The Optimal design of Climate Agreements
Inequality, Trade and Incentives for carbon policy

WORK IN PROGRESS

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Introduction

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$\Rightarrow$ Designing a climate agreement entails determining *jointly* the level of carbon tax and the club of participating countries

Introduction – this project

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  - Analyze the strategic implications of joining climate agreements
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- Preview of the result :
  - Differing incentives to join club through exposure to GE effects on energy & good prices
  - Benefit : change in climate, cost through taxation, energy rent & GE effect on price
  - Trade matters as it creates interdependence across countries
    - Damages of climate change propagate across countries : SCC become correlated
    - Leakage effect of carbon taxation : reallocate activity to outside the club
    - Generate policy leverage for making the climate club sustainable

Literature

- Climate change & optimal carbon taxation
  - RA model : Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
  - HA model : Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
  - Spatial models : Cruz, Rossi-Hansberg (2022, 2023)

  $\Rightarrow$ *Optimal and constrained policy with heterogeneous countries & trade*
- Unilateral vs. climate club policies :
  - Climate clubs : Nordhaus (2015), Non-cooperative taxation : Chari, Kehoe (1990), Suboptimal policy : Hassler, Krusell, Olovsson (2019)
  - Trade policy : Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021), Hsiao (2022), Costinot, Donalson, Vogel, Werning (2015), Adao, Costinot (2022), Antras, Fort, Gutierrez, Tintelnot (2022)

  $\Rightarrow$ *Climate cooperation and optimal design of climate club*
- Optimal policy in heterogeneous agents models
  - Policy with limited instruments : Diamond (1973), Davila, Walther (2022)
  - Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) . . .

  $\Rightarrow$ *Application to climate and carbon taxation policy*

Model – Household & Firms

- Static deterministic Neoclassical economy (for today)
  - countries $i \in \mathbb{I}$, heterogeneous in productivity z_i, temperature τ_i, energy extraction cost $\mathcal{C}_i$
  - In each country, 3 agents :
    (i) HtM household, (ii) homogeneous good firm, (iii) (fossil) energy producer

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- Representative household problem in each country i :

$$\mathcal{V}_i = u(c_i) \qquad \mathbb{P}_i c_i = w_i + \pi_i^f + \mathrm{t}_i^{ls} \qquad c_i = \begin{cases} c_{ii} & \mathbb{P}_i = \mathrm{p}_i = 1 \\ \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} & \text{price} \quad (1+\mathrm{t}_{ij}^b) d_{ij} \mathrm{p}_j \end{cases}$$

  - Labor income w_i from final good firm (labor norm. to 1), profit π_i^f from fossil firm

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- Competitive homogeneous good producer in country i

$$\max_{e_i^f} \mathrm{p}_i \, \mathcal{D}(\tau_i) z_i f(e_i^f) - w_i - (q^f + \mathrm{t}_i^e) e_i^f$$

  - Fossil energy consumption e_i^f – emitting carbon – subject to price q^f and tax/subsidy t^f.
  - Climate externality : effect of temperature on damage/TFP, $\mathcal{D}(\tau) \in (0, 1)$

Model – Energy markets & Emissions

- Competitive fossil fuels energy producer :
  - Extraction of fossil energy e^x_{it} at cost $\mathcal{C}^f_i$

$$\pi^f_i = \max_{e^x_i} q^f e^x_i - \mathcal{C}^f_i(e^x_i, \mathcal{R}_i)$$

  - Energy traded in international markets, at price q^f

$$E = \sum_{\mathbb{I}} e^f_i = \sum_{\mathbb{I}} e^x_i$$

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- Climate system
  - Fossil energy e^f releases GHGs in the atmosphere

$$\mathcal{E} = \sum_{\mathbb{I}} e^f_i$$

  - Country's local temperature :

$$\tau_i = \bar{\tau}_{i0} + \Delta_i \mathcal{E}$$

    - Linear model + Climate sensitivity/pattern scaling factor Δ_i

Model – Equilibrium

- Equilibrium
  - Given policies $\{\mathrm{t}_i^e, \mathrm{t}_{ij}^b, \mathrm{t}_i^{ls}\}$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e_i^f, e_i^x\}_{ij}$, states $\{\tau_i\}_i$ and prices $\{\mathrm{p}_i, q^f\}$ such that :

  - Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint
  - Firm choose policies $\{e_i^f\}_i$ to max. profit and Fossil firms extract/produce $\{e_i^x\}_i$ to max. profit.
  - Emissions $\mathcal{E}_t$ affects climate $\{\tau_i\}_i$.
  - Prices $\{\mathrm{p}_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$ and for each good $y_i := z_i \mathcal{D}(\tau_i) f(e_i^f) = \sum_k d_{ki} c_{ki} + \frac{q^e}{\mathrm{p}_i}(e_i^f - e_i^x) + \mathcal{C}(e_i^x, \mathcal{R}_i)$

Model – Dynamics & extensions

1. Energy market
    - Renewable energy firm in each country
    - Price of clean energy trending down
    - Fossil energy extraction/depleting reserves ⇒ Hotelling problem
2. Firm
    - Use capital as well to produce
    - Use an energy bundle of renewable and fossil energy
3. Households
    - Consumption / saving in bonds / in capital ⇒ Keynes-Ramsey rule
    - International markets to borrow bonds (in zero net supply)
4. Climate system with (short) inertia / closer to Integrated assessments models
5. Population growth dynamics for each country
6. (Exogenous) growth : TFP change and Energy-augmenting Directed TC.

