The Inequality of Climate Change

Heterogeneity, optimal energy policy and uncertainty

WORK IN PROGRESS

Thomas Bourany

THE UNIVERSITY OF CHICAGO

Econ Dynamics

May 2023

Introduction – this project

- Marginal damages of climate & temperature varies across countries
  - Vary with the damage function : non-linearity matters a lot !
- What is the optimal taxation of energy in the presence of climate externality *and* heterogeneities ?
  - In context where fossil fuels taxation and climate policy redistributes across countries

Introduction – this project

- Marginal damages of climate & temperature varies across countries
  - Vary with the damage function : non-linearity matters a lot !
- What is the optimal taxation of energy in the presence of climate externality *and* heterogeneities ?
  - In context where fossil fuels taxation and climate policy redistributes across countries
- Develop a simple and flexible model of climate economics
  - Standard NCG – IAM model with heterogeneous regions
  - Normative implications : Ramsey policy + possibility to study uncertainty

Introduction – this project

- Marginal damages of climate & temperature varies across countries
  - Vary with the damage function : non-linearity matters a lot !
- What is the optimal taxation of energy in the presence of climate externality *and* heterogeneities ?
  - In context where fossil fuels taxation and climate policy redistributes across countries
- Develop a simple and flexible model of climate economics
  - Standard NCG – IAM model with heterogeneous regions
  - Normative implications : Ramsey policy + possibility to study uncertainty

◦ Evaluate the heterogeneous welfare costs of global warming

◦ Provide analytical formulas and a numerical methodology to compute the cost of carbon
  - Heterogeneity increases the welfare cost of carbon

◦ Solve world optimal carbon policy with heterogeneous regions
  - Does the optimal carbon tax coincide with the social cost of carbon ?

  $\Rightarrow$ Maybe not, depend on transfer policy : need to adjust for inequality level
  - What are the welfare gains of suboptimal policies ?

Toy model

- Consider two countries $i = N, S$, (North/South)
  - HH consuming good c_i and producing with energy e_i and productivity z_i
  - Energy producer with profit $\pi(E)$ owned by country i with share θ_i
- Household problem :

$$\mathcal{V}_i = \max_{c_i, e_i} U(c_i)$$

$$c_i + q^e e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) \qquad [\lambda_i^k]$$

- Subject to damage $\mathcal{D}_i(\mathcal{S})$ and climate externalities :

$$\mathcal{S} = \mathcal{S}_0 + \overbrace{\xi_N e_N + \xi_S e_S}^{=\text{GHG emissions}}$$

- And consuming energy in a single energy market with price q^e

$$E = e_N + e_S \qquad q^e = c'(E) \qquad \pi(E) = q^e E - c(E)$$

Toy model – Competitive equilibrium

- Three dimensions of heterogeneity :
  1. Different levels of productivity z_i : $z_N > z_S$
  2. Different climate damage $\gamma_i = -\frac{\mathcal{D}_i'(\mathcal{S})}{\mathcal{S}\mathcal{D}_i(\mathcal{S})}$, $\gamma_S > \gamma_N$
  3. Different energy rent θ_i : $\theta_N > \theta_S$

  $\Rightarrow$ Yields heterogeneity in consumption $c_N > c_S$

- Competitive equilibrium Result :
  - Marginal Product of Energy = Energy Cost

$$MPe_i = q^e = c'(E) \qquad \textit{with} \qquad MPe_i := \mathcal{D}_i(\mathcal{S})z_i F'(e_i)$$

  - Inequality

$$\lambda_i^k = U'(c_i) \qquad\qquad c_i = \mathcal{D}_i(\mathcal{S})z_i F(e_i) + \theta_i \pi(E) - q^e e_i$$

Toy model – First Best and Decentralization

- Comparison with Social planner with full transfers (First Best)

$$\mathbb{W} = \max_{\{c_i, e_i\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$\sum_{i=N,S} c_i + c(E) = \sum_{i=N,S} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad [\lambda]$$

$$\mathcal{S} := \mathcal{S}_0 + \xi_N e_N + \xi_S e_S \qquad E := e_N + e_S$$

- Marginal Product of Energy = Energy Cost + Social Cost of Carbon

$$MPe_i = c'(E) + \xi_i \underbrace{\overline{SCC}}_{=\mathbf{t}^e} \qquad with \qquad \overline{SCC} := \sum_{i=N,S} \mathcal{D}_i'(\mathcal{S}) z_i F(e_i)$$

