The Inequality of Climate Change & Optimal Energy policy

WORK IN PROGRESS

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Introduction

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- Develop a simple and flexible model of climate economics
  - Standard IAM model with heterogeneous regions
  - Normative implications : Optimal Ramsey policy for carbon taxation
  - Provide a numerical methodology and a quantitative model

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  - In a context where fossil fuels taxation and climate policy redistribute across countries
- Develop a simple and flexible model of climate economics
  - Standard IAM model with heterogeneous regions
  - Normative implications : Optimal Ramsey policy for carbon taxation
  - Provide a numerical methodology and a quantitative model
- ○ Evaluate the heterogeneous welfare costs of global warming
  - Climate damages & temperature varies across countries
  - $\Rightarrow$ Inequality increases the welfare cost of carbon
- ○ Solve world optimal carbon policy with heterogeneous regions
  - Does the optimal carbon tax coincide with the social cost of carbon ?
  - $\Rightarrow$ Depends on transfer policy : need to adjust the tax for inequality level

Preview of the findings

- In a large class of IAM models, optimal carbon policy goes hand in hand with the availability of redistribution instruments
- Case 1 : (First Best)
  - Energy tax common for all countries i and proportional to the Social Cost of Carbon
  - Lump-sum taxes and transfers redistribute across countries
- Case 2 :
  - Without such instruments, energy taxes are country i-specific & account for redistribution
  - Tax scales with Pareto weights ω_i and marg. utility of consumption $U'(c_i)$

    $\Rightarrow$ lower energy tax for poorer countries
  - Also accounts for redistribution through energy markets, due to changes in terms-of-trade
- Case 3 :
  - If countries can exit climate agreements, one needs to account for participation constraints
  - Tax may be lower for countries with better outside options

Model – Representative Household

- Deterministic Neoclassical economy, in continuous time
  - heterogeneous countries $i \in \mathbb{I}$
  - In each country, 4 agents : (i) representative household, (ii) homogeneous good firm, (iii) fossil and (iv) renewable energy producers.

- Representative household problem in each country i :

$$\mathcal{V}_{it_0} = \max_{\{c_{it}, k_{it}, b_{it}\}} \int_{t_0}^{\infty} e^{-\rho t}\, u(c_{it}) dt$$

- Dynamics of wealth of country i, $w_{it} = b_{it} + k_{it}$ More details

$$\dot{w}_{it} := \dot{k}_{it} + \dot{b}_{it} = y_{it} + \pi^f_{it} + \pi^r_{it} + r^\star_t b_{it} + (r^\star_t - \bar{\delta}) k_{it} - c_{it} + \mathrm{t}^{ls}_{it}$$

  - Labor income y_{it} from homogeneous good firm.
  - All the lower-case variables are expressed per unit of efficient labor $y_{it} = Y_{it}/(L_{it}A_{it})$

Model – Representative Firm

- Competitive homogeneous good producer in country i

$$\max_{k_{it}, e^f_{it}, e^r_{it}} \mathcal{D}^y(\tau_{it}) z_i f(k_{it}, e^f_{it}, e^r_{it}) - r^\star_t k_{it} - (q^f_t + \mathrm{t}^f_{it}) e^f_{it} - (q^r_t + \mathrm{t}^r_{it}) e^r_{it} - y_{it}$$

  - Energy mix with fossil e^f_{it} – emitting carbon – subject to price q^f_t and tax/subsidy t^f_{it}. Similarly "clean" renewable e^r_t, at price q^r_{it} and tax t^r_{it}.
  - No international trade in goods and Labor is immobile

Model – Energy markets

- Competitive fossil fuels energy producer :
  - Static problem (for now) extract energy e^x_{it} depleting reserves $\mathcal{R}_{it}$

$$\pi^f_{it} = \max_{e^x_{it}} q^f_t e^x_{it} - \mathcal{C}^f_i(e^x_{it}, \mathcal{R}_{it})$$

$$\dot{\mathcal{R}}_{it} = -e^x_{it} \qquad \mathcal{R}_{it_0} = \mathcal{R}_{i0} \qquad \mathcal{R}_{it} \geq 0$$

