The Optimal design of Climate Agreements

Inequality, Trade and Incentives for carbon policy

WORK IN PROGRESS

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EEE

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Introduction

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  e.g. cold, “closed” or fossil-rich countries are better off outside “climate clubs”

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⇒ Designing a climate agreement entails determining *jointly* the level of carbon tax and the club of participating countries

Introduction – this project

- Trade-off between intensive margin and extensive margin :
  - Climate club with a small number of countries, higher tax and large emissions reductions
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- Preview of the results :
  - Differing incentives to join agreements due to exposure to GE effects on energy & good markets
    - Warm/low-income countries have the most to gain from climate cooperation but have the highest payoff from unilateral deviation
    - Trade policy or funds transfers are important to ensure the stability of the agreement
    - Any combination of countries could be attained with the right combination taxes/tariffs.

Literature

- Climate change & optimal carbon taxation
  - RA model : Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
  - HA model : Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
  - Spatial models : Cruz, Rossi-Hansberg (2022, 2023) among others

  $\Rightarrow$ *Optimal and constrained policy with heterogeneous countries & trade*
- Unilateral vs. climate club policies :
  - Climate clubs : Nordhaus (2015), Non-cooperative taxation : Chari, Kehoe (1990), Suboptimal policy : Hassler, Krusell, Olovsson (2019)
  - Trade policy : Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021), Hsiao (2022), Costinot, Donalson, Vogel, Werning (2015), Adao, Costinot (2022), Antras, Fort, Gutierrez, Tintelnot (2022)

  $\Rightarrow$ *Climate cooperation and optimal design of climate club*
- Optimal policy in heterogeneous agents models
  - Policy with limited instruments : Diamond (1973), Davila, Walther (2022)
  - Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) ...

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Literature

- Nordhaus (2015)
  - Examine ”stable climate coalitions” (club imposing carbon tax) in a simple model
  - Abstract from General Equilibrium and distributional effects
  - Results : Penalty tariffs necessary to enforce a climate club
- Farrokhi, Lashkaripour (2021)
  - Study and characterize the optimal trade policy with climate externality
  - General static trade model. Results : unilateral tariffs not effective
  - Sequential search for one stable climate club if EU or US join.
- Main contribution :
  - Search for the *optimal* climate agreement
  - GE on good and energy market and redistribution effects are first-order
  - Cost of climate change is endogenous to policy (damages are non-linear)
  - Possibility of analyzing other distributional policies (transfers, *loss and damage funds*)
  - General framework for analyzing macrodynamics

Model – Household & Firms

- Static deterministic Neoclassical economy (for today)
  - countries $i \in \mathbb{I}$, heterogeneous in productivity z_i, temperature τ_i, energy extraction cost $\mathcal{C}_i$
  - In each country, 3 agents :
    (i) HtM household, (ii) homogeneous good firm, (iii) (fossil) energy producer

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- Representative household problem in each country i :

$$\mathcal{V}_i = \max_{c_{ij}} u(c_i) \qquad \mathbb{P}_i c_i = w_i + \pi_i^f + \mathrm{t}_i^{ls} \qquad c_i = \Big(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\Big)^{\frac{\theta}{\theta-1}} \qquad \text{price} \quad (1+\mathrm{t}_{ij}^b) d_{ij} \mathrm{p}_j$$

  - Labor income w_i from final good firm (labor norm. to 1), profit π_i^f from fossil firm

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  - Labor income w_i from final good firm (labor norm. to 1), profit π_i^f from fossil firm
- Competitive homogeneous good producer in country i

$$\max_{e_i^f} \mathrm{p}_i \, \mathcal{D}(\tau_i) z_i f(e_i^f) - w_i - (q^f + \mathrm{t}_i^f) e_i^f$$

  - Fossil energy demand e_i^f – emitting carbon – subject to price q^f and tax/subsidy t^f.
  - Climate externality : effect of temperature on damage/TFP, $\mathcal{D}(\tau) \in (0,1)$

Model – Energy markets & Emissions

- Competitive fossil fuels energy producer :
  - Supply fossil energy e^x_{it} by extraction at cost $\mathcal{C}^f_i$

$$\pi^f_i = \max_{e^x_i} q^f e^x_i - \mathcal{C}^f_i(e^x_i)\mathbb{P}_i$$

  - Energy traded in international markets, at price q^f

$$E = \sum_{\mathbb{I}} e^f_i = \sum_{\mathbb{I}} e^x_i$$

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- Climate system
  - Fossil energy e^f releases GHGs in the atmosphere

$$\mathcal{E} = \sum_{\mathbb{I}} e^f_i$$

  - Country's local temperature :

$$\tau_i = \bar{\tau}_{i0} + \Delta_i \mathcal{E}$$

    - Linear model + Climate sensitivity/pattern scaling factor Δ_i

Model – Equilibrium

- Equilibrium
  - Given policies $\{\mathrm{t}_i^f, \mathrm{t}_{ij}^b, \mathrm{t}_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e_i^f, e_i^x\}_{ij}$, states $\{\tau_i\}_i$ and prices $\{\mathrm{p}_i\}_i, q^f$ such that :
    - Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint
    - Firm choose policies $\{e_i^f\}_i$ to max. profit
    - Fossil firms extract/produce $\{e_i^x\}_i$ to max. profit.
    - Emissions $\mathcal{E}_t$ affects climate $\{\tau_i\}_i$.
    - Government budget clear $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^f e_i^f + \sum_{i,j} \mathrm{t}_{ij}^b c_{ij} d_{ij} \mathrm{p}_j$
    - Prices $\{\mathrm{p}_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$ and for each good

$$y_i := z_i \mathcal{D}(\tau_i) f(e_i^f) = \sum_{k \in \mathbb{I}} c_{ki} d_{ki} + \sum_{k \in \mathbb{I}} \mathcal{P}_k ne_{ki}$$

with ne_{ki} net export of good i used to pay for energy use in k.
In expenditure, with import shares $s_{ij} = \frac{c_{ij} d_{ij} \mathrm{p}_j}{c_i \mathbb{P}_i}$, it yields $\mathrm{p}_i y_i = \sum_{k \in \mathbb{I}} s_{ki}\, \mathrm{p}_k y_k$

