The Inequality of Climate Change & Optimal Energy policy

WORK IN PROGRESS

Thomas Bourany
THE UNIVERSITY OF CHICAGO

JMP Proposal

June 2023

Introduction – this project

- What is the optimal taxation of energy in the presence of climate externality *and* inequality ?
  - In a context where fossil fuels taxation and climate policy redistributes across countries

Introduction – this project

- What is the optimal taxation of energy in the presence of climate externality *and* inequality ?
  - In a context where fossil fuels taxation and climate policy redistributes across countries
- Develop a simple and flexible model of climate economics
  - Standard IAM model with heterogeneous regions
  - Normative implications : Optimal Ramsey policy for carbon taxation
  - Provide a numerical methodology and a quantitative model

Introduction – this project

- What is the optimal taxation of energy in the presence of climate externality *and* inequality ?
  - In a context where fossil fuels taxation and climate policy redistributes across countries
- Develop a simple and flexible model of climate economics
  - Standard IAM model with heterogeneous regions
  - Normative implications : Optimal Ramsey policy for carbon taxation
  - Provide a numerical methodology and a quantitative model
- ○ Evaluate the heterogeneous welfare costs of global warming
  - Damages of climate & temperature varies across countries
  - $\Rightarrow$ Inequality increases the welfare cost of carbon
- ○ Solve world optimal carbon policy with heterogeneous regions
  - Does the optimal carbon tax coincide with the social cost of carbon ?
  - $\Rightarrow$ Depends on transfer policy : need to adjust the tax for inequality level

Toy model

- Consider two countries $i = N, S$, (North/South)
  - Household consumes good c_i, produced by a rep. firm with energy e_i and productivity z_i
  - Energy producers extract energy e_i^x selling it at price q^e
- Country-i planner problem :

$$\mathcal{V}_i = \max_{c_i, e_i, e_i^x} U(c_i)$$
$$c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - q^e e_i + e_i^x q^e - c_i(e_i^x) \qquad [\lambda_i]$$
$$e_N + e_S = e_N^x + e_S^x$$

- Production is subject to damage $\mathcal{D}_i(\mathcal{S})$ due to climate externalities : $\mathcal{S} = \overbrace{\xi(e_N + e_S)}^{=\text{GHG emissions}}$
- Competitive equilibrium Result :
  - Marginal Product of Energy = Energy Cost

$$MPe_i = q^e = c_i'(e_i^x) \qquad with \qquad MPe_i := \mathcal{D}_i(\mathcal{S}) z_i F'(e_i)$$

  - Inequality

$$\lambda_i = U'(c_i) \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - q^e e_i + e_i^x q^e - c_i(e_i^x)$$

Toy model – First Best and Decentralization

- Comparison with Social planner with full transfers (First Best)

$$\mathbb{W} = \max_{\{c_i, e_i, e_i^x\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$\sum_{i=N,S} c_i + c_i(e_i^x) = \sum_{i=N,S} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad [\lambda] \qquad \& \qquad \sum_{i=N,S} e_i = \sum_{i=N,S} e_i^x$$

Toy model – First Best and Decentralization

- Comparison with Social planner with full transfers (First Best)

$$\mathbb{W} = \max_{\{c_i, e_i, e_i^x\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$\sum_{i=N,S} c_i + c_i(e_i^x) = \sum_{i=N,S} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad [\lambda] \qquad \& \qquad \sum_{i=N,S} e_i = \sum_{i=N,S} e_i^x$$

- *Proposition 1 :* First-Best and decentralization :
  - Redistribution achieved with *lump-sum transfers / tax* $T_S \geq 0$ & $T_N \leq 0$

$$\omega_S U'(c_S) = \omega_N U'(c_N) = \lambda$$
$$\Rightarrow \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - (q^e + \mathbf{t}^e) e_i + e_i^x q^e - c_i(e_i^x) + T_i$$

  - Correction of the climate externality with a *common* energy tax : $\mathbf{t}_i^e \equiv \mathbf{t}^e = \xi\, \overline{SCC}$

$$MPe_i = c'(e_i^x) + \xi \underbrace{\overline{SCC}}_{=\mathbf{t}^e} \qquad with \qquad \overline{SCC} := -\sum_{i=N,S} \mathcal{D}_i'(\mathcal{S}) z_i F(e_i)$$