  - Fossil energy traded in international markets :

$$\int_{\mathbb{I}} e^f_{it}\, di = \int_{\mathbb{I}} e^x_{it}\, di$$

  - Optimal extraction

$$q^f_t = \mathcal{C}^f_e(e^x_{it}, \mathcal{R}_{it})$$

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- Renewable energy as a substitute technology in each country i (Static problem for now)

$$\pi^r_{it} = \max_{\{\bar{e}^r_{it}\}} q^r_{it}\bar{e}^r_{it} - \mathcal{C}^r_i(\bar{e}^r_{it}) \qquad \Rightarrow \qquad q^r_{it} = \mathcal{C}^r_e(\bar{e}^r_t) = z^r_{it}$$

Climate system

- Fossil energy input e_t^f causes climate externality

$$\mathcal{E}_t = \int_{\mathbb{I}} e_{it}^f \, di$$

- Cumulative GHG in atmosphere $\mathcal{S}_t$ increases temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- Country's local temperature :

$$\dot{\tau}_{it} = \zeta\left(\Delta_i \chi \mathcal{S}_t - \left(\tau_{it} - \bar{\tau}_{it_0}\right)\right)$$

  - Linear model : Climate sensitivity to carbon χ, Climate reaction/inertia ζ, Country i linear pattern scaling factor Δ_i, Carbon exit from atmosphere δ_s

Model – Solution

- Case 0 : Competitive equilibrium
  - Absence of Policies : Taxes $t_{it}^{f} = t_{it}^{r} = t_{it}^{ls} = 0$

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- Case 1 : First Best
  - Planner maximize aggregate welfare

$$\mathcal{W}_{t_0} = \max_{\{\mathrm{t}_{it}^{f},\mathrm{t}_{it}^{r},\mathrm{t}_{it}^{ls}\dots\}_{it}} \int_{\mathbb{I}} \int_{t_0}^{\infty} e^{-\bar{\rho}t}\, \omega_i\, u(c_{it})\, dt\, di$$

  - All instruments available $\mathbf{t} = \{\mathrm{t}_{it}^{f}, \mathrm{t}_{it}^{r}, \mathrm{t}_{it}^{ls}\}_{it}$, including transfers across countries.

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- Case 2 : Limited transfers
  - Ramsey policy, where lump-sum transfers across countries are prohibited
  - Country-specific energy taxes t^f_{it}, t^r_{it} and lump-sum (local) rebate $\mathrm{t}^{ls}_{it} = \mathrm{t}^f_{it} e^f_{it} + \mathrm{t}^r_{it} e^r_{it}$

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- Case 3 (Work in progress)
  - Prohibited lump-sum transfers & countries can exit climate agreements
  - Participation constraint, with $\bar{c}_{it}$ consumption in autarky

$$u(c_{it}) \geq u(\bar{c}_{it}) \qquad \forall t \geq 0$$

Model – Equilibrium

- Equilibrium
  - Given, initial conditions $\{w_{i0}, \tau_{i0}, \mathcal{R}_{i0}\}$ and country-specific policies $\{\mathrm{t}^f_{it}, \mathrm{t}^r_{it}, \mathrm{t}^{ls}_{it}\}$, a **competitive equilibrium** is a continuum of sequences of states $\{w_{it}, \tau_{it}, \mathcal{R}_{it}\}_{it}$ and $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, policies $\{c_{it}, b_{it}, k_{it}, e^f_{it}, e^r_{it}, e^x_{it}\}_{it}$ and price sequences $\{r^\star_t, q^f_t, q^r_t\}$ such that :
  - Households choose policies $\{c_{it}, b_{it}\}_{it}$ to max utility s.t. budget constraint, giving $\dot{w}_{it}$
  - Firm choose policies $\{k_{it}, e^f_{it}, e^r_{it}\}_{it}$ to max profit
  - Fossil and renewables firms extract/produce $\{e^x_{it}, \bar{e}^r_{it}\}_{it}$ to max static profit, yielding $\dot{\mathcal{R}}_t$
  - Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{it}\}_{it}$.
  - Prices $\{r^\star_t, q^f_t, q^r_{it}\}$ adjust to clear the markets : $\int_{\mathbb{I}} e^x_{it} di = \int_{\mathbb{I}} e^f_{it} di$ and $e^r_{it} = \bar{e}^r_{it}$, and $\int_{i\in\mathbb{I}} b_{it} di = 0$, with bonds $b_{it} = w_{it} - k_{it}$

