The Inequality of Climate Change

WORK IN PROGRESS

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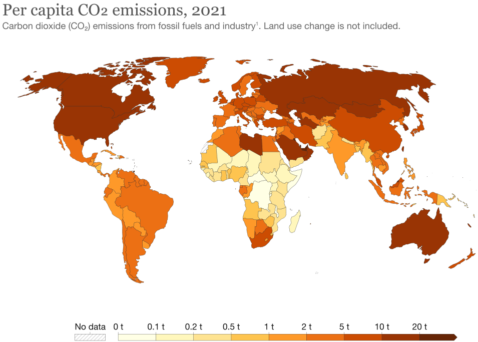
EPIC lunch

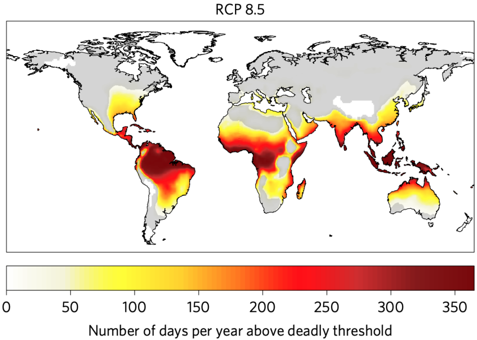
May 2023

Introduction – Motivation

Global warming is caused by greenhouse gas emissions (GHG) generated by human economic activity :

- ***Unequal causes :*** Developed economies account for over 65% of cumulative GHG emissions ($\sim$ 25% each for the EU and the US)
- ***Unequal consequences :*** Increase in temperatures disproportionately affects developing countries where the climate is already warm





Introduction – this project

- Marginal damages of climate & temperature varies across countries
- What is the optimal taxation of energy in the presence of climate externality *and* inequality ?
  - In a context, where fossil fuels taxation and climate policy redistributes across countries

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  - Standard IAM model with heterogeneous regions
  - Normative implications : Ramsey policy + possibility to study uncertainty

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- Develop a simple and flexible model of climate economics
  - Standard IAM model with heterogeneous regions
  - Normative implications : Ramsey policy + possibility to study uncertainty
- ○ Evaluate the heterogeneous welfare costs of global warming
- ○ Provide analytical formulas and a numerical methodology to compute the cost of carbon
  - Heterogeneity increases the welfare cost of carbon
- ○ Solve world optimal carbon policy with heterogeneous regions
  - Does the optimal carbon tax coincide with the social cost of carbon ?

  $\Rightarrow$ Maybe not, depending on transfer policy : need to adjust for inequality level
  - What are the welfare gains of suboptimal policies ?

Toy model

- Consider two countries $i = N, S$, (North/South)
  - HH consuming good c_i and producing with energy e_i and productivity z_i
  - Energy producer with profit $\pi(E)$ owned by country i with share θ_i
- Household problem :

$$\mathcal{V}_i = \max_{c_i, e_i} U(c_i)$$

$$c_i + q^e e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) \qquad [\lambda_i^k]$$

- Subject to damage $\mathcal{D}_i(\mathcal{S})$ and climate externalities :

$$\mathcal{S} = \mathcal{S}_0 + \overbrace{\xi_N e_N + \xi_S e_S}^{=\text{GHG emissions}}$$

- And consuming energy in a single energy market with price q^e

$$E = e_N + e_S \qquad q^e = c'(E) \qquad \pi(E) = q^e E - c(E)$$

Toy model – Competitive equilibrium

- Three dimensions of heterogeneity :
  1. Different levels of productivity z_i : $z_N > z_S$
  2. Different climate damage $\gamma_i = -\frac{\mathcal{D}_i'(\mathcal{S})}{\mathcal{S}\mathcal{D}_i(\mathcal{S})}$, $\gamma_S > \gamma_N$
  3. Different energy rent θ_i : $\theta_N > \theta_S$

  $\Rightarrow$ Yields heterogeneity in consumption $c_N > c_S$

- Competitive equilibrium Result :
  - Marginal Product of Energy $=$ Energy Cost

$$MPe_i = q^e = c'(E) \qquad with \qquad MPe_i := \mathcal{D}_i(\mathcal{S}) z_i F'(e_i)$$

  - Inequality

$$\lambda_i^k = U'(c_i) \qquad\qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) - q^e e_i$$

Toy model – First Best and Decentralization

- Comparison with Social planner with full transfers (First Best)

$$\mathbb{W} = \max_{\{c_i,e_i\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$\sum_{i=N,S} c_i + c(E) = \sum_{i=N,S} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad [\lambda]$$

$$\mathcal{S} := \mathcal{S}_0 + \xi_N e_N + \xi_S e_S \qquad E := e_N + e_S$$

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  - Marginal Product of Energy = Energy Cost + Social Cost of Carbon

$$MPe_i = c'(E) + \xi_i \underbrace{\overline{SCC}}_{=\mathbf{t}^e} \qquad with \qquad \overline{SCC} := -\sum_{i=N,S} \mathcal{D}'_i(\mathcal{S}) z_i F(e_i)$$

  - Redistribution

$$\omega_S U'(c_S) = \omega_N U'(c_N) = \lambda$$

  - Decentralization, needs to redistribute with *with lump-sum transfers* $T_S = -T_N$

$$\Rightarrow \qquad c_i + (q^e + \mathbf{t}^e) e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) + T_i$$

  - Are lump-sum transfers feasible ?

Toy model – Second Best - Ramsey Problem

- Assume now that *lump-sum transfers across countries* are prohibited
  - Allow for carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$

$$\mathcal{W} = \max_{\{c_i,e_i\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$s.t \qquad c_i + (q^e + \mathbf{t}_i^e) e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) + T^i \qquad [\phi_i]$$

$$\mathcal{S} := \mathcal{S}_0 + \xi_N e_N + \xi_S e_S \qquad E := e_N + e_S \qquad q^e = c'(E)$$

  - Ramsey policy result :
    - Planner's marginal value of wealth

$$\phi_i = \omega_i U'(c_i)$$

    - Energy decision :

$$\phi_i \left[MPe_i - c'(E) \right] + \xi_i \underbrace{\sum_j \phi_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j)}_{\propto -\text{SCC}} + \underbrace{\pi'(E) \sum_j \phi_j \theta_j}_{= \text{rent redistribution}} \underbrace{- c''(E) \sum_j \phi_j^k e_j}_{= \text{cost redistribution}} = 0$$

