

The Optimal Design of Climate Agreements

Inequality, Trade, and Incentives for Carbon Policy

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on Climate Economics*

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Motivation

- ▶ Fighting climate change requires implementing ambitious carbon reduction policies
 - The “free-riding problem” causes climate inaction:
costs of taxation are local but climate benefits are global
 - Climate policy redistributes across countries through:
(i) change in climate (ii) energy markets, and (iii) reallocation of activity through trade

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 - Trade-off:
Intensive margin: a “climate club” with few countries and large emission reductions
vs. *Extensive margin*: a larger set of countries, at the cost of lowering the carbon tax

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► Preview of the results:

- Despite complete freedom of policy instruments, **impossible** to achieve the world’s optimal policy with complete participation
- Beneficial to **leave several fossil fuels producing countries** outside of the climate agreement
- Welfare improvement with transfers, c.f. UN COP27’s “loss and damage” fund

Literature

► Climate change & optimal carbon taxation

- RA model: Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
- HA model: Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
- Spatial models: Cruz, Rossi-Hansberg (2022, 2023) among others

⇒ *Optimal and constrained policy with heterogeneous countries & trade*

► Unilateral vs. climate club policies:

- Climate clubs: Nordhaus (2015), Non-cooperative taxation: Chari, Kehoe (1990), Suboptimal policy: Hassler, Krusell, Olovsson (2019)
- Trade policy: Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021), Hsiao (2022), Costinot, Donalson, Vogel, Werning (2015), Adao, Costinot (2022), Antras, Fort, Gutierrez, Tintelnot (2022)

⇒ *Climate cooperation and optimal design of climate club*

► Optimal policy in heterogeneous agents models

- Policy with limited instruments: Diamond (1973), Davila, Walther (2022)
- Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) ...

⇒ *Application to climate and carbon taxation policy*

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Literature

► Nordhaus (2015)

- Examine "stable climate coalitions" (club imposing carbon tax) in a simple model
- Abstract from General Equilibrium and distributional effects
- Results: Penalty tariffs necessary to enforce a climate club

► Farrokhi, Lashkaripour (2021)

- Study and characterize the optimal trade policy with climate externality
- General static trade model. Results: unilateral tariffs not effective
- Sequential search for one stable climate club if EU or US join.

► Main contribution:

- Search for the *optimal* climate agreement
- GE on good and energy market and redistribution effects are first-order
- Cost of climate change is endogenous to policy (damages are non-linear)
- Possibility of analyzing other distributional policies (transfers, *loss and damage funds*)
- General framework for analyzing macrodynamics

Model – Household & Firms

► Deterministic Neoclassical economy

- countries $i \in \mathbb{I}$, heterogeneous in productivity z_i , temperature T_i , energy extraction cost C_i
- In each country, five agents:

1. Representative household $\mathcal{V}_i = \max_{c_{ij}} u(c_i)$

$$c_i = \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \quad \sum_j c_{ij} \underbrace{(1+t_{ij}^b)}_{\text{tariff}} \underbrace{\tau_{ij}}_{\text{iceberg cost}} p_j = \underbrace{w_i \ell_i}_{\text{labor income}} + \underbrace{\pi_i^f}_{\text{fossil firm profit}} + t_i^{ls}$$

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2. Competitive final good firm:

$$\max_{\ell_i, e_i^f, e_i^c, e_i^r} p_i \mathcal{D}_i(\mathcal{E}) z_i f(\ell_i, e_i^f, e_i^c, e_i^r) - w_i \ell_i - (q^f + t_i^\varepsilon) e_i^f - (q_i^c + t_i^\varepsilon) e_i^c - q_i^r e_i^r$$

- Externality: Damage function $\mathcal{D}_i(\mathcal{E})$, Income inequality from z_i , Carbon tax: t_i^ε
- Trade, à la Armington

Model – Energy markets & Emissions

3. Competitive fossil fuels (oil-gas) producer, extracting e_i^x

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - C_i^f(e_i^x) \mathbb{P}_i$$

- Energy traded in international markets, at price q^f

$$E^f = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

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$$E^f = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

4. Coal energy firm: elastic supply e_i^c at price $q_i^c = z_i^c \mathbb{P}_i$
5. Renewable energy firm: elastic supply e_i^r at price $q_i^r = z_i^r \mathbb{P}_i$

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- Climate system: mapping from emission $\mathcal{E} = \sum_{\mathbb{I}} e_i^f + e_i^c$ to damage $\mathcal{D}_i(\mathcal{E})$

Model – Equilibrium

- Given policies $\{t_i^\varepsilon, t_{ij}^b, t_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e_i^f, e_i^c, e_i^r, e_i^x\}_{ij}$, emission $\{\mathcal{E}\}_i$ changing climate and prices $\{p_i, w_i, q_i^c, q_i^r\}_i, q^f$ such that:
 - Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint
 - Firm choose inputs $\{e_i^f, e_i^c, e_i^r\}_i$ to max. profit
 - Oil-gas firms extract/produce $\{e_i^x\}_i$ to max. profit. + Elastic renewable, coal supplies $\{e_i^c, e_i^r\}$
 - Emissions \mathcal{E} affects climate and damages $\mathcal{D}_i(\mathcal{E})$
 - Government budget clear $\sum_i t_i^{ls} = \sum_i t_i^\varepsilon (e_i^f + e_i^c) + \sum_{i,j} t_{ij}^b c_{ij} \tau_{ij} p_j$
 - Prices $\{p_i, w_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$ and for each good

$$y_i := \mathcal{D}_i(\mathcal{E}) z_i f(\ell_i, e_i^f, e_i^r, e_i^x) = \sum_{k \in \mathbb{I}} \tau_{ki} c_{ki} + \sum_{k \in \mathbb{I}} \tau_{ki} (x_{ki}^f + x_{ki}^c + x_{ki}^r)$$

with x_{ki} export of good i as input in energy production in k

In expenditure, with import shares $s_{ij} = \frac{c_{ij} \tau_{ij} p_j}{c_i \mathbb{P}_i}$, it yields

$$p_i y_i = \sum_{k \in \mathbb{I}} \frac{s_{ki}}{1 + t_{ki}^b} (p_k y_k + q^f (e_k^x - e_k^f) + \tilde{t}_k^{ls})$$

