The Inequality of Climate Change

Preliminary – Work in progress

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Thomas Bourany*
The University of Chicago

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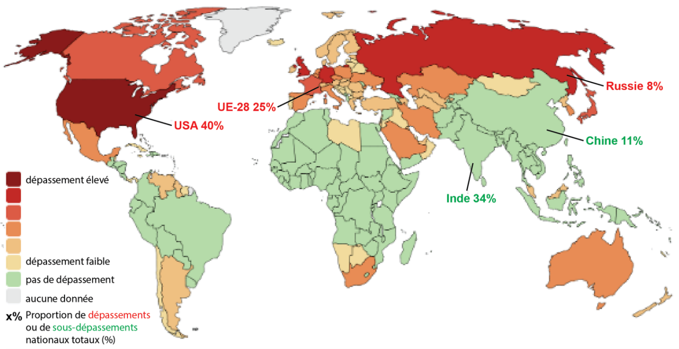
Abstract

Climate change disproportionately affects developing economies while hurting relatively less the higher-income countries responsible for a large share of greenhouse gas emissions. In this context, what is the optimal policy design for energy taxation when there is inequality in impacts and levels of development? Through the lens of an Integrated Assessment Model with heterogeneous countries, I characterize the Social Cost of Carbon (SCC) and the second-best Ramsey policy when lump-sum transfers across countries are not allowed. In contrast to the standard representative country model, both the level and the distribution of optimal carbon taxes change. In particular, the carbon tax is higher for low-marginal utility, and thus higher-income countries. This qualitative finding is general and does not depend on the market structure or the energy market's characteristics. I also propose a new numerical method relying on the sequential formulation to simulate the model globally, solve for optimal policy and potentially handle aggregate uncertainty in this heterogeneous agents model.

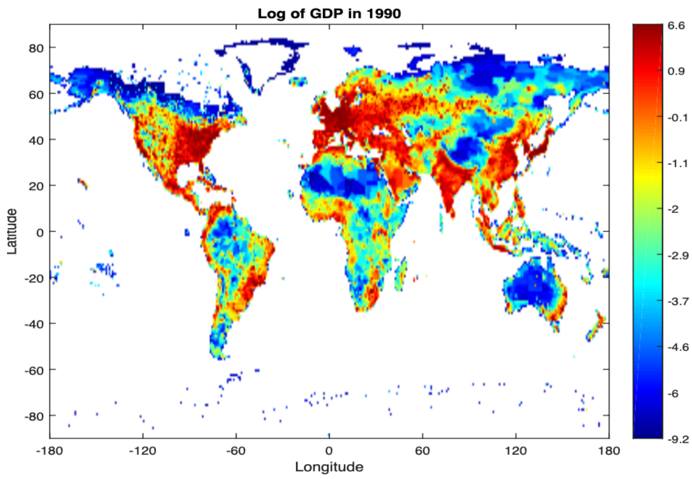
*Thomas Bourany, *thomasbourany@uchicago.edu*. I thank my advisors Mikhail Golosov, Lars Hansen and Esteban Rossi-Hansberg for valuable guidance and advice. I also thank Aditya Bhandari, Jordan Rosenthal-Kay, and other seminar participants at UChicago & Booth for stimulating discussions. All errors are mine.

1 Introduction

The climate is warming due to greenhouse gas emissions generated by economic activity. More than 500 Giga tons of Carbon have been emitted through the burning of fossil fuels in different countries, and global atmospheric temperatures have increased by more than $1.1°C$ since the end of the 19th century and the industrial revolution. The causes of these emissions are unequal: Developed economies account for over 65% of cumulative greenhouse gas (GHG) emissions – $\sim$ 25% each for the European Union countries and the United States, while some developing countries have barely emitted anything compared to their population level. In the following map, on the left, is displayed how much individuals in each country have exceeded their carbon budget – a fixed number of gigatonnes of CO_2 per inhabitant: countries in red have emitted cumulated emissions per capita much higher than their allocated budget. One can observe that this measure of emission excess is highly correlated with local development and GDP and production in each region, as we see on the map on the right.

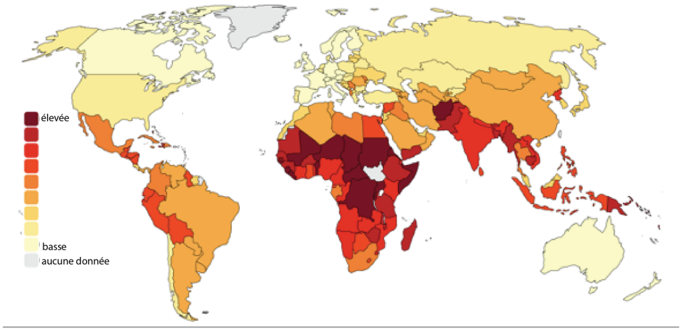


Source : Lancet planetary Health - Quantifying national responsibility for climate breakdown: an equality-based attribution approach for carbon dioxide emissions in excess of the planetary boundary - Jason Hickel 2020

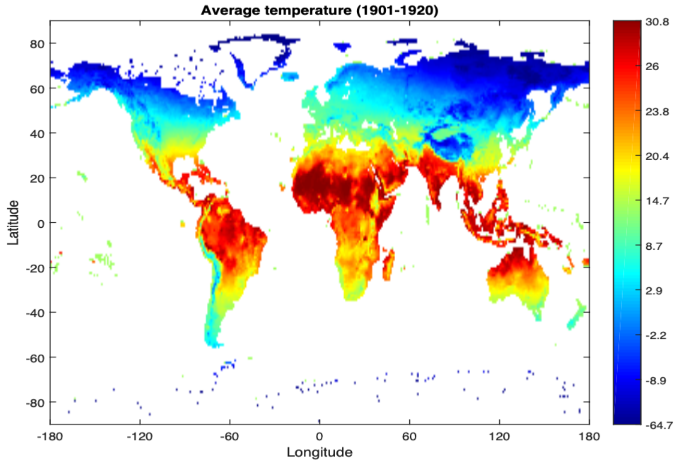


However, the consequences of global warming are also unequal: the increase in temperatures will disproportionately affect developing countries where the climate is already warm. Most emerging and low-income economies lie geographically closer to the tropics and the equator and tend to have higher levels of temperature as well as high probabilities of extreme heat events. One can argue that these countries are most vulnerable to warming. In the following map on the left is displayed an adaptation index that compiles different measures of likelihood and vulnerability of the region to extreme events, loss in biodiversity, drought and heatwaves, or sea level rising among other factors. We observe that this correlates very closely with local temperatures. Moreover, these indices covary negatively with the region's GDP as seen in the previous map.

These two layers of inequality at the world level raise the question: which countries will be affected the most by climate change? Do these different dimensions of heterogeneity matter when measuring the future costs of global warming? In such a context, how to design optimal policy in the presence of externalities and inequality? Moreover, carbon and fossil fuel taxation have strong



Source : Notre Dame Global Adaptation Initiative



redistributive effect across countries and should that be internalized when implementing climate policy.

To answer these classical questions in climate economics, I develop a simple yet flexible model of climate economics. This extends the standard Neoclassical Growth – Integrated Assessment model to include heterogeneous regions. These regions – or individual countries – are (i) heterogeneous in income and level of development, (ii) could be affected differently by the global climate and (iii) are interacting with each other through energy markets and the path of emissions and temperature. This theoretical framework is of the same family as heterogeneous agent models.

Since the quantitative framework is very general, I first provide an extremely simple toy model. Keeping the same notion of externality and interactions in an energy market, we see that most of the intuitions on the design of optimal policy and the computation of the social cost of carbon carry through. The main result is the following. In presence of inequality and climate externality, a world social planner would both issue at once: They would impose a carbon tax that accounts for the climate externality, i.e. the Social Cost of Carbon (SCC) in the Pigouvian fashion, and would redistribute across countries using lump-sum transfers, for example taxing lump-sum European and American countries and transferring to South Asian and African Countries. However, are such transfers feasible and achievable politically. It is well-known that in addition to the tragedy of the commons, there are strong policy constraints that prevent perfect redistribution even in the case of optimal taxation policy.

As a result, I consider a Second-Best Ramsey policy and a larger set of suboptimal policies that would search an alternative way to fight climate change. Despite being unable to redistribute freely across countries due to limitations on transfers, a planner could adapt its tax policy. I show that taxation changes in three ways compared to the standard Pigouvian result in Representative Agents models: (i) the level of the Social Cost of Carbon account for inequality and the correlation between poverty and vulnerability to climate change, (ii) the taxation of energy should also account for redistributive effects of the energy price – due to change in terms-of-trade between exporters and importers and (iii) that the distribution of carbon tax is correlated with the level of development: richer/advanced economies should be imposed a higher tax simply because they have lower marginal utilities of consumption and can hence "afford" to pay higher taxes without being excessively affected. These finding are very general and I develop a dynamic model with several market forces and frictions.

First, using this quantitative model, we are able to evaluate the heterogeneous welfare costs of global warming in a panel of 40 countries. In this framework, countries are heterogeneous in many dimensions – population, productivity, temperature, etc. – and in each of them a household consumes, borrows subject to credit constraints, invests in physical capital, produces homogenous goods using capital and energy, and chooses between carbon-intensive fossil fuels and carbon-neutral clean energy. Moreover, different countries are interacting on the world market for fossil energy where a representative competitive supplier extracts fossil fuels and redistribute energy rent to individual countries. The different countries are also interacting through the global climate: both atmospheric and local temperatures rise when the cumulative stock of emissions rises. However, climate damage is an externality and there are no incentives to curb emissions. This model is very general and is flexible to add numerous extensions. Simulating the model sequentially in continuous time amounts to solving differential equations, and I develop a new methodology to handle the solution of this infinite-dimensional system.

Second, in this framework, I design the optimal Ramsey policy. Using advances in public finance and optimal taxation in heterogeneous agents modeling, I adapt my sequential method to the planner problem and show how to design and decentralize the optimal taxes, with heterogeneous regions. I show how optimal Pigouvian taxes should be adapted to account for (i) redistribution effects of fossil fuel taxations, (ii) the social cost of carbon due to climate externalities, (ii) the effect of these taxes on energy markets and on the redistribution of the fossil energy rent and (iv) the distortion of energy choice both in level and in composition between different sources. As a result, the world optimal carbon policy may not be as simple as *Carbon tax* $=$ *Social Cost of Carbon*, and the taxation should be adapted to the specific situation of each country.

Third, using this theoretical model, I am able to derive several closed-form results to inform on the various mechanisms at hand in this environment. (i) How inequality affects the Social Cost of Carbon and how one can express the world Cost of Carbon as a weighted average of local marginal damages, where the weights represent the distributional effects: with the actual distribution of temperatures and outputs, the SCC is higher in this heterogeneous agent world than in a representative agent one. (ii) On which factors does the Social Cost of Carbon depend? I derive a simple yet general formula and show that the price of carbon is linear in GDP/level of development of the country and in the temperature gap from optimal climate, where proportionality constants depend on climate and damage parameters. These results contrast with the recent developments of this literature which rely on computational models that tend to be opaque and sometimes theoretically untractable.

Related literature

This paper stands at the intersection of several subfields of macroeconomics, climate economics and computational and mathematical economics.

First, since this project considers an Integrated Assessment model (IAM) with heterogeneous countries, this is naturally related to the classical approach of IAM by Nordhaus. I use a similar neoclassical model with a climate block and damage of higher temperatures, as in the DICE model, Nordhaus (1993) Nordhaus (2017). Regarding policy design, few papers actually build optimal taxation policies to tackle climate change. Golosov, Hassler, Krusell and Tsyvinski (2014) is the major exception and develops the first-best policy in a representative agent model and optimal tax as a function of the SCC and a closed-form formula of the climate parameters. Moreover, Hillebrand and Hillebrand (2019) develop several transfer policies that are Pareto improvement to the competitive.

Second, I also relate to the scientific literature that has reexamined the empirical performance of IAMs, review the calibration, and derive analytical formula as in Dietz, van der Ploeg, Rezai and Venmans (2021), Dietz and Venmans (2019), Ricke and Caldeira (2014) or Folini et al. (2021). Adopting the best practice from this literature, I consider a macroeconomic model where I derive closed form expressions for the social cost of carbon and the asymptotics of this general class of model.

Third, and importantly, handling country heterogeneity, I also relate to a booming literature on computational climate economy models, such as Hassler, Krusell, Olovsson and Reiter (2020), Krusell and Smith (2022) and Kotlikoff, Kubler, Polbin and Scheidegger (2021*b*). In a model that is extremely related, I adopt a different methodology – using the sequential formulation – and I study the optimal policy when heterogeneity and externality matter for the price of carbon.

Fourth, in a related field, the spatial-economic geography literature has done important advances in studying the heterogeneous impact of climate change. In this field, important frictions and adaptation mechanisms have been studied, such as migration, international trade or sector reallocation, such as Cruz and Rossi-Hansberg (2021), Cruz Álvarez and Rossi-Hansberg (2022), Rudik et al. (2021) or Bilal and Rossi-Hansberg (2023). In comparison, I assume away strategic complementarities such as migration or trade, as it would not be tractable in this sequential formulation. However, I do consider forward-looking heterogeneous agents and design optimal policy in this context.

Fifth, I also consider specific details to the energy markets that borrow from a literature that studies market frictions such as exhaustible resources and market power, such as Hotelling (1931), Heal and Schlenker (2019) and Bornstein, Krusell and Rebelo (2023). I keep the energy market simple, but I show that the path of emissions depends greatly on the details of the pricing of fossil energy.

Sixth, I develop a framework that is flexible enough to handle aggregate uncertainty, such as climate risk and business cycle fluctuation. The Stochastic DICE model of Cai and Lontzek (2019) and Lontzek, Cai, Judd and Lenton (2015) or the general approach to study model uncertainty and ambiguity aversion applied to climate change in Barnett, Brock and Hansen (2020), Barnett, Brock and Hansen (2022) are particularly related. If I do not include aggregate risk in the present paper, I provide intuitions in the toy model and will integrate this in forthcoming works.

Seventh, I also relate to a thriving literature that studies optimal policy design in Heterogeneous Agents models. Solving Ramsey policy, Le Grand et al. (2021), Bhandari et al. (2021*a*), Dávila and Schaab (2023) or McKay and Wolf (2022) propose different approach to conduct monetary and fiscal policy in HANK models. In my framework, I solve the Ramsey policy sequentially and climate externalities and Pigouvian taxation in presence of heterogeneity rather than managing business cycle fluctuations.

Eighth and lastly, I also integrate advances from the mathematical literature on the Probabilistic Formulation of Mean Field Games. Classical references such as Lasry-Lions approach of the PDE system, Cardaliaguet (2013/2018) or even Pham and Wei (2017) all rely on Dynamic Programming principle. Recently, the solution of the master equation has been very fruitful as in Cardaliaguet et al. (2015) or Bilal (2021). However, a probabilistic approach has realized that the Pontryagin maximum principle extends to the stochastic case, as in Yong and Zhou (1999) or the Mean-Field / McKean Vlasov infinite dimensional case, as in Carmona et al. (2015), Carmona and Delarue (2018) or Carmona and Laurière (2022). Using this approach in the deterministic case with shooting algorithms in large dimension, I solve the model, compute the social cost of carbon and design optimal policy. For the case with aggregate risk, I borrow intuitions from Carmona et al. (2016), Bourany (2018) and Carmona and Delarue (2018) to solve the Stochastic FBSDE system.

2 Toy model

In this section, we develop the simplest version of the quantitative model covered in the next section. The goal is to provide intuitions on the effects of heterogeneity across countries, the source of climate externality related to energy markets, and the implementation of optimal policy.

The model is static and all the decisions are taken in one period. Consider two countries $i = N, S$, for *North* and *South* that are heterogeneous in three dimensions that will be detailed below. A unique household in each country consumes the good c_i that is produced with energy e_i with the production function $y_i = F(e_i)$.[1] Moreover, in this world, outside of the two countries, there is an energy producer producing energy at cost. It sells this energy input at a price q^e to both countries. Due to decreasing return to scale, this competitive firm still makes profit $\pi(E)$. This producer is owned by country i with share θ_i, and the profit are redistributed according to this ownership share. Finally, the countries are subject to climate damage represented by the productivity term $\mathcal{D}_i(\mathcal{S})$ as in Nordhaus DICE models.

The Household maximization problem is the following:

$$\max_{c_i, e_i} U(c_i)$$
$$c_i + q^e e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) \qquad [\lambda_i^w]$$

where the production function $F(e_i)$ is increasing and concave in e_i, i.e. $F'(e) > 0$ and $F''(e) < 0$ and features Inada conditions.

Both countries are subject to climate damages $\mathcal{D}_i(\mathcal{S})$ caused by climate externalities related to energy consumption e_i:

$$\mathcal{S} = \mathcal{S}_0 + \overbrace{\xi_S e_S + \xi_N e_N}^{=\text{GHG emissions}}$$

where ξ_i is the conversion factor between energy use e_i in physical units – e.g. in Joule, Tons Oil Equivalent, kWh or Thermal units – and emissions measured in Tons of Carbon or CO_2. This obviously depends on the energy mix between fossil fuels used for energy and renewables. However, this is taken as given in the short run in our static equilibrium. The quantitative model introduces this endogenous channel of energy choice.

The global carbon emission stock is not internalized by households in their energy consumption decision leading to damage $\mathcal{D}_i(\mathcal{S})$ that affects country i's effective productivity, as in standard Integrated Assessment models, e.g. Nordhaus DICE models.

Each household consumes energy in a single international market, where the energy price q^e

[1] Note that one could make the model slightly more general by considering continents of differing populations $p_N \neq p_S$. One could also add additional inputs in the production function, for example, capital k_i or labor ℓ_i, and make the endowment of these inputs vary across locations. As we will show in the quantitative model, these features do not change the qualitative implication of this framework.

is set to clear the supply and demand.

$$E = e_N + e_S$$

The energy supply E – for example oil and gas extraction or nuclear, solar, and wind power – is provided by a single energy producer maximizing its profit, subject to convex cost $c(E)$, i.e. $c'(E) > 0$ and $c''(E) > 0$ that is paid in the homogenous goods.

$$\begin{aligned} &\max_E \; q^e E - c(E) \\ \Rightarrow \qquad & q^e = c'(E) \qquad \& \qquad \pi(E) := c'(E)E - c(E) \end{aligned}$$

Heterogeneity *North* and *South* are symmetric in all regards, except for differences in three parameters. First, the South and the North are different in terms of productivity z_i: $z_S < z_N$. Here, we consider a wide definition of z_i as productivity residuals that can account for technology, efficiency, market frictions, and institutions. Second, we can furthermore assume that $\theta_N > \theta_S$ for illustration purposes, as countries with large energy rent tend to have higher GDP per capita. Third, we consider that the Southern country is subject to stronger damages of climate, in the sense that the damage parameter $\gamma_i = -\frac{\mathcal{D}_i'(\mathcal{S})}{\mathcal{S}\mathcal{D}_i(\mathcal{S})}$ is higher in the South such that $\gamma_S > \gamma_N$. All these differences will yield heterogeneity in consumption in the competitive equilibrium and motives for redistribution.

The Competitive Equilibrium is a system of price q^e and allocation $\{c_i, e_i\}_i$ such that (i) the Household maximizes utility, i.e. chooses c_i and e_i to maximize utility and (ii) the energy producers choose production E to maximize profit, and market clear for both goods and energy:

$$\sum_{i=N,S} c_i + c(E) = \sum_{i=N,S} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad\qquad E = e_N + e_S$$

The competitive equilibrium results in the following optimality conditions, first for consumption :

$$\lambda_i = U'(c_i) \qquad \textit{with} \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) - q^e e_i$$

where λ_i represents the marginal value of wealth, i.e. the marginal utility of consumption. The second optimality for energy use for production writes as follow:

$$MPe_i = q^e \qquad \textit{with} \qquad MPe_i := \mathcal{D}_i(\mathcal{S}) z_i F'(e_i)$$

This corresponds to the standard tradeoff Marginal Product of Energy = Energy Price.

For illustration purposes, we assume that the North is richer, having access to superior technology $z_N > z_S$ and higher production and consumption $c_N > c_S$ for a given price q^e of energy, and that the South is subject to higher damages $\mathcal{D}_S(\mathcal{S}) < \mathcal{D}_N(\mathcal{S})$ for all $\mathcal{S}$ the stock of carbon emissions[2].

This competitive equilibrium is inefficient for two reasons: First, (i) the climate damages $\mathcal{D}_i(\mathcal{S})$ are not internalized, and energy consumption might be too high depending on the economic cost of global warming $\mathcal{D}_i(\mathcal{S})$.

In addition (ii) the redistribution of the energy rent $\pi(E)$ is not internalized either: choosing e_i affects the amounts of profit the energy firms make and redistributes as share θ_i to households.

Moreover, economic inequality results from the heterogeneity in productivity and climate damage since $c_N > c_S$ we have $\lambda_S > \lambda_N$. Redistribution from the North to the South could be desirable from a utilitarian point of view. This inequality in consumption and damages arises despite trade openness[3].

We explore how the Ramsey planner would allocate consumption and energy in such an environment.

2.1 Social planner allocation with full transfers

Consider a Social Planner who could take the agent's decisions, subject to the resource constraints in goods and energy as well as the climate externality.

$$
\begin{aligned}
&\max_{\{c_i,e_i\}_{i=N,S}} \sum_{i=N,S} \omega_i U(c_i) \\
&\sum_{i=N,S} c_i + c(E) = \sum_{i=N,S} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad [\lambda] \\
&E := e_N + e_S \\
&\mathcal{S} := \mathcal{S}_0 + \xi_S e_S + \xi_N e_N
\end{aligned}
$$

where λ is the shadow value of the good market clearing. We consider a welfare function, where the countries are weighted with Pareto weights ω_i. In the following, we denote the social planner allocation $\{\hat{c}_i, \hat{e}_i\}_{i=N,S}$ to distinguish it from the competitive equilibrium.

Choosing the consumption on behalf of the agents yields a redistribution motive:

$$
[c_i] \qquad \lambda = \omega_i U'(c_i) \qquad \Rightarrow \qquad \omega_N U'(\hat{c}_N) = \omega_S U'(\hat{c}_S)
$$

Depending on the Pareto weights there is a motive for transferring consumption across countries. Regarding the choice of energy inputs:

[2]Indeed, assuming $F(e)$ is Cobb Douglas $F(e) = \bar{k}^{1-\alpha} e^{\alpha}$, with $\bar{k} = 1$, we obtain $\alpha \mathcal{D}_i(\mathcal{S}) z_i e_i^{\alpha-1} = q^e$ leading to

$$
e_i = \big(\alpha \mathcal{D}_i(\mathcal{S}) z_i / q^e\big)^{1/(1-\alpha)} \qquad y_i - q^e e_i = \big(\mathcal{D}_i(\mathcal{S}) z_i\big)^{1/(1-\alpha)} (q^e)^{-\alpha/(1-\alpha)} \big[\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}\big]
$$

which is increasing in z_i and $\mathcal{D}_i(\mathcal{S})$.