Model – Solution

- Step 0 : Competitive equilibrium / Business as usual : No policy $\mathrm{t}_i = 0$
- Step 1 : Optimal (Ramsey) policy for the world
- Step 2 : Countries decide whether to join the climate club : participation constraints
- Step 3 : Optimal design of size $\mathbb{J}$ and countries $j \in \mathbb{J}$ in the climate agreement

Step 0 : Competitive equilibrium

- Key objects :
  - Marginal value of wealth $\lambda_i = \frac{u'(c_i)}{\mathbb{P}_i}$ with price index $\mathbb{P}_i = \left(\sum_j a_{ij}(d_{ij}\mathrm{p}_j)^{1-\theta}\right)^{\frac{1}{1-\theta}}$
  - Marginal value of carbon $\psi_i^{\mathcal{E}}$ for country i
  - "Local social cost of carbon" (LCC) for region i :

$$LCC_i := \frac{\partial \mathcal{V}_i / \partial \mathcal{E}}{\partial \mathcal{V}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(\tau_i) z_i f(e_i^f)\mathrm{p}_i \qquad (> 0 \text{ if heat causes losses})$$

Step 1 : World optimal Ramsey policy

- Maximizing welfare of the world Social Planner :

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - One single instrument : carbon tax t^f on energy e_i^f, rebated lump-sum to HHs $\mathrm{t}_i^{ls} = \mathrm{t}^f e_i^f$

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  - One single instrument : carbon tax t^f on energy e^f_i, rebated lump-sum to HHs $\mathrm{t}^{ls}_i = \mathrm{t}^f e^f_i$
  - The planner takes into account (i) the marginal value of wealth λ_i as well as (ii) the shadow value of good i, from market clearing, μ_i :

w/ trade : $$\omega_i u'(c_i) = \Big(\sum_{j\in\mathbb{I}} a_{ij}(d_{ij}\mathrm{p}_j)^{1-\theta}\Big[\omega_i\lambda_i + \omega_j\mu_j + \eta_{ij}(1-s_{ij})\Big]^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

vs. w/o trade $$\omega_i u'(c_i) = \omega_i \lambda_i$$

Step 1 : Optimal policy – Social Cost of Carbon

- Key objects : Local vs. Global Social Cost of Carbon :

$$SCC := \frac{\partial \mathcal{W}/\partial \mathcal{E}}{\partial \mathcal{W}/\partial w} = \frac{\psi^{\mathcal{E}}}{\lambda} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I}\sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

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- Inequality across regions :

$$\text{w/o trade : } \widehat{\lambda}_i = \frac{\omega_i \lambda_i}{\lambda} = \frac{\omega_i u'(c_i)}{\frac{1}{I}\sum_{\mathbb{I}} \omega_j u'(c_j)} \lesseqgtr 1$$

$$\text{w/ trade : } \widehat{\lambda}_i = \frac{\omega_i(\lambda_i + \mu_i)}{\frac{1}{I}\sum_{\mathbb{I}} \omega_i(\lambda_i + \mu_i)} \lesseqgtr 1 \qquad \Rightarrow$$

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- Social Cost of Carbon integrates these inequalities :

$$SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, LCC_i\big)$$

Step 1 : Optimal policy – Other motives

- Taxing fossil energy has additional redistributive effects :
  1. Through energy markets : distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy
- New measure : Social Cost of Fossil (SCF)

$$SCF := \frac{\partial \mathcal{W}/\partial E_t^f}{\partial \mathcal{W}_t/\partial w_t} = \mathcal{C}_{EE}^f \,\mathbb{C}\mathrm{ov}_i\Big(\widehat{\lambda}_i, e_{it}^f - e_{it}^x\Big) \; - \; \mathbb{C}\mathrm{ov}_i\Big(\widehat{\upsilon}_i, \frac{q^e(1-s_i^f)}{\sigma e_i^f}\Big)$$

  – with $\mathcal{C}_{EE}^f$ fossil energy supply elasticity, s_i^f energy cost share and σ energy demand elasticity

- *Proposition 2 :* Optimal fossil energy tax :

$$\Rightarrow \qquad t^f = SCC + SCF$$

  – Social cost of carbon : $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i$

Step 2 : Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.
- For a climate "club" of $\mathbb{J} \subset \mathbb{I}$ countries :
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $t^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ can still trade in goods s.t. tariff/tax t^b with club members and countries outside the club

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  - Countries $i \notin \mathbb{J}$ can still trade in goods s.t. tariff/tax t^b with club members and countries outside the club
- Participation constraints :