Model – Dynamics & extensions

1. Energy market
   - Renewable energy firm in each country
   - Price of clean energy trending down
   - Fossil energy extraction/depleting reserves $\Rightarrow$ Hotelling problem
2. Firm
   - Use capital as well to produce
   - Use an energy bundle of renewable and fossil energy
3. Households
   - Consumption / saving in bonds / in capital $\Rightarrow$ Keynes-Ramsey rule
   - International markets to borrow bonds (in zero net supply)
4. Climate system with inertia / closer to standard IAMs
5. Population growth dynamics for each country
6. (Exogenous) growth : TFP change and Energy-augmenting Directed TC.

Optimal world policy – Summary of results

- ***Equilibrium 0 :*** Competitive equilibrium Details eq 0
  - Trade as a powerful adaptation mechanism
  - Local social cost of carbon depends on trade integration
- ***Equilibrium 1 :*** First-Best, with unlimited instruments Details eq 1
  - Welfare : $\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{c},\boldsymbol{e}\}_i} \sum_{i\in\mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$
  - Social Planner redistribute across countries with lump-sum transfers t_i^{ls}
  - Set the optimal Pigouvian carbon tax to $\mathrm{t}^f = SCC$
- ***Equilibrium 2 :*** Second-best Ramsey policy, with limited instruments Details eq 2
  - Optimal carbon tax accounts for (i) inequality and local climate damage, (ii) energy supply elasticities, (iii) energy terms-of-trade redistribution effects, (iv) energy demand distortions
  - Optimal tariffs also account for redistributive terms-of-trade effects on goods

$\Rightarrow$ Planner takes into account trade patterns in the design of all its instruments

Different policies – Summary of results

- ***Equilibrium 3 :*** Countries decide whether to join the climate club : participation constraints
  - Almost same results as Equilibrium 2
  - – All the tax formulas are corrected for the participation constraints (Lagrange multipliers affect redistribution weights)
  - – Taxation with imperfect instruments : rescale tax rate for the missing tax base.
- ***Equilibrium 4 :*** Optimal design of size $\mathbb{J}$ and countries $j \in \mathbb{J}$ in the climate agreement
  - Object of the rest of this project/presentation !

Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.
- Design a climate "club" of $\mathbb{J} \subseteq \mathbb{I}$ countries :
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f
  - Countries can unilaterally leave : in that case, they are subject to tariffs t^b on goods from the club members. They still trade with the club members in energy at price q^f. They obtain $\tilde{c}_i$

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- Participation constraints :

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

- Welfare, for $\mathbb{J}$, weighting all countries $i \in \mathbb{I}$

$$\mathcal{W}(\mathbb{J}) = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i)$$

Optimal Design of a Climate Agreement – Naive approach

- Naive approach :
  - Combinatorial problem : $\mathcal{P}(\mathbb{I})$ with $2^{|\mathbb{I}|}$ choices

$$\max_{\mathbb{J}\in\mathcal{P}(\mathbb{I})} \mathcal{W}(\mathbb{J}) \qquad s.t. \qquad u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

  - Choice of countries $\mathbb{J}$ yields optimal carbon taxes and tariffs $\{\mathrm{t}^f(\mathbb{J}), \mathrm{t}^b(\mathbb{J})\}$
  - Hard (impossible ?) problem :
    - Combinatorial problem on $\mathbb{J} \in \mathcal{P}(\mathbb{I})$
    - Optimal club policies $\{\mathrm{t}^f(\mathbb{J}), \mathrm{t}^b(\mathbb{J})\}$ depend on unknown multiplier ν_i :
    - Depends on "effective Pareto weights" : $\omega_i(1+\nu_i)$ if $i \in \mathbb{J}$, and $\omega_i(\alpha - \nu_i)$ if $i \notin \mathbb{J}$

Optimal Design – Alternative approach

- Alternative approach : choosing policy first
  - From a level of the tax t^f and t^b imposed on club $\mathbb{J}$, we can deduce the set of countries $\widetilde{\mathbb{J}}$ with binding participation constraints

$$\widetilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\widetilde{c}_i) \quad \forall i \in \widetilde{\mathbb{J}}$$

  - Search for the couple $\{\mathrm{t}^f, \mathrm{t}^b\}$ such that $\mathbb{J} = \widetilde{\mathbb{J}}$
    - Difficult (but feasible)
    - Heuristics for algorithms (Next slides)
    - Can be extended to dynamic settings :
      choose a path of $\{\mathrm{t}^f, \mathrm{t}^b\}$ instead of a path of combinations of clubs

Numerical approach for stable coalition

- Algorithm 1 : sequential approach in $\mathbb{J}$
  1. Start from the second-best optimal policy $\{t^{f\star}, t^{b\star}\}$, on the world $\mathbb{J} = \mathbb{I}$
  2. From tax levels imposed on club $\mathbb{J}$, deduce the number of countries $\widetilde{\mathbb{J}}$ with binding participation constraints
$$\widetilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\widetilde{c}_i) \quad \forall i \in \widetilde{\mathbb{J}}$$
  – Search for $\{t^f, t^b\} \in [0, t^{f\star}] \times [0, 1]$ that yield $\mathbb{J} = f(\mathbb{J}, t^f, t^b)$

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  3. If $Im\big(f(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b)\big) \subsetneq \mathbb{J}$, remove countries one-by-one
  4. Repeat (2-3) until convergence – fixed point of $\widetilde{\mathbb{J}} = f(\mathbb{J}, \mathrm{t}^f, \mathrm{t}^b)$ – or unraveling

– Fixed point on $\mathbb{J}$ could be costly (may still need to solve the combinatorial problem)
  - With complementarities or substitution, can develop a squeezing procedure as in Jia (2008) or Arkolakis et al (2023)