[3]One could also consider trade and financial autarky and lack of redistribution across countries: production in one country can not be exported or transferred to another country. That would strengthen that heterogeneity

$$[e_i] \qquad c'(E) = \mathcal{D}_i(\mathcal{S}) z_i F'(e_i) + \xi_i \underbrace{\sum_{j=N,S} \mathcal{D}'_j(\mathcal{S}) z_i F(e_i)}_{=\overline{SCC}}$$

we see an additional term that represents the cost of emitting one ton of carbon in terms of forgone production. This term is the social cost of carbon (SCC) in the social planner allocation.

We turn now to how to decentralize such allocation. We consider a planner who has access to all instruments, and in particular lump-sum transfers across countries that we will denote T_i. The energy optimality rewrites:

$$MPe_i := \mathcal{D}_i(\mathcal{S}) z_i F'(\hat{e}_i) = q^e + \xi_i \tau^e$$

$$\text{with} \qquad q^e = c'(E) \qquad \tau^e = \overline{SCC} = \sum_{j=N,S} \mathcal{D}'_j(\mathcal{S}) z_i F(\hat{e}_j)$$

Importantly, the carbon tax τ^e is equal to the social cost of carbon. We see that this result relies on the existence of lump-sum transfers. Indeed, the budget constraint in this equilibrium allocation writes

$$\hat{c}_i + (q^e + \xi_i \tau^e)\hat{e}_i = \mathcal{D}_i(\mathcal{S}) z_i F(\hat{e}_i) + \theta_i \pi(E) + T_i$$

where the transfers are such that $\omega_N U'(\hat{c}_N) = \omega_S U'(\hat{c}_S)$. In particular, summing the two budget constraints[4] yields:

$$T_N + T_S = 0 \qquad T_S = -T_N > 0$$

implying it is lump-sum redistribution from North to South as we assumed[5] $z_S < z_N$ and $\theta_S < \theta_N$, under reasonable parametrization for the Pareto weight[6]

In the following, we forbid this assumption of lump-sum transfer: indeed if development aid exists, in practice full redistribution with lump-sum transfers and taxes is politically unfeasible.

2.2 Comparison with the Ramsey Problem with limited transfers

Consider now a Social Planner that take into account the constraint that prevent the full lump-sum redistribution. Subject to competitive equilibrium optimality conditions, and the same market frictions – climate externality and the absence of financial instruments for transfers across countries, the planner takes the decisions of consumption and energy to maximize the welfare function with weights ω_i for each country. We denote the Ramsey allocation $\{\tilde{c}_i, \tilde{e}_i\}_i$ to distinguish

[4]
$$\sum_i \hat{c}_i + q^e E + \tau^e \sum_i \xi_i \hat{e}_i = \sum_i \mathcal{D}_i(\mathcal{S}) z_i F(\hat{e}_i) + \underbrace{\sum_i \theta_i}_{=1} \left(q^e E - c(E)\right) + \sum_i T_i$$

[5] Given that $T_i = u'^{-1}(\frac{\lambda}{\omega_i}) - \mathcal{D}_i(\mathcal{S}) z_i F(\hat{e}_i) - \theta_i \pi(E) - (q^e + \xi_i \tau^e)\hat{e}_i$

[6] In particular, if the Pareto weight are large enough, i.e. $\omega_S \geq \lambda / u'(c_S)$ i.e. more than the weight that

it from the competitive equilibrium $\{c_i, e_i\}$ and the First-best planner allocation $\{\hat{c}_i, \hat{e}_i\}$.

$$\mathbb{W} = \max_{\{\tilde{c}_i, \tilde{e}_i\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

The consumption and energy allocation are subject to the budget constraint, where the household is imposed a country-specific energy tax $\tilde{\tau}_i^e$ and its amount is redistributed lump-sum $\tilde{T}_i = \tilde{\tau}_i^e e_i$

$$\tilde{c}_i + (q^e + \tilde{\tau}_i^e)\tilde{e}_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) + T_i$$

Moreover, one can show that energy is still priced at competitive price by the energy firm and profit redistributed as before:

$$q^e = c'(E) \qquad \pi(E) = c'(E)E - c(E) \qquad \text{with} \qquad E = e_N + e_S$$

As a result, the Ramsey maximization problem states

$$\begin{aligned} & \mathbb{W} = \max_{\{\tilde{c}_i, \tilde{e}_i,\}_i} \sum_{i=N,S} \omega_i \, U(c_i) \\ s.t \qquad & c_i + c'(E)e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) \qquad [\phi_i] \qquad \forall \, i = N, S \\ & \mathcal{S} := \mathcal{S}_0 + \xi_N e_N + \xi_S e_S \qquad E := e_N + e_S \end{aligned}$$

Moreover, the Lagrange Multipliers ϕ_i represent the Social Value of relaxing the budget constraint. The consumption allocation yield simply:

$$\omega_i U'(c_i) = \phi_i$$

Now the choice of energy integrates all of the distortions of that model, using the definition for marginal product of energy $MPe_i = \mathcal{D}_i(\mathcal{S}) z_i F'(e_i)$. The optimality writes:

$$\begin{aligned} \phi_i \big[MPe_i - c'(E) \big] + \underbrace{\xi_i \sum_j \phi_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j)}_{\propto-\text{SCC}} \\ + \underbrace{\pi'(E) \sum_j \phi_j \theta_j}_{= \text{rent redistribution}} - \underbrace{c''(E) \sum_j \phi_j^w e_j}_{= \text{cost redistribution}} = 0 \end{aligned}$$

We see that the energy choice of the planner can into account several forms of redistribution.

First, climate change affects countries differently according to their marginal damages $\mathcal{D}_j(\mathcal{S})$, but this damage is now scales by the marginal utility $\phi_i = \omega_i U'(c_i)$ since the planner doesn't implement full redistribution. However, this social cost of carbon needs to be rescaled in monetary unit. If many conventions could be chosen, dividing it by the average marginal utility $\bar{\phi} = \frac{1}{2}\big(\phi_N +$

ϕ_S) allows us to take into account the representative world consumer. As a result, the SCC writes:

$$SCC_t := -\frac{\partial \mathbb{W}/\partial \mathcal{S}}{\partial \mathbb{W}/\partial c} = -\frac{1}{\bar{\phi}}\sum_j \phi_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j) = -\sum_j \frac{\phi_j}{\bar{\phi}}\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)$$

Note that now, the SCC is reweighted by an inequality factor:

$$\widehat{\phi}_i = \frac{\phi_i}{\bar{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{2}(\omega_N U'(c_N) + \omega_S U'(c_S))} \lesseqgtr 1$$

In particular, in this heterogenous countries model with limited redistribution, we have that low-income countries have a higher marginal utility of consumption. Moreover, we assumed stronger damages $\mathcal{D}_S'(\mathcal{S}) > \mathcal{D}_N'(\mathcal{S})$. As result, $c_S < c_N$ and $\widehat{\phi}_S > \widehat{\phi}_N$

$$\begin{aligned} SCC &:= -\sum_j \widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j) = -\mathbb{E}_j\left(\widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j)\right) \\ &= -\mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}_j'(\mathcal{S}) z_j F(e_j)\Big) - \mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)] > -\mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)] = \overline{SCC} \end{aligned}$$

Second, one can define similarly the social value of rent (exporters) that accounts for the redistributive effect of the energy supply on price and profit

$$\begin{aligned} SVR &= \pi'(E)\frac{1}{\bar{\phi}}\sum_j \phi_j \theta_j = \pi'(E)\mathbb{E}_j\left(\widehat{\phi}_j\, \theta_j\right) \\ &= \mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \theta_j \pi_j'(E)\Big) + \pi'(E) < \pi'(E) \end{aligned}$$

where the last inequality comes from the assumption that North has a larger endowment in energy resources and hence energy rent. Note that this term would be zero if the energy firm profits are zero, which corresponds to a constant return to scale extraction function for energy.

Third, changing the energy price and quantity redistributes across energy users/importers through the change in cost along the supply curve.

$$\begin{aligned} SCE &= c''(E)\frac{1}{\bar{\phi}}\sum_j \phi_j^w e_j = \mathbb{E}_j\left(\widehat{\phi}_j\, e_j\right) \\ &= \mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, e_j\, c''(E)\Big) + c''(E) < c''(E) = \pi'(E) \end{aligned}$$

where the last inequality comes from the fact that heavy energy users – per capita – are mostly advanced economies which have larger output and consumption. Indeed, if $z_N > z_S$ then we obtain both that $e_N > e_S$ and $c_N > c_S$. Note that again that term is null if the energy production is constant return to scale and the cost function is linear $c''(E) = 0$.

As a result, the *level* of the optimal energy taxation policy account for these three distributional motives (i) climate damage in SCC, (ii) energy rent in SVR and (iii) energy spending in SCE. For (ii) and (iii), taxation is isomorphic to a terms-of-trade manipulation between the exporters and the importers in trade theory.

$$
\begin{aligned}
MPe_i &= c'(E) + \xi_i \mathbf{t}_i^e \\
\mathbf{t}_i^e &= \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[SCC - SVR + SCE \right]
\end{aligned}
$$

These three motives are with a single tax and lump-sum rebate. However, to account for the presence of inequality, the *distribution* of the tax changes.

The tax is lower for poorer countries $\omega_S U'(c_S) > \omega_N U'(c_N) \;\Rightarrow\; \mathbf{t}_S^e < \mathbf{t}_N^e$.

This main finding is very general and will hold in a dynamic quantitative model that I develop in the next sections.

3 Quantitative model

We develop a framework with neoclassical foundations and rich heterogeneity across regions. The time is continuous $t \in [t_0, \infty)$, where[7] $t_0 = 2000$. The countries/regions are indexed by $i \in \mathbb{I}$. They can be heterogenous in an arbitrary number of dimensions[8] s.

As of now, this model includes several individual states $s_i = \{z_i, p_i, \theta_i, \gamma_i, \Delta_i, \xi_i, w_i, \tau_i\}$, respectively productivity z, population p, θ share of energy rent from the fossil fuel company, climate vulnerability γ_i, geographic factors for temperature scaling Δ, and fossil energy mix ξ, which are six dimensions of heterogeneity that are time-invariant. In addition, country wealth w, and local temperature τ change over time. Moreover, the world is subject to global states which can also time-varying $S = \{\mathcal{T}, \mathcal{S}, \mathcal{R}\}$ which are respectively world atmospheric temperature $\mathcal{T}$, world atmospheric carbon concentration $\mathcal{S}$ and reserve of fossil fuel energy sources $\mathcal{R}$. All these variables will be explained in turn below.

Countries interact with the rest of the world through several channels: (i) Each country can trade financial assets b_i in world markets, with $b_{it} > 0$ for saving and $b_{it} < 0$ for borrowing. (ii) The consumption of fossil-fuel energy is traded in a single world energy market at price q_t^f and (iii) Fossil consumption releases carbon emissions in the atmosphere $\mathcal{S}_t$ which increase world temperatures $\mathcal{T}_t$ and local temperature τ_{it}. Moreover, in the baseline model, no migration and no bilateral trade or capital flows are allowed between countries.

3.1 Country Household and firm problem

At each instant t, each region $i \in \mathbb{I}$ is populated by a representative household of population size p_{it}. This population is increasing at a growth rate exogenously determined n, and $\dot{p}_{it} = np_{it}$. As a result, the population is given as $p_{it} = p_{i0}e^{nt}$.

This representative household owns the representative firm that is producing output with total factor productivity z_{it}. This total factor productivity also grows with a deterministic growth rate $\bar{g}$, giving a TFP level of $z_{it} = z_{i0}e^{\bar{g}t}$. In the tradition of the Neoclassical model, we normalize all the economic variables of the model by the rate of effective population $z_t p_t = e^{(n+barg)t}$, leaving only the relative difference between countries' population $p \equiv p_{i0}$ and productivity $z \equiv z_{i0}$. In the following, we remove the countries i indices for ease in notations and in the absence of ambiguity: each country solves an independent dynamic control problem and is subject to global variables that we shall denote with capital letters – for example, $\mathcal{T}_t$ for global temperature or $\mathcal{E}_t$ for global emissions explained below.

The household consumes the homogeneous good $c_t \equiv c_{it}$ and is subject to the temperature

[7] In the application we will consider an interval $t \in [t_0, t_T]$ with $t_0 = 2000$ and $t_T = 2100$.

[8] More precisely, state variables of heterogeneity can be split in two, $s = \{\underline{s}, \overline{s}\}$, where ex-ante heterogeneity is constant over time or relate to initial conditions and is denoted $\underline{s}$, while ex-post heterogeneity $\overline{s}$ changes over time depending on the fluctuations of the regions variables. In practice, with the method used, $\underline{s}$ can be arbitrarily large, but the size of ex-post heterogeneity $\overline{s}$ needs to be controlled, as we will explained in the computational section below.

of the region $\tau_t \equiv \tau_{it}$. It also chooses the firms inputs in the production function[9], yielding the output per capita:

$$y_{it} = \mathcal{D}_y(\tau_{it}) z_i f(k_{it}, e_{it})$$

where temperature τ, relative productivity z, capital stock per effective capita k and energy input per effective capita e all affect production. The gross production function is a CES aggregate between the capital-labor bundle k and energy e:

$$f(k_{it}, e_{it}) = \left[(1-\varepsilon)^{\frac{1}{\sigma}} k_{it}^{\alpha\frac{\sigma-1}{\sigma}} + \varepsilon^{\frac{1}{\sigma}} \left(z_t^e e_{it}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with $\sigma < 1$, such as energy is complementary in production[10] and where directed technical change z_t^e is exogenous and deterministic. This directed – energy augmenting – technical change allows an increase in output for a given energy consumption mix. An upward trend in such technology is sometimes argued to be behind the "relative decoupling" of developed economies: an increase in production and value-added simultaneous to a decline in energy consumption and carbon emissions. In this version of the model, this trend is taken exogenously increasing at rate $z_t^e = \bar{z}^e e^{g_e t}$, but in an extension of the model, we consider an endogenous directed technical change. Moreover, energy used in production comes from two sources: either fossil e_{it}^f and renewable e_{it}^r for every country i. The production of these two sources is detailed below.

Moreover, the temperature τ_{it} affects the productivity through damages $\mathcal{D}_y(\tau_{it})$. This is the source of climate externality as will detailed below.

The Household in the country $i \in \mathbb{I}$ owns the firms and hence solves the following intertemporal problem. They maximize present discounted utility, with the discount rate ρ, by choosing consumption c_{it}, energy inputs e_{it} – bought at price q_t^e.

$$v_{it_0} = \max_{\{c_{it}, e_{it}^f, e_{it}^r\}} \int_{t_0}^{\infty} e^{-(\rho-n)t} u_i(c_{it}, \tau_{it})\, dt$$

The utility that households receive from consumption is also scaled by a damage function, which represents the direct impact of temperature.

$$u_i(c_{it}, \tau_{it}) = u(\mathcal{D}_u(\tau_{it}) c_{it}) \qquad\qquad u(\mathcal{D}\, c) = \frac{(\mathcal{D}c)^{1-\eta}}{1-\eta}$$

To sum up, beside the cost of energy $e_{it} q_{it}^e$and expenditure in consumption c_t, the production can be invested to increase capital stock and cover depreciation $\tilde{i} = \dot{k} + \delta k$. Moreover, the country i

[9]The original – unnormalized – production function:

$$Y_t = F(K_t, E_t, L_t) = \mathcal{D}(\tau_t) z_t \left[(1-\varepsilon)^{\frac{1}{\sigma}} \left(K_t^{\alpha} L_t^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + \varepsilon^{\frac{1}{\sigma}} \left(z_t^e E_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

We divide the output level Y_t by the growth trend in population and TFP $e^{(n+\bar{g})t}$ and by initial population $p_0 \equiv L_t$ to obtain output per effective capita.

[10]If $\sigma = 1$ we have the Cobb Douglas : $f(k_t, e_t) = \bar{\varepsilon} z_t^{e\ \varepsilon} k_t^{\alpha} e_t^{\varepsilon}$

receives a share of profit θ_i that the fossil sector generates $\pi(E_t^f, \mathcal{R}_t)$ and that will be explained below.

The representative agent in each country can save and borrow in a liquid financial asset b_t at a world interest rate $r_t^\star$. Moreover, they can invest an amount $\tilde{i}_t$ and hold that wealth in capital k_t that produces the homogenous good with production function $f(k_t, e_t)$. However, we had a potential friction that countries may be financially constraints on world financial market and can only borrow again a fraction ϑ of their capital.

As a result, the accumulation of liquid bonds and capital is the following:

$$
\begin{aligned}
\dot{k}_{it} &= \mathcal{D}_y(\tau_{it}) z_i f(k_{it}, e_{it}) - (n + \bar{g} + \delta) k_{it} + \tilde{i}_{it} \\
\dot{b}_{it} &= r_t^\star b_t + \theta_i \pi_t(E_t^f, \mathcal{R}_t) - q_t^e e_{it} - \tilde{i}_{it} - c_{it} \\
b_{it} &\geq -\vartheta k_{it}
\end{aligned}
$$

with k_t the capital per effective capita – covering population n and TFP growth $\bar{g}$ rates.

As a result, one can aggregate the bond and capital of the individual country as a single wealth variable $w_{it} = k_{it} + b_{it}$ yielding the dynamics:

$$
\begin{aligned}
\dot{w}_{it} &= r_t^\star w_{it} + \mathcal{D}_y(\tau_{it}) z f(k_{it}, e_{it}) - (n + \bar{g} + \delta + r_t^\star) k_{it} + \theta_i \pi_t(E_t^f, \mathcal{R}_t) - q_t^e e_{it} - c_{it} \\
k_{it} &\leq \frac{1}{1 - \vartheta} w_{it}
\end{aligned}
$$

on $t \in [t_0, t_T]$ where the dynamics of wealth starts from initial condition $w_{t_0} = k_0 + b_0$. The return on capital is $r_{it}^k = MPk_{it} := \partial_k f(k_{it}, e_{it})$, and capital is now a control variable chosen by the individual agent. This wealth level constitutes the first dimension of ex-post heterogeneity.

We have two pathological cases where ϑ can take two extreme values: (i) first, financial autarky corresponds to the case where $\vartheta = 0$. The countries are forbidden to borrow $b_{it} \geq 0$, and as a result, are constrained to hold all their wealth in capital $w_{it} = k_{it}$. Indeed, in such case the marginal product of capital is higher than world interest rate $r_{it}^k = MPk_{it} := \partial_k f(k_{it}, e_{it}) > r_t^\star$ as the capital holding is limited.

In the second extreme case, (ii) full financial integration allows $\vartheta \to 1$. As a result, borrowing is unrestricted and capital holding is not bounded. We see below that in such case, capital holding is such that the capital return is equalized to the world interest rate $r_{it}^k = MPk_{it} = r_t^\star$. This case corresponds to the assumption made in Krusell and Smith (2022). Setting $\vartheta < 1$ allows us to test the quantitative implication of financial constraints for world production and consumption inequality, as this will have an influence on optimal policy as we analyze in the toy model of section 2

Climate damage and externality

Change in temperatures τ_{it} in each country $i \in \mathbb{I}$ – given in degree Celsius, $^\circ C$ – affects the productivity with a Damage function $\mathcal{D}_y(\tau_t)$. This scaler increase with $\tau < \tau_i^\star$ and decreases when $\tau < \tau_i^\star$, where the "optimal temperature" $\tau_i^\star$ such that $\mathcal{D}_y(\tau_i^\star) = 1$. We consider the “optimal” temperature as:

$$\tau_i^\star = \alpha^\tau \tau_{it_0} + (1 - \alpha^\tau)\tau^\star$$

where τ_{it_0} is the initial temperature in country i and $\tau^\star = 15.5°C$ is an optimal level of yearly temperature for temperate climates, as used in Kotlikoff et al. (2021*b*). This flexible formulation allows for differing degrees of adaptability depending on the value of α^τ. Hot temperatures do not affect countries with long histories of cold vs. hot climates in the same way, due to the presence of adaptation structures – i.e. air conditioning vs. heating infrastructures.

Productivity decays to zero when temperatures are extremely cold or hot $\lim_{\tau \to -\infty} \mathcal{D}_y(\tau) = \lim_{\tau \to \infty} \mathcal{D}_y(\tau) = 0$. We follow Nordhaus formalism and use a quadratic function for the damage function:

$$\mathcal{D}_y(\tau) = \begin{cases} e^{-\gamma_y^{\oplus} \frac{1}{2}(\tau - \tau_i^\star)^2} & \text{if } \tau > \tau_i^\star \\ e^{-\gamma_y^{\ominus} \frac{1}{2}(\tau - \tau_i^\star)^2} & \text{if } \tau < \tau_i^\star \end{cases}$$

where $\gamma_y^{\oplus}$ and $\gamma_y^{\ominus}$ represent damage parameters on output respectively for hot v.s. cold temperatures – and they are different to allow for asymmetry on climate impact.

The utility that households receive from consumption is also scaled by a similar damage function, which represents the direct impact on population likelihood of mortality – for example, due to heatwaves or extreme weather events – as a direct scaler of consumption.

$$\mathcal{D}_u(\tau) = \begin{cases} e^{-\gamma_u^{\oplus} \frac{1}{2}(\tau - \tau_i^\star)^2} & \text{if } \tau > \tau_i^\star \\ e^{-\gamma_u^{\ominus} \frac{1}{2}(\tau - \tau_i^\star)^2} & \text{if } \tau < \tau_i^\star \end{cases}$$

where $\gamma_u^{\oplus}$ and $\gamma_u^{\ominus}$ represent also the damage parameters, but on the direct impact on utility and mortality, respectively, for hot v.s. cold temperatures.