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

- Welfare :

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{\mathbb{J}} \omega_i \, u(c_i) + \sum_{\mathbb{J}^c} \alpha\omega_i \, u(c_i)$$

Step 2 : Ramsey Problem with participation constraints

- Participation constraints
$$u(c_{it}) \geq u(\tilde{c}_{it}) \quad [\nu_{it}]$$
- *Proposition 3.1 :* Second-Best social valuation with participation constraints
  - Participation incentives change our measure of inequality

$$\text{w/ trade :} \qquad \omega_i(1+\nu_i)u'(c_i) = \Big(\sum_{j\in\mathbb{I}} a_{ij}(d_{ij}\mathrm{p}_j)^{1-\theta}\Big[\omega_i\widetilde{\lambda}_i + \omega_j\widetilde{\mu}_j + \widetilde{\eta}_{ij}(1-s_{ij})\Big]^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \qquad \widehat{\widehat{\lambda}}_i = \frac{\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)} \neq \widehat{\lambda}_i$$

$$\text{vs. w/o trade} \qquad \widehat{\widehat{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_j(1+\nu_j)u'(c_j)} \neq \widehat{\lambda}_i$$

  - Similarly, the "effective Pareto weights" are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

Step 2 : Participation constraints & Optimal policy

- *Proposition 3.2 :* Second-Best taxes :
  - Taxation with imperfect instruments :
    - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
    - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu\sigma}{q^e(1-s^f)}$
      with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
  - Optimal fossil energy tax $\mathrm{t}^f(\mathbb{J})$:

$$\Rightarrow \quad \mathrm{t}^f(\mathbb{J}) = SCC + SCF$$

$$= \frac{1}{1-\vartheta_{\mathbb{J}^c}} \sum_{i\in\mathbb{I}} \tilde{\lambda}_i LCC_i + \frac{1}{1-\vartheta_{\mathbb{J}^c}} \mathcal{C}^f_{EE} \sum_{i\in\mathbb{I}} \tilde{\lambda}_i (e^f_i - e^x_i) - \sum_{i\in\mathbb{J}} \tilde{\upsilon}_i \frac{q^f(1-s^f_i)}{\sigma e^f_i}$$

  - Optimal tariffs/export taxes $\mathrm{t}^{b,r}(\mathbb{J})$ and $\mathrm{t}^b(\mathbb{J})$: In search for a closed-form expression
    As of now, only opaque system of equations (fixed point w/ demand/multipliers)

Step 3 : Optimal Design of a Climate Agreement – Naive approach

- Tradeoff extensive/intensive margin
- Reduction in emissions $\mathcal{E} = \sum_{i \in \mathbb{I}} e_i^f$ depends both on :
  - The level of tax t^f, since high $t^f \Leftrightarrow$ large change in emissions $\Delta\mathcal{E}(\mathbb{J})$
  - The *number* of countries $\mathbb{J}$ in a stable coalition

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- Naive approach :
  - Combinatorial problem : $\mathcal{P}(\mathbb{I})$ with $2^{|\mathbb{I}|}$ choices

$$\max_{\mathbb{J}\in\mathcal{P}(\mathbb{I})} \mathcal{W}(\mathbb{J})$$

  - Choice of countries $\mathbb{J}$ yields optimal taxes $\{\mathrm{t}^f(\mathbb{J}), \mathrm{t}^{b,r}(\mathbb{J}), \mathrm{t}^b(\mathbb{J})\}$
  - Search for complementarity

$$\Delta\mathcal{W}(\mathbb{J}',j) := \mathcal{W}(\mathbb{J}'\cup j) - \mathcal{W}(\mathbb{J}') > \Delta\mathcal{W}(\mathbb{J},j) \qquad \text{when } \mathbb{J}'\supset\mathbb{J} \qquad \text{for all } j\in\mathbb{I}$$

Step 3 : Optimal Design – Alternative approach

- Alternative approach : choosing policy first
  - From a level of the tax t^f and t^b imposed on club $\mathbb{J}$, we can deduce the number of countries $\widetilde{\mathbb{J}}$ with binding participation constraints

$$\widetilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\widetilde{c}_i) \quad \forall i \in \widetilde{\mathbb{J}}$$

  - Search for the couple $\{\mathrm{t}^f, \mathrm{t}^b\}$ such that $\mathbb{J} = \widetilde{\mathbb{J}}$

- What determines the choice of a country to join the climate agreement ?
  - Benefit : lower temperature path τ_i, reduce in energy price q^e, increase in domestic good price p_i, etc.
  - Costs : carbon tax, tariffs on countries outside the club, decrease in fossil rent

Step 3 : Countries' incentives – Model w/o trade in goods

- Experiment : Model with trade in energy but not in "goods"
  - Start from the equilibrium where carbon tax $t^f(\mathbb{J}) = 0$,
    $\Rightarrow$ country i is indifferent to join the club $\mathbb{J}$ or not
  - Linear approximation around that point for small changes in the carbon tax dt^f

