The Optimal Design of Climate Agreements
Inequality, Trade and Incentives for Climate Policy

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Abstract

Fighting climate change requires ambitious global policies, which are undermined by free-riding incentives. Multilateral agreements and trade policies are usually proposed to address this issue, c.f. Nordhaus (2015). Moreover, climate policy has strong redistributive effects across countries due to inequality in income, climate impacts, effects on energy markets, and trade leakage, which exacerbate non-cooperation. In this context, how can we design a climate agreement that accounts for all these different channels to fight climate change? Through the lens of an Integrated Assessment Model (IAM) with heterogeneous countries and international trade, I study the "climate club" design to maximize world's welfare, choosing carbon and trade policies when countries can exit climate agreements. Participation constraints create a policy tradeoff between an intensive margin – a climate club with few countries implementing large emission reductions – and an extensive margin – accommodating a larger number of countries at the cost of lowering the carbon tax. I solve for the optimal climate club, which consists of all the countries at the exception of Russia, where the members impose a \$100 tax per ton of CO_2 and a 50% tariff on goods from non-members. Despite full discretion in the choice of carbon tax and tariffs, one cannot achieve the world's optimal policy, \$150 tax/ton CO_2, with complete participation. The tax is reduced to encourage participation but lowering it to include Russia would compromise climate action. I explore how additional instruments, such as transfers, as proposed in the COP's "loss-and-damage fund", or fossil-fuel specific tariffs, can increase abatement and welfare by mitigating the adverse redistributive effects of climate change and carbon taxation.

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1 Introduction

Fighting climate change requires ambitious global policies. Carbon emissions must reach net zero in the next decades, and our economies need to phase out fossil fuels in a concerted effort to keep temperature under $2°C$ and avoid catastrophic consequences of global warming c.f. IPCC et al. (2022). However, we are currently facing climate inaction, and the main reason behind this lack of cooperation is the presence of free-riding in climate policy. The benefits of fighting climate change are global while the costs of reducing emissions, in particular with carbon taxation, are local. Individual countries have incentives to free-ride on the rest of the world's reduction in emissions without implementing costly carbon abatement themselves.

Moreover, taxation of carbon and fossil fuels have strong redistributive effects, changing the willingness of countries to implement climate policy. First, emerging economies may face challenges in reducing fossil fuel consumption necessary to continue their economic development. Second, carbon taxation has substantial impacts on energy markets, between of fossil fuels exporters and importers. Finally, imposing a carbon tax in one country reallocates economic activity, and carbon emissions, toward other countries through international trade – known as the "carbon leakage". All these effects reinforce free-riding incentives and climate inactions.

Multilateral climate agreements have been the traditional answer to address climate inaction, as the United Nations Conference of the Parties (COP) for example. More recently, trade instruments have been the focus of policy discussions as trade policy offers the potential to give incentives to other countries to reduce emissions. In particular, Nordhaus (2015) proposes the idea of "climate club", which are voluntary agreements where members implement carbon taxation as well as retaliatory tariffs on countries that do not participate in the club. In this context, trade sanctions are necessary to foster participation to a club and reduce free-riding incentives.[1]

In this context, what should be the design of a climate agreement that accounts for free-riding incentives as well as redistributive effects? What is the optimal climate club? This paper addresses this question by examining the conditions necessary to construct a universal climate agreement with globally optimal carbon tax and tariffs. I explore which factors incentivize countries to join such an agreement, and I investigate how carbon and trade policy needs to be implemented to promote participation, maximize welfare and fight climate change.

In this project, I tackle these policy questions in a Climate-Economy framework augmented with heterogeneous countries and international trade. I build a multi-country Integrated Assessment Model (IAM), extended with trade in goods à la Armington and energy markets in fossil fuels. Individual countries differ in their vulnerability to climate change, income levels, energy mix as well as their positions as exporters or importers of goods and energy. In this framework, I account

[1] Another notable example is the European Union's Carbon Border Adjustment Mechanism (CBAM) which is proposed to address the climate leakage. This policy is a "carbon tariff" – i.e. a tariff whose rate increases with the carbon content of the good imported. This also has the potential to generate incentives for trade partners to implement climate policy in order to lower the carbon footprint of their exports.

for the multifaceted redistribution and leakage effects that arise in general equilibrium as a result of climate policy. This serves as a laboratory for evaluating the welfare effects of different agreement designs.

With endogenous participation, countries have differing incentives to join a climate agreement. As a result, the decisions on the optimal levels of carbon tax and trade tariffs, as well as the choice of participants in the club should be made jointly. Indeed, the optimal design reveals a tradeoff between an intensive and an extensive margin. At the intensive margin, an agreement could gather a small set of countries, which can individually implement large emissions reductions with high carbon taxes. However, this may be hardly satisfying to reduce global emissions and combat climate change effectively. In contrast, building a more extensive climate club requires to accommodate the participation of a larger number of countries, which can only be done at the cost of lowering the carbon tax.

In this context, I address the policy problem where a global social planner maximizes the world's welfare by designing a climate agreement that consists of three elements: (1) the set of countries that are included in the agreement – also called "climate club" or "climate coalition" – and that are subject to the climate and trade policies, (2) the level of the carbon tax that club members set on their oil, gas and coal energy consumption and (3) the level of the trade tariffs that the members impose on the goods imported from non-member countries – either uniform tariffs as in Nordhaus (2015) or carbon-border adjustment mechanisms (CBAM) as proposed in the European Union.

Countries make an individual choice to join or leave the agreement, and such strategic participation needs to be accounted for in the design of the agreement. I consider Nash equilibria where countries take participation decisions either unilaterally, or with "coalition deviations", i.e. when a subset of countries decide jointly to deviate and leave the agreement. The policy thus mirrors an optimal taxation and trade policy problem with limited instruments, assorted with a choice of countries. Given policy instruments, the coalition choice resembles a combinatorial discrete choice problem (CDCP) that can arise in trade economics. I propose different numerical solution methods to tackle this problem in presence of participation constraints.

I contrast this framework with global policy benchmarks absent endogenous participation. First, I consider the optimal carbon policy when the coalition gathers the entire world, without participation constraints. I show that the choice of the carbon tax depends crucially on the availability of redistribution instruments in the First-Best allocation. Without such instruments, such as lump-sum transfers, I show how the choice of the carbon tax accounts for distributional motives. Indeed, the carbon tax accounts for income inequality and its effect on trade leakage, as well as demand distortions and supply redistributions through fossil fuels energy markets. As a result, the optimal carbon tax is \$150 per ton of CO_2, and is lower than the Social Cost of Carbon, i.e. the marginal cost of climate change, a result that contrast with the standard Pigouvian recommendation.[2]. Second, I also compare the "climate club" framework to the non-cooperative Nash equilibrium, in which each individual countries choose their "unilaterally optimal" carbon taxation and trade tariffs. The unilateral carbon tax policy becomes a subsidy to increase production and revenues and tariffs are used for terms-of-trade manipulation.

In comparison, climate agreements provide an "issue linkage", c.f. Maggi (2016) by coupling the implementation of climate policy with a reduction in tariffs, as there would be free trade among coalition members. We see that the countries' participation choice depends on the balance between two effects: the distortionary cost of carbon tax against the cost of tariffs, which is related to gains from trade. To choose whether to exit the agreement, individual countries consider if the first outweighs the second. This is the case for fossil-fuel producers or several developing economies. Consequently, they would not participate in a climate agreement unless the carbon tax is decreased.

As a result, the optimal climate club consists of all countries at the exception of Russia and the agreement imposes a moderate carbon tax of \$100 per ton of CO_2, and a 50% tariff on traded goods of non-participants. The optimal climate agreement cannot achieve the world's optimal policy with complete participation – an agreement with a \$150 carbon tax and all the countries – despite full discretion on the choice of the carbon tax and tariffs.

First, to increase participation, it is beneficial to reduce the carbon tax. Several Middle Eastern countries, and several developing economies in South-Asia and Africa would not join an agreement with high carbon tax, and this for any tariffs. It is therefore optimal to lower the tax by 35% to include those countries, and share the "burden" of carbon abatement across more countries.

Second, it is beneficial to leave several fossil fuels producing like Russia and former soviet countries outside of the climate agreement. Indeed, they suffer large welfare costs from carbon taxation, being relatively cold, closed and exporters of oil and gas. They would never join an agreement, unless the carbon tax would be very small, which is not optimal from a global perspective.

Third, trade policy is a key strategic instrument to undermine free-riding and incentivize countries to join the agreement. All the countries for which the cost of large tariffs outweighs the distortionary cost of carbon taxation are willing to participate in such climate club. That is especially the case for countries in Europe, East Asia and South-East Asia, like China, which trade internationally a large share of goods production, and have hence large gains from trade. Absent tariff retaliation, free-riding prevails over the cost of climate actions, as discussed in Nordhaus (2015). However, if moderate tariffs spur participation for low carbon tax, this incentive effect vanishes quickly as the carbon tax increases and larger emissions reduction are required. The gains from trade are bounded – and small for some countries like Middle-East and Russia – and therefore, there is a limit to what carbon policy can achieve.

[2]The optimal policy problem with limited instrument is treated extensively in Bourany (2024) in a large class of climate-macroeconomic models. In the present paper, I draw a particular emphasis on international trade and leakage effects, a novel channel that needs to be accounted for in optimal carbon taxation.

Additional policy instruments – such as transfers with a "loss and damage" fund, or fossil-fuel-specific tariffs – improve the climate agreements and increase the carbon tax closer to the second-Best allocation. Indeed, these two instruments addresses two channels to incentivize participation. First, redistributing part of revenues from the carbon tax to poorer economies like South-Asia and Africa – who consume less fossil fuel per capita – improve their welfare much more than the loss incurred by the richer economies of North America, Europe and East-Asia. Second, fossil-fuel-specific tariffs have strong effects on the energy rents of oil-exporting countries. This increases the retaliatory power of the climate club and is more influential to promote participation.

Lastly, I compare how these results on the optimal agreement can change depending on the impact of climate change or the gains from trade. First, I change the damage function and follow the specification of Weitzman (2012), which features larger curvature in temperature warming. This raises the cost of climate change, especially for very warm countries, increases the optimal carbon tax, but also amplifies free-riding incentives. Second, I change elasticity of substitution across goods, which decreases the gains from trade. This reinforces the trade leakage effects and dampens the power of trade instruments as a policy channel for participation. Both reasons lower the optimal carbon tax that can be achieved with an agreement. In this context, additional instruments like transfers or fossil-fuel-specific tariffs are particularly important to broaden participation.

Literature

This works relates to a large literature on the economics of climate change and bridges a gap with both the international trade policy and the game theoretical literature. First, I contribute to the debate on the formation of Climate Club, following the pioneering contribution of Nordhaus (2015). The implementation of climate policy suffers from a free-riding problem and Nordhaus proposed a simple framework to evaluate the principle of issue linkage, i.e. linking the enforcement of a climate policy with trade tariffs. He shows with the C-DICE model that for different – exogenously set – carbon prices and tariffs rates, we can achieve varying participation to a climate club. With low carbon price – up to $25\$/tCO_2$ – and high tariffs – above 10%, the climate club can achieve a club with all the 15 regions he considers.

I depart from Nordhaus' Climate Club framework in three directions. First, I show that when a Social Planner chooses endogenously and optimally both the carbon tax, the tariffs and the club members, we observe an intensive margin - extensive margin tradeoff. Lower tax and higher tariffs increase participation, and conversely. Second, I depart from the C-DICE model that use ad-hoc functions for the carbon abatement – inspired from DICE – and the gain from trade and costs of tariffs – a quadratic approximation of the results of Ossa (2014). I show that modeling the energy market – both with heterogeneity in demand and supply of fossil energy – and trade in goods, accounting for leakage effects and terms-of-trade manipulation, highlight the tradeoff between the cost of carbon taxation and the cost of tariffs. In particular, in this micro-funded setting, gains from trade are bounded, which makes some countries unwilling to join an agreement, if the loss from phasing out fossil fuels is too large, and this *for any tariffs*. Third, I model the cost of climate on production as endogenous to policy, which makes the optimal carbon tax account for redistributive effects through income inequality, trade leakage and energy markets.

Farrokhi and Lashkaripour (2024) also study how climate policy can be conducted with trade instruments. They solve for the optimal trade policy in a rich multi-industry trade model, inspired by Copeland and Taylor (2004), and show that unilateral policy accounts for carbon leakage when setting tariffs. In this setting, they explore the sequential construction of a climate club, where European Union starts a coalition, implements the unilaterally optimal trade-climate policy and iteratively grows the participation to the club. In contrast, I show how should the club *design* the trade-climate policy *strategically* to spur participation. My framework also incorporates several redistribution channels absent from their framework, related to non-linear damage, making the cost of climate change endogenous to policy, or inequality across countries that create differences between policies maximizing output, reducing emissions and improving welfare.

This project lies at the intersection of three literatures, one on trade policy, one the game-theoretical aspects of climate policy cooperation, and one macroeconomic models of climate change,

First, the interdependence between climate, environmental and trade policies is explored extensively in Kortum and Weisbach (2021), Barrett (2001), Bohringer et al. (2016), Bohringer et al. (2012) or Hsiao (2022).These articles explore the differences between unilateral policies implemented at the country level and the potential for climate cooperation using trade policies. Other articles in this trade literature explore the underpinnings of optimal trade policies, e.g. Costinot et al. (2015), Ossa (2014), Adao et al. (2023), Antras et al. (2024), more specifically the choice of trade tariffs for different objectives, like terms of trade manipulation for example. I show how these policy instruments can be used for issue linkage and climate policy.

Moreover, I also borrow from the theoretical literature on climate cooperation, with classical references such as Barrett (1994), Harstad (2012), Barrett (2003), Barrett (2013), Nordhaus (2015) or the older literature collected in Batabyal (2000) or summarized in Maggi (2016). There is also a large literature on dynamic games and coalition formation games, that focus on the building of agreements, either through coordination games or through bargaining procedures, or summarized in Ray and Vohra (2015), or Okada (2023) more recently. Iverson (2024) and Hagen and Schneider (2021) are more recent references in static settings, exploring the stability result and the set of supportable climate coalitions with different feature of climate clubs, where trade policies are crucial. Similarly, Nordhaus (2021), Harstad (2023), or Maggi and Staiger (2022) study those questions as well as other dynamic features, such as technical change, the path of climate dynamics or intertemporal decision-making. I draw inspiration from many of these references. I allow the trade framework I study to be extended to dynamic settings, and I explore the dynamic implications of climate agreement in the near future.

Third, I also draw heavily on a more quantitative literature on the macroeconomic implications of climate change and carbon policy. Indeed, in the continuity of Bourany (2024), I show that the optimal carbon policy should account several general equilibrium channels, as well as macroeconomic dynamics – in the spirit of IAMs. Starting from a static version of the classical DICE/RICE models, c.f. Nordhaus and Yang (1996), Barrage and Nordhaus (2024), I study an extension with optimal fossil fuel taxation, as in Golosov et al. (2014) and with heterogeneous countries/regions, c.f. Krusell and Smith (2022), Cruz and Rossi-Hansberg (2024, 2022), Kotlikoff, Kubler, Polbin, Sachs and Scheidegger (2021), Kotlikoff, Kubler, Polbin and Scheidegger (2021).

When the policy instruments are limited – as in our setting we only consider one uniform level of carbon and penalty tariffs against non-members and no transfer instruments – we need to change endogenously the choice of these instruments, like Hassler et al. (2021), Belfiori et al. (2024), or Douenne et al. (2023), in the case of climate policy, Davila and Walther (2022) or Chari et al. (2023) in more general cases. This follows a large literature on optimal policy in heterogeneous agents models, c.f. Bhandari et al. (2021*a*,*b*), Le Grand et al. (2024) or Davila and Schaab (2023) among many others.

2 An Integrated Assessment Model with inequality and trade

I build a climate-economy model – also called Integrated-Assessment Model (IAM) – that incorporate various dimensions of heterogeneity influencing the incentives of individual countries to join climate agreements. This framework is the simplest model that includes both climate externality, a non-trivial energy market for fossil energy, and a realistic trade structure that reproduces the leakage effects of taxation.

We study a static economy with I countries indexed by $i \in \mathbb{I}$, each with population[3] $\mathcal{P}_i$. Each country is composed of five agents: (i) a representative household that consumes the final goods, (ii) a final-good firm that produces using labor and energy, (iii-v), three energy firms: (iii) a fossil energy firm that extract oil and gas, (iv) a firm producing coal energy, and (v) a firm producing renewable/non-carbon energy. Moreover, each country has a government that sets taxes and tariffs. In the next section, we will develop in detail the choice of those policy instruments considered in the design of the climate agreement.

2.1 Household problem

The representative household in country i imports from country $j \in \mathbb{I}$ and consumes the quantity c_i. We consider an Armington structure, c.f. Anderson (1979), Arkolakis, Costinot and Rodriguez-Clare (2012), where each country produces its own variety. The utility of consumption has a structure with Constant Elasticity of Substitution (CES) θ over goods from different countries.

$$\mathcal{U}_i = \max_{\{c_{ij}\}} u\big(\{c_{ij}\}_j\big) = u(c_i) \qquad\qquad c_i = \Big(\sum_{j\in\mathbb{I}} a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\Big)^{\frac{\theta}{\theta-1}} \tag{1}$$

where a_{ij} are the preference of country i on the good purchased from country j, which also include the home-bias a_{ii}.[4] They also earn labor income and energy rent, and the budget constraint writes:

$$\sum_{j\in\mathbb{I}} c_{ij}\,(1+\mathrm{t}_{ij}^{b})\tau_{ij}\mathrm{p}_j = w_i\ell_i + \pi_i^f + \ \mathrm{t}_i^{ls} \tag{2}$$

where w_i is the wage rate, ℓ_i the exogenous labor supply, π_i^f the profit earned from the ownership of the energy firms and t_i^{ls} the lump-sum transfer received from the government. On the expenditure side, the household in i imports quantities c_{ij} from j, purchased at price p_j, and subject to iceberg cost τ_{ij} and to trade-tariffs $1+\mathrm{t}_{ij}^b$. The choice of trade policy will be made explicit below.

The optimal consumption choice of the household yields the following quantities and Arm-

[3] Countries are heterogeneous in size $\mathcal{P}_i$ and all the economic variables are expressed per "effective"-capita. For example, y_i or e_i^f are final output and fossil energy use respectively, and $\mathcal{P}_i y_i$ and $\mathcal{P}_i e_i^f$ represent the total quantities produced/consumed in the country. We allow for population and TFP growth in the dynamic climate model and in the dynamic version of this framework

[4] We assume that preferences $\{a_{ij}\}$ and iceberg trade costs $\{\tau_{ij}\}$ are policy-invariant, in particular, they are not sensitive to price changes and tariffs.

ington trade shares:

$$
\begin{aligned}
c_{ij} &= a_{ij} c_i \Big(\frac{(1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j}{\mathbb{P}_i}\Big)^{-\theta} \\
s_{ij} \equiv \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_i} &= a_{ij}\frac{((1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k)^{1-\theta}}
\end{aligned}
\tag{3}
$$

where $p_{ij} = (1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j$ is the effective price for variety from country j sold in country i, and $\mathbb{P}_i$ is the price index of country i:

$$
\mathbb{P}_i = \Big(\sum_{k\in\mathbb{I}} a_{ik}((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k)^{1-\theta}\Big)^{\frac{1}{1-\theta}}
$$

As a result, we summarize the budget as $c_i\mathbb{P}_i = \sum_{j\in\mathbb{I}} c_{ij}\,(1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j$, and the per-capita welfare of country i is summarized with the indirect utility as the income discounted by the price level:

$$
\mathcal{U}_i = u(c_i) = u\Big(\frac{w_i\ell_i+\pi_i^f+\mathrm{t}_i^{ls}}{\mathbb{P}_i}\Big) \tag{4}
$$

2.2 Final good firm problem

The final good producer in country i is producing competitively the domestic variety at price p_i. The firm solves the following maximization of profits:

$$
\max_{\ell_i,e_i^f,e_i^c,e_i^r} \mathrm{p}_i\,\mathcal{D}_i(\mathcal{E})\,z_i\,F(\ell_i,e_i^f,e_i^c,e_i^r) - w_i\ell_i - (q^f+\xi^f\mathrm{t}_i^\varepsilon)e_i^f - (q_i^c+\xi^c\mathrm{t}_i^\varepsilon)e_i^c - q_i^r e_i^r
$$

where the production function $\bar{y}_i = F(\ell_i,e_i^f,e_i^c,e_i^r)$ is Constant Return to Scale and concave in all inputs, and uses labor ℓ_i, at wage w_i, fossil energy e_i^f purchased at price q^f, coal e^c at price q_i^c and renewable energy e_i^r at price q_i^r. Energy from oil-gas e_i^f and coal e_i^c differs from renewable in the sense that emits greenhouse gases, with respective carbon concentration ξ^f and ξ^c, as we will see in the next section 2.4. As a result, there is a motive for taxing oil, gas and coal energy with the carbon tax t^ε.

The productivity of the domestic good firm $y_i = \mathcal{D}_i(\mathcal{E})\,z_i\,\bar{y}_i$ can be decomposed in two terms. First, the TFP residual z_i represents productivity as well as institutional/efficiency differences between countries. Invariant to prices and policy, this technology wedge explains the existence of income inequalities across countries. This translates into differences in consumption that create redistribution motives for taxation policy.

The second difference in productivity comes from the climate externality summarized by the net-of-damage function $\mathcal{D}_i(\mathcal{E})$, for world emissions $\mathcal{E}$. This function is a reduced-form representation of the climate system and the damage from temperature increase. It is country-specific due to differences in the vulnerability and costs of climate change. In the quantification section, we detail how we calibrate this function for each country i.

The firm input decisions solve the optimality conditions, where we define the marginal product of an input x as $MPx_i \equiv \mathcal{D}(T_i)\,z_iF_x(\ell_i,e_i^f,e_i^c,e_i^r)$ for $x\in\{\ell_i,e_i^f,e_i^c,e_i^r\}$, for example in the case of oil and gas e_i^f:

$$\mathrm{p}_i \mathcal{D}_i(\mathcal{E})\, z_i\, F_{e^f}(\ell_i, e_i^f, e_i^c, e_i^r) =: \mathrm{p}_i MPe_i^f = q^f + \xi^f \mathrm{t}^\varepsilon \tag{5}$$

and similarly for other inputs $x \in \{\ell_i, e_i^c, e_i^r\}$, with $\mathrm{p}_i MPx_i = q^x$. Moreover, the private decision of the firms does not internalize the climate externality of its own fossil-fuel energy use, except for the carbon tax t^ε.

2.3 Energy markets

The final-good firm is consuming three kind of energy sources – oil-gas, coal or renewable (non-carbon) – which are supplied by three energy firms in each country. Oil-gas are traded internationally, and countries can be exporters or importers. Coal and renewable are both traded locally, an empirically relevant assumption given the substantial trade cost in the international transfers of electricity.

2.3.1 Fossil firm

In each country $i \in \mathbb{I}$, a competitive energy producer extracts fossil fuels – oil and gas – e_i^x and sells it to the international market at price q^f. The energy is extracted at convex cost $\mathcal{C}_i^f(e_i^x)$, where the convex costs are paid in the unit of the consumption bundle of the household.[5]. The energy firm's profit maximization solves:

$$\mathcal{P}_i \pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x)\mathbb{P}_i \tag{6}$$

where $\mathcal{P}_i \pi_i^f$ is the total energy rent of country i. Since the extraction costs are convex, the production function has decreasing return to scale[6], and hence, even with competitive firms, taking the fossil price as given, a positive energy rent exists. Moreover, for the sake of simplicity, we do not consider that energy firms have market power in the setting of energy prices – for example in the case of OPEC – even though this framework could easily allow for such extension. Any sources of misallocation – in the sense of Hsieh and Klenow (2009) – is accounted for in the calibration of the cost function $\mathcal{C}_i^f(\cdot)$ as we see in the quantification section.

Naturally, the optimal extraction decision follows from the optimality condition:

$$q^f = \mathcal{C}_i^{f\prime}(e_i^x)\mathbb{P}_i \tag{7}$$

which yields the implicit function $e^{x\star} = e^x(q^f/\mathbb{P}_i) = \mathcal{C}_i^{f\prime -1}(q^f/\mathbb{P}_i)$. Finally, the energy rent comes from fossil firms' profits $\pi^f(q^f, \mathbb{P}_i) = q^f e^x(q^f/\mathbb{P}_i) - \mathcal{C}_i^f\big(e^x(q^f/\mathbb{P}_i)\big)\mathbb{P}_i > 0$ and depends on the marginal costs as well as the elasticity $\nu_i = \frac{\mathcal{C}_i^{f\prime\prime}(e^x)}{\mathcal{C}_i^{f\prime}(e^x)e^x}$.

[5]This allows to account for international inputs in goods and services for building capital for resources extraction

[6]We can also define a fossil production function with inputs x_i^f such that $e^x = g(x_i^f)$ and profit $\pi = q^f g(x) - x\mathbb{P}_i$ instead of $\pi = q^f e^x - \mathcal{C}(e^x)\mathbb{P}_i$, in which case $g(x) = \mathcal{C}^{-1}(x)$

As we will see below, the profit $\pi^f(q^f, \mathbb{P}_i)$ and its share in income $\eta_i^{\pi f} = \frac{\pi_i{}^f}{y_i \mathrm{p}_i + \pi_i{}^f}$ are key indicators for the vulnerability of a country to carbon taxation. Indeed, reducing carbon emission and the phasing out of fossil fuels reduces demand and price q^f and hence affect the welfare of large oil and gas producers.

2.3.2 International fossil energy markets

In the previous sections, we outlaid both the demand and supply of the fossil energy market. We consider that oil and gas are traded frictionlessly on international markets.[7] The market clears such that:

$$E^f = \sum_{i \in \mathbb{I}} \mathcal{P}_i e_i^f = \sum_{i \in \mathbb{I}} e_i^x \tag{8}$$

In particular, countries have different exposure to energy markets, as importers or exporters $e_i^x - e_i^f$. First, larger, richer and more productive countries use larger quantities of energy and tend to have higher energy share $\frac{q^e e_i}{\mathrm{p}_i y_i}$. Moreover, countries i that export final goods and are central in international trade networks – due to low iceberg cost τ_{ki} or high preferences a_{ki} for all $k \in \mathbb{I}$ – would have higher prices p_i and hence use more energy.

Second, countries export energy if their cost of extraction is low $\mathcal{C}_i'$ which can represent either (i) low marginal cost of production per unit of fossil reserve, (ii) large reserves of fossil fuels lowering the cost of the endowment or (iii) institutional factors that allow for higher production. These factors will be crucial in the participation of countries in climate agreements.

2.3.3 Coal firm

The final good firm uses coal. We differentiate coal for other fossil fuels like oil and gas, since coal production typically does not generate large energy rent for producing countries, which consume a large fraction of it locally. We make this empirically-grounded assumption that the supply of coal is entirely local at country-level i.

The production $\bar{e}_i^c$ from the representative clean energy firm is Constant Return to Scale and uses final good inputs, paid in the consumption bundle at price $\mathbb{P}_i$.

$$\pi_i^c = \max_{e_i^c} q_i^c \bar{e}_i^c - \mathcal{C}_i^c \mathbb{P}_i$$

where $\mathcal{C}_i^c$ is now simply a constant, as implied by constant return (CRS). This implies that there is no coal profit in equilibrium, i.e. $\pi_i^c = 0$. This is motivated by empirical evidence that even the largest coal producers never generate coal rents above 1%. As a result, the price of renewable and

[7] For the sake of simplicity, I refrain from considering a general Armington structure, where fossil demand combines varieties from different sources $e_i^f = \left(\sum_j (e_{ij}^f)^{\frac{\Theta-1}{\Theta}} \right)^{\frac{\Theta}{\Theta-1}}$. I make the simplifying assumption that fossil fuels produced in different countries are not distinguishable – crude oil from Nigeria, Saudi Arabia or Russia are not differentiated varieties – and consider the limiting case $\Theta \to \infty$

the market clearing write:

$$q_i^c = \mathcal{C}_i^c \mathbb{P}_i \qquad\qquad \bar{e}_i^c = e^c \tag{9}$$

This implies – for a given price index of inputs $\mathbb{P}_i$ – an elastic supply curve for coal energy, something we observe in practice as coal production is easily scalable in response to oil and gas price fluctuations.

2.3.4 Renewable, non-carbon, firm

The final good firm also uses renewable and other low-carbon energy sources, such as solar, wind or nuclear electricity. This provides a way of substituting away from fossil fuel in the production function $F(\ell, e^f, e^c, e^r)$. As mentioned above, the demand for renewable energy e_i^r follows from the First-Order Condition $\mathrm{p}_i MPe^r = q_i^r$.

The supply of renewable energy is local at country-level i. This assumption follows from the fact that electricity is rarely traded across countries – and if it is, it only suggests temporary differences in electricity production due to intermittency, rather than large structural imbalances. The production $\bar{e}_i^r$ from the representative clean energy firm is also Constant Return to Scale and uses final good inputs, paid in the consumption bundle at price $\mathbb{P}_i$.

$$\pi_i^r = \max_{e_i^r} q_i^r \bar{e}_i^r - \mathcal{C}_i^r \mathbb{P}_i$$

where $\mathcal{C}_i^r$ is a constant, again implied by constant return (CRS), resulting in zero profit $\pi_i^r = 0$. As a result, the price of renewable and the market clearing write

$$q_i^r = \mathcal{C}_i^r \mathbb{P}_i \qquad\qquad \bar{e}_i^r = e^r \tag{10}$$

The same elastic supply curve result applies here. This is a stronger assumption in the context of renewable energy. Indeed, in the short run renewable energy requires investments in capacity, implying a steep /inelastic supply curve, especially considering the intermittency problems of wind and solar, c.f. Gentile (2024). However, in the long-run, technological progress and learning-by-doing create positive externality decreasing substantially the cost of clean energy, which results a decreasing supply-curve, c.f. Arkolakis and Walsh (2023). We take the intermediary conservative assumption that the supply curve is flat, and explore robustness of this assumption in future extensions.

2.4 Climate system

Carbon emissions released from the burning of fossil fuels creates a climate externality as they feed back into the atmosphere, increasing temperature, and affecting damages.

We describe the damage function $\mathcal{D}_i(\mathcal{E})$ affecting country i productivity as a reduced-form summary of the impact of climate change. We develop a standard dynamic climate system that can be summarized in a static form in a simple way. It express the mapping from (i) emissions $\mathcal{E}$ to a path a atmospheric carbon concentration $\mathcal{S}_t$, (ii) from carbon concentration to a path of global and local temperatures T_{it}, and then from temperature to damage $\mathcal{D}_i(T_{it})$, and (iv) finally summarizes it in present discounted value.

First, the static model represents stationary decisions on energy choices taken "once and for all". These yearly emissions from fossil fuels sums up to

$$\mathcal{E} = \sum_{i \in \mathbb{I}} \mathcal{P}_i (\xi^f e_i^f + \xi^c e_i^c)$$

where ξ^f and ξ^c represents the carbon concentration of oil-gas and coal respectively. This leads to a constant[8] path of emissions $\mathcal{E}_t = \mathcal{E}, \forall t$. They represent trajectories of emissions or "pledges" taken in the context of climate agreement policy.

Second, we consider a dynamic system – in continuous time – for carbon concentration in the atmosphere:

$$\dot{\mathcal{S}}_t = \zeta_t \mathcal{E}_t - \delta_s \mathcal{S}_t \qquad\qquad \mathcal{S}_{t_0} = \mathcal{S}_{2023}$$

where δ_s is the rate of decay of carbon out of the atmosphere, which is typically small for typical calibrations. To make the carbon concentration bounded and non-exploding – given the constant path of emissions – I follow Krusell and Smith (2022) by assuming that part of emissions $\mathcal{E}_t$ is abated via carbon capture and storage (CCS) modeled by the exogenous parameter ζ_t. The share of emissions abated raised to 100% in the long-run, implying that $\zeta_t \rightarrow_{t\rightarrow\infty} 0$. Increasing CCS allows to reach net-zero in several centuries, stabilizing cumulative carbon emissions and temperature.

Third, we consider a linear relationship between the cumulative CO_2 emissions $\mathcal{S}$, and global temperature $\mathcal{T}$ anomaly compared to preindustrial level.

$$\mathcal{T}_t = \chi \mathcal{S}_t = \chi \Big(\mathcal{S}_{t_0} + \int_{t_0}^{\infty} e^{-\delta_s (t - t_0)} \zeta_t \mathcal{E}_t dt \Big)$$

where χ is the climate sensitivity parameters, i.e. how much warming does a ton of CO_2 cause, and where e_i^f are measured in carbon units, and the $\mathcal{S}_0$ is the initial stock of carbon before all the policy decision are made – i.e. in 2020. This specification is rationalized by a large climate-sciences literature that shows that both historical data and a large class of climate model display an approximately linear relationship between $\mathcal{S}$ and $\mathcal{T}$, as is shown is the following figure:

Fourth, we consider linear relation between global temperature and local temperature, which

[8] In the quantitative section, we also consider country-specific growth rates for TFP $\bar{g}_i$ and population n_i. This leads to path of emissions that writes as follow:

$$\mathcal{E}_t = \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(\bar{g}_i + n_i)t} (\xi^f e_i^f + \xi^c e_i^c)$$

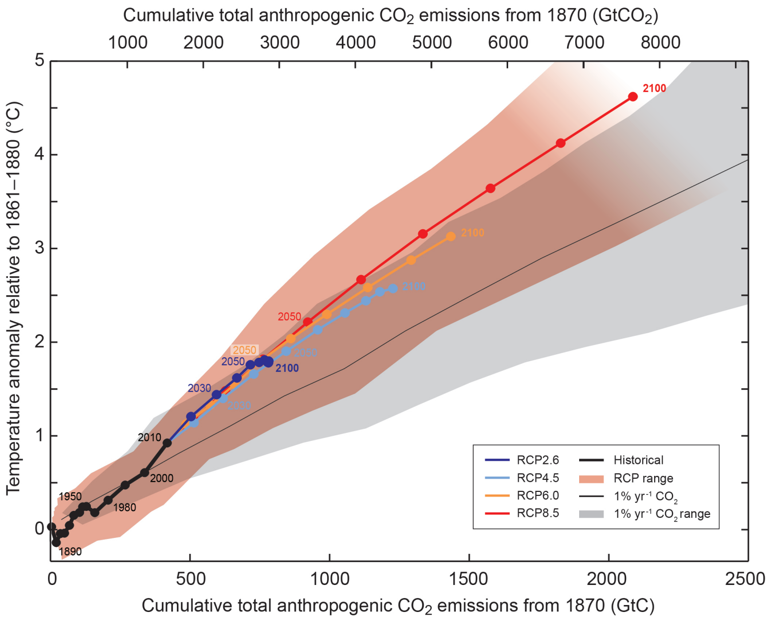

Figure 1: Linearity – Cumulative emissions and temperature

affect the local damages:

$$T_{it} = \bar{T}_{i0} + \Delta_i \mathcal{T}_t = \bar{T}_{i0} + \Delta_i \chi \mathcal{S}_t$$

where Δ_i is a linear pattern scaling parameter that depends on geographical factor such as albedo or latitude. In the quantification section, this pattern scaling is estimated regressing local temperature on global temperature.

Fifth, we consider a period damage function $\mathcal{D}(T_{it} - T_i^\star)$ where $T_i^\star$ is the "optimal" temperature for country i. The function $\mathcal{D}(\hat{T})$ is a reduced-form representation of the economic damage to productivity. In our baseline quantification, damage is considered quadratic as in standard Integrated Assessment Models. This methodology follows Krusell and Smith (2022), Kotlikoff, Kubler, Polbin, Sachs and Scheidegger (2021) and Burke et al. (2015). Such damage create winners and losers: countries that are substantially warmer than a target temperature $T_i^\star$ will be extremely affected by increases in temperature due to climate change. In contrast, regions with negative $T_{it} - T_i^\star$ will benefit – at least in the short-run – from a warmer climate. I provide a slight deviation to the above articles by assuming that the target temperature $T_i^\star$ might be different across countries: an already warm regions have different adaptation costs compared to a country which is historically cold. As a result, the target temperature $T_i^\star = \alpha T^\star + (1-\alpha)\bar{T}_{i0}$ can be more or less tilted toward historical temperature.

However, since this damage function has considerable uncertainty, we test the sensitivity of the results to different functional forms in the later section.

Finally, to obtain a reduced-form static damage function $\mathcal{D}_i(\mathcal{E})$ we summarize the future costs of climate change in present-discounted value

$$\mathcal{D}_i(\mathcal{E}) = \int_{t_0}^{\infty} e^{-\rho t}\mathcal{D}\big(T_{it} - T_i^{\star}\big)dt$$

where ρ is the household discount factor.