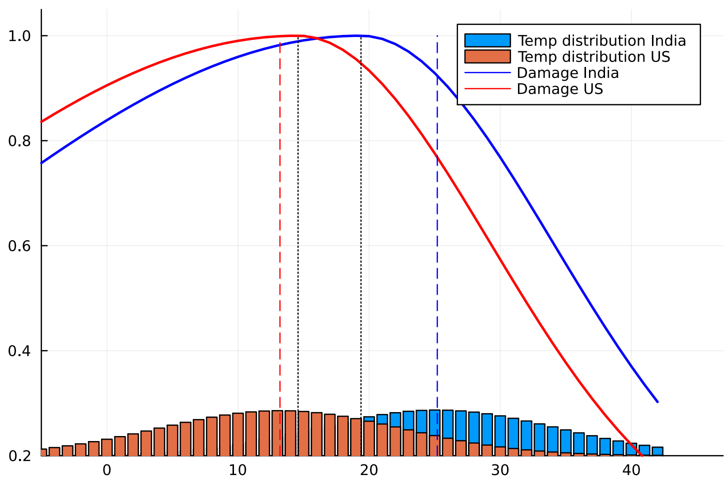


Figure 1: Damage function for two example countries, US and India

In the previous graph, we present an example of such damage function for two countries, USA and India, with the distribution of temperature (approximated by a normal distribution), their average yearly temperature (respectively $13.5°C$ and $25°C$) in dashed lines and their optimal temperature in dotted black lines (respectively $15°C$ and $20°C$)

3.2 Energy sector

Given the demand for energy inputs e_t in each country, the firm has the choice among two sources of energy: one fossil-fuel source in finite supply e_t^f and one renewable source e_t^r. We consider that these two sources are substitutable, and total energy inputs quantity e_t is given by the CES aggregator, where σ_e represents the elasticity of substitution.

$$
\begin{aligned}
e_t &= \left(\omega_f^{\frac{1}{\sigma_e}} (e_t^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\omega_f)^{\frac{1}{\sigma_e}} (e_t^r)^{\frac{\sigma_e - 1}{\sigma_e}}\right)^{\frac{\sigma_e}{\sigma_e - 1}} \qquad \text{if} \qquad \sigma_e \in (1, \infty) \\
e_t &= e_t^f + e_t^r \qquad \text{if} \qquad \sigma_e \to \infty
\end{aligned}
$$

subject to the budget for energy expenditures:

$$
e_t^f q_t^{e,f} + e_t^r q_t^{e,r} = q_t^e e_t
$$

As a result, the demand curve for both fossil and renewable energies are given by usual demands:

$$
\begin{aligned}
\frac{e_t^f}{e_t} &= \omega^f \left(\frac{q_t^{e,f}}{q^e}\right)^{-\sigma_e} \qquad \& \qquad \frac{e_t^r}{e_t} = (1-\omega^f)\left(\frac{q_t^{e,r}}{q_t^e}\right)^{-\sigma_e} \\
q_t^e &= \left(\omega_f (q_t^{e,r})^{1-\sigma_e} + (1-\omega_f)(e_t^r)^{1-\sigma_e}\right)^{\frac{1}{1-\sigma_e}} \qquad \text{if} \qquad \sigma_e \in (1, \infty) \\
q_t^e &= \min\{q_t^{e,f}, q_t^{e,r}\} \qquad \text{if} \qquad \sigma_e \to \infty
\end{aligned}
$$

where the price of the energy bundle q_t is some weighted sum of the energy price of fossil fuel $q_t^{e,f}$ and renewable $q_t^{e,r}$.

Fossil fuel extraction and exploration

Fossil energy is produced and sold in a centralized market at the world level. The single competitive producer is extracting the fuel quantity E_t^f from a single pool of resources $\mathcal{R}_t$, with production cost $\nu(E_t^f, \mathcal{R}_t)$. The assumption of a single producer is made for simplicity. In appendix, we show how we can consider a continuum of producing countries with

Fossil energy can be shipped costlessly around the world, where the global market in energy clears:

$$
E_t^f = \int_{\mathbb{I}} e^{(n+\bar{g})t} p_i \, e_{it}^f di
$$

where the demand comes from the aggregation of individual energy per capita inputs in each country $i \in \mathbb{I}$ and energy input is rescaled by the population and technology exponential trends $e^{(n+\bar{g})t}$.

Moreover, the fossil-fuel reserves $\mathcal{R}_t$ are depleted with extraction E_t^f, but can be regenerated by exploration, which require investment $\mathcal{I}_t^f$ to obtain $\delta^R \mathcal{I}_t^f$ additional reserves for an exploration cost $\mu(\mathcal{I}_t^f, \mathcal{R}_t)$

$$\dot{\mathcal{R}}_t = -E_t^f + \delta^R \mathcal{I}_t^f$$

The parameter δ^R can be interpreted in two ways: first, it can represent the probability intensity $\delta^R \mathcal{I}_t^f$ of finding developable reserves among possible reserves $\mathcal{I}_t^f$ in a continuum of fossil fuel fields and mines. Second, it can also represent the fraction of individual producers discovering developable reserves, aggregating up a representative producer. This stylized model is a simplified version of the rich framework developed in Bornstein et al. (2023).

Moreover, the fossil-fuel producer hence faces a modified Hotelling finite-resources problem – c.f. Heal and Schlenker – allowing for exploration of additional reserves. As a result, its dynamic problem is given by :

$$v^e(\mathcal{R}_t) = \max_{\{E_t^f, \mathcal{I}_t^f\}_{t \geq t_0}} \int_0^\infty e^{-\rho t} \pi(\mathcal{R}_t, E_t^f, \mathcal{I}_t^f) dt$$

$$\text{with} \qquad \pi_t(\mathcal{R}_t, E_t^f, \mathcal{I}_t^f) = q_t^{e,f} E_t^f - \nu(E_t^f, \mathcal{R}_t) - \mu(\mathcal{I}_t^f, \mathcal{R}_t)$$

$$s.t. \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta^R \mathcal{I}_t^f \qquad E_t^f = \int_{\mathbb{I}} p_{i0} e^{(n+\bar{g})t} e_{it}^f di$$

This can be solved using the Pontryagin maximum principle, where we denote λ_t^R the Hotelling rent, which is the costate of the resource depletion dynamics. The price of the fossil energy supplied and the optimal exploration are given by optimality conditions:

$$[E_t^\star] \qquad q_t^{e,f} = \nu_E(E_t^\star, \mathcal{R}_t) + \lambda_t^R$$

$$[\mathcal{I}_t^\star] \qquad \delta^R \lambda_t^R = \mu_E(\mathcal{I}_t^{e\ \star}, \mathcal{R}_t)$$

Price is hence the sum of marginal cost, plus an additional rent meant to price the finiteness of the resource. Moreover, the dynamics of that Hotelling rent are given by the equation:

$$\dot{\lambda}_t^R = \rho \lambda_t^R + \partial_R \nu(E_t^\star, \mathcal{R}_t) + \partial_R \mu(\mathcal{I}_t^\star, \mathcal{R}_t)$$

In standard Hotelling model without stock effects – i.e. where $\partial_R \nu(E^\star, \mathcal{R}) = 0$ and no exploration $\mu(\mathcal{I}^\star, \mathcal{R}) = 0$ – we have the standard expression for the finite resource rent $\dot{\lambda}_t^R = \rho \lambda_t^R$ and $\lambda_t^R = e^{\rho t} \lambda_{t_0}^R$, and $R_t \to 0$ as $t \to \infty$. In our context, the rent grows less fast because (i) the producer anticipate that the depletion of reserves will increase marginal cost in the future $\partial_R \nu(E^\star, \mathcal{R}) < 0$ and (ii) it can invest in exploration, increasing future reserves which can lower even further the future cost of exploring $\partial_R \mu(\mathcal{I}^\star, \mathcal{R}) < 0$.

As a result, even with simple functional forms that yield isoelastic supply curves for fossil energy extraction and exploration, we can solve the dynamics of the rent price.[11]

Note that this centralized market for fossil fuels is in equilibrium: the supply curve $(q^{e,f}, E_t^f)$

[11] Details of the fossil energy producers can be found in appendix .

determined by the fossil-fuel producers meets the demand coming from the aggregation of all individual countries $(q^{e,f}, e_t^f)$. Moreover, fossil fuels emit CO_2 and other GHG emissions, as we will see in the next section.

Renewable energy production

Renewable energy is not subject to the finiteness of the stock of reserves and is produced with capital k_t^e.

$$e_t^r = z_t^r f(k_t^r)$$

Furthermore, carbon emissions associated with renewable energy are null, minimizing the externality on the climate when the energy transition is complete. We assume that capital k_t^r is fungible with the capital k_t that produces the homogeneous good and is hence subject to the same interest $r_t^\star$ on the common world capital market

$$q_t^r z_t^r f'(k_t^r) = r_t$$

where q_t^r is the price of that renewable energy demanded. We make these stylized assumptions to keep the model tractable.

For now, renewable energy production is assumed constant return to scale, i.e. $f^r(k_t^r) = k_t^r$. As a result, the price of non-fossil energy $q_t^{e,r}$ is given exogenously by:

$$q_t^{e,r} = \frac{r_t^\star}{z_t^r}$$

Moreover, if the two sources of energy are perfectly substitutable, i.e. $\sigma_e \to \infty$, then we obtain that renewables act as a perfect "backstop" technology to fossil fuel. If $q_t^{e,f}$ grows up to then all the energy is produced using renewable $e_t = e_t^r$ and the emissions collapse to zeros. This example is analyzed in Heal and Schlenker (2019) in a simpler model.

3.3 Climate system, emissions and externality

Economic activity are emitting carbon and other greenhouse gas emissions, which change the climate and increase the temperature of the atmosphere. Due to these activities coming from the energy sector, each country is emitting the amount:

$$\epsilon_{it} = \xi_i^f e_{it}^f p_i$$

where ξ^f denote the carbon content of fossil fuels[12]. As a result, since the energy use is normalized by growth of TFP and population, the absolute amount of global emissions aggregates to:

$$\mathcal{E}_t = \int_{i\in\mathbb{I}} e^{(n+\bar{g})t}\epsilon_{it} di = e^{(n+\bar{g})t}\int_{\mathbb{I}} \xi_i^f e_{it}^f p_i di$$

These emissions are released in the atmosphere, adding up to the cumulative stock of greenhouse gas $\mathcal{S}_t$.

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

However, a part of these emissions exit the atmosphere and can be stored in oceans or the biosphere, discounting the current stocks by an amount δ_s. Moreover, these cumulative emissions push the global atmospheric temperature $\mathcal{T}_t$ upward linearly with parameter χ with some inertia and delay represented by parameter ζ

$$\dot{\mathcal{T}}_t = \zeta\big(\chi\mathcal{S}_t - (\mathcal{T}_t - \bar{\mathcal{T}}_{t_0})\big)$$

This simple two-equations climate system is a good approximation of large-scale climate models[13] with a small set of parameters $\xi^f, \delta_s, \zeta, \chi$.

More particularly, ζ is the inverse of persistence, and modern calibrations set $\zeta \approx 0.1$ is such that the pick of emissions happens after 10 years. Dietz et al (2021) show that classical IAM models such at Nordhaus' DICE tend to set ζ too low, generating a too large inertia of the climate system, as shown in the figure below. Moreover, if $\zeta \to \infty$, temperature reacts immediately and we obtain a linear model – which is a good long-run approximation:

$$\mathcal{T}_t = \bar{\mathcal{T}}_{t_0} + \chi\mathcal{S}_t = \bar{\mathcal{T}}_{t_0} + \chi\int_{t_0}^{t}\int_{\mathbb{I}} e^{(n+\bar{g})t}\epsilon_{it}\, di\ ds\Big|_{GtC}$$

As we see, the global externality depends on the path of individual policies $\epsilon_{it} \propto e_{it}^f$ as of function of states of the country $\{z_i, p_i, k_i, \tau_i\}$, as well as the growth rates $\bar{g} + n$ of the economy, i.e. TFP and population.

---

[12]We can consider an alternative, like in Nordhaus' DICE model, with

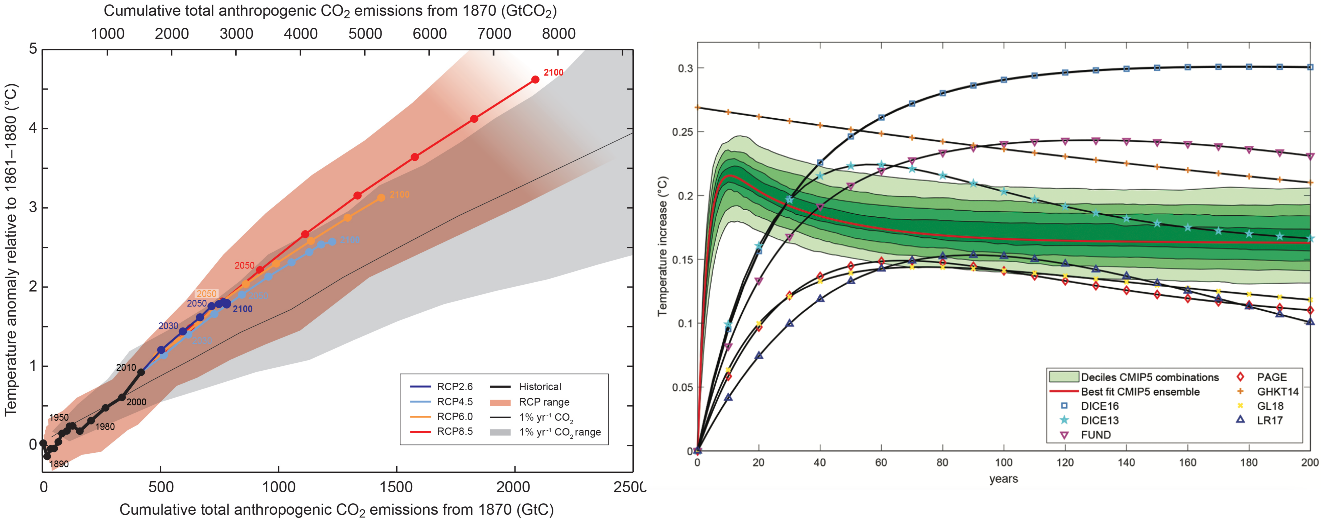
$$\epsilon_{it} = \xi^f(1-\vartheta_{it})e_{it}^f p_i \qquad \& \qquad \mathcal{E}_t = e^{(n+\bar{g})t}\int_{\mathbb{I}} \xi^f(1-\vartheta_{it})e_{it}^f p_i di$$

where ϑ_t represents the abatement policy taken in country i. It represents all the policies that allow reducing the emissions for a given choice of the energy mix – for example, additional environmental regulations or investment in carbon capture technology – with a convex cost $c(\vartheta_{it})e_{it}^f$. Its optimal choice can be determined as solution of the FOC $c'(\vartheta_i)e_{it}^f = 0 \ \Rightarrow\ \vartheta_i = 0$ (business as usual) or $c'(\vartheta_i) = -\xi_i \mathbf{t}_{it}^f$ (second best with carbon tax).

[13]These climate models have typically much more complex climate block, adding 3 to 4 more state variables, with $\mathbf{J}$ the vector of carbon "boxes": layers of the atmosphere and sinks such as layers of oceans:

$$\dot{\mathbf{J}}_t = \Phi^J \mathbf{J}_t + \rho^e \int_{\mathbb{I}} \xi^f e_i^f p_i di$$
$$F_t = \mathcal{F}(\mathbf{J}_t) \qquad \dot{\mathcal{T}}_t = \Phi^T \mathcal{T} + \eta F_t$$

with F_t Carbon forcing and ρ^e, vector of parameters, Φ^J and Φ^T Markovian transition matrices and $\mathcal{F}(\cdot)$ a non-linear function.

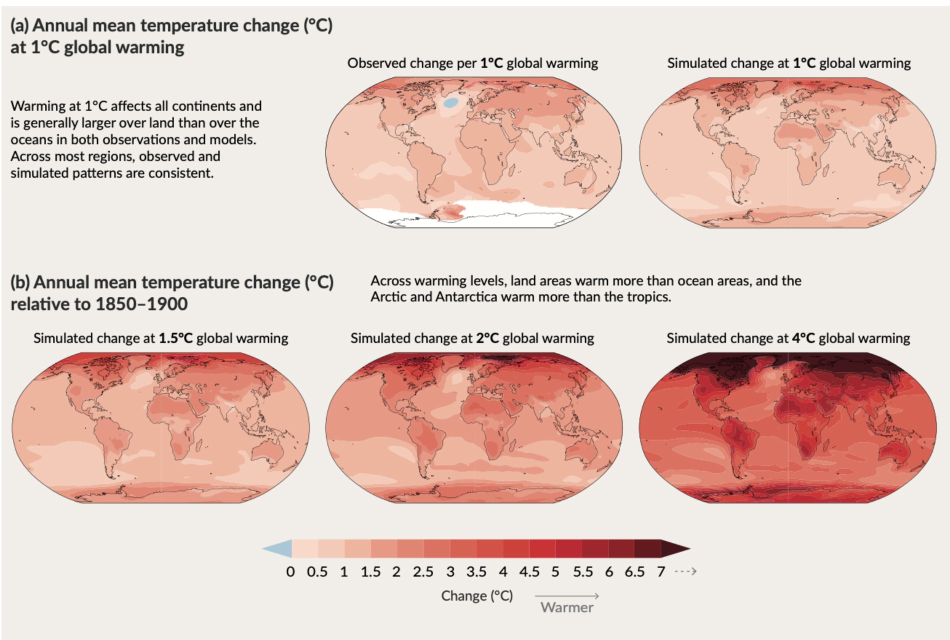


Linear temperature model - IPCC report / Dietz, van der Ploeg, Rezai, Venmans (2021)

The temperature in country i is affected by global warming of the atmosphere $\mathcal{T}_t$ with sensitivity Δ_i

$$\dot{\tau}_{it} = \Delta_i \, \dot{\mathcal{T}}_t$$

Atmospheric temperature $\mathcal{T}_t$ translates into local temperature τ_{it} according to a pattern scaler Δ_i that depends on the geographic properties of country i – like temperature, latitude, longitude, elevation, distance from coasts and water bodies, vegetation, and albedo (sunlight reflexivity due to ice, vegetation and soil properties)[14]. Evidence of this temperature scaling is displayed in the following map from the IPCC report.



[14] This pattern scaling could be simplified with a simple linear equation as a first-order approximation $\Delta_i = 1.537 - 0.0288 \times \tau_{it_0}$. Moreover, this scaling could be made more realistic and time-varying using a non-linear function of temperature $\Delta_i \equiv \Delta(\tau_{it})$.

4 Competitive equilibrium and Business as usual

4.1 Household

To solve for the competitive equilibrium and the optimal decision of the Household, we use the Pontryagin Maximum Principle. The Hamiltonian of the individual country with individual states $\{s_i\}_i = \{z_i, p_i, \theta_i, \gamma_i, \Delta_i, \xi_i, w_i, \tau_i\}_i$ and aggregate states $S = \{\mathcal{T}, \mathcal{S}, \mathcal{R}\}$, writes as follow:

$$\mathcal{H}(\{s\}_i, S, \{c, k, e^f, e^r, \lambda^w, \lambda^\tau, \lambda^S\}_i) = u(c, \tau)p + \lambda^w \dot{w} + \lambda^\tau \dot{\tau} + \lambda^S \dot{S}$$

The equilibrium relations for the household consumption/saving problem boil down to the standard neoclassical model dynamics and for each country $i \in \mathbb{I}$, we obtain a system of coupled ODEs.

$$\begin{cases} \dot{\lambda}^w_{it} &= \lambda^w_{it}(\rho - r^\star_t) \qquad\qquad \lambda^w_{it} = \partial_c u(c_{it}, \tau_{it}) \\ \dot{w}_{it} &= r^\star_t w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e^f_{it}, e^r_{it}) + \theta^R_i \pi^R_t - (n + \bar{g} + \delta) k_{it} - q^f_t e^f_{it} - q^r_{it} e^r_{it} - c_{it} \end{cases}$$

where λ^w_{it} is the costate for the wealth w_{it} of country i, i.e. the marginal value of an additional unit of wealth optimal should be increasing if the world interest rate exceeds the discount factor ρ. Moreover, the capital and energy choices are simply resulting from static optimization between price/cost and marginal return of those inputs in the production. Moreover, the choice of capital is constrained by the borrowing constraint in each country $b_{it} > -\vartheta k_{it}$, creating potential misallocation over countries.

$$\begin{cases} q^f_t &= MPe^f_{it} \\ r^\star_{it} &= MPk_{it} \end{cases} \qquad \text{or} \qquad \begin{aligned} q^r_{it} &= MPe^r_{it} \\ k_{it} &= \tfrac{1}{1-\vartheta} w_{it} \end{aligned}$$

where $MPx_{it} = \partial_x \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e^f_{it}, e^r_{it})$ for $x \in \{k, e^f, e^r\}$. Moreover, using the law of motion and the definition of the marginal value of wealth, we obtain the Euler equation:

$$\frac{\dot{c}_{it}}{c_{it}} = \frac{1}{\eta}\left(r^\star_t - \rho\right) + \gamma_i(\tau_{it} - \tau^\star_i)\dot{\tau}_{it}$$

The dynamics of local temperature appear in the Euler equation. Indeed, because the marginal utility of consumption is affected directly by changes in temperature, an increase in temperature in the future triggers substitution from present to future consumption through saving.

Moreover, the bonds are in zero net supply, and hence the aggregate wealth should equal the aggregate capital stock

$$\int_{i \in \mathbb{I}} w_{it} p_{it} di = \int_{i \in \mathbb{I}} k_{it} p_{it} di$$

4.2 Fossil energy market

The dynamics of Hoteling rents λ_t^R for the fossil energy price q_t^f are described above and listed here for completeness:

$$\begin{cases} q_t^f &= \nu_E(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad\qquad\qquad\qquad \delta^R \lambda_t^R = \mu_E(\mathcal{I}_t^{e\,\star}, \mathcal{R}_t) \\ \dot{\lambda}_t^R &= \rho \lambda_t^R + \partial_R \nu(E_t^f, \mathcal{R}_t) + \partial_R \mu(\mathcal{I}_t^\star, \mathcal{R}_t) \\ \dot{\mathcal{R}}_t &= -E_t^f + \delta^R \mathcal{I}_t^f \end{cases}$$

where the optimal extraction and exploration depend on the dynamic Hoteling rent λ_t^R that varies with stock effects due to depleting reserves $\mathcal{R}_t$.