Case 0 : Competitive equilibrium

- Household consumption/saving problem
  - Using Pontryagin Max. Principle : states $\{x\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}\}$, controls $\{c\} = \{c_{it}, b_{it}, k_{it}\}$ and costates $\{\lambda\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\} \Rightarrow$ system of coupled ODEs.

$$\mathcal{H}^{hh}(\{x\}, \{c\}, \{\lambda\}) = u(c_i) + \lambda^w_{it}\dot{w}_{it} + \lambda^\tau_{it}\dot{\tau}_{it} + \lambda^S_{it}\dot{\mathcal{S}}_t$$

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  - ODE for the costate for wealth $\lambda^w_{it} = u'(c_{it}) \;\Rightarrow$ Euler equation
  - The "local social cost of carbon" (LCC) for region i :

$$LCC_{it} := -\frac{\partial \mathcal{V}_{it}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{it}/\partial w_{it}} = -\frac{\lambda^S_{it}}{\lambda^w_{it}}$$

    - ODEs for Costates : temperature λ^τ_{it} and carbon λ^S_{it}, More details
    - Stationary equilibrium closed-form formula, analogous to GHKT (2014) Here

Case 1 : First-Best, Optimal policy with transfers

- First-Best, Maximizing welfare of the Social Planner :

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t},\boldsymbol{x},\boldsymbol{c},\boldsymbol{q}\}_{it}} \int_{\mathbb{I}} \int_{t_0}^{\infty} e^{-\bar{\rho}t} \, \omega_i \, u(c_{it}) \, dt \, di = \int_{\mathbb{I}} \mathcal{W}_{it_0} di$$

- Full set of instruments $\mathbf{t} = \{\mathrm{t}_{it}^{f}, \mathrm{t}_{it}^{r}, \mathrm{t}_{it}^{ls}\}$, including transfers *across countries*

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- Full set of instruments $\mathbf{t} = \{\mathrm{t}^f_{it}, \mathrm{t}^r_{it}, \mathrm{t}^{ls}_{it}\}$, including transfers *across countries*
- Social Planner Hamiltonian
States $\{x\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}\}_{it}$, controls $\{c\} = \{c_{it}, b_{it}, k_{it}, e^f_{it}, e^r_{it}, e^x_{it}\}_{it}$ and costates $\{\psi\} = \{\psi^w_{it}, \psi^\tau_{it}, \psi^s_{it}\}_{it} \Rightarrow$ system of coupled ODEs.

$$\mathcal{H}^{sp}(\{x\},\{c\},\{\psi\}) = \int_{i\in\mathbb{I}} \omega_i u(c_i) di + \int_{i\in\mathbb{I}} \Big(\psi^w_{it}\dot{w}_{it} + \psi^\tau_{it}\dot{\tau}_{it} + \psi^S_{it}\dot{\mathcal{S}}_t\Big) di$$

Social Cost of Carbon :

- Key Objects : Social Cost of Carbon

- Local :

$$LCC_{it} := -\frac{\partial \mathcal{W}_{it} / \partial \mathcal{S}_t}{\partial \mathcal{W}_{it} / \partial w_{it}} = -\frac{\psi_{it}^S}{\psi_{it}^w}$$

  - Intuitives ODEs for costates ψ_{it}^S and ψ_{it}^w More details

- Global :

$$SCC_t := -\frac{\partial \mathcal{W}_t / \partial \mathcal{S}_t}{\partial \mathcal{W}_t / \partial w_t} = -\frac{\psi_t^S}{\psi_t^w} = -\frac{\int_{i \in \mathbb{I}} \psi_{it}^S di}{\int_{i \in \mathbb{I}} \psi_{it}^w di}$$