We see that despite the model being static we can incorporate climate system dynamics[9] as in standard Integrated Assessment Models. The future damages are summarized in the stationary equilibrium and affect the Social Cost of Carbon and the Pigouvian level of carbon taxation, c.f. section 5

2.5 Equilibrium

To close the model, we need determine the final good prices for each country p_i and we consider the market clearing for each good i

$$\begin{aligned} y_i &= \sum_{k\in\mathbb{I}} \tau_{ki} c_{ki} + \sum_{k\in\mathbb{I}} \tau_{ki}(x_{ki}^f + x_{ki}^c + x_{ki}^r) \\ \mathrm{p}_i \underbrace{y_i}_{=\mathcal{D}_i(\mathcal{E})z_i F(\cdot)} &= \sum_{k\in\mathbb{I}} \frac{s_{ki}}{1+\mathrm{t}_{ki}^b} \left(\mathrm{p}_k y_k + q^f(e_k^x - e_k^f) + \mathrm{t}_k^{ls}\right) \end{aligned} \tag{11}$$

where x_{ki}^f, x_{ki}^c and x_{ki}^r are the good inputs used by country k and imported from country i to produce fossil and renewable energy respectively.

To summarize, the competitive equilibrium of this economy is defined as follow:

Definition. Competitive equilibrium (C.E.):
For a set of policies $\{\mathrm{t}_i^{\varepsilon}, \mathrm{t}_{ij}^b, \mathrm{t}_i^{ls}\}_i$ across countries, a C.E. is a set of decisions $\{c_{ij}, e_i^f, e_i^c e_i^r, e_i^x, \bar{e}_i^c, \bar{e}_i^r\}_{ij}$, and prices $q^f, \{\mathrm{p}_i, w_i, q_i^c, q_i^r\}_i$ such that
(i) Household choose consumption $\{c_{ij}\}_{ij}$ maximizing utility eq. (1) s.t. budget constraint eq. (2), giving trade shares eq. (3)
(ii) Final good firm choose inputs $\{\ell_i, e_i^f, e_i^r\}_i$ to maximize profit, resulting in decisions eq. (5)
(iii) Fossil energy firm maximize profit eq. (6) and extract/produce $\{e_i^x\}_i$ given by eq. (7)
(iv) Renewable and coal energy supplies $\{\bar{e}_i^c, \bar{e}_i^r\}$ is given respectively by eq. (9) and eq. (10) by firms maximizing profit
(v) Energy market clear for fossils as in eq. (8) and for coal and renewable in eq. (9) and eq. (10)
(vi) Good market clear for final good for each country as in eq. (11)

[9] It would not be computationally to incorporate a more realistic climate system as in state-of-the-art IAM or Dietz et al. (2021) or Folini et al. (2024), for example with a 3 atmospheric blocks and 2 oceanic blocks.

3 The Optimal Agreement Design with endogenous participation

Because of unequal exposure to climate change and carbon policy, countries have different incentives in enforcing climate policy, exacerbating the free-riding problem. Therefore, the optimal carbon tax needs to account for endogenous participation. Designing a climate agreement reveals a trade-off between an intensive-margin – associated with the choice of the policy instruments – and an extensive margin – related to the extent of participation in the agreement.

3.1 Agreement design and participation constraints

The social planner solves a Ramsey problem, choosing the optimal agreement, which boils down to a carbon tax, retaliatory tariffs on non-participants, and a set of countries participating in the agreement, subject to participation constraints. We first design the set of climate agreement considered and then define the planner objective.

Definition – climate agreements

A climate agreement is a set $\{\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b\}$, i.e. a coalition, a carbon tax and a tariff, with $\mathbb{J} \subseteq \mathbb{I}$ countries and a competitive equilibrium ***C.E.*** such that:

- Countries $i \in \mathbb{J}$ are subject to a carbon tax t^ε on fossil energy e_i^f
- Countries can leave the climate agreement: If j exits the agreement, club members $i \in \mathbb{J}$ charges uniform tariffs $\mathrm{t}_{ij}^b = \mathrm{t}^b$ on the final good from j.
- Countries inside the club $i, j \in \mathbb{J}$ benefit from free-trade $\mathrm{t}_{ij}^b = 0$.
- Countries outside the club $k, \ell \notin \mathbb{J}$ benefit from free-trade $\mathrm{t}_{k\ell}^b = 0$. This assumption will be relaxed in **??**
- All countries – member as well as non-member – still trade in fossil (oil-gas) energy at international price q^f. This assumption will be relaxed in section 7.2

We keep the number of policy instruments $\mathbf{t} = \{\mathrm{t}^\varepsilon, \mathrm{t}^b\}$ considered in the agreement voluntarily small. This is consistent with the idea behind deviations of the Coase theorem: when bargaining over policy instruments is associated with transaction costs, a negotiation between n parties – $\mathbb{J}$ countries here – "can be prevented from attaining a socially desirable outcome", c.f. Weitzman (2015). For this reason, I refrain for considering climate agreements that bargain over a set of country-specific taxes t_i^ε, or emissions quantity targets ε_i or bilateral tariffs t_{ij}^b, since the bargaining cost would increase at least proportionally – if not exponentially – in the number of countries. The case of individual carbon taxes is analyzed in Bourany (2024).

Participation constraints

Defining indirect utility $\mathcal{U}_i(\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b) \equiv u(c_{i(\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b)})$ as in section 2.1, we can define participation constraints in two ways, depending on the equilibrium we consider.

1. *Unilateral deviation:* country i can choose to exit the agreement unilaterally. This does not affect the composition of the agreement and the decision of the other members. The participation of country i in the agreement is made if the value from staying in is larger than the value of being outside the agreement

$$\mathcal{U}_i(\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b) \geq \mathcal{U}_i(\mathbb{J}\backslash\{i\}, \mathrm{t}^\varepsilon, \mathrm{t}^b) \qquad \forall\, i \subseteq \mathbb{J} \qquad \text{[Unilateral-Nash PC]} \tag{12}$$

2. *Sub-coalition deviation:* country i can choose to exit the agreement in cooperation with other members of a potential sub-coalition $\hat{\mathbb{J}}$. All these members leave the agreement. The decision of all those countries $i \in \hat{\mathbb{J}}$ to leave is made jointly: the value of being outside is above the value of staying for all $i \in \hat{\mathbb{J}}$. This makes the participation constraints more intricate and write as follow:

$$\mathcal{U}_i(\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b) \geq \mathcal{U}_i(\mathbb{J}\backslash\hat{\mathbb{J}}, \mathrm{t}^\varepsilon, \mathrm{t}^b) \qquad \forall\, i \in \hat{\mathbb{J}} \;\&\; \forall\, \hat{\mathbb{J}} \subseteq \mathbb{J} \qquad \text{[Coalition-Nash PC]} \tag{13}$$

The optimal agreement needs to account for these participation constraints, and be robust to unilateral or sub-coalition deviations.

Welfare criterion and planner's objective

We consider a global social planner maximizing the world welfare:

$$\max_{\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b} \mathcal{W}(\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b) = \max_{\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b} \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \, \mathcal{U}_i(\mathbb{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b) \tag{14}$$

subject to participation constraint – Unilateral Nash, robust to deviation

$$\mathbb{J} \in \mathbb{S}(\mathrm{t}^\varepsilon, \mathrm{t}^b) = \big\{ \mathcal{J} \mid \mathcal{U}_i(\mathcal{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b) \geq \; \mathcal{U}_i(\mathcal{J}\backslash\{i\}, \mathrm{t}^\varepsilon, \mathrm{t}^b), \quad \forall\, i \in \mathcal{J} \big\} \tag{15}$$

or Coalitional-Nash, robust to sub-coalition $\hat{\mathbb{J}}$ deviations:

$$\mathbb{J} \in \mathbb{C}(\mathrm{t}^\varepsilon, \mathrm{t}^b) = \big\{ \mathcal{J} \mid \mathcal{U}_i(\mathcal{J}, \mathrm{t}^\varepsilon, \mathrm{t}^b) \geq \mathcal{U}_i(\mathcal{J}\backslash\hat{\mathbb{J}}, \mathrm{t}^\varepsilon, \mathrm{t}^b), \quad \forall\, i \in \mathcal{J},\; \forall\, i \in \hat{\mathbb{J}} \;\&\; \forall\, \hat{\mathbb{J}} \subseteq \mathcal{J} \big\} \tag{16}$$

where ω_i are the Pareto weights, and $\mathcal{P}_i$ the population size of country i. The social planner seeks to maximizes world welfare, ideally with the goal to fight climate change and solve this global externality. As a result, the planner maximizes the sum over $\mathbb{I}$ instead of over $\mathbb{J}$. In such case, only maximizing the coalition welfare would yield unintended consequence that the optimal climate agreement would be restricted to a subset of rich, cold, high value $\mathcal{U}_i$ who would manipulate terms-of-trade and potentially subsidize fossil fuels. This is obviously not the usual goal of climate agreements, which aim at maximizing the world welfare.

The set of agreement stable under Coalition-Nash resembles the concept of "core" in general equilibrium theory. Both of these sets $\mathbb{S}(\mathrm{t}^\varepsilon, \mathrm{t}^b)$, $\mathbb{C}(\mathrm{t}^\varepsilon, \mathrm{t}^b)$ could be empty: it is possible that no country find it beneficial to be part of the agreement for a given policy $\mathrm{t}^\varepsilon, \mathrm{t}^b$.

Extensive margin vs. intensive margin tradeoff

The problem is mixed between a choice of the policy instruments $\mathrm{t}^{\varepsilon}, \mathrm{t}^{b}$ and the choice of the country subject to participation constraints $\mathbb{J} \in \mathbb{S}(\mathrm{t}^{\varepsilon}, \mathrm{t}^{b})$ or $\mathbb{C}(\mathrm{t}^{\varepsilon}, \mathrm{t}^{b})$. As a result, this reveals an extensive-intensive margin trade-off. Indeed, for a given set of participants $\mathbb{J}$, higher carbon tax t^{ε} and lower tariffs t^{b} increase global welfare. The planner would like to reduce carbon emissions and promote free-trade at the intensive margin to maximize welfare. However, this choice of instruments also affect countries participation: higher taxes t^{ε} and lower tariffs t^{b} reduces incentive for countries to participate. If a country deviate by exiting the agreement, they increase their emissions and international trade is reduced, lowering welfare at the extensive margin. As a result, the planner would like to balance these two countervailing effects. This tradeoff will be analyzed in details in the context of this climate-economy model in section 6.3.

3.2 Optimal design and solution method

This policy problem combines a choice of instruments and a choice of countries and is therefore difficult to solve. I provide two methods to handle the joint optimal policy/combinatorial discrete choice problem.

3.2.1 Framework for the optimal design

We formalize the policy problem under the different participation constraints – Unilateral deviation vs. Coalition deviation – subject to the allocation being a competitive equilibrium. For the sake a notation, we consider a general class of policy instruments $\mathbf{t}$ that encompass carbon tax $\mathrm{t}_i^{\varepsilon} = \mathrm{t}^{\varepsilon}\mathbb{1}_{\{i \in \mathbb{J}\}}$, uniform tariffs on non-members $\mathrm{t}_{ij}^{b} = \mathrm{t}^{b}\mathbb{1}_{\{i \in \mathbb{J}, j \notin \mathbb{J}\}}$ as well as potential additional instruments as analyzed in section 7.

$$\max_{\mathbb{J}, \mathbf{t}} \mathcal{W}(\mathbb{J}, \mathbf{t}) = \max_{\mathbb{J}, \mathbf{t}} \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \, \mathcal{U}_i(\mathbb{J}, \mathbf{t})$$
$$s.t. \qquad \mathbb{J} \in \mathbb{S}(\mathbf{t}) \quad or \quad \mathbb{J} \in \mathbb{C}(\mathbf{t})$$

Participation constraints $\mathbb{S}(\mathbf{t})$ or $\mathbb{C}(\mathbf{t})$, as defined in eq. (15) and eq. (16), makes the problem intricate as it limits the instruments the planner can use for each set of countries in the agreements.

I take the following approach: I split the problem into an inner problem and an outer problem. First, the planner chooses the policy instruments $\mathbf{t}$, and then the participation constraints yields a set of possible coalition achievable. If no coalition is achievable, then the welfare for those instruments is $-\infty$.

$$\max_{\substack{\mathbb{J}, \mathbf{t} \\ \mathbb{J} \in \mathbb{S}(\mathbf{t})}} \mathcal{W}(\mathbb{J}, \mathbf{t}) = \max_{\mathbf{t}} \max_{\mathbb{J} \,|\, \mathbb{J} \in \mathbb{S}(\mathbf{t})} \mathcal{W}(\mathbb{J}, \mathbf{t})$$

This separates the difficulty of the problem into an outer policy problem – instruments $\mathbf{t}$ choice – and an inner combinatorial discrete choice problem (CDCP) for coalition $\mathbb{J}$ taking as given the policy $\mathbf{t}$.

We explain in appendix why the opposite approach – solving for the coalition as an outer problem and for policy instruments in the inner problem – is intractable.[10].

In the approach presented here, the outer problem for the choice of instrument $\mathbf{t}$ is solved with a simple grid search since the indirect welfare is now typically discontinuous and non-convex in practice.

$$\max_{\mathbf{t}} \widehat{\mathcal{W}}(\mathbf{t}) \qquad \widehat{\mathcal{W}}(\mathbf{t}) = \max_{\mathbb{J}\,|\,\mathbb{J}\in\mathbb{S}(\mathbf{t})} \mathcal{W}(\mathbb{J},\mathbf{t})$$

For this reason, we keep the number of instruments small $\mathbf{t} = \{\mathrm{t}^{\varepsilon}, \mathrm{t}^{b}\}$. This aligns with standard principles of parsimony, and the arguments behind deviations of the Coase theorem when bargaining over policy instruments is associated with transaction costs.

3.2.2 Solution methods

I propose two methods to solve for the inner problem, which is still a combinatorial discrete choice problem. Indeed, choosing the optimal choice of countries $\mathbb{J}^{\star}$ out of all the possible combinations $\mathcal{P}(\mathbb{I})$ can be prohibitive numerically. To handle this challenge, I first use an exhaustive search method, and then I propose a squeezing procedure inspired from tools from the trade literature as a more efficient alternative in the unilateral-Nash case. I introduce the problem we are facing first before presenting each method in turn.

The combinatorial discrete choice problem we are facing, for a given policy $\mathbf{t}$, writes:

$$\max_{\mathbb{J}\in\mathcal{P}(\mathbb{I})} \mathcal{W}(\mathbb{J},\mathbf{t})$$
$$s.t. \qquad \mathbb{J}\in\mathbb{S}(\mathbf{t})$$

Expressing the Lagrangian of the constrained optimization, with multiplier ν_i for country i's participation, we obtain – with a slight abuse of notation[11] – the problem

$$\max_{\mathbb{J}\in\mathcal{P}(\mathbb{I})} \mathcal{W}(\mathbb{J},\mathbf{t}) + \sum_{i\in\mathbb{J}} \nu_{i,\mathbb{J}}\Big(\mathcal{U}_i(\mathbb{J},\mathbf{t}) - \mathcal{U}_i(\mathbb{J}\backslash\{i\},\mathbf{t})\Big) =: \max_{\mathbb{J}\in\mathcal{P}(\mathbb{I})} \widetilde{\mathcal{W}}(\mathbb{J},\mathbf{t}) \tag{17}$$

in the case where the participation constraint only consider unilateral deviation[12].

First method: Exhaustive enumeration

First, when the number of countries $I = \#\mathbb{I}$ is small, one obvious yet costly solution is to perform an exhaustive search over $\mathcal{P}(\mathbb{I})$. The idea is to enumerate all the combinations $\mathbb{J}\in\mathcal{P}(\mathbb{I})$, and evaluate

[10] In a few words, in subgame perfect equilibria, it would makes the Lagrange multipliers on the participation constraints ν_i depends not only on the coalition considered $\mathbb{J}$, but all the coalition in every subgame, e.g. $\mathbb{J}\backslash\{i\}$ etc. These multipliers obviously affects the policy choice and makes the problem unsolvable

[11] Since the function $\mathcal{W}(\mathbb{J},\mathbf{t}) = \mathcal{W}(I_1, I_2, \dots, I_n, \mathbf{t})$ for $I_j = 1$ for $j\in\mathbb{J}$ and 0 otherwise, we see that the objective is not continuous, nor differentiable, nor even convex. The handling of the inequality constraints could not in theory rely on KKT theorem which applies in the $\mathcal{C}^1$ and convex case. However, we can consider that $\nu_{i,\mathbb{J}} = \begin{cases} 0 & \text{if } \mathbb{J}\in\mathbb{S}(\mathbf{t}) \\ +\infty & \text{if } \mathbb{J}\notin\mathbb{S}(\mathbf{t}) \end{cases}$ to make the problem well defined

[12] A longer list of constraints need to be included if we consider coalition-deviations

welfare $\mathcal{W}(\mathbb{J}, \mathbf{t})$. This has evidently a computational cost proportional to $2^{\#\mathbb{I}}$, i.e. the number of potential combinations.

This has however the advantage of considering the participation constraints – including the coalition-robust agreements – "for free". Indeed, it allows to assess if the participation constraints holds both in the case of unilateral-Nash $\mathbb{J} \in \mathbb{S}(\mathbf{t})$ and coalitional-Nash $\mathbb{J} \in \mathbb{C}(\mathbf{t})$, for all sets $\mathbb{J}$. This is feasible because every possible deviation of sub-groups $\hat{\mathbb{J}}$ yields a new agreement $\mathbb{J}' = \mathbb{J} \backslash \hat{\mathbb{J}}$ that is already computed as one of the other coalition $\mathbb{J}' \in \mathcal{P}(\mathbb{I})$. In such case, if one of the participation constraints is violated, the set considered $\mathbb{J}'$ is discarded, i.e. $\nu_{i,\mathbb{J}'} = \infty$, $\mathcal{W}(\mathbb{J}') = -\infty$. In practice, several coalitions can be stable for a given policy $\mathbf{t}$ and the exhaustive search allows to choose the one that maximizes welfare.

In practice, among all the stable coalitions – respecting participation constraints – the one that maximizes welfare is typically the largest one since for a given policy $\mathbf{t} = \{\mathrm{t}^{\varepsilon}, \mathrm{t}^{b}\}$, the larger the coalition, the higher the gains from trade and the gains from reducing emissions[13].

Second method: Squeezing procedure for CDCP with Participation Constraints
Second, since full enumeration is costly, I provide an alternative algorithm inspired from the methods used in the trade literature to solve combinatorial discrete choice problems. The additional difficulty that needs to be considered is the presence of participation constraints. In this section, we only consider unilateral deviations. The idea behind this method is greatly inspired by Arkolakis, Eckert and Shi (2023) and Farrokhi and Lashkaripour (2024)

The idea is to build iteratively sets that are lower bound $\underline{\mathcal{J}}$ and upper bound $\overline{\mathcal{J}}$ sets for the optimal coalition $\mathbb{J}$: $\underline{\mathcal{J}}$ is a subset which *includes* all the countries that we know to be part of the optimal set $\mathbb{J}$ and $\overline{\mathcal{J}}$ is a superset, such that it excludes the countries that we know are not part of the optimal set. The set $\overline{\mathcal{J}} \backslash \underline{\mathcal{J}}$ is the set of potential countries. The natural starting point is $\underline{\mathcal{J}} = \varnothing$, $\overline{\mathcal{J}} = \mathbb{I}$.

The squeezing step in standard CDCP is a mapping from $\mathcal{J}$ to members that bring a positive marginal value to the objective $\mathcal{W}(\mathbb{J}) := \mathcal{W}(\mathbb{J}, \mathbf{t})$. The modification needed in our setting with participation constraints is that the country also needs have marginal *individual* value $\mathcal{U}_i(\mathcal{J}) = \mathcal{U}_i(\mathcal{J}, \mathbf{t})$ to be part of the coalition:

$$\Phi(\mathcal{J}) \equiv \left\{ j \in \mathbb{I} \,\middle|\, \Delta_j \mathcal{W}(\mathcal{J}) > 0 \;\&\; \Delta_j \mathcal{U}_j(\mathcal{J}) > 0 \right\} \tag{18}$$

where the marginal values for global welfare and individual welfare is

$$\begin{aligned}
\Delta_j \mathcal{W}(\mathcal{J}) &\equiv \mathcal{W}(\mathcal{J} \cup \{j\}, \mathbf{t}) - \mathcal{W}(\mathcal{J} \backslash \{j\}, \mathbf{t}) = \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \big(\mathcal{U}_i(\mathcal{J} \cup \{j\}, \mathbf{t}) - \mathcal{U}_i(\mathcal{J} \backslash \{j\}, \mathbf{t}) \big) \\
\Delta_j \mathcal{U}_j(\mathcal{J}) &\equiv \mathcal{U}_j(\mathcal{J} \cup \{j\}, \mathbf{t}) - \mathcal{U}_j(\mathcal{J} \backslash \{j\}, \mathbf{t}) \big)
\end{aligned}$$

[13] As long as t^{ε} is below the globally optimal carbon tax as we derive it in section 5.2

The iterative procedure build the lower bound $\underline{\mathcal{J}}$ and upper bound $\overline{\mathcal{J}}$ by successive application of the squeezing step.

$$\underline{\mathcal{J}}^{(k+1)} = \Phi(\underline{\mathcal{J}}^{(k)}) \qquad\qquad \overline{\mathcal{J}}^{(k+1)} = \Phi(\overline{\mathcal{J}}^{(k)}) \tag{19}$$

Under some conditions – complementarity, c.f. below – this sequential procedure yields two sets $\underline{\mathcal{J}}$ and $\overline{\mathcal{J}}$ such that $\underline{\mathcal{J}} \subseteq \mathbb{J} \subseteq \overline{\mathcal{J}}$. In some cases $\underline{\mathcal{J}} = \overline{\mathcal{J}} = \mathbb{J}$ which gives the optimal coalition. If not, with $\overline{\mathcal{J}} \backslash \underline{\mathcal{J}} = \mathcal{J}^{pot}$, we find the optimal coalition by searching exhaustively over all coalitions in:

$$\mathcal{J}^{rem} = \left\{ \mathcal{J} \,\middle|\, \mathcal{J} = \underline{\mathcal{J}} \cup \hat{\mathcal{J}}, \text{ with } \hat{\mathcal{J}} \in \mathcal{P}(\mathcal{J}^{pot}) \right\}$$

Applicability of the squeezing procedure

From the combinatorial discrete choice literature, Arkolakis, Eckert and Shi (2023), we know that the squeezing procedure applies in cases where the model exhibit "complementarity" or single-crossing differences in choices.

Indeed, we say that the objective $\mathcal{W}(\mathcal{J})$ obeys the property of *single crossing differences in choice* (SCD-C) from below if:

$$\Delta_j \mathcal{W}(\mathcal{J}) \geq 0 \quad \Rightarrow \quad \Delta_j \mathcal{W}(\mathcal{J}') \geq 0 \qquad \text{for } \mathcal{J} \subset \mathcal{J}' \quad \& \quad j \in \mathbb{I}$$

A simple sufficient condition for SCD-C, from below to be respected is that the marginal value of the objective is monotone in the set $\mathcal{J}$, also called "complementarity":

$$\Delta_j \mathcal{W}(\mathcal{J}) \leq \Delta_j \mathcal{W}(\mathcal{J}') \qquad \text{for } \mathcal{J} \subseteq \mathcal{J}' \qquad \& \quad j \in \mathbb{I}$$

Theorem (Arkolakis, Eckert and Shi (2023)) The SCD-C from below is *sufficient* for the application of squeezing algorithm to yield $\underline{\mathcal{J}} \subseteq \mathbb{J} \subseteq \overline{\mathcal{J}}$ in standard CDCPs.

In our setting, considering participation constraints requires to adjust the welfare objective, from $\mathcal{W}(\mathbb{J})$ to $\widetilde{\mathcal{W}}(\mathbb{J})$ as in eq. (17). In this context, the single crossing differences in choice with participation constraints (SCD-C, PC) takes an intricate form, which we detail in appendix [APPENDIX REF HERE]. The following condition is sufficient for (SCD-C, PC) and provides intuitions on the trade-offs at play in the construction of the optimal coalition:

$$\begin{cases} \Delta_j \mathcal{W}(\mathcal{J}, \mathbf{t}) \leq \Delta_j \mathcal{W}(\mathcal{J}', \mathbf{t}) \\ 0 \leq \Delta_i \mathcal{U}_i(\mathcal{J} \cup \{j\}, \mathbf{t}) \leq \Delta_i \mathcal{U}_i(\mathcal{J}' \cup \{j\}, \mathbf{t}) \qquad \forall\, i \in \mathcal{J} \cup \{j\} \;\&\; i \in \mathcal{J}' \cup \{j\} \\ 0 \leq \Delta_i \mathcal{U}_i(\mathcal{J}, \mathbf{t}) \leq \Delta_i \mathcal{U}_i(\mathcal{J}', \mathbf{t}) \qquad \forall\, i \in \mathcal{J} \& i \in \mathcal{J}' \end{cases} \tag{20}$$

or

$$\begin{cases} 0 \leq \Delta_i \mathcal{U}_i(\mathcal{J} \cup \{j\}, \mathbf{t}) \leq \Delta_i \mathcal{U}_i(\mathcal{J}' \cup \{j\}, \mathbf{t}) \qquad \forall\, i \in \mathcal{J} \cup \{j\} \;\&\; i \in \mathcal{J}' \cup \{j\} \\ \exists i \in \mathcal{J} \Delta_i \mathcal{U}_i(\mathcal{J}, \mathbf{t}) < 0 \quad \& \quad \exists i \in \mathcal{J}' \quad \Delta_i \mathcal{U}_i(\mathcal{J}', \mathbf{t}) < 0 \end{cases} \tag{21}$$

$$\text{for } \forall\, \mathcal{J} \subseteq \mathcal{J}' \subseteq \mathbb{I} \qquad \& \quad j \in \mathbb{I}$$

This sufficient condition states either one of these cases is verified: (1) in the first case of eq. (20), the marginal welfare $\Delta_j \mathcal{W}$ is monotone in $\mathcal{J}$: the welfare gain of adding country j grows with the size of the coalition $\mathcal{J}$. Moreover, the participation constraint of each member i is still respected when we include country j, and this monotonically in the coalition, from $\mathcal{J}$ to $\mathcal{J}'$, and the coalition is also stable without j. (2) In the second case of eq. (21), we do not require any condition of global welfare, but the participation constraint of each member i is respected when including country j, while it is violated when j is not present in $\mathcal{J}$ and $\mathcal{J}'$. Either one of these two conditions need to be respected for every pairs of sets $\mathcal{J} \subset \mathcal{J}'$ and every country j.

This condition, as well as its weaker counterpart in appendix (SCD-C-PC), are sufficient condition for SCD-C from below for $\widetilde{\mathcal{W}}$. It shows that the requirements for coalition building are much stronger as they need to verify if adding marginal members still satisfies the participation constraints of all the incumber members. In this context, the modified squeezing steps account for such constraint and thus:

Theorem The SCD-C-PC from below is *sufficient* for the application of modified squeezing algorithm, i.e. successive application of eq. (18), starting from $\{\varnothing, \mathbb{I}\}$ and eq. (19), to yield bounding sets $\underline{\mathcal{J}} \subseteq \mathbb{J} \subseteq \overline{\mathcal{J}}$ in CDCPs with participation constraints.

One of the advantage of our setting is that, for a small number of countries $\#\mathbb{I} \approx 10$, we can evaluate numerically if the sufficient conditions mentioned above are satisfied. The disadvantage of the model displayed above is that the large amount of heterogeneity, general equilibrium effects through energy markets and international trade, prevent the simple evaluation of those sufficient conditions analytically.

4 Quantification

The model is calibrated on a panel of ten regions to provide realistic predictions on the impact of optimal carbon policy. We first describe the data used. I then provide details on the quantification, which functional forms are used, and how the parameters are calibrated to match the data. I summarize in table 1 the dimensions of heterogeneity of the model and in table 1 in appendix, the summary table for the calibration that is described in this section.

4.1 Data

First, I describe briefly the data used to calibrate the model. We use data from the year 2018-2023, taken the average over that period to smooth out the effect of the COVID recession on energy and macroeconomic data.

In the present paper, we use a sample of 10 "regions": (i) US and Canada, (ii) China and Hong Kong, (iii) European Union, United Kingdom, other countries of the Schengen Area, (iv) South Asia (India, Pakistan, Bangladesh, Nepal) (v) Sub-saharian Africa, (vi) Middle-East and North Africa, (vii) Russia and CIS, (viii) Japan, Korea, Australia, Taiwan and Singapore, (ix) South-East

Asia (Asean), (x) South and Central America. In a future iteration of this project, we will consider a panel of 25 countries and seven regions[14].

We use data for GDP per capita, in Purchasing Power Parity (PPP, in 2011 USD) from the World Bank, as collected and processed by the Maddison Project Bolt and van Zanden (2023). For the energy variables, we use the comprehensive data collected and processed in the Statistical Review of Energy Energy Institute (2024) that include data on production and consumption of various energy sources, including Oil, Gas and Coal. It also includes Proved reserves of those fossil fuels. For energy rent, we use the World Development Indicators that use national accounts to measure the share of GDP coming from the energy (oil, gas and coal) and natural resource rents.

Finally, for temperature, we use the same time series as Burke et al. (2015), which use the temperature at country level, averaged over the year and weighted by population across locations.

For trade variable, we take the trade flow and gravity variables compiled by the CEPII in Conte et al. (2022).

4.2 Welfare and Pareto weights

The welfare function that the climate agreement designer would maximize is the weighted sum of individual countries:

$$\mathcal{W} = \sum_{i \in \mathbb{I}} \mathcal{P}_i \, \omega_i \, \mathcal{U}_i$$

with $\mathcal{P}_i$ the population size per country, ω_i the Pareto weights and $\mathcal{U}_i$ the country indirect utility per capita. Note that the climate agreement designer maximizes the *world* welfare. If, in contrast, the designer only cares about maximizing the club welfare $\mathcal{W}(\mathbb{J}) = \sum_{i \in \mathbb{J}} \mathcal{P}_i \, \omega_i \, \mathcal{U}_i$, we would obtain unusual results. Indeed, the planner would gather a small club of rich and cold countries, that would not be affected by climate change and would subsidize carbon and fossil-fuel as well as imposing large tariffs on the Rest-of-World, to manipulate terms-of-trade. I give intuitions for such results in **??**, where countries unilaterally choose their climate and trade policy.

Following the discussion in Anthoff et al. (2009), Nordhaus (2011) and Nordhaus and Yang (1996), we would like to chose Pareto weights that eliminate redistributive effects that are orthogonal to climate change and climate policy. To that purpose, I choose the "Negishi" Pareto weights that make the preexisting competitive equilibrium efficient under that welfare metric. This implies:

$$\omega_i = \frac{1}{u'(\bar{c}_i)} \qquad \Leftrightarrow \qquad C.E.(\bar{c}_i) \in \underset{\bar{c}_i}{\operatorname{argmax}} \sum_i \mathcal{P}_i \omega_i u(\bar{c}_i)$$

$$\omega_i u'(\bar{c}_i) = \omega_j u'(\bar{c}_j) \qquad \forall i, j \;\in \mathbb{I}$$

[14] In future extension, we will consider the following countries, which gather the G20 and additional large countries: United States, Canada, Germany, France, United Kingdom, Italy, Spain, China, India, Pakistan, Nigeria, South Africa, Saudi Arabia, Iran, Egypt, Turkey, Russia, Australia, Japan, South Korea, Indonesia, Thailand, Argentina, Brazil, Mexico, and seven regions: rest of European Union, rest of East Asia, rest of South Asia, rest of Sub-saharian Africa, rest of Middle-East, other CIS countries, others Latin American countries

where $\bar{c}_i$ is the consumption level in the present competitive equilibrium – the period 2019-2023 – absent future climate damage. This implies that the climate agreement and the climate policy does not look for redistributing across countries through goods and energy general equilibrium effects.

However, global warming, carbon taxation and tariffs do have redistributive effects, as it would change the distribution of c_i. These effects are taken into consideration in the choice of the policy as we will see in section 5.2. In the following fig. 2, we show the weights ω_i, and $\omega_i \widehat{\mathcal{P}}_i$ when adjusted for population $\widehat{\mathcal{P}}_i = \mathcal{P}_i / \mathcal{P}$.

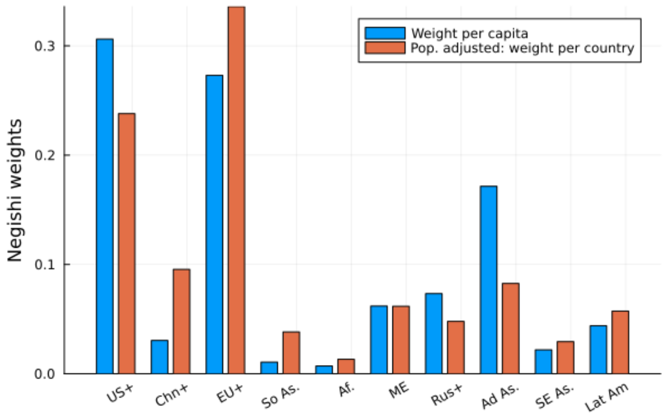

Figure 2: Pareto weights across regions
weights ω_i (blue, left), $\widehat{\mathcal{P}}_i \omega_i$ (red, right)

In **??**, we compare the baseline result to the one with utilitarian weights $\omega_i = 1$, to see how the optimal climate agreement would change.

4.3 Macroeconomy, trade and production

For the macroeconomic part of framework, we consider standard utility and production functions. First, we consider Constant Relative Risk Aversion (CRRA) function, as well as climate damage in utility function. This implies that countries that have a very low production / GDP per capita still suffer potential large losses due to climate damages:

$$\mathcal{U}_i = \frac{(c_i \widetilde{\overline{\mathcal{D}}}^u(\mathcal{E}))^{1-\eta}}{1-\eta} = \frac{c_i^{1-\eta}}{1-\eta} \overline{\mathcal{D}}^u(\mathcal{E})$$

I calibrate the CRRA/IES parameter to be $\eta = 1.5$.[15] Moreover, the damage $\overline{\mathcal{D}}^u(\mathcal{E})$ is adjusted for the curvature in the utility function.[16]

For production, I use a Nested CES framework. The firm combines a Cobb-Douglas bundle

[15] This is slightly lower than the standard value $\eta = 2$, for the reason that higher curvature would implies more inequal weights ω_i across different countries.

[16] In future research, I plan to examine the role of uncertainty, its interaction with inequality/heterogenous impacts, and the insurance effects of carbon policy, see discussion in Bourany (2024) on this topics. In the case of climate agreements, countries would also account for uncertainty when making forward looking decisions on participation.

of capital k_i and labor ℓ_i[17] with a composite of energy e_i, with elasticity σ^y. Second, the energy e_i aggregates the different energy sources: oil and gas e^f, coal e_i^c and renewable/non-carbon e_i^r, with elasticity σ^e.

$$\text{Output} \qquad \bar{y}_i = \left((1-\varepsilon)^{\frac{1}{\sigma^y}} (e_i)^{\frac{\sigma^y - 1}{\sigma^y}} + \varepsilon^{\frac{1}{\sigma^y}} (k_i^\alpha \ell_i^{1-\alpha})^{\frac{\sigma^y-1}{\sigma^y}} \right)^{\frac{\sigma^y}{\sigma^y-1}} \qquad y_i = \overline{\mathcal{D}}^y(\mathcal{E})\, z_i\, \bar{y}_i$$

$$\text{Energy} \qquad e_i = \left((\omega^f)^{\frac{1}{\sigma^e}} (e_i^f)^{\frac{\sigma^e-1}{\sigma^e}} + (\omega^c)^{\frac{1}{\sigma^e}} (e_i^c)^{\frac{\sigma^e-1}{\sigma^e}} + (\omega^r)^{\frac{1}{\sigma^e}} (e_i^r)^{\frac{\sigma^e-1}{\sigma^e}} \right)^{\frac{\sigma^e}{\sigma^e-1}}$$

To calibrate these functions, I set the capital-labor ratio to be $\alpha = 0.35$ following the literature. For energy share, I set $\varepsilon = 0.10$ to match the average energy cost share of $\frac{q_i^e e_i}{\mathrm{p}_i y_i} = 6\%$, as measured in Kotlikoff, Kubler, Polbin and Scheidegger (2021) and used in Krusell and Smith (2022). For the elasticity, I set $\sigma^y = 0.3$, following the estimation I did in Bourany (2022). This implies that capital/labor and energy are complement in production: a increase in the price of energy has a greater impact on output as it is less productive to "substitute away" to other inputs – capital, labor here, also decreasing in quantity due to complementarity. This aligns with other empirical and structural evidence on the impact of energy shocks, e.g. Hassler et al. (2019). For each energy sources, we calibrate the energy mix with oil-gas: $\omega^f = 0.56$, coal $\omega^c = 0.27$, and non-carbon $\omega^r = 0.17$, to match the aggregate shares in each of these energy sources. In the next section, I document how to match the individual countries energy mix using energy prices/costs. Finally, for the elasticity between energy inputs, I use the value $\sigma_e = 2$, following the rest of the literature, i.e. Kotlikoff, Kubler, Polbin and Scheidegger (2021) and Hillebrand and Hillebrand (2019), and empirical estimates endogenous growth work on directed technical change.