Optimal Design – Alternative approach

- Search for complementarity
  - Combinatorial discret choice lit, c.f. Antras, Fort, Tintelnot (2017), Jia (2008), Arkolakis, Eckert, Shi (2023), Alfaro-Ureña, Castro-Vincenzi, Fanelli, Morales (2024)

$$\Delta\mathcal{W}(\mathbb{J}',j) := \mathcal{W}(\mathbb{J}' \cup j) - \mathcal{W}(\mathbb{J}') > \Delta\mathcal{W}(\mathbb{J},j) \qquad \text{when } \mathbb{J}' \supseteq \mathbb{J} \qquad \text{for all } j \in \mathbb{I}$$

  - Adding an extra member j is increasingly profitable with the size of the club $\mathbb{J}$

- What determines the choice of a country to join the climate agreement ?
  - Benefit : lower temperature τ_i, reduction in energy price q^f, increase in good price p_i, etc.
  - Costs : carbon tax, tariffs on countries outside the club, decrease in fossil rent

Quantification

- Quantification and calibration More details
  - Quadratic damage as in Nordhaus DICE $y = \mathcal{D}_i(\tau)\bar{y}$ with $\mathcal{D}_i(\tau) = e^{-\frac{\gamma_i}{1+\delta}(\tau-\tau_{i0})^{(1+\delta)}}$
  - Energy parameters to match production/reserves,
    Isoelastic cost function $\mathcal{C}_i(e_i^x) = \bar{\nu}_i \left(e_i^x/\mathcal{R}_i\right)^{1+\nu}\mathcal{R}_i$
  - Armington model, distance d_{ij} and preferences a_{ij} to match import shares $s_{ij} = \frac{c_{ij} d_{ij} \mathrm{p}_j}{c_i \mathbb{P}_i}$
  - Production $\bar{y} = zf(\ell_i, e_i^f)$ with CES labor/energy σ and energy shares s_i^f.
    - Extension : $\bar{y} = zf(\ell_i, k_i, e_i^f, e_i^r)$. Nested CES energy vs. labor-capital Cobb-Douglas bundle (elasticity $\sigma_y < 1$), and fossil/renewable $\sigma_e > 1$.
    - TFP, and DTC, z_i, z_i^e, calibrated to match GDP / energy shares data.

Countries' incentives – Model w/o trade in goods

- Experiment : Model with trade in energy but not in "goods"
  - Start from the equilibrium where carbon tax $t^f(\mathbb{J}) = 0$,
    $\Rightarrow$ country i is indifferent to join the club $\mathbb{J}$ or not
  - Linear approximation around that point $\Rightarrow$ small changes in carbon tax dt^f

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  - Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})} = -e_i d\mathfrak{t}^f - \gamma_i(\tau_i - \tau_{i0})^\delta y_i \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \pi_i \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

  - Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}} = -\left(E_{\mathbb{J}} - E_{\mathbb{J}\setminus\{i\}}\right) \frac{\sigma\, d\mathfrak{t}^f}{q^f(1-s^f)} \frac{1}{1+\frac{\nu\sigma}{(1-s^f)}}$$

○ Params : σ energy demand elast^y, s^f energy cost share, ν energy supply elas^y, Climate damage γ_i and curv. δ

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Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\begin{aligned}\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = & -e_i d\mathrm{t}^f - \gamma_i(\tau_i - \tau_{i0})^\delta \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \\ & - e_i \frac{q^f \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \; + \; \eta_i^f \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \\ & + \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}}\Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij}\Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}}\Big)\end{aligned}$$

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$$+ \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}} \Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij} \Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}} \Big)$$

  - GE effect on energy markets, same as before
  - GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki} \Big(\frac{dv_k}{v_k} - \theta \frac{d\mathrm{t}^b_{ki}}{1+\mathrm{t}^b_{ki}} \Big) \qquad \alpha_{ki} = \frac{c_{ki} d_{ki} \mathrm{p}_i}{\sum_\ell c_{k\ell} d_{k\ell} \mathrm{p}_\ell} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+\mathrm{t}^b_{ki}) v_i}$$

    – Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Countries' incentives – Armington Model with trade in goods

- Trade in energy and goods *à la* Armington, Linear approx. around $\mathrm{t}^f \approx 0$ and $\mathrm{t}^b \approx 0$
  - Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i d\mathrm{t}^f - \gamma_i(\tau_i - \tau_{i0})^\delta \, \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$
$$+ \eta_i^y \Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big|_{i\notin\mathbb{J}}\Big) - s_{i\mathbb{J}^c} d\mathrm{t}^b - \sum_{j\in\mathbb{I}} s_{ij}\Big(\frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\in\mathbb{J}} - \frac{d\mathrm{p}_j}{\mathrm{p}_j}\Big|_{i\notin\mathbb{J}}\Big)$$

  - GE effect on energy markets, same as before
  - GE effect on goods markets, equilibrium on expenditure $v_i = \mathbb{P}_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki}\Big(\frac{dv_k}{v_k} - \theta \frac{d\mathrm{t}_{ki}^b}{1+\mathrm{t}_{ki}^b}\Big) \qquad \alpha_{ki} = \frac{c_{ki} d_{ki} \mathrm{p}_i}{\sum_\ell c_{k\ell} d_{k\ell} \mathrm{p}_\ell} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+\mathrm{t}_{ki}^b) v_i}$$

    – Params : σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i \mathrm{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

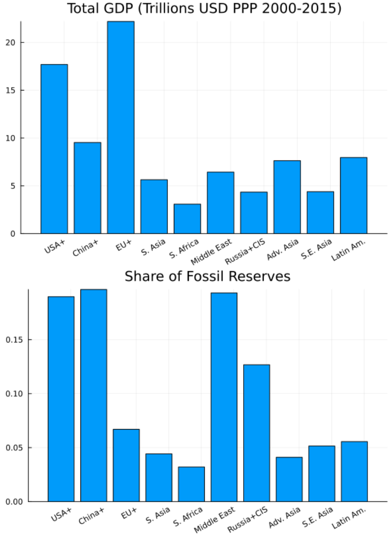
Complementarity in coalition formation – Model w/o trade in goods