Marginal Product of Energy = Energy Marg. Cost + Social Cost of Carbon

Toy model – Optimal energy policy without transfers

- Assume now that *lump-sum transfers across countries* are prohibited
  - Ramsey policy, allowing country-specific carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$

$$\mathcal{W} = \max_{\{c_i, e_i, e_i^x\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$s.t \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - (q^e + \mathbf{t}_i^e) e_i + e_i^x q^e - c_i(e_i^x) + T^i \qquad [\phi_i] \qquad \& \qquad \sum_{i=N,S} e_i = \sum_{i=N,S} e_i^x$$

Toy model – Optimal energy policy without transfers

- Assume now that *lump-sum transfers across countries* are prohibited
  - Ramsey policy, allowing country-specific carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$

$$\mathcal{W} = \max_{\{c_i, e_i, e_i^x\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$s.t \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - (q^e + \mathbf{t}_i^e) e_i + e_i^x q^e - c_i(e_i^x) + T^i \qquad [\phi_i] \qquad \& \qquad \sum_{i=N,S} e_i = \sum_{i=N,S} e_i^x$$

- *Proposition 2 :* Second-Best without transfers :
  - Inequality because of lack of redistribution

$$\omega_S U'(c_S) = \phi_S \neq \phi_N = \omega_N U'(c_N)$$

  - Energy tax integrate redistributive concerns : $\mathbf{t}_i^e \neq \mathbf{t}^e := \xi\, SCC$

$$MPe_i = c_i'(e_i^x) + \mathbf{t}_i^e$$

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[\xi\, SCC + c''(\bar{E})\, SCE\right]$$

Social Cost of Carbon (SCC) with inequality

- The Energy taxation integrates three motives :

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[\xi \, SCC + c''(\bar{E}) \, SCE\right] \qquad \Rightarrow \qquad \mathbf{t}_S^e \leq \mathbf{t}_N^e \ \underline{i.f.f.} \ \widehat{\phi}_S \geq \widehat{\phi}_N$$

  - A measure of inequality, subject to Pareto weights ω_i

$$\widehat{\phi}_i = \frac{\phi_i}{\bar{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{2}\sum_j \omega_j U'(c_j)} \lesseqgtr 1 \qquad \widehat{\phi}_S \geq \widehat{\phi}_N \quad \underline{i.f.f.} \quad \omega_S \geq \gamma \omega_N \quad \gamma \in (0,1)$$

Social Cost of Carbon (SCC) with inequality

- The Energy taxation integrates three motives :

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[\xi\, SCC + c''(\bar{E})\, SCE\right] \qquad \Rightarrow \qquad \mathbf{t}_S^e \leq \mathbf{t}_N^e \ \underline{i.f.f.}\ \widehat{\phi}_S \geq \widehat{\phi}_N$$

  - A measure of inequality, subject to Pareto weights ω_i

$$\widehat{\phi}_i = \frac{\phi_i}{\bar{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{2}\sum_j \omega_j U'(c_j)} \lessgtr 1 \qquad \widehat{\phi}_S \geq \widehat{\phi}_N \quad \underline{i.f.f.} \quad \omega_S \geq \gamma\omega_N \quad \gamma \in (0,1)$$

  - The Social Cost of Carbon (SCC) exacerbated by heterogeneity

$$\begin{aligned} SCC &= -\textstyle\sum_j \widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j) \\ &= -\mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}_j'(\mathcal{S}) z_j F(e_j)\right) - \mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)] \gtrapprox -\mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)] \end{aligned}$$

Social Cost of Carbon (SCC) with inequality

- The Energy taxation integrates three motives :

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[\xi\, SCC + c''(\bar{E})\, SCE\right] \qquad \Rightarrow \qquad \mathbf{t}_S^e \leq \mathbf{t}_N^e \;\; \underline{i.f.f.} \;\; \widehat{\phi}_S \geq \widehat{\phi}_N$$

  - A measure of inequality, subject to Pareto weights ω_i

$$\widehat{\phi}_i = \frac{\phi_i}{\overline{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{2}\sum_j \omega_j U'(c_j)} \lesseqgtr 1 \qquad \widehat{\phi}_S \geq \widehat{\phi}_N \quad \underline{i.f.f.} \quad \omega_S \geq \gamma \omega_N \quad \gamma \in (0,1)$$