Additionally, I calibrate the productivity z_i of the production function $y_i = \overline{\mathcal{D}}^y(\mathcal{E})\, z_i\, \bar{y}_i$ to match exactly GDP/output differences $y_i \mathrm{p}_i$ across countries. This parameter z_i, represent productivity residuals as well as institutional/efficiency differences across countries. In fig. 3, I show the differences across countries that are replicated with this model.

Finally, we use trade flow data to match the pattern of international trade in goods. First, I estimate a gravity regression between trade flow and geographical distance[18]– with fixed effects for importers and exporters – finding an elasticity with distance $\kappa = -1.72$. To rationalize that in the model, I project iceberg trade costs on this geographical distance $\tau_{ij} = d_{ij}^\beta$ and use the conventional trade elasticity $\theta = 5.63$, which implies $\beta = 0.375$. All the residuals difference in trade flows, not rationalized by trade costs, or differences in prices p_j or demand y_i are then explained by differences in preferences a_{ij}. We calibrate those parameters a_{ij} to minimize the distance – mean squared error – between model-generated trade shares $s_{ij} = \frac{c_{ij}\tau_{ij}\mathrm{p}_j}{c_i \mathbb{P}_i} = a_{ij} \frac{(\tau_{ij}\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}(\tau_{ik}\mathrm{p}_k)^{1-\theta}}$, and observed shares $\bar{s}_{ij}$ in the data. Since our model imposes trade balance, which is not the case

[17] Labor is inelastically supplied $\ell_i = \bar{\ell}_i$ for each country and normalized to 1 – since the country size $\mathcal{P}_i$ is already taken into account. As a result, all the variables can be seen as input per capita.

[18] The gravity regression is standard: $\log x_{ij} = \kappa \log d_{ij} + \alpha_i + \gamma_j$. In the model, $\tau_{ij} = d_{ij}^\beta$, we get $\kappa = (1-\theta)\beta$

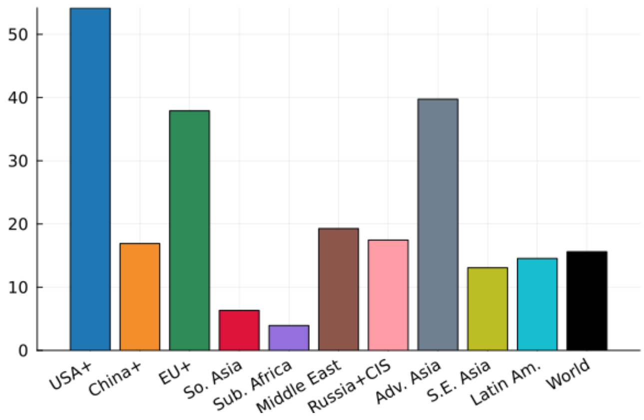

Figure 3: GDP per capita across regions
thousands of 2011-USD PPP, avg. over 2018-2023

in the data due to current account imbalances, we cannot match exactly those trade shares, only approximately. It does match the fact that some countries are relying more on trade exports and imports – like China, East Asia and South-East Asia – compared to others – Middle East, Africa and Russia and CIS.

4.4 Energy markets

For the energy market, I match the energy mix of different countries, using the CES framework displayed above, as well as differences in cost of production. For the supply side, we use iso-elastic fossil extraction cost, to replicate the oil-gas supply of fossil producers.

First, let us recall that in out model, oil and gas are traded on international market, with demand e_i^f for demand from the final good firm, and supply e_i^x from fossil energy firm, extracting oil and gas from its own reserves. We use the extraction function $\mathcal{C}_i^f$ to have the following isoelastic form:

$$\mathcal{C}_i^f(e_i^x, \mathcal{R}_i)\mathbb{P}_i = \frac{\bar{\nu}_i}{1+\nu}\Big(\frac{e_i^x}{\mathcal{R}_i}\Big)^{1+\nu}\mathcal{R}_i\mathbb{P}_i$$

with inputs paid in the price of the consumption bundle, since the input $x_i^f = \mathcal{C}_i^f(e_i^x, \mathcal{R}_i)$ take the same CES form as the consumption demand c_i. This implies[19] the profit function :

$$\mathcal{P}_i\pi_i^f = q^f e_i^x - \mathcal{C}_i^f(e_i^x, \mathcal{R}_i) = \frac{\nu\bar{\nu}_i}{1+\nu}\big(\frac{e_i^x}{\mathcal{R}_i}\big)^{1+\nu}\mathcal{R}_i\mathbb{P}_i$$

We calibrate the three parameters $\mathcal{R}_i$, ν_i and $\bar{\nu}_i$ to match two important country levels variables: e_i^x and π_i^f as observes in the data. The reserve data $\mathcal{R}_i$ are taken directly from the data on oil and

[19]We express the oil-gas extraction with a cost function $x_i^f = \mathcal{C}_i^f$. We can also express analogously with the following production function:

$$e_i^x = g(x_i^f) = \big(\tfrac{1+\nu_i}{\bar{\nu}_i}\big)^{\frac{1}{1+\nu}}\mathcal{R}_i^{\frac{\nu_i}{1+\nu_i}}(x_i^f)^{\frac{1}{1+\nu_i}}$$

where the inputs x_i^f are paid in international goods, using the same CES demand for x_i^f as c_i.

gas reserves documented by Energy Institute (2024). We calibrate the slope of this cost function $\bar{\nu}_i$ to match exactly the production of oil and gas e_i^x, as informed by that same data source. This is displayed in fig. 4 I then calibrate the curvature of the cost function to match the share $\eta_i^\pi = \frac{\pi_i^f}{y_i \mathrm{p}_i + \pi_i^f}$ of fossil energy profit as share of GDP. I calibrate ν to minimize the distance – mean squared error – between the model share η_i^π and the data, successfully matching the share within 5−10 percentage points. Differences in oil and gas energy rent across countries are not only determined by differences in cost and technology, but also in differences in trade costs and market power – by the existence of OPEC which control more than 28% of oil supply and around 15% of natural gas supply. This explains why it is difficult to match exactly the value η_i^π. However, to keep the simplicity and tractability of the model, we refrain from adding an additional Armington structure over energy sources and oligopoly power over oil and gas. While discussed in Bornstein et al. (2023) and Hassler et al. (2010), I keep the analysis of such extensions for future research.

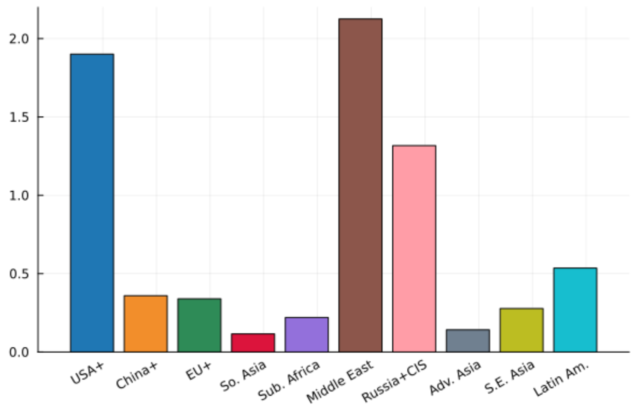

Figure 4: Share of oil and gas production across regions e_i^x in GTOE (gigatons oil equiv.) avg. over 2018-2023

Second, I match the energy mix of the different countries by relying on the two assumptions made in the model: (i) coal and renewable are only traded at the country level: $\bar{e}_i^r = e_i^r$ and $\bar{e}_i^c = e_i^c$, and (ii) the cost function is linear in goods, i.e. the production is constant return to scale, implying $q_i^c = \mathcal{C}_i^c \mathbb{P}_i$ and $q_i^r = \mathcal{C}_i^r \mathbb{P}_i$. This allows to matching the energy mix of each country by calibrating the energy costs parameters $\mathcal{C}_i^c$ and $\mathcal{C}_i^r$ for each country to match the data $\frac{e_i^c}{e_i^f + e_i^c + e_i^r}$ and $\frac{e_i^r}{e_i^f + e_i^c + e_i^r}$. Using the CES framework above, we can match exactly the energy shares, successfully identifying countries that are more reliant on coal, vs. oil and gas vs. non-carbon/renewable: for example China and India are very coal dependent and Russia, Middle-East and United-States/Canada are the biggest consumer of oil and gas.

4.5 Climate system

Finally, we calibrate the climate model described in section 2.4 to match important feature of the relationship between carbon emissions, temperatures and climate damages.

First, we calibrate the two parameters of the global climate system: first, the climate sensitivity χ representing the reaction of global temperature $\mathcal{T}_t$ to a change in $\mathcal{S}_t$ atmospheric concentration of CO_2, and second, the carbon decay rate representing the exit of carbon of the atmosphere into carbon sinks – oceans, biosphere – and out of the higher atmosphere. To this end, as is standard in the Integrated Assessment models literature, we match the pulse experiment dynamics of larger IAMs – CMIP5 in this case: for a "pulse" of $100GT$ of carbon released – corresponding to 10 years of emissions – the global temperature reaches its peak between $0.20°C$ and $0.25°C$ after 10 years and then decreases slightly to stabilize around $0.17°C$ after 200 years. I follow Dietz et al. (2021), and calibrate $\chi = 0.23$ and $\delta = 0.0004$ to match these two moments, as seen in fig. 20 displayed in appendix.

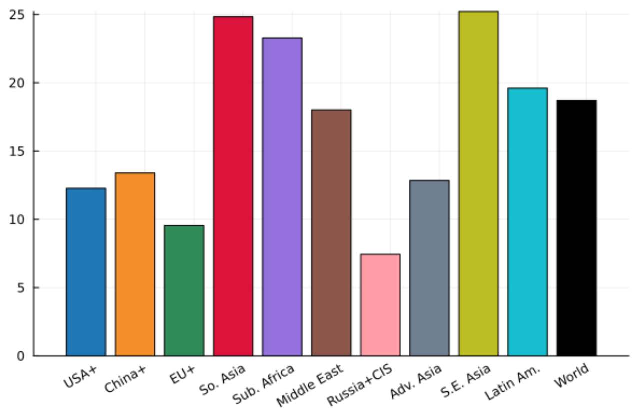

Figure 5: Temperature across countries
Avg., population weighted temperature T_{it_0} for $t_0 = 2015$

Moreover, since this climate system is inherently unstable for a given trend of emissions – given once and for all by our static economic model $\mathcal{E}_t = \sum_i \mathcal{P}_i e^{(n_i+\bar{g}_i)t}(e_i^f + e_i^c)$ with n_i the population growth and $\bar{g}_i$ the long-term GDP/TFP growth – I follow Krusell and Smith (2022) by assuming that part of emissions $\mathcal{E}_t$ are captured and stored. I assume the exponential form:

$$\mathcal{T}_t = \chi \mathcal{S}_t \qquad \dot{\mathcal{S}}_t = \zeta_t \mathcal{E}_t - \delta_s \mathcal{S}_t \qquad \zeta_t = e^{-\zeta t}$$

and match the moment suggested by Krusell and Smith (2022): 50% are captured by 2125, and 100% is by 2300 – which is $> 99.99\%$ in our model. This implies that in the Business-as-Usual scenario, global temperatures reach $\sim 5°$ by 2100 and are stabilized around $9°$ by 2400[20]. More optimistic scenarios for carbon capture and storage could be imagined, without affecting the main results, since most of the damages are discounted heavily after 2100.

Second, we calibrate the initial temperature T_{it_0} using data from Burke et al. (2015), and we display those difference across regions in fig. 5. Furthermore, we consider the linear pattern scaling

[20]Such high temperatures between 2100 and 2400 comes from our static model assumption that the model and emission decisions are made once and for all. In a dynamic model, the damages over time decreases TFP and economic activity leading to an endogenous reduction in the path of emissions and temperature. In Bourany (2024), I simulate the dynamic model over time which align with standard path of future temperatures from IAMs.

$\dot{T}_{it} = \Delta_i \mathcal{T}_t$. I identify the scaling parameter in reduced-form, by estimate this linear regression over the period $t = 1950-2015$ for each country and then aggregating by region i[21]. This procedure does not require extensive and granular data such at geographical characteristic, albedo, etc.

Third, to calibrate the damage function, I use the following quadratic function common in many Integrated Assessment Models:

$$\mathcal{D}(T_{it} - T_i^\star) = e^{\gamma^+ \mathbb{1}_{\{T_{it} > T_i^\star\}} (T_{it} - T_i^\star)^2 + \gamma^- \mathbb{1}_{\{T_{it} < T_i^\star\}} (T_{it} - T_i^\star)^2}$$

with the damage parameter $\gamma^+ = 0.00340$. This value is intermediary between the value $\gamma^+ = 0.00311$ in Krusell and Smith (2022), calibrated to match Nordhaus' DICE calibration of 6.6% of loss of global GDP when $\mathcal{T}_t - \mathcal{T}_{t_0} = 5$, and the updated calibration in Barrage and Nordhaus (2024) which calibrate it at $\gamma^+ = 0.003467$. For small values, I consider $\gamma^- = 0.3\gamma^+$, following the quantification done in Rudik et al. (2021) that find that the negative productivity impact of cold temperatures is much weaker than for hot temperature. In the robustness section ??, I also consider the functional form of Weitzman (2012), which imposes additional curvature[22], which matters when temperature reach high levels $\mathcal{T} > 3$.

Finally, to calibrate $T_i^\star$, I use also an intermediary assumption between the following two cases: (i) the representative agent economy, like Barrage and Nordhaus (2024), would assume $T_i^\star = T_{it_0}$, which implies that $T_{it} - T_i^\star = \Delta_i(\mathcal{T}_t - \mathcal{T}_{t0})$: differences in damages only comes from increases in aggregate temperature. The analysis by Bilal and Känzig (2024) shows that climate damages on GDP comes for a large part from the increase in global temperature, causing extreme events. In contrast, (ii) a different view in heterogeneous countries economies would set $T_i^\star = T^\star$ the same for all regions, at an "ideal" temperature, as in Krusell and Smith (2022) and Kotlikoff, Kubler, Polbin, Sachs and Scheidegger (2021). In this case, differences in climate damages comes essentially from differences in initial temperatures. I take the intermediary step and assume:

$$T_i^\star = \alpha^T T^\star + (1 - \alpha^T) T_{it_0}$$

where $\alpha^T = 0.5$ and $T^\star = 14.5$ is the average spring temperature of developed economies – and around the yearly average of places like California or Spain. In ??, I perform robustness checks regarding the value of α^T.

[21] To control for the fact that country j has an influence on world temperature $\mathcal{T} = \sum_i g_i T_{it}$, I estimate the linear equation with $\mathcal{T}_{t,\neq j} = \sum_{i \neq j} g_i T_{it}$ for each j, i.e. $T_{jt} = \Delta_j \mathcal{T}_{t,\neq j}$

[22] He considers a functional form that I approximate as follow: $\mathcal{D}(T_{it} - T_i^\star) = e^{\gamma^+ (T_{it} - T_i^\star)^2 + \bar{\gamma}(T_{it} - T_i^\star)^6}$ adding more curvature and damages for large temperatures.

4.6 Heterogeneity

In this section, I summarize the different dimensions of heterogeneity included in the model, and aggregate the

Table 1: Heterogeneity across countries

Dimension of heterogeneity	Model parameter	Matched variable from the data	Source of the data
Population	Country size $\mathcal{P}_i$	Population	UN Population Prospect
TFP/technology/institutions	Firm productivity z_i	GDP per capita (2011-PPP)	World Bank/Maddison project
Productivity in energy	Energy-augmenting productivity z_i^e	Energy cost share	SRE Energy Institute (2024)
Cost of coal energy	Cost of coal production $\mathcal{C}_i^c$	Energy mix/coal share e_i^c/e_i	SRE Energy Institute (2024)
Cost of non-carbon energy	Cost of non-carbon production $\mathcal{C}_i^r$	Energy mix/coal share e_i^r/e_i	SRE Energy Institute (2024)
Local temperature	Initial temperature T_{it_0}	Pop-weighted yearly temperature	Burke et al. (2015)
Pattern scaling	Pattern scaling Δ_i	Sensitivity of T_{it} to world $\mathcal{T}_t$	Burke et al. (2015)
Oil-gas reserves	Reserves $\mathcal{R}_i$	Proved Oil-gas reserves	SRE Energy Institute (2024)
Cost of oil-gas extraction	Slope of extraction cost $\bar{\nu}_i$	Oil-gas extracted/produced e_i^x	SRE Energy Institute (2024)
Cost of oil-gas extraction	Curvature of extraction cost ν_i	Profit π_i^f / energy rent	World Bank / WDI
Trade costs	Distance iceberg costs τ_{ij}	Geographical distance $\tau_{ij} = d_{ij}^\beta$	CEPII Conte et al. (2022)
Armington preferences	CES preferences a_{ij}	Trade flows	CEPII Conte et al. (2022)

5 Optimal Climate policy benchmarks with cooperation, without participation constraints

I provide two benchmarks for optimal policy when participation is exogenous. We consider a global social planner policy that maximizes aggregate welfare, representing the cooperative allocation.

The cooperative policy depends on the availability of redistribution instruments. With unlimited instruments, in particular lump-sum transfers, the optimal tax is the social cost of carbon, a measure of the marginal cost of climate change. Without transfers, the optimal tax needs to account for inequality across countries and trade leakage effects. Accounting for inequality and lack of redistribution, the optimal tax can be lower. The main lessons from this analysis can also be described in details in Bourany (2024) where I develop this argument in a large class of climate-economy, Integrated Assessment models.

5.1 First Best allocation with unlimited instruments

With unlimited instruments, the social planner uses lump-sum transfers to redistribute across countries and offset the negative effects of climate change and carbon taxation. In this context, the optimal tax is the standard Pigouvian tax, which is the social cost of carbon.

We consider a planner that maximize global welfare by choosing the allocation $\boldsymbol{x} = \{c_{ij}, x_{ij}^\ell, e_i^\ell\}$

$$\mathcal{W} = \max_{\{c_i, e_i, e_i^x, \dots\}_i} \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \; u(c_i) = \sum_{\mathbb{I}} \mathcal{P}_i \omega_i \mathcal{U}_i$$

subject to the market clearing of energy and goods.

The first lesson from this exercise is that the planner would like to equalize marginal utilities through the following condition:

$$\overline{\lambda} = \frac{\omega_i u(c_i)\overline{\mathcal{D}}_i^u(\mathcal{E})}{\mathbb{P}_i} = \frac{\omega_j u(c_i)\overline{\mathcal{D}}_j^u(\mathcal{E})}{\mathbb{P}_j} \qquad \forall\, i,j \in \mathbb{I}$$

This implies arbitrary large redistribution, using lump-sum transfers, such that:

$$\begin{aligned} c_i &= u'^{-1}\big(\overline{\lambda}\mathbb{P}_i/\overline{\mathcal{D}}_i^u(\mathcal{E})\big), \forall i \in \mathbb{I} \\ &= w_i\ell_i + \pi_i^f + \mathrm{t}_i^{ls} \end{aligned}$$

In that cases, the transfers t_i^{ls} are designed, such that the consumptions are equalized. This implies redistribution, as $\mathrm{t}_i^{ls} < 0$ for some countries and $\mathrm{t}_i^{ls} > 0$ for some other countries.

Second, the Social Cost of Carbon defined as the ratio of marginal value of emissions over marginal utility of consumption can then be simply reformulated with the following multipliers:

$$SCC = -\frac{\frac{\partial \mathcal{W}}{\partial \mathcal{E}}}{\frac{\partial \mathcal{W}}{c_i}} = \frac{\phi^{\mathcal{E}}}{\overline{\lambda}}$$

where we recognize that the multiplier $\phi^{\mathcal{E}}$ is the welfare value of one additional ton of carbon (the welfare cost comes from the minus sign) which correspond to the constraint $\mathcal{E} = \sum_i \xi^f e_i^f + \xi^c e_i^c$, and $\overline{\lambda}$ the average marginal utility of consumption – or marginal value of wealth.

In that context, the optimal tax is simply the Social Cost of Carbon (SCC)

$$\mathrm{t}^{\varepsilon} = \frac{\phi^{\mathcal{E}}}{\overline{\lambda}} = -\sum_{\mathbb{I}} \mathcal{P}_i\omega_i\Big[\frac{u(c_i)}{u'(c_i)}\mathbb{P}_i\overline{\mathcal{D}}^{u'}(\mathcal{E}) + \mathcal{D}_i^{y\prime}(\mathcal{E})z_iF(e_i,\ell_i)\mathrm{p}_i\Big] > 0$$

The optimal carbon tax is the Pigouvian level that summarizes the marginal cost of climate change for all countries i.

This recovers the result in Golosov et al. (2014) that prevail in representative agents economies: absent redistributive motive, the optimal tax is the Pigouvian level that summarize the marginal cost of climate change.

In the next section, we see how that results changes when the transfers and other instruments, like tariffs or subsidies, are constrained and prevented to do redistribution.

5.2 Second-Best Ramsey problem without transfers

When the social planner does not have access to transfers, the carbon tax needs to account for redistributive effects. The optimal tax needs to be corrected for (i) the heterogeneous effects of climate change, (ii) the redistributive effects on energy markets, (iii) the distortion of demand, (iv) the leakage effects of trade.

We now consider the Ramsey planner again chooses $\boldsymbol{x} = \{c_{ij}, x_{ij}^{\ell}, e_i^{\ell}\}$, i.e. the traded good for consumption c_{ij}, for energy inputs for the production of in fossil x^f_{ij}, coal x^c_{ij}, non-carbon x^r_{ij} or capital x^k_{ij}, and the energy demand, in fossil e^f_i, coal e^c_i and non-carbon e^r_i, as well as the carbon tax t^ε and the prices $\boldsymbol{p} = \{\mathrm{p}_i, q^f, q^c_i, q^r_i\}_i$. However, the allocation and prices are constrained to be a competitive equilibrium: in that case, the planner is restricted to choose controls that respect the individual optimality conditions.

In the Primal approach, these optimality conditions are internalized by the planner through a large array of Multipliers for all these constraints which summarize the distortionary effects – through the multipliers on inputs and energy choice of households and firms – the general equilibrium effects – through the multipliers on market clearing and – the redistributive effects – through the multipliers on the household constraints for example. The optimality condition of the planners are numerous and technical and we restrict their complete exposition to appendix C.2.

The optimal carbon tax t^ε corrects climate externality, but also need to account for: (i) Redistribution motives, through the multiplier λ_i, (ii) G.E. effects on energy markets, both of the supply – which affect energy rent – and the demand through the distortion of the optimality condition of energy choice, with multipliers υ^f_i for fossils for example and (iii) G.E. effects on good markets, or trade leakage, through multipliers of the market clearing μ_i for good from country i.

The optimal tax formula for carbon – in the case of fossil for example – can be summarized as:

$$\xi^f \mathrm{t}^\varepsilon = \xi^f \underbrace{\textstyle\sum_i \widehat{\lambda}_i \, LCC_i}_{=SCC} + \textstyle\sum_i \widehat{\lambda}_i \text{ Supply Redistrib}^\circ_i + \sum_i \widehat{\lambda}_i \text{ Demand Distort}^\circ_i - \sum_i \text{Trade Redistrib}^\circ_i$$
$$\text{with} \qquad \widehat{\lambda}_i \propto \mathcal{P}_i \omega_i u'(c_i)$$

which can be unpacked as :

$$\xi^f \mathrm{t}^\varepsilon = \xi^f \mathcal{P} \sum_i \widehat{\lambda}_i LCC_i + q^f \mathcal{P} \frac{\bar{\nu}}{E^f} \sum_i \widehat{\lambda}_i \big(e^f_i - \frac{e^x_i}{\mathcal{P}_i}\big) - (q^f + \xi^f \mathrm{t}^\varepsilon) \sum_i \widehat{\mu}_i - \sum_i \widehat{\upsilon}^f_i$$
$$\xi^f \mathrm{t}^\varepsilon = \xi^f \underbrace{\mathcal{P}\mathbb{E}_i\big[LCC_i\big] + \mathcal{P}\mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, LCC_i\big)}_{=\text{Social Cost of Carbon}} + \underbrace{q^f \mathcal{P} \frac{\bar{\nu}}{E^f} \mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, e^f_i - \tfrac{e^x_i}{\mathcal{P}_i}\big)}_{\substack{E^f \text{ supply} \\ \text{redistribut}^\circ}} - (q^f + \xi^f \mathrm{t}^\varepsilon) \underbrace{\mathbb{E}_i\big[\widehat{\mu}_i\big]}_{\substack{y_i \text{Trade} \\ \text{redistribut}^\circ}} - \underbrace{\mathbb{E}_i\big[\widehat{\upsilon}^f_i\big]}_{\substack{e^{\ell'}_i \text{ demand} \\ \text{distort}^\circ}}$$

with the aggregate inverse supply elasticity for fossil $\bar{\nu} = \big(\sum_i \lambda^x_i \nu_i^{-1}\big)^{-1}$, and the social welfare weights", which are the rescaled multipliers for the budget constraint: $\widehat{\lambda}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\sum_i \omega_i \widehat{\mathcal{P}}_i \lambda_i}$, the multiplier the FOC demand $\widehat{\upsilon}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \upsilon_i}{\bar{\lambda}}$, for the multiplier for market clearing for good: $\widehat{\mu}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \mu_i}{\bar{\lambda}}$. In appendix, we further unpack the distortion terms $\widehat{\upsilon}_i$ to understand where this distortion can be expressed as function of the elasticities of demand for fossil fuels.

This implies that the carbon tax can be different from the Social Cost of Carbon when the planner has redistributive motives. This is described in more details in the companion paper Bourany (2024). This matters quantitatively for our model as we show in fig. 6.

We see that the endogenous cost of climate change is very large in the competitive – Business-as-Usual – equilibrium due to climate inaction. However, in the First-Best, thanks to large redistribution, the planner lower the marginal value of wealth for the "average household" $\bar{\lambda}$. This increases the Social Cost of Carbon and allow to set a higher carbon tax as we see here.

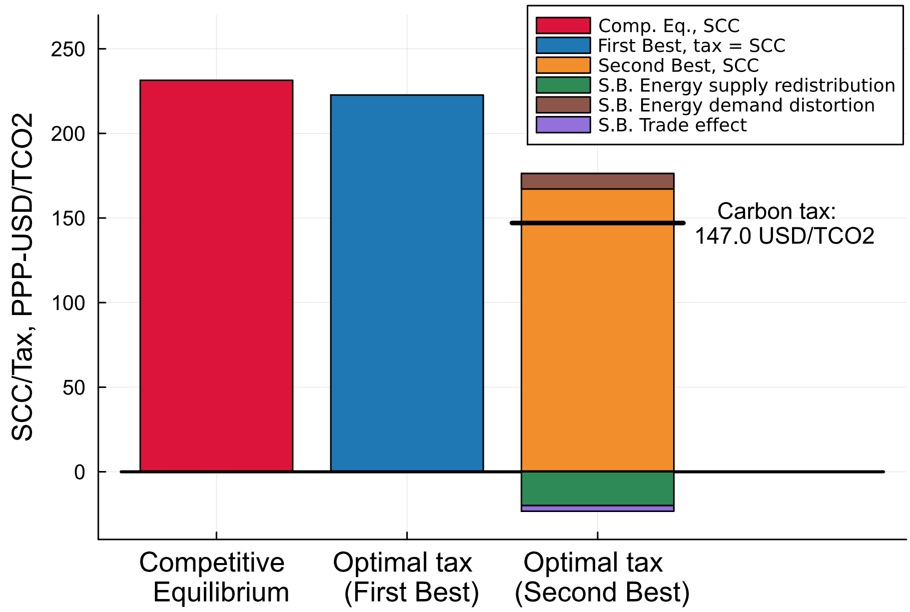

Figure 6: Social Cost of Carbon and optimal carbon taxation

However, in the Second-Best allocation, the social planner cannot redistribute easily and therefore account for the redistributive effects in the carbon tax. The average household is now "poorer", increasing the aggregate weight $\bar{\lambda}$, lowering the Social Cost of Carbon from \$220 to approximately \$170 per ton of CO_2. Moreover, the redistributive effect through the energy supply dries part of the energy rent of large oil and gas producers like the US, Middle East and Russia. This is accounted for in the optimal policy which is now set at \$147 per ton of CO_2.

Winners and losers from cooperative carbon taxation

This optimal second-best Carbon Tax, despite accounting for redistributive effects, still have heterogeneous impact across countries. In fig. 7 we display the welfare change from this policy $\mathcal{U}_i(\mathbb{I}, \mathrm{t}^\varepsilon)/\mathcal{U}_i(\mathbb{I}, 0)$ in consumption equivalent, in comparison to the competitive equilibrium.

We first see that most regions gain large benefit from cooperation, with an aggregate effect of 5% for global welfare. However, this hides a large heterogeneity across countries.

The biggest winners are without contest the countries that are the most affected by climate change: South-East Asia, Africa and South Asia, which gains between 6% and 13% of consumption change. However, countries who consume a large share of coal like China and India are not gaining as much because of the large redistributive effect imposed by carbon taxation. Finally, large fossil fuels – oil and gas – exporters like Russia and Middle-East are losers because they see a drying up of their energy rent. In addition, Russia is also a cold countries that do not gain anything for slowing down climate change.

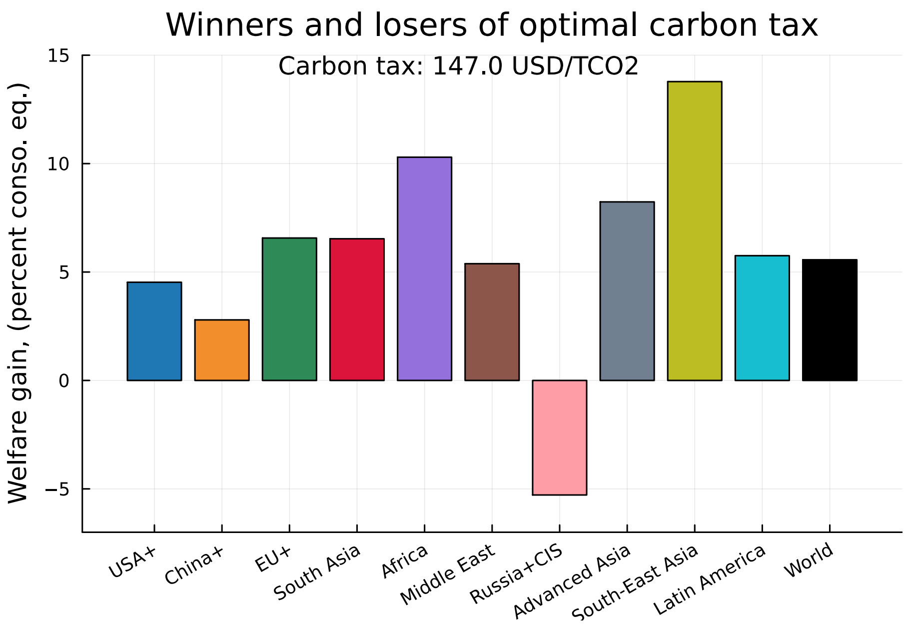

Figure 7: Welfare gains across countries

6 Optimal Climate agreement

We know turn to our main result. The optimal design of climate agreement is a climate club that consists of all the countries at the exception of Russia and CIS countries. Members of the club impose a \$98 carbon tax per ton of CO_2 and a 50% tariff on goods from non-members. The intuition behind this result can be summarized by the tradeoff at the heart of countries' club participation between the distortionary effect of the carbon tax, and the cost of tariffs, related to gains from trade. We note that for several countries like fossil-fuel producers or developing economies, the first outweighs the second, implying that they would not participate in a climate agreement unless the tax is decreased from \$147 to \$98. This encourages the participation of Middle-East and South Asia, but the optimal agreement does not include the entire world. Indeed, lowering the tax so low to be able to incentivize Russia to participation would compromise climate action.

We first provide details on the trade-off behind country participation. We then present the main result of the optimal climate club.

6.1 Trade-off: distortionary effects of carbon taxation vs. gains from trade

The following two effects influence the design of the agreement. First, the distortionary effect of the carbon tax differs across countries, and some countries – poor, closed to trade or cold countries and fossil-fuel producers have very large gains from free-riding. Second, the cost of tariffs of other trade partners, related to gains from trade, also differs across countries.

In the following fig. 8, I present an experiment where all countries $j \in \mathbb{I}$ are implementing the optimal level of fossil-fuel taxation $\mathrm{t}^{\varepsilon} = \$147/tCO_2$, except for country i, which deviates from that policy, setting the tax to zero. In this experiment, other countries j do not impose retaliatory tariffs on country i, and continue to implement the optimal policy. For each country i, I plot, in consumption-equivalent units, the welfare gain of such "deviation" compared to the case where the country i stays in this "agreement". This represents the gains from free-riding, while the Rest of the World, or equivalently, the cost of the distortionary taxation of carbon and fossil fuels.

We see first that such gains from "deviating" range from 1.5% – for Europe, which uses more renewable energy and less coal – to close to 4% for Russia, former soviet countries and Middle-Eastern countries, whose economies are relying on oil and gas both for good production and energy exports. These distortionary costs are also relatively high for developed economies like South Asia, Sub-Saharian Africa and Latin America, for the reason that energy, and fossil and coal in particular, are necessary input in production and the welfare cost scale with marginal utility of consumption $u'(c_i)$. We provide a welfare decomposition in section 6.2 to show the sources of these welfare costs and how it differ across regions.

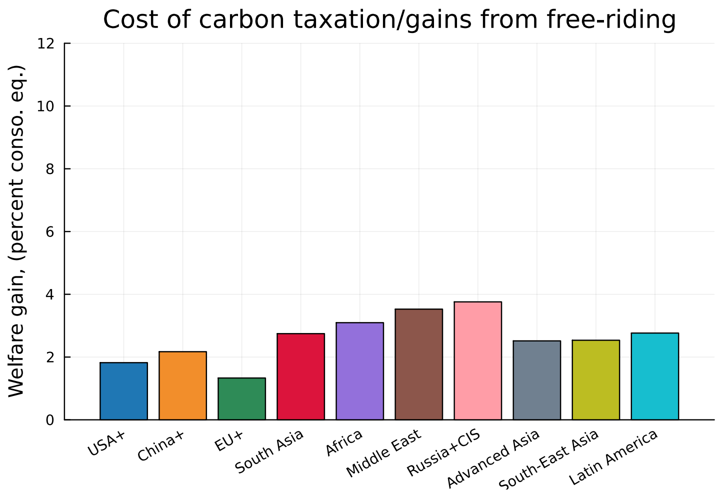

Figure 8: Welfare gain for country i of unilaterally deviating from a world agreement setting a \$147 carbon tax

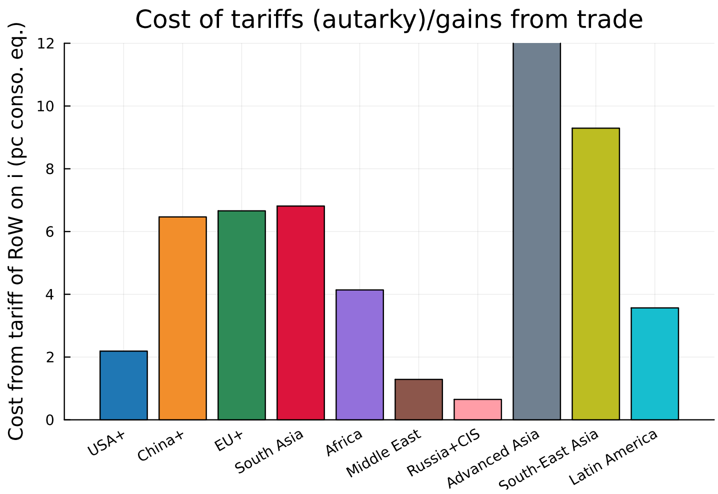

Figure 9: Losses for country i of country $j \neq i$ imposing a 500% tariff on country i

Now, let us compare that cost of energy taxation to the cost of trade tariffs. In fig. 9, we measure the welfare costs of tariffs in the following experiment: all countries $j \neq i$ impose a very large tariff – 500% – on country i. For each country i, we display the welfare loss, in consumption equivalent percentage changes in the figure. This is a good representation of the upper bound on welfare cost of tariffs – as those welfare costs are virtually identical for higher values of tariffs, e.g. 1000%. This is closely related to the cost of autarky, or gains-from-trade which are bounded and relatively small in standard trade model, c.f. Arkolakis et al. (2012).[23]

We see that Asian countries – advanced economies like Japan and Korea, as well as China and South-East Asian economies – have the most to lose to be in subject to tariffs – respectively around 12%, 8% and 6% welfare equivalent consumption losses. This is in part due to the large

[23]In this experiment, autarky would be the case when both countries j impose large tariffs on i and i imposes tariffs on countries j as well. We consider the experiment of one-side tariffs, which is closer to the policy implemented in our climate club

trade shares of these countries with each other and with Europe. In comparison, countries like the US, Middle-East and Russia, which are much more closed to international trade, suffer less from tariffs, which change their willing to join a climate club.

