^b, \forall j, k$

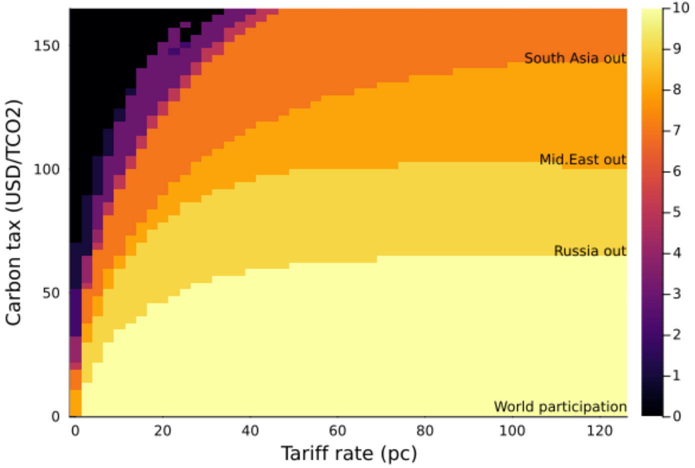
$$\frac{d\mathrm{p}}{\mathrm{p}} = \Big[\mathbf{I}-\mathbf{T}-(\theta-1)\big[\mathbf{T}\odot\mathbf{S}-(\mathbf{T}\odot\mathbf{I})'\big]\Big]^{-1}\Big((\mathbf{T}-\mathbf{I})\frac{dy}{y} + \big(\mathbf{T}[(\theta-1)\mathbf{I}-\theta\mathbf{S}]\odot\frac{d\mathrm{t}^b}{\mathrm{t}^b}\big)\mathbb{1}\Big)$$

$$\frac{dy_i}{y_i} = \alpha_i^{p,qr}\frac{d\mathrm{p}_i}{\mathrm{p}_i} - \alpha_i^{qf}\frac{d\mathrm{t}_i^f}{\mathrm{t}_i^f} + \ldots$$

∘ Params: **S** Trade share matrix, **T** income flow matrix, $\theta$, Armington CES

# Climate Agreements: Intensive vs. Extensive Margin

- Intensive margin:
  higher tax, emissions ↓, welfare ↑
- Extensive margin:
  higher tax, participation ↓,
  free-riding and emissions ↑