Ramsey Problem with endogenous participation

- **Definition:** A climate agreement is a set $\{\mathbb{J}, \mathbf{t}^\varepsilon, \mathbf{t}^b\}$ of $\mathbb{J} \subseteq \mathbb{I}$ countries and a C.E. s.t.:
- Countries $i \in \mathbb{J}$ pay carbon tax \mathbf{t}^ε
 - If j **exits** the agreement, club members $i \in \mathbb{J}$ impose uniform tariffs $\mathbf{t}_{ij}^b = \mathbf{t}^b$ on goods from j
They still trade with club members in oil-gas at price q^f
 - Exit: unilateral deviation $\mathbb{J} \setminus \{j\}$, \Rightarrow **Nash equilibrium**
- Participation constraints, given indirect utility $U_i(\mathbb{J}, \mathbf{t}^\varepsilon, \mathbf{t}^b) \equiv u(c_i(\mathbb{J}, \mathbf{t}^\varepsilon, \mathbf{t}^b))$

$$U_i(\mathbb{J}, \mathbf{t}^\varepsilon, \mathbf{t}^b) \geq U_i(\mathbb{J} \setminus \{i\}, \mathbf{t}^\varepsilon, \mathbf{t}^b) \quad [\text{Nash equilibrium}]$$

- Objective: search for the optimal *and stable* climate agreement

$$\begin{aligned} \max_{\mathbb{J}, \mathbf{t}^\varepsilon, \mathbf{t}^b} \mathcal{W}(\mathbb{J}, \mathbf{t}^\varepsilon, \mathbf{t}^b) &= \max_{\mathbf{t}^\varepsilon, \mathbf{t}^b} \max_{\mathbb{J}} \sum_{i \in \mathbb{I}} \omega_i U_i(\mathbb{J}, \mathbf{t}^\varepsilon, \mathbf{t}^b) \\ \text{s.t.} \quad \mathbb{J} \in \mathbb{S}(\mathbf{t}^\varepsilon, \mathbf{t}^b) &= \left\{ \mathcal{J} \mid U_i(\mathbb{J}, \mathbf{t}^\varepsilon, \mathbf{t}^b) \geq U_i(\mathbb{J} \setminus \{i\}, \mathbf{t}^\varepsilon, \mathbf{t}^b) \ \forall i \in \mathcal{J} \right\} \end{aligned}$$

Ramsey Problem with endogenous participation

- **Objective:** optimal *and stable* climate agreement $\{\mathbb{J}, t^e, t^b\}$

$$\max_{\mathbb{J}, t^e, t^b} \mathcal{W}(\mathbb{J}, t^e, t^b) = \max_{t^e, t^b} \max_{\mathbb{J}} \sum_{i \in \mathbb{I}} \omega_i U_i(\mathbb{J}, t^e, t^b)$$

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- Alternative: **Coalitional Nash-equilibrium** $\mathbb{C}(t^f, t^b)$: robust of sub-coalitions deviations:

$$\mathbb{J} \in \mathbb{C}(t^f, t^b) = \left\{ \mathcal{J} \mid U_i(\mathbb{J}, t^f, t^b) \geq U_i(\mathbb{J} \setminus \hat{\mathbb{J}}, t^f, t^b) \ \forall i \in \hat{\mathbb{J}} \ \& \ \forall \hat{\mathbb{J}} \subseteq \mathcal{J} \cup \{i\} \right\}$$

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- No country i and subcoalition $\hat{\mathbb{J}}$ would be better off than in the current agreement \mathbb{J}

- Current design: (i) choose taxes $\{t^f, t^b\}$,
(ii) choose the coalition \mathbb{J} s.t. participation constraints hold
- Solution method (Nash equilibrium):
 - relies on the complementarity of the combinatorial discrete choice problem and use a “squeezing procedure”, c.f. Jia (2008), Arkolakis, Eckert, Shi (2023), to handle the problem

Quantification

- ▶ Energy parameters to match production/reserves,
 - Isoelastic cost function $C_i(e_i^x) = \bar{\nu}_i (e_i^x / \mathcal{R}_i)^{1+\nu_i} \mathcal{R}_i$
 - Use $\bar{\nu}_i, \nu_i$ to match e_i^x and π_i^f ,
- ▶ Armington model,
 - Iceberg cost τ_{ij} projected on distance and preferences a_{ij} to match import shares $s_{ij} = \frac{c_{ij} \tau_{ij} p_j}{c_i \mathbb{P}_i}$
- ▶ Production $\bar{y} = z_f(\ell_i, k_i, e_i^f, e_i^r)$
 - Nested CES energy vs. labor-capital Cobb-Douglas bundle $k_i^\alpha \ell_i^{1-\alpha}$ (elasticity $\sigma_y < 1$), and fossil/renewable $\sigma_e > 1$, $CES(e_i^f, e_i^r)$
 - TFP, and DTC, z_i, z_i^e , calibrated to match GDP / energy shares data.
- ▶ Pareto weights ω_i :
 - Imply no redistribution motive, \bar{c}_i consumption in initial equilibrium $t = 2020$

$$\omega_i = \frac{1}{u'(\bar{c}_i)}$$

- ▶ Details [More details](#) [Details Pareto weights](#)

Quantification – Climate system and damage

► Static economic model:

decisions $e_i^f + e_i^c$ taken “once and for all”, $\mathcal{E} = \sum_i e_i^f + e_i^c$

- Climate system:

$$\dot{S}_t = \mathcal{E} - \delta_s S_t$$

$$T_{it} = \bar{T}_{i0} + \Delta_i S_t$$

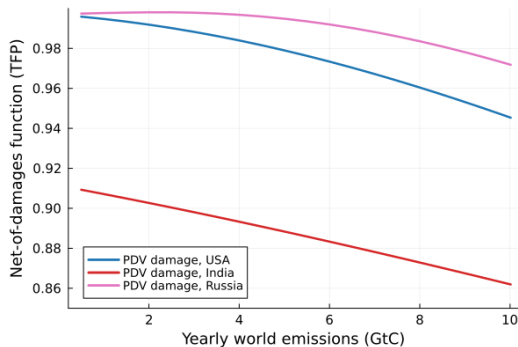
- Path of period damages heterogeneous across countries. Quadratic (c.f. Nordhaus-DICE)

$$\mathcal{D}_i(T_{it}) = e^{-\gamma(T_{it} - T_i^*)^2}$$

- Economic feedback in Present discounted value

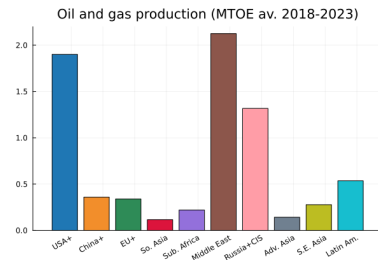
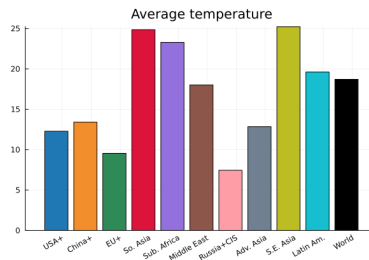
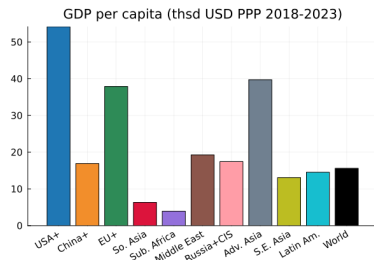
$$\mathcal{D}_i(\mathcal{E}) = \int_0^\infty e^{-\rho t} \mathcal{D}(T_{it}) dt$$