  - The Social Cost of Carbon (SCC) exacerbated by heterogeneity

$$\begin{aligned} SCC &= -\textstyle\sum_j \widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j) \\ &= -\mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}_j'(\mathcal{S}) z_j F(e_j)\right) - \mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)] \gtrapprox -\mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)] \end{aligned}$$

  - The Social cost of energy (SCE) depending on the redistribution btw importer and exporter

$$SCE = \mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, e_j - e_j^x\right) \lessapprox 0 \qquad c''(\bar{E}) := \left(\textstyle\sum_i \frac{1}{c_i''(e_i^x)}\right)^{-1} \geq 0$$

Toy model – Optimal policy : no transfers & participation constraint

- Assume that lump-sum transfers are prohibited & countries can exit climate agreements
  - Ramsey policy, allowing country-specific carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$

$$\widetilde{\mathcal{W}} = \max_{\{c_i, e_i, e_i^x\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$s.t \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - (q^e + \mathbf{t}_i^e) e_i + e_i^x q^e - c_i(e_i^x) + T^i \qquad [\phi_i] \qquad \& \qquad \sum_{i=N,S} e_i = \sum_{i=N,S} e_i^x$$

$$U(c_i) \geq U(\bar{c}_i) \qquad [\nu_i] \qquad \qquad \bar{c}_i = \mathcal{D}_i(\mathcal{S}) z_i F(\bar{e}_i) - c_i(\bar{e}_i)$$

Toy model – Optimal policy : no transfers & participation constraint

- Assume that lump-sum transfers are prohibited & countries can exit climate agreements
  - Ramsey policy, allowing country-specific carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$

$$\widetilde{\mathcal{W}} = \max_{\{c_i, e_i, e_i^x\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$s.t \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - (q^e + \mathbf{t}_i^e) e_i + e_i^x q^e - c_i(e_i^x) + T^i \qquad [\phi_i] \qquad \& \qquad \sum_{i=N,S} e_i = \sum_{i=N,S} e_i^x$$

$$U(c_i) \geq U(\bar{c}_i) \qquad [\nu_i] \qquad \bar{c}_i = \mathcal{D}_i(\mathcal{S}) z_i F(\bar{e}_i) - c_i(\bar{e}_i)$$

- *Proposition 3 :* Second-Best without transfers & participation constraints
  - Inequality because of lack of redistribution & participation incentive

$$\phi_N = \omega_N U'(c_N) + \nu_N U'(c_N) = \widetilde{\omega}_N U'(c_N) \lesseqgtr \phi_S \qquad \widetilde{\omega}_N = \omega_N + \nu_N \geq \omega_N$$

  - Energy tax integrate both redistributive and participation concerns : $\mathbf{t}_i^e \neq \mathbf{t}^e := \xi \, SCC$

$$MPe_i = c_i'(e_i^x) + \mathbf{t}_i^e$$

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \widetilde{\omega}_j U'(c_j)}{\widetilde{\omega}_i U'(c_i)} \left[\xi \, SCC + c''(\bar{E}) \, SCE\right] \qquad \Rightarrow \qquad \mathbf{t}_N^e \lesseqgtr \mathbf{t}_S^e$$

Toy model – Summary

- In a large class of IAM models, optimal carbon policy goes hand in hand with the availability of redistribution instruments

Toy model – Summary

- In a large class of IAM models, optimal carbon policy goes hand in hand with the availability of redistribution instruments
- With lump-sum tax, fossil energy tax is proportional to the Social Cost of Carbon
  - Optimal fossil tax : $\mathbf{t}^f = \xi\, SCC$
  - Redistribution is taken care of by transfers T_i across locations i

Toy model – Summary

- In a large class of IAM models, optimal carbon policy goes hand in hand with the availability of redistribution instruments
- With lump-sum tax, fossil energy tax is proportional to the Social Cost of Carbon
  - Optimal fossil tax : $\mathbf{t}^f = \xi \, SCC$
  - Redistribution is taken care of by transfers T_i across locations i
- Without such instruments, energy taxes are country i-specific & accounts for 3 motives :
  - $\mathbf{t}_i^e = \frac{\overline{\omega U'(c)}}{\omega_i U'(c_i)} [\xi \, SCC + c''(E) \, SCE]$
  - Adjusted for Pareto weights ω_i and marg. utility of consumption $U'(c_i)$
    $\Rightarrow$ lower energy tax for poorer countries