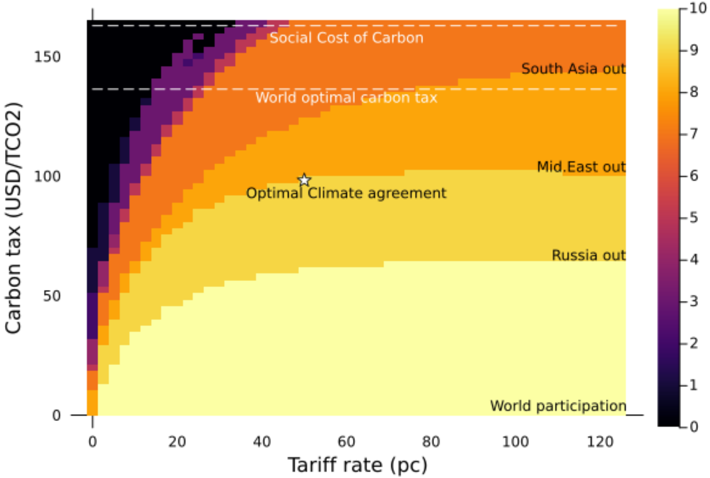
# Optimal Climate Agreement

- Despite full freedom of instruments $(t^{\varepsilon}, t^{b})$

  $\Rightarrow$ can not sustain an agreement with Russia & Middle East

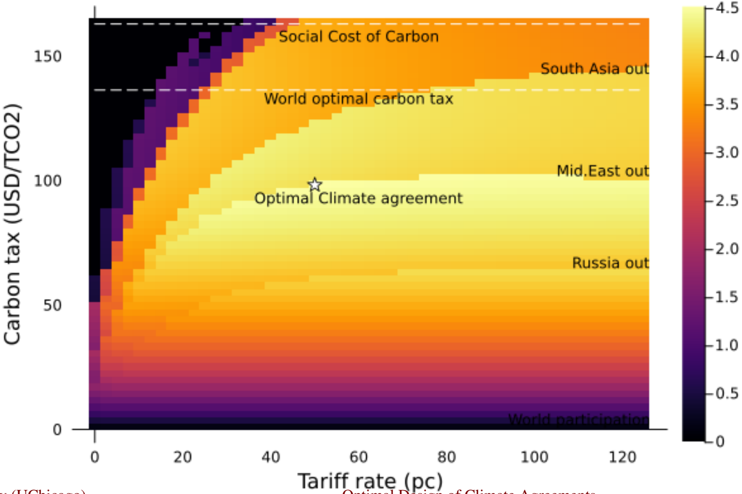
  $\Rightarrow$ need to reduce carbon tax from \$136 to \$98

- Intuition:
  relatively cold and closed economy, and fossil-fuel producers



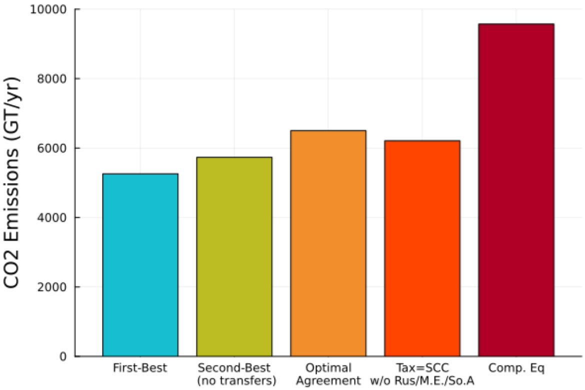
# Climate agreement and welfare

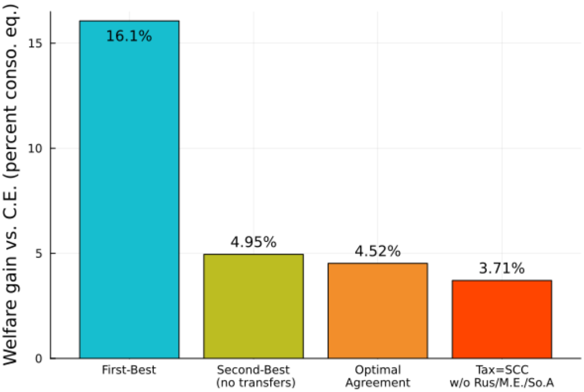
Recover 91% of welfare gains, i.e. 4.5% out of 5% conso equivalent.



# Welfare and emission reduction: Different metrics!

- Agreements with tariffs recover 91% of welfare gains from the Second-Best – optimal carbon tax without transfers – at a cost of increasing emissions by 13%
- First-best allocation relies heavily on transfers to be able to impose a higher carbon tax



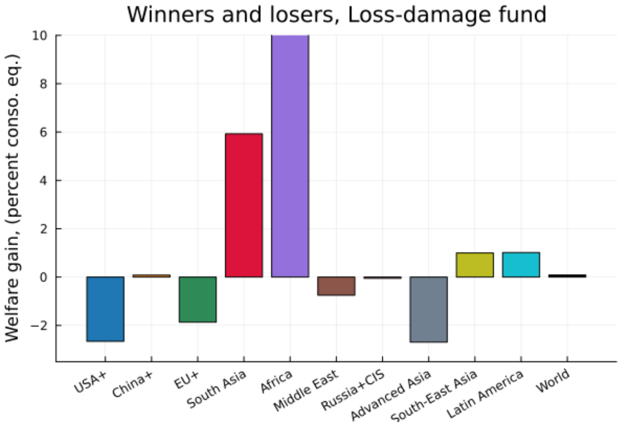



# Transfers – Loss and damage funds

- COP28 Major policy proposal: *Loss and damage funds* for countries vulnerable to the effects of climate change
- Simple implementation in our context: lump-sum receipts of carbon tax revenues:

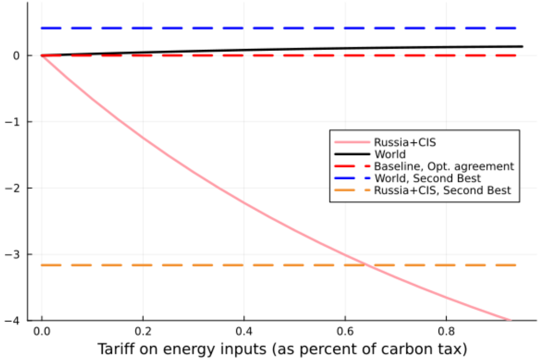
$$t_i^{ls} = (1-\alpha)t^{\varepsilon}\varepsilon_i + \alpha\frac{1}{\mathcal{P}}\sum_j t^{\varepsilon}\varepsilon_j$$