- Similarly for $LCC_i, SCC_i \dots$



Quantitative application – Sample of 10 “regions”

- ▶ Sample of 10 “regions”: (i) US+Canada, (ii) China+HK, (iii) UE+UK+Schengen, (iv) South Asia, (v) Sub-saharian Africa, (vi) Middle-East+Maghreb, (vii) Russia+CIS, (viii) Japan+Korean+Australia+Asian Dragons, (ix) South-East Asia (Asean), (x) Latin America **WIP: 25 countries + 5 regions**
- ▶ Data (Avg. 2018-2023) on macro variables, energy markets, trade shares $s_{ij} = \frac{c_{ij}\tau_{ij}p_j}{c_i p_i}$, etc.



Details [Trade shares – details](#)

Optimal policy : benchmarks

- ▶ Three policy benchmarks, c.f. Bourany (2024), without endogenous participation
 - ***First-Best***, Social planner maximizing global welfare with unlimited instruments
 - Pigouvian result: Carbon tax = Social Cost of Carbon
 - Relies heavily on cross-country transfers to offset redistributive effects

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 - Relies heavily on cross-country transfers to offset redistributive effects
- **Second-Best**: Social planner, single carbon tax without transfers
 - Optimal carbon tax t^e correct climate externality, but also accounts for:
 - (i) Redistribution motives, G.E. effects on (ii) energy markets and (iii) trade leakage

$$t^e = \underbrace{\sum_i \phi_i LCC_i}_{=SCC} + \sum_i \phi_i \text{Supply Redistribution}_i + \sum_i \phi_i \text{Demand Distortion}_i \quad \phi_i \propto \omega_i(u'(c_i) + \mu_i)$$

- Details: *Competitive equilibrium* [Details eq 0](#), *First-Best*, with unlimited instruments [Details eq 1](#), *Second-best*, Ramsey policy with limited instruments [Details eq 2](#)

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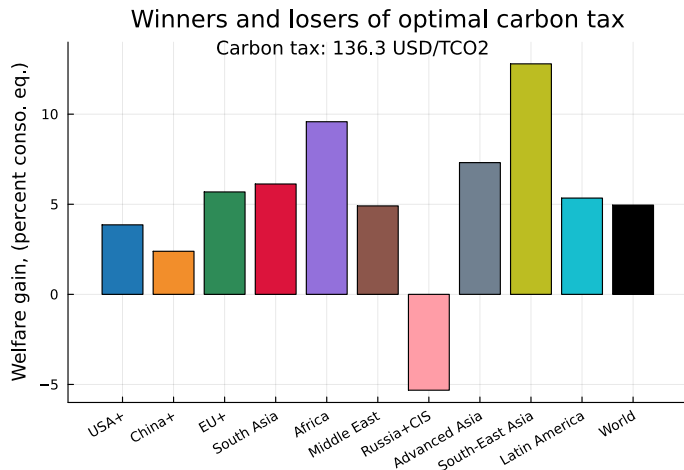
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Second-best, Ramsey policy with limited instruments [Details eq 2](#)

- **Unilateral policy**: Local planner in country i unilaterally choosing t_i^ε and t_{ij}^b
 - Optimal unilateral carbon tax (subsidy!) and tariffs for terms-of-trade manipulations
 - Nash equilibrium of \mathbb{I} countries choosing individually unilateral policies

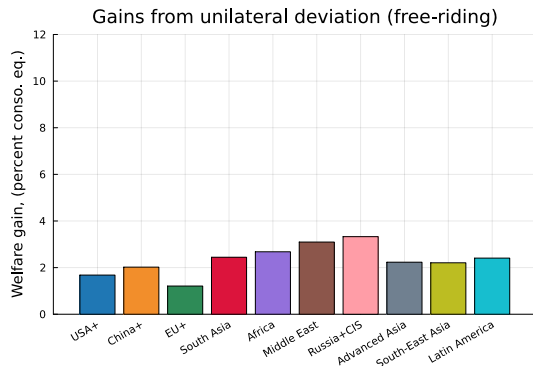
Gains from cooperation – World Optimal policy

- ▶ Optimal carbon tax
(Second Best): $\sim \$136/tCO_2$
- ▶ Reduce fossil fuels / CO_2 emissions by 40% compared to Business as Usual (BAU)
- ▶ Welfare difference btw world optimal policy w/o participation constraints vs BAU (Comp. Eq.)



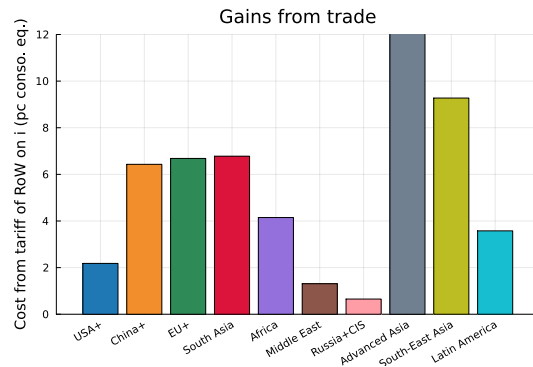
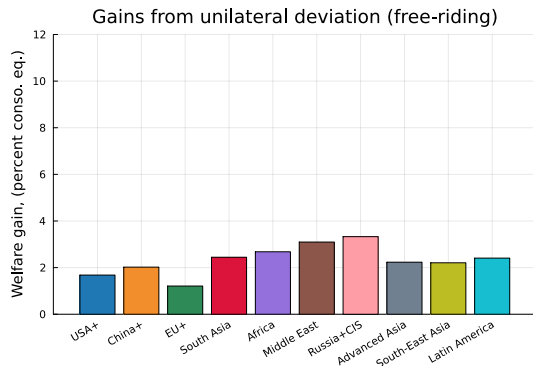
Trade-off – Cost of Carbon Taxation vs. Gains from trade