Case 1 : First-Best, Optimal policy with transfers

- *Proposition 1 :* Optimal carbon tax :

$$\mathrm{t}_t^S = -\frac{\psi_t^S}{\psi_t^w} =: SCC_t$$

  - Same as in Representative Agent economy, c.f. GHKT (2014)

- Implies lump-sum transfers to redistribution, s.t.

$$\omega_i u'(c_{it}) = \psi_{it}^w = \psi_t^w = \psi_{jt}^w = \omega_j u'(c_{jt}) \;\; \forall i,j \in \mathbb{I}$$

  - Imply cross-countries lump-sum transfers $\exists\, i \;\; s.t.\; T_i \geq 0$ or $\exists\, j \;\; s.t.\; T_j \leq 0$
  - There exist Pareto weights $\{\omega_i\}$ shutting down redistribution $T_i = 0$, e.g. $\omega_i = 1/u'(c_{it})$

Case 2 : Ramsey policy with limited transfers

- Second best without access to lump-sum transfers
  - Only region-i-specific distortive energy taxes : $\{\mathrm{t}^f_{it}, \mathrm{t}^r_{it}\}$.
    $\Rightarrow$ Tax receipts redistributed lump-sum : $\mathrm{t}^{ls}_{it} = \mathrm{t}^f_{it} e^f_{it} + \mathrm{t}^r_{it} e^r_{it}$
  - Implies inequality across regions :

$$\widehat{\psi}^w_{it} = \frac{\psi^w_{it}}{\psi^w_t} = \frac{\omega_i u'(c_{it})}{\int_{j\in\mathbb{I}} \omega_j u'(c_{jt}) dj} \lessgtr 1$$

$\Rightarrow$ ceteris paribus, poorer countries have higher $\widehat{\psi}^w_{it}$

  - Social Cost of Carbon integrates these inequalities :

$$SCC_t = -\int_{i\in\mathbb{I}} \widehat{\psi}^w_{it} \underbrace{\frac{\psi^S_{it}}{\psi^w_{it}}}_{=-LCC_{it}} di$$

$$SCC_t = \mathbb{C}\mathrm{ov}_i\left(\widehat{\psi}^w_{it}, LCC_{it}\right) + \mathbb{E}_i[LCC_{it}]$$

Case 2 : Ramsey Problem – Optimal Carbon & Energy Policy

- Optimal Pigouvian carbon tax :

$$\mathrm{t}^S_{it} = \frac{1}{\widehat{\psi}^w_{it}} SCC_t$$

  - Integrate redistribution motives, both :
    - for the distribution of tax : countries with higher $\widehat{\psi}^w_{it} \propto \omega_i u'(c_{it})$ have lower tax t^S_{it}
    - for the level : $SCC_t = \mathbb{C}\mathrm{ov}_i\left(\widehat{\psi}^w_{it}, LCC_{it}\right) + \mathbb{E}_i[LCC_{it}]$
  - Implementation : taxing carbon amounts to taxing fossil fuels/energy

Case 2 : Ramsey Problem – Optimal Carbon & Energy Policy

- Taxing fossil energy has additional redistributive effects :
  - Lowering the equilibrium price of fossil fuels benefit importers and hurt exporters
  - New measure of this effect : Social Cost of Fossil (SCF)

$$SCF_t := \frac{\partial \mathcal{W}_t / \partial E_t^f}{\partial \mathcal{W}_t / \partial w_t} = \mathcal{C}_{EE}^f \, \mathbb{C}\text{ov}_i\left(\widehat{\psi}_{it}^w, e_{it}^f - e_{it}^x\right) \qquad \mathcal{C}_{EE}^f = \left(\int_{i \in \mathbb{I}} \frac{1}{\mathcal{C}_{i,e^x e^x}^f} dj\right)^{-1}$$

  – with $\mathcal{C}_{EE}^f$ and $\mathcal{C}_{i,e^x e^x}^f \propto$ fossil energy supply elasticity

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  – with $\mathcal{C}_{EE}^f$ and $\mathcal{C}_{i,e^x e^x}^f \propto$ fossil energy supply elasticity

- *Proposition 2 :* Optimal fossil energy tax :

$$\Rightarrow \qquad \mathrm{t}_{it}^f = \frac{1}{\widehat{\psi}_{it}^w}\left[SCC_t + SCF_t\right]$$