- In practice: transfers from large emitters to low emitters



# Taxation of fossil fuels energy inputs

- Current climate club:
  only imposes penalty tariffs on final goods, not on energy imports
  - Empirically relevant, c.f. Shapiro (2021): inputs are more emission-intensives but trade policy is biased against final goods output
- Alternative: tax energy import from non-participants $t_{ij}^{bf} = \beta t^b \mathbb{1}\{i \in \mathbb{J}, j \notin \mathbb{J}\}$



# Dynamic coalition formation

- Current "equilibrium": $\mathrm{t}_i^{\varepsilon} = 0$, $\mathrm{t}_{ij}^{b} = 0$
- Optimal club equilibrium $\mathrm{t}_i^{\varepsilon} = \mathrm{t}^{\varepsilon\star}$, $\mathrm{t}_{ij}^{b} = \mathrm{t}^{b\star}\mathbb{1}\{i \in \mathbb{J}, j \notin \mathbb{J}\}$
- Optimal agreement follows the planner taxes and participation decision:

$$\mathbb{J}^{\star} = \mathbb{J}\left(\mathrm{t}^{\varepsilon\star}, \mathrm{t}^{b\star}\right)$$

▶ What is driving the coordination failure?

- Possible explanation: coalition building and *bargaining* may never reach such equilibrium:

$$\bar{\mathbb{J}}_{t_0}(0,0) = \mathbb{I} \quad \xrightarrow[t \to T]{?} \quad \bar{\mathbb{J}}_T\left(\mathrm{t}^{\varepsilon\star}, \mathrm{t}^{b\star}\right) = \mathbb{J}^{\star}$$

- Can we find a sequence $\mathbb{J}_t, \mathrm{t}_t^{f}, \mathrm{t}_t^{b}$ such that

$$\{\mathbb{J}_{t_0}, \mathrm{t}_{t_0}^{f}, \mathrm{t}_{t_0}^{b}\} = \{\bar{\mathbb{J}}_{t_0}, 0, 0\} \qquad \{\mathbb{J}_T, \mathrm{t}_T^{\varepsilon}, \mathrm{t}_T^{b}\} = \{\bar{\mathbb{J}}_T, \mathrm{t}^{f\star}, \mathrm{t}^{b\star}\}$$

- Instruments used by leader countries (e.g. E.U., U.S. or China?) to reach such agreement?

# Conclusion

- In this project, I solve for the optimal design of climate agreements
  - Correcting for inequality, redistribution effects through energy markets and trade leakage, as well as free-riding incentives
- Climate agreement design jointly solves for:
  - The optimal choice of countries participating
  - The carbon tax and tariff levels, accounting for both the climate externality, redistributive effects and the participation constraints
- Optimal coalition depends on the trade-off between
  - the gain from cooperation and free riding incentives
  - the gain from trade, i.e. the cost of retaliatory tariffs

  $\Rightarrow$ Need a large coalition and a carbon at 70% of the world optimum
- Extensions:
  - Extend this to dynamic settings: coalition building
  - Explore additional policy proposal to improve the optimal agreement