Gains from **unilateral exit** from agreement vs. **Gains from trade**, i.e. loss from tariffs/autarky



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Theoretical investigation: decomposing the welfare effects

► Experiment:

- Start from the equilibrium where carbon tax $\tau_j^f = 0$, $\tau_{jk}^b = 0, \forall j$,
- Change in welfare: Linear approximation around that point \Rightarrow small changes in carbon tax $d\tau_j^f, \forall j$ and tariffs $d\tau_{j,k}^b, \forall j, k$

$$\frac{d\mathcal{V}_i}{u'(c_i)} = \eta_i^c \frac{dp_i}{p_i} + \left[\eta_i^c \gamma_i \frac{1}{\bar{\nu}} - \eta_i^c s_i^e + \eta_i^\pi \left(1 + \frac{1}{\bar{\nu}}\right) \right] \frac{dq^f}{q^f} + \dots$$

- Difference in the GE effect on energy markets $\frac{dq^f}{q^f} \approx \bar{\nu} \frac{dE^f}{E^f} + \dots$, due to taxation

$$\frac{dq^f}{q^f} = -\sum_j \nu_j^f \frac{d\tau_j^f}{\tau_j^f} + \sum_i \nu_j^{p,R} \frac{dp_i}{p_i} + \sum_{j,k} \nu_j^{R,f,z,qr} s_{j,k} \frac{d\tau_{jk}^b}{\tau_{jk}^b}$$

- Trade and leakage effect: GE impact of τ_j^f and τ_{jk}^b on y_i and p_i
- Simplifying assumption: no renewable
- Params: σ energy demand elast^y, s^e energy cost share, $\bar{\nu}$ energy supply inverse elas^y, Climate damage γ_i

Decomposing the welfare effects: gains from trade

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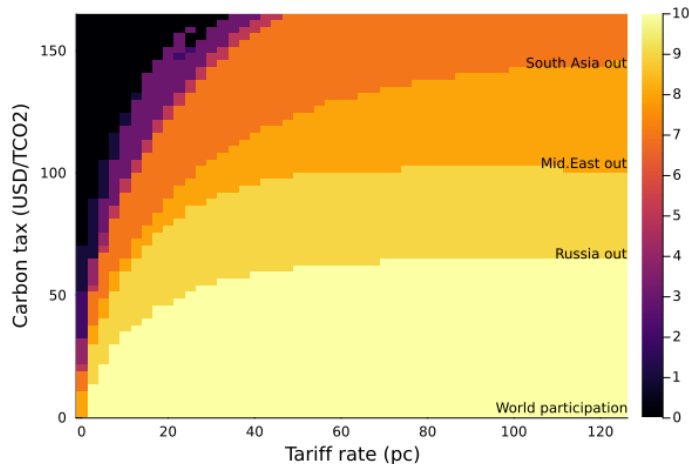
$$\frac{dp}{p} = \left[\mathbf{I} - \mathbf{T} - (\theta - 1) [\mathbf{T} \odot \mathbf{S} - (\mathbf{T} \odot \mathbf{I})'] \right]^{-1} \left((\mathbf{T} - \mathbf{I}) \frac{dy}{y} + (\mathbf{T} [(\theta - 1) \mathbf{I} - \theta \mathbf{S}] \odot \frac{dt^b}{t^b}) \mathbf{1} \right)$$

$$\frac{dy_i}{y_i} = \alpha_i^{p,qr} \frac{dp_i}{p_i} - \alpha_i^{qf} \frac{dt_i^f}{t_i^f} + \dots$$

- Params: \mathbf{S} Trade share matrix, \mathbf{T} income flow matrix, θ , Armington CES

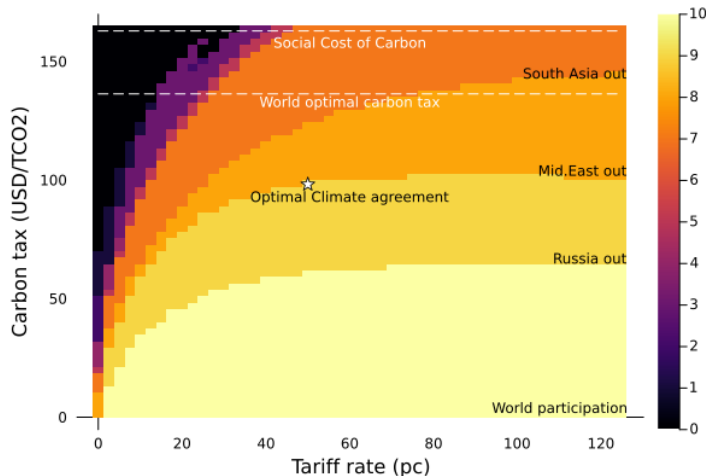
Climate Agreements: Intensive vs. Extensive Margin

- **Intensive margin:**
higher tax, emissions ↓, welfare ↑
- **Extensive margin:**
higher tax, participation ↓,
free-riding and emissions ↑



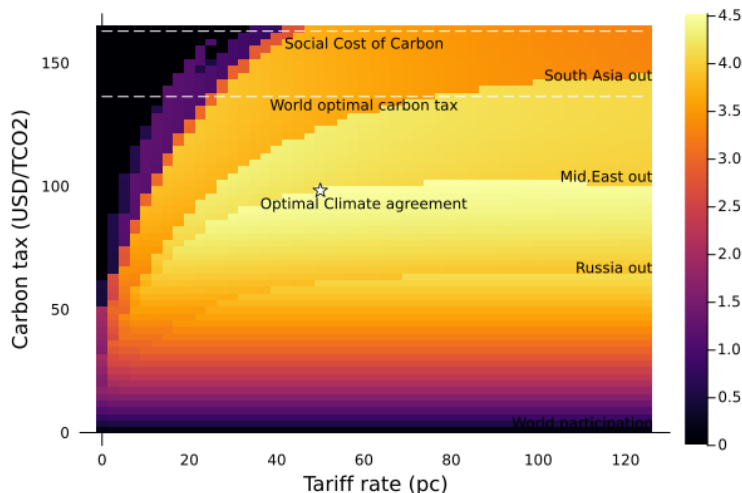
Optimal Climate Agreement

- ▶ Despite full freedom of instruments (t^e, t^b)
 ⇒ can not sustain an agreement with Russia & Middle East
 ⇒ need to reduce carbon tax from \$136 to \$98
- ▶ Intuition:
 relatively cold and closed economy, and fossil-fuel producers