Toy model – Summary

- In a large class of IAM models, optimal carbon policy goes hand in hand with the availability of redistribution instruments
- With lump-sum tax, fossil energy tax is proportional to the Social Cost of Carbon
  - Optimal fossil tax : $\mathbf{t}^f = \xi \, SCC$
  - Redistribution is taken care of by transfers T_i across locations i
- Without such instruments, energy taxes are country i-specific & accounts for 3 motives :
  - $\mathbf{t}_i^e = \frac{\overline{\omega U'(c)}}{\omega_i U'(c_i)} [\xi \, SCC + c''(E) \, SCE]$
  - Adjusted for Pareto weights ω_i and marg. utility of consumption $U'(c_i)$
    $\Rightarrow$ lower energy tax for poorer countries
  - Heterogeneity can increase the level of the Social Cost of Carbon SCC

Toy model – Summary

- In a large class of IAM models, optimal carbon policy goes hand in hand with the availability of redistribution instruments
- With lump-sum tax, fossil energy tax is proportional to the Social Cost of Carbon
  - Optimal fossil tax : $\mathbf{t}^f = \xi\, SCC$
  - Redistribution is taken care of by transfers T_i across locations i
- Without such instruments, energy taxes are country i-specific & accounts for 3 motives :
  - $\mathbf{t}_i^e = \frac{\overline{\omega U'(c)}}{\omega_i U'(c_i)} [\xi\, SCC + c''(E)\, SCE]$
  - Adjusted for Pareto weights ω_i and marg. utility of consumption $U'(c_i)$
    $\Rightarrow$ lower energy tax for poorer countries
  - Heterogeneity can increase the level of the Social Cost of Carbon SCC
  - Account for the terms-of-trade redistribution $SCE \propto \mathbb{C}\text{ov}_j(\omega_j U'(c_j), ne_i)$

Toy model – Summary

- In a large class of IAM models, optimal carbon policy goes hand in hand with the availability of redistribution instruments
- With lump-sum tax, fossil energy tax is proportional to the Social Cost of Carbon
  - Optimal fossil tax : $\mathbf{t}^f = \xi \, SCC$
  - Redistribution is taken care of by transfers T_i across locations i
- Without such instruments, energy taxes are country i-specific & accounts for 3 motives :
  - $\mathbf{t}_i^e = \frac{\overline{\omega U'(c)}}{\omega_i U'(c_i)} \left[\xi \, SCC + c''(E) \, SCE\right]$
  - Adjusted for Pareto weights ω_i and marg. utility of consumption $U'(c_i)$
    $\Rightarrow$ lower energy tax for poorer countries
  - Heterogeneity can increase the level of the Social Cost of Carbon SCC
  - Account for the terms-of-trade redistribution $SCE \propto \mathbb{C}\text{ov}_j(\omega_j U'(c_j), ne_i)$
  - Participation constraints change the distribution of tax $\{\mathbf{t}_i^f\}$:
    $\Rightarrow$ lower tax for richer countries as an incentive to join climate agreements

Model – Representative Household

- Deterministic Neoclassical economy, in continuous time
  - heterogeneous countries $i \in \mathbb{I}$
  - In each country, 4 agents : (i) representative household, (ii) homogeneous good firm, (iii) fossil and (iv) renewable energy producers.

- Representative household problem in each country i :

$$\mathcal{V}_{it_0} = \max_{\{c_{it}, k_{it}, b_{it}\}} \int_{t_0}^{\infty} e^{-\rho t} \, u_i(c_{it}, \tau_{it}) dt$$

- Dynamics of wealth of country i, $w_{it} = b_{it} + k_{it}$ More details

$$\dot{w}_{it} := \dot{k}_{it} + \dot{b}_{it} = y_{it} + \pi^{f}_{it} + \pi^{r}_{it} + r^{\star}_{t} b_{it} + (r^{\star}_{t} - \bar{\delta}) k_{it} - c_{it} + \mathbf{t}^{ls}_{it}$$

  - Labor income y_{it} from homogeneous good firm.
  - All the lower-case variables are expressed per unit of efficient labor $y_{it} = Y_{it}/(L_{it} A_{it})$