# Conclusion

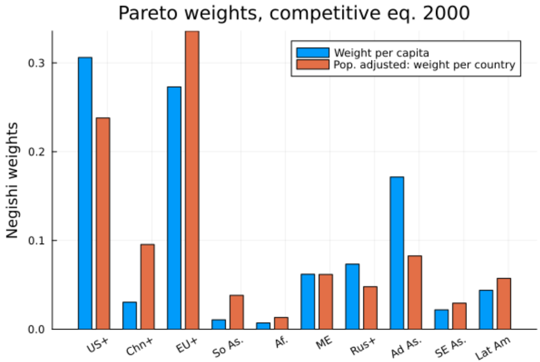
**Thank you!**

**thomasbourany@uchicago.edu**

# Appendices

# Welfare and Pareto weights

- Welfare:
$$\mathcal{W}(\mathbb{J}) = \sum_{i \in \mathbb{I}} \omega_i \, u(c_i)$$
- Pareto weights $\omega_i$:
$$\omega_i = \frac{1}{u'(\bar{c}_i)}$$
for $\bar{c}_i$ consumption in initial equilibrium "without climate change", i.e. year $= 2020$
- Imply no redistribution motive in $t = 2020$
$$\omega_i u'(\bar{c}_i) = \omega_j u'(\bar{c}_j) \qquad \forall i, j \in \mathbb{I}$$
- Climate change, taxation, and climate agreement (tax + tariffs) have redistributive effects
$\Rightarrow$ change distribution of $c_i$

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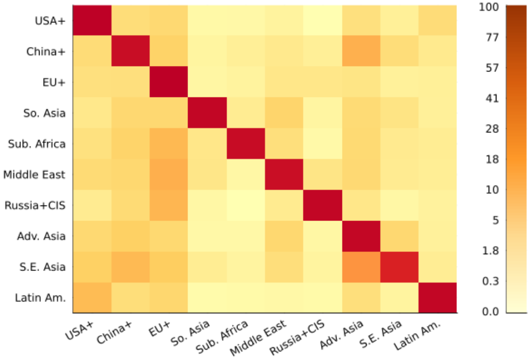
# Quantification – Trade model

- Armington Trade model:

$$s_{ij} \equiv \frac{c_{ij} p_{ij}}{c_i \mathbb{P}_i} = a_{ij} \frac{((1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k)^{1-\theta}}$$

- CES $\theta = 5.63$ estimated from a gravity regression
- Iceberg cost $\tau_{ij}$ as projection of distance
  $\log \tau_{ij} = \beta \log d_{ij}$
- Preference parameters $a_{ij}$ identified as remaining variation in the trade share $s_{ij}$
  $\Rightarrow$ policy invariant

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# Step 0: Competitive equilibrium & Trade

- Each household in country $i$ maximize utility and firms maximize profit
- Standard trade model results:
  - Consumption and trade:

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_i} = a_{ij}\frac{(\tau_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}(\tau_{ik}(1+\mathrm{t}^b_{ik})\mathrm{p}_k)^{1-\theta}} \qquad \& \qquad \mathbb{P}_i = \left(\sum_j a_{ij}(\tau_{ij}\mathrm{p}_j)^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

  - Energy consumption doesn't internalize climate damage:

$$\mathrm{p}_i MPe_i = q^e$$

  - Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

  - "Local Social Cost of Carbon", for region $i$

$$LCC_i = \frac{\partial\mathcal{W}_i/\partial\mathcal{E}}{\partial\mathcal{W}_i/\partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i\mathcal{D}'(T_i)z_i f(e_i^f)\frac{\mathrm{p}_i}{\mathbb{P}_i} \qquad (> 0 \text{ if heat causes losses})$$

back

# Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - Full array of instruments: cross-countries lump-sum transfers $\mathrm{t}_i^{ls}$, individual carbon taxes $\mathrm{t}_i^f$ on energy $e_i^f$, bilateral tariffs $\mathrm{t}_{ij}^b$
  - Budget constraint: $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^f e_i^f + \sum_{i,j} \mathrm{t}_{ij}^b c_{ij} \tau_{ij} \mathrm{p}_j$

- Maximize welfare subject to
  - Market clearing for good $[\mu_i]$, market clearing for energy $\mu^e$

back

# Step 1: World First-best policy

- Social planner results:
  - Consumption:

$$\omega_i u'(c_i) = \left[\sum_j a_{ij}(\tau_{ij}\omega_j\mu_j)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

  - Energy use:

$$\omega_i\mu_i MPe_i = \mu^e + SCC$$

  - Social cost of carbon:

$$SCC = -\frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}'_j(T_j)\bar{y}_j}{\frac{1}{I}\sum_j \omega_j\mu_j}$$

back

# Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - One single instrument: uniform carbon tax $\mathrm{t}^f$ on energy $e_i^f$
  - Rebate tax lump-sum to HHs $\mathrm{t}_i^{ls} = \mathrm{t}^f e_i^f$