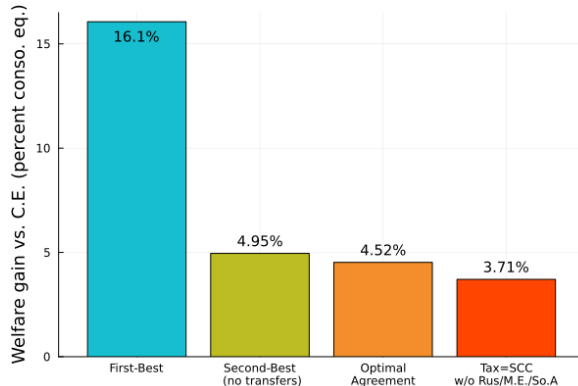
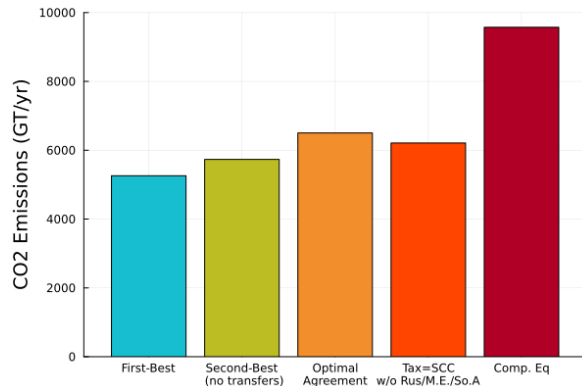
Climate agreement and welfare

Recover 91% of welfare gains, i.e. 4.5% out of 5% conso equivalent.



Welfare and emission reduction: Different metrics!

- Agreements with tariffs recover 91% of welfare gains from the Second-Best – optimal carbon tax without transfers – at a cost of increasing emissions by 13%
- First-best allocation relies heavily on transfers to be able to impose a higher carbon tax

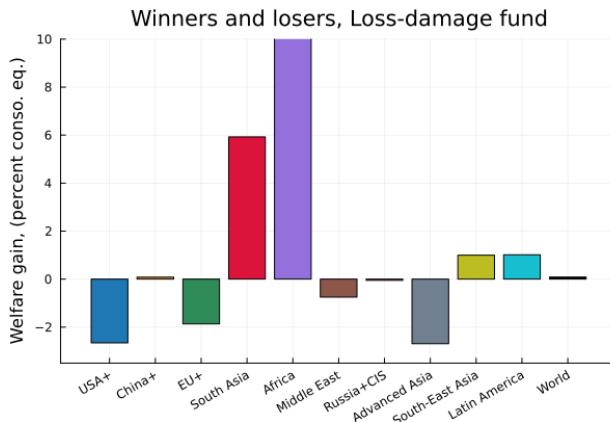


Transfers – Loss and damage funds

- ▶ COP28 Major policy proposal:
Loss and damage funds for countries vulnerable to the effects of climate change
- ▶ Simple implementation in our context: lump-sum receipts of carbon tax revenues:

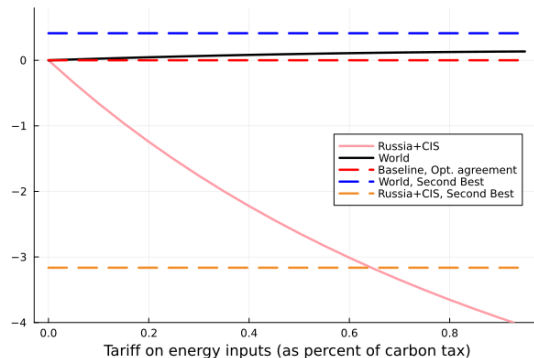
$$t_i^{ls} = (1 - \alpha)t^\varepsilon \varepsilon_i + \alpha \frac{1}{P} \sum_j t^\varepsilon \varepsilon_j$$

- ▶ In practice: transfers from large emitters to low emitters



Taxation of fossil fuels energy inputs

- ▶ Current climate club:
only imposes penalty tariffs on final goods,
not on energy imports
 - Empirically relevant, c.f. Shapiro (2021):
inputs are more emission-intensives but trade
policy is biased against final goods output
- ▶ Alternative: tax energy import from
non-participants $t_{ij}^{bf} = \beta t^b \mathbb{1}\{i \in \mathbb{J}, j \notin \mathbb{J}\}$



Dynamic coalition formation

- Current “equilibrium”: $t_i^\varepsilon = 0, t_{ij}^b = 0$
- Optimal club equilibrium $t_i^\varepsilon = t^{\varepsilon*}, t_{ij}^b = t^{b*} \mathbb{1}\{i \in \mathbb{J}, j \notin \mathbb{J}\}$
- Optimal agreement follows the planner taxes and participation decision:

$$\mathbb{J}^* = \mathbb{J}(t^{\varepsilon*}, t^{b*})$$

► What is driving the coordination failure?

- Possible explanation: coalition building and *bargaining* may never reach such equilibrium:

$$\bar{\mathbb{J}}_{t_0}(0, 0) = \mathbb{I} \quad \xrightarrow[t \rightarrow T]{?} \quad \bar{\mathbb{J}}_T(t^{\varepsilon*}, t^{b*}) = \mathbb{J}^*$$

- Can we find a sequence $\bar{\mathbb{J}}_t, t_t^\varepsilon, t_t^b$ such that

$$\{\bar{\mathbb{J}}_{t_0}, t_{t_0}^\varepsilon, t_{t_0}^b\} = \{\bar{\mathbb{J}}_{t_0}, 0, 0\} \quad \{\bar{\mathbb{J}}_T, t_T^\varepsilon, t_T^b\} = \{\bar{\mathbb{J}}_T, t^{\varepsilon*}, t^{b*}\}$$

- Instruments used by leader countries (e.g. E.U., U.S. or China?) to reach such agreement?

Conclusion

- ▶ In this project, I solve for the optimal design of climate agreements
 - Correcting for inequality, redistribution effects through energy markets and trade leakage, as well as free-riding incentives
- ▶ Climate agreement design jointly solves for:
 - The optimal choice of countries participating
 - The carbon tax and tariff levels, accounting for both the climate externality, redistributive effects and the participation constraints
- ▶ Optimal coalition depends on the trade-off between
 - the gain from cooperation and free riding incentives
 - the gain from trade, i.e. the cost of retaliatory tariffs

⇒ Need a large coalition and a carbon at 70% of the world optimum
- ▶ Extensions:
 - Extend this to dynamic settings: coalition building
 - Explore additional policy proposal to improve the optimal agreement

Conclusion

Thank you!