- Redistribution

$$\omega_S U(c_S) = \omega_N U(c_N) = \lambda$$

- Decentralization, needs to redistribute with *with lump-sum transfers* $T_S = -T_N$

$$\Rightarrow \qquad c_i + (q^e + \mathbf{t}^e) e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) + T_i$$

- Are lump-sum transfers feasible ?

Toy model – Second Best - Ramsey Problem

- Assume now that *lump-sum transfers across countries* are prohibited
  - Allow for carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$

$$\mathcal{W} = \max_{\{c_i, e_i\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$s.t \qquad c_i + (q^e + \mathbf{t}_i^e) e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) + T^i \qquad [\phi_i]$$

$$\mathcal{S} := \mathcal{S}_0 + \xi_N e_N + \xi_S e_S \qquad E := e_N + e_S \qquad q^e = c'(E)$$

  - Ramsey policy result :
    - Planner's marginal value of wealth

$$\phi_i = \omega_i U'(c_i)$$

    - Energy decision :

$$\phi_i \left[MPe_i - c'(E) \right] + \xi_i \underbrace{\sum_j \phi_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j)}_{\propto -\text{SCC}} + \underbrace{\pi'(E) \sum_j \phi_j \theta_j}_{= \text{rent redistribution}} \underbrace{- c''(E) \sum_j \phi_j^k e_j}_{= \text{cost redistribution}} = 0$$

Social Cost of Carbon with inequality

- Measure of inequality

$$\widehat{\phi}_i = \frac{\phi_i}{\overline{\phi}} = \frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)} \lesseqgtr 1 \qquad \overline{\phi} = \frac{1}{2}\big(\omega_N U'(c_N) + \omega_S U'(c_S)\big)$$

- The SCC is exacerbated by heterogeneity

$$\begin{aligned} SCC &= -\sum_j \widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) y_j \\ &= -\mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}_j'(\mathcal{S}) y_j\right) - \mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) y_j] > -\mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) y_j] = \overline{SCC} \end{aligned}$$

  - Why ? Low-income countries tend to be warmer/more vulnerable to climate change

Social Cost of Carbon with inequality

- Measure of inequality

$$\widehat{\phi}_i = \frac{\phi_i}{\overline{\phi}} = \frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)} \lessgtr 1 \qquad \overline{\phi} = \frac{1}{2}\big(\omega_N U'(c_N) + \omega_S U'(c_S)\big)$$

- The SCC is exacerbated by heterogeneity

$$\begin{aligned} SCC &= -\sum_j \widehat{\phi}_j \mathcal{D}'_j(\mathcal{S}) y_j \\ &= -\mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}'_j(\mathcal{S}) y_j\right) - \mathbb{E}_j[\mathcal{D}'_j(\mathcal{S}) y_j] > -\mathbb{E}_j[\mathcal{D}'_j(\mathcal{S}) y_j] = \overline{SCC} \end{aligned}$$

  - Why ? Low-income countries tend to be warmer/more vulnerable to climate change

- For the social value of rent (exporters) and energy cost (importer), it's the contrary !

$$\begin{aligned} SVR &= \mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \theta_j \pi'_j(E)\right) + \pi'(E) < \pi'(E) \\ SCE &= \mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, e_j\, c''(E)\right) + c''(E) < c''(E) = \pi'(E) \end{aligned}$$

Optimal energy policy

- Energy taxation :

$$MPe_i = c'(E) + \xi_i \mathbf{t}_i^e$$
$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[SCC - SVR + SCE\right]$$

- Four motives with a single tax and lump-sum rebate
  - Distribution : Tax is higher for poorer countries $\omega_S U'(c_S) > \omega_N U'(c_N) \Rightarrow \mathbf{t}_S^e > \mathbf{t}_N^e$
  - Optimal tax level : Depends on
    - Distribution of climate damage in SCC
    - Distribution of energy rent in SVR
    - Distribution of energy spending in SCE