Social Cost of Carbon with inequality

- Measure of inequality

$$\widehat{\phi}_i = \frac{\phi_i}{\overline{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{2}\sum_j \omega_j U'(c_j)} \lesseqgtr 1 \qquad \overline{\phi} = \frac{1}{2}\big(\omega_N U'(c_N) + \omega_S U'(c_S)\big)$$

- The SCC is exacerbated by heterogeneity

$$SCC = -\sum_j \widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) y_j$$
$$= -\mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}_j'(\mathcal{S}) y_j\right) - \mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) y_j] > -\mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) y_j] = \overline{SCC}$$

  - Why ? Low-income countries tend to be warmer/more vulnerable to climate change

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$$\begin{aligned} SCC &= -\sum_j \widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) y_j \\ &= -\mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}_j'(\mathcal{S}) y_j\Big) - \mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) y_j] > -\mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) y_j] = \overline{SCC} \end{aligned}$$

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- For the social value of rent (exporters) and energy cost (importer), it's the contrary !

$$\begin{aligned} SVR &= \mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \theta_j \pi_j'(E)\Big) + \pi'(E) < \pi'(E) \\ SCE &= \mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, e_j\, c''(E)\Big) + c''(E) < c''(E) = \pi'(E) \end{aligned}$$

Optimal energy policy

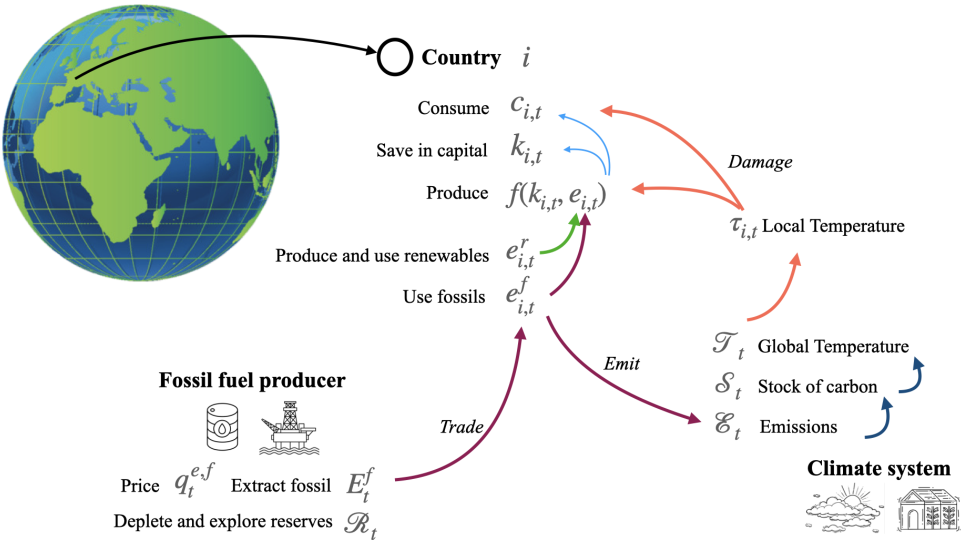
- Energy taxation :

$$MPe_i = c'(E) + \xi_i \mathbf{t}_i^e$$

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[SCC - SVR + SCE \right]$$

- Four motives with a single tax and lump-sum rebate
  - Distribution : Tax is higher for poorer countries $\omega_S U'(c_S) > \omega_N U'(c_N) \Rightarrow \mathbf{t}_S^e > \mathbf{t}_N^e$
  - Optimal tax level : Depends on
    - Distribution of climate damage in SCC
    - Distribution of energy rent in SVR
    - Distribution of energy spending in SCE

Summary of the quantitative dynamic model



Summary of the Model Environment

1. Households in individual countries $i \in \mathbb{I}$ consuming c_{it} Household/Firms HH Solution
   - Markets : borrow/save on world bond markets b_{it} / invest in productive capital k_{it}
   - Energy spending : fossil energy e^f_{it} and renewable e^r_{it}
   - Taxation, fossil, $\mathbf{t}^f_{it}$ and renewable $\mathbf{t}^r_{it}$
2. Energy markets Energy
   - Representative (Competitive) Fossil Fuel producer making profit $\pi^f_t(q^f_t, E^f_t, \mathcal{R}_t)$
   - Extended Hotelling problem : Extraction E^f_t vs. Exploration $\mathcal{I}_t$
   - Redistribute share θ_i to household of country i
   - Renewables with price q^r_t
3. Climate system Climate
   - Linear dynamics : emissions $\mathcal{E}_t$ to atm. carbon $\mathcal{S}_t$ to temperature $\mathcal{T}$, cf Dietz Venmans (19)
   - Damage function on TFP (c.f. DICE) $\mathcal{D}_i(\tau_{it})$ and utility $u(c_{it}, \tau_{it}) = U\big(\mathcal{D}_i(\tau_{it})c_{it}\big)$, U CRRA

- Heterogeneity :

  1. Productivity z_i
  2. Population p_i
  3. Temperature scaling Δ_i
  4. Fossil energy rent θ_i
  5. Carbon intensity of energy mix ξ_i
  6. Local damage γ_i
  7. Capital stock k_{it}
  8. Local temperature τ_{it}

  $\Rightarrow$ Yield inequality in consumption c_{it}

Model – Solution

- Main model equations and equilibrium Equilibrium
    1. Household problem $\mathcal{V}_i(w_{it_0}, \tau_{it_0}) = \max_{\{c,k,e^f,e^r\}} \int e^{-\rho t} u(c_{it}, \tau_{it}) dt$

$$\dot{w}_{it} = r_t^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{it}^{ls}$$

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  2. Energy Markets

$$E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \begin{aligned} \dot{\mathcal{R}}_t &= -E_t^f + \delta_R \mathcal{I}_t \\ \pi_t^f &= q_t^{e,f} E_t^f - \nu(E_t^f, \mathcal{R}_t) - \mu(\mathcal{I}_t, \mathcal{R}_t) \end{aligned}$$

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  3. Climate system

$$\mathcal{E}_t = \int_{\mathbb{I}} \xi_i \, e_{it}^f \, di \qquad \dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t \qquad \dot{\tau}_{it} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \bar{\tau}_{it_0})\big)$$

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  4. Household Decisions, consumption/saving, and energy

$$\dot{c}_{it} = c_{it} \frac{1}{\eta}(r_t^\star - \rho) \qquad MPk_{it} - \bar{\delta} = r_t^\star \qquad MPe_{it} = \mathcal{Q}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r)$$

$$\Rightarrow \qquad e_{it}^f = \mathcal{Q}_{q^f}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r) e_{it} \qquad e_{it}^r = \mathcal{Q}_{q^r}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r) e_{it}$$