- Is marginal gain $\Delta\mathcal{W}(\mathbb{J},j) := \mathcal{W}(\mathbb{J}\cup j) - \mathcal{W}(\mathbb{J})$ "growing" in $\mathbb{J}$?
  - Linear approximation for small $\{\mathrm{t}^f, \mathrm{t}^b\}$

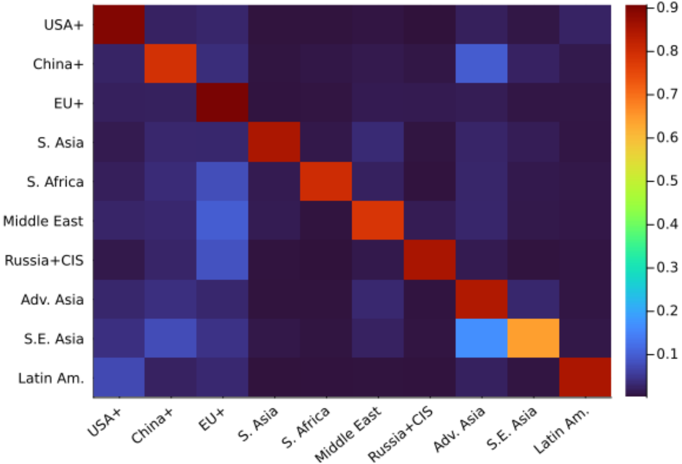
$$\Delta\mathcal{W}(\mathbb{J},j) = -\omega_j u'(c_j) e_j d\mathrm{t}^f + \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i)\Delta_i \gamma_i (\tau_i - \tau_{i0})^\delta y_i\Big] \frac{\sigma\, e_j d\mathrm{t}^f}{q^f(1-s^f+\nu\sigma)}$$
$$+ \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i) e_i\Big] \frac{1}{1+\frac{1-s^f}{\nu\sigma}} \frac{e_j d\mathrm{t}^f}{E_{\mathbb{I}}} - \Big[\sum_{i\in\mathbb{I}} \omega_i u'(c_i)\pi_i\Big] \frac{(1+\nu)}{E_{\mathbb{I}}} \frac{\sigma\, e_j d\mathrm{t}^f}{q^f(1-s^f+\nu\sigma)}$$

  - Free-riding problem : $\Delta\mathcal{W}(\mathbb{J},j)$ could be negative
  - If $\Delta\mathcal{W}(\mathbb{J},j) > 0$, what effects does $\mathbb{J}$ have on marginal gain ?
    - Marginal climate benefit decreases in $\mathbb{J}$, since temperature τ_i declines !
    - G.E. effect on energy price : $E_{\mathbb{I}}$, q and π^f decreases with $\mathbb{J}$, effect on demand ambiguous
    - Similar formula for the case with trade tariffs : Work in progress.