Model – Representative Firm

- Competitive homogeneous good producer in country i

$$\max_{k_{it}, e^f_{it}, e^r_{it}} \mathcal{D}^y(\tau_{it}) z_i f(k_{it}, e^f_{it}, e^r_{it}) - r^\star_t k_{it} - (q^f_t + \mathbf{t}^f_{it}) e^f_{it} - (q^r_t + \mathbf{t}^r_{it}) e^r_{it} - y_{it}$$

  - Energy mix with fossil e^f_{it} – emitting carbon – subject to price q^f_t and tax/subsidy $\mathbf{t}^f_{it}$. Similarly "clean" renewable e^r_t, at price q^r_{it} and tax $\mathbf{t}^r_{it}$.
  - No international trade in goods and Labor is immobile

Model – Energy markets

- Competitive fossil fuels energy producer :
  - Static problem (for now) extract energy e^x_{it} depleting reserves $\mathcal{R}_{it}$

$$\pi^f_{it} = \max_{e^x_{it}} q^f_t e^x_{it} - \mathcal{C}^f_i(e^x_{it}, \mathcal{R}_{it})$$

$$\dot{\mathcal{R}}_{it} = -e^x_{it} \qquad \mathcal{R}_{it_0} = \mathcal{R}_{i0} \qquad \mathcal{R}_{it} \geq 0$$

  - Fossil energy traded in international markets :

$$\int_{\mathbb{I}} e^f_{it}\, p_i\, di = \int_{\mathbb{I}} e^x_{it}\, di$$

  - Optimal extraction

$$q^f_t = \mathcal{C}^f_e(e^x_{it}, \mathcal{R}_{it})$$

Model – Energy markets

- Competitive fossil fuels energy producer :
  - Static problem (for now) extract energy e^x_{it} depleting reserves $\mathcal{R}_{it}$

$$\pi^f_{it} = \max_{e^x_{it}} q^f_t e^x_{it} - \mathcal{C}^f_i(e^x_{it}, \mathcal{R}_{it})$$

$$\dot{\mathcal{R}}_{it} = -e^x_{it} \qquad \mathcal{R}_{it_0} = \mathcal{R}_{i0} \qquad \mathcal{R}_{it} \geq 0$$

  - Fossil energy traded in international markets :

$$\int_{\mathbb{I}} e^f_{it} p_i \, di = \int_{\mathbb{I}} e^x_{it} \, di$$

  - Optimal extraction

$$q^f_t = \mathcal{C}^f_e(e^x_{it}, \mathcal{R}_{it})$$

- Renewable energy as a substitute technology in each country i (Static problem for now)

$$\pi^r_{it} = \max_{\{\bar{e}^r_{it}\}} q^r_{it}\bar{e}^r_{it} - \mathcal{C}^r_i(\bar{e}^r_{it}) \qquad \Rightarrow \qquad q^r_{it} = \mathcal{C}^r_e(\bar{e}^r_t) = z^r_{it}$$

Climate system

- Fossil energy input e_t^f causes climate externality

$$\mathcal{E}_t = \xi \int_{\mathbb{I}} e_{it}^f \, p_i \, di$$

- Cumulative GHG in atmosphere $\mathcal{S}_t$ increases temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- Country's local temperature :

$$\dot{\tau}_{it} = \zeta \big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \bar{\tau}_{it_0}) \big)$$

  - Linear model : Climate sensitivity to carbon χ, Climate reaction/inertia ζ, Carbon content of fossils ξ, Country i linear pattern scaling factor Δ_i, Carbon exit from atmosphere δ_s

Model – Equilibrium

- Equilibrium
  - Given, initial conditions $\{w_0, \tau_0\}$ and country-specific policies $\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, \mathbf{t}^{ls}_{it}\}$, a competitive equilibrium is a continuum of sequences of states $\{w_{it}, \tau_{it}\}_{it}$ and $\{\mathcal{S}_t, \mathcal{T}_t, \mathcal{R}_t\}_t$ and policies $\{c_{it}, b_{it}, k_{it}, e^f_{it}, e^r_{it}, e^x_{it}\}_{it}$ and price sequences $\{r^\star_t, q^f_t, q^r_t\}$ such that :