- What about renewable energy e_t^r ?
  - Not traded, with constant return to scale, and not carbon intensive, hence :

$$\mathrm{t}_{it}^r = 0$$

Case 3 : Ramsey Problem with participation constraints

- Assume that lump-sum transfers are prohibited & countries can exit climate agreements
  - Participation constraint, with $\bar{c}_i$ autarky consumption (no trade in energy/assets)

$$u(c_{it}) \geq u(\bar{c}_{it}) \qquad [\nu_{it}]$$

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- *Proposition 3 :* Second-Best without transfers & participation constraints
  - Participation incentive change our measure of inequality

$$\widetilde{\psi}^w_{it} \propto \omega_i u'(c_{it}) + \nu_{it} u'(c_{it}) \neq \widehat{\psi}^w_{it}$$

  - Optimal fossil energy tax :

$$\Rightarrow \qquad \mathrm{t}^f_{it} = \frac{1}{\widetilde{\psi}^w_{it}} \left[SCC_t + SCF_t \right]$$

    – With levels changing

$$SCC_t = \mathbb{C}\mathrm{ov}_j\left(\widetilde{\psi}^w_{it}, LSCC_{jt}\right) + \mathbb{E}_j[LSCC_{jt}]$$

$$SCF_t = \mathcal{C}^f_{EE}\mathbb{C}\mathrm{ov}_j\left(\widetilde{\psi}^w_{it}, e^f_{jt} - e^x_{jt}\right)$$

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \varepsilon(e^f, e^r)) = \left[(1-\epsilon)^{\frac{1}{\sigma_y}} k^{\alpha \frac{\sigma_y - 1}{\sigma_y}} + \epsilon^{\frac{1}{\sigma_y}} \left(z_i^e \, \varepsilon(e^f, e^r) \right)^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\omega)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

  - Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
  - Today : $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
  - Future : $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

- Damage functions in production function y :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm,y}(\tau - \tau_i^\star)^2}$$

  - Asymmetry in damage to match empirical evidence, with $\gamma^y = \gamma^{+,y} \mathbb{1}_{\{\tau > \tau_i^\star\}} + \gamma^{-,y} \mathbb{1}_{\{\tau < \tau_i^\star\}}$
  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $\tau_i^\star = \bar{\alpha}\tau_{it_0} + (1-\bar{\alpha})\tau^\star$

Quantification – Energy markets

- Fossil production e^x_{it} and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now : $\bar{\nu}_i = \bar{\nu}$ and $\nu_i = \nu$ and $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction level data e^x_i (BP, IEA)

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  - Now : $\bar{\nu}_i = \bar{\nu}$ and $\nu_i = \nu$ and $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction level data e^x_i (BP, IEA)
- Renewable : Production $\bar{e}^r_{it}$ and price q^r_{it}
  - Now : $q^r_{it} = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
  - Future : Choose z^r_i to match the energy mix (e^f_i, e^r_i)

Quantification – Future Extensions :

- Damage parameters :
  - $\gamma_i^{\pm,y}$ depends on daily temperature distribution $\tau \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
  - Use Climate Lab's (Greenstone et al) estimates for damage γ_i
- Fossil Energy markets :
  - Divide fossils e^f_{it}/e^x_{it} into oil/gas/coal
  - Match the production/cost/reserves data across countries
  - Use a dynamic model : extraction/exploration a la Hotelling
- Renewables Energy markets :
  - Make the problem dynamic with investment in capacity C^r_{it}
- Population dynamics
  - Match UN forecast for growth rate / fertility

Calibration

TABLE – Baseline calibration ($\star$ = subject to future changes)

Technology & Energy markets			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
g_e	$0.01^\star$	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	$-0.01^\star$	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	$2^\star$	Extraction elasticity of fossil energy	Cubic extraction cost
Preferences & Time horizon			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	$0.01^\star$	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

TABLE – Baseline calibration ($\star$ = subject to future changes)