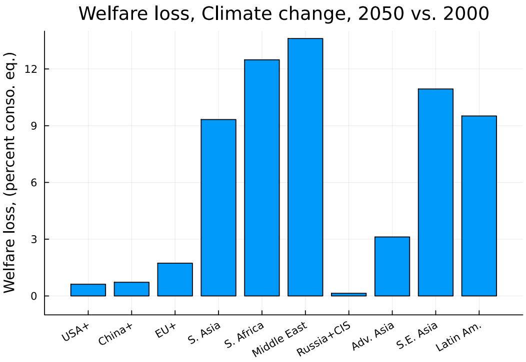
Numerical Application - Sample



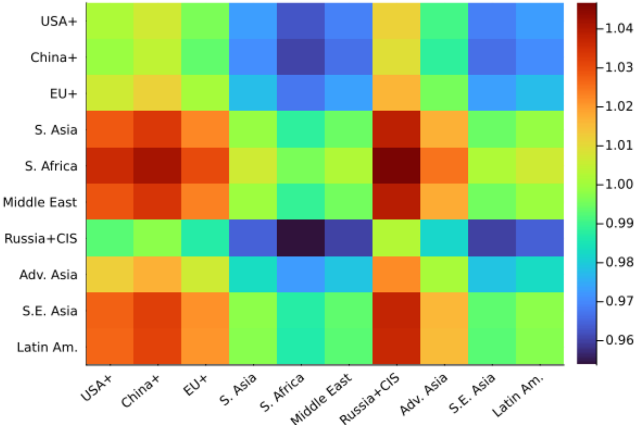
- Data : 10 regions, Average 2000-2015



Cost of Climate Change

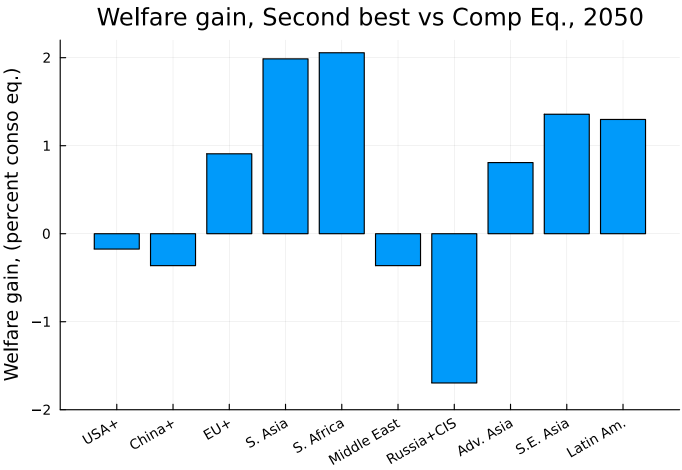


- Trade reallocation – climate change

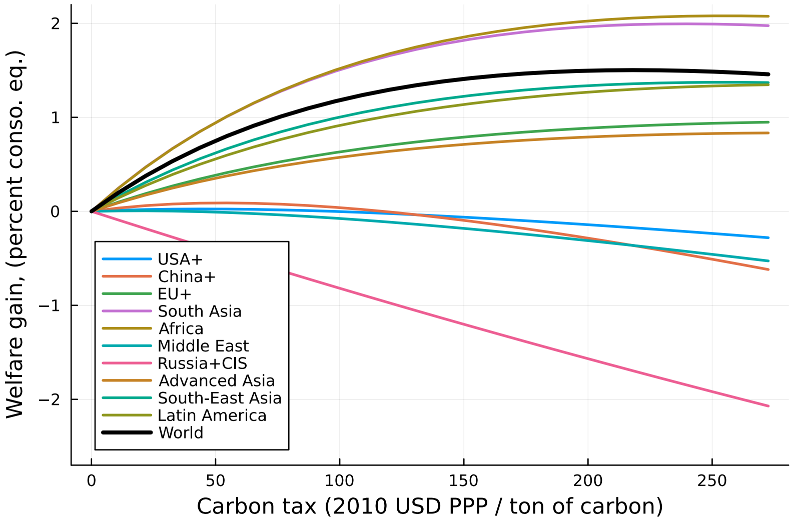


Gains from cooperation – Second Best

- Static problem : Calibration subject to changes
- Optimal carbon tax, Second Best : $\sim \$215/tC \;\; (\sim \$880/tCO_2)$
- Reduce fossil fuels / CO_2 emissions by 24% compared to Business as Usual (BAU)
- Welfare difference between World Second-Best Policy and BAU (Comp. Eq.)

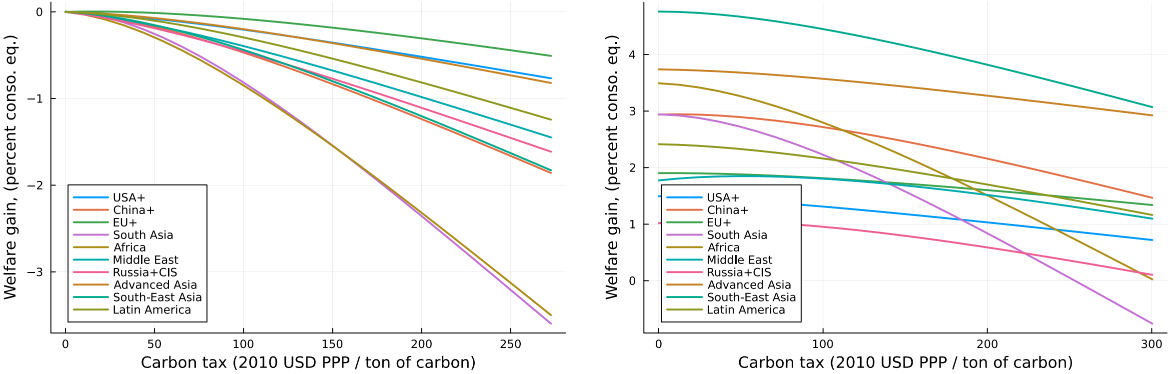


Gains from cooperation – Second Best – Tax variation



Free riding problem ... solved with trade tariffs

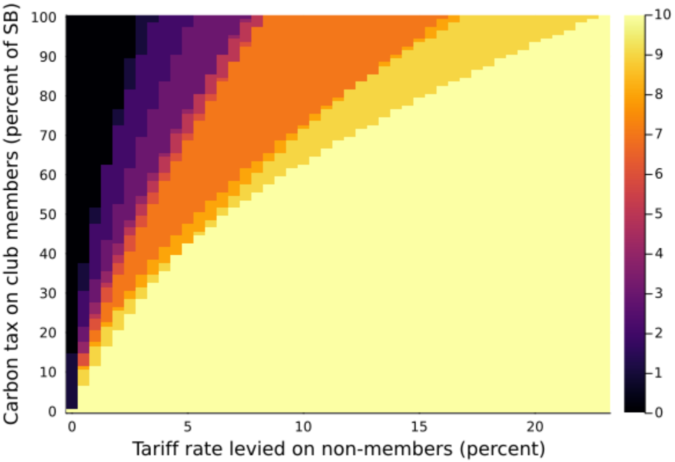
- Welfare difference between World Second-Best Policy and Unilateral deviation
  - Recover Nordhaus (2015) result



Taxes combination can recover any climate coalition

- Choice of any couples $(t^f, t^b) \in \mathbb{R}^2_+$ allow to enforce any coalitions (any number of countries)
- Trade penalties change the country's outside options, ruling out unilateral deviations

$\Rightarrow$ One can reproduce the second-best : full-cooperation, high-tax and maximum welfare



(Obvious ?) conclusion – Optimal climate agreement is the Second-Best

- With flexible t^f, t^b, the ”agreement designer” can reproduce the world’s optimal policy
  - Carbon taxation corrects externality, accounting for terms-of-trade / redistribution effects
  - Trade tariffs serve as penalties to enforce the stability of the club

(Obvious ?) conclusion – Optimal climate agreement is the Second-Best

- With flexible t^f, t^b, the "agreement designer" can reproduce the world's optimal policy
  - Carbon taxation corrects externality, accounting for terms-of-trade / redistribution effects
  - Trade tariffs serve as penalties to enforce the stability of the club
- Same mechanisms with conditional transfers :
  - Ensure stability (change one side of the participation constraint)
  - Harder to decide/rationalize : who "deserve" the funds ? the poorest ? the most vulnerable ? or the countries with the highest outside options ?
  - Coase type of arguments : harder to bargain on I-instruments (c.f. Weitzman 2014)
- Obvious conclusion ?
  - With enough instruments, easy to reach full coordination
  - In practice, coordination failure to implement binding agreements