Toy model – Effect of uncertainty

- Consider risks related to both
  (i) Climate damage $\mathcal{D}_i(\mathcal{S}|\epsilon_d)$
  (ii) Could also consider economic growth $z_i(\epsilon_z)$
  – Probability distribution $(\epsilon_z, \epsilon_d) =: \boldsymbol{\epsilon} \sim \varphi(\boldsymbol{\epsilon})$

$$\max_{e_i} \int_{\mathcal{E}} \max_{c_i(\boldsymbol{\epsilon})} U(c_{i(\boldsymbol{\epsilon})}) d\varphi(\boldsymbol{\epsilon}) \qquad \text{vs.} \qquad \max_{\{e_j\}_j} \int_{\mathcal{E}} \max_{\{c_j(\boldsymbol{\epsilon})\}_j} \sum_{j=N,S} \omega_j U(c_{j(\boldsymbol{\epsilon})}) d\varphi(\boldsymbol{\epsilon})$$

Toy model – Effect of uncertainty

- Consider risks related to both
  - (i) Climate damage $\mathcal{D}_i(\mathcal{S}|\epsilon_d)$
  - (ii) Could also consider economic growth $z_i(\epsilon_z)$
  - – Probability distribution $(\epsilon_z, \epsilon_d) =: \boldsymbol{\epsilon} \sim \varphi(\boldsymbol{\epsilon})$

$$\max_{e_i} \int_{\mathcal{E}} \max_{c_i(\boldsymbol{\epsilon})} U(c_{i(\boldsymbol{\epsilon})}) d\varphi(\boldsymbol{\epsilon}) \qquad \text{vs.} \qquad \max_{\{e_j\}_j} \int_{\mathcal{E}} \max_{\{c_j(\boldsymbol{\epsilon})\}_j} \sum_{j=N,S} \omega_j U(c_{j(\boldsymbol{\epsilon})}) d\varphi(\boldsymbol{\epsilon})$$

- Competitive equilibrium :
  - Almost no change in behavior : Expected Marginal Product of Energy = Energy Price

$$\int_{\mathcal{E}} MPe_{i(\boldsymbol{\epsilon})}\, d\varphi(\boldsymbol{\epsilon}) = q^e$$

Toy model – Effect of uncertainty

- Consider risks related to both
  - (i) Climate damage $\mathcal{D}_i(\mathcal{S}|\epsilon_d)$
  - (ii) Could also consider economic growth $z_i(\epsilon_z)$
  - – Probability distribution $(\epsilon_z, \epsilon_d) =: \boldsymbol{\epsilon} \sim \varphi(\boldsymbol{\epsilon})$

$$\max_{e_i} \int_{\mathcal{E}} \max_{c_i(\boldsymbol{\epsilon})} U(c_i(\boldsymbol{\epsilon})) d\varphi(\boldsymbol{\epsilon}) \qquad \text{vs.} \qquad \max_{\{e_j\}_j} \int_{\mathcal{E}} \max_{\{c_j(\boldsymbol{\epsilon})\}_j} \sum_{j=N,S} \omega_j U(c_j(\boldsymbol{\epsilon})) d\varphi(\boldsymbol{\epsilon})$$

- Competitive equilibrium :
  - Almost no change in behavior : Expected Marginal Product of Energy = Energy Price

$$\int_{\mathcal{E}} MPe_{i(\boldsymbol{\epsilon})}\, d\varphi(\boldsymbol{\epsilon}) = q^e$$

- Ramsey planner :
  - Taxes take uncertainty into account :