Moreover, the energy market clears between the demand of individual countries and supply from the fossil energy firm:

$$\int_{i \in \mathbb{I}} e^{(n+\bar{g})t} e_{it}^f p_{it} di = E_t^f$$

4.3 Local cost of carbon and Climate system

In addition, the climate block for carbon stock $\mathcal{S}_t$ and temperature τ_{it} are valued with the costates λ_{it}^S and λ_{it}^τ, representing respectively the marginal value of adding an additional unit of carbon in the atmosphere $\mathcal{S}_t$ and the marginal value of increasing local temperature by an additional degree. Recalling the dynamics of the climate system,

$$\begin{cases} \mathcal{E}_t &= \int_{i \in \mathbb{I}} \epsilon_{it} di = \int_{\mathbb{I}} e^{(n+\bar{g})t} \xi_i e_{it}^f p_i di \\ \dot{\mathcal{S}}_t &= \mathcal{E}_t - \delta^s \mathcal{S}_t \\ \dot{\tau}_{it} &= \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})\big) \end{cases}$$

we can use the Pontryagin principle to pin down the dynamics of the local cost of carbon. First, the shadow value of increasing temperatures is affected by the cost of climate on both the productivity effect $\mathcal{D}^y(\tau) z f(k, e)$ and the utility effect $u(\mathcal{D}^u(\tau) c)$.

$$\dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho + \zeta) + \underbrace{\gamma_i^y(\tau_{it} - \tau_i^\star)\mathcal{D}^y(\tau_{it})}_{-\partial_\tau \mathcal{D}^y} f(k_{it}, e_{it}) \lambda_{it}^w + \underbrace{\gamma_i^u(\tau_{it} - \tau_i^\star)\mathcal{D}^u(\tau_{it})}_{-\partial_\tau \mathcal{D}^u} u'(\mathcal{D}^u(\tau) c_{it}) c_{it}$$

Indeed, this shadow value increases with marginal damages, scaled by both marginal utility of wealth λ_{it}^w and consumption $u'(\mathcal{D}^u(\tau) c_{it})$. This change in the marginal value of temperature affects directly the shadow value of adding carbon in the atmosphere according to the dynamics of λ_{it}^S:

$$\dot{\lambda}_{it}^S = \lambda_{it}^S(\rho + \delta^s) - \zeta\, \chi\, \Delta_i\, \lambda_{it}^\tau$$

We hence see why adding an extra unit of carbon in the atmosphere has a differential impact of different regions due to heterogeneous costs of temperature and vulnerability to climate synthesized by the pattern scaling parameters Δ_i and marginal damages γ_i^y and γ_i^u.

The Local Cost of carbon is a common measure used by climate scientists and climate economists to summarize the marginal welfare cost of carbon in monetary terms. The Cost of Carbon is an equilibrium concept, in the sense that it depends on the trajectories of temperatures but also on production and consumption. In the competitive equilibrium, the climate externality of fossil fuel use is not internalized and households do not take climate damage into account for choosing consumption, production, and energy decisions. A typical microfoundation of such an assumption is to consider infinitesimal agents and regions, such that $\partial_{e_i^f} \mathcal{E}_t = 0$. However, one doesn't need such an assumption to analyze the cost of externality, especially when looking at large countries with the United States, China or India that have large carbon footprints.

Moreover, with or without infinitesimal agents, it doesn't prevent the households to be rational and to anticipate perfectly the evolution of climate in the region. The Local Cost of Carbon (LCC) represents such a welfare measure that is normalized into monetary units according to the marginal utility of wealth/consumption in the region, as indeed the monetary value of one unit of welfare is different across regions due to inequality in consumption $\frac{\partial v_{it}}{\partial c_{it}} = \lambda_{it}^w = u_c(c_{it}) \neq u_c(c_{jt}) = \lambda_{jt}^w$.

In continuous time, and using our framework of the Pontryagin Maximum Principle, this local cost of carbon rewrite easily as the ratio of the two costates:

$$LCC_{it} := \frac{\frac{\partial v_{it}}{\partial \mathcal{S}_t}}{\frac{\partial v_{it}}{\partial c_{it}}} = -\frac{\lambda_{it}^S}{\lambda_{it}^w}$$

In the competitive equilibrium, this measure integrates the cost of climate on locality i even in any suboptimal policy. Note that is *not* the social cost of carbon (SCC) as the SCC would integrate spillovers of each country on the rest of the world and a potentially optimal path of consumption. However, this notion is exactly analogous to the Local Cost of Carbon concept developed in Cruz Álvarez and Rossi-Hansberg (2022).

As a result, following the dynamics of the LCC amounts to solve for the dynamics of both costates λ_{it}^w and λ_{it}^S.

4.4 General Equilibrium

A complete description of the system can be found in appendix D. In this framework, there are types of interaction mechanisms between the different countries $i \in \mathbb{I}$.

First, the emissions from each country affect the global climate and local temperatures, creating these heterogeneous impacts and costs of climate change λ. Second, fossil energy markets clear such that the energy demand from all the individual countries impact the fossil fuel price q_t^f and has redistributive effects on the fossil energy rent $\pi_t(E^f, \mathcal{R})$. Third, the bonds market also clears as assets are in zero net supply, and individual savings and consumptions depend on the path of world interest rate as well as collateral constraints. However, there are no bilateral flows between individual countries, such as migration or bilateral trade and capital flow.

This makes this system of ordinary differential equations (ODEs) the specificity of being strongly coupled. Despite the infinite dimensionality of this system, this problem is well-posed, as it is the solution of Forward Backward McKean Vlasov system of ordinary differential equations. Despite the possibility many global interactions, i.e. each country interacts with global variables affected by the entire distribution of agents – atmospheric temperature $\mathcal{T}_t$, fossil energy price q_t^f, world interest rate $r_t^\star$ – one can not add bilateral flow. Allowing bilateral/local interaction may make the problem ill-posed, as explained in Boucekkine, Camacho and Zou (2009) and in the sense that there is no existence of solutions to the problem. We hence assume solely global interactions in the scope of this paper. The definition of competitive equilibrium is as follows:

Definition 4.1. *Given, ex-ante heterogeneity $\{z_i, p_i, \theta_i, \gamma_i, \Delta_i, \xi_i\}$ and initial conditions $\{w_{it_0}, \tau_{it_0}\}$ and $\{\mathcal{S}_{t_0}, \mathcal{T}_{t_0}, \mathcal{R}_{t_0}\}$a competitive equilibrium is a continuum of sequences of states $\{w_{it}, \tau_{it}\}_{it}$ and $\{\mathcal{S}_t, \mathcal{T}_t, \mathcal{R}_t\}_t$, policies $\{c_{it}, k_{it}, e^f_{it}, e^r_{it}\}_{it}$ and $\{E^f_t, \mathcal{E}_t, \mathcal{I}_t\}_t$, and price sequences $\{q^f_t, q^r_t, r^\star_t\}$ such that:*

- *Individual households choose policies $\{c_{it}, k_{it}, e^f_{it}, e^r_{it}\}_{it}$ to maximize their utility subject to budget and credit constraints*
- *Individual renewable energy firm produce $\{e^r_{it}\}$ to maximize static profits*
- *The global fossil fuel firm extracts and explores $\{E^f_t, \mathcal{I}_t\}$ to maximize profit*
- *Emissions $\mathcal{E}_t$ affect climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{it}\}_{it}$ following the climate system dynamics.*
- *Prices $\{q^f_t, q^r_{it}, r^\star_t\}$ adjust to clear the markets for fossil and renewable energy and bonds,*

$$E^f_t = \int_{\mathbb{I}} e^{(\bar{g}+n)t} e^f_{it} p_i di \qquad e^r_{it} = z^r_i f(k^r_{it}) \qquad \int_{i\in\mathbb{I}} b_{it} p_i di = 0$$

while the last good market clear by Walras law

$$\int_{\mathbb{I}} c_{it} p_i di + \nu(E^f_t, \mathcal{R}_t) + \mu(\mathcal{I}^f_t, \mathcal{R}_t) + \int_{\mathbb{I}} [\dot{k}_{it} + (n + \bar{g} + \delta) k_{it}] p_i di = \int_{\mathbb{I}} \mathcal{D}^y_i(\tau_{it}) z_i f(k_{it}, e_{it})\, p_i\, di$$

This Business as Usual scenario features unrestricted use of fossil energy until its price increase when resources are depleted. In particular, temperature increase to high levels, and climate damages are large. We will analyze the result in the quantitative section below. We now turn to the optimal policy to take into account the climate externalities.

5 Optimal policy and first-best climate policy

We consider the optimal policy of a social planner that maximizes the weighted sum of the Household utility, where the Pareto weights ω_i are arbitrary[15], and subject to the resource constraints of the economy.

[15]The only constraint we impose is that they integrate to one $\int_{\mathbb{I}} \omega_i di = 1$

By choosing all the agent decisions, consumption c_{it}, energy e^f_{it} and e^r_{it}, it would internalize the climate externality due to emissions $\mathcal{E}_t$ and increase in temperature τ_{it}. We denote by $\mathcal{V}_t$ the aggregate welfare in this social planner equilibrium.

$$\mathcal{V}_{t_0} = \max_{\{c,e^f,e^r,k,E^f,\mathcal{E},\mathcal{I}\}} \int_{t_0}^{\infty} \int_{\mathbb{I}} e^{-(\widetilde{\rho}+n)t}\ \omega_i\ u\big(\mathcal{D}^u(\tau_{it})\, c_{it}\big) p_i\ di\ dt$$

subject to the resources constraints of the economy and the energy and climate system:

$$\int_{\mathbb{I}} c_{it} p_i di + \nu(E^f_t, \mathcal{R}_t) + \mu(\mathcal{I}^f_t, \mathcal{R}_t) + \int_{\mathbb{I}} [\dot{k}_{it} + (n+\bar{g}+\delta)k_{it}] p_i di = \int_{\mathbb{I}} \mathcal{D}^y_i(\tau_{it}) z_i f(k_{it}, e_{it})\, p_i\, di \qquad [\widehat{\lambda}_t]$$

$$E^f_t = \int_{\mathbb{I}} e^{(\bar{g}+n)t} e^f_{it} p_i di \qquad e^r_{it} = z^r_i f(k^r_{it}) \qquad [\widehat{\lambda}^r_{it}]$$

$$\mathcal{E}_t = \int_{\mathbb{I}} e^{(n+\bar{g})t} \xi_i e^f_{it} p_i di \qquad \dot{\mathcal{S}}_t = \mathcal{E}_t - \delta^s \mathcal{S}_t \qquad [\widehat{\lambda}^S_t]$$

$$\dot{\tau}_{it} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})\big) \qquad [\widehat{\lambda}^\tau_{it}]$$

$$\dot{\mathcal{R}}_t = -E^f_t + \delta^R \mathcal{I}^f_t \qquad [\widehat{\lambda}^R_t]$$

The Social planner choses, consumption/saving c_{it}, energy mix e^f_{it}, extraction E^f_t and exploration $\mathcal{I}^f_t$, as well as the trajectories of dynamic states $(k, \tau, \mathcal{S}, \mathcal{R})$. Note that the planner has discount factor $\widetilde{\rho}$ which might be different than the agent discount parameter ρ, and notably smaller, if we believe the planner could be more patient. Moreover, we denote the Lagrange multiplier of the Social Planner allocation by $\widehat{\lambda}$'s. We observe now that the market clearing for goods has a common shadow value $\widehat{\lambda}_t$ for all locations $i \in \mathbb{I}$ at the difference to the competitive equilibrium.

The result is analogous to the toy model example. The choice of consumption solves for redistribution motive, as the planner searches for equalizing marginal utility, subject to the Pareto weights:

$$\omega_i \partial_c u(c_{it}, \tau_{it}) = \widehat{\lambda}_t = \omega_j \partial_c u(c_{jt}, \tau_{jt})$$

with marginal utility $\partial_c u(c_{it}, \tau_{it}) = \mathcal{D}^u(\tau_{it}) u'(\mathcal{D}^u(\tau_{it})c) = \mathcal{D}^u(\tau_{it})^{1-\eta} c_{it}^{-\eta}$, with the CRRA functional form. Despite the possibility, in the competitive equilibrium, to trade in goods, bonds, and energy, strong inequality exists due to differences in productivity, energy rents or climate damage. As a result, the social planner, would like to redistribute consumption and this would be done using lump-cum transfers in the decentralized equilibrium.

The fossil energy choice is similar to the toy model since the marginal utility of consumption are equalized to $\widehat{\lambda}$ across countries.

$$MPe^f_{it}\, \widehat{\lambda}_t = \partial_E \nu(E^f_t, \mathcal{R}_t)\, \widehat{\lambda}_t\ +\ \widehat{\lambda}^R_t\ - \xi_i\, \widehat{\lambda}^S_t$$

We see that the planner equalizes the marginal product of fossil energy $MPe^f_{it} = \partial_{e^f} \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e^f_{it}, e^r_{it})$ to its shadow cost. This marginal cost is the sum of different channels: first, the marginal extraction cost $\nu(\cdot)$, second, the social Hoteling rent $\widehat{\lambda}^R_t / \widehat{\lambda}_t$ and third integrates the climate damage $\widehat{\lambda}^S_t / \widehat{\lambda}_t$ as we will see in the next section.

The conditions for the choice of renewable energy and capital are standard in the neoclassical model:

$$\widehat{\lambda}^r_{it}/\widehat{\lambda}_t = MPe^r_{it}$$

$$\dot{\widehat{\lambda}}_t = (\widehat{r}_t - \widehat{\rho})\widehat{\lambda}_t \qquad \widehat{r}_t = MPk_{it}$$

where $\widehat{r}_t$ is the shadow price of capital which is equalized across countries. Moreover, the capital choice is not constrained by borrowing limits, because goods can be allocated and transferred freely between regions and time periods.

5.1 Social Cost of Carbon

In this optimal allocation, the marginal cost of adding one unit of carbon in the atmosphere $\mathcal{S}_t$ can be summarized by the Social Cost of Carbon:

$$\overline{SCC}_t := -\frac{\frac{\partial \mathcal{V}_t}{\partial \mathcal{S}_t}}{\frac{\partial \mathcal{V}_t}{\partial \hat{c}_{it}}} = -\frac{\widehat{\lambda}^S_t}{\widehat{\lambda}_t}$$

We see that since the marginal utility of consumption/ marginal value of wealth is equalized across countries $\partial \mathcal{V}_t/\partial \hat{c}_{it} = \omega_i u_c(c_{it}, \tau_{it}) = \widehat{\lambda}_t$, the normalization of the welfare cost $\widehat{\lambda}^S_t$ into monetary unit is not ambiguous, and doesn't depend on the country one chooses. The welfare cost of carbon evolve again with the marginal damage of temperature:

$$\dot{\widehat{\lambda}}^\tau_{it} = \widehat{\lambda}^\tau_{it}(\rho + \zeta) + \underbrace{\gamma^y_i(\tau_{it} - \tau^\star_i)\mathcal{D}^y(\tau_{it})}_{-\partial_\tau \mathcal{D}^y} f(k_{it}, e_{it})\lambda_t p_i + \underbrace{\gamma^u_i(\tau_{it} - \tau^\star_i)\mathcal{D}^u(\tau_{it})}_{-\partial_\tau \mathcal{D}^u} u'(\mathcal{D}^u(\tau)c_{it})c_{it}p_i$$

Again, the shadow value increases with marginal damages, scaled by the common marginal value of wealth $\widehat{\lambda}_{it}$ and consumption $u'(\mathcal{D}^u(\tau)c_{it})$. The marginal value of temperature again affects the shadow value of carbon, but this time in an aggregate fashion, where all of the costs for all countries $\lambda^\tau_{it}, \forall i \in \mathbb{I}$:

$$\dot{\widehat{\lambda}}^S_{it} = \widehat{\lambda}^S_{it}(\rho + \delta^s) - \zeta\,\chi \int_{\mathbb{I}} \Delta_i\,\widehat{\lambda}^\tau_{it} di$$

Given the dynamics of this welfare cost of carbon, the fossil energy choice boils down

$$MPe^f_{it} = \partial_E \nu(E^f_t, \mathcal{R}_t) + \frac{\widehat{\lambda}^R_t}{\widehat{\lambda}_t} \underbrace{-\xi_i \frac{\widehat{\lambda}^S_t}{\widehat{\lambda}_t}}_{=\xi_i SCC_t}$$

with the conversion parameter "energy to carbon" ξ_i for each country.

From this optimality condition, we recover the standard Representative agent's result that Pigouvian Taxation should equal the marginal damage from the externality, exactly as in the result of Golosov et al. (2014). In particular, the carbon tax is equal across countries. To see that taxation result, let us analyze the decentralization of such allocation in the competitive equilibrium.

5.2 Decentralization, taxation, and transfers

We recall the budget constraint of each agent and augment it with two tax instruments that will be necessary for the planner to decentralize the optimal allocation: first, a fossil fuel tax $\mathbf{t}^f_{it}$ is used to account for the climate externality, second, lump-sum transfers are used to tax or transfers lump-sum to each country.

$$\dot{w}_{it} = r^\star_t w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e^f_{it}, e^r_{it}) + \theta^R_i \pi^R_t - (n + \bar{g} + \delta) k_{it} - (q^f_t + \mathbf{t}^f_{it}) e^f_{it} - q^r_{it} e^r_{it} - c_{it} + T_{it}$$

First, turning to the energy tax, we see how the planner's first-order condition can be decentralized:

$$MPe^f_{it} = \partial_E \nu(E^f_t, \mathcal{R}_t) + \frac{\widehat{\lambda}^R_t}{\widehat{\lambda}_t} - \xi_i \underbrace{\frac{\widehat{\lambda}^S_t}{\widehat{\lambda}_t}}_{=-\overline{SCC}_t}$$

$$MPe^f_{it} = q^f_t + \xi_i \mathbf{t}^S_t \qquad\qquad \mathbf{t}^S_t = \overline{SCC}_t$$

In particular, the carbon tax is equal across countries, thanks to the adjacent equalization of marginal utility of consumption / marginal value of wealth. To achieve such equalization in the decentralization, the planner needs to use lump-sum transfers:

$$\omega_i \partial_c u(c_{it}, \tau_{it}) = \widehat{\lambda}_t = \omega_j \partial_c u(c_{jt}, \tau_{jt}) \Rightarrow \qquad\qquad c_{it} = u_c^{-1}(\widehat{\lambda}_t | \tau_{it})$$

and, using the budget constraint above, one obtains such consumption levels using lump-sum transfers:

$$c_{it} = r^\star_t w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e^f_{it}, e^r_{it}) + \theta^R_i \pi^R_t - (n + \bar{g} + \delta) k_{it} - (q^f_t + \xi_i \mathbf{t}^S_t) e^f_{it} - q^r_{it} e^r_{it} - \dot{w}_{it} + T_{it}$$

In particular, lump-sum transfers (per efficient unit of population) allow redistributing across countries and across time:

$$\int_{t_0}^{\infty} e^{(n+\bar{g})t} \int_{\mathbb{I}} T_{it} p_i \, di \, dt = 0$$

In particular, in situations where the technology difference z_i, rent distribution θ_i, vulnerability to climate γ_i or Pareto weights ω_i are very heterogeneous such that consumption differentials in the equilibrium without policy intervention are large, one can show that some countries would receive positive lump-sum transfers $\exists j, s.t.\ T_j > 0$ and some would have to pay lump-sum taxes $\exists j', s.t.\ T_{j'} < 0$. This implies that such decentralized allocation features direct lump-sum transfers across countries.

The question is whether such lump-sum transfers are feasible politically. Would a world central planner be able to solve world inequality by imposing lump-sum transfers, for example taxing North America and Europe and rebating it lump-sum to Africa or South Asia? The representative agent framework such as Golosov et al. (2014) or heterogeneous agent models with unrestricted redistribution such as Hillebrand and Hillebrand (2019) all assume the availability of such lump-sum transfers.

In the next section, we will analyze the policies where this family of policies is not feasible for political, governance, or economic reasons. Imposing such constraints prevents redistribution and equalization of marginal utilities across countries, and requires to solve for different kinds of optimal policy problems.

6 Ramsey problem and optimal energy policy

We again consider the optimal policy of a social planner that maximizes the weighted sum of the Household utility, now subject to the optimality conditions of the agents. In this context, it would not only internalize all the dynamics of economic variables, the climate, and energy markets but also the decisions that households and firms take.

The Ramsey planner chooses consumption/saving c_{it}, energy mix e^f_{it} and e^r_{it}, the extraction and exploration E^f_t and $\mathcal{I}_t$ as well as the trajectories of dynamic states $(\tau, \mathcal{S}, \mathcal{R})$ indirectly:

$$\mathcal{W}_{t_0} = \max_{\{c, e^f, e^r, k, \mathcal{I}\}} \int_{t_0}^{\infty} \int_{\mathbb{I}} e^{-(\widetilde{\rho}+n)t} \; \omega_i \; u\big(\mathcal{D}(\tau_{it}) c_{it}\big) p_i \; di \; dt$$

subject to (i) the optimality conditions of households, for c_i, e^f_i, e^r_i and k_i, (ii) the optimality conditions of the Fossil fuel producers for E^f, $\mathcal{I}$ and $\mathcal{R}$ and (iii) the Climate and temperature dynamics τ_i and $\mathcal{S}$. The discount factor of the planner $\widetilde{\rho}$ could again be different than the one of the agents ρ. We apply the Pontryagin Maximum Principle in infinite dimension and the resulting system of McKean Vlasov differential equations is very large – the details of the entire system can be found in appendix appendix E. The Lagrange multipliers corresponding to states dynamics equations are denoted ψ's and the ones corresponding to agents' optimality conditions are named with ϕ's. Note that the social planner has full commitment, in the sense that decisions taken in the initial period t_0 are binding until the end of times and there is no time inconsistency.

We provide some intuitions of the most important results and those that connect with the rest of the literature.

First, the optimality for consumption yields the marginal value of wealth ψ^w_{it}. This multiplier informs on the value of consumption in country i and measures directly the extent of inequality across countries. This is directly related to the marginal utility of consumption and the distortion of the saving decisions:

$$[c_{it}] \qquad\qquad \psi^w_{it} = \underbrace{\omega_i \partial_c u(c_i, \tau_{it})}_{=\text{direct effect}} + \underbrace{\phi^c_{it} \partial_{cc} u(c_i, \tau_i)}_{=\text{effect on savings}}$$

This expression for the social shadow value of wealth is analogous to the "marginal value of liquidity" in heterogenous agents analysis like Le Grand, Martin-Baillon and Ragot (2021) and Dávila and Schaab (2023). Unlike the previous analysis in section 5, there is inequality in consumption, and the planner can not equalize marginal utilities:

$$\omega_i \partial_c u(c_{it}, \tau_{it}) \neq \omega_j \partial_c u(c_{jt}, \tau_{jt})$$
$$\psi^w_{it} \neq \psi^w_{jt}$$

Moreover, one can show, c.f. appendix E, that if the discount factor of planner and agents coincide $\rho = \widetilde{\rho}$ and the credit constraint is not binding, with $\vartheta \to 1$, then the consumption/saving decisions of the planner and the household coincide and we obtain that ϕ^c_{it} and there is no time-varying difference between household's marginal value of wealth $\lambda^w_{it} = u_c(c,\tau)$ and social planner marginal value of wealth $\psi^w_{it} = \omega_i \partial_c u(c,\tau)$, except for the Pareto weight that is a normalization constant. As a result, the agent and the planner would take the same consumption/saving decisions along the transition period.