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  - Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})} = -e_i d\mathfrak{t}^f - \gamma_i(\tau_i - \tau_{i0})y_i\Delta_i(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^e \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \pi_i \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

  - Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}} = -\left(E_{\mathbb{J}} - E_{\mathbb{J}\setminus\{i\}}\right)\frac{\sigma\, d\mathfrak{t}^f}{q^e(1 - s^f)}\frac{1}{1 + \frac{\nu\sigma}{(1-s^f)}}$$

∘ Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, Climate damage γ_i

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Step 3 : Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned}
\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} &= -e_i d\mathrm{t}^f - \gamma_i(\tau_i - \tau_{i0})\eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \\
&- e_i \frac{q^e \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \\
&+ \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}}\Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij}\Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}}\Big)
\end{aligned}$$

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- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i d\mathrm{t}^f - \gamma_i(\tau_i - \tau_{i0})\eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^e \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$+ \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}}\Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij}\Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}}\Big)$$

  - GE effect on energy markets, same as before
  - GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki}\Big(\frac{dv_k}{v_k} - \theta \frac{d\mathrm{t}_{ki}^b}{1+\mathrm{t}_{ki}^b}\Big) \qquad \alpha_{ki} = \frac{c_{ki} d_{ki} \mathrm{p}_i}{\sum_\ell c_{k\ell} d_{k\ell} \mathrm{p}_\ell} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+\mathrm{t}_{ki}^b) v_i}$$

    – Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Step 3 : Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i d\mathrm{t}^f - \gamma_i(\tau_i - \tau_{i0})\eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^e \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$+ \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}}\Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij}\Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}}\Big)$$

  - GE effect on energy markets, same as before
  - GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki}\Big(\frac{dv_k}{v_k} - \theta \frac{d\mathrm{t}_{ki}^b}{1+\mathrm{t}_{ki}^b}\Big) \qquad \alpha_{ki} = \frac{c_{ki} d_{ki} \mathrm{p}_i}{\sum_\ell c_{k\ell} d_{k\ell} \mathrm{p}_\ell} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+\mathrm{t}_{ki}^b) v_i}$$

    – Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Step 3 : Numerical approach

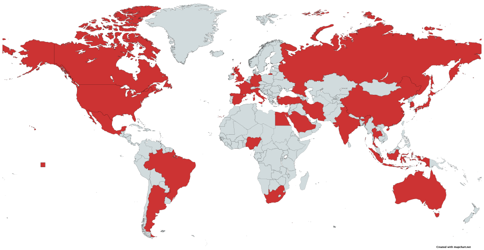
- Algorithm : sequential approach
  1. Start from the second-best optimal policy $\{\mathfrak{t}^f, \mathfrak{t}^b\}$, on the world $\mathbb{J} = \mathbb{I}$
  2. From tax levels imposed on club $\mathbb{J}$, deduce the number of countries $\widetilde{\mathbb{J}}$ with binding participation constraints
  $$\widetilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\widetilde{c}_i) \quad \forall i \in \widetilde{\mathbb{J}}$$
  – Search for $\mathfrak{t}^f$ that yield $\mathbb{J} = f(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b)$
  3. If $Im\big(f(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b)\big) \subsetneq \mathbb{J}$, remove countries one-by-one
  4. Repeat (2-3) until convergence – fixed point of $\widetilde{\mathbb{J}} = f(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b)$ – or unraveling

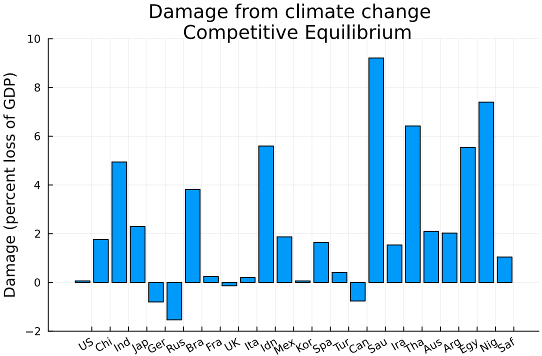
Quantification and numerical method

- Quantification More details
  - Production $\bar{y} = zf(k, e^f, e^r)$ with Nested CES capital/energy $\sigma_y < 0$ and fossil/renewable $\sigma_e > 1$. Calibrate parameters to match GDP / energy shares data.
  - Quadratic damage as in Nordhaus DICE $y = \mathcal{D}_i(\tau)\bar{y}$ with $\mathcal{D}_i(\tau) = e^{-\gamma(\tau-\tau_i)^2}$
  - Energy parameters to match production/reserves
- Numerical method More details
  - Sequential approach : rely on Pontryagin Maximum Principle
  - Can simulate models with arbitrary numbers of dimensions of heterogeneity