6.2 Welfare decomposition

To understand the mechanisms through which climate change, carbon taxation and tariffs affect welfare, I provide a first-order approximation of welfare to shed light on different mechanisms. This welfare decomposition is described in thorough details in appendix D, and this exercise is inspired by Kleinman et al. (2020),

I compute the change in welfare, linearizing the model around the competitive equilibrium where $\mathrm{t}^{\varepsilon} = \bar{\mathrm{t}}^{\varepsilon} = 0$ and $\mathrm{t}^{b}_{ij} = \bar{\mathrm{t}}^{b}_{ij} = 0$, where policies are identical to the "status-quo". I consider a climate agreement $\mathcal{J}$ of J countries, which are indifferent between being in the club or not, since the policy $(\mathrm{t}^{\varepsilon}_{i}, \mathrm{t}^{b}_{ij}) = (0,0)$ does not change the equilibrium.

I consider a perturbation where those policy instruments are increased by a small amount, denoting $d\ln z_i = \frac{dz_i}{z_i}$:

$$\mathbf{J}d\mathbf{t}^{\varepsilon} = \left\{\mathbb{1}_{\{i\in\mathcal{J}\}}\, d\ln \mathrm{t}^{\varepsilon}_{i}\right\}_{i} \qquad\qquad \overline{\mathbf{J}}\odot d\mathbf{t}^{b} = \left\{\mathbb{1}_{\{i\in\mathcal{J},j\notin\mathcal{J}\}} d\mathrm{t}^{b}_{ij}\right\}_{ij}$$

with $\mathbf{J} = \mathrm{J}_i = \mathbb{1}_{\{i\in\mathcal{J}\}}$, and $\overline{\mathbf{J}} \equiv \mathrm{J}_{ki} = \mathbb{1}_{\{i\in\mathcal{J},j\notin\mathcal{J}\}}$.

We compute the welfare decomposition of individual country i, defined as the indirect utility $\mathcal{U}_i = u(\{c_{ij}\}_j)$ as:

$$\frac{d\mathcal{U}_i}{u'(c_i)} = \eta^{c}_{i} d\ln \mathrm{p}_i + \Big[-\eta^{c}_{i}\bar{\gamma}_i \tfrac{1}{\bar{\nu}} - \eta^{c}_{i} s^{e}_{i} s^{f}_{i} + \eta^{\pi}_{i}(1+\tfrac{1}{\bar{\nu}})\Big] d\ln q^{f} - \Big[\eta^{c}_{i} s^{e}_{i}(s^{c}_{i}+s^{r}_{i}) + \eta^{\pi}_{i}\tfrac{1}{\nu} + 1\Big] d\ln \mathbb{P}_i$$

where $\eta^{c}_{i} = \frac{y_i \mathrm{p}_i}{x_i}$, with $x_i = c_i\mathbb{P}_i$ is the ratio of final good output in comparison to consumption – which can also come from energy rent. The counterpart is $\eta^{\pi}_{i} = \frac{\pi^{f}_{i}}{x_i}$. The energy share $s^{e}_{i} = \frac{e_i q^{e}_{i}}{y_i \mathrm{p}_i}$ and the share of oil-gas/coal/renewable $s^{\ell}_{i} = \frac{e^{\ell}_{i} q^{\ell}_{i}}{e_i q^{e}_{i}}$ governs the impact of energy prices. The aggregate supply elasticity $\bar{\nu} = \left(\sum_i \lambda^{x}_{i} \nu^{-1}_{i}\right)^{-1}$ represents the oil-gas supply curve, and the climate damage $\bar{\gamma}_i = \gamma(T_i - T^{\star}_{i}) T_i\, s^{E/S}$ is represented in a static fashion – with $\mathcal{E}$ the emission of that period and $s^{E/S} = \mathcal{E}/\mathcal{S}$ with $\mathcal{S}$ the carbon concentration in the atmosphere.

We see that most of the impacts arise through aggregate quantity of emissions and fossil fuels consumption, which then affect world prices q^{f}. For conciseness, we express all the General Equilibrium effects on fossil quantities as a function of price q^{f}: $d\ln q^{f} \approx \bar{\nu} d\ln E^{f} + \dots$.

The countries affected the most by a change in equilibrium quantity of fossil fuels consumed E^{f}, price q^{f}, and thus by carbon taxation, are the countries that have a high sensitivity to $d\ln q^{f}$ here. A reduction in fossil demand benefits the countries that have large damages from climate changes $\bar{\gamma}_i$, as well as large energy share from fossil s^{f}_{i}: this latter effect dampens the cost of taxation: if a larger coalition lower energy demand, it benefits other countries through a reduction in fossil price. This is sometimes called in the literature the "energy price leakage effect". However, this decrease in price hurt the fossil fuel producers as it dries out their energy rent as summarized by $\eta_i^\pi(1+\frac{1}{\bar{\nu}})$. Moreover, there are many additional equilibrium effects through trade and good prices p_i and $\mathbb{P}_i$ as we see below.

To see the direct effect of carbon taxation – at the intensive margin – and the extensive margin effect of the size of the club J_i, we simplify the model further to obtain an analytical formula for the fossil price. In the following, we assume that the energy mix is concentrated on oil and gas $s_i^f = 1, s_i^c = s_i^r = 0$. The details of the derivation are provided in appendix D:

$$d\ln q^f = -\frac{\bar{\nu}}{1+\bar{\gamma}+\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i) + \bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \widetilde{\lambda}_i^f \mathrm{J}_i d\mathrm{t}^\varepsilon + \sum_i \beta_i d\ln \mathrm{p}_i$$

with the market share $\lambda_i^f = \frac{\mathcal{P}_i e_i^f}{E^f}$ for fossil, and weighted by elasticity $\widetilde{\lambda}_i^f = \lambda_i^f \frac{\sigma^y}{1-s_i^e}$ and its average $\overline{\lambda}^{\sigma,f} = \sum_i \widetilde{\lambda}_i^f \frac{\sigma^y}{1-s_i^e}$.

The energy curve expressed here q^f is affected by climate change: more emissions imply larger damages $\bar{\gamma} = \sum_i \bar{\gamma}_i$, which in turn reduce energy demand and hence emissions. The price impact of taxation is higher – analogous to the slope of the demand curve – as seen in the denominator of the first term. Moreover, the covariance term indicates that if the large energy producers – with a larger share of the market $\lambda_i^f = \widetilde{\lambda}_i^f = \lambda_i^f \frac{\sigma^y}{1-s_i^e}$ and high elasticity σ – are also the most affected by climate change, this effect is stronger and the demand curve is even steeper and more inelastic.

Moreover, carbon taxation t_j^ε and tariffs t_{ij}^b have large trade and leakage effect, through general equilibrium impact of on y_i and p_i

We can compute the change in price in general equilibrium:

$$\begin{aligned} d\ln \mathrm{p} = \mathbf{A}^{-1}\big[&-(\mathbf{I}-\mathbf{T}\odot v^y)\alpha^{y,qf} + \mathbf{T}(v^{e^x}\odot\tfrac{1}{\nu}+v^{e^f}\tfrac{\sigma^y}{1-s^e}+v^{ne}) - \Big((\mathbf{I}-\mathbf{T}\odot v^y)\alpha^{y,z}-\tfrac{\sigma^y}{1-s^e}\Big)\bar{\gamma}\tfrac{1}{\bar{\nu}}\big] d\ln q^f \\ &+\big[-(\mathbf{I}-\mathbf{T}\odot v^y)\alpha^{y,qf} + \mathbf{T}(v^{e^f}\odot\tfrac{\sigma^y}{1-s^e})\big]\odot \mathrm{J} d\ln \mathrm{t}^\varepsilon + \theta\big(\mathbf{TS}\odot\mathbf{J}\odot d\ln \mathrm{t}^b - \mathbf{T}(\mathbb{1}+\mathbf{S}')\odot(\mathbf{J}\odot d\ln \mathrm{t}^b)'\big) \end{aligned}$$

with parameters: $\mathbf{S}$ for the trade share matrix, $\mathbf{T}$ income flow matrix, θ, Armington CES. Moreover, the general equilibrium (and leakage) effects are summarized in a complicated matrix $\mathbf{A}$ that summarize the fact that the price p_i also affects energy demand, oil-gas extraction, energy trade balance and output. Further description can be found in the appendix.

6.3 Optimal climate agreement

In this section, we describe the design of the optimal climate agreement. The climate club that maximizes world's welfare is a large coalition with all the country at the exception of Russia. Moreover, the carbon tax for member of the club is lowered below \$100 and tariffs are set at moderate rate of 50% for non-members. This outcome balances the intensive margin-extensive margin tradeoff of this policy design.

At the intensive margin, increasing carbon taxation reduces fossil fuel use and emissions for the countries participating in a climate agreement. As a result, aggregate welfare is increased until the optimal carbon tax $t^{\varepsilon} = \$147$ is reached. However, at the extensive margin, a higher tax reduces participation as free-riding incentives increase with the cost of taxation. If the tax becomes too high, individual countries deviate and leave the agreement, which raises world's emissions. In fig. 10, I show this phenomenon, where I plot the maximum participation that can be achieved depending on the choice of the levels of carbon tax for club members on the y-axis and the tariffs that are imposed on non-members on the x-axis.

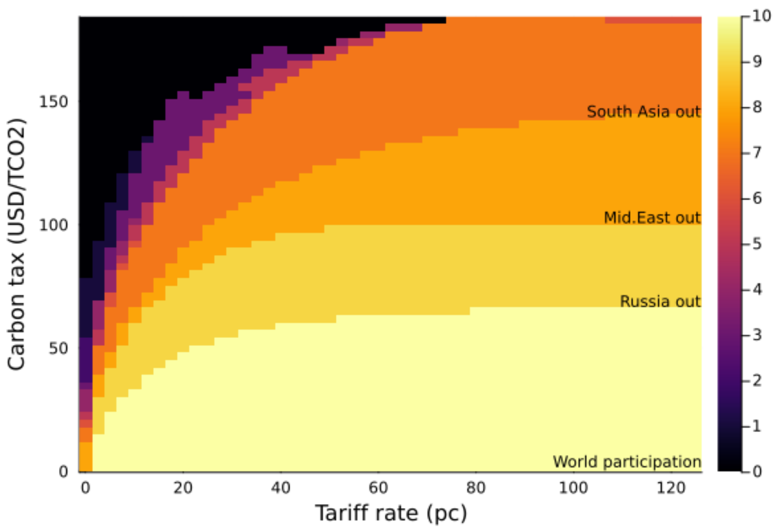

Figure 10: Participation: Intensive and extensive margin trade-off for agreement design: t^{ε} on y-axis, t^{b} on x-axis

For tax under \$50, the cost of carbon abatement is low, and it is relatively costless for countries to participate in an agreement. For higher taxes, we see that the first region to deviate is Russia and former Soviet Republics. Then leaves Middle-Eastern countries and South Asia. For even larger tax, closer to \$200, Sub-Saharian Africa would also exit the climate agreement. These decisions originate from the tradeoff explained in section 6.1. Indeed, those countries have a large cost of distortionary carbon taxation, either because they are producers of oil and gas, like Russia and Gulf countries, or because they consume a significant part of their energy mix in coal, like India and Africa. This compare to the cost of tariffs, which are relatively small for these four regions, at least for tariffs below 150% in the case of Africa and South Asia.

Another lesson from this analysis is that trade policy is a key strategic instrument to undermine free-riding. Indeed, absent tariff retaliation, with $t^{b} < 5\%$, the gains from unilateral deviation prevail over the costs of climate actions, and no carbon tax above $t^{\varepsilon} > \$50$ could be implemented for a large enough set of country. This result is discussed in Nordhaus (2015) and we recover this effect in our quantitative model. However, if moderate tariffs spur participation for low carbon tax, this incentive effect vanishes quickly as the carbon tax increases. Since the gains from trade are bounded – and small for some countries like Middle-East and Russia – there is a limit to what carbon policy can achieve.

Now, we turn to the design of optimal climate agreement, and the decision of carbon and tariffs. We see that the optimal Second-Best policy with a large carbon tax $t^{\varepsilon} = \$147$ and complete participation is not achievable in this context. As shown in the dotted lines in fig. 11 that it corresponds to an area where South-Asia, Middle-East, Russia and former soviet countries would all exit the agreement. As a result, the optimal agreement that would maximize welfare is such that the carbon tax is lowered from \$147 to \$98: this incentivizes the participation of South Asia and the Middle-East.

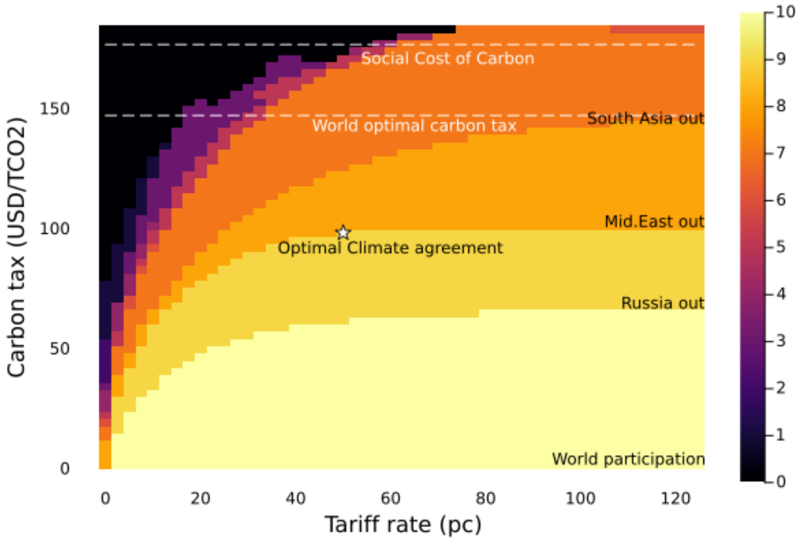

Figure 11: Optimal climate agreement
t^{ε} on y-axis, t^{b} on x-axis

However, it is optimal to leave Russia outside the agreement. Reducing the carbon tax to accommodate Russia's participation to the agreement necessitates a large fall in climate effort. A decrease of the tax from \$98 to around \$50 increases emissions of the entire world by that will compromise the implementation of effective solutions for global warming.

Our optimal climate agreement realize close to 90% of the welfare gains of the optimal-policy without endogenous participation, as seen in fig. 7 of section 5.2. In the next graph, we plot global welfare for the different values of carbon tax and tariffs (t^{ε}, t^{b}) , in consumption equivalent, as a difference to welfare in the competitive equilibrium $(t^{\varepsilon}, t^{b}) = (0, 0)$. We see that welfare increases non-monotonically in the carbon tax as carbon emissions and global temperature are reduced. However, when participation declines because countries unilaterally exit the agreement, the deviating countries go back to their status-quo policies, raising their emissions, which decreases discontinuously global welfare. The optimal agreement achieves almost 5% of consumption equivalent welfare gains, close to 90% of the welfare gains attained in the Second-Best when world participation is supposed exogenous.

However, when tariffs increases, this has a very moderate impact on welfare. Indeed, it has

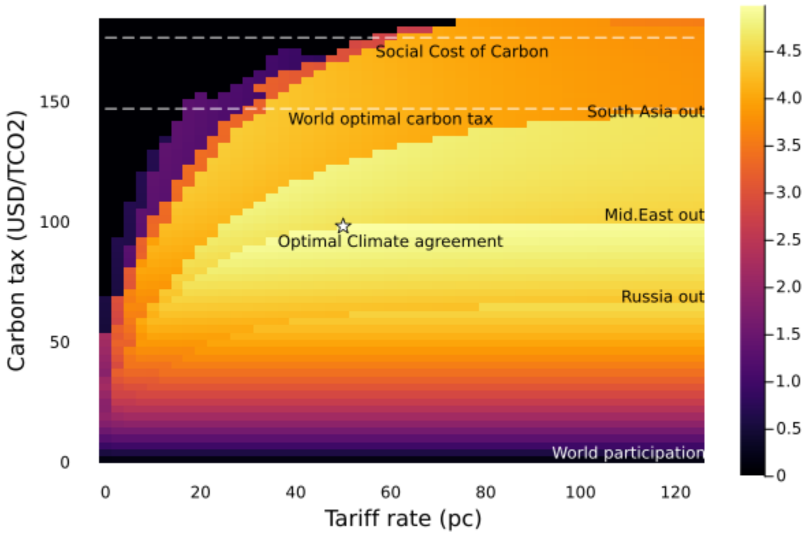

Figure 12: Welfare over different climate agreements
t^{ε} on y-axis, t^b on x-axis

a strong impact on individual countries utility if they are outside the club. However, since such countries – e.g. Russia, Middle-East and South-Asia – do not have much weights in the welfare criterion we used, especially considering the Negishi weights in fig. 2, this has a limited influence on global welfare. It mainly has a strong influence on participation through the impact on country outside option and welfare.

Changing the level of the carbon tax is fundamental for participation and the optimal design of the agreement, creating a *Laffer curve* for emissions and welfare. We plot in the following two graphs the change in global welfare – fig. 14 – and carbon emissions – fig. 13 – varying the carbon tax, and keeping the tariffs fixed at the optimum $t^b = 50\%$. Raising the carbon tax reduces emissions and improve welfare up to the point where participation declines. It is therefore optimal to "share the burden" of the carbon tax on a larger set of countries. In the optimal agreement, where all the countries in the world except Russia are included, we reduce emissions from 35 to below $24\,GtCO_2$ per year, a decline of 32% compared to the competitive – Business-as-Usual – equilibrium.

However, with the optimal climate agreement, emission reduction is not the maximum emission that can be achieved. If we choose a carbon tax of $\$170/tCO_2$, which corresponds roughly to the Social Cost of Carbon of the Second-Best equilibrium, we can shrink emissions further by more than 35% from 35 to around $22.5\,GtCO_2$. This implies that the very affected countries – Russia, Middle-East, South-Asia – all exit the agreements. The remaining countries, which are the developed economies, Europe, the Americas and East-Asia, all have to be a much larger cost of taxation. This agreement is still stable due to the enforcement power of tariffs, but the negative welfare impact of taxation for those countries is much larger now.

The difference of welfare between those two cases is sizable: our optimal climate agreement achieve a 5% welfare gain while a club with a more restricted set of countries and larger tax reaches

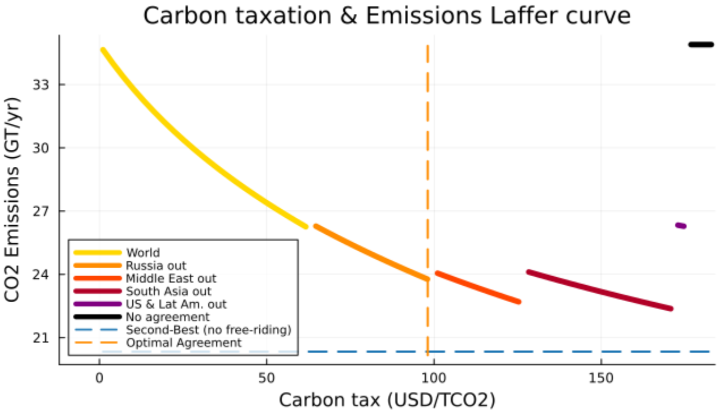

Figure 13: Global Emissions (yearly)
for different carbon tax a given tariff $t^b = 50\%$

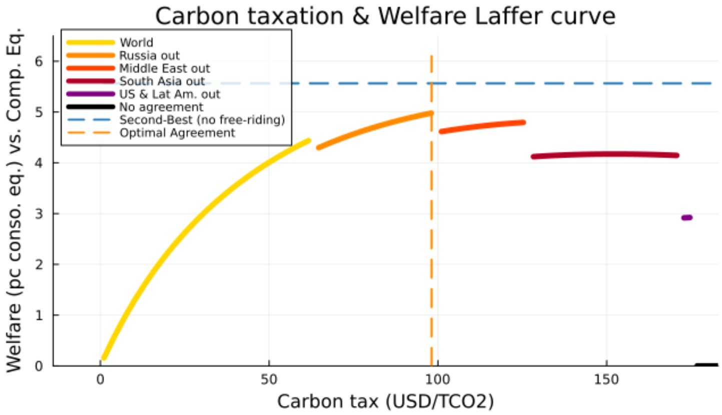

Figure 14: Global welfare (percent consumption eq.),
compared to the Competitive Equilibrium
for a given tariff $t^b = 50\%$

only a 4.2% welfare gain. Making a smaller set of countries bear the cost of taxation is detrimental for their welfare: developed countries typically consume larger quantities of energy, and developing economies have a larger cost of the distortion as their production and consumption is scarce – especially if they are affected by climate change. The agreement is stable because the cost of tariffs is larger, enforcing this cooperation.

However, it is beneficial to work at the extensive margin, reduce the distortionary carbon tax, foster participation and thus reduce the tariffs that are imposed to non-members, promoting free-trade. This shares the cost of fighting climate change over a larger set of countries without being too harmful for economic activity and welfare.

Because of endogenous participation, welfare and emissions are indeed different metrics that provide contrasting insights on what should be the optimal policy. In the next graphs, fig. 15 and fig. 16, we respectively summarize, for the different equilibria we considered above, the global emissions in Gigatons of CO_2 and welfare in consumption equivalent difference compared to the competitive equilibrium. We see, without surprise that the First Best has the lowest emissions – $18.5 GtCO_2$, a reduction of 47% compared to the Business-as-Usual scenario – and the maximum welfare – 16% of consumption equivalent change. In this case, the planner has access to unlimited instruments: it uses transfers to redistribute across countries, which offset the negative general equilibrium effects of taxation, and allows to set a higher carbon tax and lowering further the carbon emissions. In contrast, in the Second-Best, these redistributive instruments are not available, which make the welfare gains much smaller at 5.6% and emissions are slightly higher.

In these two benchmarks, we assume away free-riding, which constrains the achievable policy and carbon reduction. We now compare the two equilibria considered above: the optimal agreement with all the countries except Russia reaches only a reduction of 32% of carbon emissions. This reaches the maximum welfare but not the minimum emissions, while a club with larger tax loses on welfare by increasing the tax burden because of lower participation in the agreement.

The analysis of the potential welfare gains of the First-Best highlight that transfers can serve as a particularly strong instruments to offset the negative effects of the uniform carbon taxation and tariffs, and we investigate if we can provide such welfare improvements with transfers section 7.1 and with fossil-fuels specific tariffs in section 7.2

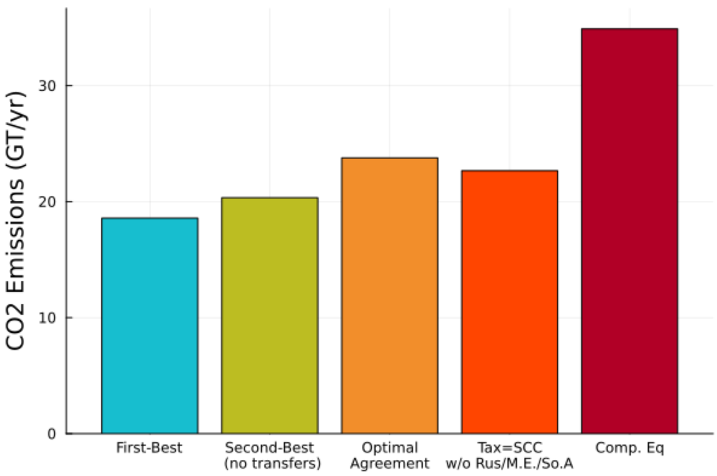

Figure 15: Global Emissions (yearly) comparison across equilibria

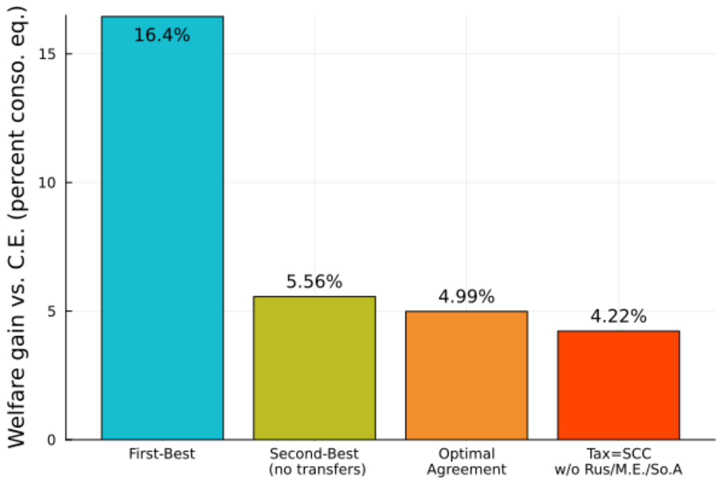

Figure 16: Global welfare (percent consumption eq.), comparison across equilibria

6.4 Coalition building

We proposed an optimal agreement chosen by the social planner/designer. However, can this agreement be achieved by coalition building? Can a sequence of countries joining the climate agreement in turn reach this agreement? This relates to the question of which country has the most interest in joining such a club.

We investigate if this climate agreement can be constructed, with a sequence of "rounds" of our static equilibrium: At each round (n), each country decide to enter or not depending on the welfare gain:

$$\Delta_i \mathcal{U}_i(\mathbb{J}^{(n)}) = \mathcal{U}_i(\mathbb{J}^{(n)} \cup \{i\}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b}) - \mathcal{U}_i(\mathbb{J}^{(n)} \backslash \{i\}, \mathrm{t}^{\varepsilon}, \mathrm{t}^{b})$$

For now, the construction is evaluated at the optimal carbon tax $\mathrm{t}^{\varepsilon} = 98\$$, and tariff $\mathrm{t}^{b} = 50\%$ and perform this sequential procedure – coming “for free” from our CDCP algorithm / squeezing procedure section 3.2. This experiment is inspired by an analogous exercise in Farrokhi and Lashkaripour (2024).

The result of this exercise is a sequence building up to the optimal climate agreement:

- Round 1: European Union
- Round 2: China, South East Asia (Asean)
- Round 3: North-America, South-Asia, Subsaharian Africa,
  Advanced East Asia, Latin America

– Round 4: Middle-East

$\notin$ Stay out of the agreement: Russia+CIS

The European Union has the best-interest in both reducing climate change, being relatively little affected by a change in fossil-fuel price, consuming a small share of coal in their energy mix, and being wealthy enough to suffer less from a cost on energy for their production. In the second round, China, an important trading partner of European countries and South-East-Asia which has one of the highest gain of fighting global warming, in turn join the climate agreement. In the next round, most other countries which have large gains-from-trade join the climate club to avoid retaliatory tariffs. Lastly, Middle-East also join to be able to trade with the rest of the world.

7 Extensions: Additional instruments - Impact of additional policy instruments

In this section, we propose extensions to our baseline climate agreement by suggesting additional instruments to improve the allocation. By proposing simple policy that could be achievable in practice, we investigate if we can improve on the optimal climate agreement presented above.

7.1 Transfers and Loss and damage funds

One of the major policy proposal of the COP28 in Dubai is the idea of *loss and damage funds* to compensate the countries particularly affected by global warming.

I propose a simple implementation in the context of our model. Given that the club is implementing a substantial carbon tax t^{ε}, one practical proposal is to redistribute a share of the revenues of that tax, with lump-sum transfer across countries.

In the baseline agreement, the revenue of that tax are redistributed to the household of the country that pays the tax: $\mathrm{t}_i^{ls} = \mathrm{t}^{\varepsilon}(\xi^f e_i^f + \xi^c e_i^c)$. The exercise proposed here is to redistribute a share α^{ε} to a "loss-and-damage fund" which would then share those revenues equally across countries, with a simple rule:

$$\mathrm{t}_i^{ls} = (1-\alpha^{\varepsilon})\mathrm{t}^{\varepsilon}\varepsilon_i + \alpha^{\varepsilon}\frac{1}{\mathcal{P}}\sum_j \mathcal{P}_i \mathrm{t}^{\varepsilon}(\xi^f e_h^f + \xi^c e_j^c)$$

In practice, it would transfers from large emitters – which tend to be developed economies – to low emitters – developing economies that tend to be more vulnerable to climate change.

We then choose the optimal share α^{ε} to maximize global welfare $\mathcal{W}$, which is computed to be $\alpha^{\varepsilon,\star} = 15\%$. We show the result of that experiment in fig. 17.

Unfortunately, the optimal loss-and-damages fund proposed does not provide particularly large welfare gain in aggregate – around 0.006%, since this policy only redistribute lump-sum from large energy users to low energy users. The welfare cost are around 0.6% for Europe, the United-States and Advanced countries in Asia and Oceania. Because our welfare function is particularly biased toward the welfare of these advanced countries, this is an explanation why the effort that

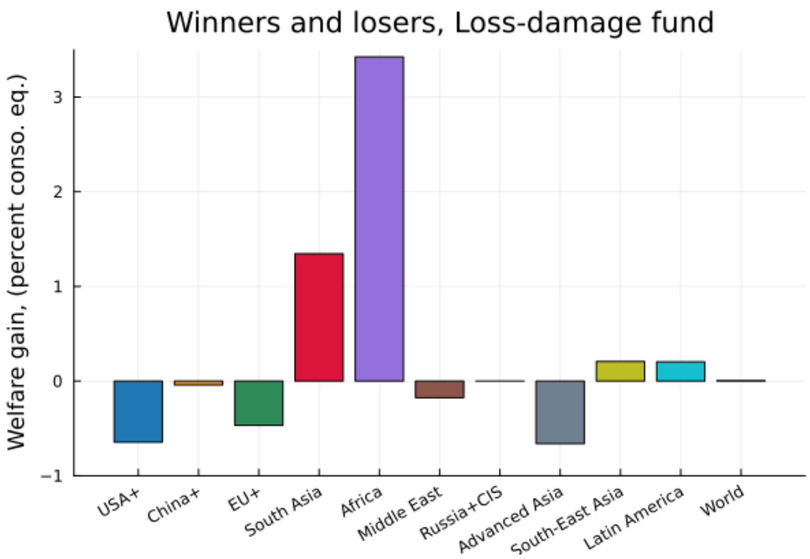

Figure 17: Welfare across countres
Optimal loss and damage fund: $\alpha^{\varepsilon,\star} = 15\%$

is optimal is quite low at $\alpha^{\varepsilon,\star} = 15\%$. Larger contribution would be detrimental to their welfare which is not optimal for the planner.

However, even small loss-and-damage funds redistribution is particularly welfare-improving for South-Asia and Africa who gain respectively 1.4 and 3.3% welfare gains – and South-East Asia and Latin America to a smaller magnitude. Those regions are particularly smaller contributors of the global climate externality and such transfers would allow to lower the cost of climate change through adaptation and to dampen the redistributive cost of carbon taxation.

7.2 Fossil-fuels specific tariffs

In the current climate club, the members of the club only impose penalty tariffs on the final goods traded by the firm, and not on energy imports.

This is empirically relevant, c.f. Shapiro (2021) and Copeland et al. (2021): Inputs are usually more emission-intensives but trade policy is biased against final goods output. Moreover, in our context, fossil-fuel energy inputs are not carbon-intensive *per-se*, it is the use – i.e. the burning – of those fossil fuels in the production process that is carbon-intensive. As a result, Carbon-Border-Adjustment mechanisms would typically only impose a tariff on the "scope-1" (or scope-2) carbon footprint of the production of the fossil fuel – and not the "scope-3" of the downstream supply chain.

However, in our climate club setting, these tariffs are also strategic to incentivize participation. We therefore propose an alternative mechanism where the club members impose a tariff on the fossil fuel export of the countries outside the club.

The tariff is a import tax on the energy import from non-participants of the form:

$$\mathrm{t}_{ij}^{bf} = \beta \xi^f \mathrm{t}^{\varepsilon} \mathbb{1}\{i \in \mathbb{J}, j \notin \mathbb{J}\}$$

In the following graph fig. 18, we plot over different values of β the welfare impact for members of the club and the non-members – which is Russia here – to see if this strategic tariff can provide enough incentive for Russia to join the club with a carbon-tax as in the second best.

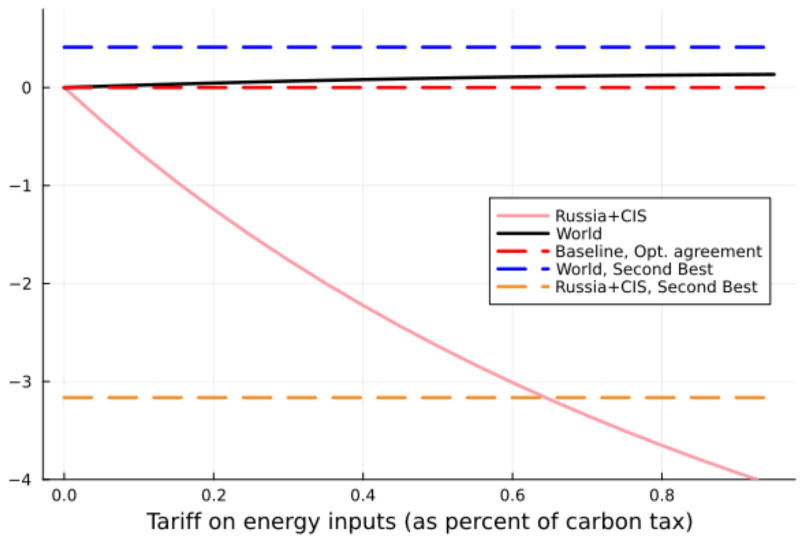

Figure 18: Welfare across countres
Optimal loss and damage fund: $\alpha^{\varepsilon,\star} = 15\%$

We see that with a fossil-fuel specific tariffs of $\beta = 60\%$ of the carbon-tax, the second-best allocation can be achieved, through incentive effect and welfare can be maximized.

8 Conclusion

This paper examines the design of an optimal climate agreement in the presence of free-riding incentives and redistributive effects. I develop a multi-country Integrated Assessment Model (IAM) that incorporates international trade in goods and energy markets for fossil fuels. This model accounts for heterogeneity across countries in terms of their vulnerability to climate change, income levels, energy mix, and positions as exporters or importers of goods and energy.

This analysis focuses on a global social planner's problem of maximizing world welfare through a climate agreement comprising three key elements: (1) the set of countries included in the agreement (the "climate club"), (2) an optimal level of carbon tax imposed on club members, and (3) an optimal level of trade tariffs imposed on non-member countries. I consider Nash equilibria where countries make strategic decisions about their participation, either unilaterally or through coalition deviations.

This study reveals a crucial trade-off between intensive and extensive margins in designing the optimal climate agreement. A small coalition of countries can implement high carbon taxes, achieving significant emissions reductions. However, a more extensive club with broader participation may be necessary for effectively combating global climate change, albeit at the cost of lower carbon taxes.

The main findings is first that the optimal climate club includes all countries except Russia, with a moderate carbon tax of $100 per ton of CO_2 and a 50% tariff on goods from non-participants. To increase participation, it is beneficial to reduce the carbon tax by 35% from the globally optimal level of $150 per ton of CO_2. This allows for the inclusion of Middle Eastern countries and several developing economies in South Asia and Africa. Excluding fossil fuel producers like Russia from the agreement is optimal, as their welfare costs from carbon taxation are too high to justify inclusion at any reasonable tax rate. Trade policy, particularly the threat of tariffs, is a key strategic instrument for undermining free-riding and incentivizing participation. However, its effectiveness diminishes as carbon taxes increase. Additional policy instruments, such as transfers through a "loss and damage" fund or fossil-fuel-specific tariffs, can improve the climate agreement and push the carbon tax closer to the second-best allocation.

In conclusion, this research underscores the complexity of designing effective climate agreements in a world of heterogeneous countries with divergent interests. It demonstrates that while a universal agreement with globally optimal carbon taxation may be unattainable due to free-riding incentives and redistributive effects, carefully designed climate clubs with strategic use of trade policy can achieve significant progress in global climate action.

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Appendix

A Calibration

Table 2: Baseline calibration

Technology & Energy markets			
α	0.35	Capital share in $F(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $F(\cdot)$	Energy cost share (8.5%)
σ^y	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2022)
ω^f	0.56	Fossil energy share in $e(\cdot)$	Oil-gas/Energy ratio
ω^c	0.27	Coal energy share in $e(\cdot)$	Coal/Energy ratio
ω^r	0.17	Non-carbon energy share in $e(\cdot)$	Non-carbon/Energy ratio
σ^e	2.0	Elasticity fossil-coal-non-carbon	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
Preferences & Time horizon			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	1.5	IES / Risk aversion	Standard calibration
n	0.0035	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
ω_i	$1/u'(c_i)$	Pareto weights	Negishi / Status-quo Social Planner
T	400	Time horizon	Time for climate system to stabilize
Climate parameters			
ξ^f	2.761	Emission factor – Oil & natural gas	Conversion 1 $MTOE$ $\Rightarrow$ 1 $MT\ CO_2$
ξ^c	3.961	Emission factor – Oil & natural gas	Conversion 1 $MTOE$ $\Rightarrow$ 1 $MT\ CO_2$
χ	$2.3/1e6$	Climate sensitivity	Pulse experiment: $100\,GtC \equiv 0.23°C$ medium-term warming
δ_s	0.0004	Carbon exit from atmosphere	Pulse experiment: $100\,GtC \equiv 0.15°C$ long-term warming
ζ	0.027	Growth rate, Carbon Capture and Storage	Starting after 2100, Follows Krusell Smith (2022)
$\gamma^\oplus$	0.003406	Damage sensitivity	Nordhaus' DICE
$\gamma^\ominus$	$0.3\times\gamma^\oplus$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^T	0.5	Weight historical climate for optimal temp.	Marginal damage correlated with initial temp.
$T^\star$	14.5	Optimal yearly temperature	Average spring temperature / Developed economies

A.1 Additional calibration graphs

A.1.1 Quantification – Trade shares

We displayed the trade share from the data in fig. 19 and how we calibrate the trade model. Armington Trade model and trade shares:

$$s_{ij} \equiv \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_i} = a_{ij}\frac{((1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k)^{1-\theta}}$$

We estimate a gravity regression, and CES $\theta = 5.63$. The Iceberg cost τ_{ij} are projection of geographical distance $\log\tau_{ij} = \beta\log d_{ij}$. The preference parameters a_{ij} identified as remaining variation in the trade share s_{ij}. As a results, both τ_{ij} and a_{ij} are policy invariant in our climate agreement setting. The description of the procedure is detailed in section 4.3.