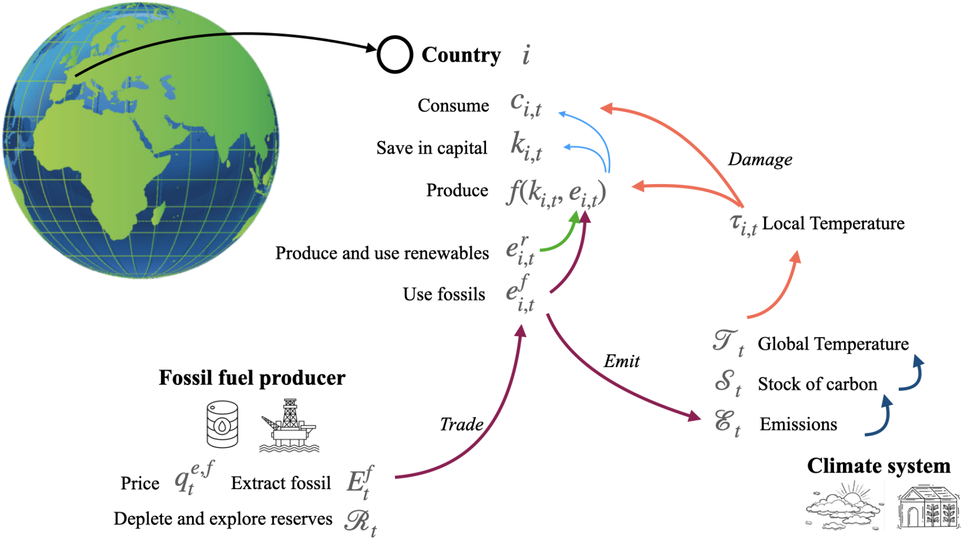
$$\mathbb{E}_{\boldsymbol{\epsilon}}\left(MPe_{i(\boldsymbol{\epsilon})}\right) = q^e + \underbrace{\frac{\mathbb{E}_{k,\boldsymbol{\epsilon}}[\omega_k U'(c_k)]}{\mathbb{E}_{\boldsymbol{\epsilon}}\left(\omega_i U'(c_{j(\boldsymbol{\epsilon})})\right)}}_{=\text{redistributive effect w/ risk}} \left[\underbrace{-\mathbb{C}\mathrm{ov}_{\boldsymbol{\epsilon}}\left(\frac{\omega_i U'(c_{i(\boldsymbol{\epsilon})})}{\mathbb{E}_{k,\boldsymbol{\epsilon}}[\omega_k U'(c_k)]}, MPe_{i(\boldsymbol{\epsilon})}\right)}_{=\text{effect of agg. risk } \boldsymbol{\epsilon} \text{ on energy choice}} + \mathbb{E}_{\boldsymbol{\epsilon}}\left[SCC(\boldsymbol{\epsilon}) - SVR(\boldsymbol{\epsilon}) + SCE(\boldsymbol{\epsilon})\right]\right]$$

Toy model – Social cost of carbon and Uncertainty

- Social cost of carbon

$$
\begin{aligned}
\mathbb{E}_{\epsilon}[SCC] &= \int_{\mathcal{E}} \sum_{j=N,S} \frac{\omega_i U'(c_i(\epsilon))}{\mathbb{E}_{k,\epsilon}[\omega_k U'(c_k)]} \mathcal{D}_j'(\mathcal{S}, \epsilon_d) y_j(\epsilon_z) d\varphi(\epsilon) \\
&= -\mathbb{C}\mathrm{ov}_{j,\epsilon}\left(\frac{\omega_j U'(c_j(\epsilon))}{\mathbb{E}_{k,\epsilon}[\omega_k U'(c_k)]}, \mathcal{D}_j'(\mathcal{S}, \epsilon_d) y_j(\epsilon_z) \right) - \mathbb{E}_{j,\epsilon}[\mathcal{D}_j'(\mathcal{S}) y_k] \\
&= -\mathbb{E}_j \Bigg[\underbrace{\mathbb{C}\mathrm{ov}_{\epsilon}\left(\frac{\omega_j U'(c_j(\epsilon))}{\mathbb{E}_{k,\epsilon}[\omega_k U'(c_k)]}, \mathcal{D}_j'(\mathcal{S}, \epsilon_d) y_j(\epsilon_z) \right)}_{=\text{effect of agg. risk } \epsilon} \Bigg] \\
&\qquad \underbrace{- \mathbb{C}\mathrm{ov}_j \left[\frac{\mathbb{E}_{\epsilon}\left(\omega_j U'(c_j(\epsilon))\right)}{\mathbb{E}_{k,\epsilon}[\omega_k U'(c_k)]}, \mathbb{E}_{\epsilon}\left(\mathcal{D}_j'(\mathcal{S}, \epsilon_d) y_j(\epsilon_z) \right) \right]}_{=\text{effect of heterogeneity across } j} - \underbrace{\mathbb{E}_{j,\epsilon}[\mathcal{D}_j'(\mathcal{S}) y_k]}_{=\text{average exp. damage}} \\
&> \mathbb{E}_{\epsilon}[\overline{SCC}(\epsilon)]
\end{aligned}
$$