- Ramsey policy: Primal approach, maximize welfare subject to
  - Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
  - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
  - Trade-off faced by the planner:
    - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

back

# Step 2: World optimal Ramsey policy

- The planner takes into account
  (i) the marginal value of wealth $\lambda_i$
  (ii) the shadow value of good $i$, from market clearing, $\mu_i$:

$$\text{w/o trade} \qquad \omega_i u'(c_i) = \omega_i \lambda_i$$

$$\text{vs. w/ trade in goods:} \qquad \omega_i u'(c_i) = \left( \sum_{j \in \mathbb{I}} a_{ij} (\tau_{ij} \mathrm{p}_j)^{1-\theta} \Big[ \omega_i \lambda_i + \omega_j \mu_j + \eta_{ij}(1 - s_{ij}) \Big]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Relative welfare weights, representing inequality

$$\text{w/o trade:} \qquad \widehat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \lessgtr 1 \qquad \Rightarrow$$

ceteris paribus, poorer countries have higher $\widehat{\lambda}_i$

$$\text{vs. w/ trade:} \qquad \widehat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)} \lessgtr 1$$

# Step 2: Optimal policy – Social Cost of Carbon

- Key objects: Local vs. Global Social Cost of Carbon:
  - Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country $i$
  - "Local social cost of carbon" (LCC) for region $i$:

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e_i^f) \mathrm{p}_i \qquad (> 0 \text{ if heat causes losses})$$

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  - Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W}/\partial \mathcal{E}}{\partial \mathcal{W}/\partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I}\sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

  - Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, LCC_i\right)$$

# Step 2: Optimal policy – Other motives

- Taxing fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy
- New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W} / \partial E}{\partial \mathcal{W} / \partial w} = \mathcal{C}^{f}_{EE}\, \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right) \; - \; \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\right)$$

◦ Params: $\mathcal{C}^{f}_{EE}$ agg. fossil supply elasticity, $s^f_i$ energy cost share and $\sigma$ energy demand elasticity

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◦ Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, $s^f_i$ energy cost share and $\sigma$ energy demand elasticity

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∘ Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, $s^f_i$ energy cost share and $\sigma$ energy demand elasticity

- *Proposition 2:* Optimal fossil energy tax:

$$\Rightarrow \qquad t^f = SCC + SVF$$

  – Social cost of carbon: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i$

back

# Step 3: Ramsey Problem with participation constraints

- Consider that countries can “exit” climate agreement.
- For a climate “club” of $\mathbb{J} \subset \mathbb{I}$ countries:
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax $\mathrm{t}^f$
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $\mathrm{t}^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff $\mathrm{t}^b$ with club members and countries outside the club. They still trade with the club members in energy at price $q^f$

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  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff $\mathrm{t}^b$ with club members and countries outside the club. They still trade with the club members in energy at price $q^f$
- Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

- Welfare:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{\mathbb{J}} \omega_i \, u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i \, u(c_i)$$

# Step 3: Ramsey Problem with participation constraints

- Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- *Proposition 3.1:* Second-Best social valuation with participation constraints
  - Participation incentives change our measure of inequality

$$\text{w/ trade:} \qquad \omega_i(1+\nu_i)u'(c_i) = \Big(\sum_{j\in\mathbb{I}} a_{ij}(\tau_{ij}\mathrm{p}_j)^{1-\theta}\Big[\omega_i\widetilde{\lambda}_i + \omega_j\widetilde{\mu}_j + \widetilde{\eta}_{ij}(1-s_{ij})\Big]^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)} \neq \widehat{\lambda}_i$$

$$\text{vs. w/o trade} \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_j(1+\nu_j)u'(c_j)} \neq \widehat{\lambda}_i$$

  - Similarly, the "effective Pareto weights" are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

# Step 3: Participation constraints & Optimal policy

- *Proposition 3.2:* Second-Best taxes:
  - Taxation with imperfect instruments:
    - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
    - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu\sigma}{q^f(1-s^f)}$
      with $\nu$ fossil supply elasticity, $\sigma$ energy demand elasticity and $s^f$ energy cost share.
  - Optimal fossil energy tax $\mathfrak{t}^f(\mathbb{J})$:

$$\Rightarrow \qquad \mathfrak{t}^f(\mathbb{J}) = SCC + SVF$$

$$= \frac{1}{1-\vartheta_{\mathbb{J}^c}} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i LCC_i + \frac{1}{1-\vartheta_{\mathbb{J}^c}} \mathcal{C}^f_{EE} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i (e^f_i - e^x_i) - \sum_{i\in\mathbb{J}} \widetilde{\lambda}_i \frac{q^f(1-s^f_i)}{\sigma}$$