Dynamic game & coalition formation

- Large literature on coalition formation in game theory
  - Cooperative approach : decisions are made in groups, but sub-coalition can block the group decisions $\Rightarrow$ concept of core
  - Bargaining approach : proposal and negotiation made by individual agents $\Rightarrow$ question of reversibility of agreements
- In both cases, dynamic structure of the group formation
- Objective as of now :
  - Are the incentives to join stronger as climate change becomes more severe ? $\Rightarrow$ could exploit the monotonicity of the climate system.
  - Potential increasing sequence of club members, starting with Europe, with increasing carbon tax rate

Conclusion

- In this project, I solve for the optimal climate policy
  - Accounting for inequality as it depends on the availability of transfer mechanisms
  - Redistributing through GE effects on energy and good markets $\Rightarrow$ terms-of-trade effects
- Climate agreement design jointly solves for :
  - The optimal choice of countries participating
  - The carbon tax level, both for correcting externality & respecting participation constraints
- Can reproduce any coalition with arbitrary trade tariffs or conditional transfers
  - Can achieve the second-best, world climate agreement and largest emission reductions
  - Objective to extend this to dynamic settings where the tradeoffs are less obvious/more realistic.

Appendices

Step 0 : Competitive equilibrium & Trade

- Each household in country i maximize utility and firms maximize profit
- Standard trade model results :
  - Consumption and trade :

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_i} = a_{ij}\frac{(d_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}(d_{ik}(1+\mathrm{t}^b_{ik})\mathrm{p}_k)^{1-\theta}} \qquad \& \qquad \mathbb{P}_i = \Big(\sum_j a_{ij}(d_{ij}\mathrm{p}_j)^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

  - Energy consumption doesn't internalize climate damage :

$$\mathrm{p}_i MPe_i = q^e$$

  - Inequality, as measured in local welfare units :

$$\lambda_i = u'(c_i)$$

  - "Local Social Cost of Carbon", for region i

$$LCC_i = \frac{\partial \mathcal{W}_i/\partial \mathcal{E}}{\partial \mathcal{W}_i/\partial w_i} = \frac{\psi^{\mathcal{E}}_i}{\lambda_i} = -\Delta_i \mathcal{D}'(\tau_i) z_i f(e^f_i)\frac{\mathrm{p}_i}{\mathbb{P}_i} \qquad (> 0 \text{ if heat causes losses})$$

back

Step 1 : World First-best policy

- Maximizing welfare of the world Social Planner :

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{i\in\mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - Full array of instruments : cross-countries lump-sum transfers t_i^{ls}, individual carbon taxes t_i^f on energy e_i^f, bilateral tariffs t_{ij}^b
  - Budget constraint : $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^f e_i^f + \sum_{i,j} \mathrm{t}_{ij}^b c_{ij} d_{ij} \mathrm{p}_j$

- Maximize welfare subject to
  - Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1 : World First-best policy

- Social planner results :
  - Consumption :

$$\omega_i u'(c_i) = \Big[\sum_j a_{ij} (d_{ij} \omega_j \mu_j)^{1-\theta} \Big]^{\frac{1}{1-\theta}}$$

  - Energy use :

$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

  - Social cost of carbon :

$$SCC = -\frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}'_j(\tau_j) \bar{y}_j}{\frac{1}{I} \sum_j \omega_j \mu_j}$$

back

Step 2 : World optimal Ramsey policy

- Maximizing welfare of the world Social Planner :

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - One single instrument : uniform carbon tax t^f on energy e_i^f
  - Rebate tax lump-sum to HHs $\mathrm{t}_i^{ls} = \mathrm{t}^f e_i^f$

- Ramsey policy : Primal approach, maximize welfare subject to
  - Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
  - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
  - Trade-off faced by the planner :
    - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

back

Step 2 : World optimal Ramsey policy

- The planner takes into account
  (i) the marginal value of wealth λ_i
  (ii) the shadow value of good i, from market clearing, μ_i :

$$\text{w/o trade} \qquad \omega_i u'(c_i) = \omega_i \lambda_i$$

$$\text{vs. w/ trade in goods :} \qquad \omega_i u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij} (d_{ij} \mathrm{p}_j)^{1-\theta} \Big[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij} (1 - s_{ij}) \Big]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Relative welfare weights, representing inequality

$$\text{w/o trade :} \qquad \widehat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \lessgtr 1 \qquad \Rightarrow$$

ceteris paribus, poorer countries have higher $\widehat{\lambda}_i$

$$\text{vs. w/ trade :} \qquad \widehat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)} \lessgtr 1$$

Step 2 : Optimal policy – Social Cost of Carbon

- Key objects : Local vs. Global Social Cost of Carbon :
  - Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i
  - "Local social cost of carbon" (LCC) for region i :

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(\tau_i) z_i f(e_i^f) \mathrm{p}_i \qquad (> 0 \text{ if heat causes losses})$$

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  - Social Cost of Carbon for the planner :

$$SCC := \frac{\partial \mathcal{W} / \partial \mathcal{E}}{\partial \mathcal{W} / \partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

  - Social Cost of Carbon integrates these inequalities :

$$SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, LCC_i\big)$$

Step 2 : Optimal policy – Other motives

- Taxing fossil energy has additional redistributive effects :
  1. Through energy markets : distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy
- New measure : Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \mathcal{C}^f_{EE}\, \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right) - \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\right)$$

∘ Params : $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, s^f_i energy cost share and σ energy demand elasticity

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$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \underbrace{\mathcal{C}^f_{EE}}_{\substack{\text{agg. supply}\\ \text{distortion}}} \underbrace{\mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right)}_{\substack{\text{terms-of-trade}\\ \text{redistribution}}} - \underbrace{\mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\right)}_{\substack{\text{demand}\\ \text{distortion}}}$$

∘ Params : $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, s^f_i energy cost share and σ energy demand elasticity

Step 2 : Optimal policy – Other motives

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  1. Through energy markets : distort supply, lowers eq. fossil price, benefit net importers
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∘ Params : $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, s^f_i energy cost share and σ energy demand elasticity