Climate parameters			
ξ	0.81	Emission factor	Conversion 1 $MTOE \Rightarrow 1\ MT\ CO_2$
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature $\sim 11-15$ years
χ	$2.1/1e6$	Climate sensitivity	Pulse experiment : $100\ GtC \equiv 0.21^{\circ}C$ medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment : $100\ GtC \equiv 0.16^{\circ}C$ long-term warming
$\gamma^{\oplus}$	$0.00234^{\star}$	Damage sensitivity	Nordhaus' DICE
$\gamma^{\ominus}$	$0.2 \times \gamma^{\oplus\ \star}$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
$\alpha^{\mathcal{T}}$	$0.2^{\star}$	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
$\tau^{\star}$	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
Parameters calibrated to match data			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
τ_i		Local Temperature	To match temperature of largest city
$\mathcal{R}_i$		Local Fossil reserves	To match data from BP Energy review

Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
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- Global Numerical solution :
  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

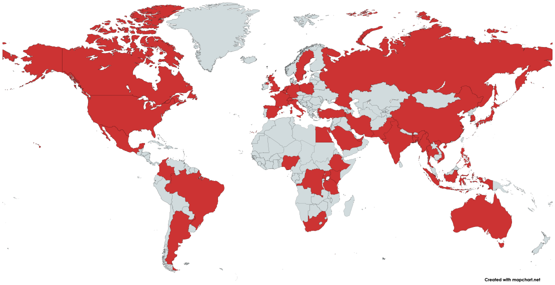
- Why use a sequential approach ?
  - *Global approach* : *Only* need to follow the trajectories for i agents :
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
    Productivity z_i *Population* p_i, *Temperature scaling* Δ_i, *Fossil energy cost* $\bar{\nu}_i$, *Energy mix* $\epsilon_i, \omega_i, z_i^r$, *Local damage* $\gamma_i^y, \gamma_i^u, \tau_i^\star$, *Directed Technical Change* z_i^e
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
    For now : Wealth w_{it}, *temperature* τ_{it}, *reserves* $\mathcal{R}_{it}$, *Carbon* $\mathcal{S}_t$
    Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
  - Newton method & Non-linear solvers very efficient

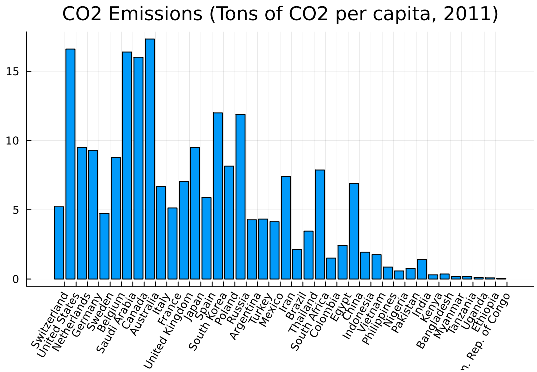
- Why not :
  - Numerical constraint to solve a large system of ODEs and non-linear equations :

  $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

Numerical Application – Competitive equilibrium

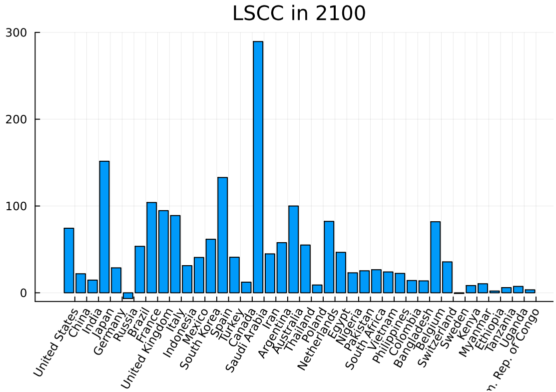
- Data : 40 countries, 25 largest countries either both GDP and population
- Work in progress (quantification/algorithm) subject to changes