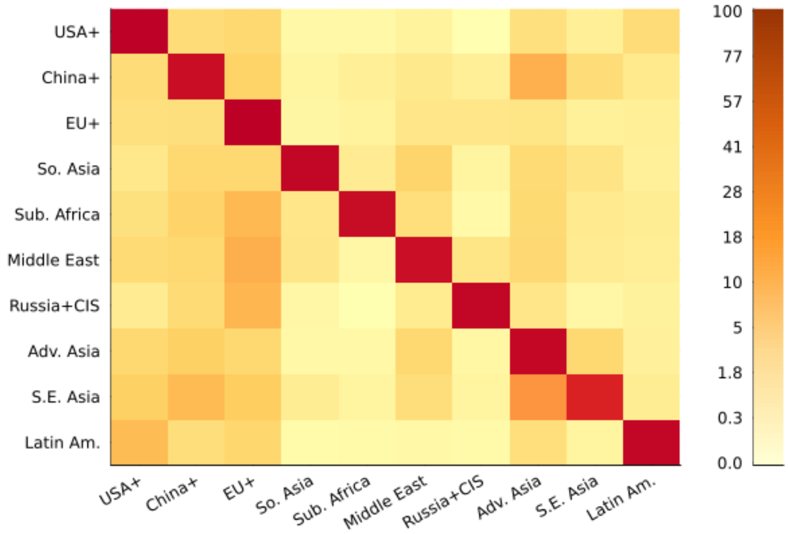

Figure 19: Trade shares as mesured in Conte et al. (2022)

A.1.2 Climate system and pulse experiment

This pulse experiment, from Dietz et al. (2021), summarizes how our climate model should be calibrated to replicate larger scale IAMs like CMIP5.

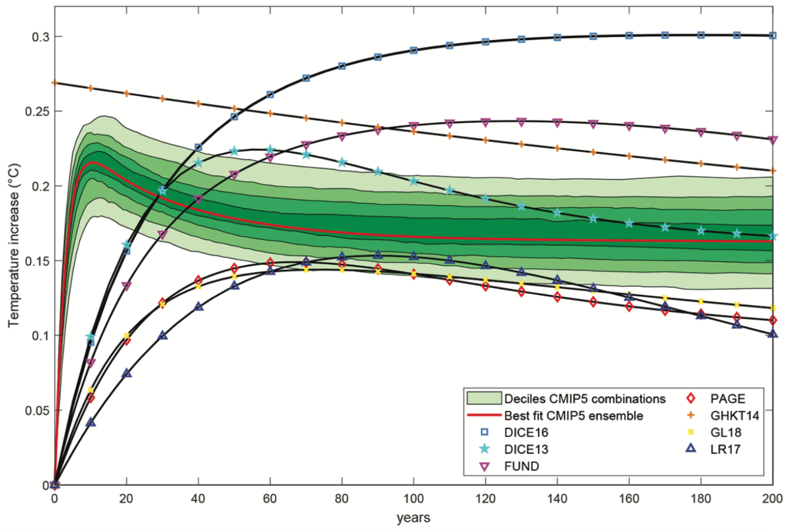

Figure 20: Pulse experiment: $\mathcal{E} = 100GtCO_2$ at $t = 0$
Comparison across different IAMs, Dietz et al. (2021)

B Model

B.1 Model environment

- Gov. policies $t_i = \{\mathrm{t}^{\varepsilon}_{it}, \mathrm{t}^{b}_{ijt}, \mathrm{t}^{ls}_{it}\}$
- State: $s_i = \{k_{it_0}, T_{it}, \mathcal{R}_{it}\}_t$,
- Agents (HH/firms) controls $c_i = \{c_{it}, c_{ijt}, k_{it}, e^f_{it}, e^c_{it}, e^r_{it}, e^x_{it}\}_t$
- Eq prices: $p = \mathrm{p}_{it}, w_{it}, q^f_t, q^c_{it}, q^r_{it}$
- Lagrange multipliers / costates: $\lambda_i = \{\lambda^w_{it}, \lambda^S_{it}, \lambda^T_{it}\}$
- Local welfare vs Global welfare

$$\mathcal{U}_i = \max_{c_i} \int_0^T e^{-\bar{\rho}_i t} \mathcal{P}_i u(c_{it}, T_{it}) dt$$

$$\mathcal{W} = \max_{\{c_i\}_i} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i \int_0^T e^{-\bar{\rho}_i t} u(c_{it}, T_{it}) dt$$

- Assumptions:
  - weak separability of utility

$$u(c_{it}, T_{it}) = \tilde{u}(c_i)\mathcal{D}^u(T_{it}) = \frac{c_{it}^{1-\eta}}{1-\eta}\big(\tilde{\mathcal{D}}^u(T_{it})\big)^{1-\eta}$$

  - DICE damage functions $\mathcal{D}^u_i(T)$ and $\mathcal{D}^y_i(T)$

$$\mathcal{D}^u_i(T) = e^{-\frac{\gamma^c}{2}(T-T^\star_i)^2} \qquad \Rightarrow \qquad \mathcal{D}^{u\prime}_i(T) = -\mathcal{D}^u_i(T)\gamma^c(T - T^\star_i)$$

  - CES demand / Armington structure, price of imports $p_{ij} = \tau_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_j$

$$u(\{c_{ij}\}_j, T_i)) = u(c_i, T_i) \qquad c_i = \Big(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\Big)^{\frac{\theta}{\theta-1}}$$

- Heterogenous discount rate: $\bar{\rho}_i = \rho - n_i - (1-\eta)\bar{g}_i$
- Climate system:

$$\dot{\mathcal{S}}_t = \zeta_t \mathcal{E}_t - \delta_s \mathcal{S}_t = e^{-ot}[\xi^f e^f_{it} + \xi^c e^c_{it}] - \delta_s \mathcal{S}_t$$

$$\mathcal{S}_t = \mathcal{S}_0 e^{-\delta_s t} + \int_0^t e^{-\delta_s(t-u)} e^{(n+\bar{g}-o)u}\big(\xi^f e^f_{iu} + \xi^c e^c_{iu}\big) du$$

$$T_{it} = T_{it_0} + \Delta_i \chi \mathcal{S}_t$$

with $o = g_\zeta \mathbb{1}\{t > 2100\}$ the rate of growth of additional abatement due to CCS after 2100.

B.2 Summary, model setting

- Expenditure by household:

$$\sum_j c_{ijt}\tau_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_{jt} = c_{it}\mathbb{P}_{it}$$

- Final good firm problem: pay for labor and capital and buys three energy inputs:

$$\pi^g_{it} = \mathrm{p}_i\mathcal{D}^y(T_{it})z_{it}F(\ell_i, k_{it}, e^f_{it}, e^c_{it}, e^r_{it}) - w_{it}\ell_i - r_t k_{it} - (q^f_t+\xi^f\mathrm{t}^s_{it})e^f_{it} - (q^c_t+\xi^c\mathrm{t}^s_{it})e^c_{it} - q^r_{it}e^r_{it} = 0$$

- Fossil Energy firm profit:

$$\mathcal{P}_i\pi^f_{it} = q^f_t e^x_{it} - \mathcal{C}^f_i(e^x_{it}, \mathcal{R}_{it})\mathcal{P}_i$$

- Budget constraint for the household:
  replace labor income, and divide by price index (analog of "real" quantities). Reminder that capital expenditure are made in the final consumption good bundle.

$$\begin{aligned}
c_{it}\mathbb{P}_{it} + \big(\dot{k}_{it} + (n_i + \bar{g}_i + \delta)\big)\mathbb{P}_{it} &= w_{it}\ell_i + r_t k_{it} + \pi^f_t + \mathrm{t}^{ls}_i \\
0 &= \frac{\mathrm{p}_{it}}{\mathbb{P}_{it}}\mathcal{D}^y(T_{it})z_{it}F(k_{it}, e^f_{it}, e^c_{it}, e^r_{it}) - (n_i + \bar{g}_i + \delta)k_{it} \\
&\quad + \frac{1}{\mathcal{P}_i\mathbb{P}_{it}}\Big[q^f_t e^x_{it} - \nu(e^x_{it}, \mathcal{R}_{it})\Big] - \frac{(q^f_t+\xi^f\mathrm{t}^s_{it})}{\mathbb{P}_{it}}e^f_{it} - \frac{(q^c_t+\xi^c\mathrm{t}^s_{it})}{\mathbb{P}_{it}}e^c_{it} - \frac{q^r_{it}}{\mathbb{P}_{it}}e^r_{it} - c_{it} + \frac{\mathrm{t}^{ls}_{it}}{\mathbb{P}_{it}} - \dot{k}_{it}
\end{aligned}$$

- CES / Armington trade model, with price of imports $p_{ij} = \tau_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_j$

$$u(\{c_{ij}\}_j, T_i)) = u(c_i)\,\mathcal{D}^u_i(T_i) \qquad c_i = \Big(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\Big)^{\frac{\theta}{\theta-1}}$$

$$FOC \qquad [c_{ij}] \qquad u'(c_i)\mathcal{D}^u_i(T_i)\, c_i^{\frac{1}{\theta}} a_{ij}^{\frac{1}{\theta}} c_{ij}^{-\frac{1}{\theta}} = p_{ij}\lambda^w_i$$

Price index:

$$\mathbb{P}_i = \Big(\sum_j a_{ij}p_{ij}^{1-\theta}\Big)^{\frac{1}{1-\theta}} = \Big(\sum_j a_{ij}(\tau_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_j)^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

Demand system:

$$\Rightarrow \qquad \frac{c_{ij}}{c_i} = a_{ij}\big(\frac{\mathbb{P}_i}{p_{ij}}\big)^{\theta} \qquad \Rightarrow \qquad \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_i} = a_{ij}\big(\frac{p_{ij}}{\mathbb{P}_i}\big)^{1-\theta}$$

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_{it}} = a_{ij}\frac{p_{ij}^{1-\theta}}{\sum_k a_{ik}p_{ik}^{1-\theta}} = a_{ij}\frac{((1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k)^{1-\theta}}$$

Aggregating, we obtain the marginal value of "wealth":

$$\lambda^w_i = \frac{u'(c_i)\mathcal{D}^u_i(T_i)}{\mathbb{P}_i}$$

B.2.1 Market clearing

We reexpress the market clearing, for good i in expenditure terms. The time subscripts are removed for conciseness.

$$\mathcal{P}_i y_i = \overline{\mathcal{D}}_i(\{T_{it}\}) z_i F(k_i, e_i) = \sum_{k\in\mathbb{I}} \tau_{ki}\mathcal{P}_k c_{ki} + \sum_{k\in\mathbb{I}} \mathcal{P}_k \tau_{ki}(x^f_{ki} + x^c_{ki} + x^r_{ki} + x^k_{ki})$$

Rewriting in expenditure, multiplying by p_j, and using the fact that the input choice is identical between

$$\begin{aligned}
\mathcal{P}_i y_i \mathrm{p}_i &= \sum_{k\in\mathbb{I}} \mathcal{P}_k \tau_{ki}(x^f_{ki} + x^c_{ki} + x^r_{ki} + x^k_{ki})\mathrm{p}_i \\
&= \sum_{k\in\mathbb{I}} \mathcal{P}_k \frac{1}{1+\mathrm{t}^b_{ki}} \tau_{ki}(1+\mathrm{t}^b_{ki})\mathrm{p}_i(c_{ki} + x^f_{ki} + x^c_{ki} + x^r_{ki} + x^k_{ki}) \\
&= \sum_{k\in\mathbb{I}} \mathcal{P}_k \frac{1}{1+\mathrm{t}^b_{ki}} s_{ki}(c_i + \mathcal{C}^f_i(e^x_i) + z^c_i e^c_i + z^r_i e^r_i + (n_k + \bar{g}_k + \delta)k_k)\mathbb{P}_k
\end{aligned}$$

Using the budget constraint to replace c_i

$$\begin{aligned}
\mathcal{P}_i y_i \mathrm{p}_i = \sum_{k\in\mathbb{I}} \frac{\mathcal{P}_k s_{ki}}{1+\mathrm{t}^b_{ki}} \Big(& y_k \mathrm{p}_k - (q^f + \xi^f \mathrm{t}^\varepsilon_k)e^f_k - (q^c_k + \xi^c \mathrm{t}^\varepsilon) - q^r_k e^r_k - (n_i + \bar{g}_i + \delta)k_{it}\mathbb{P}_k + \Big[q^f e^x_k - \mathcal{C}^f_k(e^x_k)\mathbb{P}_k\Big] + \mathrm{t}^{ls}_k \\
& + \mathcal{C}^f_i(e^x_k)\mathbb{P}_k + z^c_k e^c_k \mathbb{P}_k + z^r_k e^r_k \mathbb{P}_k + (n_k + \bar{g}_k + \delta)k_k\mathbb{P}_k \Big) \\
\mathcal{P}_i y_i \mathrm{p}_i = \sum_{k\in\mathbb{I}} \frac{\mathcal{P}_k s_{ki}}{1+\mathrm{t}^b_{ki}} \Big(& y_k \mathrm{p}_k + q^f\left(e^x_k - e^f_k\right) + \widetilde{\mathrm{t}}^{ls}_k \Big) = \sum_{k\in\mathbb{I}} \frac{s_{ki}}{1+\mathrm{t}^b_{ki}} \mathcal{P}_k(\widetilde{v}_k + \widetilde{\mathrm{t}}^{ls}_k)
\end{aligned}$$

where $\widetilde{v}_k = y_k \mathrm{p}_k + q^f\left(e^x_k - e^f_k\right)$ represent the revenues of country k in terms of production and energy export and the lump-sum transfers of the tariffs: $\widetilde{\mathrm{t}}^{ls}_k = \sum_j \mathrm{t}^b_{kj}\tau_{kj}(c_{kj} + x^f_{kj} + x^c_{kj} + x^r_{kj} + x^k_{kj})$.

We see that the lump-sum transfer also depends on the quantities. To be able to express the market in expenditure, we solve:

$$\begin{aligned}
\mathcal{P}_i p_i y_i &= \sum_i \frac{s_{ki}}{1+\mathrm{t}^b_{ki}} \mathcal{P}_k[\widetilde{v}_k + \widetilde{\mathrm{t}}^{ls}_k] \\
v_k &:= \widetilde{v}_k + \widetilde{\mathrm{t}}^{ls}_k \\
\widetilde{\mathrm{t}}^{ls}_k &= \sum_j \mathrm{t}^b_{kj}\tau_{kj}(\underbrace{c_{kj} + x^f_{kj} + x^r_{kj} + x^c_{kj} + x^k_{kj}}_{=:x_{kj}})\mathrm{p}_k \qquad\qquad x_{kj} = \frac{s_{kj} v_k}{(1+\mathrm{t}_{kj})\tau_{kj}\mathrm{p}_k}
\end{aligned}$$

As a result, we solve the "fixed point" for v_i as follow:

$$\begin{aligned}
v_i &= \widetilde{v}_i + v_i \sum_j \frac{\mathrm{t}^b_{ij}}{1+\mathrm{t}^b_{ij}} s_{ij} \\
v_i &= \frac{1}{1 - \sum_j \frac{\mathrm{t}^b_{ij}}{1+\mathrm{t}^b_{ij}} s_{ij}} \widetilde{v}_i = m_i \widetilde{v}_i \qquad \text{with} \qquad m_i = \frac{1}{1 - \sum_j \frac{\mathrm{t}^b_{ij}}{1+\mathrm{t}^b_{ij}} s_{ij}}
\end{aligned}$$

To conclude, the market clearing writes:

$$\mathcal{P}_i p_i y_i = \sum_i \frac{s_{ki}}{1+\mathrm{t}^b_{ki}} \mathcal{P}_k m_k \Big[y_k \mathrm{p}_k + q^f\left(e^x_k - e^f_k\right)\Big]$$

B.3 Making the dynamic model stationary

We solve the optimization problem of the household – who own the firms. This is a dynamic problem, since climate changes over time, with emissions $\mathcal{E}_t$. We express the Lagrangian for the problem as a finite-horizon problem, and we take the finite horizon $T \to \infty$.

$$\begin{aligned}\mathcal{L}(s_i, c_i, \lambda_i) = &\int_0^T e^{-\bar{\rho}_i t}\mathcal{P}_i u(c_{it}, T_{it})dt + \int_0^T e^{-\bar{\rho}_i t}\mathcal{P}_i \lambda^w_{it}\Big(\mathrm{p}_i \mathcal{D}^y(T_{it}) z_{it} F(k_{it}, e^f_{it}, e^c_{it}, e^r_{it}) - (n_i + \bar{g}_i + \delta)k_{it} \\ &\frac{1}{\mathcal{P}_i}\Big[q^f_t e^x_{it} - \nu(e^x_{it}, \mathcal{R}_{it})\Big] - (q^f_t + \xi^f \mathrm{t}^s_{it})e^f_{it} - (q^c_t + \xi^c \mathrm{t}^s_{it})e^c_{it} - q^r_{it} e^r_{it} - c_{it}\mathbb{P}_{it} + \mathrm{t}^{ls}_i - \dot{k}_{it}\Big)dt \\ &+ \int_0^T e^{-\bar{\rho}_i t}\mathcal{P}_i \lambda^S_{it}\Big[\mathcal{S}_0 e^{-\delta_s t} + \int_0^t e^{-\delta_s(t-u)} e^{(n+\bar{g}-o)u}\big(\xi^f e^f_{iu} + \xi^c e^c_{iu}\big)du - \mathcal{S}_t\Big]dt \\ &+ \int_0^T e^{-\bar{\rho}_i t}\mathcal{P}_i \lambda^T_{it}\Big(\Delta_i \chi \mathcal{S}_t - (T_{it} - T_{it_0})\Big)dt\end{aligned}$$

We impose the "constraint" that all the economic controls need to be constant over time, $c_{it} = c_i$ $\forall i$. As a result, all the equilibrium prices are also constant over time $p_{it} = p_i$ $\forall i$.

$$\begin{aligned}\mathcal{L}(s_i, c_i, \lambda_i) = &\mathcal{P}_i u(c_i) \underbrace{\int_0^T e^{-\bar{\rho}_i t}\mathcal{D}^u(T_{it})dt}_{=\overline{\mathcal{D}}^u(\{T_{it}\}_t)} + \mathcal{P}_i\Big(\int_0^T e^{-\bar{\rho}_i t}\lambda^w_{it}\mathcal{D}^y(T_{it})dt\Big)\mathrm{p}_i z_{it} F(k_i, e^f_i, e^c_i, e^r_i) \; - \int_0^T e^{-\bar{\rho}_i t}\dot{k}_{it}\lambda^w_{it}dt \\ &+ \frac{1}{\mathcal{P}_i}\left[q^f e^x_i - \nu(e^x_i, \mathcal{R}_i)\right]\underbrace{\int_0^T e^{-\bar{\rho}_i t}\lambda^w_{it}dt}_{=\overline{\lambda}^w_i} - \mathcal{P}_i\overline{\lambda^w_i}\Big((q^f + \xi^f \mathrm{t}^\varepsilon_i)e^f_i + (q^c + \xi^c \mathrm{t}^\varepsilon_i)e^c_i + q^r_i e^r_i + (n_i + \bar{g}_i + \delta)k_i + c_i\mathbb{P}_i + \mathrm{t}^{ls}_i\Big) \\ &+ \mathcal{P}_i \int_0^T e^{-\bar{\rho}_i t}\lambda^S_{it}\mathcal{S}_t dt + \mathcal{P}_i \int_0^T e^{-\bar{\rho}_i t}\lambda^T_{it}\Big(\Delta_i \chi \mathcal{S}_t - (T_{it} - T_{it_0})\Big)dt\end{aligned}$$

with $\mathcal{S}_t = \mathcal{S}_0 e^{-\delta_s t} + \int_0^t e^{-\delta_s(t-u)} e^{(n+\bar{g}-o)u}\big(\xi^f e^f_{iu} + \xi^c e^c_{iu}\big)du$

Optimality conditions:

- Consumption $[c_i]$:

$$u'(c_i)\overline{\mathcal{D}}^u(\{T_{it}\}_t) = \overline{\lambda}^w_{it}\mathbb{P}_i$$

- Energy choices $[e^k_i]$, for $k \in \{f, c, r\}$:

$$\begin{aligned}\overline{\lambda}^w_i\big(\mathrm{p}_i MPe^f_i - q^f + \xi^f \mathrm{t}^\varepsilon_i)\big) &= 0 \\ \overline{\lambda}^w_i\big(\mathrm{p}_i MPe^c_i - q^c + \xi^c \mathrm{t}^\varepsilon_i\big) &= 0 \\ \overline{\lambda}^w_i\big(\mathrm{p}_i MPe^r_i - q^r\big) &= 0\end{aligned}$$

- Capital choice $[k_{it}]$

$$\dot{\lambda}^w_{it} = \lambda^w_{it}\big(\mathrm{p}_i MPk_i - \delta - \eta\bar{g}_i - \rho\big)$$
$$\mathrm{p}_i MPk_i - \delta = \bar{r} = \eta\bar{g}_i + \rho \qquad \Rightarrow \qquad \dot{\lambda}^w_{it} = 0 \quad \& \quad \lambda^w_{it} = \lambda^w_{it'} = \lambda^w_i$$
$$\overline{\lambda}^w_{iT} = \int_0^T e^{-\bar{\rho}_i t}\lambda^w_{it}dt = \frac{1}{\bar{\rho}_i}(1 - e^{-\bar{\rho}_i T})\lambda^w_i$$

- Fossil production choice $[e^x_i]$:

$$\overline{\lambda}^w_i\big(q^f - \mathcal{C}_{e^x}(e^x_i, \mathcal{R}_i)\mathbb{P}_i\big) = 0$$

- Stock of carbon in the atmosphere $[\mathcal{S}_t]$

$$\lambda^S_{it} = \Delta_i\chi\lambda^T_{it}$$

- Local temperatures $[T_i]$

$$\begin{aligned}\lambda^T_{it} &= \mathcal{D}^{u'}(T_{it})u(c_i) + \mathcal{D}^{y'}(T_{it})z_iF(k_i, e_i)\lambda^w_i \\ &= -(T_{it} - T^\star_i)\big(\gamma^c_i c_{it} + \gamma^y_i y_{it}\big)\lambda^w_i \qquad [w/\text{DICE damage fcts + CRRA pref}]\end{aligned}$$

- Market clearing, energy

$$\sum_{i\in\mathbb{I}} e^x_i = \sum_{i\in\mathbb{I}} \mathcal{P}_i e^f_i \qquad e^c_i = \bar{e}^c_i \qquad e^r_i = \bar{e}^r_i$$

- Market clearing, good i

$$y_i = \overline{\mathcal{D}}_i(\{T_{it}\})z_iF(k_i, e_i) = \sum_{k\in\mathbb{I}}\tau_{ki}c_{ki} + \sum_{k\in\mathbb{I}}\tau_{ki}(x^f_{ki} + x^c_{ki} + x^r_{ki})$$
$$\mathrm{p}_i \underbrace{y_i}_{=\mathcal{D}(T_i)z_iF(\cdot)} = \sum_{k\in\mathbb{I}}\frac{s_{ki}}{1+\mathrm{t}^b_{ki}}\left(\mathrm{p}_k y_k + q^f(e^x_k - e^f_k) + \mathrm{t}^{ls}_k\right)$$

B.3.1 Social Cost of Carbon and present discounted value of damages

In Integrated Assessment, we want to measure the present-discounted value of damages – for country i – of one ton of carbon emitted in the atmosphere at time $\mathcal{S}_u$ over all its "lifetime" in the atmosphere $t \in [u, T]$

As a result, with discounting :

$$\begin{aligned}\lambda^{S,pv}_{iu} &= \int_u^T e^{-\bar{\rho}_i(t-u)}e^{-\delta_s(t-u)}\lambda^S_{it}dt = \int_u^T e^{-(\bar{\rho}_i+\delta_s)(t-u)}\Delta_i\chi\lambda^T_{it}dt \\ &= u(c_i)\Delta_i\chi\int_u^T e^{-(\bar{\rho}_i+\delta_s)(t-u)}\mathcal{D}^{u'}(T_{it})dt + \lambda^w_i z_iF(k_i, e_i)\Delta_i\chi\int_u^T e^{-(\bar{\rho}_i+\delta_s)(t-u)}\mathcal{D}^{y'}(T_{it})dt\end{aligned}$$

If the cost of carbon is constant (supposing that temperature is stable $T_{it} \to \bar{T}_i$) then the welfare cost of one ton of carbon writes:

$$\lim_{T_{it}\to\bar{T}_i} \lambda_{iu}^{S,pv} = u(c_i)\frac{\chi\Delta_i}{\bar{\rho}_i+\delta_s}(1-e^{-(\bar{\rho}_i+\delta_s)(T-u)})\mathcal{D}^{u'}(\bar{T}_i) + \lambda_i^w z_i F(k_i,e_i)\frac{\chi\Delta_i}{\bar{\rho}_i+\delta_s}\mathcal{D}^{y'}(\bar{T}_i)(1-e^{-(\bar{\rho}_i+\delta_s)(T-u)})$$

$$\lim_{T_{it}\to\bar{T}_i, T\to\infty} \lambda_{iu}^{S,pv} = \bar{\lambda}_i^S = \frac{\chi\Delta_i}{\bar{\rho}_i+\delta_s}\Big(u(c_i)\mathcal{D}^{u'}(\bar{T}_i) + \mathcal{D}^{y'}(\bar{T}_i)\lambda_i^w z_i F(k_i,e_i)\Big)$$

The local cost of carbon LCC_i of emitting one ton $[\varepsilon_{it}]$ summarizes the damages of a ton emitted – per effective capita unit! – at time t by country i. It accounts for the damages occurred between t and T. We measure it in monetary unit by dividing it by marginal value of wealth λ_{it}^w at time t – the time of the emission.

$$\begin{aligned} LCC_{it} &= -\frac{\frac{\partial\mathcal{V}_{it}}{\partial\varepsilon_{it}}}{\frac{\partial\mathcal{V}_{it}}{\partial c_{it}}} = -\frac{1}{\lambda_{it}^w} e^{(n+\bar{g})t}\int_t^T e^{-(\bar{\rho}_i+\delta_s)(s-t)}\mathcal{P}_i\lambda_{is}^S ds \\ &= -e^{(n+\bar{g})t}\frac{\lambda_{it}^{S,pv}}{\lambda_{it}^w} \end{aligned}$$

Now, we were trying to measure the model in stationary form, by taking the present discounted value of the welfare costs and the marginal value of wealth. The stationary local cost of carbon LCC_i writes:

$$LCC_i = -\frac{\overline{\lambda}_i^s}{\overline{\lambda}_i^w} = -\frac{\int_0^T e^{-\bar{\rho}_i t}e^{(n+\bar{g})t}\lambda_{it}^{S,pv}dt}{\int_0^T e^{-\bar{\rho}_i t}\lambda_{it}^w dt}$$

The numerator can be rearranged:

$$\begin{aligned} \int_0^T e^{-\bar{\rho}_i t}\lambda_{it}^{S,pv}dt &= \int_0^T e^{-\bar{\rho}_i t}\int_t^T e^{-\bar{\rho}_i(s-t)}e^{-\delta_s(s-t)}\lambda_{is}^S ds\ dt \\ &= \int_0^T e^{-\bar{\rho}_i t}\int_t^T e^{-(\bar{\rho}_i+\delta_s)(s-t)}\Delta_i\chi\lambda_{is}^T ds\ dt \\ &= \Delta_i\chi\mathcal{P}_i\int_0^T\int_t^T e^{-\bar{\rho}_i s}e^{-\delta_s(s-t)}\lambda_{is}^T ds\ dt \\ &= \Delta_i\chi\mathcal{P}_i\Big[u(c_i)\int_0^T\int_t^T e^{-\bar{\rho}_i s}e^{-\delta_s(s-t)}\mathcal{D}^{u'}(T_{it})ds\ dt + \lambda_i^w z_i F(k_i,e_i)\int_0^T\int_t^T e^{-\bar{\rho}_i s}e^{-\delta_s(s-t)}\mathcal{D}^{y'}(T_{it})ds\ dt\Big] \end{aligned}$$

We see that the "dynamic marginal cost" can be isolated from the other economic variables $y_i, c_i, e_i, \lambda_i^w$. These are the object we will use when considering optimal climate policy.

Example for policy

To give an example for policy, remember that the LCC_i summarize the future cost of climate change:

$$LCC_i = -\frac{\overline{\lambda}_i^s}{\overline{\lambda}_i^w} = -\frac{\int_0^T e^{-\bar{\rho}_i t}e^{(n+\bar{g})t}\lambda_{it}^{S,pv}dt}{\int_0^T e^{-\bar{\rho}_i t}\lambda_{it}^w dt}$$

Suppose one conduct the unilateral climate policy, choosing yearly oil consumption per (effective) capita $[e_i^f]$, internalizing the climate externality ε_u at every period u, and considering that the revenue of the carbon tax is redistributed lump-sum $\mathrm{t}_i^{ls} = \xi^f \mathrm{t}_i^s e_i^f$. The FOC for e_i^f becomes:

$$\begin{aligned}
& \mathcal{P}_i \overline{\lambda}_i^w \left(\xi^f \mathrm{t}_i^s\right) + \mathcal{P}_i \int_0^T \int_u^T e^{-\bar{\rho}_i t} \lambda_{it}^S e^{-\delta_s (t-u)} e^{(n_i+\bar{g}_i)u} \xi^f dt\ du = 0 \\
\Rightarrow \qquad & \mathrm{t}_i^s = LCC_i
\end{aligned}$$

the optimal unilateral carbon tax is the local cost of carbon for country i. This is the standard Pigouvian result and we will see how to conduct the policy at the global level and accounting for redistribution effects and endogenous participation.

B.3.2 Summary: Climate model

Here, we summarize the climate model, and express the present-discounted damages $\overline{\mathcal{D}}(\mathcal{E})$, normalized by discounting $\bar{\rho}_i = \rho - n - (1-\eta)\bar{g}_i$

$$\begin{aligned}
\mathcal{S}_t &= \mathcal{S}_0 e^{-\delta_s t} + \int_0^t e^{-\delta_s (t-u)} e^{(n+\bar{g})u} \mathcal{E} du & T_{it} &= T_{it_0} + \Delta_i \chi \mathcal{S}_t \\
\overline{\mathcal{D}}^u(\mathcal{E}) &= \frac{\bar{\rho}_i}{1-e^{-\bar{\rho}_i T}} \int_0^T e^{-\bar{\rho}_i t} \mathcal{D}^u(T_{it}) dt & \overline{\mathcal{D}}^y(\mathcal{E}) &= \frac{\bar{\rho}_i}{1-e^{-\bar{\rho}_i T}} \int_0^T e^{-\bar{\rho}_i t} \mathcal{D}^y(T_{it}) dt \\
\lambda_i^w &= \frac{\bar{\rho}_i}{1-e^{-\bar{\rho}_i T}} \overline{\lambda}_{iT}^w = \frac{\bar{\rho}_i}{1-e^{-\bar{\rho}_i T}} \int_0^T e^{-\bar{\rho}_i t} \lambda_{it}^w dt
\end{aligned}$$

$$\overline{\lambda}_{iT}^w LCC_i = \Delta_i \chi \mathcal{P}_i \Big[u(c_i) \int_0^T \int_t^T e^{-\bar{\rho}_i s} e^{-\delta_s (s-t)} \mathcal{D}^{u\prime}(T_{it}) ds\ dt + \lambda_i^w z_i F(k_i, e_i) \int_0^T \int_t^T e^{-\bar{\rho}_i s} e^{-\delta_s (s-t)} \mathcal{D}^{y\prime}(T_{it}) ds\ dt \Big]$$

$$\mathcal{E} = \sum_{i \in \mathbb{I}} \mathcal{P}_i (\xi^f e_i^f + \xi^c e_i^c)$$

B.3.3 Summary: Economic model

$$\begin{aligned}
u'(c_i) \overline{\mathcal{D}}^u(\mathcal{E}) &= \overline{\lambda}_{it}^w \mathbb{P}_i \\
\mathrm{p}_i MPe_i^f &= q^f + \xi^f \mathrm{t}_i^\varepsilon & \mathrm{p}_i MPe_i^c &= q^c + \xi^c \mathrm{t}_i^\varepsilon \\
\mathrm{p}_i MPe_i^r &= q^r & \mathrm{p}_i MPk_i - \delta &= \bar{r} = \eta \bar{g}_i + \rho \\
q^f &= \mathcal{C}_{e^x}(e_i^x, \mathcal{R}_i) \mathbb{P}_i \\
\sum_{i \in \mathbb{I}} e_i^x &= \sum_{i \in \mathbb{I}} \mathcal{P}_i e_i^f & e_i^c = \bar{e}_i^c &= z_i^c \mathbb{P}_i \qquad e_i^r = \bar{e}_i^r = z_i^r \mathbb{P}_i \\
\overline{\mathcal{D}}_i(\mathcal{E}) z_i F(k_i, e_i) &= \sum_{k \in \mathbb{I}} \tau_{ki} c_{ki} + \sum_{k \in \mathbb{I}} \tau_{ki} (x_{ki}^f + x_{ki}^c + x_{ki}^r + x_{ki}^k) \\
\mathcal{E} &= \sum_i \mathcal{P}_i (\xi^f e_i^f + \xi^c e_i^c)
\end{aligned}$$

C Policy

In this section, we provide details on the three policy benchmark, considered in section section 5 and ?? of the main text. We cover first the optimal allocation when the planner only accounts for resources constraints – the First-Best. Then, we turn to the Ramsey allocation when the planner is constrained and is not allowed cross-countries transfers nor bilateral tariffs, and can only choose carbon taxation. In the last section, we consider unilateral policy, which is a benchmark policy in Nash equilibrium when countries do not cooperate and choose their carbon taxation and trade tariffs to maximize their country's utility.

C.1 First Best

In this allocation, the planner chooses $\boldsymbol{x} = \{c_{ij}, x^{\ell}_{ij}, e^{\ell}_i\}$, i.e. the traded good for consumption c_{ij}, for energy inputs for the production of in fossil x^f_{ij}, coal x^c_{ij}, non-carbon x^r_{ij} or capital x^k_{ij}, and the energy demand, in fossil e^f_i, coal e^c_i and non-carbon e^r_i.

The welfare criterion the planner maximizes is:

$$\mathcal{W} = \sum_i \omega_i \mathcal{P}_i u(\{c_{ij}\}_j)\overline{\mathcal{D}}^u(\mathcal{E})$$

The Planner Lagrangian – in the First-Best allocation – writes:

$$\begin{aligned}\mathcal{L}(\boldsymbol{x},\boldsymbol{\lambda}) = &\sum_i \omega_i \mathcal{P}_i u(\{c_{ij}\}_j)\overline{\mathcal{D}}^u(\mathcal{E}) + \bar{\lambda}\mu^f\big[\sum_{i\in\mathbb{I}} e^x_i - \mathcal{P}_i e^f_i\big] + \sum_{\mathbb{I}} \bar{\lambda}\mu^c_i[\bar{e}^c_i - \mathcal{P}_i e^c_i] + \sum_{\mathbb{I}} \bar{\lambda}\mu^r_i[\bar{e}^r_i - \mathcal{P}_i e^r_i] \\ &+ \sum_{\mathbb{I}} \mathcal{P}_i\omega_i\phi^{\varepsilon}_i\big(\mathcal{E} - \sum_{i\in\mathbb{I}}\mathcal{P}_i(\xi^f e^f_i + \xi^c e^c_i)\big) + \sum_i \omega_i\mu_i\bar{\lambda}\big[\mathcal{P}_i z_i \overline{\mathcal{D}}^y(\mathcal{E})F(\ell_i, k_i, e_i) - \sum_{k\in\mathbb{I}}\mathcal{P}_k\tau_{ki}(c_{ki} + x^f_{ki} + x^c_{ki} + x^r_{ki})\big]\end{aligned}$$

where we rescale the multipliers for the market clearing for good $\bar{\lambda}\mu_i$, for fossil energy $\bar{\lambda}\mu^f_i$, coal energy $\bar{\lambda}\mu^c_i$ and non-carbon energy $\bar{\lambda}\mu^r_i$ by the constant $\bar{\lambda}$ to simplify the comparison with the decentralized equilibrium.