- *Proposition 2 :* Optimal fossil energy tax :

$$\Rightarrow \qquad t^f = SCC + SVF$$

– Social cost of carbon : $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i\, LCC_i$

back

Step 3 : Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.
- For a climate "club" of $\mathbb{J} \subset \mathbb{I}$ countries :
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^{f}
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $\mathrm{t}^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^{b} with club members and countries outside the club. They still trade with the club members in energy at price q^{f}

Step 3 : Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.
- For a climate "club" of $\mathbb{J} \subset \mathbb{I}$ countries :
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $\mathrm{t}^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f
- Participation constraints :

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

- Welfare :

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{\mathbb{J}} \omega_i \, u(c_i) + \sum_{\mathbb{J}^c} \alpha\omega_i \, u(c_i)$$

Step 3 : Ramsey Problem with participation constraints

- Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- <u>*Proposition 3.1 :*</u> Second-Best social valuation with participation constraints
  - Participation incentives change our measure of inequality

$$\text{w/ trade :} \qquad \omega_i(1+\nu_i)u'(c_i) = \Big(\sum_{j\in\mathbb{I}} a_{ij}(d_{ij}\mathrm{p}_j)^{1-\theta} \big[\omega_i\widetilde{\lambda}_i + \omega_j\widetilde{\mu}_j + \widetilde{\eta}_{ij}(1-s_{ij})\big]^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \qquad \widehat{\widehat{\lambda}}_i = \frac{\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_i(\widetilde{\lambda}_i+\widetilde{\mu}_i)} \neq \widehat{\lambda}_i$$

$$\text{vs. w/o trade} \qquad \widehat{\widehat{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_j(1+\nu_j)u'(c_j)} \neq \widehat{\lambda}_i$$

  - Similarly, the "effective Pareto weights" are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

Step 3 : Participation constraints & Optimal policy

- *Proposition 3.2 :* Second-Best taxes :
  - Taxation with imperfect instruments :
    - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
    - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu\sigma}{q^f(1-s^f)}$
      with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
  - Optimal fossil energy tax $\mathrm{t}^f(\mathbb{J})$:

$$\Rightarrow \qquad \mathrm{t}^f(\mathbb{J}) = SCC + SVF$$

$$= \frac{1}{1-\vartheta_{\mathbb{J}^c}} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i LCC_i + \frac{1}{1-\vartheta_{\mathbb{J}^c}} \mathcal{C}^f_{EE} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i (e^f_i - e^x_i) - \sum_{i\in\mathbb{J}} \widetilde{\lambda}_i \frac{q^f(1-s^f_i)}{\sigma}$$

  - Optimal tariffs/export taxes $\mathrm{t}^{b,r}(\mathbb{J})$ and $\mathrm{t}^b(\mathbb{J})$: In search for a closed-form expression
    As of now, only opaque system of equations (fixed point w/ demand/multipliers)

back

More details – Capital market

- In each countries, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} & = \mathcal{D}_i^y(\tau_{it}) z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g}) k_{it} + \iota_{it} \\ \dot{b}_{it} & = r^\star b_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathrm{t}_{it}^f) e_{it}^f - (q_t^r + \mathrm{t}_{it}^r) e_{it}^r - \iota_{it} - c_{it} + \mathrm{t}_{it}^{ls} \\ b_{it} & \geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathrm{t}_{it}^f) e_{it}^f - (q_t^r + \mathrm{t}_{it}^r) e_{it}^r - c_{it} + \mathrm{t}_{it}^{ls}$$

$$k_{it} \leq \frac{1}{1-\vartheta} w_{it}$$

- Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it}) z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r_t^\star$$

back

Impact of increase in temperature

- Marginal values of the climate variables : λ^s_{it} and λ^τ_{it}

$$\dot{\lambda}^\tau_{it} = \lambda^\tau_{it}(\rho+\zeta) + \overbrace{\gamma_i(\tau_{it}-\tau^\star_i)\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it},e_{it})\lambda^k_{it} + \overbrace{\phi_i(\tau_{it}-\tau^\star_i)\mathcal{D}^u(\tau_{it})^{1-\eta}c^{1-\eta}_{it}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}^S_{it} = \lambda^S_{it}(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{it}$$

- Costate λ^S_{it} : marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{it}-\tau^\star_i$ & damage sensitivity of TFP γ^y_i and utility γ^u_i
  - Development level $f(k_{it},e_{it})$ and c_{it}
  - Climate params : χ climate sensitivity, Δ_i "catching up" of τ_i and ζ reaction speed
  - back

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}^{\tau}_{it} = \lambda^{\tau}_t(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^{\star})\mathcal{D}^y(\tau)f(k,e)\lambda^k_t + \phi(\tau - \tau^{\star})\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}^{S}_t = \lambda^{S}{}_t(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda^{\tau}_{it}$$

- Solving for λ^{τ}_t and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^S_t = \dot{\lambda}^{\tau}_t = 0$

$$\lambda^{\tau}_{it} = -\int_t^{\infty} e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^{\star})\Big(\gamma\mathcal{D}^y(\tau_u)y_{\tau}\lambda^k_u + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda^{\tau}_{it} = -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_{\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^y(\tau_{\infty})y_{\infty}\lambda^k_{\infty} + \phi\mathcal{D}^u(\tau_{\infty})u(c_{\infty})\Big)$$

$$\begin{aligned}
\lambda^S_t &= -\int_t^{\infty} e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\tau}_{j,u}dj\,du \\
&= \frac{1}{\widetilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\tau}_{j,\infty} \\
&= -\frac{\chi}{\widetilde{\rho}+\delta^s}\frac{\zeta}{\widetilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{\infty}\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

$$\lambda^S_t \xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj$$

Cost of carbon / Marginal value of temperature

- ***Proposition (Stationary LSCC)*** :
  When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ_i^y, γ_i^u, temperature, and output, consumption.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s}(\tau_\infty - \tau^\star)\Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty\Big)$$

  - Stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - Back

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega \int_{t_0}^{T} e^{(n+\bar{g})t-\delta_s(T-t)} q_t^{f\;-\sigma_e} \int_{j\in\mathbb{I}} \left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e-\sigma} dj\, dt$$

  - Geographical factors determining warming Δ_i
  - Climate sensitivity χ & carbon exit from atmosphere δ_s
  - Growth of population n, aggregate productivity $\bar{g}$
  - Deviation of output from trend y_i & relative TFP z_j
  - Directed technical change z_t^e, elasticity of energy in output σ
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}_t^R/\lambda_t^R = \rho$
  - Change in energy mix, renewable share ω, price q_t^r & elasticity of source σ_e
- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \tilde{\gamma}\right) + (\sigma_e - \sigma)(1-\omega)g^{q^r} - (\sigma_e(1-\omega) + \sigma\omega)g^{q^f}$$

Back

Equilibrium – Mean Field Games

- Uniqueness : Lasry-Lions monotonicity on the function :

$$\int_{\mathbb{I}} [f(t, x, \mu', \alpha) - f(t, x, \mu, \alpha)] d(\mu' - \mu)(x) \geq 0 \qquad \mu, \mu' \in \mathcal{P}(\mathbb{X})$$

  - Work in progress : checking such conditions along the transition

$$\sum_{i \in \mathbb{I}} \Big(u(c^\star(w, \tau, p')) - u(c^\star(w, \tau, p)) \Big) [p'(w, \tau) - p'(w, \tau)] \geq 0$$

  with $p'(w, \tau)$ empirical distribution $p'(w, \tau) = \frac{1}{\mathbb{I}} \sum_{i \in \mathbb{I}} \delta_{\{x_i = (w, \tau)\}} \equiv$ population distribution !

- Mean Field approximation & Carmona Delarue (2013)
  - Mean-Field is an ε-equilibrium of the N-player game when $N \to \infty$
  - Require symmetry and invariance under permutation
  - Back

Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness More details

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  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
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  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

- Why use a sequential approach ?
  - *Global approach* : *Only* need to follow the trajectories for i agents :
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
    Productivity z_i Population p_i, Temperature scaling Δ_i, Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$, Local damage $\gamma_i^y, \gamma_i^u, \tau_i^\star$, Directed Technical Change z_i^e
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
    For now : Wealth w_{it}, temperature τ_{it}, reserves $\mathcal{R}_{it}$, Carbon $\mathcal{S}_t$
    Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
  - Newton method & Non-linear solvers very efficient
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back

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \ell, \varepsilon(e^f, e^r)) = \left[(1-\epsilon_i)^{\frac{1}{\sigma_y}} \left(k^\alpha \ell^{1-\alpha}\right)^{\frac{\sigma_y - 1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} \left(z_i^e \; \varepsilon_i(e^f, e^r)\right)^{\frac{\sigma_y - 1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega_i^{\frac{1}{\sigma_e}} \left(e^f\right)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\omega_i)^{\frac{1}{\sigma_e}} \left(e^r\right)^{\frac{\sigma_e - 1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

  - Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
  - Today : $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
  - Future : $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

- Damage functions in production function y :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm,y}(\tau - \tau_i^\star)^2}$$

  - Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y} \mathbb{1}_{\{\tau > \tau_i^\star\}} + \gamma^{-,y} \mathbb{1}_{\{\tau < \tau_i^\star\}}$
  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $\tau_i^\star = \bar{\alpha}\tau_{it_0} + (1-\bar{\alpha})\tau^\star$

Quantification – Energy markets

- Fossil production e^x_{it} and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now : $\bar{\nu}_i$ to match extraction data e^x_i, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. $\nu_i = \nu = 1$ quadratic extraction cost.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e^x_i (BP, IEA)

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- Renewable : Production $\bar{e}^r_{it}$ and price q^r_{it}
  - Now : $q^r_{it} = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
  - Future : Choose z^r_i to match the energy mix (e^f_i, e^r_i)

back

Quantification – Future Extensions :

- Damage parameters :
  - $\gamma_i^{\pm,y}$ depends on daily temperature distribution $\tau \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
  - Use Climate Lab's (Greenstone et al) estimates for damage γ_i ?
- Fossil Energy markets :
  - Divide fossils e_{it}^f / e_{it}^x into oil/gas/coal
  - Match the production/cost/reserves data across countries
  - Use a dynamic model : extraction/exploration a la Hotelling
- Renewables Energy markets :
  - Make the problem dynamic with investment in capacity C_{it}^r
- Population dynamics
  - Match UN forecast for growth rate / fertility

Calibration

TABLE – Baseline calibration ($\star$ = subject to future changes)

Technology & Energy markets			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
g_e	$0.01^\star$	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	$-0.01^\star$	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	$2^\star$	Extraction elasticity of fossil energy	Cubic extraction cost
Preferences & Time horizon			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	$0.01^\star$	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

TABLE – Baseline calibration ($\star$ = subject to future changes)

Climate parameters			
ξ	0.81	Emission factor	Conversion 1 $MTOE \Rightarrow 1\ MT\ CO_2$
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature $\sim 11-15$ years
χ	$2.1/1e6$	Climate sensitivity	Pulse experiment : 100 $GtC \equiv 0.21^\circ C$ medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment : 100 $GtC \equiv 0.16^\circ C$ long-term warming
$\gamma^\oplus$	$0.00234^\star$	Damage sensitivity	Nordhaus' DICE
$\gamma^\ominus$	$0.2 \times \gamma^{\oplus\ \star}$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
$\alpha^\mathcal{T}$	$0.2^\star$	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
$\tau^\star$	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
Parameters calibrated to match data			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
τ_i		Local Temperature	To match temperature of largest city
$\mathcal{R}_i$		Local Fossil reserves	To match data from BP Energy review

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