  - Optimal tariffs/export taxes $\mathfrak{t}^{b,r}(\mathbb{J})$ and $\mathfrak{t}^b(\mathbb{J})$: In search for a closed-form expression
    As of now, only opaque system of equations (fixed point w/ demand/multipliers)

back

# Countries' incentives – Model w/o trade in goods

- Experiment: Model with trade in energy but not in "goods"
  - Start from the equilibrium where carbon tax $t^f(\mathbb{J}) = 0$,
    $\Rightarrow$ country $i$ is indifferent to join the club $\mathbb{J}$ or not
  - Linear approximation around that point $\Rightarrow$ small changes in carbon tax $dt^f$

# Countries' incentives – Model w/o trade in goods

- Experiment: Model with trade in energy but not in "goods"
  - Start from the equilibrium where carbon tax $\mathfrak{t}^f(\mathbb{J}) = 0$,
    $\Rightarrow$ country $i$ is indifferent to join the club $\mathbb{J}$ or not
  - Linear approximation around that point $\Rightarrow$ small changes in carbon tax $d\mathfrak{t}^f$
  - Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})} = -e_i d\mathfrak{t}^f - \gamma_i(T_i - T_{i0})^\delta y_i \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \pi_i \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

  - Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}} = -\left(E_{\mathbb{J}} - E_{\mathbb{J}\setminus\{i\}}\right) \frac{\sigma \, d\mathfrak{t}^f}{q^f(1-s^f)} \frac{1}{1 + \frac{\nu\sigma}{(1-s^f)}}$$

○ Params: $\sigma$ energy demand elast$^y$, $s^f$ energy cost share, $\nu$ energy supply elas$^y$, Climate damage $\gamma_i$ and curv. $\delta$

# Countries' incentives – Model w/o trade in goods

- Experiment: Model with trade in energy but not in "goods"
  - Start from the equilibrium where carbon tax $t^f(\mathbb{J}) = 0$,
    $\Rightarrow$ country $i$ is indifferent to join the club $\mathbb{J}$ or not
  - Linear approximation around that point $\Rightarrow$ small changes in carbon tax $dt^f$
  - Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})} = -e_i dt^f - \gamma_i (T_i - T_{i0})^{\delta} y_i \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \pi_i \frac{(1+\nu)}{E} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

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○ Params: $\sigma$ energy demand elast$^y$, $s^f$ energy cost share, $\nu$ energy supply elas$^y$, Climate damage $\gamma_i$ and curv. $\delta$

# Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$
\begin{aligned}
\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = & -e_i d\mathrm{t}^f - \gamma_i(T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \\
& - e_i \frac{q^f \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \; + \; \eta_i^f \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \\
& + \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}}\Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij}\Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}}\Big)
\end{aligned}
$$

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  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i d\mathrm{t}^f - \gamma_i (T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$+ \eta_i^y \Big( \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}} \Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij} \Big( \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}} \Big)$$

  - GE effect on energy markets, same as before
  - GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki} \left( \frac{dv_k}{v_k} - \theta \frac{d\mathrm{t}_{ki}^b}{1+\mathrm{t}_{ki}^b} \right) \qquad \alpha_{ki} = \frac{c_{ki}\tau_{ki}\mathrm{p}_i}{\sum_\ell c_{k\ell}\tau_{k\ell}\mathrm{p}_\ell} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+\mathrm{t}_{ki}^b)v_i}$$

    – Params: $\sigma$ energy demand elasticity, $s^f$ energy cost share, $\nu$ energy supply elasticity, share of output $y$ in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

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# Complementarity in coalition formation – Model w/o trade in goods