Numerical Application – Competitive equilibrium

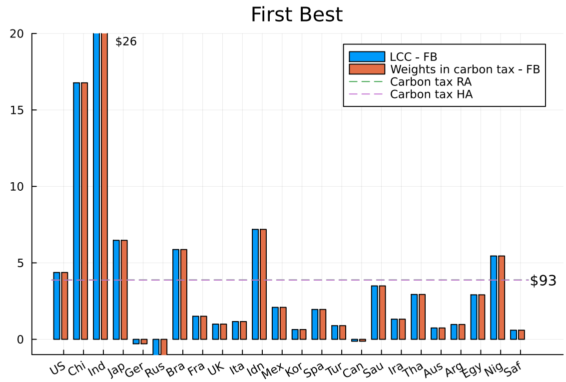
- Data : 24 countries, (G20+4 large countries)

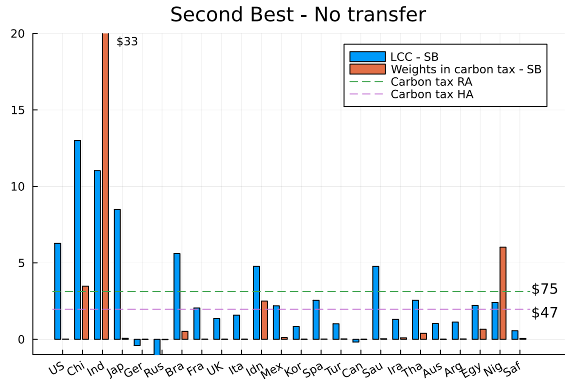




Local Cost of Carbon & Carbon Tax – First and Second Best

- Difference $LCC_i = \frac{\psi_i^{\mathcal{E}}}{\lambda_i^w}$ vs. $\widehat{\lambda}_i^w LCC_i = \frac{\psi_i^{\mathcal{E}}}{\bar{\lambda}^w}$ since $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i^w \, LCC_i$





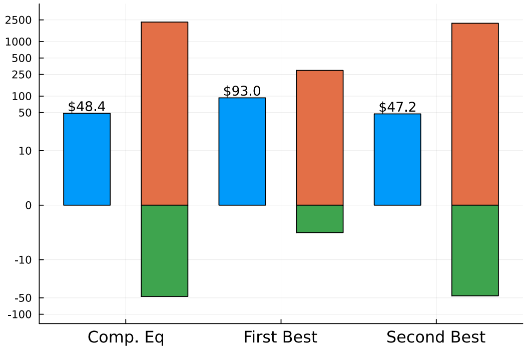
Comparison - Value of wealth vs. Social Cost of Carbon

- Social Cost of Carbon can be decomposed :

$$SCC := -\frac{\partial \mathcal{W}/\partial \mathcal{E}}{\partial \mathcal{W}/\partial c} = \frac{\psi^{\mathcal{E}}}{\lambda} = \frac{\sum_{\mathbb{I}} \psi_i^{\mathcal{E}}}{\frac{1}{I}\sum_{\mathbb{I}} \lambda_i}$$

- Here plot that decomposition :

$$\log(SCC) = \log(\psi^{\mathcal{E}}) - \log(\lambda)$$



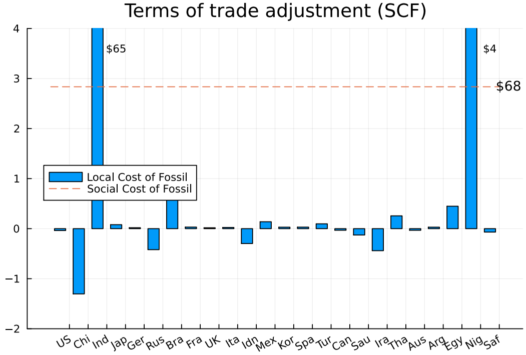
Local Cost of Fossil and Terms of Trade Adjustment

- Social Cost of Fossil Energy :

$$SCF = \mathcal{C}_{EE} \sum_{\mathbb{I}} \widehat{\lambda}_i (e_i^f - e_i^x) \qquad \mathcal{C}_{EE}^{f\ -1} = \sum_{\mathbb{I}} \mathcal{C}_{i,e^x e^x}^{f\ -1}$$

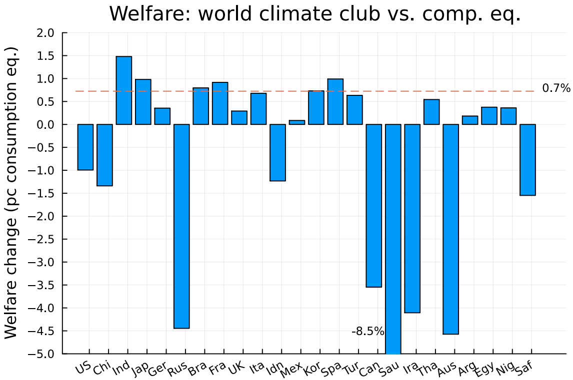
- Here plotting local cost of fossil :

$$LCF_i = \widehat{\lambda}_i (e_i^f - e_i^x)$$



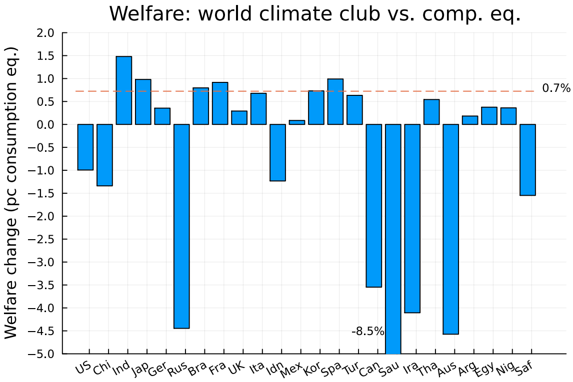
Winner and losers – Second Best vs. Competitive equilibrium