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Appendices

Welfare and Pareto weights

- Welfare:

$$\mathcal{W}(\mathbb{J}) = \sum_{i \in \mathbb{I}} \omega_i u(c_i)$$

- Pareto weights ω_i :

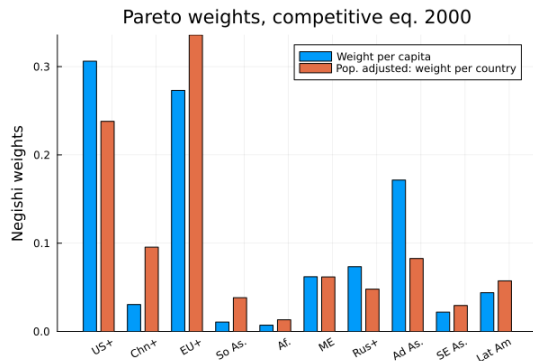
$$\omega_i = \frac{1}{u'(\bar{c}_i)}$$

for \bar{c}_i consumption in initial equilibrium
“without climate change“, i.e. year = 2020

- Imply no redistribution motive in $t = 2020$

$$\omega_i u'(\bar{c}_i) = \omega_j u'(\bar{c}_j) \quad \forall i, j \in \mathbb{I}$$

- Climate change, taxation, and climate agreement (tax + tariffs) have redistributive effects
 \Rightarrow change distribution of c_i



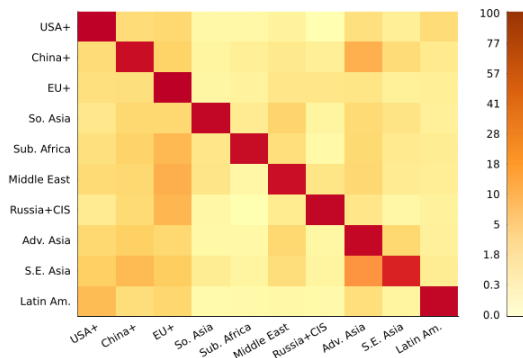
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Quantification – Trade model

- Armington Trade model:

$$s_{ij} \equiv \frac{c_{ij}p_{ij}}{c_i p_i} = a_{ij} \frac{((1+t_{ij})\tau_{ij}p_j)^{1-\theta}}{\sum_k a_{ik}((1+t_{ik})\tau_{ik}p_k)^{1-\theta}}$$

- CES $\theta = 5.63$ estimated from a gravity regression
- Iceberg cost τ_{ij} as projection of distance
 $\log \tau_{ij} = \beta \log d_{ij}$
- Preference parameters a_{ij} identified as remaining variation in the trade share s_{ij}
 \Rightarrow policy invariant



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Step 0: Competitive equilibrium & Trade

- ▶ Each household in country i maximize utility and firms maximize profit
- ▶ Standard trade model results:
 - Consumption and trade:

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i \mathbb{P}_i} = a_{ij} \frac{(\tau_{ij}(1+t_{ij}^b)p_j)^{1-\theta}}{\sum_k a_{ik}(\tau_{ik}(1+t_{ik}^b)p_k)^{1-\theta}} \quad \& \quad \mathbb{P}_i = \left(\sum_j a_{ij}(\tau_{ij}p_j)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Energy consumption doesn't internalize climate damage:

$$p_i MPe_i = q^e$$

- Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

- “Local Social Cost of Carbon”, for region i

$$LCC_i = \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_{if}(e_i^f) \frac{p_i}{\mathbb{P}_i} \quad (> 0 \text{ if heat causes losses})$$

Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- Full array of instruments: cross-countries lump-sum transfers \mathbf{t}_i^{ls} , individual carbon taxes \mathbf{t}_i^f on energy \mathbf{e}_i^f , bilateral tariffs \mathbf{t}_{ij}^b
 - Budget constraint: $\sum_i \mathbf{t}_i^{ls} = \sum_i \mathbf{t}_i^f \mathbf{e}_i^f + \sum_{i,j} \mathbf{t}_{ij}^b c_{ij} \tau_{ij} \mathbf{p}_j$
- Maximize welfare subject to
- Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1: World First-best policy

► Social planner results:

- Consumption:

$$\omega_i u'(c_i) = \left[\sum_j a_{ij} (\tau_{ij} \omega_j \mu_j)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

- Energy use:

$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

- Social cost of carbon:

$$SCC = - \frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}'_j(T_j) \bar{y}_j}{\frac{1}{I} \sum_j \omega_j \mu_j}$$

back

Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- One single instrument: uniform carbon tax \mathfrak{t}^f on energy e_i^f
 - Rebate tax lump-sum to HHs $\mathfrak{t}_i^{ls} = \mathfrak{t}^f e_i^f$
- Ramsey policy: Primal approach, maximize welfare subject to
- Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
 - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
 - Trade-off faced by the planner:
 - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

back

Step 2: World optimal Ramsey policy

- The planner takes into account

- (i) the marginal value of wealth λ_i
- (ii) the shadow value of good i , from market clearing, μ_i :

w/o trade $\omega_i u'(c_i) = \omega_i \lambda_i$

vs. w/ trade in goods: $\omega_i u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij} (\tau_{ij} p_j)^{1-\theta} \left[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij} (1 - s_{ij}) \right] \right)^{\frac{1}{1-\theta}}$

- Relative welfare weights, representing inequality

w/o trade: $\hat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \leq 1 \quad \Rightarrow \quad \text{ceteris paribus, poorer countries have higher } \hat{\lambda}_i$

vs. w/ trade: $\hat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)} \leq 1$

Step 2: Optimal policy – Social Cost of Carbon

► Key objects: Local vs. Global Social Cost of Carbon:

- Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i
- “Local social cost of carbon” (LCC) for region i :

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_{if}(e_i^f) p_i \quad (> 0 \text{ if heat causes losses})$$

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- Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i
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- Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W} / \partial \mathcal{E}}{\partial \mathcal{W} / \partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

- Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{Cov}_i(\hat{\lambda}_i, LCC_i)$$

Step 2: Optimal policy – Other motives

- ▶ Taxing fossil energy has additional redistributive effects:
 1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
 2. Distort energy demand, of countries that need more or less energy
- ▶ New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W} / \partial E}{\partial \mathcal{W} / \partial w} = \mathcal{C}_{EE}^f \mathbb{Cov}_i \left(\hat{\lambda}_i, e_i^f - e_i^x \right) - \mathbb{Cov}_i \left(\hat{\lambda}_i, \frac{q^f (1 - s_i^f)}{\sigma} \right)$$

- Params: \mathcal{C}_{EE}^f agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