Summary of the quantitative model



Summary of the Model Environment

1. Households in individual countries $i \in \mathbb{I}$ consuming c_{it} Household/Firms HH Solution
   - Markets : borrow/save on world bond markets b_{it} / invest in productive capital k_{it}
   - Energy spending : fossil energy e^f_{it} and renewable e^r_{it}
   - Taxation, fossil, $\mathbf{t}^f_{it}$ and renewable $\mathbf{t}^r_{it}$
2. Energy markets Energy
   - Representative (Competitive) Fossil Fuel producer making profit $\pi^f_t(q^f_t, E^f_t, \mathcal{R}_t)$
   - Extended Hotelling problem : Extraction E^f_t vs. Exploration $\mathcal{I}_t$
   - Redistribute share θ_i to household of country i
   - Renewables with price q^r_t
3. Climate system Climate
   - Linear dynamics : emissions $\mathcal{E}_t$ to atm. carbon $\mathcal{S}_t$ to temperature $\mathcal{T}$, cf Dietz Venmans (19)
   - Damage function on productivity as in DICE $\mathcal{D}_i(\tau_{it})$ and in utility $u(c_{it}, \tau_{it}) = u\big(\mathcal{D}_i(\tau_{it})c_{it}\big)$ with $u(\cdot)$ CRRA

- Heterogeneity :
  1. Productivity z_i
  2. Population p_i
  3. Temperature scaling Δ_i
  4. Local damage γ_i
  5. Capital stock k_{it}
  6. Local temperature τ_{it}

  $\Rightarrow$ Yield inequality in consumption c_{it}

Model – Solution

- Main model equations and equilibrium Equilibrium
  1. Household problem

$$\dot{w}_{it} = r^{\star} w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^{\star}) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{it}^{ls}$$

Model – Solution

- Main model equations and equilibrium Equilibrium
  1. Household problem

$$\dot{w}_{it} = r^{\star} w_{it} + \mathcal{D}^{y}(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^{\star}) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{it}^{ls}$$

  2. Energy Markets

$$E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta_R \mathcal{I}_t$$

Model – Solution

- Main model equations and equilibrium Equilibrium
  1. Household problem

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{it}^{ls}$$

  2. Energy Markets

$$E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta_R \mathcal{I}_t$$

  3. Climate system

$$\mathcal{E}_t = \int_{\mathbb{I}} \xi_i \, e_{it}^f \, di \qquad \dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t \qquad \dot{\tau}_{i,t} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{i,t} - \bar{\tau}_{i,t_0})\big)$$

Model – Solution

- Main model equations and equilibrium Equilibrium
  1. Household problem

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{it}^{ls}$$

  2. Energy Markets

$$E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta_R \mathcal{I}_t$$

  3. Climate system

$$\mathcal{E}_t = \int_{\mathbb{I}} \xi_i \, e_{it}^f \, di \qquad \dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t \qquad \dot{\tau}_{i,t} = \zeta\Big(\Delta_i \chi \mathcal{S}_t - (\tau_{i,t} - \bar{\tau}_{i,t_0})\Big)$$