Optimal policy

- Social planner, First best with a full set of instruments :
  - Solves world's inequality, using lump-sum transfers such that $\lambda_t = \omega_i u'(c_{it}) = \omega_j u'(c_{jt})\ \forall i, j \in \mathbb{I}$
  - Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^S}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)
  - Imply cross-countries lump-sum transfers

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- Second best / Ramsey planner :
  - Doesn't have access to redistribution / lump-sum transfers
  - Can only use region-i-specific distortive energy taxes : $\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r\}$
  - Redistribute lump sum the tax revenues : $\mathbf{t}_{it}^{ls} = \mathbf{t}_{it}^f e_{it}^f + \mathbf{t}_{it}^r e_{it}^r$

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- Questions :
  - Is the level of energy tax regions specific ?
    - $\Rightarrow$ Yes ! depends on the distribution of wealth/consumption
  - What is the level of the Pigouvian tax ?
    - $\Rightarrow$ $\propto$ Welfare cost/climate damage : "social costate" for carbon $\mathcal{S}$, i.e. ψ^S
    - $\Rightarrow$ Inequality/Heterogeneity in damage change the *level* of this tax

The Ramsey Problem

- Consider a Social Planner that care about aggregate welfare :

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, c_{it}, e^f_{it}, e^r_{it}, k_{it}, \lambda^k_{it}, \tau_{it}, \mathcal{S}_t, \mathcal{R}_t, \mathcal{I}_t, \lambda^{\mathcal{R}}_t\}_{it}} \int_{t_0}^{\infty} \int_{\mathbb{I}} e^{-\bar{\rho}t}\, \omega_i\, u(c_{it}, \tau_{it})\, di\, dt$$

subject to

- Optimality conditions of households, for c_i, e^f_i, e^r_i and k_i
- Optimality conditions of the Fossil firm, for E^f, $\mathcal{I}$ and $\mathcal{R}$
- Optimality condition of the renewable firm, for e^r_i
- Climate and temperature dynamics τ_i and $\mathcal{S}$
- Given Pareto weights ω_i

$\Rightarrow$ Large scale system of ODE More details - Hamiltonian

- A Ramsey plan is a set $\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, \mathbf{t}^{ls}_{it}\}_{it}$ s.t. the competitive equilibrium is maximizing welfare
- States $\{w_{it}, \tau_{it}, c_{it}, k_{it}, e^f_{it}, e^r_{it}, E^f_t, \mathcal{E}_t, \mathcal{S}_t, \mathcal{R}_t\}$
- Costates $\{\psi^w_{it}, \psi^\tau_{it}, \psi^{\mathcal{S}}_{it}, \psi^{\mathcal{R}}_t\}$

The Ramsey Problem – Solution 1

- Measure of inequality : given by the shadow value of wealth

$$\widehat{\psi}^w_{it} = \frac{\psi^w_{it}}{\overline{\psi}^w_t} = \frac{\omega_i u_c(c_i, \tau_{it}) p_i}{\int_{j \in \mathbb{I}} \omega_j u_c(c_{jt}, \tau_{jt}) p_j dj} \lesseqgtr 1 \qquad \text{low } z_i, k_i/\text{high } \tau_{it} \;\Rightarrow\; \text{low } c_i, \text{high } \psi^w_{it} \propto \omega_i u'(c_i) p_i$$

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- SCC : Shadow values of carbon $\psi^{\mathcal{S}}_{it}$ give measures for local/global cost of carbon SCC

$$\psi^{\mathcal{S}}_t = \frac{\partial \mathcal{W}_t}{\partial \mathcal{S}_t} = \int_{j\in\mathbb{I}} \psi^{\mathcal{S}}_{jt} dj \qquad\qquad LSCC^{sp}_{it} := -\frac{\psi^{\mathcal{S}}_{it}}{\psi^w_{it}}$$

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- Expression for the social cost of carbon SCC_t

$$SCC_t = -\frac{\psi^{\mathcal{S}}_t}{\overline{\psi}^w_t} = \mathbb{C}\text{ov}_j\left(\widehat{\psi}^w_{it}, LSCC^{sp}_{j,t}\right) + \mathbb{E}_j[LSCC^{sp}_{j,t}] > \mathbb{E}_j[LSCC^{sp}_{j,t}] = \overline{SCC}_t$$

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- SVF : Energy taxation redistribution

$$SVF_t = \frac{\phi^{Ef}_t}{\overline{\psi}^k_t} = \int_{\mathbb{I}} \widehat{\psi}^k_{jt} e^f_{jt} dj - \partial_{q^f} \pi^f \int_{\mathbb{I}} \widehat{\psi}^k_{jt} \theta_j dj = \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}^k_{jt}, e^f_{jt}\right) - E^f_t\, \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}^k_{jt}, \theta^f_{jt}\right)$$

The Ramsey Problem – Solution 2

- Optimal policy for fossil energy, FOC of Ramsey planner :

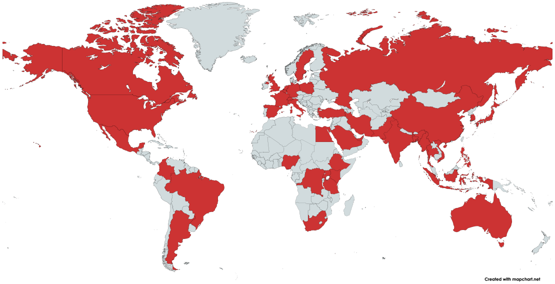
$$\Rightarrow \quad \widehat{\psi}^{w}_{it}\, \mathbf{t}^{f}_{it} = \xi_i\, SCC_t + SVF_t\, \mathcal{C}^{f}_{EE} \qquad \& \qquad \mathbf{t}^{r}_{it} = 0$$

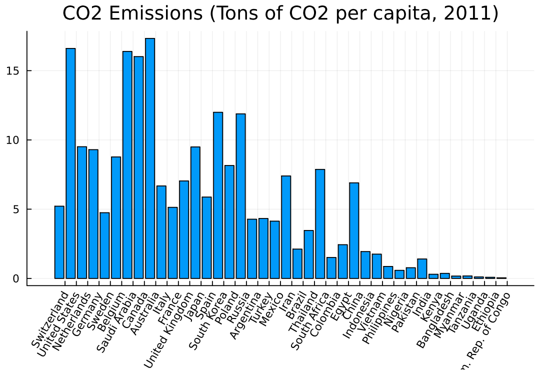
- Pigouvian tax :
  - Integrate several redistribution motives : Climate SCC_t & fossil price redistribution SVF
  - ***Depends*** on country's consumption level $\widehat{\psi}^{w}_{it}$: lower tax on poorer/high $\widehat{\psi}^{w}_{it}$ countries