- Is marginal gain $\Delta\mathcal{W}(\mathbb{J},j) := \mathcal{W}(\mathbb{J}\cup j) - \mathcal{W}(\mathbb{J})$ "growing" in $\mathbb{J}$ ?
  - Linear approximation for small $\{\mathfrak{t}^f, \mathfrak{t}^b\}$

$$\Delta\mathcal{W}(\mathbb{J},j) = -\omega_j u'(c_j) e_j d\mathfrak{t}^f + \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i)\Delta_i\gamma_i(T_i - T_{i0})^\delta y_i\Big] \frac{\sigma\, e_j d\mathfrak{t}^f}{q^f(1-s^f+\nu\sigma)}$$
$$+ \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i) e_i\Big] \frac{1}{1+\frac{1-s^f}{\nu\sigma}} \frac{e_j d\mathfrak{t}^f}{E_{\mathbb{I}}} - \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i)\pi_i\Big] \frac{(1+\nu)}{E_{\mathbb{I}}} \frac{\sigma\, e_j d\mathfrak{t}^f}{q^f(1-s^f+\nu\sigma)}$$

  - Free-riding problem: $\Delta\mathcal{W}(\mathbb{J},j)$ could be negative
  - If $\Delta\mathcal{W}(\mathbb{J},j) > 0$, what effects does $\mathbb{J}$ have on marginal gain?
    - Marginal climate benefit decreases in $\mathbb{J}$, since temperature $T_i$ declines!
    - G.E. effect on energy price: $E_{\mathbb{I}}$, $q$ and $\pi^f$ decreases with $\mathbb{J}$, effect on demand ambiguous
    - Similar formula for the case with trade tariffs: Work in progress.

# Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(T_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \ell, \varepsilon(e^f, e^r)) = \left[(1-\epsilon_i)^{\frac{1}{\sigma_y}} \left(k^\alpha \ell^{1-\alpha}\right)^{\frac{\sigma_y - 1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} \left(z_i^e \, \varepsilon_i(e^f, e^r)\right)^{\frac{\sigma_y - 1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega_i^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\omega_i)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

  - Calibrate TFP $z_i$ to match $y_i = GDP_i$ per capita in 2011 (PPP).
  - Today: $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all $i$
  - Future: $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix $(e_i^f, e_i^r)$

- Damage functions in production function $y$:

$$\mathcal{D}_i^y(T) = e^{-\gamma_i^{\pm,y}(T - T_i^\star)^2}$$

  - Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y} \mathbb{1}_{\{T > T_i^\star\}} + \gamma^{-,y} \mathbb{1}_{\{T < T_i^\star\}}$
  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $T_i^\star = \bar{\alpha} T_{it_0} + (1-\bar{\alpha}) T^\star$

# Quantification – Energy markets

- Fossil production $e^x_{it}$ and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now: $\bar{\nu}_i$ to match extraction data $e^x_i$, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. $\nu_i$ extraction cost curvature to match profit $\pi^f_i = \frac{\bar{\nu}_i \nu_i}{1+\nu_i}(\frac{e^x_i}{\mathcal{R}_i})^{\nu_i}\mathcal{R}_i\mathbb{P}_i$
  - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data $e^x_i$ (BP, IEA)

# Quantification – Energy markets

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  - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data $e^x_i$ (BP, IEA)
- Renewable: Production $\bar{e}^r_{it}$ and price $q^r_{it}$
  - Now: $q^r_{it} = z^r e^{-g_r t}$, with $g_r$ growth rate in renewable energy price decreases.
  - Future: Choose $z^r_i$ to match the energy mix $(e^f_i, e^r_i)$

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# Quantification – Future Extensions:

- Damage parameters:
  - $\gamma_i^{\pm,y}$ depends on daily temperature distribution $T \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
  - Use Climate Lab's (Greenstone et al) estimates for damage $\gamma_i$ ?
- Fossil Energy markets:
  - Divide fossils $e_{it}^f / e_{it}^x$ into oil/gas/coal
  - Match the production/cost/reserves data across countries
  - Use a dynamic model: extraction/exploration a la Hotelling
- Renewables Energy markets:
  - Make the problem dynamic with investment in capacity $C_{it}^r$
- Population dynamics
  - Match UN forecast for growth rate / fertility