  4. Household Decisions, consumption/saving, and energy

$$\dot{c}_{it} = c_{it} \frac{1}{\eta}(r_t^\star - \rho) \qquad MPk_{it} - \bar{\delta} = r_t^\star \qquad MPe_{it} = \mathcal{Q}(q_{i,t}^f + \mathbf{t}_{it}^f, q_{i,t}^r + \mathbf{t}_{it}^r)$$

$$\Rightarrow \qquad e_{i,t}^f = \mathcal{Q}_{q^f}(q_{i,t}^f + \mathbf{t}_{it}^f, q_{i,t}^r + \mathbf{t}_{it}^r) e_{it} \qquad e_{i,t}^r = \mathcal{Q}_{q^r}(q_{i,t}^f + \mathbf{t}_{it}^f, q_{i,t}^r + \mathbf{t}_{it}^r) e_{i,t}$$

Optimal policy

- Social planner, First best with a full set of instruments :
  - Solves world's inequality, using lump-sum transfers such that $\lambda_t = u'(c_{it}) = u'(c_{jt}), \forall i,j \in \mathbb{I}$
  - Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^S}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)
  - Imply cross-countries lump-sum transfers

Optimal policy

- Social planner, First best with a full set of instruments :
  - Solves world's inequality, using lump-sum transfers such that $\lambda_t = u'(c_{it}) = u'(c_{jt}), \forall i,j \in \mathbb{I}$
  - Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^S}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)
  - Imply cross-countries lump-sum transfers

- Second best / Ramsey planner :
  - Doesn't have access to redistribution / lump-sum transfers
  - Can only use region-i-specific distortive energy taxes : $\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r\}$
  - Redistribute lump sum the tax revenues : $\mathbf{t}_{it}^{ls} = \mathbf{t}_{it}^f e_{it}^f + \mathbf{t}_{it}^r e_{it}^r$

Optimal policy

- Social planner, First best with a full set of instruments :
  - Solves world's inequality, using lump-sum transfers such that $\lambda_t = u'(c_{it}) = u'(c_{jt}), \forall i,j \in \mathbb{I}$
  - Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^S}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)
  - Imply cross-countries lump-sum transfers
- Second best / Ramsey planner :
  - Doesn't have access to redistribution / lump-sum transfers
  - Can only use region-i-specific distortive energy taxes : $\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r\}$
  - Redistribute lump sum the tax revenues : $\mathbf{t}_{it}^{ls} = \mathbf{t}_{it}^f e_{it}^f + \mathbf{t}_{it}^r e_{it}^r$
- Questions :
  - Is the level of energy tax regions specific ?

    $\Rightarrow$ Yes ! depends on the distribution of wealth/consumption
  - What is the level of the Pigouvian tax ?

    $\Rightarrow$ $\propto$ Welfare cost/climate damage : "social costate" for carbon $\mathcal{S}$, i.e. ψ^S

    $\Rightarrow$ Inequality/Heterogeneity in damage change the *level* of this tax

The Ramsey Problem

- Consider a Social Planner that care about aggregate welfare :

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t}^f_{it},\mathbf{t}^r_{it},c_{it},e^f_{it},e^r_{it},k_{it},\lambda^k_{it},\tau_{it},\mathcal{S}_t,\mathcal{R}_t,\mathcal{I}_t,\lambda^{\mathcal{R}}_t\}_{i,t}} \int_{t_0}^{\infty}\int_{\mathbb{I}} e^{-\bar{\rho}t}\,\omega_i\, u(c_{i,t},\tau_{i,t})\,di\,dt$$

subject to

  - Optimality conditions of households, for c_i, e^f_i, e^r_i and k_i
  - Optimality conditions of the Fossil firm, for E^f, $\mathcal{I}$ and $\mathcal{R}$
  - Optimality condition of the renewable firm, for e^r_i
  - Climate and temperature dynamics τ_i and $\mathcal{S}$
  - Given Pareto weights ω_i

$\Rightarrow$ Large scale system of ODE More details - Hamiltonian

  - A Ramsey plan is a set $\{\mathbf{t}^f_{it},\mathbf{t}^r_{it},\mathbf{t}^{ls}_{it}\}_{it}$ s.t. the competitive equilibrium is maximizing welfare
  - States $\{w_{it},\tau_{it},c_{it},k_{it},e^f_{it},e^r_{it},E^f_t,\mathcal{E}_t,\mathcal{S}_t,\mathcal{R}_t\}$
  - Costates $\{\psi^w_{it},\psi^\tau_{it},\psi^{\mathcal{S}}_{it},\psi^{\mathcal{R}}_t\}$