- Welfare costs of suboptimal taxes :
  - Depends on the curvature of the production function $f(k,e)$ and substitution e^{f}_{it} and e^{r}_{it}

Numerical Application

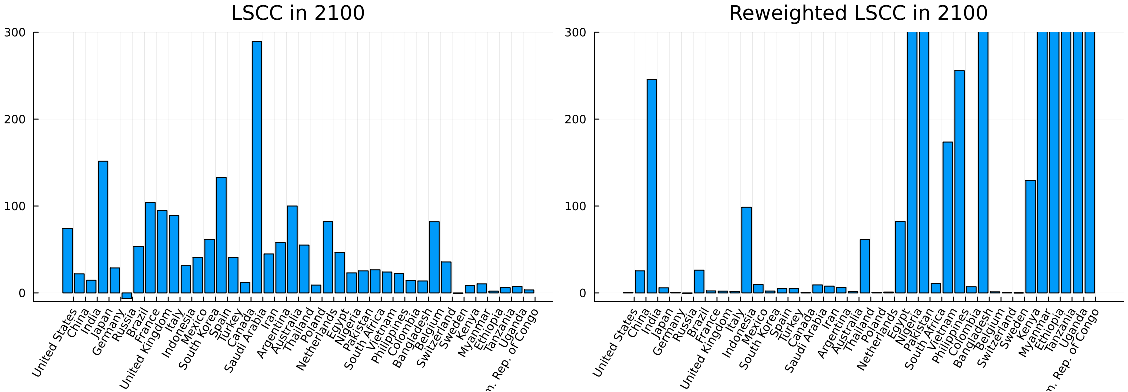
- Data : 40 countries
- Temperature (of the *largest city*), GDP, energy, population
- Calibrate z to match the distribution of output per capita at steady state





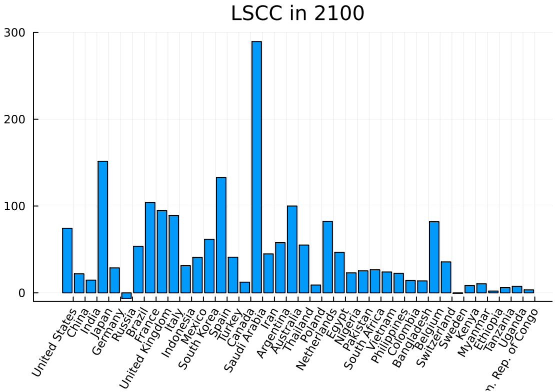
Local Cost of Carbon

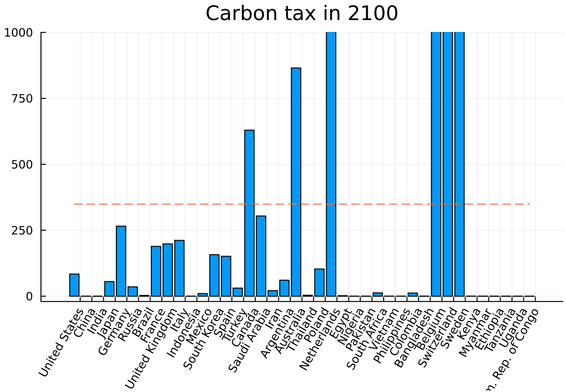
- Difference $LSCC_i = \psi^{\mathcal{S}}_{it} / \psi^{w}_{it}$ and $LWCC_{it} = \widehat{\psi}^{w}_{it} LSCC_{it} = \psi^{\mathcal{S}}_{it} / \bar{\psi}^{k}_{t}$



Social Cost of Carbon and Carbon Tax

- Difference $LSCC_i = \psi_{it}^{S} / \psi_{it}^{w}$ and $\mathbf{t}_{it} = (1/\widehat{\psi}_{it}^{w})SCC$





Conclusion

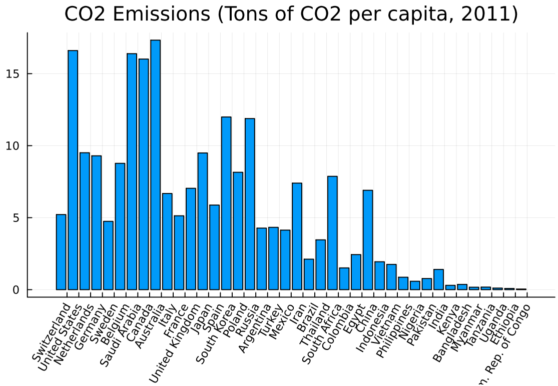
- Climate change has redistributive effects & heterogeneous impacts
- Redistributive effects of policy
  - Pigouvian tax that covers aggregate marginal damages
  - Can account for inequality both for heterogeneous welfare costs of climate and redistributive effects of energy price, for importers and exporters
- Study suboptimal policies
  - If carbon taxes are unfeasible : renewable subsidy ?
- Future plans
  - Dynamics on the capacity of renewable ?
  - Endogenous growth in TFP/energy saving technology
    Learning-by-doing : positive externality ?
  - Uncertainty : HA model with aggregate risk hard to solve

Appendices

Numerical Applications

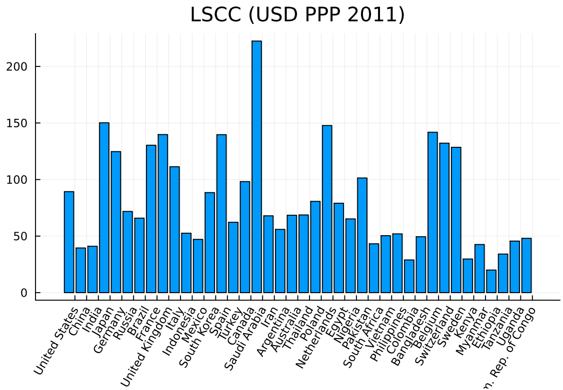
Numerical Application

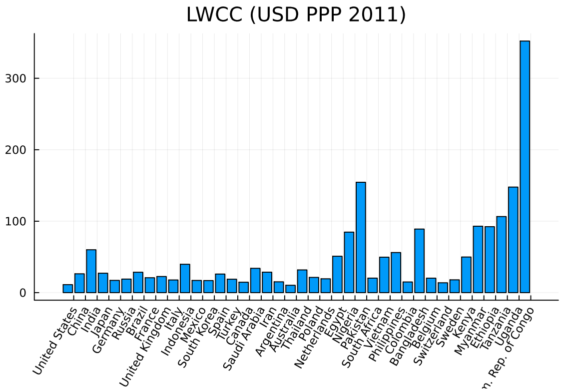
- Data : 40 countries
- Temperature (of the *largest city*), GDP, energy, population
- Calibrate z to match the distribution of output per capita at steady state