The problem being convex, we write the optimality conditions for each of the controls:

- Consumption

$$[c_{ij}] \qquad \omega_i\mathcal{P}_i u'(c_i) c_i^{1/\theta} a_{ij}^{1/\theta} c_{ij}^{-1/\theta} = \mathcal{P}_i \tau_{ij}\omega_j\mu_j\bar{\lambda}$$

$$c_{ij} = a_{ij} c_i \Big(\tau_{ij}\omega_j\mu_j \frac{\bar{\lambda}}{\omega_i u'(c_i)}\Big)^{-\theta}$$

To get the ideal "price" index, we aggregate:

$$\Rightarrow \qquad c_{ij}^{(\theta-1)/\theta} = [\omega_i u'(c_i)]^{\theta-1} c_i^{(\theta-1)/\theta} a_{ij}^{(\theta-1)/\theta} (\tau_{ij}\omega_j \mu_j \overline{\lambda})^{1-\theta}$$

$$\Rightarrow \qquad c_i^{\frac{\theta-1}{\theta}} = \sum_j a_{ij}^{1/\theta} c_{ij}^{(\theta-1)/\theta} = [\omega_i u'(c_i)]^{\theta-1} c_i^{(\theta-1)/\theta} \sum_j a_{ij} (\tau_{ij}\omega_j \mu_j \overline{\lambda})^{1-\theta}$$

$$\omega_i u'(c_i) = \overline{\lambda} \big[\sum_j a_{ij} (\tau_{ij}\omega_j \mu_j)^{1-\theta} \big]^{\frac{1}{1-\theta}}$$

- Energy inputs $\bar{e}_i^\ell$ and x_{ij}^ℓ

$$[x_{ij}] \qquad \overline{\lambda}\mu_i^\ell g'(x_i^\ell) x_i^{1/\theta} a_{ij}^{1/\theta} x_{ij}^{-1/\theta} = \tau_{ij}\omega_j\mu_j\overline{\lambda}$$

$$\Rightarrow \qquad x_{ij}^{(\theta-1)/\theta} = [\mu_i^\ell g'(x_i^\ell)]^{\theta-1} (x_i^\ell)^{(\theta-1)/\theta} a_{ij}^{(\theta-1)/\theta} (\tau_{ij}\omega_j\mu_j)^{1-\theta}$$

$$\Rightarrow \qquad (x_i^\ell)^{\frac{\theta-1}{\theta}} = \sum_j a_{ij}^{1/\theta} (x_{ij}^\ell)^{(\theta-1)/\theta} = [\mu_i^\ell]^{\theta-1} (x_i^\ell)^{(\theta-1)/\theta} \sum_j a_{ij} (\tau_{ij}\omega_j\mu_j)^{1-\theta}$$

$$\mu_i^\ell g'(x_i^\ell) = \big[\sum_j a_{ij} (\tau_{ij}\omega_j\mu_j)^{1-\theta} \big]^{\frac{1}{1-\theta}}$$

- Energy demand e_i^ℓ

$$[e_i^f] \qquad \omega_i \mathcal{P}_i \mu_i \overline{\lambda} MPe_i^f = \mathcal{P}_i \overline{\lambda} \mu^f + \mathcal{P}_i \xi^f \lambda^{\mathcal{S}}$$

$$\Rightarrow \qquad \omega_i \mu_i MPe_i = \mu^f + \xi^\ell \frac{\phi^{\mathcal{E}}}{\overline{\lambda}}$$

$$[e_i^c] \qquad \omega_i \mu_i MPe_i^f = \mu_i^c + \xi^c \frac{\phi^{\mathcal{E}}}{\overline{\lambda}}$$

$$[e_i^r] \qquad \omega_i \mu_i MPe_i^f = \mu_i^r$$

- Climate damage through carbon emissions $\mathcal{E}$

$$[\mathcal{E}] \qquad \phi^\varepsilon = \sum_{\mathbb{I}} \mathcal{P}_i \omega_i \phi_i^\varepsilon = -\sum_{\mathbb{I}} \mathcal{P}_i \omega_i \Big[u(c_i) \overline{\mathcal{D}}^{u\prime}(\mathcal{E}) + \overline{\lambda}\mu_i \mathcal{D}_i^{y\prime}(\mathcal{E}) z_i F(e_i, \ell_i) \Big]$$

Decentralization

We now look at how this planner allocation can be decentralized in the competitive equilibrium.

First, we note the that social cost of carbon is formulated with the multipliers:

$$SCC = -\frac{\frac{\partial \mathcal{W}}{\partial \mathcal{E}}}{\frac{\partial \mathcal{W}}{c_i}} = \frac{\phi^{\mathcal{E}}}{\overline{\lambda}}$$

where we recognize that the multiplier $\phi^{\mathcal{E}}$ is the welfare value of one additional ton of carbon (the welfare cost comes from the minus sign), and $\overline{\lambda}$ the average marginal utility of consumption – or marginal value of wealth.

Indeed, the First-Best allocation equalizes marginal utilities through the condition:

$$\overline{\lambda} = \frac{\omega_i u(c_i)\overline{\mathcal{D}}_i^u(\mathcal{E})}{\mathbb{P}_i} = \frac{\omega_j u(c_i)\overline{\mathcal{D}}_j^u(\mathcal{E})}{\mathbb{P}_j} \qquad \forall\ i, j \in \mathbb{I}$$

This implies large redistribution, using lump-sum transfers, such that

$$\begin{aligned} c_i &= u'^{-1}\big(\overline{\lambda}\mathbb{P}_i/\overline{\mathcal{D}}_i^u(\mathcal{E})\big), \forall i \in \mathbb{I} \\ &= w_i \ell_i + \pi_i^f + \mathrm{t}_i^{ls} \end{aligned}$$

In that cases, the transfers t_i^{ls} are designed, such that the consumptions are equalized. This implies redistribution, as $\mathrm{t}_i^{ls} < 0$ for some countries and $\mathrm{t}_i^{ls} > 0$ for some other countries.

The price p_i, output subsidy t_i^y (or inputs subsidy) and tariffs t_{ij}^b in the allocation are determined such that the FOC in the goods demand (for consumption and energy inputs) are satisfied:

$$\begin{aligned} (1+\mathrm{t}_i^y)\mathrm{p}_i &= \omega_i \mu_i \\ (1+\mathrm{t}_{ij}^b)\mathrm{p}_j &= \omega_j \mu_j \\ \mathbb{P}_i &= \big[\sum_j a_{ij}(\tau_{ij}(1+\mathrm{t}_{ij}^b)\mathrm{p}_j)^{1-\theta}\big]^{\frac{1}{1-\theta}} \\ \mathbb{P}_i &= \big[\sum_j a_{ij}(\tau_{ij}\omega_j\mu_j)^{1-\theta}\big]^{\frac{1}{1-\theta}} \end{aligned}$$

A priori, there could be multiple sets of $\{\mathrm{t}_i^y, \mathrm{t}_{ij}^b\}$ such that these conditions are met. It can not be characterized further, because the prices p_i in the Armington model are endogenous objects that depend on the demand and market clearing of each good, and can not be expressed analytically. If the conditions above are satisfied, the energy prices are simply the multipliers :

$$q^f = \mu^f \qquad q_i^c = \mu_i^c \qquad q_i^r = \mu_i^r$$

Finally, the optimal tax is simply the Social Cost of Carbon (SCC)

$$\mathrm{t}^{\varepsilon} = \frac{\phi^{\mathcal{E}}}{\overline{\lambda}} = -\sum_{\mathbb{I}} \mathcal{P}_i \omega_i \Big[\frac{u(c_i)}{u'(c_i)}\mathbb{P}_i \overline{\mathcal{D}}^{u\prime}(\mathcal{E}) + \mathcal{D}_i^{y\prime}(\mathcal{E}) z_i F(e_i, \ell_i)\mathrm{p}_i\Big] > 0$$

The optimal carbon tax is the Pigouvian level that summarizes the marginal cost of climate change for all countries i.

We will see now how that results changes when the transfers and other instruments (like tariffs or subsidies) are constrained and prevented to do redistribution.

C.2 Second best: Ramsey policy with constrained instruments

In this allocation, the Ramsey planner again chooses $\boldsymbol{x} = \{c_{ij}, x_{ij}^{\ell}, e_i^{\ell}\}$, i.e. the traded good for consumption c_{ij}, for energy inputs for the production of in fossil x_{ij}^f, coal x_{ij}^c, non-carbon x_{ij}^r or capital x_{ij}^k, and the energy demand, in fossil e_i^f, coal e_i^c and non-carbon e_i^r, as well as the carbon tax t^{ε} and the prices $\boldsymbol{p} = \{\mathrm{p}_i, q^f, q_i^c, q_i^r\}_i$. However, the allocation and prices are constrained to be a competitive equilibrium: in that case, the planner is restricted to choose controls that respect the individual optimality conditions.

We use the same multipliers: $\boldsymbol{\lambda} = \{\mu_i, \mu_i^c, \mu_i^r\}$ and $\mu^f, \phi^{\varepsilon}$ for the market clearing clearing of the final goods, the coal, renewable and fossil energy, and the carbon emissions. We add the constraints that are satisfied in competitive equilibria: λ_i for the budget constraint, ϕ^c for the consumption decision, θ_i^{ℓ} for the production quantity (supply) choice of energy firms $\ell = f, c, r$ for fossil, coal and renewable of country i, υ_i^{ℓ} for the quantity (demand) of energy ℓ chosen by the good firm, η_{ij} for the consumption choice for imports j by the household in i, ϑ_{ij}^{ℓ} for the import choice for inputs from j for the energy firm j. Not that the all the multipliers are normalized by ω_i, $\mathcal{P}_i$, and prices or quantity, to simplify optimal policies formulas.

As a result, the controls are $\boldsymbol{x} = \{c_{ij}, x_{ij}^{\ell}, e_i^{\ell}, \mathrm{p}_i, q^f, q_i^c, q_i^r, \mathrm{t}^{\varepsilon}\}_i$ and the multipliers are $\boldsymbol{\lambda} = \{\lambda_i, \mu_i, \mu_i^c, \mu_i^r, \mu^f, \phi_i^c, \theta_i^{\ell}, \upsilon_i^{\ell}, \phi^{\varepsilon}, \eta_{ij}, \vartheta_{ij}^{\ell}\}_{\ell,i,j}$.

We see that, the Ramsey planner, in choosing t^{ε}, with other instruments fixed at baseline value t_{ij}^b need to account for many redistributive effects through all the agents decisions.

$$
\begin{aligned}
\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}) = & \sum_{\mathbb{I}} \omega_i \mathcal{P}_i u(c_i) \overline{\mathcal{D}}^u(\mathcal{E}) + \sum_{\mathbb{I}} \omega_i \mathcal{P}_i \lambda_i \Big(\mathrm{p}_i \overline{\mathcal{D}}^y(\mathcal{E}) F(\ell_i, k_i, e_i^f, e_i^c, e_i^r) + \frac{1}{\mathcal{P}_i} \big[q^f g^f(x_i^{\ell}) - \sum_j x_{ij}^{\ell} \tau_{ij} \mathrm{p}_j (1+\mathrm{t}_{ij}^b) \big] \\
& + \sum_{\ell} \big\{ q^{\ell} g^{\ell}(x_i^{\ell}) - \sum_j x_{ij}^{\ell} \tau_{ij} \mathrm{p}_j (1+\mathrm{t}_{ij}^b) \big\} - \big((q^f + \xi^f \mathrm{t}_i^{\varepsilon}) e_i^f + (q^c + \xi^c \mathrm{t}_i^{\varepsilon}) e_i^c + q_i^r e_i^r + (n_i + \bar{g}_i + \delta) k_i + c_i \mathbb{P}_i + \mathrm{t}_i^{ls} \big) \Big) \\
& + \sum_{\mathbb{I}} \omega_i \mathrm{p}_i \mu_i \Big(\mathcal{P}_i \overline{\mathcal{D}}^y(\mathcal{E}) z_i F(\ell_i, k_i, e^i) - \sum_{k \in \mathbb{I}} \mathcal{P}_k \tau_{ki} c_{ki} + \sum_{k \in \mathbb{I}} \tau_{ki} (x_{ki}^f + x_{ki}^c + x_{ki}^r) \Big) \\
& + \mu^f q^f \big[\sum_{i \in \mathbb{I}} e_i^x - \mathcal{P}_i e_i^f \big] + \sum_{\mathbb{I}} \omega_i \mu_i^c q_i^c (\bar{e}_i^c - \mathcal{P}_i e_i^c) + \sum_{\mathbb{I}} \omega_i \mu_i^r q_i^r (\bar{e}_i^r - \mathcal{P}_i e_i^r) \\
& + \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i \phi_i^{\varepsilon} [\mathcal{E} - \sum_{i \in \mathbb{I}} \mathcal{P}_i (\xi^f e_i^f + \xi^c e_i^c)] + \sum_{\mathbb{I}} \omega_i \mathcal{P}_i \phi_i^c \big(\mathbb{P}_i \lambda_i^h - u'(c_i) \overline{\mathcal{D}}^u(\mathcal{E}) \big) + \sum_{\ell \in \{f,c,r\}} \sum_{\mathbb{I}} \omega_i \theta_{it}^{\ell} \big(\mathbb{P}_{it} - q_{it}^{\ell} g'(x_{it}^{\ell}) \big) \\
& + \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{P}_i \Big(\upsilon_i^f \big[q^f + \xi^f \mathrm{t}^{\varepsilon} - \mathrm{p}_i MPe_i^f \big] + \upsilon_i^c \big[q_i^c + \xi^c \mathrm{t}^{\varepsilon} - \mathrm{p}_i MPe_i^c \big] + \upsilon_i^r \big[q^r - \mathrm{p}_i MPe_i^r \big] + \upsilon_i^k \big[\underbrace{\rho + \eta \bar{g}_i}_{r_i + \bar{g} + n} + \delta - \mathrm{p}_i MPk_i \big] \Big) \\
& + \sum_{i,j \in \mathbb{I}} \omega_i \mathcal{P}_i \eta_{ij} c_{ij} [(1+\mathrm{t}_{ij}) \tau_{ij} \mathrm{p}_j - \mathbb{P}_i c_i^{\frac{1}{\theta}} a_{ij}^{\frac{1}{\theta}} c_{ij}^{-\frac{1}{\theta}}] \\
& + \sum_{\ell \in \{f,c,r,k\}} \sum_{i,j \in \mathbb{I}} \omega_i \vartheta_{ij}^{\ell} x_{ij}^{\ell} [(1+\mathrm{t}_{ij}) \tau_{ij} \mathrm{p}_j - \mathbb{P}_i (x_i^{\ell})^{\frac{1}{\theta}} a_{ij}^{\frac{1}{\theta}} (x_{ij}^{\ell})^{-\frac{1}{\theta}}]
\end{aligned}
$$

Let us go over the optimality conditions of the planner. Note, that – the problem being statics/stationary – the planner does not distort the consumption/saving decision of the household, which implies $\phi_i^c = 0$ – as that can be seen by optimizing over the household marginal value of wealth λ_i^h.

The optimality conditions writes:

- Consumption: c_{ij}

$$\omega_i \mathcal{P}_i u'(c_i) c_i^{\frac{1}{\theta}} a_{ij}^{\frac{1}{\theta}} c_{ij}^{-\frac{1}{\theta}} - \omega_i \mathcal{P}_i \lambda_i \tau_{ij} \mathrm{p}_j - \omega_j \mathcal{P}_i \mu_j \tau_{ij} \mathrm{p}_j + \omega_i \mathcal{P}_i c_{ij} \eta_{ij} \frac{1}{\theta} \frac{\tau_{ij}(1+\mathrm{t}_{ij})\mathrm{p}_j}{c_{ij}} (1 - s_{ij}) = 0$$

$$c_{ij} = a_{ij} c_i \Big((\tau_{ij}\mathrm{p}_j)[1 + \frac{\omega_j}{\omega_i}\frac{\mu_j}{\lambda_i} - \frac{\eta_{ij}}{\lambda_i}\frac{1}{\theta}(1+\mathrm{t}_{ij}^b)(1-s_{ij})] \Big)^{-\theta} \big(\underbrace{\frac{u'(c_i)}{\lambda_i}}_{=\mathbb{P}_i} \big)^{\theta}$$

$$u'(c_i) = \lambda_i \Big(\sum_j a_{ij} (\tau_{ij}\mathrm{p}_j)^{1-\theta} \underbrace{[1 + \frac{\omega_j}{\omega_i}\frac{\mu_j}{\lambda_i} - \frac{\eta_{ij}}{\lambda_i}\frac{1}{\theta}(1+\mathrm{t}_{ij})(1-s_{ij})]^{1-\theta}}_{=1+\mathrm{t}_{ij}} \Big)^{\frac{1}{1-\theta}} = \lambda_i \mathbb{P}_i$$

We see that the the consumption choice c_{ij} is distorted due to (i) the fact that demand for good j change the market clearing of country j, hence with shadow value μ_j, and (ii) the FOC is distorted with value η_{ij}.

If η_{ij} is positive, planner would like to relax the FOC $\mathrm{p}_j(1+\mathrm{t}_{ij}) - u'(c_{ij})$ implying it would like to increase the price.

To give intuition for the good demand distortion, let us give an expression for η_{ij}:

$$\eta_{ij}\frac{1}{\theta}\tau_{ij}(1+\mathrm{t}_{ij})\mathrm{p}_j(1-s_{ij}) = u'(c_i) c_i^{\frac{1}{\theta}} a_{ij}^{\frac{1}{\theta}} c_{ij}^{-\frac{1}{\theta}} - \lambda_i \tau_{ij}\mathrm{p}_j - \frac{\omega_j}{\omega_i}\mathcal{P}_i \mu_j \tau_{ij}\mathrm{p}_j$$

$$\Rightarrow \qquad \eta_{ij} = \frac{\theta}{(1-s_{ij})}\Big(\frac{u'(c_i)}{\mathbb{P}_i}\frac{1}{\lambda_i} - \frac{1+\frac{\omega_j}{\omega_i}\frac{\mu_j}{\lambda_i}}{1+\mathrm{t}_{ij}^b} \Big)$$

The distortion is positive $\eta_{ij} > 0$ for redistributive reasons, related to the budget of i and the market clearing of j. If $u'(c_i)/\mathbb{P}_i > \lambda_i$ and $\mathrm{t}_{ij}^b < \frac{\omega_j}{\omega_i}\frac{\mu_j}{\lambda_i}$, then the planner would like to distort the FOC by increasing the bilateral cost $(1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j$.

If tariffs are set optimally, we have $\eta_{ij} = 0$, – from the above equation – we obtain that $\lambda_i + \frac{\omega_j}{\omega_i}\mu_j = \frac{u'(c_i)}{\mathbb{P}_i}(1+\mathrm{t}_{ij})$ and hence

$$1+\mathrm{t}_{ij}^b = 1 + \frac{\omega_j}{\omega_i}\frac{\mu_j}{\lambda_i}$$

for a hypothetical optimal tariffs on consumption imports. By consequence, we would also obtain naturally that $\frac{u'(c_i)}{\mathbb{P}_i} = \lambda_i$. However, for arbitrary policies t_{ij}^b, the FOC of the household is distorted and $\eta_{ij} \neq 0$.

- Price p_i

$$\omega_i \mathcal{P}_i \lambda_i \overline{\mathcal{D}}^y(\mathcal{E}) F(\ell_i, k_i, e_i) - \sum_k \omega_k \mathcal{P}_k \lambda_k \tau_{ki} c_{ki} - \sum_\ell \sum_k \omega_k \mathcal{P}_k \lambda_k \tau_{ki} x^\ell_{ki}$$
$$- \sum_{i\in\mathbb{I}} \omega_i \, \mathcal{P}_i \big[\upsilon_i^f MPe_i^f + \upsilon_i^c MPe_i^c + \upsilon_i^r MPe_i^r \big] + \sum_{\ell\in\{f,c,r\}} \sum_k \omega_k \theta^\ell_{kt} \big(\frac{\tau_{ki}(1+\mathrm{t}_{ki})\mathrm{p}_i}{\mathbb{P}_k} \frac{\partial \mathbb{P}_k}{\partial \mathrm{p}_i} \big)$$
$$+ \sum_k \omega_k \mathcal{P}_k c_{ki} \eta_{ki} \big[\tau_{ki}(1+\mathrm{t}_{ki}) - \frac{\tau_{ki}(1+\mathrm{t}_{ki})\mathrm{p}_i}{\mathbb{P}_k} \frac{\partial \mathbb{P}_k}{\partial \mathrm{p}_i} \big]$$
$$+ \sum_{\ell\in\{f,c,r,k\}} \sum_k \omega_k \mathcal{P}_k x^\ell_{ki} \vartheta^\ell_{ki} \tau_{ki}(1+\mathrm{t}_{ki})[1 - s_{ki}] = 0$$

$$\omega_i \mathcal{P}_i \lambda_i \overline{\mathcal{D}}^y(\mathcal{E}) F(k_i, e_i) - \sum_k \omega_k \mathcal{P}_k \lambda_k \big[\tau_{ki} c_{ki} + \sum_\ell \tau_{ki} x^\ell_{ki} \big] - \sum_{i\in\mathbb{I}} \omega_i \, \mathcal{P}_i \big[\upsilon_i^f MPe_i^f + \upsilon_i^c MPe_i^c + \upsilon_i^r MPe_i^r + \upsilon_i^k MPe_i^k \big]$$
$$+ \sum_{\ell\in\{f,c,r\}} \sum_k \omega_k \theta^\ell_{kt} \big(\tau_{ki}(1+\mathrm{t}_{ki}) s_{ki} \big) + \sum_k \omega_k \mathcal{P}_k \tau_{ki}(1+\mathrm{t}_{ki})[1-s_{ki}] \Big(c_{ki}\eta_{ki} + \sum_{\ell\in\{f,c,r,k\}} x^\ell_{ki} \vartheta^\ell_{ki} \Big) = 0$$

This balances out all the redistributive effects (through λ_i on supply from i and on k's demand λ_k, the distortionary effects on energy choice υ^ℓ and energy production θ^ℓ_i, and the distortion on the bilateral import good choice η_{ki} for consumption and ϑ^ℓ_{ki} for inputs in energy inputs.

- Energy inputs: x^ℓ_{ij}, for $\ell = \{f, c, r, k\}$

$$\omega_i [\lambda_i + \mu^\ell_i] q^\ell_i g'(x^\ell_i)(x^\ell_i)^{\frac{1}{\theta}} a^{\frac{1}{\theta}}_{ij} (x^\ell_{ij})^{-\frac{1}{\theta}} - \omega_i \theta^\ell_i q^\ell_i g''(x^\ell_i)(x^\ell_i)^{\frac{1}{\theta}} a^{\frac{1}{\theta}}_{ij} (x^\ell_{ij})^{-\frac{1}{\theta}}$$
$$- \omega_i \lambda_i \tau_{ij} \mathrm{p}_j - \omega_j \mu_j \tau_{ij} \mathrm{p}_j - \omega_i \vartheta_{ij} \frac{1}{\theta} \tau_{ij}(1+\mathrm{t}_{ij}) \mathrm{p}_j (s_{ij}-1) = 0$$
$$\{\mu^\ell_i - \theta^\ell_i \frac{g''(x^\ell_i)}{g'(x^\ell_i)}\} q^\ell_i g'(x^\ell_i)(x^\ell_i)^{\frac{1}{\theta}} a^{\frac{1}{\theta}}_{ij} (x^\ell_{ij})^{-\frac{1}{\theta}} = \tau_{ij} \mathrm{p}_j \big[\mathrm{t}^b_{ij} \lambda_i + \frac{\omega_j}{\omega_i} \mu_j - \vartheta_{ij} \frac{1}{\theta} (1+\mathrm{t}^b_{ij})(1 - s_{ij}) \big]$$
$$\{\mu^\ell_i - \theta^\ell_i \frac{g''(x^\ell_i)}{g'(x^\ell_i)}\} = \frac{1}{1+\mathrm{t}^b_{ij}} \big[\mathrm{t}^b_{ij} \lambda_i + \frac{\omega_j}{\omega_i} \mu_j - \vartheta_{ij} \frac{1}{\theta} (1+\mathrm{t}^b_{ij})(1 - s_{ij}) \big]$$

As for the consumption good above, this input choice x^ℓ_{ij} for energy production – which resembles a production networks/supply chain problem – bring additional distortions for each energy price ℓ, i.e. ϑ^ℓ_{ij}, which we can reexpress:

$$\vartheta^\ell_{ij} = \frac{\theta}{1-s_{ij}} \Big[\{\mu^\ell_i - \theta^\ell_i \frac{g''(x^\ell_i)}{g'(x^\ell_i)}\} - \frac{1 + \frac{\omega_j}{\omega_i} \frac{\mu_j}{\lambda_i}}{1+\mathrm{t}^b_{ij}} \Big]$$

which resemble the expression for η_{ij} for consumption good. This time the distortion for energy inputs ℓ from j are distorted if the shadow value of the market clearing for that energy sources μ^ℓ_i outweighs the distortion from the supply of that energy θ^ℓ_i – weighted by supply elasticity, related to g''/g', in the case tariffs are such that $\mathrm{t}^b_{ij} < \frac{\omega_j}{\omega_i} \frac{\mu_j}{\lambda_i}$.

Again, in the hypothetical case, where tariffs are set optimally, we obtain $\vartheta^{\ell}_{ij}=0$ and thus:

$$1+\mathrm{t}^{b}_{ij}=1+\frac{\omega_j}{\omega_i}\frac{\mu_j}{\lambda_i}$$

and therefore: $\mu^{\ell}_i=\theta^{\ell}_i\frac{g''(x^{\ell}_i)}{g'(x^{\ell}_i)}$. However, in the standard case where tariffs are set suboptimally, we have $\vartheta^{\ell}_{ij}\neq 0$.

- Price q^{ℓ}_i, for coal and renewable $\ell=r,c$

$$\omega_i\mathcal{P}_i\lambda_i\Big[\frac{1}{\mathcal{P}_i}g(x^{\ell}_i)-e^{\ell}_i\Big]+\omega_i(\mathcal{P}_i\upsilon^{\ell}_i-g'(x^{\ell}_i)\theta^{\ell}_i)=0 \qquad\Rightarrow\qquad \mathcal{P}_i\upsilon^{\ell}_i=g'(x^{\ell}_i)\theta^{\ell}_i$$

since the market clearing is local at the country level, there are no redistributive effect across countries and the distortion of demand υ^{ℓ}_i equates the distortion of supply θ^{ℓ}_i.

- Price q^{f}, for oil/gas

$$\sum_{\mathbb{I}}\omega_i\mathcal{P}_i\lambda_i[\frac{1}{\mathcal{P}_i}g(x^{f}_i)-e^{f}_i]+\sum_i\omega_i(\mathcal{P}_i\upsilon^{f}_i-g'(x^{f}_i)\theta^{f}_i)=0$$

At the difference of the FOC for q^{ℓ}_i, for local energy sources, the oil-gas is traded internationally and therefore, changing its price has redistributive effects between countries depending on net-exports $g(x^{\ell}_i)-\mathcal{P}_ie^{f}_i$, through the covariance between those net-exports and the marginal value of income λ_i:

$$\sum_{\mathbb{I}}\omega_i\mathcal{P}_i\lambda_i[\frac{1}{\mathcal{P}_i}g(x^{f}_i)-e^{f}_i]=\mathbb{C}\mathrm{ov}(\omega_i\lambda_i,g(x^{f}_i)-\mathcal{P}_ie^{f}_i)$$

- Energy demand e^{ℓ}_i

$$\begin{aligned}\omega_i\mathcal{P}_i\lambda_i(\mathrm{p}_iMPe^{\ell}_i-q^{\ell})+\omega_i\mathcal{P}_i\mu_i\mathrm{p}_iMPe^{\ell}_i-\omega_iq^{\ell}_i\mu^{\ell}_i\mathcal{P}_i-\phi^{\varepsilon}\mathcal{P}_i\xi^{\ell}&\\ -\sum_{\ell'}\omega_i\mathcal{P}_i\mathrm{p}_i\upsilon^{\ell'}_i\partial_{e^{\ell}_i}MPe^{\ell'}_i&=0\end{aligned}$$

We see that the energy demand choice by the planner internalize multiple effects that will be key in the formulation of the carbon tax: First it internalizes the climate externality, as summarized by the multiplier ϕ^{ε}. Second, it also accounts for the redistributive effect through the change on the energy market clearing μ^{ℓ}_i for that particular energy source. Third, it also distorts the FOC of the firm in all its energy and input sourcing ℓ', as summarized by the multipliers $\upsilon^{\ell'}_i$, and weighted by the terms $\partial_{e^{\ell}_i}MPe^{\ell'}_i$ which relates to the cross elasticity between energies ℓ and ℓ'. Moreover, it internalizes the effects that energy use has on good production, through multiplier μ_i. All these effects are detailed in more details below.
Let us more specific about each energy sources.

Fossil:

$$\omega_i\lambda_i\xi^f \mathrm{t}^\varepsilon + \omega_i\mu_i\mathrm{p}_i MPe_i^f - q^f\mu^f - \phi^\varepsilon\xi^f - \sum_{\ell'}\omega_i\mathrm{p}_i\upsilon_i^{\ell'}\partial_{e_i^f}MPe_i^{\ell'} = 0$$

Coal:

$$\omega_i\lambda_i\xi^c \mathrm{t}^\varepsilon + \omega_i\mu_i\mathrm{p}_i MPe_i^c - \omega_i q_i^c\mu_i^c - \phi^\varepsilon\xi^c - \sum_{\ell'}\omega_i\mathrm{p}_i\upsilon_i^{\ell'}\partial_{e_i^c}MPe_i^{\ell'} = 0$$

Renewable / non-carbon

$$\omega_i\mu_i\mathrm{p}_i MPe_i^r - \omega_i q_i^r\mu^f - \sum_{\ell'}\omega_i\mathrm{p}_i\upsilon_i^{\ell'}\partial_{e_i^r}MPe_i^{\ell'} = 0$$

- Carbon tax t_i^ε:

$$\sum_{\mathbb{I}}\omega_i\mathcal{P}_i(\upsilon_i^f\xi^f + \upsilon_i^c\xi^c) = 0$$

The choice of the optimal carbon tax is a uniform tax that does not impose any additional aggregate distortion on the world economy. As a result, the sum of the country-levels distortions sum to zero: a positive distortion – multiplier $\upsilon_i^f > 0$ – need to be compensated by a negative distortion $\upsilon_{i'}^c < 0$ for another country, or across energy sources.

- Climate damage:

$$\begin{aligned}\phi^\varepsilon &= \sum_{\mathbb{I}}\mathcal{P}_i\omega_i\phi_i^\varepsilon \\ &= -\sum_{\mathbb{I}}\mathcal{P}_i\omega_i\Big[u(c_i)\overline{\mathcal{D}}^{u\prime}(\mathcal{E}) + (\lambda_i+\mu_i)\mathcal{D}_i^{y\prime}(\mathcal{E})\mathrm{p}_i z_i F(e_i,\ell_i,k_i) - \mathcal{D}_i^{y\prime}(\mathcal{E})\mathrm{p}_i\sum_{\ell'}\upsilon_i^{\ell'}MPe_i^{\ell'}\Big]\end{aligned}$$

The marginal cost of climate change can be summarized by ϕ^ε and it internalizes the direct cost $\mathcal{D}_i^{y\prime}(\mathcal{E})$ and $\mathcal{D}_i^{u\prime}(\mathcal{E})$ of climate change on income – hence the multiplier λ_i – but also the effects of climate on good production μ_i and on the distortion of the energy demand optimality υ_i^ℓ.

Reformulation of the carbon tax

We take the example of the carbon tax on fossil fuels (oil-gas) to provide details the formulation of the tax:

$$\omega_i\lambda_i\xi^f \mathrm{t}^\varepsilon + \omega_i\mu_i\mathrm{p}_i MPe_i^f - q^f\mu^f - \phi^\varepsilon\xi^f - \sum_{\ell'}\omega_i\mathrm{p}_i\upsilon_i^{\ell'}\partial_{e_i^f}MPe_i^{\ell'} = 0$$

$$\underbrace{\sum_i\omega_i\widehat{\mathcal{P}}_i\lambda_i}_{=\overline{\lambda}}\ \xi^f\mathrm{t}^\varepsilon = \xi^f\phi^\varepsilon + q^f\mu^f - (q^f+\xi^f\mathrm{t}^\varepsilon)\sum_i\omega_i\widehat{\mathcal{P}}_i\mu_i + \sum_{\ell'}\sum_i\omega_i\widehat{\mathcal{P}}_i\upsilon_i^{\ell'}\ q_i^{\ell'}\frac{\partial_{e_i^f}MPe_i^{\ell'}}{MPe_i^{\ell'}}$$

Which gives, when aggregating over all countries i and rescaling the multipliers for the good market clearing $\widehat{\mu}_i$, the ones for energy $e_i^{\ell'}$ distortion $\widehat{\upsilon}_i^{\ell'}$, a formula for the carbon tax:

$$\xi^f \mathrm{t}^\varepsilon = \xi^f \underbrace{\frac{\phi^\varepsilon}{\overline{\lambda}}}_{=\text{SCC}} + \underbrace{q^f \frac{\mu^f}{\overline{\lambda}}}_{\substack{E^f \text{ supply} \\ \text{redistribut}^\circ}} - (q^f + \xi^f \mathrm{t}^\varepsilon) \sum_i \underbrace{\widehat{\mu}_i}_{\substack{y_i\text{Trade} \\ \text{redistribut}^\circ}} - \sum_{\ell'} \sum_i \underbrace{\widehat{\upsilon}_i^{\ell'} \; q_i^{\ell'} \frac{\partial_{e_i^f} MPe_i^{\ell'}}{MPe_i^{\ell'}}}_{\substack{e_i^{\ell'} \text{ demand} \\ \text{distort}^\circ}}$$

We now use the functional forms assumptions in our model to simplify this formula further.