That shadow value for wealth ψ^w_{it} – whatever our assumptions on discounting or borrowing constraints – allows us to build a measure of inequality, by comparing the individual value with the average value:

$$\widehat{\psi}^w_{it} = \frac{\psi^w_{it}}{\overline{\psi}^w_t} \lesseqgtr 1 \qquad \text{with} \qquad \overline{\psi}^w_t = \int_{\mathbb{I}} \psi^w_{jt} dj$$

If the ratio is higher than 1, we can argue that the country is relatively poorer, with a lower welfare than the average household.

Second, let us turn toward the social value of fossil energy supply, called *SVF*. We denote it by $\widehat{\phi}^{Ef}_t$, and it represent the global value of changing marginally the global fossil market, by manipulating supply, price and profits:

$$\begin{aligned}
\phi^{Ef}_t &= \int_{\mathbb{I}} e^f_{jt} \psi^w_{jt} p_j dj - \partial_q \pi^f(\cdot) \int_{\mathbb{I}} \theta_j \psi^w_{jt} p_j dj \\
SVF &= \frac{\phi^{Ef}_t}{\overline{\psi}^w_t} = \int_{\mathbb{I}} e^f_{jt} \widehat{\psi}^w_{jt} p_j dj - \partial_q \pi^f(\cdot) \int_{\mathbb{I}} \theta_j \widehat{\psi}^w_{jt} p_j dj \\
SVF &= \mathbb{C}\mathrm{ov}^{\mathbb{I}}\big(e^f_{jt}, \widehat{\psi}^w_{jt}\big) - E^f_t \mathbb{C}\mathrm{ov}^{\mathbb{I}}\big(\theta_j, \widehat{\psi}^w_{jt}\big)
\end{aligned}$$

where we used that the product decomposition $\int_{\mathbb{I}} x_{it} y_{it} di = \mathbb{C}\mathrm{ov}^{\mathbb{I}}(x_{it}, y_{it}) + \mathbb{E}^{\mathbb{I}}[x_{it}]\mathbb{E}^{\mathbb{I}}[y_{it}]$ and the facts that $\partial_q \pi^f = E^f_t$, $\mathbb{E}^{\mathbb{I}}[\theta_i] = 1$ and $\mathbb{E}^{\mathbb{I}}[\widehat{\psi}^w_{it}] = 1$ by definition

This Social Value of Fossil (SVF) accounts for the two different redistributive effects of increasing energy price: (i) increasing the price hurts importers that are using a large amount of fossil energy per capita. That terms in high if the importer have a large marginal value of wealth $\widehat{\psi}^w_{it}$, i.e. a low value of consumption c_{it} or high temperature τ_{it} since $\widehat{\psi}^w_{it} \propto u_c(c_{it}, \tau_{it})$. This term is reminiscent of the cost redistribution term in the toy model. Empirically, $\mathbb{C}\mathrm{ov}^{\mathbb{I}}\big(e^f_{jt}, \widehat{\psi}^w_{jt}\big)$ tends to be small, if not negative, since the richer countries are using large quantities of fossil fuels per capita – and not that here the relative share in the energy mix $e^f/(e^f + e^r)$ does not matter, only the absolute quantity used. In the quantitative section, we explore a measure of such covariance.

Moreover, the energy firm is making profits from the sales of fossil fuels that need to be redistributed. As a result, (ii) increasing fossil price benefits fossil energy exporters, who own a large share of the reserves and earn a large share of the profits. However, this covariance term is smaller if earning a large share of fossil correlates with a low marginal value of wealth i.e. high consumption. As a result, empirically, $\mathbb{C}\text{ov}^{\mathbb{I}}\big(\theta_{jt}, \widehat{\psi}^w_{jt}\big)$ tends to be strongly negative as the country rich in fossil and reserve have a large energy rent in GDP and are thus richer and have lower marginal utility of consumption.

As a result, the social value of fossil supply SVF is slightly positive we will see in the next section what it entails for energy taxation.

Third, the costate for climate and marginal value of carbon is the last important planner costate that matters for the Ramsey energy policy ψ^S_t and we will study it in a separate section.

6.1 Second Best – Social cost(s) of carbon

In this model, the social cost of carbon writes very simply:

$$SCC_t := -\frac{\frac{\partial \mathcal{W}_t}{\partial \mathcal{S}_t}}{\frac{\partial \mathcal{W}_t}{\partial c_t}} = -\frac{\psi^S_t}{\overline{\psi}^w_t}$$

The costate for the stock of carbon $\mathcal{S}_t$ measures the social shadow value of an additional ton of GHG emitted in the atmosphere. To convert this welfare measure into monetary units, one should renormalize it using the marginal value of wealth or capital $\partial \mathcal{W}_t / \partial c_t \equiv \partial \mathcal{W}_t / \partial k_t$. As the cost of climate is a global measure, the standard naive intuition from the "representative agent" framework is to use the average marginal value $\overline{\psi}^w_t$. This allows us to consider an average SCC, but we will see that redistribution terms need to be accounted for in the optimal taxation results.

To measure the welfare cost of climate damage, one can follow the dynamics of ψ^S_t along the trajectories of climate and aggregate temperatures. Applying the Pontryagin Max Principle in this Ramsey problem – or using integration by part as in the proof of the PMP – we can follow this costate for carbon $\mathcal{S}$ that depends on the costate for local temperatures.

$$\begin{aligned}
\dot{\psi}^{\tau}_{it} = \psi^{\tau}_{it}(\widetilde{\rho} + \zeta) + \underbrace{\gamma^y_i(\tau_{it} - \tau^\star_i)\mathcal{D}^y(\tau_{it})}_{-\partial_\tau \mathcal{D}^y} f(k_{it}, e_{it})\psi^w_{it} + \underbrace{\gamma^u_i(\tau_{it} - \tau^\star_i)\mathcal{D}^u(\tau_{it})}_{\partial_\tau \mathcal{D}^u} u'(\mathcal{D}^u c_{it})c_{it} \\
+ \gamma^y_i(\tau_{it} - \tau^\star_i)\mathcal{D}^y(\tau_{it}) z_i \big[\phi^k_{it}\partial_k f(k_{it}, e_{it}) + \phi^e_{it}\partial_e f(k_{it}, e_{it})\big] \\
+ \gamma^u_i(\tau_{it} - \tau^\star_i)\mathcal{D}^u(\tau_{it}) u'(\mathcal{D}^u c)(1-\gamma)\phi^c_{it}
\end{aligned}$$

The marginal value for country i of being subject to an increase in local temperature is measured by ψ^{τ}_t. It increases with different terms: the temperature gap $\tau_{it} - \tau^\star_i$, due to the convexity of the damage function, the damage sensitivity to temperature for TFP γ^y_i and utility/mortality γ^u_i. Moreover, in contrast to the costate in the competitive equilibrium and in the first-best allocations, the Ramsey planner needs to take into account the optimal decisions of agents, and how changes in temperature distort the first-order conditions of the optimizing agents. These terms depends how "binding" the optimality conditions are, and scale with the multipliers ϕ_{it}^k, ϕ_{it}^e and ϕ_{it}^c respectively for the capital, energy, and consumption choices. If the Ramsey planner would make the same decisions as the agents – for example if they have the same preference and there aren't any market frictions, then some of these Lagrange are null and we recover the same Social Cost of Carbon as in the second best allocation.

Furthermore, as before, the marginal cost for country i of releasing carbon in atmosphere ψ_t^S is directly and globally affected by the marginal value of temperatures:

$$\dot{\psi}_t^S = \psi_t^S(\widetilde{\rho} + \delta^s) - \zeta\,\chi \int_{\mathbb{I}} \Delta_i\, \psi_{it}^{\tau}\, di$$

through the climate parameters: ζ the climate inverse persistence (e.g. lags), χ the climate sensitivity and Δ_i the catching up effect" of temperature at the cold location.

Moreover, the marginal damage affects all the countries locally and symmetrically through a value ψ_{it}^{τ}. These gain/costs are cumulated additively, as we see from the previous ODE for ψ_t^S, and we can perform this (exact) decomposition:

$$\psi_t^S = \int_{\mathbb{I}} \psi_{it}^S di \qquad\qquad \dot{\psi}_{it}^S = \psi_{it}^S(\widetilde{\rho} + \delta^s) - \zeta\,\chi\,\Delta_i\,\psi_{it}^{\tau}$$

where the local costate ψ_{it}^S follow an analogous ODE for each location $i \in \mathbb{I}$.

More particularly, the Social Cost of Carbon can hence be reexpressed as a weighted sum of this local measure that we denote Local Social Cost of carbon $LSCC_{it}$. This cost is local as it takes into account the individual damages in location $i \in \mathbb{I}$, and it is normalized in monetary unit ψ_{it}^w, which is the marginal value of wealth/income in location i. However, it is also social because the Ramsey planner is choosing the optimal energy, emissions, and temperature paths internalizing the global damages across countries.

$$\begin{aligned}
LSCC_{it} &= -\frac{\psi_{it}^S}{\psi_{it}^w} \\
SCC_t &= -\frac{\psi_t^S}{\overline{\psi}_t^w} = -\int_{\mathbb{I}} \overbrace{\frac{\psi_{it}^w}{\overline{\psi}_t^w}}^{=\substack{\text{inequality}\\ \text{measure}}} \quad \overbrace{\frac{\psi_{it}^S}{\psi_{it}^w}}^{=-LSCC_{it}} \quad di \\
SCC_t &= -\int_{\mathbb{I}} \widehat{\psi}_{it}^w \;\; LSCC_{it} di
\end{aligned}$$

As a result, we can express the social cost of carbon emissions as:

$$\begin{aligned}
SCC_t &= -\int_{\mathbb{I}} \widehat{\psi}_{it}^w \;\; LSCC_{it} di \\
&= \mathbb{E}^{\mathbb{I}}\big[LSCC_{it}\big] + \mathbb{C}\mathrm{ov}^{\mathbb{I}}\Big(\widehat{\psi}_{it}^w, LSCC_{it}\Big) \qquad > \mathbb{E}^{\mathbb{I}}\big[LSCC_{it}\big] =: \overline{SCC}_t
\end{aligned}$$

where the last inequality follows the empirical observation that marginal damage – i.e. high local temperature τ_{it} – tends to be negatively correlated with development levels y_i, i.e. lower production, consumption and hence a higher marginal utility of consumption.

To conclude, the presence of heterogeneity and the correlation between local damage and poverty increases the Social Cost of Carbon from the Social Planner perspective. In the following section, we summarize the different concepts of social cost of carbon and we solve closed-form for the SCC in the long-run in appendix F.

6.2 Optimal energy policy and decentralization

In this section, we uncover our main result that derives the optimal policy for energy. We derive the optimality conditions for the Ramsey planner, in particular for fossil energy e^f_{it} and the other equilibrium relations are detailed in appendix E. We will see that it integrates the different redistribution motives that we detailed above. However, the Ramsey planner, by internalizing these externalities, would like to distort agents' optimality condition that we denote, in the general fashion with the aggregators $\mathscr{Q}(\widetilde{q}^f, \widetilde{q}^r)$, with $\widetilde{q}^f = q^f + \mathbf{t}^f$ and the production function $\bar{f}_{it} = \mathcal{D}^y_i(\tau_{it}) z_i f(k_{it}, e_{it})$

$$\begin{aligned} MPe_{it} = \bar{f}_{e,it} = \mathcal{D}^y_i(\tau_{it}) z_i \partial_e f(k_{it}, e_{it}) = \mathscr{Q}(q^f_t + \mathbf{t}^f_{it}, q^r_t + \mathbf{t}^r_{it}) \\ e^f_{it} = \mathscr{Q}_{q^f}(\widetilde{q}^f_{it}, \widetilde{q}^r_{it}) e_{it} \qquad\qquad e^r_{it} = \mathscr{Q}_{q^r}(\widetilde{q}^f_{it}, \widetilde{q}^r_{it}) \end{aligned}$$

Taking this optimality condition into account as well as the different sources of externality, the fossil energy choice the Ramsey planner writes as follows:

$$\Big(\frac{\mathscr{Q}^2_{q^f}}{\bar{f}_{ee,it}} + \mathscr{Q}_{q^f q^f}\Big)\big[\psi^w_{it} \underbrace{(MPe^f_{it} - q^f_t)}_{=\mathbf{t}^f_{it}} + \xi_i \psi^S_t p_i - \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot)\big] + \Big(\frac{\mathscr{Q}_{q^f}\mathscr{Q}_{q^r}}{\bar{f}_{ee,it}} + \mathscr{Q}_{q^r q^f}\Big)\big[\psi^w_{it} \underbrace{(MPe^r_{it} - q^r_t)}_{=\mathbf{t}^r_{it}} - \phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot)\big]$$

with $\bar{f}_{ee,it}$ the second-order derivative/curvature of the production function, $\mathscr{Q}_{q^f}$ is related to the fossil demand function $e^f_{it} = \mathscr{Q}_{q^f} e_{it}$, and $\mathscr{Q}_{q^f q^f}$ is related to the curvature of that demand, which is related to the cross elasticity of e^f and e^r in the CES framework $\mathscr{Q}_{q^f q^f}/\mathscr{Q}_{q^f} \propto \sigma^e$, and similarly for $\mathscr{Q}_{q^f q^r}$. Moreover, the cost function $\mathcal{C}^f(\cdot)$ for fuel extraction has curvature $\mathcal{C}^f_{EE}(\cdot)$ for non-linear extraction cost, which is zero in the constant return to scale (CRS) setting $\mathcal{C}^f_{EE}(\cdot) = 0$. Finally, we similarly have the curvature of the cost function $\mathcal{C}^r_{e^r e^r}(\cdot)$ for renewable energy. Earlier in section 3.2, we assumed that this cost function is null, because of constant return to scale $e^r = f^r(k^r) = k^r$, which implies $\mathcal{C}^r_{e^r e^r}(\cdot) = 0$.

Despite these numerous notations, a sufficient optimal policy for satisfying this condition is the following:

$$
\begin{aligned}
& \mathbf{t}_{it}^{f} = \frac{1}{\psi_{it}^{w}} \big[\phi_t^{Ef} \mathcal{C}_{EE}^{f}(E_t^f) - \xi_i \, \psi_t^S \, p_i \big] && \mathbf{t}_{it}^{r} = 0 \\
\Rightarrow \qquad & \mathbf{t}_{it}^{f} = \frac{1}{\widehat{\psi}_{it}^{w}} \big[SVF_t \; \mathcal{C}_{EE}^{f}(E_t^f) + \xi_i p_i SCC_t \big] && \mathbf{t}_{it}^{r} = 0 \\
& \text{with} \\
& SVF_i = \mathbb{C}\mathrm{ov}^{\mathbb{I}}\big(e_{jt}^{f}, \widehat{\psi}_{jt}^{w}\big) - E_t^f \mathbb{C}\mathrm{ov}^{\mathbb{I}}\big(\theta_j, \widehat{\psi}_{jt}^{w}\big) \\
& SCC_t = \mathbb{E}^{\mathbb{I}}\big[LSCC_{it}\big] + \mathbb{C}\mathrm{ov}^{\mathbb{I}}\Big(\widehat{\psi}_{it}^{w}\,, LSCC_{it}\Big)
\end{aligned}
$$

Where the SCC the social value of carbon, and $LSCC_{it}$ the local social cost of carbon studied in section 6.1 and SVF the social value of fossil fuel supply are detailed in section above. As in the Toy model of section 1, these terms account for both externality and redistribution effect.

In that formula, similarly to the Toy model of section 2, we see that the *level* of the carbon tax is common for all countries and accounts for (i) the climate externality, (ii) the fossil cost redistribution for importers and (iii) the fossil rent redistribution for exporters/reserve owners. As a result, even without climate externality $SCC_t = 0$, the fossil fuel tax $\mathbf{t}_{it}^{f}$ would not be zero. It accounts for the manipulation of terms of trade because of the wealthy exporters and the relatively poorer importers. Such a result holds as long as the fossil production is traded internationally in a market where agents have different marginal utilities of consumption, and if there are redistributive effects of rents and moving along the curvature of the supply curve. These motives would be absent in models like Golosov et al. (2014) where the production function is constant return to scale.

Moreover, and very importantly, not only the *level* but also the *distribution* of the fossil fuel/carbon tax is affected by redistribution motives. Indeed the ratio $1/\widehat{\psi}_{it}^{w}$ is the inverse of our inequality measure developed at the beginning of this section. It implies that richer/colder countries, which have higher consumption and lower marginal utilities will be charged a higher carbon tax, and conversely for poorer countries that will be charged a lower tax:

$$
\text{low} \;\; c_{it} \;\; \text{high} \;\; \tau_{it} \qquad \Rightarrow \qquad \text{high} \;\; \widehat{\psi}_{it}^{w} \;\propto\; \partial_c u(c_{it}, \tau_{it}) \qquad \Rightarrow \qquad \text{low} \;\; \mathbf{t}_{it}^{f}
$$

everything else being constant, in particular SCC_t and SVF.

6.3 Suboptimal policies

We saw that in the previous section, we uncovered the optimal policy by solving the equation:

$$
\Big(\frac{\mathscr{Q}_{q^f}^{2}}{\bar{f}_{ee,it}} + \mathscr{Q}_{q^f q^f}\Big)\big[\widehat{\psi}_{it}^{w}\mathbf{t}_{it}^{f} - \xi_i p_i SSC_t - SVF_t\, \mathcal{C}_{EE}^{f}(\cdot)\big] + \Big(\frac{\mathscr{Q}_{q^f}\mathscr{Q}_{q^r}}{\bar{f}_{ee,it}} + \mathscr{Q}_{q^r q^f}\Big)\,\big[\widehat{\psi}_{it}^{w}\mathbf{t}_{it}^{r} - \widehat{\psi}_{it}^{w}\, LSVR_t\, \mathcal{C}_{e^r e^r}^{r}(\cdot)\big]
$$

where the local social value of renewable supply $LSVR_t = \phi_{it}^{Er}/\psi_{it}^{w}$ which is the welfare gain in country i of increasing the supply of renewable energy, again renormalized in monetary unit based in the same country $i \in \mathbb{I}$.

As a result, we see that the policy developed in section 6.2 is sufficient, but not necessary. In particular, one can build policies that feature lower carbon taxes and higher renewable subsidy. As we see in appendix E, the size of the family of necessary optimal policies $(\mathbf{t}^f_{it}, \mathbf{t}^r_{it})$ depends on the invertibility of the following matrix:

$$M^{\mathbf{t}} = \begin{bmatrix} \left(\frac{\mathscr{Q}^2_{q^f}}{f_{ee,it}} + \mathscr{Q}_{q^f q^f}\right) & \left(\frac{\mathscr{Q}_{q^f}\mathscr{Q}_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^r q^f}\right) \\ \left(\frac{\mathscr{Q}_{q^f}\mathscr{Q}_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^f q^r}\right) & \left(\frac{\mathscr{Q}^2_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^r q^r}\right) \end{bmatrix}$$

In particular, if the matrix is invertible, there exists only one policy. However, if it is singular, it is possible to find a continuum a couples $(\mathbf{t}^f_{it}, \mathbf{t}^r_{it})$ that satisfy the equation.

The rest of this section, which is inspired by the analysis Hassler et al. (2021*b*) is forthcoming.

7 Unilateral climate policy

We consider a continuum of unilateral social planners, one for each country $i \in \mathbb{I}$, and would take decisions for consumption, capital, and energy. The problem is very similar to the competitive equilibrium of section 4. The main difference is the internalization of the climate externality in each country: the planner would take into of its own country's energy impact on the world climate. The main assumption for this reasoning to hold is that the mass of each country j is not zero: $\int_{\mathbb{I}} \mathbb{1}\{i = j\} di > 0$, i.e. if the regions are not infinitesimal. As a result, in each country, the planner would solve the following problem, with individual welfare $\mathcal{W}_{it}$

$$\mathcal{W}_{it_0} = \max_{c_i, e^f_i, e^r_i, k_i} \int_{t_0}^{\infty} e^{-(\tilde{\rho}+n)t}\, u\big(\mathcal{D}^u(\tau_{it})\, c_{it}\big) p_i \; di \; dt$$

Note that this problem corresponds to the planner of section section 6, with Pareto weight distribution to be degenerate at $\omega_i = 1$ and $\omega_j = 0 \; \forall j \neq i$.

In the rest of this section, which is forthcoming, we consider an optimal allocation full commitment, where no planner has any incentive to deviate for the plan set up in period t_0. Assuming away full-commitment implies strategic interactions between countries, and since the mass of agents is not zero, this $N-$player differential game can prove hard to solve both mathematically and computationally.

8 Long-run analysis

In this section, we provide analytical results of the Competitive equilibrium, First-Best and Ramsey allocations on the cost of carbon, the path of emissions, and temperature in the asymptotic stationary equilibrium.

8.1 The Social Cost of Carbon

Given the path for the costate that informs on the social value of carbon emission, we canÂ find a balance-growth path that keeps the SCC stationary. We consider the long-run equilibrium where the terminal time horizon $T \to \infty$. In this context, only a stable temperature makes the system stationary, such that the emissions entering the atmosphere $\mathcal{E}_t$ are exactly offset by the one rejected outside the climate system δ_i

$$\mathcal{E}_t = \delta_s \mathcal{S}_t \qquad \text{and} \qquad \tau_t \to \tau_\infty$$

Depending on the trajectory of emissions between t_0 and t_T – when $\mathcal{E}_t \approx \delta_s \mathcal{S}_t$ – there are different cumulative emission/atmospheric carbon level $\mathcal{S}_t$ possible and hence different distribution of temperature $\mathcal{T}_T$ and $\{\tau_i\}_i$.

In particular, it is not difficult to guess the ordering between Competitive equilibrium (CE), Unilateral policy (UP), Ramsey allocation (RA), and First Best Allocation (FB) :

$$\mathcal{T}_T^{FB} < \mathcal{T}_T^{RA} < \mathcal{T}_T^{UP} < \mathcal{T}_T^{CE}$$

Solving the stationary differential equations at the limit $t \to T \to \infty$, we find an analytical characterization for the Social Cost of Carbon.