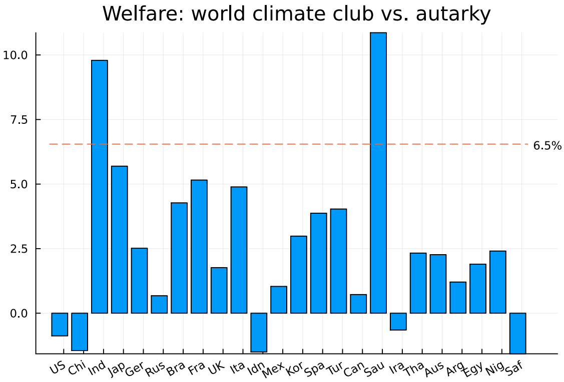
- Difference $\mathcal{W}_i(\mathbb{I})$ (second-best climate club) vs. $\mathcal{V}_i$ (no climate club)
- .



Winner and losers – Second Best vs. Outside options

- Difference $\mathcal{W}_i(\mathbb{I})$ (second-best climate club) vs. $\mathcal{V}_i$ (no climate club)
- Difference $\mathcal{W}_i(\mathbb{I})$ (second-best climate club) vs. $\mathcal{W}_i(\mathbb{I}\backslash\{i\})$ (outside options)





Conclusion

- In this project, I solve for the optimal climate policy
  - Accounting for inequality as it depends on the availability of transfer mechanisms
  - Redistributing through general equilibrium on energy and good markets $\Rightarrow$ terms-of-trade effects
- Climate agreement design jointly solves for :
  - The optimal choice of countries participating
  - The carbon tax level, both for correcting externality & respecting participation constraints
- Differing incentive to join
  - Benefit : change in climate due to participation,
    cost through taxation, loss in energy rent, GE effect on price
  - Complementarity : the larger the group, the higher the effect on (1) climate, (2) energy price, (3) price of outside countries $i \notin \mathbb{J}$

Appendices

More details – Capital market

- In each countries, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} & = \mathcal{D}_i^y(\tau_{it}) z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g}) k_{it} + \iota_{it} \\ \dot{b}_{it} & = r^\star b_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathrm{t}_{it}^f) e_{it}^f - (q_t^r + \mathrm{t}_{it}^r) e_{it}^r - \iota_{it} - c_{it} + \mathrm{t}_{it}^{ls} \\ b_{it} & \geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathrm{t}_{it}^f) e_{it}^f - (q_t^r + \mathrm{t}_{it}^r) e_{it}^r - c_{it} + \mathrm{t}_{i}^{ls}$$

$$k_{it} \leq \frac{1}{1-\vartheta} w_{it}$$

- Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it}) z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r_t^\star$$

back

Impact of increase in temperature

- Marginal values of the climate variables : λ^s_{it} and λ^τ_{it}

$$\dot\lambda^\tau_{it} = \lambda^\tau_{it}(\rho+\zeta) + \overbrace{\gamma_i(\tau_{it}-\tau_i^\star)\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it},e_{it})\lambda^k_{it} + \overbrace{\phi_i(\tau_{it}-\tau_i^\star)\mathcal{D}^u(\tau_{it})^{1-\eta}c_{it}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot\lambda^S_{it} = \lambda^S_{it}(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{it}$$

- Costate λ^S_{it} : marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{it}-\tau_i^\star$ & damage sensitivity of TFP γ^y_i and utility γ^u_i
  - Development level $f(k_{it},e_{it})$ and c_{it}
  - Climate params : χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
  - back

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}^{\mathcal{T}}_{it} = \lambda^{\mathcal{T}}_t(\tilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\lambda^k_t + \phi(\tau - \tau^\star)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}^S_t = \lambda^S{}_t(\tilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i\zeta\chi\lambda^{\mathcal{T}}_{it}$$

- Solving for $\lambda^{\mathcal{T}}_t$ and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^S_t = \dot{\lambda}^{\mathcal{T}}_t = 0$

$$\lambda^{\mathcal{T}}_{it} = -\int_t^\infty e^{-(\tilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_u)y_{\mathcal{T}}\lambda^k_u + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda^{\mathcal{T}}_{it} = -\frac{1}{\tilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_\infty)y_\infty\lambda^k_\infty + \phi\mathcal{D}^u(\tau_\infty)u(c_\infty)\Big)$$

$$\begin{aligned}
\lambda^S_t &= -\int_t^\infty e^{-(\tilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,u}dj\,du \\
&= \frac{1}{\tilde{\rho} + \delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,\infty} \\
&= -\frac{\chi}{\tilde{\rho} + \delta^s}\frac{\zeta}{\tilde{\rho} + \zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_\infty\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj \\
\lambda^S_t &\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\tilde{\rho} + \delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