  - Households choose policies $\{c_{it}, b_{it}\}_{it}$ to max utility s.t. budget constraint, giving $\dot{w}_{it}$
  - Firm choose policies $\{k_{it}, e^f_{it}, e^r_{it}\}_{it}$ to max profit
  - Fossil and renewables firms extract/produce $\{e^x_{it}, \bar{e}^r_{it}\}_{it}$ to max static profit, yielding $\dot{\mathcal{R}}_t$
  - Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{it}\}_{it}$.
  - Prices $\{r^\star_t, q^f_t, q^r_{it}\}$ adjust to clear the markets : $\int_{\mathbb{I}} e^x_{it} di = \int_{\mathbb{I}} e^f_{it} di$ and $e^r_{it} = \bar{e}^r_{it}$, and $\int_{i \in \mathbb{I}} b_{it} di = 0$, with bonds $b_{it} = w_{it} - k_{it}$

Calibration – Household

- Household utility $u_i(c,\tau) = U\big(\mathcal{D}_i^u(\tau)c\big)$ with CRRA $U(\tilde{c}) = \frac{\tilde{c}^{1-\eta}}{1-\eta}$
- Damage functions in utility u or production function y :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm,y}(\tau-\tau_i^\star)^2}$$

and similarly for $\mathcal{D}_i^u(\tau)$, with $\gamma^{\pm,y} = \gamma^{\oplus,y}\mathbb{1}_{\{\tau>\tau_i^\star\}} + \gamma^{\ominus,y}\mathbb{1}_{\{\tau<\tau_i^\star\}}$

  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $\tau_i^\star = \bar{\alpha}\tau_{it_0} + (1-\bar{\alpha})\tau^\star$.
  - Future : $\gamma_i^{\pm,y}$ depends on daily temperature distribution $\tau \sim \mathcal{T}_i(\bar{T},\sigma^T)$ following Rudik et al. (2022)

Calibration – Firms

- Production function $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \varepsilon(e^f, e^r)) = \left[(1-\epsilon)^{\frac{1}{\sigma_y}} k^{\alpha \frac{\sigma_y - 1}{\sigma_y}} + \epsilon^{\frac{1}{\sigma_y}} \left(z_i^e \ \varepsilon(e^f, e^r)\right)^{\frac{\sigma_y - 1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\omega)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

  - Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
  - Now : $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
  - Future : $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

Calibration – Energy markets

- Fossil production e^x_{it} and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now : $\bar{\nu}_i = \bar{\nu}$ and $\nu_i = \nu$ and $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction level data e^x_i (BP, IEA)
  - Extension : Divide fossils into oil/gas/coal, and match the production/cost/reserves data across countries + use a dynamic model (extraction/exploration Hotelling problem).

Calibration – Energy markets

- Fossil production e^x_{it} and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now : $\bar{\nu}_i = \bar{\nu}$ and $\nu_i = \nu$ and $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction level data e^x_i (BP, IEA)
  - Extension : Divide fossils into oil/gas/coal, and match the production/cost/reserves data across countries + use a dynamic model (extraction/exploration Hotelling problem).
- Renewable : Production $\bar{e}^r_{it}$ and price q^r_{it}
  - Now : $q^r_{it} = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
  - Future : Choose z^r_i to match the energy mix (e^f_i, e^r_i)
  - Extension : make the problem dynamic with capacity C^r_{it}

Model Solution

- Household consumption/saving problem
  - Using Pontryagin Max. Principle : states $\{x\} = \{w_{it}, \tau_{it}\}$, controls $\{c\} = \{c_{it}, b_{it}, k_{it}\}$ and costates $\{\lambda\} = \{\lambda_{it}^{w}, \lambda_{it}^{\tau}, \lambda_{it}^{s}\} \Rightarrow$ system of coupled ODEs.

$$\mathcal{H}^{hh}(\{x\},\{c\},\{\lambda\}) = u(c_i, \tau_i) + \lambda_{it}^{w}\dot{w}_{it} + \lambda_{it}^{\tau}\dot{\tau}_{it} + \lambda_{it}^{S}\dot{S}_t$$

Model Solution

- Household consumption/saving problem
  - Using Pontryagin Max. Principle : states $\{x\} = \{w_{it}, \tau_{it}\}$, controls $\{c\} = \{c_{it}, b_{it}, k_{it}\}$ and costates $\{\lambda\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\} \Rightarrow$ system of coupled ODEs.