Proposition:
In the stationary competitive equilibrium, the Ramsey or the First Best allocations, the Social Cost of Carbon can be expressed as:

$$SCC_t \equiv \frac{1}{\overline{\psi}_t^w} \frac{\chi}{\widetilde{\rho} + \delta^s} \int_{\mathbb{I}} \Delta_i(\tau_{i,\infty} - \tau_i^\star) \Big(\gamma_i^y \mathcal{D}^y(\tau_{i,\infty}) y_{i,\infty} \psi_{i,\infty}^k + \gamma_i^u \mathcal{D}^u(\tau_{i,\infty})\, \omega_i\, u'(\mathcal{D}^u c_{i,\infty})\, c_i \Big) di$$

This formula is analogous to the Social Cost of Carbon expressed in Golosov et al. (2014). Considering a linear instead of quadratic damage function – and only applied to TFP, without direct effects on mortality, would yield an exactly identical expression. We rely on a different set of assumptions – stationarity and continuous time – while the analysis in Golosov et al. (2014) relies on a representative agent, full depreciation every discrete period, and log-utility assumptions such that income and substitution forces in consumption/saving offset each other to yield such formula.

In particular, the noticeable feature is the proportionality of the SCC with $y_{i,\infty}$ and $c_{i\infty}$ and the temperature gap $(\tau_{i,\infty} - \tau_i^\star)$. If countries are richer, and more developed, the marginal damage has a larger economic impact. Moreover, due to the convexity of the damage function, the cost of carbon increases with temperature: hotter countries have more to lose from an additional increase in temperature. The extent of this proportionality depends on the exact calibration of the damage parameters $\gamma_i^y = \gamma_i^{\oplus}$ or $\gamma_i^{\ominus}$ for productivity impact and $\gamma_i^u = \gamma_{u,i}^{\oplus}$ or $\gamma_{u,i}^{\ominus}$ for mortality effects. More work is needed to make these damage parameters empirically grounded, as studied in Carleton et al. (2022)

Moreover, the SCC is proportional to the extent that the country is warming faster than the world's atmosphere due to geographical factors Δ_i.

Finally, these different effects are scaled with the effective discount factor – the rate of the social planner and including the depreciating of carbon due to the exit of the greenhouse gas from the atmosphere. This highlight in a very clear fashion how the discount factor affects the Social Cost of Carbon, as raised in the debate Stern and Stern (2007) and Nordhaus (2007).

Moreover, the ratio $1/\overline{\psi}_t^w$ and ψ_{it}^w in the expression of the Social Cost of Carbon highlight the importance of inequality for the computation of carbon price.

To study this, one could also consider the *"Local cost of carbon"* as the marginal damage for the region $i \in \mathbb{I}$:

$$LCC_{it} = \frac{\chi}{\widetilde{\rho} + \delta^s} \Delta_i (\tau_{i,\infty} - \tau_i^\star) \Big(\gamma_i^y y_{i,\infty} + \gamma_i^u c_{i,\infty} \Big)$$

with output $y_{i,\infty} = \mathcal{D}_i^y(\tau_{it}) z_i f(k_{it}, e_{it})$. Again, considering a single country, this formula boils down to the SCC for a representative country. Taking heterogeneous countries and following the same logic as above, we observe that:

$$\begin{aligned} SCC_t &= \int_{\mathbb{I}} \widehat{\psi}_{it}^w \, LCC_{it} \, di \\ &= \mathbb{E}^{\mathbb{I}} \big[LCC_{it} \big] + \mathbb{C}\mathrm{ov}^{\mathbb{I}} \Big(\widehat{\psi}_{it}^w, LCC_{it} \Big) > \mathbb{E}^{\mathbb{I}} \big[LCC_{it} \big] =: \overline{SCC}_t \end{aligned}$$

This covariance between $\widehat{\psi}_{it}^w = \psi_{it}^w / \overline{\psi}_t^w$ and the LCC_i that is proportional to y_i and $\tau_{i,\infty} - \tau_i^\star$ is clearly positive as we will explore in our quantitative experiments. This is obviously identical to the theoretical result we showed above in the non-stationary path. In this long-run context, the covariance is easier to compute as it relies on less assumptions on preferences and technology as it can be directly measured from the data on τ_{it}, y_{it} and c_{it}.

8.2 Green Growth and decoupling from energy

Empirically, energy use has correlated strongly with GDP levels and industrial production in the last century, as seen in figures in appendix A. However, lowering GHG emissions tend to go hand in hand with reducing energy consumption. This asks the question of the possibility of decoupling between economic growth and energy supply, and fossils in particular.

To examine this in our framework, let us study the optimality conditions for energy and express the energy share in the final output.

$$\begin{cases} MPe_i = {z_i}^{1-\frac{1}{\sigma}} {y_{it}}^{\frac{1}{\sigma}} \varepsilon^{\frac{1}{\sigma}} (z_{it}^e)^{1-\frac{1}{\sigma}} {e_{it}}^{-\frac{1}{\sigma}} & = q_t^e \\ MPe_i \big(\frac{e_t^f}{\omega e_t}\big)^{-\frac{1}{\sigma_e}} & = q_t^{e,f} \\ MPe_i \big(\frac{e_t^r}{(1-\omega) e_t}\big)^{-\frac{1}{\sigma_e}} & = q_t^{e,r} \end{cases}$$

As a result, the total energy share writes:

$$s_{e,t} := \frac{e_{it} q_t^e}{y_{it}} = (q_t^e)^{1-\sigma} z_i^{\sigma-1} (z_t^e)^{\sigma-1} \varepsilon$$

Since all the variable are already expressed in efficient unit per capita, accounting for the trend in population n and TFP growth $\bar{g}$, we have z_i constant and all the variables growth in absolute value. However, all the other variables can feature additional long-run trends, such as energy price $\dot{q}_t^e / q_t^e = g_q$ or directed technical change $\dot{z}_t^e / z_t^e = g_e$.

We consider two case: (i) the cost share of energy stays stable in output and (ii) this share falls to zeros.

$$\begin{aligned} &(i) \qquad s_{e,t} \rightarrow_{t\rightarrow\infty} \bar{s}_e && \Leftrightarrow \qquad g_q(1-\sigma) + g_e(\sigma - 1) = 0 \\ &(ii) \qquad s_{e,t} \rightarrow_{t\rightarrow\infty} 0 && \Leftrightarrow \qquad g_q - g_e < 0 \end{aligned}$$

In our quantitative exercise, following empirical evidence that energy share $s_{e,t}$ tends to comove strongly with energy price q_t^e, we assume that $\sigma < 1$ and energy is a complementary factor in production. As result, $g_e = g_q$ for (i) and $g_e > g_q$ for (ii). For the energy share to stay stable or decline, directed technical change should at least compensate for the increase in price.

To determine the path of price in our context, recall the supply side of the energy market, we have:

$$\frac{\dot{q}_t^e}{q_t^e} = s_{ef,t} \frac{\dot{q}_t^{e,f}}{q_t^{e,f}} + s_{er,t} \frac{\dot{q}_t^{e,r}}{q_t^{e,r}}$$

where $s_{ef,t} = \frac{e_t^f q_t^{e,f}}{e_t q_t}$ is the expenditure share in fossil and $s_{er,t} = 1 - s_{ef,t}$ the share in renewable. Recall that in our context,

$$q_t^f = \Big(\frac{E_t^f}{\mathcal{R}_t}\Big)^{\nu} + \lambda_t^R \qquad\qquad \Rightarrow \qquad \frac{\dot{q}_t^f}{q_t^f} = s_{\mathcal{C}} \nu \Big(\frac{\dot{E}_t^f}{E_t^f} - \frac{\dot{\mathcal{R}}_t}{\mathcal{R}_t}\Big) + (1 - s_{\mathcal{C}}) \frac{\dot{\lambda}_t^R}{\lambda_t^R}$$

where $s_{\mathcal{C}} = \frac{\mathcal{C}_E(\cdot)}{q_t^f}$ is the share of marginal in the fossil price, and $\frac{\dot{\lambda}_t^R}{\lambda_t^R}$ is the growth of the Hotelling rent, which is ρ at the first order. Obviously if extraction rate is faster than exploration of new reserves, the price will grow to infinity. Moreover, the rent of the monopolist will at least grow at the speed ρ in the first order,

Similarly, to get decoupling from fossils in the energy mix, we must have $g_r = \frac{\dot{q}_t^{e,r}}{q_t^{e,r}} < \frac{\dot{q}_t^{e,f}}{q_t^{e,f}} = g_f$. In this case, $g_q \rightarrow g_r$.

To conclude, to obtain a balance green growth equilibrium in our context, we need: (i) fossil prices to grow sufficiently fast due to extraction or rise in Hotelling rents, (ii) the price of renewables to grow less fast than fossils and (iii) that the directed technical change grows at a rate at least faster than the growth in the relative price of the resulting energy.

8.3 Path of emissions and temperature

We saw that the cost of carbon depends mostly on the resulting final temperatures once the economy and climate reach a stationary path where temperatures stay constant. This level matters and varies enormously as it depends linearly on the path of emissions:

$$\tau_{it} - \tau_{it_0} = \Delta_i \, \chi \int_{t_0}^{T} e^{-\delta_s (T-t)} \mathcal{E}_t dt$$

As a result, replacing the aggregate emissions, we obtain:

$$\tau_{it} - \tau_{it_0} = \Delta_i \, \chi \, \xi \, \omega \int_{t_0}^{T} e^{(n+\bar{g})t - \delta_s (T-t)} q_t^{f \ -\sigma_e} \int_{j \in \mathbb{I}} \big(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t}) \big)^{\sigma - 1} y_{j,t} \, q_{j,t}^{\sigma_e - \sigma} dj \ dt$$

where the path of world emissions $\{\epsilon_j\}_j$ has been expressed by fossil energy demand $e_j^f(q_t^f, z_j, z_{j,t}^e)$. In the long-run, the local temperature will uniquely be affected by the externality of the world economy, along with geographical factors determining warming Δ_i, the climate sensitivity parameter χ and the carbon exit from atmosphere δ_s,

We observe that the path of emissions depends positively on the growth of population n and aggregate productivity $\bar{g}$, the deviation of output from trend y_j & relative TFP z_j, the directed technical change z_t^e. Fossil demand is also shaped by the elasticity of energy in output σ, the Fossil energy price $q^{e,f}$ and its long run growth rate g^{q^f}, as expressed above. Finally, the change in energy mix, renewable share ω and price q_t^r & elasticity of the energy source σ_e are factors that would help reduce these paths of emissions.

To analyze this asymptotic behavior, we perform an approximation of this resulting temperature at terminal time. T.

$$\frac{\dot{\tau}_T}{\tau_T} \propto \ n \, + \, \bar{g}^y - (1-\sigma)\big(g_e - \widetilde{\gamma}\big) + (\sigma_e - \sigma)(1-\omega) g^{q^r} - (\sigma_e(1-\omega) + \sigma\omega) g^{q^f}$$

This decomposition is reminiscent of a Generalized Kaya (or $I = PAT$) identity, where Emission growth can be decomposed as

$$\varepsilon_{it} = \frac{\epsilon_{it}}{e_{it}} \frac{e_{it}}{y_{it}} \frac{y_{it}}{p_{it}} p_{it}$$

where y_{it} is already the output per capita. Taking the growth rate of this decomposition, we obtain the formula above. This show how important the path of energy prices g^{q^f} and g^{q^r} and technology g_e matter for future path of emissions and climate.

9 Calibration

The calibration of this model is preliminary, and will be updated to match (i) empirical moments on output growth, production, population demographic and energy markets (ii) reasonable estimates of the SCC. In particular, parameters denoted by $\star$ are subject to future changes. As of now, this calibration is aimed at simulate a first version of the model to provide intuitions of economic and climate mechanisms. Many of the parameters are taken or inspired by the rest of the literature.

Table 1: Baseline calibration

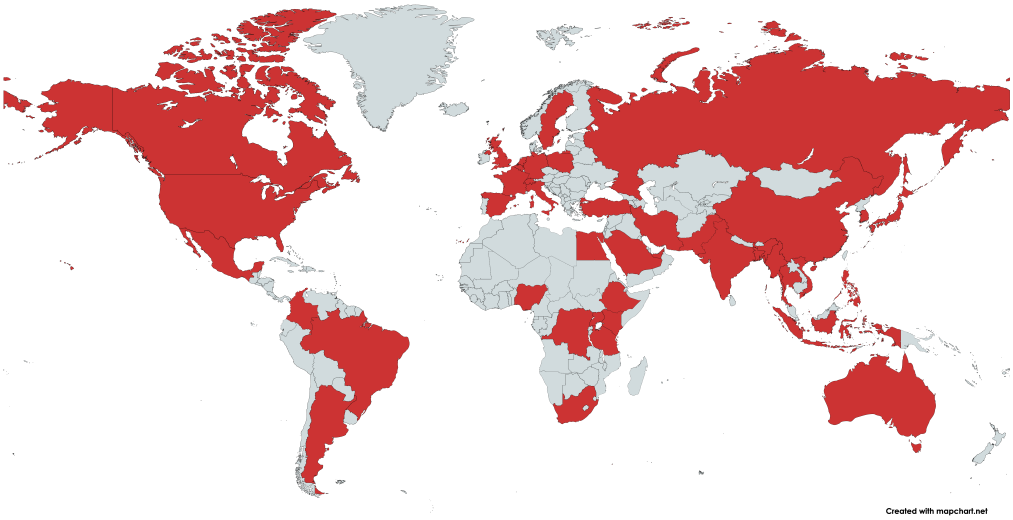
Technology & Energy markets			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy/Output ratio
σ	0.3	Elasticity capital-labor vs. energy	Slight complementarity in production
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
g_e	$0.01^\star$	Long run energy directed technical change	Conservative estimate for growth
g_r	$0.01^\star$	Long run renewable price increase	Conservative estimate for growth
ν	$2^\star$	Extraction elasticity of fossil energy	Cubic extraction cost
μ	$2^\star$	Exploration elasticity of fossil energy	Cubic exploration cost
δ^R	$0.45^\star$	Probability of new reserves discovery	Conservative estimate for energy growth
Preferences & Time horizon			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
$\widetilde{\rho}$	$0.03^\star$	SP Discount factor	Planner as patient as Households
η	2.5	Risk aversion	Positive utility in steady state
n	$0.01^\star$	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	100	Time horizon	Horizon 2100 years since 1950
Climate parameters			
ξ	0.81	Emission factor	Conversion 1 $MTOE \Rightarrow 1\ MT\ CO_2$
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature $\sim 10-15$ years
χ	$2.1/1e6$	Climate sensitivity	Pulse experiment: $100\,GtC \equiv 0.21^\circ C$ medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment: $100\,GtC \equiv 0.16^\circ C$ long-term warming
$\gamma^\oplus$	$0.00234^\star$	Damage sensitivity	Conservative estimate: Nordhaus' DICE
$\gamma^\ominus$	$0.2\times\gamma^{\oplus\ \star}$	Damage sensitivity	Conservative estimate: Nordhaus' DICE
α^τ	$0.2^\star$	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
$\tau^\star$	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
Parameters calibrated to match data			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
τ_i		Local Temperature	To match temperature of largest city

10 Quantitative Experiment

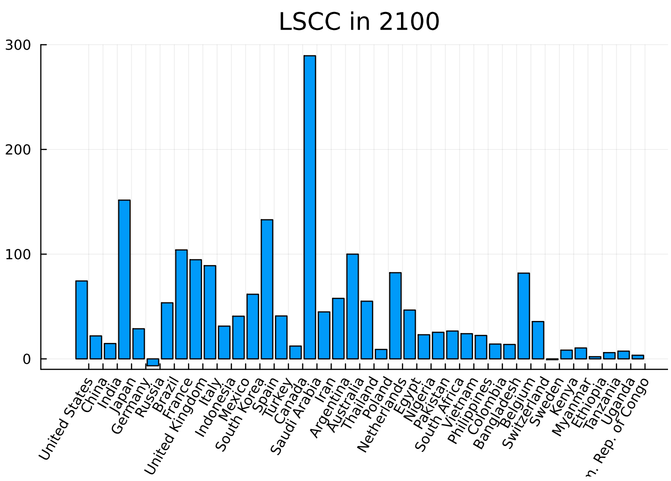
We collect data on 40 countries, selected as the union of the 25 largest in terms of population and the 25 largest in terms of total GDP. As a result, it includes both small but rich countries as well as large but lower-income economies.

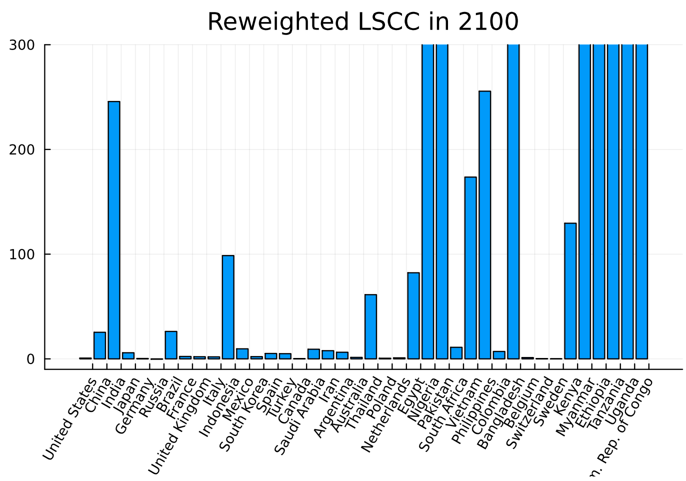
We use the local temperatures of the largest city as well as GDP, energy use, CO_2 emissions, population from international data from the World Bank. In particular, I calibrate productivity residual z to match the distribution of output per capita at the steady state, assumed to be the mean over the years 2000-2011.

More work is needed to match the data and to make the model empirically grounded.

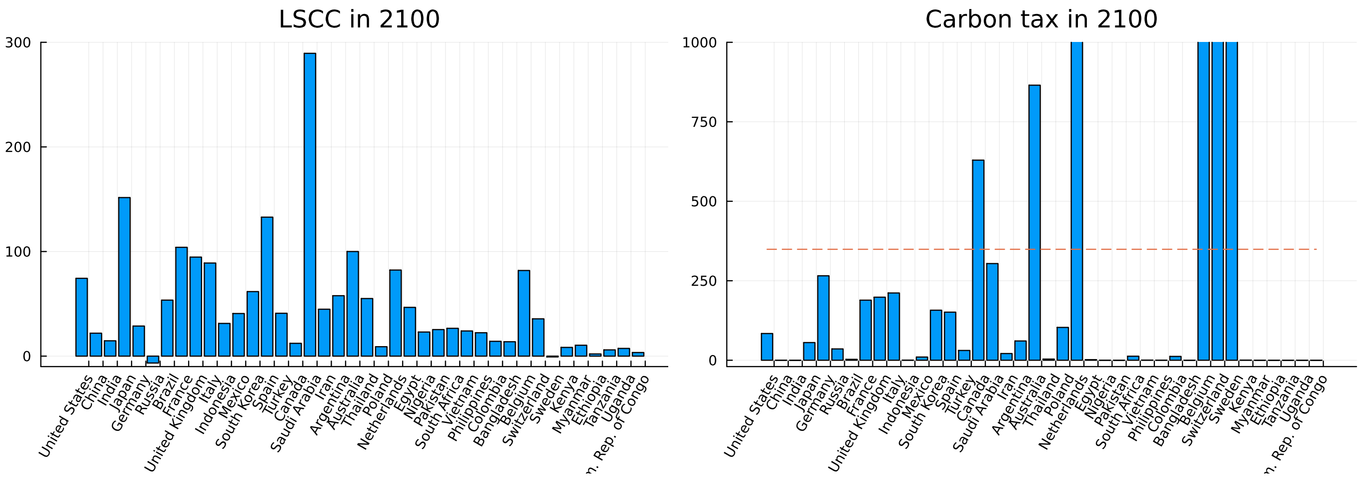


In the following two graphs, I plot the difference, in the long run stationary competitive equilibrium, between the distribution of local cost of carbon $LCC_i = \lambda^{\mathcal{S}}_{it}/\lambda^{w}_{it}$ and the local cost of carbon reweighted by our measure of inequality $\widehat{\lambda}^{w}_{it} = \lambda^{w}_{it}/\overline{\lambda}^{w}_{it}$, and hence $LWCC_{it} = \widehat{\lambda}^{w}_{it} LCC_{it} = \lambda^{\mathcal{S}}_{it}/\bar{\lambda}^{k}_{t}$





In the following two graphs, I plot the difference, again in the long run stationary competitive equilibrium between the $LCC_i = \lambda^{\mathcal{S}}_{it}/\lambda^{w}_{it}$ and the optimal carbon tax $\mathbf{t}^{S}_{it} = (1/\widehat{\lambda}^{w}_{it})SCC_t$ in the context where the social value of fossil supply is null (constant return to scale case).



The rest of this section is forthcoming

11 Conclusion

This section is forthcoming

References

Acemoglu, Daron, Philippe Aghion and David Hémous (2014), 'The environment and directed technical change in a north–south model', *Oxford Review of Economic Policy* **30**(3), 513–530.

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn and David Hemous (2012), 'The environment and directed technical change', *American economic review* **102**(1), 131–166.

Acemoglu, Daron, Ufuk Akcigit, Douglas Hanley and William Kerr (2016), 'Transition to clean technology', *Journal of Political Economy* **124**(1), 52–104.

Anderson, Soren T, Ryan Kellogg and Stephen W Salant (2018), 'Hotelling under pressure', *Journal of Political Economy* **126**(3), 984–1026.

Bardi, Ugo (2011), *The limits to growth revisited*, Springer Science & Business Media.

Barnett, Michael, William Brock and Lars Peter Hansen (2020), 'Pricing uncertainty induced by climate change', *The Review of Financial Studies* **33**(3), 1024–1066.

Barnett, Michael, William Brock and Lars Peter Hansen (2022), 'Climate change uncertainty spillover in the macroeconomy', *NBER Macroeconomics Annual* **36**(1), 253–320.

Bhandari, Anmol, David Evans, Mikhail Golosov and Thomas J Sargent (2021*a*), 'Inequality, business cycles, and monetary-fiscal policy', *Econometrica* **89**(6), 2559–2599.

Bhandari, Anmol, David Evans, Mikhail Golosov and Thomas Sargent (2021*b*), Efficiency, insurance, and redistribution effects of government policies, Technical report, Working paper.

Bilal, Adrien (2021), Solving heterogeneous agent models with the master equation, Technical report, Technical report, University of Chicago.

Bilal, Adrien and Esteban Rossi-Hansberg (2023), Anticipating climate change risk across the united states, Technical report, Technical report, University of Chicago.

Bornstein, Gideon, Per Krusell and Sergio Rebelo (2023), 'A world equilibrium model of the oil market', *The Review of Economic Studies* **90**(1), 132–164.

Boucekkine, Raouf, Carmen Camacho and Benteng Zou (2009), 'Bridging the gap between growth theory and the new economic geography: The spatial ramsey model', *Macroeconomic Dynamics* **13**(1), 20–45.

Bourany, Thomas (2018), The wealth distribution over the business cycle, a mean field game with common noise, Technical report, Technical report, Paris Diderot University.