Local Cost of Carbon

- Difference $LSCC_i = \psi_{it}^{S} / \psi_{it}^{k}$ and $LWCC_{it} = \widehat{\psi}_{it}^{k} LSCC_{it} = \psi_{it}^{S} / \bar{\psi}_{it}^{k}$





Model 1

- Neoclassical economy, in continuous time Back
  - countries/regions $i \in \mathbb{I}$: ex-ante heterogeneous : productivity z_i and more
  - ex-post heterogeneity in capital and temperature $\{k_i, \tau_i\}$
- Representative household problem in each country i :

$$\mathcal{V}_{it_0} = \max_{\{c_{it}, e^f_{it}, e^r_{it}\}} \int_{t_0}^{\infty} e^{-\rho t}\, u(c_{it}, \tau_{it}) dt$$

- Dynamics of wealth of country i, More details with wealth $w_{it} = b_{it} + k_{it}$:

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r^\star_t) k_{it} + \theta_i \pi^f_t + \pi^r_{it} - (q^f_t + \mathbf{t}^f_{it}) e^f_{it} - (q^r_t + \mathbf{t}^r_{it}) e^r_{it} - c_{it} + \mathbf{t}^{ls}_{it}$$

  - Damage $\mathcal{D}^y(\tau_{it})$ affect country's production and consumption $u(\cdot, \tau_{it})$
  - Energy mix : $e_{it} = \mathscr{E}(e^f_{it}, e^r_{it} | \sigma_e)$ with fossil e^f_{it} – emitting carbon – vs. renewable e^r_t
  - Energy rents redistributed : share θ_i for fossils / fully for local renew. firm.
  - Prices, fossil q^f_t and non-carbon q^r_t (c.f. next slides)

Model 2 – Energy markets

- Fossil fuels energy producer :
  - Extended-Hotelling problem (depleting reserves with stock effects and exploration)

$$\max_{\{E_t^f, \mathcal{I}_t\}_t} \int_0^\infty e^{-\rho t} \pi_t^f(q_t^f, E_t^f, \mathcal{R}_t) dt \qquad \text{with } \pi_t(E_t^f, \mathcal{R}_t) = q_t^{e,f} E_t^f - \mathcal{C}^f(E_t^f, \mathcal{R}_t) - \mathcal{C}^i(\mathcal{I}_t, \mathcal{R}_t)$$

$$s.t. \qquad E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta_R \mathcal{I}_t$$

  - Optimal pricing with finite-resources rents More details

$$q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \mathcal{C}_{\mathcal{I}}^i(\mathcal{I}_t, \mathcal{R}_t) = \delta_R \lambda_t^R$$

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- Renewable energy as a substitute technology *for each country* i (Static problem)

$$\pi_{it}^r = \max_{\{e_{it}^r\}} q_{it}^r e_{it}^r - \mathcal{C}^r(e_{it}^r) \qquad \Rightarrow \qquad q_{it}^r = \mathcal{C}_E^r(e_t^r)$$

Back

Model 3 - Climate model :

- Fossil energy input e^f_t causes climate externality

$$\mathcal{E}_t = \int_{\mathbb{I}} \xi_i \, e^f_{it} \, di$$

- World climate – cumulative GHG in atmosphere $\mathcal{S}_t$ leads to increase in temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- Impact of climate on country's local temperature :

$$\dot{\tau}_{it} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \bar{\tau}_{it_0})\big)$$

  - Simple model : Climate sensitivity to carbon χ, Climate reaction/inertia ζ, Carbon content of fossils ξ, Country scaling factor Δ_i, Carbon exit for atmosphere δ_s
  - Back

Model 4 – Household Solution

- Household solves a consumption/saving/energy decision, as in the NCG More details
  - Using Pontryagin (PMP), we obtain a system of coupled ODEs More details

Model 4 – Household Solution

- Household solves a consumption/saving/energy decision, as in the NCG More details
  - Using Pontryagin (PMP), we obtain a system of coupled ODEs More details
  - Consumption/Saving Euler equation (financial integration) :

$$\dot{c}_{it} = c_{it}\frac{1}{\eta}(r_t^\star - \rho) \qquad\qquad MPk_{it} - \bar{\delta} = r_t^\star$$

  - Energy decisions :
    Static demand for the two sources of energy : fossil e_{it}^f and renewable e_{it}^r for every i, taking prices $\{q^f, q^r\}$ as given

$$\begin{aligned} & MPe_{it} = \mathcal{Q}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r) \\ \Rightarrow \qquad & e_{it}^f = \mathcal{Q}_{q^f}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r)e_{it} \qquad e_{it}^r = \mathcal{Q}_{q^r}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r)e_{it} \end{aligned}$$

  - with $MPk_{it} = \mathcal{D}(\tau_{it})z_i f_{k,it}$ and $MPe_{it} = \mathcal{D}^y(\tau_{it})z_i f_e(k_{it}, e_{it})$, and $\mathcal{Q}(\cdot)$ are aggregators functions (e.g. CES) and $\mathcal{Q}_{q^f}(\cdot)$ demand for fossil.

Model – Equilibrium

- Three types of interactions Equilibrium
  - On climate (externality) + heterogeneous effects of temperatures
  - On bonds markets + capital constraints
  - On energy market + redistribution effects of energy rent
  - No bilateral flows (eq. doesn't exist with continuum and trade or migration)

Model – Equilibrium

- Three types of interactions Equilibrium
  - On climate (externality) + heterogeneous effects of temperatures
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  - No bilateral flows (eq. doesn't exist with continuum and trade or migration)
- Equilibrium
  - Given, initial conditions $\{k_0, \tau_0\}$ and country-specific policies $\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, \mathbf{t}^{ls}_{it}\}$, a competitive equilibrium is a continuum of sequences of states $\{k_{it}, \tau_{it}\}_{it}$ and $\{\mathcal{S}_t, \mathcal{T}_t, \mathcal{R}_t\}_t$ and policies $\{c_{it}, e^f_{it}, e^r_{it}\}_{it}$ and $\{E^f_t, \mathcal{E}_t, \mathcal{I}_t\}_t$, and price sequences $\{q^f_t, q^r_t\}$ such that :