The Ramsey Problem – Solution 1

- Shadow value of wealth gives a measure of inequality

$$\widehat{\psi}_{it}^{w} = \frac{\psi_{it}^{w}}{\overline{\psi}_{t}^{w}} = \frac{\omega_i u_c(c_i, \tau_{it})}{\int_{j\in\mathbb{I}} \omega_j u_c(c_{jt}, \tau_{jt}) dj} \lesseqgtr 1$$

$$\text{low } z_i, k_i, \quad \text{high } \tau_{it} \qquad \Rightarrow \quad \text{low } c_i, \text{ high } \psi_{i,t}^{w} \approx \omega_i u'(c_i) p_i > \overline{\psi}_{t}^{w} = \int_{\mathbb{I}} u_c(c_{jt}, \tau_{jt}) dj$$

The Ramsey Problem – Solution 1

- Shadow value of wealth gives a measure of inequality

$$\widehat{\psi}^w_{it} = \frac{\psi^w_{it}}{\overline{\psi}^w_t} = \frac{\omega_i u_c(c_i, \tau_{it})}{\int_{j\in\mathbb{I}} \omega_j u_c(c_{jt}, \tau_{jt}) dj} \lesseqgtr 1$$

$$\text{low } z_i, k_i, \quad \text{high } \tau_{it} \qquad \Rightarrow \quad \text{low } c_i, \text{ high } \psi^w_{i,t} \approx \omega_i u'(c_i)\, p_i > \overline{\psi}^w_t = \int_{\mathbb{I}} u_c(c_{jt}, \tau_{jt}) dj$$

- Shadow value of carbon and temperature gives a measure of welfare cost of carbon :

$$\psi^{\mathcal{S}}_t = \frac{\partial \mathcal{W}_t}{\partial \mathcal{S}} = \int_{j\in\mathbb{I}} \psi^{\mathcal{S}}_{jt} dj \qquad LSCC^{sp}_{it} := \frac{\psi^{\mathcal{S}}_{it}}{\psi^w_{it}} \qquad \dot{\psi}^{\mathcal{S}}_{it} = (\tilde{\rho} + \delta^s)\psi^{\mathcal{S}}_{it} - \Delta_i \zeta \chi \psi^{\mathcal{T}}_{it}$$

The Ramsey Problem – Solution 1

- Shadow value of wealth gives a measure of inequality

$$\widehat{\psi}_{it}^{w} = \frac{\psi_{it}^{w}}{\overline{\psi}_{t}^{w}} = \frac{\omega_i u_c(c_i, \tau_{it})}{\int_{j\in\mathbb{I}} \omega_j u_c(c_{jt}, \tau_{jt}) dj} \lesseqgtr 1$$

$$\text{low } z_i, k_i, \quad \text{high } \tau_{it} \qquad \Rightarrow \qquad \text{low } c_i, \text{ high } \psi_{i,t}^{w} \approx \omega_i u'(c_i)\, p_i > \overline{\psi}_t^{w} = \int_{\mathbb{I}} u_c(c_{jt}, \tau_{jt}) dj$$

- Shadow value of carbon and temperature gives a measure of welfare cost of carbon :

$$\psi_t^{\mathcal{S}} = \frac{\partial \mathcal{W}_t}{\partial \mathcal{S}} = \int_{j\in\mathbb{I}} \psi_{jt}^{\mathcal{S}} dj \qquad LSCC_{it}^{sp} := \frac{\psi_{it}^{\mathcal{S}}}{\psi_{it}^{w}} \qquad \dot{\psi}_{it}^{\mathcal{S}} = (\tilde{\rho} + \delta^s)\psi_{it}^{\mathcal{S}} - \Delta_i \zeta \chi \psi_{it}^{\mathcal{T}}$$