Cai, Yongyang, Kenneth L Judd and Thomas S Lontzek (2012*a*), Continuous-time methods for integrated assessment models, Technical report, National Bureau of Economic Research.

Cai, Yongyang, Kenneth L Judd and Thomas S Lontzek (2012*b*), 'Dsice: A dynamic stochastic integrated model of climate and economy'.

Cai, Yongyang and Thomas S Lontzek (2019), 'The social cost of carbon with economic and climate risks', *Journal of Political Economy* **127**(6), 2684–2734.

Cardaliaguet, Pierre (2013/2018), 'Notes on mean field games.', *Lecture notes from P.L. Lions' lectures at College de France and P. Cardaliaguet at Paris Dauphine* .

Cardaliaguet, Pierre, François Delarue, Jean-Michel Lasry and Pierre-Louis Lions (2015), 'The master equation and the convergence problem in mean field games', *arXiv preprint 1509.02505* .

Carleton, Tamma, Amir Jina, Michael Delgado, Michael Greenstone, Trevor Houser, Solomon Hsiang, Andrew Hultgren, Robert E Kopp, Kelly E McCusker, Ishan Nath, James Rising, Ashwin Rode, Hee Kwon Seo, Arvid Viaene, Jiacan Yuan and Alice Tianbo Zhang (2022), 'Valuing the Global Mortality Consequences of Climate Change Accounting for Adaptation Costs and Benefits*', *The Quarterly Journal of Economics* **137**(4), 2037–2105.

Carleton, Tamma and Michael Greenstone (2021), 'Updating the united states government's social cost of carbon', *University of Chicago, Becker Friedman Institute for Economics Working Paper* (2021-04).

Carmona, Rene and François Delarue (2018), *Probabilistic Theory of Mean Field Games with Applications I-II*, Springer.

Carmona, René, François Delarue and Aimé Lachapelle (2013), ‘Control of mckean–vlasov dynamics versus mean field games’, *Mathematics and Financial Economics* **7**(2), 131–166.

Carmona, René, François Delarue and Daniel Lacker (2016), ‘Mean field games with common noise’.

Carmona, René, François Delarue et al. (2015), ‘Forward–backward stochastic differential equations and controlled mckean–vlasov dynamics’, *The Annals of Probability* **43**(5), 2647–2700.

Carmona, René, Gökçe Dayanıklı and Mathieu Laurière (2022), ‘Mean field models to regulate carbon emissions in electricity production’, *Dynamic Games and Applications* **12**(3), 897–928.

Carmona, René and Mathieu Laurière (2022), ‘Convergence analysis of machine learning algorithms for the numerical solution of mean field control and games: Ii – the finite horizon case’, *The Annals of Applied Probability* **32**(6), 4065–4105.

Cruz Álvarez, José Luis and Esteban Rossi-Hansberg (2022), ‘Local carbon policy’, *NBER Working Paper* (w30027).

Cruz, José-Luis and Esteban Rossi-Hansberg (2021), The economic geography of global warming, Technical report, National Bureau of Economic Research.

Dávila, Eduardo and Andreas Schaab (2023), Optimal monetary policy with heterogeneous agents: Discretion, commitment, and timeless policy, Technical report, National Bureau of Economic Research.

Dietz, Simon and Frank Venmans (2019), ‘Cumulative carbon emissions and economic policy: in search of general principles’, *Journal of Environmental Economics and Management* **96**, 108–129.

Dietz, Simon, Frederick van der Ploeg, Armon Rezai and Frank Venmans (2021), ‘Are economists getting climate dynamics right and does it matter?’, *Journal of the Association of Environmental and Resource Economists* **8**(5), 895–921.

Folini, Doris, Felix Kübler, Aleksandra Malova and Simon Scheidegger (2021), ‘The climate in climate economics’, *arXiv preprint arXiv:2107.06162* .

Golosov, Mikhail, John Hassler, Per Krusell and Aleh Tsyvinski (2014), ‘Optimal taxes on fossil fuel in general equilibrium’, *Econometrica* **82**(1), 41–88.

Grossman, Gene M, Elhanan Helpman, Ezra Oberfield and Thomas Sampson (2017), ‘Balanced growth despite uzawa’, *American Economic Review* **107**(4), 1293–1312.

Hansen, Lars Peter and Thomas J Sargent (2001), ‘Robust control and model uncertainty’, *American Economic Review* **91**(2), 60–66.

Hassler, John, Per Krusell and Anthony A Smith Jr (2016), Environmental macroeconomics, *in* ‘Handbook of macroeconomics’, Vol. 2, Elsevier, pp. 1893–2008.

Hassler, John, Per Krusell and Conny Olovsson (2010), ‘Oil monopoly and the climate’, *American Economic Review* **100**(2), 460–64.

Hassler, John, Per Krusell and Conny Olovsson (2021*a*), ‘Directed technical change as a response to natural resource scarcity’, *Journal of Political Economy* **129**(11), 3039–3072.

Hassler, John, Per Krusell and Conny Olovsson (2021*b*), ‘Presidential Address 2020 Suboptimal Climate Policy’, *Journal of the European Economic Association* **19**(6), 2895–2928.

Hassler, John, Per Krusell, Conny Olovsson and Michael Reiter (2020), 'On the effectiveness of climate policies', *IIES WP* **53**, 54.

Heal, Geoffrey and Wolfram Schlenker (2019), Coase, hotelling and pigou: The incidence of a carbon tax and co 2 emissions, Technical report, National Bureau of Economic Research.

Hillebrand, Elmar and Marten Hillebrand (2019), 'Optimal climate policies in a dynamic multi-country equilibrium model', *Journal of Economic Theory* **179**, 200–239.

Hotelling, Harold (1931), 'The economics of exhaustible resources', *Journal of political Economy* **39**(2), 137–175.

Kellogg, Ryan (2014), 'The effect of uncertainty on investment: Evidence from texas oil drilling', *American Economic Review* **104**(6), 1698–1734.

Kilian, Lutz (2009), 'Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market', *American Economic Review* **99**(3), 1053–69.

Koeste, Mark J., Henri L.F. de Groot and Raymond J.G.M. Florax (2008), 'Capital-energy substitution and shifts in factor demand: A meta-analysis', *Energy Economics* **30**(5), 2236–2251.

Kotlikoff, Laurence, Felix Kubler, Andrey Polbin and Simon Scheidegger (2021*a*), 'Pareto-improving carbon-risk taxation', *Economic Policy* **36**(107), 551–589.

Kotlikoff, Laurence J, Felix Kubler, Andrey Polbin and Simon Scheidegger (2021*b*), Can today's and tomorrow's world uniformly gain from carbon taxation?, Technical report, National Bureau of Economic Research.

Krusell, Per and Anthony A Smith (2022), Climate change around the world, Technical report, National Bureau of Economic Research.

Le Grand, François, Alaïs Martin-Baillon and Xavier Ragot (2021), Should monetary policy care about redistribution? optimal fiscal and monetary policy with heterogeneous agents, Technical report, Working Paper, SciencesPo.

LeGrand, François and Xavier Ragot (2022), Optimal policies with heterogeneous agents: Truncation and transitions, Technical report, Working Paper, SciencesPo.

Lemoine, Derek and Ivan Rudik (2017*a*), 'Managing climate change under uncertainty: Recursive integrated assessment at an inflection point', *Annual Review of Resource Economics* **9**, 117–142.

Lemoine, Derek and Ivan Rudik (2017*b*), 'Steering the climate system: using inertia to lower the cost of policy', *American Economic Review* **107**(10), 2947–2957.

Lenton, Timothy M (2011), 'Early warning of climate tipping points', *Nature climate change* **1**(4), 201–209.

Lontzek, Thomas S, Yongyang Cai, Kenneth L Judd and Timothy M Lenton (2015), 'Stochastic integrated assessment of climate tipping points indicates the need for strict climate policy', *Nature Climate Change* **5**(5), 441–444.

McKay, Alisdair and Christian K Wolf (2022), Optimal policy rules in HANK, Technical report, Working Paper, FRB Minneapolis.

Meadows, Donella H, Dennis L Meadows, Jorgen Randers and William W Behrens (1972), 'The limits to growth', *New York* **102**(1972), 27.

Nakov, Anton and Galo Nuno (2013), 'Saudi arabia and the oil market', *The Economic Journal* **123**(573), 1333–1362.

Nordhaus, William D (1993), 'Optimal greenhouse-gas reductions and tax policy in the" dice" model', *The American Economic Review* **83**(2), 313–317.

Nordhaus, William D (2007), 'A review of the stern review on the economics of climate change', *Journal of economic literature* **45**(3), 686–702.

Nordhaus, William D (2017), 'Revisiting the social cost of carbon', *Proceedings of the National Academy of Sciences* **114**(7), 1518–1523.

Nordhaus, William D and Joseph Boyer (2000), *Warming the world: economic models of global warming*, MIT press.

Pham, Huyên and Xiaoli Wei (2017), 'Dynamic programming for optimal control of stochastic mckean–vlasov dynamics', *SIAM Journal on Control and Optimization* **55**(2), 1069–1101.

Ricke, Katharine L and Ken Caldeira (2014), 'Maximum warming occurs about one decade after a carbon dioxide emission', *Environmental Research Letters* **9**(12), 124002.

Ricke, Katharine, Laurent Drouet, Ken Caldeira and Massimo Tavoni (2018), 'Country-level social cost of carbon', *Nature Climate Change* **8**(10), 895–900.

Rode, Ashwin, Tamma Carleton, Michael Delgado, Michael Greenstone, Trevor Houser, Solomon Hsiang, Andrew Hultgren, Amir Jina, Robert E Kopp, Kelly E McCusker et al. (2021), 'Estimating a social cost of carbon for global energy consumption', *Nature* **598**(7880), 308–314.

Rudik, Ivan, Gary Lyn, Weiliang Tan and Ariel Ortiz-Bobea (2021), 'The economic effects of climate change in dynamic spatial equilibrium'.

Stern, Nicholas and Nicholas Herbert Stern (2007), *The economics of climate change: the Stern review*, cambridge University press.

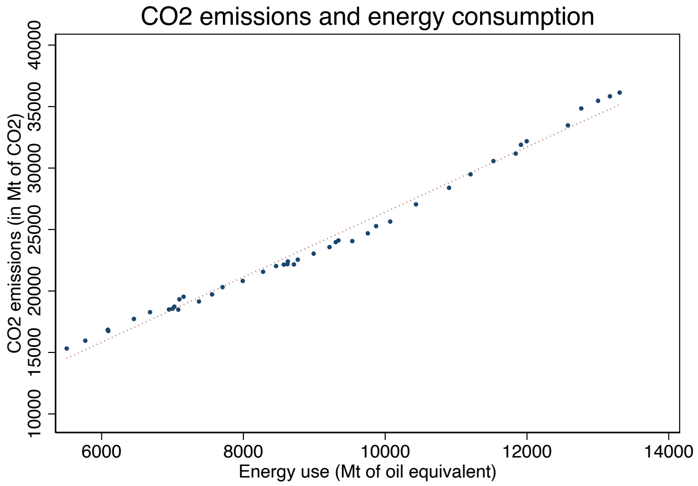
Stoddard, Isak, Kevin Anderson, Stuart Capstick, Wim Carton, Joanna Depledge, Keri Facer, Clair Gough, Frederic Hache, Claire Hoolohan, Martin Hultman et al. (2021), 'Three decades of climate mitigation: why haven't we bent the global emissions curve?', *Annual Review of Environment and Resources* **46**, 653–689.

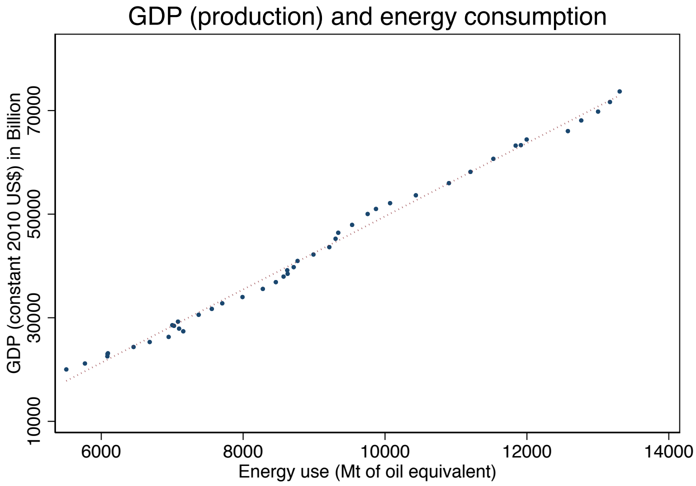
Van den Bremer, Ton S and Frederick Van der Ploeg (2021), 'The risk-adjusted carbon price', *American Economic Review* **111**(9), 2782–2810.

Yong, Jiongmin and Xun Yu Zhou (1999), *Stochastic controls: Hamiltonian systems and HJB equations*, Vol. 43, Springer Science & Business Media.

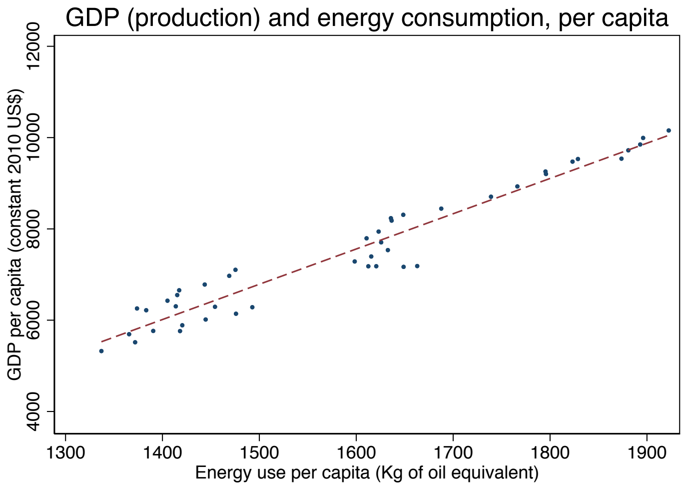
A Coupling GDP, Energy and Emissions

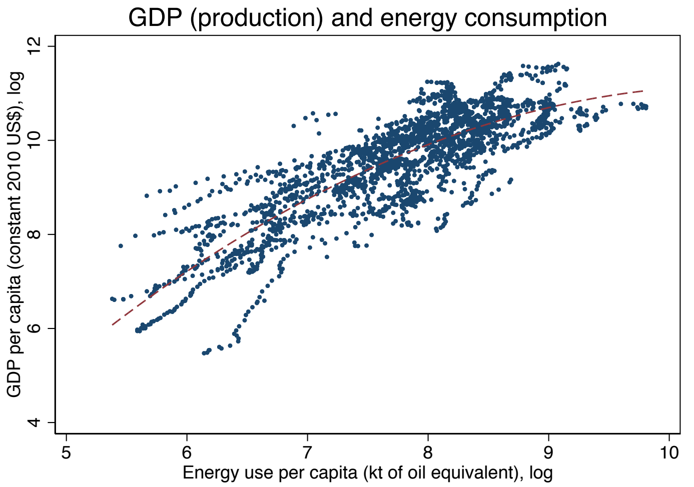
CO_2 emissions correlate linearly with energy use. Energy use (including 85% from fossil fuels sources) correlates with output/growth





This trend is also true per capita and for the trajectory of individual countries





B Energy producers – fossil fuel company

We consider the simplest functional forms, yielding isoelastic supply curves for fossil energy extraction and exploration:

$$\nu(E, \mathcal{R}) = \frac{\bar{\nu}}{1+\nu}\left(\frac{E}{\mathcal{R}}\right)^{1+\nu}\mathcal{R} \qquad \mu(\mathcal{I}^e, \mathcal{R}) = \frac{\bar{\mu}}{1+\mu}\left(\frac{\mathcal{I}^e}{\mathcal{R}}\right)^{1+\mu}\mathcal{R}$$

Setting up the Hamiltonian,

$$\mathcal{H}(\mathcal{R}_t, \lambda_t^R, E_t, \mathcal{I}_t^f) = \pi_t(\mathcal{R}_t, E_t, \mathcal{I}_t^f) + \lambda_t^R(\delta^R \mathcal{I}_t^f - E_t)$$

The optimal decisions are given by:

$$[E_t] \qquad q_t^{e,f} = \nu_E(E, R) + \lambda_t^R = \bar{\nu}\Big(\frac{E_t}{\mathcal{R}_t}\Big)^{\nu} + \lambda_t^R$$

$$[\mathcal{I}_t] \qquad \lambda_t^R \delta^R = \mu_I(\mathcal{I}_t, \mathcal{R}_t) = \bar{\mu}\Big(\frac{\mathcal{I}_t}{\mathcal{R}_t}\Big)^{\mu} \qquad \mathcal{I}_t = \mathcal{R}_t\big(\frac{\lambda_t^R \delta}{\bar{\mu}}\big)^{1/\mu}$$

The Pontryagin Maximum Principle yields the dynamics of the costate :

$$\begin{aligned}
-\dot{\lambda}_t^R + \rho \lambda_t^R &= \partial_R \mathcal{H}(R, E^\star, I^\star) \\
\dot{\lambda}_t^R &= \rho \lambda_t^R + \partial_R \nu(E_t^\star, \mathcal{R}_t) + \partial_R \mu(\mathcal{I}_t^\star, \mathcal{R}_t) \\
\dot{\lambda}_t^R &= \rho \lambda_t^R - \frac{\bar{\nu}\nu}{1+\nu}\Big(\frac{E_t^\star}{\mathcal{R}_t}\Big)^{1+\nu} - \frac{\bar{\mu}\mu}{1+\mu}\Big(\frac{I_t^\star}{\mathcal{R}_t}\Big)^{1+\mu} \\
\dot{\lambda}_t^R &= \rho \lambda_t^R - \frac{\bar{\nu}\nu}{1+\nu}\Big(\frac{E_t^\star}{\mathcal{R}_t}\Big)^{1+\nu} - \frac{\bar{\mu}\mu}{1+\mu}\Big(\frac{I_t^\star}{\mathcal{R}_t}\Big)^{1+\mu}
\end{aligned}$$

Replacing it with the optimal decisions, we obtain a non-linear equation for the Hotelling rent:

$$\dot{\lambda}_t^R = \rho \lambda_t^R - \frac{\bar{\nu}^{-1/\nu}\nu}{1+\nu}\big(q^{e,f} - \lambda_t^R\big)^{1+1/\nu} - \frac{\bar{\mu}^{-1/\mu}\mu}{1+\mu}\big(\delta^R \lambda_t^R\big)^{1+1/\mu}$$

Moreover, we should add the transversality conditions

$$\lim_{t \to \infty} e^{-\rho t} \lambda_t^R \mathcal{R}_t = 0$$

and since we know that λ_t^R grows less fast than $e^{\rho t}$, we have the transversality respected even if $\mathcal{R}_t \not\to 0$ when $t \to \infty$.

This implies a (highly!) non-linear ODE for the Hotelling rent λ_t^R, where λ_0^R is chosen such that $\mathcal{R}_t = 0$ by terminal time $t = \bar{t}$. We can "simplify" the ODE, in the case where the cost are quadratic $\mu = \nu = 1$ and

$$\dot{\lambda}_t^R = \rho \lambda_t^R + \frac{1}{2\bar{\nu}}\big(q_t^{e,f} - \lambda_t^R\big)^2 + \frac{1}{2\bar{\mu}}\big(\delta^R \lambda_t^R\big)^2$$

We see that the Hotelling rent account for the extraction cost (scaled by $\bar{\nu}$) and the exploration cost (scaling in $\bar{\mu}$) and depend on the price/inverse demand for determining the quantity produced in equilibrium.

A stationary solution can be found in the case where $\dot{\lambda}_t^R = 0$

$$\rho\lambda_t^R + \frac{1}{2\bar{\nu}}\big(q_t^{e,f} - \lambda_t^R\big)^2 + \frac{1}{2\bar{\mu}}\big(\delta^R\lambda_t^R\big)^2 = 0$$

$$\rho\lambda_t^R - \frac{1}{\bar{\nu}}q_t^{e,f}\lambda_t^R + \frac{1}{2\bar{\nu}}(\lambda_t^R)^2 + \frac{1}{2\bar{\nu}}(q_t^{e,f})^2 + \frac{1}{2\bar{\mu}}(\delta^R)^2(\lambda_t^R)^2 = 0$$

$$\lambda_\infty^R = \frac{\frac{q_t^{e,f}}{\bar{\nu}} - \rho \pm \sqrt{(\frac{q_t^{e,f}}{\bar{\nu}} - \rho)^2 - (\frac{1}{\bar{\nu}} + \frac{\delta^2}{\bar{\mu}})\frac{1}{\bar{\nu}}(q_t^{e,f})^2}}{\frac{1}{\bar{\nu}} + \frac{\delta^2}{\bar{\mu}}}$$

We obtain two stationary positive solutions: for a given energy price (demanded) $q^{e,f}$, in one equilibrium, the rent is very high, incentiving a lot of exploration as a share of reserve ($\mathcal{I}/\mathcal{R}$ is high) but the production is relatively low ($q^{e,f} - \lambda^R$ is low and so is the marginal cost and quantity $E/\mathcal{R}$). In a second stationary equilibrium, the rent is lower and the marginal cost is higher since the extraction is larger as a share of reserves. Note, that this stationary equilibrium is not consistent with state $\mathcal{R}_t$ dynamics since the reserves are depleting at different rates: only the first case is consistent with a sustainable level of extraction and exploration.

C Energy producers – continuum fossil fuel producers

Static problem:

$$\pi(q^f, \mathcal{R}_{it}) = \max_{\bar{e}_{it}^f} q_t^f \bar{e}_{it}^f - \frac{\nu_i}{1+\nu}\Big(\frac{\bar{e}_{it}^f}{\mathcal{R}_{it}}\Big)^{1+\nu}\mathcal{R}_{it}$$

Each firm in country has its own reserve $\mathcal{R}_i$ and it's own marginal cost ν_i. The firm solves the problem at each period giving rise to the optimal extraction:

$$q_t^f = \nu_i\Big(\frac{\bar{e}_{it}^f}{\mathcal{R}_{it}}\Big)^{\nu} \qquad\qquad \Rightarrow \bar{e}_{it}^{\star} = \mathcal{R}_{it}\Big(\frac{q_t^f}{\nu_i}\Big)^{1/\nu}$$

Extraction increases one-for-one with reserves and with elasticity $1/\nu$ with marginal cost (1% decrease in cost increase extraction by $1/\nu$) and with inverse supply elasticity $1/\nu$ (1% increase in cost increase extraction by $1/\nu$)
As a result, the profit of country i writes:

$$\pi(q^f, \mathcal{R}_{it}) = \big(q_t^f\big)^{1+\frac{1}{\nu}}\mathcal{R}_{it}\nu_i^{-1/\nu}\frac{\nu}{1+\nu}$$

Which writes as an individual characteristics $r_{it} = \mathcal{R}_{it}\nu_i^{-1/\nu}$.