Simplifying the formula

Assumptions, for formula in the paper

- Rewriting the social cost of carbon (SCC):

$$SCC = \frac{\phi^\varepsilon}{\overline{\lambda}} = \frac{\mathcal{P}\sum_i \omega_i \widehat{\mathcal{P}}_i \phi_i^\varepsilon}{\sum_i \omega_i \widehat{\mathcal{P}}_i \lambda_i} = \mathcal{P}\sum_i \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\sum_i \omega_i \widehat{\mathcal{P}}_i \lambda_i} \frac{\phi_i^\varepsilon}{\lambda_i} = \mathcal{P}\sum_i \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\sum_i \omega_i \widehat{\mathcal{P}}_i \lambda_i} LCC_i$$

$$\phi_i^\varepsilon = -\Big[u(c_i)\overline{\mathcal{D}}^{u\prime}(\mathcal{E}) + (\lambda_i + \mu_i)\mathcal{D}_i^{y\prime}(\mathcal{E})\mathrm{p}_i z_i F(e_i, \ell_i)\ldots\Big]$$

$$\text{with} \qquad \lambda_i = \frac{u'(c_i)\overline{\mathcal{D}}^u(\mathcal{E})}{\mathbb{P}_i} \qquad\qquad \text{and CRRA} \qquad u'(c_i) = \frac{c_i^{1-\eta}}{1-\eta}$$

$$\text{damage} \qquad \overline{\mathcal{D}}^u(\mathcal{E}) = \Big(\overline{\mathcal{D}}^{\widetilde{u}\prime}(\mathcal{E})\Big)^{1-\eta} \qquad\qquad \overline{\mathcal{D}}^{u\prime}(\mathcal{E}) = (1-\eta)(\overline{\mathcal{D}}^{\widetilde{u}}(\mathcal{E}))^{-\eta}\overline{\mathcal{D}}^{\widetilde{u}\prime}(\mathcal{E})$$

$$\frac{u(c_i)\overline{\mathcal{D}}^{u\prime}(\mathcal{E})}{u'(c_i)\overline{\mathcal{D}}^{u}(\mathcal{E})} = \frac{c_i^{1-\eta}}{1-\eta} \frac{1}{\overline{\mathcal{D}}^{\widetilde{u}}(\mathcal{E})^{1-\eta}} (1-\eta)(\overline{\mathcal{D}}^{\widetilde{u}}(\mathcal{E}))^{-\eta}\overline{\mathcal{D}}^{\widetilde{u}\prime}(\mathcal{E}) = \frac{c_i \overline{\mathcal{D}}^{\widetilde{u}\prime}(\mathcal{E})}{\overline{\mathcal{D}}^{\widetilde{u}}(\mathcal{E})}$$

$$\frac{\phi_i^\varepsilon}{\overline{\lambda}} = -\Big[\frac{\lambda_i \mathbb{P}_i c_i}{\overline{\lambda}} \frac{\overline{\mathcal{D}}^{\widetilde{u}\prime}(\mathcal{E})}{\overline{\mathcal{D}}^{\widetilde{u}}(\mathcal{E})} + \frac{\lambda_i + \mu_i}{\overline{\lambda}} \frac{\overline{\mathcal{D}}^{y\prime}(\mathcal{E})}{\overline{\mathcal{D}}^{y}(\mathcal{E})} \mathrm{p}_i \mathcal{D}_i^y(\mathcal{E}) z_i F(e_i, \ell_i)\ldots\Big]$$

- Nordhaus DICE quadratic damage function and simple climate system: c.f. above.

$$\mathcal{D}^y(T - T^\star) = e^{-\frac{\gamma^y}{2}(T - T_i^\star)^2} \qquad \Rightarrow \qquad \mathcal{D}_i^{y\prime}(T - T^\star) = -\mathcal{D}_i^y(T - T^\star)\gamma^y(T - T_i^\star)$$

$$\dot{\mathcal{S}}_t = \mathcal{E} - \delta_s \mathcal{S}_t$$

$$T_{it} = T_{it_0} + \Delta\chi\mathcal{S}_t$$

$$\frac{\overline{\mathcal{D}}^{y\prime}(\mathcal{E})}{\overline{\mathcal{D}}^{y}(\mathcal{E})} \to_{t\to\infty, T_{it}\to T_i} -\frac{\Delta\chi}{\rho - n + (1-\eta)\bar{g} + \delta_s}\gamma^y(T_i - T_i^\star)$$

$$\frac{\phi_i^\varepsilon}{\overline{\lambda}} = \frac{\Delta\chi(T_i - T_i^\star)}{\rho - n + (1-\eta)\bar{g} + \delta_s}\Big[\frac{\lambda_i}{\overline{\lambda}}\mathbb{P}_i c_i \gamma^c + \frac{\lambda_i + \mu_i}{\overline{\lambda}}\mathrm{p}_i y_i \gamma^y - \gamma^y\big(\frac{\upsilon_i^f}{\overline{\lambda}}(q^f + \xi^f \mathrm{t}^\varepsilon) + \frac{\upsilon_i^c}{\overline{\lambda}}(q_i^c + \xi^c \mathrm{t}^\varepsilon) + \frac{\upsilon_i^r}{\overline{\lambda}} q_i^r + \frac{\upsilon_i^k}{\overline{\lambda}}(\rho + \eta\bar{g})\big)\Big]$$

The Local cost of carbon, for country i if $\omega_i = 1, \omega_j = 0$

$$LCC_i = \frac{\phi_i^\varepsilon}{\lambda_i} = \frac{\Delta\chi(T_i - T_i^\star)}{\rho - n + (1-\eta)\bar{g} + \delta_s}\Big[\mathbb{P}_i c_i \gamma^c + (1 + \frac{\mu_i}{\lambda_i})\mathrm{p}_i y_i \gamma^y - \gamma^y\big(\frac{\upsilon_i^f}{\lambda_i}(q^f + \xi^f \mathrm{t}^\varepsilon) + \frac{\upsilon_i^c}{\lambda_i}(q_i^c + \xi^c \mathrm{t}^\varepsilon) + \frac{\upsilon_i^r}{\lambda_i} q_i^r + \frac{\upsilon_i^k}{\lambda_i}(\rho + \eta\bar{g})\big)\Big]$$

Reexpressing the total global social cost of carbon:

$$SCC = \mathcal{P}\sum_i \omega_i \widehat{\mathcal{P}}_i \frac{\phi_i^\varepsilon}{\bar{\lambda}} = \mathcal{P}\sum_i \widehat{\lambda}_i LCC_i$$
$$= \mathcal{P}\sum_i \frac{\Delta\chi(T_i - T_i^\star)}{\rho - n + (1-\eta)\bar{g} + \delta_s}\Big[\widehat{\lambda}_i \mathbb{P}_i c_i \gamma^c + (\widehat{\lambda}_i + \widehat{\mu}_i)\mathrm{p}_i y_i \gamma^y$$
$$- \gamma^y\big(\widehat{\upsilon}_i^f(q^f + \xi^f \mathrm{t}^\varepsilon) + \widehat{\upsilon}_i^c(q_i^c + \xi^c \mathrm{t}^\varepsilon) + \widehat{\upsilon}_i^r q_i^r + \widehat{\upsilon}_i^k(\rho + \eta\bar{g})\big)\Big]$$

with the rescaled multipliers for the budget constraint: $\widehat{\lambda}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\sum_i \omega_i \widehat{\mathcal{P}}_i \lambda_i}$, the multiplier the FOC demand $\widehat{\upsilon}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \upsilon_i}{\bar{\lambda}}$, for the multiplier for market clearing for good: $\widehat{\mu}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \mu_i}{\bar{\lambda}}$ and population share $\widehat{\mathcal{P}}_i = \frac{\mathcal{P}_i}{\mathcal{P}}$

- Isoelastic energy supply curve: $x^f = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e_i^x}{\mathcal{R}_i}\right)^{1+\nu_i}\mathcal{R}_i$

$$g_i(x) = \mathcal{R}_i^{\frac{\nu_i}{1+\nu_i}} x^{\frac{1}{1+\nu_i}} \qquad\qquad \frac{g''(x^f)}{g'(x^f)} = -\frac{\nu_i}{1+\nu_i}\frac{1}{x_i}$$

As a result, FOC of energy inputs $[x_i^f]$ becomes:

$$\mu^f = \omega_i \theta_i^f \frac{g''(x^f)}{g'(x^f)} = \omega_i \theta_i^f \frac{-\nu_i}{1+\nu_i}\frac{1}{x_i^f} \qquad\qquad \omega_i \theta_i^f = -\frac{1+\nu_i}{\nu_i} x_i^f \mu^f$$

Moreover, the FOC of energy price $[q^f]$ becomes:

$$\sum_i \omega_i \theta_i^f g'(x^f) = -\mu^f \sum_i \frac{1+\nu_i}{\nu_i} x_i^f g'(x_i^f)$$
$$= -\mu^f \sum_i \frac{1+\nu_i}{\nu_i}\frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e_i^x}{\mathcal{R}_i}\right)^{1+\nu_i}\mathcal{R}_i\left(\frac{e_i^x}{\mathcal{R}_i}\right)^{-\nu_i}\bar{\nu}_i^{-1}$$
$$\sum_i \omega_i \theta_i^f g'(x^f) = -\mu^f \sum_i \frac{e_i^x}{\nu_i} = -\mu^f / \Big(\frac{1}{E^f}\underbrace{\big(\sum_i \frac{e_i^x}{E^f}\frac{1}{\nu_i}\big)^{-1}}_{=\bar{\nu}}\Big)$$

As a result, we have, with the aggregate elasticity $\bar{\nu}$

$$\mu^f = \frac{\bar{\nu}}{E^f}\sum_i \omega_i \mathcal{P}_i \lambda_i \big(e_i^f - \frac{e_i^x}{\mathcal{P}_i}\big) \qquad\qquad \bar{\nu} = \big(\sum_i \tfrac{e_i^x}{E^f}\nu_i^{-1}\big)^{-1}$$
$$\frac{\mu^f}{\bar{\lambda}} = \mathcal{P}\frac{\bar{\nu}}{E^f}\sum_i \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\bar{\lambda}}\big(e_i^f - \frac{e_i^x}{\mathcal{P}_i}\big)$$

- Nested CES framework:

Energy $\quad e_i = \Big(\sum_\ell (\omega^\ell)^{\frac{1}{\sigma_e}}(e_i^\ell)^{\frac{\sigma_e - 1}{\sigma_e}}\Big)^{\frac{\sigma_e}{\sigma_e - 1}} \qquad$ Output $\quad y_i = \Big((1-\varepsilon)^{\frac{1}{\sigma}}(e_i)^{\frac{\sigma-1}{\sigma}} + \varepsilon^{\frac{1}{\sigma}}(k_i^\alpha \ell_i^{1-\alpha})^{\frac{\sigma-1}{\sigma}}\Big)^{\frac{\sigma}{\sigma-1}}$

FOC for fossil energy demand:

$$\bar{\upsilon}_i^f = \left[\upsilon_i^f \partial_{e^f} MPe_i^f + \upsilon_i^c \partial_{e^f} MPe_i^c + \upsilon_i^r \partial_{e^f} MPe_i^r + \upsilon_i^k \partial_{e^f} MPk_i\right]$$

$$= \frac{1}{e_i^f}\Big[-\upsilon_i^f(q^f+\xi^f \mathrm{t}^\varepsilon)\big[\frac{1-s^f}{\sigma^e}+s^f\frac{1-s^e}{\sigma^y}\big]+\upsilon_i^c(q_i^c+\xi^f \mathrm{t}^\varepsilon)s_i^f\big[\frac{1}{\sigma^e}-\frac{1-s^e}{\sigma^y}\big]+\upsilon_i^r q_i^r s_i^f\big[\frac{1}{\sigma^e}-\frac{1-s^e}{\sigma^y}\big]+\upsilon_i^k(r^\star+\bar{\delta})\frac{s_i^{e^r/y}}{\sigma^y}\Big]$$

and when normalizing by $\overline{\lambda}$

$$\widehat{\upsilon}_i^f = \frac{1}{e_i^f}\Big[-\widehat{\upsilon}_i^f(q^f+\xi^f \mathrm{t}^\varepsilon)\big[\frac{1-s^f}{\sigma^e}+s^f\frac{1-s^e}{\sigma^y}\big]+\widehat{\upsilon}_i^c(q_i^c+\xi^f \mathrm{t}^\varepsilon)s_i^f\big[\frac{1}{\sigma^e}-\frac{1-s^e}{\sigma^y}\big]+\widehat{\upsilon}_i^r q_i^r s_i^f\big[\frac{1}{\sigma^e}-\frac{1-s^e}{\sigma^y}\big]+\widehat{\upsilon}_i^k(r^\star+\bar{\delta})\frac{s_i^{e^r/y}}{\sigma^y}\Big]$$

If the production function only contains fossil (oil-gas) in energy $\omega^f = 1$, then we obtain:

$$\bar{\upsilon}_i^f = -\frac{q^f+\xi^f \mathrm{t}^\varepsilon}{e_i^f}\upsilon_i^f\big[\frac{1-s^e}{\sigma^y}\big] \qquad s_i^e = \frac{q^e e_i}{\mathrm{p}_i y_i}$$

Proposition: Using these assumptions, we can reexpress the carbon tax:

$$\xi^f \mathrm{t}^\varepsilon = \xi^f \mathcal{P}\sum_i \widehat{\lambda}_i LCC_i + q^f \mathcal{P}\frac{\bar{\nu}}{E^f}\sum_i \widehat{\lambda}_i\big(e_i^f - \frac{e_i^x}{\mathcal{P}_i}\big) - (q^f+\xi^f \mathrm{t}^\varepsilon)\sum_i \widehat{\mu}_i - \sum_i \widehat{\bar{\upsilon}}_i^f$$

$$\xi^f \mathrm{t}^\varepsilon = \xi^f \underbrace{\mathcal{P}\mathbb{E}_i\big[LCC_i\big] + \mathcal{P}\mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, LCC_i\big)}_{=\text{Social Cost of Carbon}} + \underbrace{q^f \mathcal{P}\frac{\bar{\nu}}{E^f}\mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, e_i^f - \frac{e_i^x}{\mathcal{P}_i}\big)}_{E^f \text{ supply redistribut}^\circ} - (q^f+\xi^f \mathrm{t}^\varepsilon)\underbrace{\mathbb{E}_i\big[\widehat{\mu}_i\big]}_{y_i \text{Trade redistribut}^\circ} - \underbrace{\mathbb{E}_i\big[\widehat{\bar{\upsilon}}_i^f\big]}_{e_i^{\ell'} \text{ demand distort}^\circ}$$

With the demand distortion of fossil fuels $\widehat{\bar{\upsilon}}_i^f$

$$\widehat{\bar{\upsilon}}_i^f = \frac{1}{e_i^f}\Big[-\widehat{\upsilon}_i^f(q^f+\xi^f \mathrm{t}^\varepsilon)\big[\frac{1-s^f}{\sigma^e}+s^f\frac{1-s^e}{\sigma^y}\big]+\widehat{\upsilon}_i^c(q_i^c+\xi^f \mathrm{t}^\varepsilon)s_i^f\big[\frac{1}{\sigma^e}-\frac{1-s^e}{\sigma^y}\big]$$
$$+\widehat{\upsilon}_i^r q_i^r s_i^f\big[\frac{1}{\sigma^e}-\frac{1-s^e}{\sigma^y}\big]+\widehat{\upsilon}_i^k(r^\star+\bar{\delta})\frac{s_i^{e^r/y}}{\sigma^y}\Big]$$
$$\widehat{\bar{\upsilon}}_i^f = -\frac{q^f+\xi^f \mathrm{t}^\varepsilon}{e_i^f}\widehat{\upsilon}_i^f\big[\frac{1-s_i^e}{\sigma^y}\big] \qquad \text{if } s^f = 1, s^r = s^c = 0$$

and the social cost of carbon SCC as:

$$SCC = \mathcal{P}\sum_i \widehat{\lambda}_i LCC_i = \mathcal{P}\mathbb{E}_i\big[LCC_i\big] + \mathcal{P}\mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, LCC_i\big)$$
$$= \mathcal{P}\sum_i \frac{\Delta\chi(T_i - T_i^\star)}{\rho - n + (1-\eta)\bar{g} + \delta_s}\Big[\widehat{\lambda}_i \mathbb{P}_i c_i \gamma^c + (\widehat{\lambda}_i + \widehat{\mu}_i)\mathrm{p}_i y_i \gamma^y$$
$$- \gamma^y\big(\widehat{\upsilon}_i^f(q^f+\xi^f \mathrm{t}^\varepsilon) + \widehat{\upsilon}_i^c(q_i^c+\xi^c \mathrm{t}^\varepsilon) + \widehat{\upsilon}_i^r q_i^r + \widehat{\upsilon}_i^k(\rho + \eta\bar{g})\big)\Big]$$

with the rescaled multipliers for the budget constraint: $\widehat{\lambda}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\overline{\lambda}} = \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\sum_i \omega_i \widehat{\mathcal{P}}_i \lambda_i}$, the multiplier the FOC demand $\widehat{\upsilon}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \upsilon_i}{\overline{\lambda}}$, for the multiplier for market clearing for good: $\widehat{\mu}_i = \frac{\omega_i \widehat{\mathcal{P}}_i \mu_i}{\overline{\lambda}}$

Simplifying the multiplier for the FOC for energy demand

$$\xi^f \mathrm{t}^\varepsilon = \xi^f \mathcal{P}\Big(\mathbb{E}_j[LCC_j] + \mathbb{C}\mathrm{ov}_j(\widehat{\lambda}_j, LCC_j)\Big) + \mathcal{P}\frac{q^f \bar{\nu}}{E}\mathbb{C}\mathrm{ov}_j(\widehat{\lambda}_j, e^f_j - \frac{e^x_j}{\mathcal{P}_j})$$
$$- (q^f + \xi^f \mathrm{t}^\varepsilon)\mathbb{E}_j[\widehat{\mu}_j] - (q^f + \xi^f \mathrm{t}^\varepsilon)\mathbb{C}\mathrm{ov}(\widehat{\upsilon}^f_i, \frac{1-s^e}{\sigma e^f_i})$$

where the last equality comes from the fact that $\mathbb{E}_i[\widehat{\upsilon}^f_i] = \sum_i \widehat{\upsilon}^f_i = 0$ by the assumption that there is no aggregate distortion – only individual distortion – when the uniform carbon tax is set at the world level and $\mathbb{E}_i[e^f_i - \frac{e^x_i}{\mathcal{P}_i}] = \sum_i e^f_i - \frac{e^x_i}{\mathcal{P}_i} = 0$ by market clearing on the fossil energy market.

To investigate the demand distortion $\widehat{\upsilon}^f_i$ further, we can see that, with the planner's FOC for energy demand, the individual distortion becomes:

$$\omega_i \mathcal{P}_i \upsilon^f_i = \frac{\mathcal{P}_i}{q^f + \xi^f \mathrm{t}^\varepsilon} \frac{\sigma^y e^f_i}{1-s^e_i}\Big[\xi^f \phi^S + q^f \mu^f - \omega_i \mu_i (q^f + \xi^f \mathrm{t}^\varepsilon) - \omega_i \lambda_i \xi^f \mathrm{t}^\varepsilon\Big]$$

The distortion is higher if the welfare motives in terms of cost of climate change ϕ^S, supply redistribution μ^f and trade effect $-\mu_i$ etc. outweighs the welfare cost of the carbon tax $\lambda_i \mathrm{t}^\varepsilon$. As mentioned earlier, the averate/aggregate distortion is null, so – in the case where fossil (oil-gas) is the only energy – we obtain:

$$\sum_i \omega_i \mathcal{P}_i \upsilon^f_i = 0$$
$$\frac{1}{q^f + \xi^f \mathrm{t}^\varepsilon}\big(\xi^f \phi^S + q^f \mu^f\big)\sum_i \mathcal{P}_i \frac{\sigma^y e^f_i}{1-s^e_i} - \frac{1}{q^f + \xi^f \mathrm{t}^\varepsilon}\xi^f \mathrm{t}^\varepsilon \sum_i \omega_i \mathcal{P}_i \lambda_i \frac{\sigma^y e^f_i}{1-s^e_i} - \frac{q^f + \xi^f \mathrm{t}^\varepsilon}{q^f + \xi^f \mathrm{t}^\varepsilon}\sum_i \omega_i \mathcal{P}_i \mu_i \frac{\sigma^y e^f_i}{1-s^e_i} = 0$$
$$\xi^f \mathrm{t}^\varepsilon \sum_i \omega_i \mathcal{P}_i \lambda_i \frac{\sigma^y e^f_i}{1-s^e_i} = \big(\xi^f \phi^S + q^f \mu^f\big)\sum_i \mathcal{P}_i \frac{\sigma^y e^f_i}{1-s^e_i} - (q^f + \xi^f \mathrm{t}^\varepsilon)\sum_i \omega_i \mathcal{P}_i \mu_i \frac{\sigma^y e^f_i}{1-s^e_i}$$

Divide both side by $\overline{\lambda} = \sum_i \omega_i \widehat{\mathcal{P}}_i \lambda_i$ and by $E^{s,\sigma} = \sum_i \widehat{\mathcal{P}}_i \frac{\sigma^y e^f_i}{1-s^e_i}$ and $\widehat{e}^{s,\sigma}_i = \frac{\frac{\sigma^y e^f_i}{1-s^e_i}}{\sum_i \widehat{\mathcal{P}}_i \frac{\sigma^y e^f_i}{1-s^e_i}}$, it becomes;

$$\xi^f \mathrm{t}^\varepsilon \sum_i \frac{\omega_i \widehat{\mathcal{P}}_i \lambda_i}{\overline{\lambda}} \frac{1}{E^{s,\sigma}} \frac{\sigma^y e^f_i}{1-s^e_i} = \big(\xi^f \frac{\phi^S}{\overline{\lambda}} + q^f \frac{\mu^f}{\overline{\lambda}}\big)\frac{1}{E^{s,\sigma}}\sum_i \widehat{\mathcal{P}}_i \frac{\sigma^y e^f_i}{1-s^e_i} - (q^f + \xi^f \mathrm{t}^\varepsilon)\sum_i \frac{\omega_i \widehat{\mathcal{P}}_i \mu_i}{\overline{\lambda}} \frac{1}{E^{s,\sigma}} \frac{\sigma^y e^f_i}{1-s^e_i}$$

$$\xi^f \mathrm{t}^\varepsilon = \frac{1}{1 + \mathbb{C}\mathrm{ov}_j(\widehat{\lambda}_j, \widehat{e}^{s,\sigma}_i)}\Big[\xi^f \mathcal{P} SCC + \mathcal{P}\frac{q^f \bar{\nu}}{E}\mathbb{C}\mathrm{ov}_j(\widehat{\lambda}_j, e^f_j - \frac{e^x_j}{\mathcal{P}_j}) - (q^f + \xi^f \mathrm{t}^\varepsilon)\mathbb{C}\mathrm{ov}_j(\widehat{\mu}_j, \widehat{e}^{s,\sigma}_i)\Big]$$

It implies that the carbon tax is dampened if the Planner put larger social weights on countries that use a lot of energy e^f_i, with a higher elasticity σ^y, and as a larger share of production s^e_i, resulting in $\mathbb{C}\mathrm{ov}_j(\widehat{\lambda}_j, \widehat{e}^{s,\sigma}_i) > 0$. If this covariance is negative, then the carbon tax is amplified and the planner would optimally choose a higher carbon tax.

D Welfare decomposition

D.1 Change in welfare – experiment

We compute the change in welfare, linearizing the model around an equilibrium where $\mathrm{t}^{\varepsilon} = \bar{\mathrm{t}}^{\varepsilon} = 0$ and $\mathrm{t}^{b}_{ij} = \bar{\mathrm{t}}^{b}_{ij} = 0$. This corresponds to the competitive equilibrium, since policies are identical to the "status-quo". We consider a climate agreement $\mathcal{J}$ of J countries, which are indifferent of being in the club or not, since the policy $(\mathrm{t}^{\varepsilon}_i, \mathrm{t}^{b}_{ij}) = (0,0)$ does not change the equilibrium. We consider a deviation where we increase those policy instruments by a small amount. To save on notation, we denote $d\ln z_i = \frac{dz_i}{z_i}$ – with a slight abuse of notation.

As a result, the policy change we consider is $d\ln \mathrm{t}^{\varepsilon}_i = \frac{d\mathrm{t}^{\varepsilon}_i}{1+\mathrm{t}^{\varepsilon}_i} = d\mathrm{t}^{f}_i$ where we consider a multiplicative carbon tax on fossil fuel $q^{f}_i(1+\mathrm{t}^{\varepsilon}_i)$. Similarly we consider a tariff change : $d\ln \mathrm{t}^{b}_{ij} = \frac{d\mathrm{t}^{b}_{ij}}{1+\mathrm{t}^{b}_{ij}} = d\mathrm{t}^{b}_{ij}$. In matrix notation, these changes in carbon tax are noted

$$\mathbf{J}d\mathbf{t}^{\varepsilon} = \left\{\mathbb{1}_{\{i\in\mathcal{J}\}}\, d\ln \mathrm{t}^{\varepsilon}_i\right\}_i \qquad\qquad \overline{\mathbf{J}}\odot d\mathbf{t}^{b} = \left\{\mathbb{1}_{\{i\in\mathcal{J}, j\notin\mathcal{J}\}} d\mathrm{t}^{b}_{ij}\right\}_{ij}$$

with $\mathbf{J} = \mathrm{J}_i = \mathbb{1}_{\{i\in\mathcal{J}\}}$, and $\overline{\mathbf{J}} \equiv \mathrm{J}_{ki} = \mathbb{1}_{\{i\in\mathcal{J}, j\notin\mathcal{J}\}}$.

This section is inspired Kleinman et al. (2020), where we follow the same steps, using a richer model, with trade a la Armington, energy in production and carbon and trade policy instruments.

We compute the welfare of individual country i, defined as the indirect utility $\mathcal{U}_i = u(\{c_{ij}\}_j)$, changes as:

$$d\mathcal{U}_i = du\Big(\frac{x_i}{\mathbb{P}_i}\Big) = u'(c_i)\Big(\frac{dx_i}{\mathbb{P}_i} - \frac{x_i}{\mathbb{P}_i}\frac{d\mathbb{P}_i}{\mathbb{P}_i}\Big) = u'(c_i)c_i\Big(\frac{dx_i}{x_i} - \frac{d\mathbb{P}_i}{\mathbb{P}_i}\Big)$$

with $x_i = c_i\mathbb{P}_i$ the consumption expenditure.

D.2 Model summary

First let us summarize the model, as presented above

$$\begin{aligned}
c_i\mathbb{P}_i = x_i &= w_i\ell_i + \pi^{x}_i + \mathrm{t}^{ls}_i = \mathrm{p}_i z_i \mathcal{D}_i(T_i)F(e_i,\ell_i) - q^{e}_t e_i + \frac{1}{\mathcal{P}_i}\Big(q^{e}e^{x}_i - \mathbb{P}_i\mathcal{C}^{f}(e^{x}_i,\mathcal{R}_i)\Big) + \mathrm{t}^{ls}_i\\
\mathcal{P}_i\mathrm{p}_i y_i &= \sum_{k\in\mathbb{I}} \mathcal{P}_k s_{ki}\frac{v_k}{1+\mathrm{t}_{ki}}\\
\widetilde{v}_i &= \mathrm{p}_i y_i + q^{f}(e^{x}_i/\mathcal{P}_i - e^{f}_i)\\
v_i &= m_i\widetilde{v}_i = \mathrm{p}_i y_i + q^{f}(e^{x}_i/\mathcal{P}_i - e^{f}_i) + \mathrm{t}^{ls}_i\\
\pi^{f}_i &= \frac{1}{\mathcal{P}_i}\frac{\nu\bar{\nu}^{-1/\nu}}{1+\nu}\mathcal{R}_i(q^{f})^{1+\frac{1}{\nu}}\mathbb{P}_i^{-1/\nu}\\
\sum_k \mathcal{P}_i e^{f}_i &= \sum_k e^{x}_i = (q^{f})^{1/\nu}\sum_k \mathcal{R}_i\bar{\nu}_i^{-1/\nu}\mathbb{P}_i^{-1/\nu}\\
F_i(\varepsilon(e^f,e^c,e^r),\ell) &= \Big[(1-\epsilon)^{\frac{1}{\sigma_y}}(\bar{k}^{\alpha}\ell^{1-\alpha})^{\frac{\sigma_y-1}{\sigma_y}} + \epsilon^{\frac{1}{\sigma_y}}\Big(z^{e}_i\ \varepsilon_i(e^f,e^c,e^r)\Big)^{\frac{\sigma_y-1}{\sigma_y}}\Big]^{\frac{\sigma_y}{\sigma_y-1}}\\
\varepsilon(e^f,e^c,e^r) &= \Big[(\omega^{f}_i)^{\frac{1}{\sigma_e}}(e^f)^{\frac{\sigma_e-1}{\sigma_e}} + (\omega^{c}_i)^{\frac{1}{\sigma_e}}(e^c)^{\frac{\sigma_e-1}{\sigma_e}} + (\omega^{r}_i)^{\frac{1}{\sigma_e}}(e^r)^{\frac{\sigma_e-1}{\sigma_e}}\Big]^{\frac{\sigma_e}{\sigma_e-1}}
\end{aligned}$$

D.3 Production

Starting from the budget constraint:

$$c_i\mathbb{P}_i = x_i = \mathrm{p}_i z_i \mathcal{D}_i(T_i)F(e_i,\ell_i) - q^e e_i + \frac{1}{\mathcal{P}_i}\Big(q^f e_i^x - \mathbb{P}_i\mathcal{C}^f(e_i^x,\mathcal{R}_i)\Big) + \mathrm{t}_i^{ls}$$
$$= \mathrm{p}_i z_i \mathcal{D}_i(T_i)F(e_i,\ell_i) - \big(q^f(1+\mathrm{t}_i^\varepsilon)e_i^f + q_i^c(1+\mathrm{t}_i^\varepsilon)e_i^c + q_i^r e_i^r\big) + \frac{1}{\mathcal{P}_i}\Big(q^e e_i^x - \mathbb{P}_i\mathcal{C}^f(e_i^x,\mathcal{R}_i)\Big) + \widetilde{\mathrm{t}}_i^{ls} + q^f\mathrm{t}_i^\varepsilon e_i^f + q_i^c\mathrm{t}_i^\varepsilon e_i^c$$

Since, the revenues of the carbon-tax are redistributed lump-sum to the Household, we do not see any direct redistributive effect of carbon taxation, as the terms $q^f\mathrm{t}_i^\varepsilon e_i^f$ cancel out. Moreover,

We define the shares what are relevant for the decomposition:

- Energy share in production: $s_i^e = \frac{e_i q_i^e}{y_i \mathrm{p}_i}$
- Fossil share in energy mix $s_i^f = \frac{e_i^f q^f}{e_i q_i^e}$ and similarly $s_i^c = \frac{e_i^c q_i^c}{e_i q_i^e}$ and $s_i^r = \frac{e_i^r q_i^r}{e_i q_i^e}$
- Production share/rent share in GDP: $\eta_i^y = \frac{y_i\mathrm{p}_i}{y_i\mathrm{p}_i+\pi_i^f} = 1-\eta_i^\pi$
- Consumption share in GDP: $\eta_i^c = \frac{x_i}{y_i\mathrm{p}_i+\pi_i^f}$
- Consumption as a ratio of output: $s_i^c = \frac{c_i\mathbb{P}_i}{y_i\mathrm{p}_i} = \frac{x_i}{y_i\mathrm{p}_i+\pi_i^f}\frac{y_i\mathrm{p}_i+\pi_i^f}{y_i\mathrm{p}_i} = \frac{\eta_i^c}{1-\eta_i^\pi} = \frac{\eta_i^c}{\eta_i^y}$,
- Energy share as a ratio of consumption: $\frac{e_i q_i^e}{x_i} = \frac{e_i q_i^e}{y_i\mathrm{p}_i}\frac{y_i\mathrm{p}_i}{y_i\mathrm{p}_i+\pi_i^f}\frac{y_i\mathrm{p}_i+\pi_i^f}{x_i} = s_i^e\frac{\eta_i^y}{\eta_i^c}$
- Profit share as a ratio of consumption: $\frac{\pi_i^f}{x_i} = \frac{\pi_i^f}{y_i\mathrm{p}_i+\pi_i^f}\frac{y_i\mathrm{p}_i+\pi_i^f}{x_i} = \frac{\eta_i^\pi}{\eta_i^c}$

Taking the first-order expansion of the budget constraint, we obtain:

$$\frac{dc_i}{c_i} = \frac{dx_i}{x_i} - \frac{d\mathbb{P}_i}{\mathbb{P}_i} = \frac{\mathrm{p}_i y_i}{x_i}\Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{dy_i}{y_i}\Big) - \frac{e_i q_i^e}{x_i}\Big(\frac{e_i^f q^f}{e_i q_i^e}\Big(\frac{de^f}{e^f}+\frac{dq^f}{q^f}\Big) + \frac{e_i^c q^c}{e_i q_i^e}\Big(\frac{de^c}{e^c}+\frac{dq^c}{q^c}\Big) + \frac{e_i^r q^r}{e_i q_i^e}\Big(\frac{de^r}{e^r}+\frac{dq^r}{q^r}\Big)\Big)$$
$$+ \frac{\pi^f}{x_i}\frac{d\pi_i^f}{\pi^f} + \frac{\widetilde{\mathrm{t}}_i^{ls}}{x_i}\Big(\frac{d\widetilde{\mathrm{t}}_i^{ls}}{\widetilde{\mathrm{t}}_i^{ls}}\Big) \quad - \frac{d\mathbb{P}_i}{\mathbb{P}_i}$$

$$\frac{dc_i}{c_i} = \frac{\eta_i^y}{\eta_i^c}\Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{dy_i}{y_i}\Big) - s_i^e\frac{\eta_i^y}{\eta_i^c}\Big(s_i^f\Big(\frac{de^f}{e^f}+\frac{dq^f}{q^f}\Big)+s_i^c\Big(\frac{de^c}{e^c}+\frac{dq^c}{q^c}\Big)+s_i^r\Big(\frac{de^r}{e^r}+\frac{dq^r}{q^r}\Big)\Big)$$
$$+ \frac{\eta_i^\pi}{\eta_i^c}\frac{d\pi_i^f}{\pi^f} + \frac{\widetilde{\mathrm{t}}_i^{ls}}{x_i}\Big(\frac{d\widetilde{\mathrm{t}}_i^{ls}}{\widetilde{\mathrm{t}}_i^{ls}}\Big) \quad - \frac{d\mathbb{P}_i}{\mathbb{P}_i}\Big)$$

First, using output changes, approximating the production function – c.f. the logic in Farhi-Baqaee (2021):

$$\frac{dy_i}{y_i} = \frac{d\mathcal{D}_i}{\mathcal{D}_i} + \frac{MPe_i e_i}{y_i}\frac{de_i}{e_i} = \frac{d\mathcal{D}_i}{\mathcal{D}_i} + s_i^e\Big[s_i^f\frac{de_i^f}{e_i^f} + s_i^r\frac{de_i^r}{e_i^r}\Big]$$

we see that the Hulten's theorem imply an first-order impact of a change in energy price that scale with the share of energy in production s_i^e, which is typically around $5-10\%$ and the share of fossils in the energy mix s_i^f, s_i^c, which sum to above 85%.

Second, using fossil energy firm problem and the profit change,

$$\frac{d\pi_i^f}{\pi^f} = \Big((1+\frac{1}{\nu_i})\frac{dq^f}{q^f} - \frac{1}{\nu_i}\frac{d\mathbb{P}_i}{\mathbb{P}_i}\Big)$$

The energy rent is affected by changes in the aggregate fossil energy price dq^f. Since the cost also depends on imported inputs, the prices of goods $\mathbb{P}_i$ also matter for profit and welfare.

Third, using the production of coal and renewable, which are simply $q_i^c = \upsilon_i^c \mathbb{P}_i$ and $q_i^r = \upsilon_i^r \mathbb{P}_i$, we get

$$\frac{dq^r}{q^r} = \frac{d\mathbb{P}_i}{\mathbb{P}_i} \qquad \text{and} \qquad \frac{dq^c}{q^c} = \frac{d\mathbb{P}_i}{\mathbb{P}_i}$$

the price of both coal and renewable energy are directly exposed to changes in the price of imports.

Accounting for these different effects simplify dramatically the change in consumption:

$$\begin{aligned}
\frac{dc_i}{c_i} &= \frac{\eta_i^y}{\eta_i^c}\Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{dy_i}{y_i}\Big) - s_i^e \frac{\eta_i^y}{\eta_i^c}\Big(s_i^f\big(\frac{de^f}{e^f} + \frac{dq^f}{q^f}\big) + s_i^c\big(\frac{de^c}{e^c} + \frac{dq^c}{q^c}\big) + s_i^r\big(\frac{de^r}{e^r} + \frac{dq^r}{q^r}\big)\Big) \\
&\quad + \frac{\eta_i^\pi}{\eta_i^c}\Big(1 + \frac{1}{\nu}\Big)\Big(\frac{dq^f}{q^f} - \frac{1}{1+\nu}\frac{d\mathbb{P}_i}{\mathbb{P}_i}\Big) + \frac{d\widetilde{\mathrm{t}}_i^{ls}}{x_i}\Big) \quad - \frac{d\mathbb{P}_i}{\mathbb{P}_i} \\
&= \frac{\eta_i^y}{\eta_i^c}\Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{d\mathcal{D}_i}{\mathcal{D}_i}\Big) - \frac{\eta_i^y}{\eta_i^c} s_i^e \Big[s_i^f \frac{dq^f}{q^f} + s_i^c \frac{dq^c}{q^c} + s_i^r \frac{dq^r}{q^r}\Big] + \frac{\eta_i^\pi}{\eta_i^c}\Big(1 + \frac{1}{\nu}\Big)\Big(\frac{dq^f}{q^f} - \frac{1}{1+\nu}\frac{d\mathbb{P}_i}{\mathbb{P}_i}\Big) + \frac{d\widetilde{\mathrm{t}}_i^{ls}}{x_i} \quad - \frac{d\mathbb{P}_i}{\mathbb{P}_i} \\
\frac{dc_i}{c_i} &= \frac{\eta_i^y}{\eta_i^c}\Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{d\mathcal{D}_i}{\mathcal{D}_i}\Big) - \frac{\eta_i^y}{\eta_i^c} s_i^e s_i^f \frac{dq^f}{q^f} + \frac{\eta_i^\pi}{\eta_i^c}\Big(1 + \frac{1}{\nu}\Big)\frac{dq^f}{q^f} - \Big[\frac{\eta_i^y}{\eta_i^c} s_i^e (s_i^c + s_i^r) + \frac{\eta_i^\pi}{\eta_i^c}\frac{1}{\nu} + 1\Big]\frac{d\mathbb{P}_i}{\mathbb{P}_i} + \frac{d\widetilde{\mathrm{t}}_i^{ls}}{x_i}
\end{aligned}$$

D.4 Climate externality

To see the positive influence of carbon taxation on climate, we unpack the damage $d\mathcal{D}_i$. In this section, we simplify the climate system by considering the following static model:

$$\begin{aligned}
T_i &= \Delta_i \mathcal{T} = \Delta_i \chi \mathcal{S} \\
\mathcal{S} &= \mathcal{S}_0 + \mathcal{E} = \mathcal{S}_0 + E^f + \hat{\xi}^c E^c
\end{aligned}$$

where $\mathcal{E} = E^f + \xi^c E^c$ is a representation of long-term emission. If the static model represent a long-period of time – say 50 or 100 years – we consider $\mathcal{E}$ scale up the annual emissions $\sum_i e_i^f + e_i^c$ due to oil-gas and coal. Moreover, the premium ξ^c represents the premium in carbon emission due to coal $\hat{\xi}^c = \xi^c - \xi^f$, and we therefore normalize the unit of energy to the carbon content of oil and gas.