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- Params: \mathcal{C}_{EE}^f agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity
- ▶ Proposition 2: Optimal fossil energy tax:

$$\Rightarrow \quad t^f = SCC + SVF$$

- Social cost of carbon: $SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i$

Step 3: Ramsey Problem with participation constraints

- ▶ Consider that countries can “exit” climate agreement.
- ▶ For a climate “club” of $\mathbb{J} \subset \mathbb{I}$ countries:
 - Countries $i \in \mathbb{J}$ are subject to a carbon tax τ^f
 - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $\tau^{b,r}$ on goods and get consumption \tilde{c}_i
 - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff τ^b with club members and countries outside the club. They still trade with the club members in energy at price q^f

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 - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f
- ▶ Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- ▶ Welfare:

$$\mathcal{W} = \max_{\{t, e, q\}_i} \sum_{\mathbb{J}} \omega_i u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i u(c_i)$$

Step 3: Ramsey Problem with participation constraints

► Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

► Proposition 3.1: Second-Best social valuation with participation constraints

- Participation incentives change our measure of inequality

$$\text{w/ trade:} \quad \omega_i(1+\nu_i)u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij}(\tau_{ij}p_j)^{1-\theta} \left[\omega_i \tilde{\lambda}_i + \omega_j \tilde{\mu}_j + \tilde{\eta}_{ij}(1-s_{ij}) \right] \right)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(\tilde{\lambda}_i + \tilde{\mu}_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(\tilde{\lambda}_i + \tilde{\mu}_i)} \neq \hat{\lambda}_i$$

$$\text{vs. w/o trade} \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(1+\nu_j)u'(c_j)} \neq \hat{\lambda}_i$$

- Similarly, the “effective Pareto weights” are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

Step 3: Participation constraints & Optimal policy

► Proposition 3.2: Second-Best taxes:

- Taxation with imperfect instruments:
 - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
 - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu \sigma}{q^f (1-s^f)}$
with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
- Optimal fossil energy tax $t^f(\mathbb{J})$:

$$\begin{aligned} \Rightarrow \quad t^f(\mathbb{J}) &= \text{SCC} + \text{SVF} \\ &= \frac{1}{1 - \vartheta_{\mathbb{J}^c}} \sum_{i \in \mathbb{I}} \tilde{\lambda}_i \text{LCC}_i + \frac{1}{1 - \vartheta_{\mathbb{J}^c}} C_{EE}^f \sum_{i \in \mathbb{I}} \tilde{\lambda}_i (e_i^f - e_i^x) - \sum_{i \in \mathbb{J}} \tilde{\lambda}_i \frac{q^f (1-s_i^f)}{\sigma} \end{aligned}$$

- Optimal tariffs/export taxes $t^{b,r}(\mathbb{J})$ and $t^b(\mathbb{J})$: In search for a closed-form expression
As of now, only opaque system of equations (fixed point w/ demand/multipliers)

Countries' incentives – Model w/o trade in goods

- ▶ Experiment: Model with trade in energy but not in “goods”
 - Start from the equilibrium where carbon tax $\tau^f(\mathbb{J}) = 0$,
 \Rightarrow country i is indifferent to join the club \mathbb{J} or not
 - Linear approximation around that point \Rightarrow small changes in carbon tax $d\tau^f$

Countries' incentives – Model w/o trade in goods

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 - Linear approximation around that point \Rightarrow small changes in carbon tax dt^f
 - Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})} = & -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta y_i \Delta_i (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \pi_i \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \end{aligned}$$

- Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}} = - \left(E_{\mathbb{J}} - E_{\mathbb{J} \setminus \{i\}} \right) \frac{\sigma dt^f}{q^f (1 - s^f)} \frac{1}{1 + \frac{\nu \sigma}{(1 - s^f)}}$$

- Params: σ energy demand elast^y, s^f energy cost share, ν energy supply elas^y, Climate damage γ_i and curv. δ

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- Params: σ energy demand elast^y, s^f energy cost share, ν energy supply elas^y, Climate damage γ_i and curv. δ

Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$
 - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})c_i} = & -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & + \eta_i^y \left(\frac{dp_i}{p_i} \Big|_{i \in \mathbb{J}} - \frac{dp_i}{p_i} \Big|_{i \notin \mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j \in \mathbb{I}} s_{ij} \left(\frac{dp_j}{p_j} \Big|_{i \in \mathbb{J}} - \frac{dp_j}{p_j} \Big|_{i \notin \mathbb{J}} \right) \end{aligned}$$

Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$

- Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})c_i} = & -e_i dt^f - \gamma_i(T_i - T_{i0})^\delta \eta_i^y \Delta_i(dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^f \nu}{E_{\mathbb{J}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & + \eta_i^y \left(\frac{dp_i}{p_i} \Big|_{i \in \mathbb{J}} - \frac{dp_i}{p_i} \Big|_{i \notin \mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j \in \mathbb{I}} s_{ij} \left(\frac{dp_j}{p_j} \Big|_{i \in \mathbb{J}} - \frac{dp_j}{p_j} \Big|_{i \notin \mathbb{J}} \right) \end{aligned}$$

- GE effect on energy markets, same as before
- GE effect on goods markets, equilibrium on expenditure $v_i = p_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k \in \mathbb{I}} \mathcal{P}_k \alpha_{ki} \left(\frac{dv_k}{v_k} - \theta \frac{dt_{ki}^b}{1+t_{ki}^b} \right) \quad \alpha_{ki} = \frac{c_{ki} \tau_{ki} p_i}{\sum_{\ell} c_{k\ell} \tau_{k\ell} p_{\ell}} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+t_{ki}^b) v_i}$$

- Params: σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i p_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$

- Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned} \frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})c_i} = & -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & - e_i \frac{q^f \nu}{E_{\mathbb{J}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) \\ & + \eta_i^y \left(\frac{dp_i}{p_i} \Big|_{i \in \mathbb{J}} - \frac{dp_i}{p_i} \Big|_{i \notin \mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j \in \mathbb{I}} s_{ij} \left(\frac{dp_j}{p_j} \Big|_{i \in \mathbb{J}} - \frac{dp_j}{p_j} \Big|_{i \notin \mathbb{J}} \right) \end{aligned}$$

- GE effect on energy markets, same as before
- GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k \in \mathbb{I}} \mathcal{P}_k \alpha_{ki} \left(\frac{dv_k}{v_k} - \theta \frac{dt_{ki}^b}{1+t_{ki}^b} \right) \quad \alpha_{ki} = \frac{c_{ki} \tau_{ki} p_i}{\sum_{\ell} c_{k\ell} \tau_{k\ell} p_{\ell}} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+t_{ki}^b) v_i}$$