$$\mathcal{H}^{hh}(\{x\}, \{c\}, \{\lambda\}) = u(c_i, \tau_i) + \lambda^w_{it}\dot{w}_{it} + \lambda^\tau_{it}\dot{\tau}_{it} + \lambda^S_{it}\dot{S}_t$$

  - ODE for the costate for wealth $\lambda^w_{it} = u_c(c_{it}, \tau_{it}) \;\Rightarrow$ Euler equation
  - The "local social cost of carbon" (SCC) for region i :

$$LSCC_{it} := -\frac{\partial \mathcal{V}_{it}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{it}/\partial c_{it}} = -\frac{\lambda^S_{it}}{\lambda^w_{it}}$$

    - ODEs for Costates : temperature λ^τ_{it} and carbon λ^S_{it}, More details
    - Stationary equilibrium closed-form formula, analogous to GHKT (2014) Here

Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$

Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
- Global Numerical solution :
  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

- Why use a sequential approach ?
  - *Global approach* : *Only* need to follow the trajectories for i agents :
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
    Productivity z_i *Population* p_i, *Temperature scaling* Δ_i, *Fossil energy cost* $\bar{\nu}_i$, *Energy mix* $\epsilon_i, \omega_i, z_i^r$, *Local damage* $\gamma_i^y, \gamma_i^u, \tau_i^\star$, *Directed Technical Change* z_i^e
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
    For now : Wealth w_{it}, *temperature* τ_{it}, *reserves* $\mathcal{R}_{it}$, *Carbon* $\mathcal{S}_t$
    Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
  - Newton method & Non-linear solvers very efficient

- Why not :
  - Numerical constraint to solve a large system of ODEs and non-linear equations :

  $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

Competitive equilibrium

- Simulation for $M =$ countries and $T = 20$.
- Result of CE to show here

Optimal policy

- Social planner, First best with a full set of instruments :
  - Lump-sum transfers to solve inequality, s.t.

$$\lambda_t = \omega_i u'(c_{it}) = \omega_j u'(c_{jt}) \ \ \forall i, j \in \mathbb{I}$$

  - Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^S}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)
  - Imply cross-countries lump-sum transfers $\exists\ i \ \ s.t.\ T_i > 0$ and $\exists\ j \ \ s.t.\ T_j < 0$

Optimal policy

- Social planner, First best with a full set of instruments :
  - Lump-sum transfers to solve inequality, s.t.

$$\lambda_t = \omega_i u'(c_{it}) = \omega_j u'(c_{jt}) \;\; \forall i,j \in \mathbb{I}$$

  - Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^S}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)
  - Imply cross-countries lump-sum transfers $\exists\, i \;\; s.t. \; T_i > 0$ and $\exists\, j \;\; s.t. \; T_j < 0$
- Second best without access to lump-sum transfers
  - Only region-i-specific distortive energy taxes : $\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r\}$. Tax receipts redistributed lump-sum : $\mathbf{t}_{it}^{ls} = \mathbf{t}_{it}^f e_{it}^f + \mathbf{t}_{it}^r e_{it}^r$
  - Welfare of the Ramsey planner :

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t},\boldsymbol{x},\boldsymbol{\lambda},\boldsymbol{c},\boldsymbol{q}\}_{it}} \int_{t_0}^{\infty} \int_{\mathbb{I}} e^{-\bar{\rho}t} \, \omega_i \, u(c_{it}, \tau_{it}) p_i \, di \, dt$$

The Ramsey Problem – Optimal Energy Policy

- Optimal Pigouvian tax for fossil energy :

$$\Rightarrow \quad \widehat{\psi}^w_{it}\, \mathbf{t}^f_{it} = p_i \xi\, SCC_t + SCF_t\, \mathcal{C}^f_{EE} \qquad \& \qquad \mathbf{t}^r_{it} = 0$$

  - Integrate several redistribution motives :

$$\widehat{\psi}^w_{it} = \frac{\psi^w_{it}}{\overline{\psi}^w_t} = \frac{\omega_i u_c(c_{it}, \tau_{it}) p_i}{\int_{j\in\mathbb{I}} \omega_j u_c(c_{jt}, \tau_{jt}) p_j dj} \lesseqgtr 1$$