Cost of carbon / Marginal value of temperature

- ***Proposition (Stationary LSCC)*** :
  When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ_i^y, γ_i^u, temperature, and output, consumption.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s}(\tau_\infty - \tau^\star)\Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty\Big)$$

  - Stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - Back

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega \int_{t_0}^{T} e^{(n+\bar{g})t-\delta_s(T-t)}\, q_t^{f\;-\sigma_e} \int_{j\in\mathbb{I}} \left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e-\sigma} dj\, dt$$

  - Geographical factors determining warming Δ_i
  - Climate sensitivity χ & carbon exit from atmosphere δ_s
  - Growth of population n, aggregate productivity $\bar{g}$
  - Deviation of output from trend y_i & relative TFP z_j
  - Directed technical change z_t^e, elasticity of energy in output σ
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}_t^R/\lambda_t^R = \rho$
  - Change in energy mix, renewable share ω, price q_t^r & elasticity of source σ_e

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto\; n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \tilde{\gamma}\right) + (\sigma_e-\sigma)(1-\omega)g^{q^r} - \left(\sigma_e(1-\omega)+\sigma\omega\right)g^{q^f}$$

Back

Equilibrium – Mean Field Games

- Uniqueness : Lasry-Lions monotonicity on the function :

$$\int_{\mathbb{I}} [f(t,x,\mu',\alpha) - f(t,x,\mu,\alpha)] d(\mu'-\mu)(x) \geq 0 \qquad \mu,\mu' \in \mathcal{P}(\mathbb{X})$$

  - Work in progress : checking such conditions along the transition

$$\sum_{i\in\mathbb{I}} \Big(u(c^\star(w,\tau,p')) - u(c^\star(w,\tau,p))\Big)[p'(w,\tau) - p'(w,\tau)] \geq 0$$

  with $p'(w,\tau)$ empirical distribution $p'(w,\tau) = \frac{1}{\mathbb{I}}\sum_{i\in\mathbb{I}} \delta_{\{x_i=(w,\tau)\}} \equiv$ population distribution !

- Mean Field approximation & Carmona Delarue (2013)
  - Mean-Field is an ε-equilibrium of the N-player game when $N \to \infty$
  - Require symmetry and invariance under permutation
  - Back

Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness More details

Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
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  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness More details
- Global Numerical solution :
  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \mathbf{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

- Why use a sequential approach ?
  - *Global approach* : *Only* need to follow the trajectories for i agents :
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
    Productivity z_i Population p_i, Temperature scaling Δ_i, Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$, Local damage $\gamma_i^y, \gamma_i^u, \tau_i^\star$, Directed Technical Change z_i^e
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
    For now : Wealth w_{it}, temperature τ_{it}, reserves $\mathcal{R}_{it}$, Carbon $\mathcal{S}_t$
    Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
  - Newton method & Non-linear solvers very efficient
- Why not :
  - Numerical constraint to solve a large system of ODEs and non-linear equations :

  $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

back

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \varepsilon(e^f, e^r)) = \left[(1-\epsilon_i)^{\frac{1}{\sigma_y}} k^{\alpha \frac{\sigma_y - 1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} \left(z_i^e \, \varepsilon_i(e^f, e^r)\right)^{\frac{\sigma_y - 1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega_i^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\omega_i)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

  - Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
  - Today : $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
  - Future : $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

- Damage functions in production function y :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm,y}(\tau - \tau_i^\star)^2}$$

  - Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y} \mathbb{1}_{\{\tau > \tau_i^\star\}} + \gamma^{-,y} \mathbb{1}_{\{\tau < \tau_i^\star\}}$
  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $\tau_i^\star = \bar{\alpha}\tau_{it_0} + (1-\bar{\alpha})\tau^\star$

Quantification – Energy markets

- Fossil production e^x_{it} and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now : $\bar{\nu}_i$ to match extraction data e^x_i, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. $\nu_i = \nu = 1$ quadratic extraction cost.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e^x_i (BP, IEA)

Quantification – Energy markets

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  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now : $\bar{\nu}_i$ to match extraction data e^x_i, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. $\nu_i = \nu = 1$ quadratic extraction cost.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e^x_i (BP, IEA)
- Renewable : Production $\bar{e}^r_{it}$ and price q^r_{it}
  - Now : $q^r_{it} = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
  - Future : Choose z^r_i to match the energy mix (e^f_i, e^r_i)

back

Quantification – Future Extensions :

- Damage parameters :
  - $\gamma_i^{\pm,y}$ depends on daily temperature distribution $\tau \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
  - Use Climate Lab's (Greenstone et al) estimates for damage γ_i ?
- Fossil Energy markets :
  - Divide fossils e^f_{it}/e^x_{it} into oil/gas/coal
  - Match the production/cost/reserves data across countries
  - Use a dynamic model : extraction/exploration a la Hotelling
- Renewables Energy markets :
  - Make the problem dynamic with investment in capacity C^r_{it}
- Population dynamics
  - Match UN forecast for growth rate / fertility