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Complementarity in coalition formation – Model w/o trade in goods

- Is marginal gain $\Delta\mathcal{W}(\mathbb{J}, j) := \mathcal{W}(\mathbb{J} \cup j) - \mathcal{W}(\mathbb{J})$ “growing” in \mathbb{J} ?
- Linear approximation for small $\{t^f, t^b\}$

$$\begin{aligned} \Delta\mathcal{W}(\mathbb{J}, j) = & -\omega_j u'(c_j) e_j dt^f + \left[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) \Delta_i \gamma_i (T_i - T_{i0})^\delta y_i \right] \frac{\sigma e_j dt^f}{q^f (1 - s^f + \nu \sigma)} \\ & + \left[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) e_i \right] \frac{1}{1 + \frac{1-s^f}{\nu \sigma}} \frac{e_j dt^f}{E_{\mathbb{I}}} - \left[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) \pi_i \right] \frac{(1+\nu)}{E_{\mathbb{I}}} \frac{\sigma e_j dt^f}{q^f (1 - s^f + \nu \sigma)} \end{aligned}$$

- Free-riding problem: $\Delta\mathcal{W}(\mathbb{J}, j)$ could be negative
- If $\Delta\mathcal{W}(\mathbb{J}, j) > 0$, what effects does \mathbb{J} have on marginal gain?
 - Marginal climate benefit decreases in \mathbb{J} , since temperature T_i declines!
 - G.E. effect on energy price: $E_{\mathbb{I}}$, q and π^f decreases with \mathbb{J} , effect on demand ambiguous
 - Similar formula for the case with trade tariffs: Work in progress.

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(T_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \ell, \varepsilon(e^f, e^r)) = \left[(1 - \epsilon_i)^{\frac{1}{\sigma_y}} (k^\alpha \ell^{1-\alpha})^{\frac{\sigma_y-1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} (z_i^e \varepsilon_i(e^f, e^r))^{\frac{\sigma_y-1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y-1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega_i^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e-1}{\sigma_e}} + (1 - \omega_i)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e-1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e-1}}$$

- Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
- Today: $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
- Future: $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

- Damage functions in production function y :

$$\mathcal{D}_i^y(T) = e^{-\gamma_i^{\pm,y}(T-T_i^*)^2}$$

- Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y} \mathbb{1}_{\{T > T_i^*\}} + \gamma^{-,y} \mathbb{1}_{\{T < T_i^*\}}$
- Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $T_i^* = \bar{\alpha} T_{it0} + (1 - \bar{\alpha}) T^*$

Quantification – Energy markets

► Fossil production e_{it}^x and reserve \mathcal{R}_{it}

- Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i} \left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i} \mathcal{R}$
- Now: $\bar{\nu}_i$ to match extraction data e_i^x , \mathcal{R}_{it} calibrated to *proven reserves* data from BP. ν_i extraction cost curvature to match profit $\pi_i^f = \frac{\bar{\nu}_i \nu_i}{1+\nu_i} \left(\frac{e_i^x}{\mathcal{R}_i}\right)^{\nu_i} \mathcal{R}_i \mathbb{P}_i$
- Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost \mathcal{C}_e & extraction data e_i^x (BP, IEA)

Quantification – Energy markets

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- Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost \mathcal{C}_e & extraction data e_i^x (BP, IEA)

► Renewable: Production \bar{e}_{it}^r and price q_{it}^r

- Now: $q_{it}^r = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
- Future: Choose z_i^r to match the energy mix (e_i^f, e_i^r)

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Quantification – Future Extensions:

► Damage parameters:

- $\gamma_i^{\pm,y}$ depends on daily temperature distribution $T \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
- Use Climate Lab's (Greenstone et al) estimates for damage γ_i ?

► Fossil Energy markets:

- Divide fossils e_{it}^f / e_{it}^x into oil/gas/coal
- Match the production/cost/reserves data across countries
- Use a dynamic model: extraction/exploration a la Hotelling

► Renewables Energy markets:

- Make the problem dynamic with investment in capacity C_{it}^r

► Population dynamics

- Match UN forecast for growth rate / fertility

Calibration

Table: Baseline calibration (★ = subject to future changes)

<i>Technology & Energy markets</i>			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
\bar{g}	0.01★	Long run TFP growth	Conservative estimate for growth
g_e	0.01★	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	-0.01★	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	2★	Extraction elasticity of fossil energy	Cubic extraction cost
<i>Preferences & Time horizon</i>			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	0.01★	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

Table: Baseline calibration (★ = subject to future changes)

<i>Climate parameters</i>			
ξ	0.81	Emission factor	Conversion 1 <i>MTOE</i> \Rightarrow 1 <i>MT CO₂</i>
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature \sim 11–15 years
χ	2.1/1e6	Climate sensitivity	Pulse experiment: 100 <i>GtC</i> \equiv 0.21°C medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment: 100 <i>GtC</i> \equiv 0.16°C long-term warming
γ^{\oplus}	0.00234★	Damage sensitivity	Nordhaus' DICE
γ^{\ominus}	$0.2 \times \gamma^{\oplus}$ ★	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^T	0.2★	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
T^{\star}	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
<i>Parameters calibrated to match data</i>			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
T_i		Local Temperature	To match temperature of largest city
\mathcal{R}_i		Local Fossil reserves	To match data from BP Energy review

Sequential solution method

► Summary of the model:

- ODEs for states $\{\mathbf{x}\} = \{w_{it}, T_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^T, \lambda_t^S, \lambda_{it}^{\mathcal{R}}\}_{it}$
- Non-linear equations (FOCs) for household controls $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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► Global Numerical solution:

- Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
- Express as a large vector $\mathbf{y} = \{\mathbf{x}, \boldsymbol{\lambda}, \mathbf{c}, \mathbf{q}\}$ in a large non-linear function

$$F(\mathbf{y}) = \mathbf{0}$$

- Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method: Pros and Cons

► Why use a sequential approach?

- *Global approach: Only need to follow the trajectories for i agents:*
 - Arbitrary (!) number of dimension of *ex-ante* heterogeneity:
*Productivity z_i Population p_i , Temperature scaling Δ_i , Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$,
 Local damage $\gamma_i^y, \gamma_i^u, T_i^*$, Directed Technical Change z_i^e*
 - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables:
For now: Wealth w_{it} , temperature T_{it} , reserves \mathcal{R}_{it} , Carbon S_t
Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
 - Newton method & Non-linear solvers very efficient

► Why not:

- Numerical constraint to solve a large system of ODEs and non-linear equations:
 ⇒ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
- Relying on numerical solvers/structure of the problem can be opaque

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