  - Households choose policies $\{c_{it}, e^f_{it}, e^r_{it}\}_{it}$ to max utility s.t. budget constraint, giving $\dot{k}_{it}$
  - Renewable energy firm produce $\{e^r_{it}\}$ to max static profit
  - Fossil fuel firm extract and explore $\{E^f_t, \mathcal{I}_t\}$ to max profit, yielding $\dot{\mathcal{R}}_t$
  - Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{it}\}_{it}$.
  - Prices $\{q^f_t, q^r_{it}, r^\star_t\}$ adjust to clear the markets : $E^f_t = \int_{\mathbb{I}} e^f_{it} di$ and $e^r_{it} = e^r_{it}$, and $\int_{i\in\mathbb{I}} b_{it} di = 0$

Impact of increase in temperature

- Using Damage fct $\mathcal{D}^y(\tau_{it}) = e^{-\frac{1}{2}\gamma_i(\tau_{it}-\tau_i^\star)^2}$ and $u(c,\tau) = u(\mathcal{D}^u(\tau_{it})c)$, w/ $u(\hat{c}) = \frac{c^{1-\eta}}{1-\eta}$
- Marginal values of the climate variables : λ_{it}^S and λ_{it}^τ

$$\dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho+\zeta) + \overbrace{\gamma_i(\tau_{it}-\tau_i^\star)\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it},e_{it})\lambda_{it}^k + \overbrace{\phi_i(\tau_{it}-\tau_i^\star)\mathcal{D}^u(\tau_{it})^{1-\eta}c_{it}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}_{it}^S = \lambda_{it}^S(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda_{it}^\tau$$

- Costate λ_{it}^S : marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{it}-\tau_i^\star$ & damage sensitivity of TFP γ_i and utility ϕ_i
  - Development level $f(k_{it},e_{it})$ and c_{it}
  - Climate params : χ climate sensitivity, Δ_i "catching up" of τ_i and ζ reaction speed
  - back

Local Social cost of carbon

- The marginal "externality damage" or "local social cost of carbon" (SCC) for region i :

$$LSCC_{it} := -\frac{\partial \mathcal{V}_{it}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{it}/\partial c_{it}} = -\frac{\lambda^S_{it}}{\lambda^k_{it}}$$

  - Ratio of marg. cost of carbon vs. the marg. value of consumption/capital
  - Theorem : ***Stationary LSCC*** :
    When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ, ϕ, temperature, and output, consumption.

$$LSCC_{it} \equiv \frac{\chi\, \Delta_i}{\tilde{\rho} + \delta^s} \left(\tau_{i,\infty} - \tau_i^\star\right)\left[\gamma_i\, y_{i,\infty} + \phi_i\, c_{i,\infty}\right]$$

    – More general formula : Here, Proof : Here + What determine temperatures ? Details Temperature

More details – Capital market

- In each countries, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} & = \mathcal{D}_i^y(\tau_{it}) z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g}) k_{it} + \iota_{it} \\ \dot{b}_{it} & = r^\star b_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - \iota_{it} - c_{it} + \mathbf{t}_{it}^{ls} \\ b_{it} & \geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{i}$$

$$k_{it} \leq \frac{1}{1 - \vartheta} w_{it}$$

- Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it}) z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r_t^\star$$

back

More details – Energy market

- Fossil fuel producer : price the Hotelling rent with the maximum principle :

$$\mathcal{H}^m(\mathcal{R}_t, \lambda_t^R, E_t, \mathcal{I}_t^e) = \pi_t(E_t^f, \mathcal{I}_t^f, \mathcal{R}_t) + \lambda_t^R(\delta^R \mathcal{I}_t^e - E_t)$$

- Rent λ_t^R grows with interest ρ and with the marginal gain of increasing reserves

$$\begin{aligned}\dot{\lambda}_t^R &= \rho\lambda_t^R - \partial_R \mathcal{C}(E_t^f, \mathcal{I}_t^f, \mathcal{R}_t) \\ &= \rho\lambda_t^R + \frac{\bar{\nu}\nu}{1+\nu}\left(\frac{E_t^\star}{R_t}\right)^{1+\nu} + \frac{\bar{\mu}\mu}{1+\mu}\left(\frac{I_t^\star}{R_t}\right)^{1+\mu} \\ \dot{\lambda}_t^R &= \rho\lambda_t^R + \frac{\bar{\nu}^{-1/\nu}\nu}{1+\nu}(q^{e,f} - \lambda_t^R)^{1+1/\nu} + \frac{\bar{\mu}^{-1/\mu}\mu}{1+\mu}(\delta^R\lambda_t^R)^{1+1/\mu}\end{aligned}$$

- Because of decreasing return to scale and Hotelling rents : profits are > 0

$$\pi_t(E_t^f, \mathcal{R}_t, \lambda_t^R) = \frac{1+\nu-\bar{\nu}}{1+\nu}\left(\frac{E_t^f}{\mathcal{R}_t}\right)^{1+\nu}\mathcal{R}_t + \lambda_t^R E_t^f - \frac{\bar{\mu}^{-1/\mu}}{1+\mu}(\delta^r\lambda_t^R)^{1+\frac{1}{\mu}}$$

back

More details – PMP – Competitive equilibrium

- Household problem : State variables $s_{it} = (k_i, \tau_i, z_i, p_i, \Delta_i)$ Back
- Pontryagin Maximum Principle

$$\mathcal{H}^{hh}(s, \{c\}, \{e^f\}, \{e^r\}, \{\lambda\}) = u(c_i, \tau_i) + \lambda^k_{it}\Big(\mathcal{D}(\tau_{it})f(k_t, e_t) - (n + \bar{g} + \delta)k_t - q^f_t e^f_{it} - q^r_{it}e^r_{it} - c_t\Big) \\ + \lambda^\tau_{it}\zeta\Big(\Delta_i\,\chi\,\mathcal{S}_t - (\tau_{it} - \tau_{i0})\Big) + \lambda^S_{it}\Big(\mathcal{E}_t - \delta^s\mathcal{S}_t\Big)$$

$$\begin{aligned} &[c_t] && u'(c_{it}) = \lambda^k_{it} \\ &[e^f_t] && MPe^f_{it} = \mathcal{D}(\tau_{it})z\,\partial_e f(k_{it}, e_{it})\Big(\frac{e^f_{it}}{\omega e_{it}}\Big)^{-\frac{1}{\sigma_e}} = q^f_t \\ &[e^r_t] && MPe^r_{it} = \mathcal{D}(\tau_{it})z\,\partial_e f(k_{it}, e_{it})\Big(\frac{e^r_{it}}{(1-\omega)e_{it}}\Big)^{-\frac{1}{\sigma_e}} = q^r_{it} \\ &[k_t] && \dot{\lambda}^k_t = -\lambda^k_t\big(\mathcal{D}(\tau_{it})\partial_k f(k_{it}, e_{it}) - \delta - \bar{g} - n - \rho\big) \end{aligned}$$