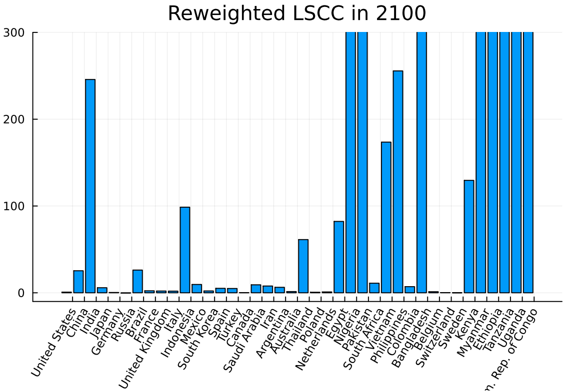




Local Cost of Carbon

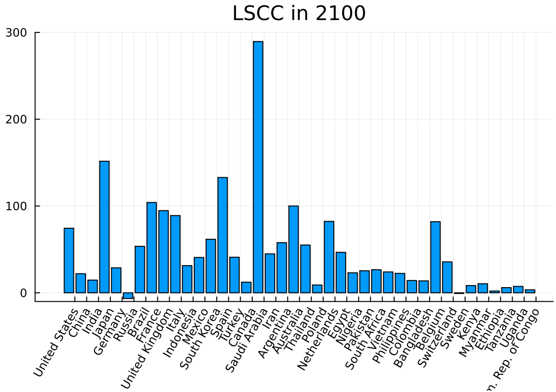
- Difference $LCC_i = \frac{\lambda^{\mathcal{S}}_{it}}{\lambda^{w}_{it}}$ and $LWCC_{it} = \widehat{\lambda}^{w}_{it} LCC_{it} = \frac{\lambda^{\mathcal{S}}_{it}}{\bar{\lambda}^{w}_{t}}$

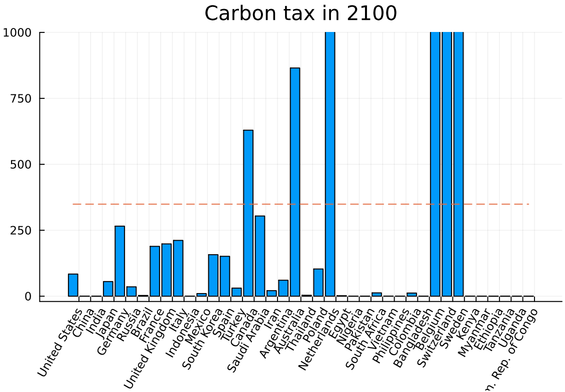




Social Cost of Carbon and Carbon Tax

- Difference $LCC_i = \frac{\lambda_{it}^S}{\lambda_{it}^w}$ and $\mathbf{t}_{it}^S = (1/\widehat{\lambda}_{it}^w)SCC$





Conclusion

- Climate change has redistributive effects & heterogeneous impacts
- Optimal carbon policy take into account inequality and redistribution
  - Depends on the availability of transfer mechanisms
  - Level of Pigouvian tax & Social Cost of Carbon exacerbated by inequality
  - Distribution of carbon & energy taxes inversely related to distribution of consumption
  - Energy tax also depends on redistribution through changes in terms-of-trade
- Future improvement in the calibration / quantification & numerical method

Appendices

More details – Capital market

- In each countries, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} & = \mathcal{D}^y_i(\tau_{it}) z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g}) k_{it} + \iota_{it} \\ \dot{b}_{it} & = r^\star b_{it} + \theta_i \pi^f_t + \pi^r_{it} - (q^f_t + \mathrm{t}^f_{it}) e^f_{it} - (q^r_t + \mathrm{t}^r_{it}) e^r_{it} - \iota_{it} - c_{it} + \mathrm{t}^{ls}_{it} \\ b_{it} & \geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r^\star_t) k_{it} + \theta_i \pi^f_t + \pi^r_{it} - (q^f_t + \mathrm{t}^f_{it}) e^f_{it} - (q^r_t + \mathrm{t}^r_{it}) e^r_{it} - c_{it} + \mathrm{t}^{ls}_{it}$$

$$k_{it} \leq \frac{1}{1-\vartheta} w_{it}$$

- Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}^y_i(\tau_{it}) z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r^\star_t$$

back

Impact of increase in temperature

- Marginal values of the climate variables : λ_{it}^{s} and λ_{it}^{τ}

$$\dot{\lambda}_{it}^{\tau} = \lambda_{it}^{\tau}(\rho + \zeta) + \overbrace{\gamma_i(\tau_{it} - \tau_i^{\star})\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it}, e_{it})\lambda_{it}^{k} + \overbrace{\phi_i(\tau_{it} - \tau_i^{\star})\mathcal{D}^u(\tau_{it})^{1-\eta} c_{it}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}_{it}^{S} = \lambda_{it}^{S}(\rho + \delta^s) - \zeta\,\chi\,\Delta_i\,\lambda_{it}^{\tau}$$