We can write the aggregate supply curve as :

$$E_t = \int_{j\in\mathbb{I}} \bar{e}_{j,t}dj = \int_{j\in\mathbb{I}} \mathcal{R}_{j,t}(q_t^f)^{1/\nu}\nu_i^{-1/\nu}dj = (q_t^f)^{1/\nu}\int_{j\in\mathbb{I}} \mathcal{R}_{it}\nu_i^{-1/\nu}dj = (q_t^f)^{1/\nu}\widetilde{\mathcal{R}}_t$$

Defining $\widetilde{\mathcal{R}}_t = \int_{j\in\mathbb{I}} \mathcal{R}_{it}\nu_i^{-1/\nu} dj$ the marg cost-weighted reserves. Since, reserves are depleted at rate $\bar{e}^f_{it}$

$$\dot{\mathcal{R}}_{it} = -\bar{e}_{j,t} = -\mathcal{R}_{it}\nu_i^{-1/\nu}(q^f_t)^{1/\nu}$$

which implies the rate of decrease in cost-adjusted reserves:

$$\dot{r}_{it} = \dot{\mathcal{R}}_{it} = -\mathcal{R}_{it}\nu_i^{-1/\nu}(q^f_t)^{1/\nu} = -r_{it}(q^f_t)^{1/\nu}$$

The rate of aggregate reserves:

$$\dot{\widetilde{\mathcal{R}}}_t = \int_{j\in\mathbb{I}} \dot{r}_{j,t}\nu_j^{-1/\nu} dj = -(q^f_t)^{1/\nu}\int_{j\in\mathbb{I}} r_{it} dj = -(q^f_t)^{1/\nu}\widetilde{\mathcal{R}}_t$$

The higher the price, the higher the extraction rate and the higher the depletion of reserves. One could also add other features, like forward-looking decisions, with a Hoteling problem, and a meaningful distribution of reserve/costate. We abstract from these considerations in the model.

Note, that the model "aggregate" and we have the share of profit

$$\widetilde{\pi}(q^f_t, \mathcal{R}_t) = \int_{j\in\mathbb{I}} \pi_j(q^f, \mathcal{R}_{j,t}) dj = \left(q^f_t\right)^{1+\frac{1}{\nu}}\frac{\nu}{1+\nu}\int_{j\in\mathbb{I}} \mathcal{R}_{j,t}\nu_j^{-1/\nu} dj = \left(q^f_t\right)^{1+\frac{1}{\nu}}\frac{\nu}{1+\nu}\widetilde{\mathcal{R}}_t$$

which give endogenously the reserve share $\theta_{it} = \frac{\mathcal{R}_{it}\nu_i^{-1/\nu}}{\widetilde{\mathcal{R}}_t} = \frac{r_{it}}{\int_j r_{j,t} dj}$. Since, we

D Competitive equilibrium

Dynamics of the individual state variables $s_{it} = (k_{it}, \tau_{it}, z_i, p_i, \theta_i, \gamma_i, \Delta_i, \xi_i)$ and aggregate ones $(\mathcal{S}_t, \mathcal{T}_t, \mathcal{R}_t)$:

$$\begin{aligned}
\dot{w}_t &= r^\star_t w_{it} + \mathcal{D}(\tau_t) f(k_t, e_t) - (n + \bar{g} + \delta + r^\star_t)k_t + \theta\pi^f_t - c_t - q^e_t e_t - c(\vartheta_t)e^f_t \\
\mathcal{E}_t &= e^{(n+\bar{g})t}\int_{\mathbb{S}} \xi(1-\vartheta_{it})e^f_{it}p_{it} ds \\
\dot{\tau}_{it} &= \zeta(\Delta_i\,\chi\mathcal{S}_t - (\tau_{it} - \tau_{it_0})) \qquad\qquad \dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s\mathcal{S}_t \\
\dot{\mathcal{R}}_t &= -E^f_t + \delta_R\mathcal{I}_t \qquad\qquad q^{e,f}_t = \bar{\nu}(E^f_t/\mathcal{R}_t)^\nu
\end{aligned}$$

Household problem: Pontryagin Maximum Principle

$$\mathcal{H}^{hh}(s, \{c, k, e^f, e^r\}, \{\lambda\}) = u(c_i, \tau_i) + \lambda^w_{it}\Big(r^\star_t w_{it} + \mathcal{D}(\tau_{it})f(k_t, e_t) - (n + \bar{g} + \delta + r^\star_t)k_t + \theta\pi^f_t - q^f_t e^f_{it} - q^r_{it}e^r_{it} - c_t\Big)$$

$$\begin{aligned}
&[c_t] &&u'(c_{it}) = \lambda^w_{it} \\
&[k_t] &&MPk_{it} = r^\star_t[e^f_t] \qquad MPe^f_{it} \\
&[e^r_t] &&MPe^r_{it} = \mathcal{D}(\tau_{it})z\ \partial_e f(k_{it}, e_{it})\Big(\frac{e^r_{it}}{(1-\omega)e_{it}}\Big)^{-\frac{1}{\sigma_e}} = q^r_{it} \\
&[k_t] &&\dot{\lambda}^w_t = \lambda^w_t(\rho - r^\star_t)
\end{aligned}$$

Fossil Energy Monopoly problem:

$$\mathcal{H}^m(\mathcal{R}_t,\lambda_t^R,E_t,\mathcal{I}_t^f)=\pi_t(E_t^f,\mathcal{I}_t^f,\mathcal{R}_t)+\lambda_t^R(\delta^R\mathcal{I}_t^f-E_t)$$

$$[\mathcal{R}_t] \qquad \dot{\lambda}_t^R=\rho\lambda_t^R+\frac{\bar{\nu}\nu}{1+\nu}\Big(\frac{E_t^\star}{R_t}\Big)^{1+\nu}+\frac{\bar{\mu}\mu}{1+\mu}\Big(\frac{I_t^\star}{R_t}\Big)^{1+\mu}$$

$$[E_t^f] \qquad q_t^{e,f}=\nu_E(E,\mathcal{R})+\lambda_t^R=\bar{\nu}\Big(\frac{E_t}{\mathcal{R}_t}\Big)^{\nu}+\lambda_t^R$$

$$[\mathcal{I}_t] \qquad \lambda_t^R\delta^R=\mu_I(\mathcal{I}_t^f,R_t)=\bar{\mu}\Big(\frac{\mathcal{I}_t^f}{\mathcal{R}_t}\Big)^{\mu} \qquad \mathcal{I}_t^f=R_t\big(\frac{\lambda_t^R\delta}{\bar{\mu}}\big)^{1/\mu}$$

E Optimal policy and Ramsey problem

The dynamic optimization problem of the Ramsey planner can be summarized by the Hamiltonian of the system, for the state $s_{it}=(z_i,p_i,\theta_i,\gamma_i,\Delta_i,\xi_i,k_{it},\tau_{it})$.

$$\begin{aligned}
\mathcal{H}^{sp}(s,\{c\},\{e^f\},\{e^r\},\{\lambda\},\{\psi\}) &= \int_{\mathbb{I}}\omega_i u(c_{it},\tau_{it})p_i di \\
&+\psi_{it}^w\Big(r_t^\star w_{it}+\mathcal{D}(\tau_{it})f(k_{it},e_{it})-(n+\bar{g}+\delta)k_t+\theta_i\pi(E_t^f,\mathcal{I}_t,\mathcal{R}_t)-(q_t^f+\mathbf{t}_{it}^f)e_{it}^f-(q_{it}^r+\mathbf{t}_{it}^r)e_{it}^r-c_t+\mathbf{t}_t^{ls}\Big) \\
&+\psi_t^S\Big(\mathcal{E}_t-\delta^s\mathcal{S}_t\Big)+\psi_{it}^\tau\zeta\Big(\Delta_i\,\chi\,\mathcal{S}_t-(\tau_{it}-\tau_{i0})\Big)+\psi_{it}^{\mathcal{R}}\Big(-E_t^f+\delta^R\mathcal{I}_t\Big) \\
&+\psi_{it}^{\lambda w}\Big(\lambda_{it}^w(\rho-r_t^\star)\Big)+\psi_t^{\lambda R}\Big(\rho\lambda_t^R+\mathcal{C}_{\mathcal{R}}^f(E_t^f,\mathcal{I}_t,\mathcal{R}_t)\Big)+\phi_{it}^c\big(u_c(c_{it},\tau_{it})-\lambda_{it}^w\big) \\
&+\phi_{it}^{ef}\Big(e_{it}^f-\mathscr{Q}_{q^f}\big(q_t^f+\mathbf{t}_{it}^f,q_t^r+\mathbf{t}_{it}^r\big)e_{it}\Big)+\phi_{it}^{er}\Big(e_{it}^r-\mathscr{Q}_{q^r}\big(q_t^f+\mathbf{t}_{it}^f,q_t^r+\mathbf{t}_{it}^r\big)e_{it}\Big) \\
&+\phi_{it}^k\Big(f_k(k_{it},e_{it})-r_t^\star\Big)+\phi_{it}^b\Big(\tfrac{1}{1-\vartheta}w_{it}-k_{it}\Big)+\phi_{it}^e\Big(f_e(k_{it},e_{it})-\mathscr{Q}\big(q_t^f+\mathbf{t}_{it}^f,q_t^r+\mathbf{t}_{it}^r\big)\Big) \\
&+\phi_t^{Ef}\big(q_t^f-\mathcal{C}_E^f(\cdot)-\lambda_t^{\mathcal{R}}\Big)+\phi_{it}^{Er}\big(q_{it}^r-\mathcal{C}_e^r(\cdot)\Big)+\phi_t^{\mathcal{I}f}\big(\delta\lambda_t^{\mathcal{R}}-\mathcal{C}_{\mathcal{I}}^f(\cdot)\big)
\end{aligned}$$

Where the marginal product $f_e(k_{it},e_{it})=MPe_{it}$, the CES price aggregator $\mathscr{Q}_{it}(q_t^f,q_{it}^r)$ in the case the energy use function in production is CES:

$$\mathscr{Q}_{it}(q_t^f,q_{it}^r)=\Big(\omega_f(q_t^f)^{1-\sigma_e}+(1-\omega_f)(q_{it}^r)^{1-\sigma_e}\Big)^{\frac{1}{1-\sigma_e}}$$

The FOCs of the Planners w.r.t. all the controls $\{c_{it}, e_{it}, e^f_{it}, e^r_{it}, \mathcal{I}_t\}$ and prices $\{q^f_t, q^r_{it}\}$ as well as taxes, and denote $\tilde{q}_{it} = q_t + \mathbf{t}_{it}$

$$[c_{it}] \qquad \psi^w_{it} = \underbrace{\omega_i u_c(c_i, \tau_{it}) p_i}_{=\text{direct effect}} + \underbrace{\phi^c_{it} u_{cc}(c_{it}, \tau_{it})}_{=\text{effect on savings}}$$

$$[e_{it}] \qquad \psi^w_{it} f_{e,it} + \phi^e_{it} f_{ee,it} - \phi^{ef}_{it} \mathscr{Q}_{q^f} - \phi^{er}_{it} \mathscr{Q}_{q^r} = 0 \qquad \Rightarrow \qquad \phi^e_{it} = \frac{1}{f_{ee,it}} \big(\phi^{ef}_{it} \mathscr{Q}_{q^f} + \phi^{er}_{it} \mathscr{Q}_{q^r} - \psi^w_{it} f_{e,it}\big)$$

$$[e^f_{it}] \qquad \phi^{ef}_{it} = \psi^w_{it} \tilde{q}^f_t - \psi^w_{it} \mathbf{t}^f_{it} - \xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot)$$

$$[e^r_{it}] \qquad \phi^{er}_{it} = \psi^w_{it} \tilde{q}^r_t - \psi^w_{it} \mathbf{t}^r_{it} + \phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot)$$

$$[\tilde{q}^f_{it}] \qquad \phi^e_{it} \mathscr{Q}_{q^f} + \phi^{ef}_{it} \mathscr{Q}_{q^f q^f} + \phi^{er}_{it} \mathscr{Q}_{q^r q^f} = 0$$

$$\Rightarrow \qquad \frac{1}{f_{ee,it}} \big(\phi^{ef}_{it} \mathscr{Q}_{q^f} + \phi^{er}_{it} \mathscr{Q}_{q^r} - \psi^w_{it} f_{e,it}\big) \mathscr{Q}_{q^f} + \big(\psi^w_{it} \tilde{q}^f_t - \psi^w_{it} \mathbf{t}^f_{it} - \xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot)\big) \mathscr{Q}_{q^f q^f} + \big(\psi^w_{it} \tilde{q}^r_t - \psi^w_{it} \mathbf{t}^r_{it} +$$

$$\Rightarrow \qquad \Big(\frac{\mathscr{Q}^2_{q^f}}{f_{ee,it}} + \mathscr{Q}_{q^f q^f}\Big) \big[- \xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^w_{it} \mathbf{t}^f_{it} \big] + \Big(\frac{\mathscr{Q}_{q^f} \mathscr{Q}_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^r q^f}\Big) \big[\phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot) - \psi^w_{it} \mathbf{t}^f_{it}\big]$$

$$[\tilde{q}^r_{it}] \qquad \phi^e_{it} \mathscr{Q}_{q^r} + \phi^{ef}_{it} \mathscr{Q}_{q^f q^r} + \phi^{er}_{it} \mathscr{Q}_{q^r q^r} = 0$$

$$\Rightarrow \qquad \Big(\frac{\mathscr{Q}_{q^f} \mathscr{Q}_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^f q^r}\Big) \big[- \xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^w_{it} \mathbf{t}^f_{it} \big] + \Big(\frac{\mathscr{Q}^2_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^r q^r}\Big) \big[\phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot) - \psi^w_{it} \mathbf{t}^f_{it}\big] = 0$$

$$[q^f_{it}] \qquad \phi^{Ef}_t = \int_{\mathbb{I}} \psi^w_{jt} e^f_{jt} dj - \partial_{q^f} \pi^f(\cdot) \int_{\mathbb{I}} \theta_j \psi^w_{jt} dj$$

$$[\mathcal{I}_t] \qquad \delta\, \psi^{\mathcal{R}}_t + \partial^2_{\mathcal{R}\mathcal{I}} \mathcal{C}(\cdot)\, \psi^{\lambda,\mathcal{R}}_t - \phi^{\mathcal{I}}_t \partial^2_{\mathcal{I}\mathcal{I}} \mathcal{C}(\cdot) = 0$$

$$[q^r_{it}] \qquad \phi^{Er}_{it} = \psi^w_{it} e^r_{it} - \psi^w_{it} \partial^r_q \pi^r_{it} = 0$$

$$[r^\star_t] \qquad \psi^w_{it} w_{it} - \psi^{\lambda w}_{it} \lambda^w_{it} + \phi^k_{it} = 0$$

$$[k_{it}] \qquad \psi^w_{it} [f_{k,it} - (n + \bar{g} + \delta)] + \phi^k_{it} f_{kk,it} - \phi^b_{it} + \phi^e_{it} f_{ek,it} = 0$$

$$\Rightarrow \qquad [\psi^{\lambda w}_{it} u_c(c_{it}, \tau_{it}) - \psi^w_{it} w_{it}] f_{kk,it} - \phi^b_{it} + \phi^e_{it} f_{ek,it} = 0$$

Moreover, noting that $\pi^r_e(\cdot) = 0$ and $\pi^f_E(\cdot) = 0$ and that $\pi^r_e(\cdot) = q^r e^r - \mathcal{C}^r(e^r_{it}, {}_{it}) = \mathcal{C}^r_e(e^r_{it}, \mathscr{C}_{it}) e^r - \mathcal{C}^r(e^r_{it}, \mathscr{C}_{it})$, for isoelastic production function, $\mathcal{C}^r(e, \mathscr{C}) = \frac{\bar{\upsilon}}{1+\upsilon} \big(\frac{e^r_{it}}{\mathscr{C}_{it}}\big)^{1+\upsilon} \mathscr{C}_{it}$, we obtain $\pi^r_q(\cdot) = e^r$ and $\pi^f_q(\cdot) = E^f$.

A simple expression for $\mathbf{t}^f_t$ and $\mathbf{t}^r_t$ are respectively

$$\begin{aligned}
\psi^w_{it} \mathbf{t}^f_{it} &= -\xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) \\
\psi^w_{it} \mathbf{t}^r_{it} &= \phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot) = 0 \\
\mathbf{t}^f_{it} &= \frac{1}{\psi^w_{it}} \Big[- \xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) \Big] \\
\mathbf{t}^r_{it} &= 0
\end{aligned}$$

if the matrix following matrix $M^{\mathbf{t}^f}$ of size $2\times\#\mathbb{I}$ is invertible:

$$M^{\mathbf{t}^f} = \begin{bmatrix} (\frac{\mathscr{Q}^2_{q^f}}{f_{ee,it}} + \mathscr{Q}_{q^f q^f}) & (\frac{\mathscr{Q}_{q^f}\mathscr{Q}_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^r q^f}) \\ (\frac{\mathscr{Q}_{q^f}\mathscr{Q}_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^f q^r}) & (\frac{\mathscr{Q}^2_{q^r}}{f_{ee,it}} + \mathscr{Q}_{q^r q^r}) \end{bmatrix}$$

Applying the Pontryagin Maximum Principle, we obtain the dynamics of the costate / Lagrange multipliers for state dynamics of the system.

$$[w_i] \qquad \dot{\psi}^w_{it} = \psi^w_{it}(\tilde{\rho} - r^\star_t) + \phi^b_{it}\frac{1}{1-\vartheta_{it}}$$

$$[\mathcal{S}_i] \qquad \dot{\psi}^{\mathcal{S}}_t = (\tilde{\rho} + \delta^s)\psi^{\mathcal{S}}_t - \int_{\mathbb{I}} \Delta_j \zeta\chi\psi^\tau_{jt} dj$$

$$[\tau_i] \qquad \dot{\psi}^\tau_t = (\tilde{\rho} + \zeta)\psi^\tau_t - \Big(\partial_{c,\tau}u(c_{it},\tau_{it}) + \psi^w_{it}\mathcal{D}^{y'}_i(\tau_{it})f(k_{it},e_{it}) + \phi^c_{it}\partial_{c,\tau}u(c_{it},\tau_{it}) + \mathcal{D}^{y'}(\tau_{it})\big[\phi^k_{it}f_k + \phi^e_{it}f_e\big]\Big)$$

$$[\mathcal{R}] \qquad \dot{\psi}^{\mathcal{R}}_t = \psi^{\mathcal{R}}_t\Big(\tilde{\rho} - \partial^2_{\mathcal{R}\mathcal{R}}\mathcal{C}(\cdot)\Big) - \phi^{Ef}_t\partial^2_{\mathcal{R}E}\mathcal{C}(\cdot)$$

$$[\lambda^w_i] \qquad \dot{\psi}^{\lambda,k}_t = \psi^{\lambda,k}_t\big[\tilde{\rho} - (\rho - r_{it})\big] + \phi^c_{it}$$

$$[\lambda^{\mathcal{R}}_i] \qquad \dot{\psi}^{\lambda,\mathcal{R}}_t = \psi^{\lambda,\mathcal{R}}_t(\tilde{\rho} - \rho) + \phi^{Ef}_t - \delta\phi^{\mathcal{I}f}_t$$

F Closed form solution for the Social Cost of Carbon

Solving for the shadow cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\psi}^\tau_{it} = \psi^\tau_t(\tilde{\rho} + \Delta\zeta) + \gamma^y_j(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\psi^w_t + \gamma^u_j(\tau-\tau^\star)\mathcal{D}^u(\tau)\underbrace{u'(\mathcal{D}^u(\tau)c)}_{=\psi^w_{it}}c$$

$$\dot{\psi}^S_t = {\psi^S}_t(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i\zeta\chi\psi^\tau_{it}$$

We need to solve for ψ^τ_t and $\psi^{\mathcal{S}}_t$. In stationary equilibrium $\dot{\psi}^S_t = \dot{\psi}^\tau_t = 0$. As a result, we obtain:

$$\begin{aligned}
\psi^\tau_{it} &= -\int_t^\infty e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma^y_j\mathcal{D}^y(\tau_u)y_\tau\psi^w_u + \gamma^u_j\mathcal{D}^u(\tau_u)u'(\mathcal{D}^u(\tau_u)c_u)c_u\Big)du \\
\psi^\tau_{it} &= -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma^y_j\mathcal{D}^y(\tau_\infty)y_\infty + \gamma^u_j\mathcal{D}^u(\tau_\infty)c_\infty\Big)\psi^w_\infty \\
\psi^S_t &= -\int_t^\infty e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\psi^\tau_{j,u}dj\ du \\
&= \frac{1}{\widetilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\psi^\tau_{j,\infty}dj \\
&= -\frac{\chi}{\widetilde{\rho}+\delta^s}\frac{\zeta}{\widetilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma^y_j\mathcal{D}^y(\tau_{j,\infty})y_\infty + \gamma^u_j\mathcal{D}^u(\tau_{j,\infty})c_{j,\infty}\Big)\psi^w_{j,\infty}dj \\
\psi^S_t &\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma^y_j\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty} + \gamma^u_j\mathcal{D}^u(\tau_{j,\infty})c_{j,\infty}\Big)\psi^w_{j,\infty}dj
\end{aligned}$$

which proves the analytical formula in the main text.

Moreover, observing that we obtained an expression for the Social Cost, we can rewrite it as the integral of Local Cost, invoking Fubini's theorem:

$$
\begin{aligned}
\psi_t^S &= -\int_t^{\infty} e^{-(\widetilde{\rho}+\delta^s)u} \zeta\chi\Delta_j\psi_{j,u}^{\tau} dj \; du \\
&= -\int_{\mathbb{I}}\int_t^{\infty} e^{-(\widetilde{\rho}+\delta^s)u} \zeta\chi\Delta_j\psi_{j,u}^{\tau} \, du \, dj \\
&= \int_{\mathbb{I}} \psi_{j,t}^S dj \\
\text{with} \qquad \psi_{j,t}^S &= \int_t^{\infty} e^{-(\widetilde{\rho}+\delta^s)u} \zeta\chi\Delta_j\psi_{j,u}^{\tau} \, du \\
&\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^s}\Delta_j(\tau_{j,\infty}-\tau^{\star})\Big(\gamma_j^y\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}+\gamma_j^u\mathcal{D}^u(\tau_{j,\infty})c_{j,\infty}\Big)\psi_{j,\infty}^w
\end{aligned}
$$