- Fossil Energy Monopoly :

$$\mathcal{H}^m(\mathcal{R}_t, \lambda^R_t, E^f_t, \mathcal{I}_t) = \pi_t(E^f_t, \mathcal{I}_t, \mathcal{R}_t) + \lambda^R_t(\delta^R\mathcal{I}_t - E^f_t)$$

$$\begin{aligned} &[\mathcal{R}_t] && \dot{\lambda}^R_t = \rho\lambda^R_t + \frac{\bar{\nu}\nu}{1+\nu}\Big(\frac{E^\star_t}{R_t}\Big)^{1+\nu} + \frac{\bar{\mu}\mu}{1+\mu}\Big(\frac{I^\star_t}{R_t}\Big)^{1+\mu} \\ &[E^f_t] && q^{e,f}_t = \nu_E(E, \mathcal{R}) + \lambda^R_t = \bar{\nu}\Big(\frac{E_t}{\mathcal{R}_t}\Big)^{\nu} + \lambda^R_t \\ &[\mathcal{I}_t] && \lambda^R_t\delta^R = \mu_I(I_t, R_t) = \bar{\mu}\Big(\frac{\mathcal{I}_t}{\mathcal{R}_t}\Big)^{\mu} \qquad \mathcal{I}_t = R_t\Big(\frac{\lambda^R_t\delta}{\bar{\mu}}\Big)^{1/\mu} \end{aligned}$$

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature ⇔ solving ODE

$$\dot{\lambda}^{\mathcal{T}}_{it} = \lambda^{\mathcal{T}}_t(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\lambda^k_t + \phi(\tau - \tau^\star)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}^S_t = {\lambda^S}_t(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta\chi\lambda^{\mathcal{T}}_{it}$$

- Solving for $\lambda^{\mathcal{T}}_t$ and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^S_t = \dot{\lambda}^{\mathcal{T}}_t = 0$

$$\lambda^{\mathcal{T}}_{it} = -\int_t^\infty e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_u)y_{\mathcal{T}}\lambda^k_u + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda^{\mathcal{T}}_{it} = -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_\infty)y_\infty\lambda^k_\infty + \phi\mathcal{D}^u(\tau_\infty)u(c_\infty)\Big)$$

$$\begin{aligned}
\lambda^S_t &= -\int_t^\infty e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,u}dj\,du \\
&= \frac{1}{\widetilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,\infty} \\
&= -\frac{\chi}{\widetilde{\rho}+\delta^s}\frac{\zeta}{\widetilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_\infty\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj \\
\lambda^S_t &\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

Cost of carbon / Marginal value of temperature

- Closed form solution for CC :
  - In stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - no internalization of externality (business as usual)

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s}(\tau_\infty - \tau^\star)\Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty\Big)$$

- Heterogeneity + uncertainty about models Back

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega\int_{t_0}^{T} e^{(n+\bar{g})t-\delta_s(T-t)}\,q_t^{f\ -\sigma_e}\int_{j\in\mathbb{I}}\left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\,q_{j,t}^{\sigma_e-\sigma}\,dj\,dt$$

  - Geographical factors determining warming Δ_i
  - Climate sensitivity χ & carbon exit from atmosphere δ_s
  - Growth of population n, aggregate productivity $\bar{g}$
  - Deviation of output from trend y_i & relative TFP z_j
  - Directed technical change z_t^e, elasticity of energy in output σ
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}_t^R/\lambda_t^R = \rho$
  - Change in energy mix, renewable share ω, price q_t^r & elasticity of source σ_e

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto\ n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \widetilde{\gamma}\right) + (\sigma_e - \sigma)(1-\omega)g^{q^r} - (\sigma_e(1-\omega) + \sigma\omega)g^{q^f}$$

Back

More details – PMP – Ramsey Optimal Allocation

- Hamiltonian :

$$\mathcal{H}^{sp}(s,\{c\},\{e^f\},\{e^r\},\{\lambda\},\{\psi\}) = \int_{\mathbb{I}} \omega_i u(c_i,\tau_i)p_i di$$
$$+\psi^k_{it}\Big(\mathcal{D}(\tau_{it})f(k_{it},e_{it}) - (n+\bar{g}+\delta)k_t + \theta_i\pi(E^f_t,\mathcal{I}_t,\mathcal{R}_t) + \pi^r_{it}(e^r_{it}) - (q^f_t+\mathbf{t}^f_{it})e^f_{it} - (q^r_{it}+\mathbf{t}^r_{it})e^r_{it} - c_t + \mathbf{t}^{ls}_t\Big)$$
$$+\psi^S_t\Big(\mathcal{E}_t - \delta^s\mathcal{S}_t\Big) + \psi^\tau_{it}\zeta\Big(\Delta_i\,\chi\,\mathcal{S}_t - (\tau_{it}-\tau_{i0})\Big) + \psi^{\mathcal{R}}_{it}\Big(-E^f_t + \delta^R\mathcal{I}_t\Big)$$
$$+\psi^{\lambda k}_{it}\Big(\lambda^k_t(\rho - r_t)\Big) + \psi^{\lambda R}_t\Big(\rho\lambda^R_t + \mathcal{C}^f_{\mathcal{R}}(E^f_t,\mathcal{I}_t,\mathcal{R}_t)\Big) + \phi^c_{it}\big(u_c(c_i,\tau_{it}) - \lambda^k_{it}\big)$$
$$+\phi^{ef}_{it}\Big(e^f_{it} - \mathcal{Q}_{q^f}(q^f_t+\mathbf{t}^f_{it},q^r_t+\mathbf{t}^r_{it})e_{it}\Big) + \phi^{er}_{it}\Big(e^r_{it} - \mathcal{Q}_{q^r}(q^f_t+\mathbf{t}^f_{it},q^r_t+\mathbf{t}^r_{it})e_{it}\Big)$$
$$+\phi^e_{it}\Big(f_e(k_{it},e_{it}) - \mathcal{Q}(q^f_t+\mathbf{t}^f_{it},q^r_t+\mathbf{t}^r_{it})\Big) + \phi^{Ef}_t\Big(q^f_t - \mathcal{C}^f_E(\cdot) - \lambda^{\mathcal{R}}_t\Big) + \phi^{Er}_{it}\Big(q^r_{it} - \mathcal{C}^r_e(\cdot)\Big) + \phi^{\mathcal{I}f}_t\big(\delta\lambda^{\mathcal{R}}_t - \mathcal{C}^f_{\mathcal{I}}(\cdot)\big)$$