- Costate λ_{it}^{S} : marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{it} - \tau_i^{\star}$ & damage sensitivity of TFP γ_i^y and utility γ_i^u
  - Development level $f(k_{it}, e_{it})$ and c_{it}
  - Climate params : χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
  - back

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}^{\mathcal{T}}_{it} = \lambda^{\mathcal{T}}_t(\tilde{\rho} + \Delta\zeta) + \gamma(\mathcal{T} - \mathcal{T}^\star)\mathcal{D}^y(\mathcal{T})f(k,e)\lambda^k_t + \phi(\mathcal{T} - \mathcal{T}^\star)\mathcal{D}^u(\mathcal{T})u(c)$$

$$\dot{\lambda}^S_t = {\lambda^S}_t(\tilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta\chi\lambda^{\mathcal{T}}_{it}$$

- Solving for $\lambda^{\mathcal{T}}_t$ and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^S_t = \dot{\lambda}^{\mathcal{T}}_t = 0$

$$\lambda^{\mathcal{T}}_{it} = -\int_t^\infty e^{-(\tilde{\rho}+\zeta)u}(\mathcal{T}_u - \mathcal{T}^\star)\Big(\gamma\mathcal{D}^y(\mathcal{T}_u)y_{\mathcal{T}}\lambda^k_u + \phi\mathcal{D}^u(\mathcal{T}_u)u(c_u)\Big)du$$

$$\lambda^{\mathcal{T}}_{it} = -\frac{1}{\tilde{\rho} + \Delta\zeta}(\mathcal{T}_\infty - \mathcal{T}^\star)\Big(\gamma\mathcal{D}^y(\mathcal{T}_\infty)y_\infty\lambda^k_\infty + \phi\mathcal{D}^u(\mathcal{T}_\infty)u(c_\infty)\Big)$$

$$\begin{aligned}
\lambda^S_t &= -\int_t^\infty e^{-(\tilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,u}dj\,du \\
&= \frac{1}{\tilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,\infty} \\
&= -\frac{\chi}{\tilde{\rho}+\delta^s}\frac{\zeta}{\tilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\mathcal{T}_{j,\infty} - \mathcal{T}^\star)\Big(\gamma\mathcal{D}^y(\mathcal{T}_{j,\infty})y_\infty\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\mathcal{T}_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

$$\lambda^S_t \xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\tilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\mathcal{T}_{j,\infty} - \mathcal{T}^\star)\Big(\gamma\mathcal{D}^y(\mathcal{T}_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\mathcal{T}_{j,\infty})u(c_{j,\infty})\Big)dj$$

Cost of carbon / Marginal value of temperature

- ***Proposition (Stationary LSCC)*** :
When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ_i^y, γ_i^u, temperature, and output, consumption.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^\star) \Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \Big)$$

  - Stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - Back

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\, \chi\, \xi\, \omega \int_{t_0}^{T} e^{(n+\bar{g})t - \delta_s(T-t)} q_t^{f\ -\sigma_e} \int_{j \in \mathbb{I}} \left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e - \sigma} dj\, dt$$

  - Geographical factors determining warming Δ_i
  - Climate sensitivity χ & carbon exit from atmosphere δ_s
  - Growth of population n, aggregate productivity $\bar{g}$
  - Deviation of output from trend y_i & relative TFP z_j
  - Directed technical change z_t^e, elasticity of energy in output σ
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}_t^R / \lambda_t^R = \rho$
  - Change in energy mix, renewable share ω, price q_t^r & elasticity of source σ_e

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto\ n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \widetilde{\gamma}\right) + (\sigma_e - \sigma)(1-\omega) g^{q^r} - \left(\sigma_e(1-\omega) + \sigma\omega\right) g^{q^f}$$

Back