$\Rightarrow$ lower tax on poorer/high $\widehat{\psi}^w_{it}$ countries

  - Level depends on SCC_t & fossil price SCF_t

$$SCC_t = \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}^w_{it}, LSCC_{jt}\right) + \mathbb{E}_j[LSCC_{jt}]$$

$$SCF_t = \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}^w_{it}, e^f_{jt} - e^x_{jt}\right) \qquad\qquad \mathcal{C}^f_{EE} = \left(\int_{j\in\mathbb{I}} \frac{1}{\mathcal{C}^f_{j,e^x e^x}} dj\right)^{-1}$$

Optimal Energy tax

- Numerical results for $M =$ countries and $T = 20$.
- Result of Optimal Policy to show here

Conclusion & Future plans

-

Appendices

More details – Capital market

- In each countries, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} & = \mathcal{D}_i^y(\tau_{it}) z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g}) k_{it} + \iota_{it} \\ \dot{b}_{it} & = r^\star b_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - \iota_{it} - c_{it} + \mathbf{t}_{it}^{ls} \\ b_{it} & \geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{i}^{l}$$

$$k_{it} \leq \frac{1}{1-\vartheta} w_{it}$$

- Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it}) z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r_t^\star$$

back

Impact of increase in temperature

- Marginal values of the climate variables : λ^s_{it} and λ^τ_{it}

$$\dot{\lambda}^\tau_{it} = \lambda^\tau_{it}(\rho + \zeta) + \overbrace{\gamma_i(\tau_{it} - \tau^\star_i)\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it}, e_{it})\lambda^k_{it} + \overbrace{\phi_i(\tau_{it} - \tau^\star_i)\mathcal{D}^u(\tau_{it})^{1-\eta} c_{it}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}^S_{it} = \lambda^S_{it}(\rho + \delta^s) - \zeta\, \chi\, \Delta_i\, \lambda^\tau_{it}$$

- Costate λ^S_{it} : marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{it} - \tau^\star_i$ & damage sensitivity of TFP γ^y_i and utility γ^u_i
  - Development level $f(k_{it}, e_{it})$ and c_{it}
  - Climate params : χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
  - back

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}^{\mathcal{T}}_{it} = \lambda^{\mathcal{T}}_t(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\lambda^k_t + \phi(\tau - \tau^\star)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}^{S}_t = \lambda^{S}{}_t(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda^{\mathcal{T}}_{it}$$

- Solving for $\lambda^{\mathcal{T}}_t$ and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^{S}_t = \dot{\lambda}^{\mathcal{T}}_t = 0$

$$\begin{aligned}
\lambda^{\mathcal{T}}_{it} &= -\int_t^\infty e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_u)y_{\mathcal{T}}\lambda^k_u + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du \\
\lambda^{\mathcal{T}}_{it} &= -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_\infty)y_\infty\lambda^k_\infty + \phi\mathcal{D}^u(\tau_\infty)u(c_\infty)\Big)
\end{aligned}$$

$$\begin{aligned}
\lambda^{S}_t &= -\int_t^\infty e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,u}dj\,du \\
&= \frac{1}{\widetilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,\infty} \\
&= -\frac{\chi}{\widetilde{\rho}+\delta^s}\frac{\zeta}{\widetilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_\infty\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj \\
\lambda^{S}_t &\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

Cost of carbon / Marginal value of temperature

- ***Proposition (Stationary LSCC)*** :
  When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ_i^y, γ_i^u, temperature, and output, consumption.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s}(\tau_\infty - \tau^\star)\Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty\Big)$$

  - Stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - Back

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega \int_{t_0}^{T} e^{(n+\bar{g})t-\delta_s(T-t)}\, q_t^{f\ -\sigma_e} \int_{j\in\mathbb{I}} \left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e-\sigma}\, dj\, dt$$

  - Geographical factors determining warming Δ_i
  - Climate sensitivity χ & carbon exit from atmosphere δ_s
  - Growth of population n, aggregate productivity $\bar{g}$
  - Deviation of output from trend y_i & relative TFP z_j
  - Directed technical change z_t^e, elasticity of energy in output σ
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}_t^R / \lambda_t^R = \rho$
  - Change in energy mix, renewable share ω, price q_t^r & elasticity of source σ_e
- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto\ n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \tilde{\gamma}\right) + (\sigma_e - \sigma)(1-\omega)g^{q^r} - (\sigma_e(1-\omega) + \sigma\omega)g^{q^f}$$

Back