Calibration

TABLE – Baseline calibration ($\star$ = subject to future changes)

Technology & Energy markets			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
g_e	$0.01^\star$	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	$-0.01^\star$	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	$2^\star$	Extraction elasticity of fossil energy	Cubic extraction cost
Preferences & Time horizon			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	$0.01^\star$	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

TABLE – Baseline calibration ($\star$ = subject to future changes)

Climate parameters			
ξ	0.81	Emission factor	Conversion 1 *MTOE* $\Rightarrow$ 1 *MT* CO_2
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature $\sim 11-15$ years
χ	$2.1/1e6$	Climate sensitivity	Pulse experiment : 100 $GtC \equiv 0.21^\circ C$ medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment : 100 $GtC \equiv 0.16^\circ C$ long-term warming
$\gamma^\oplus$	$0.00234^\star$	Damage sensitivity	Nordhaus' DICE
$\gamma^\ominus$	$0.2\times\gamma^{\oplus\,\star}$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^τ	$0.2^\star$	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
$\tau^\star$	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
Parameters calibrated to match data			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
τ_i		Local Temperature	To match temperature of largest city
$\mathcal{R}_i$		Local Fossil reserves	To match data from BP Energy review

Step 4 : the Design of a Climate agreement

- Welfare effect : 1st order :

$$\delta(\mathbb{J},j) = \mathcal{W}_{t_0}(\mathbb{J}\cup\{j\}) - \mathcal{W}_{t_0}(\mathbb{J}) = \omega_j u(c_{jt}) + \sum_{i\in\mathbb{J}} \Delta\mathcal{W}_i$$

$$\Delta\mathcal{W}_i \approx d\mathcal{W}_i = \lambda_i^w(1-\theta_i)\Big(\underbrace{\epsilon_i^e + (1-\epsilon_i^e)\alpha\sigma_i^{k/e}}_{\text{production } f(k,e)}\Big)\Big(\underbrace{-\omega_i^f\sigma_i^f + (1-\omega_i^f)\sigma_i^{r/f}}_{\text{energy use } \varepsilon(e^f,e^r)}\Big)\Big(\underbrace{\frac{\mathrm{t}^f}{q^f+\mathrm{t}^f}\frac{d\mathrm{t}^f}{\mathrm{t}^f}}_{\text{tax change}} + \underbrace{\frac{q^f}{q^f+\mathrm{t}^f}\frac{dq^f}{q^f}}_{\text{GE effect}}\Big)$$

$$+\lambda_i^w\underbrace{\theta_i\Big(1+\frac{1}{\nu_i}\Big)}_{\text{fossil rent/supply}}\underbrace{\frac{q^f}{q^f+\mathrm{t}^f}\frac{dq^f}{q^f}}_{\text{GE effect}} + \underbrace{\psi_i^S}_{\tau_i \text{ damage}}\Big[\underbrace{\chi}_{\text{climate sens}^{ty}}\sum_{j\in\mathbb{I}}\varepsilon_i\sigma_j^f\Big]\Big(\underbrace{\frac{\mathrm{t}^f}{q^f+\mathrm{t}^f}\frac{d\mathrm{t}^f}{\mathrm{t}^f}}_{\text{tax change}} + \underbrace{\frac{q^f}{q^f+\mathrm{t}^f}\frac{dq^f}{q^f}}_{\text{GE effect}}\Big)$$

- Direct effect on energy use on production and substitutability with renewable
  cost-share ϵ_e, fossil-share ω_i, elasticity σ_j^f & capital-energy cross elast^{ty}. $\sigma_{k,e}$, fossil-renewable cross elast^{ty}. $\sigma_i^{r/f}$
- Indirect effect through energy market fossil rent θ_i, supply elasticity ν_i
- Indirect climate effect of a reduction in world emissions

Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^{w}, \lambda_{it}^{\tau}, \lambda_{t}^{\mathcal{S}}, \lambda_{it}^{\mathcal{R}}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e_{it}^{f}, e_{it}^{r}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e_{it}^{x}, \bar{e}_{it}^{r}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q_t^{f}, r_t^{\star}\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)
- Global Numerical solution :
  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

- Why use a sequential approach ?
  - *Global approach* : *Only* need to follow the trajectories for i agents :
    - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
      Productivity z_i Population p_i, Temperature scaling Δ_i, Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$, Local damage $\gamma_i^y, \gamma_i^u, \tau_i^\star$, Directed Technical Change z_i^e
    - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
      For now : Wealth w_{it}, temperature τ_{it}, reserves $\mathcal{R}_{it}$, Carbon $\mathcal{S}_t$
      Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
    - Newton method & Non-linear solvers very efficient
- Why not :
  - Numerical constraint to solve a large system of ODEs and non-linear equations :

  $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

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