Using Nordhaus' damage function,

$$\mathcal{D}_i = e^{-\frac{\gamma}{2}(T_i - T_i^\star)^2} \bar{z}$$

A linear approximation implies:

$$\frac{d\mathcal{D}_i}{\mathcal{D}_i} = -\gamma(T_i - T_i^\star)dT_i = -\gamma(T_i - T_i^\star)T_i \frac{dT_i}{T_i}$$

Regarding the change in temperature caused by emissions, we get:

$$\begin{aligned}
& dT_i = \Delta_i \chi d\mathcal{S} = \Delta_i \chi \big(dE^f + \hat{\xi}^c dE^c \big) \\
\Rightarrow \qquad & \frac{dT_i}{T_i} = \frac{\Delta_i \chi d\mathcal{E}}{\Delta_i \chi (\mathcal{S}_0 + \mathcal{E})} = s^{E/S} \Big(s^{f/E} \frac{dE^f}{E^f} + s^{c/E} \frac{dE^c}{E^c} \Big) \\
& \qquad \text{with } s^{E/S} = \frac{\mathcal{E}}{\mathcal{S}_0 + \mathcal{E}} \qquad s^{f/E} = \frac{E^f}{\mathcal{E}} \qquad s^{c/E} = \hat{\xi}^c \frac{E^c}{\mathcal{E}}
\end{aligned}$$

As a result, to summarize, the change in damage depends on the total energy used in fossil (oil-gas) and coal.

$$d\ln \mathcal{D}_i = -\bar{\gamma}_i \big(s^{f/E} d\ln E^f + s^{c/E} d\ln E^c \big) \qquad \bar{\gamma}_i = \gamma (T_i - T_i^\star) T_i \, s^{E/S}$$

where $\bar{\gamma}_i$ summarize in one parameter the heterogeneous impact of climate change.

D.5 Energy markets

We now turn to energy where the demand and equilibrium effect on prices will be of first-oder importance for our welfare decomposition.

Energy demand

To examine the demand side of the market, we compute the elasticities of demand for each energy source, which are determined jointly by the firm First-Order Conditions. Thanks to our nested CES formulation, we can compute the elasticity $\varepsilon^\ell_{q^k} = \frac{\partial e^\ell_i}{\partial q^k} \frac{q^k}{e^\ell_i}$ as :

$$\begin{bmatrix} \varepsilon^f_{q^f} & \varepsilon^f_{q^c} & \varepsilon^f_{q^r} \\ \varepsilon^c_{q^f} & \varepsilon^c_{q^c} & \varepsilon^c_{q^r} \\ \varepsilon^r_{q^f} & \varepsilon^r_{q^c} & \varepsilon^r_{q^r} \end{bmatrix} = (\widetilde{H}^e)^{-1} = -\frac{\sigma^y}{1-s^e} \begin{bmatrix} s^f & s^c & s^r \\ s^f & s^c & s^r \\ s^f & s^c & s^r \end{bmatrix} + \sigma^e \begin{bmatrix} -(1-s^f) & s^c & s^r \\ s^f & -(1-s^c) & s^r \\ s^f & s^c & -(1-s^r) \end{bmatrix}$$

where the first part correspond to the change in aggregate price of energy q^e, since $\frac{\partial q^e_i}{\partial q^k} \frac{q^k}{q^e_i} = s^k_i$, which reduces demands for overall energy, according to elasticity $\frac{\sigma^y}{1-s^e_i}$ where s^e_i is the cost share of energy and σ^y the elasticity between energy and other inputs. Second, the later part summarizes the substitution effect across energy sources, negative along the diagonal and positive out of diagonal, due to positive cross-elasticity in the CES framework.

Moreover, the energy demand also depends on aggregate TFP (and hence climate damage), and the price level at which the final good is sold. As a result, the productivity elasticities and the final good price elasticity write:

$$\begin{bmatrix} \varepsilon^f_z \\ \varepsilon^c_z \\ \varepsilon^r_z \end{bmatrix} = \frac{\sigma^y}{1-s^e} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} \varepsilon^f_p \\ \varepsilon^c_p \\ \varepsilon^r_p \end{bmatrix} = \frac{\sigma^y}{1-s^e} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

which again is standard in the Nested CES framework.

As a result, we can express the energy demand as a function of the other endogenous variables:

$$
\begin{aligned}
d\ln e_i^f &= -\big(\frac{\sigma^y}{1-s_i^e}s_i^f + (1-s_i^f)\sigma^e\big)[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big)s_i^c[d\ln q_i^c + \hat{\xi}^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big)s_i^r d\ln q_i^r \\
&\quad + \frac{\sigma^y}{1-s^e}d\ln \mathcal{D}_i + \frac{\sigma^y}{1-s^e}d\ln \mathrm{p}_i \\
d\ln e_i^c &= \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big)s_i^f[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] - \big(\frac{\sigma^y}{1-s_i^e}s_i^c + (1-s_i^c)\sigma^e\big)[d\ln q_i^c + \hat{\xi}^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big)s_i^r d\ln q_i^r \\
&\quad + \frac{\sigma^y}{1-s^e}d\ln \mathcal{D}_i + \frac{\sigma^y}{1-s^e}d\ln \mathrm{p}_i \\
d\ln e_i^r &= \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big)s_i^f[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big)s_i^c[d\ln q_i^c + \hat{\xi}^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] - \big(\frac{\sigma^y}{1-s_i^e}s_i^r + (1-s_i^r)\sigma^e\big)d\ln q_i^r \\
&\quad + \frac{\sigma^y}{1-s^e}d\ln \mathcal{D}_i + \frac{\sigma^y}{1-s^e}d\ln \mathrm{p}_i
\end{aligned}
$$

Those endogenous energy demand can be reintegrated in the production function to obtain, the change in output, as function of prices of good, energies and productivity:

$$
\begin{aligned}
d\ln y_i &= d\ln \mathcal{D}_i + s_i^e\big[s_i^f d\ln e_i^f + s_i^r d\ln e_i^c + s_i^r d\ln e_i^r\big] \\
&= (1+\frac{s_i^e\sigma^y}{1-s_i^e})d\ln \mathcal{D}_i + \frac{s_i^e\sigma^y}{1-s_i^e}d\ln \mathrm{p}_i - s_i^e\frac{\sigma^y}{1-s_i^e}s_i^f[d\ln q^f + d\ln \mathrm{t}_i^\varepsilon] \\
&\qquad - s_i^e\frac{\sigma^y}{1-s_i^e}s_i^r[d\ln q_i^c + \hat{\xi}^c d\ln \mathrm{t}_i^\varepsilon] - s_i^e\frac{\sigma^y}{1-s_i^e}s_i^r d\ln q_i^r \\
d\ln y_i &= \alpha^{y,z}d\ln z_i + \alpha^{y,p}d\ln \mathrm{p}_i - \alpha^{y,qf}\big[d\ln q^f + d\ln \mathrm{t}_i^\varepsilon\big] - \alpha^{y,qc}\big[d\ln q_i^c + d\ln \mathrm{t}_i^\varepsilon\big] - \alpha^{y,qr}d\ln q_i^r \\
\alpha_i^{y,z} &= 1+\frac{s_i^e\sigma^y}{1-s_i^e} \qquad \alpha_i^{y,p} = \frac{s_i^e\sigma^y}{1-s_i^e} \\
\alpha_i^{y,qf} &= s_i^e\frac{\sigma^y}{1-s^e}s_i^f \qquad \alpha_i^{y,qc} = s_i^e\frac{\sigma^y}{1-s^e}s_i^c \qquad \alpha_i^{y,qr} = s_i^e\frac{\sigma^y}{1-s_i^e}s_i^r
\end{aligned}
$$

$$
d\ln y_i = \alpha^{y,z}d\ln z_i + \alpha^{y,p}d\ln \mathrm{p}_i - \alpha^{y,qf}d\ln q^f - (\alpha^{y,qf}+\hat{\xi}^c\alpha^{y,qc})d\ln \mathrm{t}_i^\varepsilon - (\alpha^{y,qc}+\alpha^{y,qr})d\ln \mathbb{P}_i
$$

where this last equation uses the supply curve of coal and renewable. We can see the exposure of country i's output of carbon tax: $\alpha^{y,qf}+\hat{\xi}^c\alpha^{y,qc} = s_i^e\frac{\sigma^y}{1-s^e}(s_i^f + \hat{\xi}^c s_i^c)$, through the price and substitution effect of oil, gas and coal.

Fossil energy market

The energy demand in fossil is the sum of individual countries demand, where we denote the share of country i in global production $\lambda_i^f = \frac{\mathcal{P}_i e_i^f}{E^f}$

$$
\begin{aligned}
dE^f &= \sum_i \mathcal{P}_i de_i^f \\
d\ln E^f &= \sum_i \lambda_i^f d\ln e_i^f \\
&= -\sum_i \lambda_i^f \big(\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e\big)[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \sum_i \lambda_i^f \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big) s_i^c [d\ln q_i^c + \hat{\xi}^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] \\
&\qquad + \sum_i \lambda_i^f \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big) s_i^r d\ln q_i^r + \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} d\ln \mathcal{D}_i + \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} d\ln \mathrm{p}_i
\end{aligned}
$$

We see that carbon taxation decreases demand for oil and gas by substitution, but can also increases it if the substitution away for coal is strong enough. The first effect dominate the second – up to the first order – if:

$$
\overline{\lambda}_{\mathcal{J}}^{\sigma,f} := \sum_{i\in\mathcal{J}} \lambda_i^f \big(\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e\big) > \sum_{i\in\mathcal{J}} \lambda_i^f \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big) s_i^c \hat{\xi}^c =: \overline{\lambda}_{\mathcal{J}}^{\sigma,c}
$$

which depend, among others, on the covariance $\mathbb{C}\mathrm{ov}_i(\lambda_i^f, 1-s_i^f)$ and $\mathbb{C}\mathrm{ov}_i(\lambda_i^f, s_i^c)$, since the substitution effect is stronger than the income effect $\sigma^e > \sigma^y/(1-s_i^e)$, in most empirically-relevant cases.

Now, the energy supply curve can also be recasted as the sum of individual extraction $E^f = \sum_i \mathcal{P}_i e_i^f = \sum_i e_i^x$, and, with the share of fossil production $\lambda_i^x = e_i^x/E^f$, it hence derives as follow:

$$
\begin{aligned}
e_i^x &= (q^f)^{1/\nu_i} \mathcal{R}_i \bar{\nu}_i^{-1/\nu_i} \mathbb{P}_i^{-1/\nu_i} \\
d\ln E^f &= \sum_i \lambda_i^x d\ln e_i^x = \sum_i \lambda_i^x \frac{1}{\nu_i}[d\ln q^f - d\ln \mathbb{P}_i] \\
\Rightarrow \qquad d\ln q^f &= \bar{\nu} d\ln E^f + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathbb{P}_i
\end{aligned}
$$

with the aggregate supply elasticity $\bar{\nu} = \big(\sum_i \lambda_i^x \nu_i^{-1}\big)^{-1}$, that we already encountered in the second best optimal Ramsey policy.

Now, replacing the energy demand quantity $d\ln E^f$ into the energy supply/price curve, we obtain:

$$
\begin{aligned}
d\ln q^f &= \bar{\nu} d\ln E^f + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathbb{P}_i \\
&\qquad + \bar{\nu} \sum_i \lambda_i^f \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big) s_i^r d\ln q_i^r + \bar{\nu} \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e}[d\ln \mathcal{D}_i + d\ln \mathrm{p}_i] + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathbb{P}_i \\
d\ln q^f &= -\frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^{\sigma,f} \mathrm{J}_i d\ln \mathrm{t}^\varepsilon + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big) s_i^c [d\ln q_i^c + \hat{\xi}^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] \\
&\quad + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f \big(\sigma^e - \frac{\sigma^y}{1-s_i^e}\big) s_i^r d\ln q_i^r + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e}[d\ln \mathcal{D}_i + d\ln \mathrm{p}_i] + \frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathbb{P}_i
\end{aligned}
$$

where $\overline{\lambda}^{\sigma,f} = \overline{\lambda}_{\mathbb{I}}^{\sigma,f}$, for $\mathbb{I}$ the whole world, and $\overline{\lambda}_{\mathcal{J}}^{\sigma,f} := \sum_{i\in\mathcal{J}} \lambda_i^f \big(\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e\big)$. Replacing the known terms, prices of renewable, coal, this factor, we obtain:

$$
\begin{aligned}
d\ln q^f = \; & \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\big[-\overline{\lambda}_{\mathcal{J}}^{\sigma,f} + \overline{\lambda}_{\mathcal{J}}^{\sigma,c}\big] d\ln \mathrm{t}^{\varepsilon} + \frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \big(\bar{\nu}\lambda_i^{\sigma,c} + \bar{\nu}\lambda_i^{\sigma,r} + \lambda_i^x \frac{\bar{\nu}}{\nu_i}\big) d\ln \mathbb{P}_i \\
& + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e}[d\ln \mathcal{D}_i + d\ln \mathrm{p}_i]
\end{aligned}
$$

As before we see that carbon taxation decrease the oil-gas energy price if $\overline{\lambda}_{\mathcal{J}}^{\sigma,f} > \overline{\lambda}_{\mathcal{J}}^{\sigma,c}$. Moreover, we see that a change in the price index $d\ln \mathcal{P}_i$ of all the countries change the aggregate price of oil and gas because it both increases the price of renewable and coal, increases demand for oil-gas by substitutions – the terms $\bar{\nu}\lambda_i^{\sigma,c}$ and $\bar{\nu}\lambda_i^{\sigma,r}$ – and it also increases the price of the input – through the term $\lambda_i^x \frac{\bar{\nu}}{\nu_i}$.

D.6 Trade *à la* Armington

To investigate how the price indices $\mathcal{P}_i$ and the good price p_i are determined, we should how consider the market of goods.

$$
\mathcal{P}_i \mathrm{p}_i y_i = \sum_{k\in\mathbb{I}} \mathcal{P}_k s_{ki} \frac{v_k}{1+\mathrm{t}_{ki}}
$$

Using the CES framework, we obtain that:

$$
\begin{aligned}
\mathbb{P}_i &= \Big(\sum_j a_{ij}(\tau_{ij}(1+\mathrm{t}_{ij}^b)\mathrm{p}_j)^{1-\theta}\Big)^{\frac{1}{1-\theta}} \\
\frac{d\mathbb{P}_i}{\mathbb{P}_i} &= \sum_j s_{ij}\big(\frac{d\mathrm{p}_j}{\mathrm{p}_j} + \frac{d\mathrm{t}_{ij}^b}{1+\mathrm{t}_{ij}^b}\big) \\
s_{ij} &= \frac{c_{ij}(1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j}{\sum_k c_{ik}(1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k} = a_{ij}\frac{\big((1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j\big)^{1-\theta}}{\sum_k \big((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k\big)^{1-\theta}} = \Big(\frac{(1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j}{\mathbb{P}_i}\Big)^{1-\theta} \\
\frac{ds_{ij}}{s_{ij}} &= (\theta-1)\Big(\frac{d\mathbb{P}_i}{\mathbb{P}_i} - \big(\frac{d\mathrm{p}_j}{\mathrm{p}_j} + \frac{d\mathrm{t}_{ij}^b}{1+\mathrm{t}_{ij}^b}\big)\Big) \\
\frac{ds_{ij}}{s_{ij}} &= (\theta-1)\Big(\sum_k s_{ik}\big(\frac{d\mathrm{p}_k}{\mathrm{p}_k} + \frac{d\mathrm{t}_{ik}^b}{1+\mathrm{t}_{ik}^b}\big) - \big(\frac{d\mathrm{p}_j}{\mathrm{p}_j} + \frac{d\mathrm{t}_{ij}^b}{1+\mathrm{t}_{ij}^b}\big)\Big)
\end{aligned}
$$

Using those formulas, the market clearing linearizes as follow:

$$
\frac{d[\frac{s_{ij}m_i\widetilde{v}_i}{1+t_{ij}}]}{\frac{s_{ij}m_i\widetilde{v}_i}{1+\mathrm{t}_{ij}}} = \Big[d\ln\widetilde{v}_i + \theta\sum_k \big(s_{ik}d\ln \mathrm{t}_{ik} - (1+s_{ij})d\ln \mathrm{t}_{ij}\big) + (\theta-1)\sum_{k\neq j}\big(s_{ik}d\ln \mathrm{p}_k - d\ln \mathrm{p}_j\big)\Big]
$$

$$
\text{with} \qquad \widetilde{v}_i = \mathrm{p}_i y_i + q^f(e_i^x/\mathcal{P}_i - e_i^f)
$$

This implies:

$$\mathcal{P}_i\widetilde{v}_i\Big(\frac{d\mathrm{p}_i}{d\mathrm{p}_i}+\frac{dy_i}{y_i}\Big)=\sum_k \mathcal{P}_k\frac{s_{ki}m_k\widetilde{v}_k}{1+\mathrm{t}_{ki}}d\ln[\frac{s_{ki}m_k\widetilde{v}_k}{1+t_{ki}}]$$

$$\Big(\frac{d\mathrm{p}_i}{d\mathrm{p}_i}+\frac{dy_i}{y_i}\Big)=\sum_k \frac{\mathcal{P}_k\widetilde{v}_k}{\mathcal{P}_i\widetilde{v}_i}s_{ik}\Big[d\ln\widetilde{v}_k+\theta\sum_h\big(s_{kh}d\ln\mathrm{t}_{kh}-(1+s_{ki})d\ln\mathrm{t}_{ki}\big)+(\theta-1)\sum_h\big(s_{kh}d\ln\mathrm{p}_h-d\ln\mathrm{p}_i\big)\Big]$$

$$\begin{aligned}\Big(\frac{d\mathrm{p}_i}{d\mathrm{p}_i}+\frac{dy_i}{y_i}\Big)=\sum_k \boldsymbol{t}_{ik}\Big[\Big(\frac{\mathrm{p}_k y_k}{v_k}\Big)(d\ln\mathrm{p}_k+d\ln y_k)+\frac{q^f e^x_k}{v_k}d\ln e^x_k-\frac{q^f e^f_k}{v_k}d\ln e^f_k+\frac{q^f(e^x_k-e^f_k)}{v_k}d\ln q^f\\ +\theta\sum_h\big(s_{kh}d\ln\mathrm{t}_{kh}-(1+s_{ki})d\ln\mathrm{t}_{ki}\big)+(\theta-1)\sum_h\big(s_{kh}d\ln\mathrm{p}_h-d\ln\mathrm{p}_i\big)\Big]\end{aligned}$$

with $\boldsymbol{t}_{ik}=\frac{\mathcal{P}_k\widetilde{v}_k}{\mathcal{P}_i\widetilde{v}_i}s_{ki}$, which is analogous to the same matrix in Kleinman et al. (2020). This implies, that rewritten in matrix notation, we get:

$$\begin{aligned}d\ln(\mathrm{p}y)&=\mathbf{T}v^y d\ln(\mathrm{p}y)+\mathbf{T}v^{e^x}d\ln e^x_k-\mathbf{T}v^{e^f}d\ln e^f_k+\mathbf{T}v^{ne}d\ln q^f+(\theta-1)(\mathbf{TS}-\mathbf{I})d\ln\mathrm{p}\\&\quad+\theta\big(\mathbf{TS}\odot\mathbf{J}\odot d\ln\mathrm{t}^b-\mathbf{T}(\mathbb{1}+\mathbf{S}')\odot(\mathbf{J}\odot d\ln\mathrm{t}^b)'\big)\\ \Big[(\mathbf{I}-\mathbf{T}\odot v^y)-(\theta-1)(\mathbf{TS}-\mathbf{T}')\Big]d\ln\mathrm{p}&=(\mathbf{I}-\mathbf{T}\odot v^y)d\ln y+\mathbf{T}v^{e^x}d\ln e^x_k-\mathbf{T}v^{e^f}d\ln e^f_k+\mathbf{T}v^{ne}d\ln q^f\\&\quad+\theta\big(\mathbf{TS}\odot\mathbf{J}\odot d\ln\mathrm{t}^b-\mathbf{T}(\mathbb{1}+\mathbf{S}')\odot(\mathbf{J}\odot d\ln\mathrm{t}^b)'\big)\end{aligned}$$

with $v^{e^x}=\frac{q^f e^x_i}{v_i}$, $v^{e^f}=\frac{q^f e^f_i}{v_i}$ and $v^{ne}=\frac{q^f(e^x_i-e^f_i)}{v_k}$.

D.7 Back to welfare

From above, we saw that welfare writes as:

$$\frac{d\mathcal{U}_i}{u'(c_i)c_i}=\frac{dc_i}{c_i}=\frac{\eta^y_i}{\eta^c_i}\Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}+\frac{d\mathcal{D}_i}{\mathcal{D}_i}\Big)-\frac{\eta^y_i}{\eta^c_i}s^e_i s^f_i\frac{dq^f}{q^f}+\frac{\eta^\pi_i}{\eta^c_i}\Big(1+\frac{1}{\nu}\Big)\frac{dq^f}{q^f}-\Big[\frac{\eta^y_i}{\eta^c_i}s^e_i(s^c_i+s^r_i)+\frac{\eta^\pi_i}{\eta^c_i}\frac{1}{\nu}+1\Big]\frac{d\mathbb{P}_i}{\mathbb{P}_i}+\frac{d\widetilde{\mathrm{t}}^{ls}_i}{x_i}$$

with the damage

$$d\ln\mathcal{D}_i=-\bar{\gamma}_i\big(s^{f/E}d\ln E^f+s^{c/E}d\ln E^c\big)$$

With the oil-gas energy price:

$$\begin{aligned}d\ln q^f&=\bar{\nu}d\ln E^f+\sum_i\lambda^x_i\frac{\bar{\nu}}{\nu_i}d\ln\mathbb{P}_i\\ d\ln q^f&=\frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\big[-\overline{\lambda}^{\sigma,f}_{\mathcal{J}}+\overline{\lambda}^{\sigma,c}_{\mathcal{J}}\big]d\ln\mathrm{t}^\varepsilon+\frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i\big(\lambda^{\sigma,c}_i+\lambda^{\sigma,r}_i+\lambda^x_i\frac{1}{\nu_i}\big)d\ln\mathbb{P}_i\\&\quad+\frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i\lambda^f_i\frac{\sigma^y}{1-s^e_i}[d\ln\mathcal{D}_i+d\ln\mathrm{p}_i]\\ d\ln E^f&=\frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\big[-\overline{\lambda}^{\sigma,f}_{\mathcal{J}}+\overline{\lambda}^{\sigma,c}_{\mathcal{J}}\big]d\ln\mathrm{t}^\varepsilon+\frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i\big(\lambda^{\sigma,c}_i+\lambda^{\sigma,r}_i\big)d\ln\mathbb{P}_i+\frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i\lambda^f_i\frac{\sigma^y}{1-s^e_i}[d\ln\mathcal{D}_i+d\ln\mathrm{p}_i\end{aligned}$$

D.8 Further simplification

To simplify the welfare formula even further, in the following we consider that energy is only composed of oil-gas. In practice, oil and gas compose the largest share of energy, with oil representing close to 35% of energy use and natural gas close to 20% at the world level.

We consider that $s_i^f = 1$ and $s_i^r = s_i^c = 0$ in all the formulas above. We also consider that oil-gas supply only uses the local good, which makes isolated from international trade – making the extraction $e_i^x = (q^f)^{1/\nu_i}\mathcal{R}_i\bar{\nu}_i^{-1/\nu_i}\mathrm{p}_i^{-1/\nu_i}$ instead of $\mathbb{P}_i^{-1/\nu_i}$.

Those two assumptions simplify our setting dramatically. The previous welfare decomposition reduces to :

$$\frac{d\mathcal{U}_i}{u'(c_i)c_i} = \frac{dc_i}{c_i} = \big[\frac{\eta_i^y}{\eta_i^c} - \frac{\eta_i^\pi}{\eta_i^c}\frac{1}{\nu_i}\big]\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{\eta_i^y}{\eta_i^c}\frac{d\mathcal{D}_i}{\mathcal{D}_i} - \frac{\eta_i^y}{\eta_i^c}s_i^e\frac{dq^f}{q^f} + \frac{\eta_i^\pi}{\eta_i^c}\Big(1+\frac{1}{\nu}\Big)\frac{dq^f}{q^f} - \frac{d\mathbb{P}_i}{\mathbb{P}_i} + \frac{d\widetilde{\mathrm{t}}_i^{ls}}{x_i}$$

where the damage rewrite:

$$d\ln\mathcal{D}_i = -\bar{\gamma}_i d\ln E^f$$

with the average damage is defined as $\bar{\gamma} = \sum_i \bar{\gamma}_i$. And the oil-gas demand curve write:

$$\begin{aligned}
d\ln E^f &= \sum_i \lambda_i^f d\ln e_i^f \\
&= -\sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e}[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} d\ln\mathcal{D}_i + \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} d\ln \mathrm{p}_i \\
&= -\sum_i \widetilde{\lambda}_i^f[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \sum_i \widetilde{\lambda}_i^f d\ln\mathcal{D}_i + \sum_i \widetilde{\lambda}_i^f d\ln \mathrm{p}_i
\end{aligned}$$

where, to simplify notations, we denote $\widetilde{\lambda}_i^f = \lambda_i^f \frac{\sigma^y}{1-s_i^e}$, and it's average $\overline{\lambda}^{\sigma,f} = \sum_i \widetilde{\lambda}_i^f \frac{\sigma^y}{1-s_i^e}$. As a result, the demand now rewrites:

$$d\ln E^f = \frac{1}{1+\bar{\gamma}+\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i)}\Big[-\sum_i \widetilde{\lambda}_i^f[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \sum_i \widetilde{\lambda}_i^f d\ln \mathrm{p}_i\Big]$$

We can see that the energy demand curve is affected by climate change: more emission imply larger damage, which in turn reduce energy demand and hence emissions. Moreover, the covariance term indicates that if the large energy producers (with a larger share of the market, and high elasticity σ) are also the most affected by climate change, this effect is stronger and the demand curve is even steeper / more inelastic.

Recasting this logic in the supply curve, we obtain:

$$
\begin{aligned}
d\ln q^f &= \bar{\nu} d\ln E^f + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathrm{p}_i \\
d\ln E^f &= -\sum_i \widetilde{\lambda}_i^f [d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] - \sum_i \widetilde{\lambda}_i^f \bar{\gamma}_i d\ln E^f + \sum_i \widetilde{\lambda}_i^f d\ln \mathrm{p}_i \\
[1 + \sum_i \widetilde{\lambda}_i^f \bar{\gamma}_i + \bar{\nu} \sum_i \widetilde{\lambda}_i^f] d\ln E^f &= -\sum_i \widetilde{\lambda}_i^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon + \sum_i \widetilde{\lambda}_i^f d\ln \mathrm{p}_i - \big(\sum_i \widetilde{\lambda}_i^f\big) \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathrm{p}_i \\
d\ln E^f &= \frac{1}{1+\bar{\gamma}+\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i) + \bar{\nu}\overline{\lambda}^{\sigma,f}} \Big[-\sum_i \widetilde{\lambda}_i^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon + \sum_i \big(\widetilde{\lambda}_i^f - \overline{\lambda}^{\sigma,f} \lambda_i^x \tfrac{\bar{\nu}}{\nu_i}\big) d\ln \mathrm{p}_i \Big] \\
d\ln q^f &= -\frac{\bar{\nu} \sum_i \widetilde{\lambda}_i^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon}{1+\bar{\gamma}+\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i) + \bar{\nu}\overline{\lambda}^{\sigma,f}} + \sum_i \beta_i d\ln \mathrm{p}_i \\
\text{with} \qquad \beta_i &= \lambda_i^x \frac{\bar{\nu}}{\nu_i} + \frac{\bar{\nu}}{1+\bar{\gamma}+\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i) + \bar{\nu}\overline{\lambda}^{\sigma,f}} \big(\widetilde{\lambda}_i^f - \overline{\lambda}^{\sigma,f} \lambda_i^x \tfrac{\bar{\nu}}{\nu_i}\big)
\end{aligned}
$$

Welfare rewrites:

$$
\begin{aligned}
\frac{d\mathcal{U}_i}{u'(c_i)c_i} &= \Big[\frac{\eta_i^y}{\eta_i^c} - \frac{\eta_i^\pi}{\eta_i^c}\frac{1}{\nu_i}\Big] \frac{d\mathrm{p}_i}{\mathrm{p}_i} - \frac{\eta_i^y}{\eta_i^c} \bar{\gamma}_i d\ln E_i^f - \frac{\eta_i^y}{\eta_i^c} s_i^e \frac{dq^f}{q^f} + \frac{\eta_i^\pi}{\eta_i^c}\Big(1+\frac{1}{\nu}\Big)\frac{dq^f}{q^f} - \frac{d\mathbb{P}_i}{\mathbb{P}_i} + \frac{d\widetilde{\mathrm{t}}_i^{ls}}{x_i} \\
&= \Big[\frac{\eta_i^y}{\eta_i^c} - \frac{\eta_i^\pi}{\eta_i^c}\frac{1}{\nu_i}\Big] d\ln \mathrm{p}_i + \Big[-\frac{\eta_i^y}{\eta_i^c}\bar{\gamma}_i \frac{1}{\bar{\nu}} - \frac{\eta_i^y}{\eta_i^c} s_i^e + \frac{\eta_i^\pi}{\eta_i^c}\Big(1+\frac{1}{\nu}\Big)\Big] d\ln q^f + \frac{\eta_i^y}{\eta_i^c}\bar{\gamma}_i \sum_i \lambda_i^x \frac{1}{\nu_i} d\ln \mathrm{p}_i - d\ln \mathbb{P}_i + d\widetilde{\mathrm{t}}_i^{ls}/x_i
\end{aligned}
$$

Trade :

$$
\begin{aligned}
\Big(\frac{d\mathrm{p}_i}{d\mathrm{p}_i} + \frac{dy_i}{y_i}\Big) &= \sum_k \boldsymbol{t}_{ik} \Big[\Big(\frac{\mathrm{p}_k y_k}{v_k}\Big)(d\ln \mathrm{p}_k + d\ln y_k) + \frac{q^f e_k^x}{v_k} d\ln e_k^x - \frac{q^f e_k^f}{v_k} d\ln e_k^f + \frac{q^f (e_k^x - e_k^f)}{v_k} d\ln q^f \\
&\quad + \theta \sum_h \big(s_{kh} d\ln \mathrm{t}_{kh} - (1+s_{ki}) d\ln \mathrm{t}_{ki}\big) + (\theta-1)\sum_h \big(s_{kh} d\ln \mathrm{p}_h - d\ln \mathrm{p}_i\big) \Big] \\
\Big[(\mathbf{I} - \mathbf{T}\odot v^y) - (\theta-1)(\mathbf{TS} - \mathbf{T}')\Big] d\ln \mathrm{p} &= (\mathbf{I} - \mathbf{T}\odot v^y) d\ln y + \mathbf{T} v^{e^x} d\ln \mathrm{e}_k^x - \mathbf{T} v^{e^f} d\ln e_k^f + \mathbf{T} v^{ne} d\ln q^f \\
&\quad + \theta\big(\mathbf{TS}\odot \mathbf{J} \odot d\ln \mathrm{t}^b - \mathbf{T}(\mathbb{1}+\mathbf{S}')\odot(\mathbf{J}\odot d\ln \mathrm{t}^b)'\big)
\end{aligned}
$$

Output:

$$
\begin{aligned}
d\ln y_i &= \alpha^{y,z} d\ln \mathcal{D}_i + \alpha^{y,p} d\ln \mathrm{p}_i - \alpha^{y,qf}\big[d\ln q^f + d\ln \mathrm{t}_i^\varepsilon\big] \\
\alpha_i^{y,z} &= 1+\frac{s_i^e \sigma^y}{1-s_i^e} \qquad \alpha_i^{y,p} = \frac{s_i^e \sigma^y}{1-s_i^e} \qquad \alpha_i^{y,qf} = s_i^e \frac{\sigma^y}{1-s^e} s_i^f
\end{aligned}
$$

Energy demand:

$$
d\ln e_i^f = -\frac{\sigma^y}{1-s_i^e}[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i + \frac{\sigma^y}{1-s^e} d\ln \mathrm{p}_i
$$

Energy supply:

$$
d\ln e_i^x = \frac{1}{\nu_i}[d\ln q^f - d\ln \mathrm{p}_i]
$$

$$
\begin{aligned}
&\Big[(\mathbf{I}-\mathbf{T}\odot v^{y})[\mathbf{I}-\alpha^{y,p}\odot\mathbf{I}]+\mathbf{T}(v^{e^{x}}\odot\tfrac{1}{\nu})+\mathbf{T}v^{e^{f}}\tfrac{\sigma^{y}}{1-s^{e}}-(\theta-1)(\mathbf{TS}-\mathbf{T}')\Big]d\ln \mathrm{p}=\\
&(\mathbf{I}-\mathbf{T}\odot v^{y})[\alpha^{y,z}d\ln\mathcal{D}-\alpha^{y,qf}\big[d\ln q^{f}+\mathbf{J}d\ln \mathrm{t}^{\varepsilon}\big]]+\mathbf{T}(v^{e^{x}}\odot\tfrac{1}{\nu})d\ln q^{f}+\mathbf{T}v^{e^{f}}(\tfrac{\sigma^{y}}{1-s^{e}_{i}}[d\ln q^{f}+\mathrm{J}_{i}d\ln \mathrm{t}^{\varepsilon}]-\tfrac{\sigma^{y}}{1-s^{e}})d\ln\mathcal{D}_{i}+\mathbf{T}v^{n}\\
&\qquad+\theta\big(\mathbf{TS}\odot\mathbf{J}\odot d\ln \mathrm{t}^{b}-\mathbf{T}(\mathbb{1}+\mathbf{S}')\odot(\mathbf{J}\odot d\ln \mathrm{t}^{b})'\big)\\
&=\Big((\mathbf{I}-\mathbf{T}\odot v^{y})\alpha^{y,z}-\tfrac{\sigma^{y}}{1-s^{e}}\Big)d\ln\mathcal{D}_{i}+\big[-(\mathbf{I}-\mathbf{T}\odot v^{y})\alpha^{y,qf}+\mathbf{T}(v^{e^{x}}\odot\tfrac{1}{\nu}+v^{e^{f}}\tfrac{\sigma^{y}}{1-s^{e}}+v^{ne})\big]d\ln q^{f}\\
&\qquad+\big[-(\mathbf{I}-\mathbf{T}\odot v^{y})\alpha^{y,qf}+\mathbf{T}(v^{e^{f}}\odot\tfrac{\sigma^{y}}{1-s^{e}}\odot\mathrm{J})\big]d\ln \mathrm{t}^{\varepsilon}+\theta\big(\mathbf{TS}\odot\mathbf{J}\odot d\ln \mathrm{t}^{b}-\mathbf{T}(\mathbb{1}+\mathbf{S}')\odot(\mathbf{J}\odot d\ln \mathrm{t}^{b})'\big)
\end{aligned}
$$

$$
\begin{aligned}
&\Big[(\mathbf{I}-\mathbf{T}\odot v^{y})[\mathbf{I}-\alpha^{y,p}\odot\mathbf{I}]+\mathbf{T}(v^{e^{x}}\odot\tfrac{1}{\nu})+\mathbf{T}v^{e^{f}}\tfrac{\sigma^{y}}{1-s^{e}}-(\theta-1)(\mathbf{TS}-\mathbf{T}')-\Big((\mathbf{I}-\mathbf{T}\odot v^{y})\alpha^{y,z}-\tfrac{\sigma^{y}}{1-s^{e}}\Big)\odot\bar{\gamma}\mathbf{I}\odot(\tfrac{\lambda^{x}}{\nu})'\Big]d\ln \mathrm{p}=\\
&\qquad\big[-(\mathbf{I}-\mathbf{T}\odot v^{y})\alpha^{y,qf}+\mathbf{T}(v^{e^{x}}\odot\tfrac{1}{\nu}+v^{e^{f}}\tfrac{\sigma^{y}}{1-s^{e}}+v^{ne})-\Big((\mathbf{I}-\mathbf{T}\odot v^{y})\alpha^{y,z}-\tfrac{\sigma^{y}}{1-s^{e}}\Big)\bar{\gamma}\tfrac{1}{\bar{\nu}}\big]d\ln q^{f}\\
&\qquad+\big[-(\mathbf{I}-\mathbf{T}\odot v^{y})\alpha^{y,qf}+\mathbf{T}(v^{e^{f}}\odot\tfrac{\sigma^{y}}{1-s^{e}})\big]\odot\mathrm{J}d\ln \mathrm{t}^{\varepsilon}+\theta\big(\mathbf{TS}\odot\mathbf{J}\odot d\ln \mathrm{t}^{b}-\mathbf{T}(\mathbb{1}+\mathbf{S}')\odot(\mathbf{J}\odot d\ln \mathrm{t}^{b})'\big)
\end{aligned}
$$