# Calibration

Table: Baseline calibration ($\star$ = subject to future changes)

| | | | |
|---|---|---|---|
| *Technology & Energy markets* | | | |
| $\alpha$ | 0.35 | Capital share in $f(\cdot)$ | Capital/Output ratio |
| $\epsilon$ | 0.12 | Energy share in $f(\cdot)$ | Energy cost share (8.5%) |
| $\sigma$ | 0.3 | Elasticity capital-labor vs. energy | Complementarity in production (c.f. Bourany 2020) |
| $\omega$ | 0.8 | Fossil energy share in $e(\cdot)$ | Fossil/Energy ratio |
| $\sigma_e$ | 2.0 | Elasticity fossil-renewable | Slight substitutability & Study by Stern |
| $\delta$ | 0.06 | Depreciation rate | Investment/Output ratio |
| $\bar{g}$ | $0.01^\star$ | Long run TFP growth | Conservative estimate for growth |
| $g_e$ | $0.01^\star$ | Long run energy directed technical change | Conservative / Acemoglu et al (2012) |
| $g_r$ | $-0.01^\star$ | Long run renewable price decrease | Conservative / Match price fall in R.E. |
| $\nu$ | $2^\star$ | Extraction elasticity of fossil energy | Cubic extraction cost |
| *Preferences & Time horizon* | | | |
| $\rho$ | 0.03 | HH Discount factor | Long term interest rate & usual calib. in IAMs |
| $\eta$ | 2.5 | Risk aversion | |
| $n$ | $0.01^\star$ | Long run population growth | Conservative estimate for growth |
| $\omega_i$ | 1 | Pareto weights | Uniforms / Utilitarian Social Planner |
| $T$ | 90 | Time horizon | Horizon 2100 years since 2010 |

# Calibration

Table: Baseline calibration ($\star$ = subject to future changes)

| | | | |
|---|---|---|---|
| *Climate parameters* | | | |
| $\xi$ | 0.81 | Emission factor | Conversion 1 $MTOE \Rightarrow 1\ MT\ CO_2$ |
| $\zeta$ | 0.3 | Inverse climate persistence / inertia | Sluggishness of temperature $\sim 11-15$ years |
| $\chi$ | $2.1/1e6$ | Climate sensitivity | Pulse experiment: $100\,GtC \equiv 0.21^\circ C$ medium-term warming |
| $\delta_s$ | 0.0014 | Carbon exit from atmosphere | Pulse experiment: $100\,GtC \equiv 0.16^\circ C$ long-term warming |
| $\gamma^{\oplus}$ | $0.00234^\star$ | Damage sensitivity | Nordhaus' DICE |
| $\gamma^{\ominus}$ | $0.2 \times \gamma^{\oplus\ \star}$ | Damage sensitivity | Nordhaus' DICE & Rudik et al (2022) |
| $\alpha^T$ | $0.2^\star$ | Weight historical climate for optimal temp. | Marginal damage decorrelated with initial temp. |
| $T^\star$ | 15.5 | Optimal yearly temperature | Average spring temperature / Developed economies |
| *Parameters calibrated to match data* | | | |
| $p_i$ | | Population | Data – World Bank 2011 |
| $z_i$ | | TFP | To match GDP Data – World Bank 2011 |
| $T_i$ | | Local Temperature | To match temperature of largest city |
| $\mathcal{R}_i$ | | Local Fossil reserves | To match data from BP Energy review |

# Sequential solution method

- Summary of the model:
    - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, T_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
    - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^T_{it}, \lambda^{\mathcal{S}}_t, \lambda^{\mathcal{R}}_{it}\}_{it}$
    - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
    - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
    - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)
- Global Numerical solution:
  - Discretize agents (countries) space $i \in \mathbb{I}$ with $M$ and time-space $t \in [t_0, t_T]$ with $T$ periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and $N$ equations with gradient-descent – Newton-Raphson methods.

# Sequential method: Pros and Cons

- Why use a sequential approach?
  - *Global approach*: *Only* need to follow the trajectories for $i$ agents:
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity:
    *Productivity* $z_i$ *Population* $p_i$, *Temperature scaling* $\Delta_i$, *Fossil energy cost* $\bar{\nu}_i$, *Energy mix* $\epsilon_i, \omega_i, z_i^r$, *Local damage* $\gamma_i^y, \gamma_i^u, T_i^\star$, *Directed Technical Change* $z_i^e$
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables:
    *For now: Wealth* $w_{it}$, *temperature* $T_{it}$, *reserves* $\mathcal{R}_{it}$, *Carbon* $\mathcal{S}_t$
    *Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)*
  - Newton method & Non-linear solvers very efficient
- Why not:
  - Numerical constraint to solve a large system of ODEs and non-linear equations:
  - $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either $M$ or $T$ can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

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