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Ramsey Optimal Allocation - FOCs

- FOCs w.r.t. $\{c_{it}, e_{it}, e^f_{it}, e^r_{it}, \mathcal{I}_t\}$, prices $\{q^f_t, q^r_{it}\}$ and taxes, denoting $\tilde{q}_{it} = q_t + \mathbf{t}_{it}$

$$[c_{it}] \qquad \psi^k_{it} = \underbrace{\omega_i u_c(c_i, \tau_{it}) p_i}_{=\text{direct effect}} + \underbrace{\phi^c_{it} u_{cc}(c_i, \tau_{it})}_{=\text{effect on savings}}$$

$$[e_{it}] \qquad \psi^k_{it} f_{e,it} + \phi^e_{it} f_{ee,it} - \phi^{ef}_{it}\mathcal{Q}_{q^f} - \phi^{er}_{it}\mathcal{Q}_{q^r} = 0 \qquad \Rightarrow \qquad \phi^e_{it} = \frac{1}{f_{ee,it}}\left(\phi^{ef}_{it}\mathcal{Q}_{q^f} + \phi^{er}_{it}\mathcal{Q}_{q^r} - \psi^k_{it} f_{e,it}\right)$$

$$[e^f_{it}] \qquad \phi^{ef}_{it} = \psi^k_{it}\tilde{q}^f_t - \psi^k_{it}\mathbf{t}^f_{it} - \xi\psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) \qquad [e^r_{it}] \qquad \phi^{er}_{it} = \psi^k_{it}\tilde{q}^r_t - \psi^k_{it}\mathbf{t}^r_{it} + \phi^{Er}_{it}\mathcal{C}^r_{e^r e^r}(\cdot)$$

$$[\tilde{q}^f_{it}] \qquad \phi^e_{it}\mathcal{Q}_{q^f} + \phi^{ef}_{it}\mathcal{Q}_{q^f q^f} + \phi^{er}_{it}\mathcal{Q}_{q^r q^f} = 0$$

$$\Rightarrow \qquad \Big(\frac{\mathcal{Q}^2_{q^f}}{f_{ee,it}} + \mathcal{Q}_{q^f q^f}\Big)\Big[-\xi\psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it}\mathbf{t}^f_{it}\Big] + \Big(\frac{\mathcal{Q}_{q^f}\mathcal{Q}_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^r q^f}\Big)\Big[\phi^{Er}_{it}\mathcal{C}^r_{e^r e^r}(\cdot) - \psi^k_{it}\mathbf{t}^f_{it}\Big]$$

$$[\tilde{q}^r_{it}] \qquad \phi^e_{it}\mathcal{Q}_{q^r} + \phi^{ef}_{it}\mathcal{Q}_{q^f q^r} + \phi^{er}_{it}\mathcal{Q}_{q^r q^r} = 0$$

$$\Rightarrow \qquad \Big(\frac{\mathcal{Q}_{q^f}\mathcal{Q}_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^f q^r}\Big)\Big[-\xi\psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it}\mathbf{t}^f_{it}\Big] + \Big(\frac{\mathcal{Q}^2_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^r q^r}\Big)\Big[\phi^{Er}_{it}\mathcal{C}^r_{e^r e^r}(\cdot) - \psi^k_{it}\mathbf{t}^f_{it}\Big] = 0$$

$$[q^f_{it}] \qquad \phi^{Ef}_t = \int_{\mathbb{I}} \psi^k_{jt} e^f_{jt} dj - \partial_{q^f}\pi^f(\cdot)\int_{\mathbb{I}}\theta_j \psi^k_{jt} dj \qquad [q^r_{it}] \qquad \phi^{Er}_{it} = \psi^k_{it} e^r_{it} - \psi^k_{it}\partial^r_q \pi^r_{it} = 0$$

$$[\mathcal{I}_t] \qquad \delta\,\psi^{\mathcal{R}}_t + \partial^2_{\mathcal{R}\mathcal{I}}\mathcal{C}(\cdot)\,\psi^{\lambda,\mathcal{R}}_t - \phi^{\mathcal{I}}_t \partial^2_{\mathcal{I}\mathcal{I}}\mathcal{C}(\cdot) = 0$$

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Ramsey Optimal Allocation - FOCs

- Backward equations for planner's costates

$$[k_i] \qquad \dot{\psi}^k_{it} = \psi^k_{it}(\tilde{\rho} - r_{it}) + \psi^{\lambda k}_{it}\lambda^k_{it}\partial_k MPk_i + \frac{f_{ek,it}}{f_{ee,it}}\Big[-\xi\psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it}\mathbf{t}^f_{it}\Big]$$

$$[\mathcal{S}_i] \qquad \dot{\psi}^{\mathcal{S}}_t = (\tilde{\rho} + \delta^s)\psi^{\mathcal{S}}_t - \int_{\mathbb{I}} \Delta_j \zeta\chi\psi^{\tau}_{jt}\, dj$$

$$[\tau_i] \qquad \dot{\psi}^{\tau}_t = (\tilde{\rho} + \zeta)\psi^{\tau}_t - \Big(\omega_i u_{\tau}(c_{it}, \tau_{it}) + \psi^k_{it}\mathcal{D}'(\tau_{it}) f(k_{it}, e_{it}) + \phi^c_{it} u_{c,\tau}(c_{it}, \tau_{it}) + \mathcal{D}'(\tau_{it}) f_e \phi^e_{it}\Big)$$

$$[\mathcal{R}] \qquad \dot{\psi}^{\mathcal{R}}_t = \psi^{\mathcal{R}}_t\Big(\tilde{\rho} - \partial^2_{\mathcal{R}\mathcal{R}}\mathcal{C}(\cdot)\Big) - \phi^{Ef}_t \partial^2_{\mathcal{R}E}\mathcal{C}(\cdot)$$

$$[\lambda^k_i] \qquad \dot{\psi}^{\lambda,k}_t = \psi^{\lambda,k}_t\big[\tilde{\rho} - (\rho - r_{it})\big] + \phi^c_{it}$$

$$[\lambda^{\mathcal{R}}_i] \qquad \dot{\psi}^{\lambda,\mathcal{R}}_t = \psi^{\lambda,\mathcal{R}}_t(\tilde{\rho} - \rho) + \phi^{Ef}_t - \delta\phi^{\mathcal{I}f}_t$$

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