- One can reexpress the welfare cost of carbon WCC_t

$$SCC_t = -\frac{\psi_t^{\mathcal{S}}}{\overline{\psi}_t^{w}} = \mathbb{C}\text{ov}_j\left(\widehat{\psi}_{it}^{w}, LSCC_{j,t}^{sp}\right) + \mathbb{E}_j[LSCC_{j,t}^{sp}] > \mathbb{E}_j[LSCC_{j,t}^{sp}] = \overline{SCC}_t$$

The Ramsey Problem – Solution 2

- Shadow value of fossil price ϕ_t^{Ef}

$$SVF_t = \frac{\phi_t^{Ef}}{\overline{\psi}_t^k} = \int_{\mathbb{I}} \widehat{\psi}_{jt}^k e_{jt}^f dj - \partial_{q^f} \pi^f \int_{\mathbb{I}} \widehat{\psi}_{jt}^k \theta_j dj = \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}_{jt}^k, e_{jt}^f\right) - E_t^f \, \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}_{jt}^k, \theta_{jt}^f\right)$$

  - SVF is the shadow value of changing (endogenously !) the fossil price q_t^f
    - Low price q_t^f benefit fossil consumers and hurts the fossil firm owners θ_{jt}^f
    - Especially more w/ high $\widehat{\psi}_{it}^k$. Empirically, $SVF_t > 0$

The Ramsey Problem – Solution 2

- Shadow value of fossil price ϕ_t^{Ef}

$$SVF_t = \frac{\phi_t^{Ef}}{\overline{\psi}_t^k} = \int_{\mathbb{I}} \widehat{\psi}_{jt}^k e_{jt}^f dj - \partial_{q^f} \pi^f \int_{\mathbb{I}} \widehat{\psi}_{jt}^k \theta_j dj = \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}_{jt}^k, e_{jt}^f\right) - E_t^f \,\mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}_{jt}^k, \theta_{jt}^f\right)$$

  - SVF is the shadow value of changing (endogenously !) the fossil price q_t^f
    - Low price q_t^f benefit fossil consumers and hurts the fossil firm owners θ_{jt}^f
    - Especially more w/ high $\widehat{\psi}_{it}^k$. Empirically, $SVF_t > 0$
- Optimal policy for fossil energy, FOC of Ramsey planner :

$$\Big(\frac{\mathcal{Q}_{q^f}^2}{f_{ee,it}} + \mathcal{Q}_{q^f q^f}\Big)\Big[-\xi_i \frac{\psi_t^S}{\overline{\psi}_t^k} + \frac{\phi_t^{Ef}}{\overline{\psi}_t^w} \mathcal{C}_{EE}^f - \psi_{it}^w \mathbf{t}_{it}^f\Big] + \cdots = 0$$

$$\Rightarrow \qquad \widehat{\psi}_{it}^w \,\mathbf{t}_{it}^f = \xi_i SCC_t + SVF_t \mathcal{C}_{EE}^f \qquad \& \qquad \mathbf{t}_{it}^r = 0$$

- Pigouvian tax :
  - Integrate several redistribution motives : Climate SCC_t, fossil fuel price redistribution SVF
  - ***Depends*** on country's consumption level $\widehat{\psi}_{it}^w$: lower tax on poorer/high $\widehat{\psi}_{it}^k$ countries
  - Welfare costs of suboptimal taxes : proportional to $\Big(\frac{\mathcal{Q}_{q^f}^2}{f_{ee,it}} + \mathcal{Q}_{q^f q^f}\Big)$

Conclusion

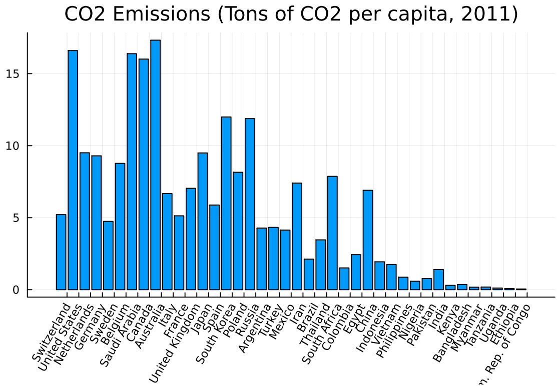
- Climate change has redistributive effects & heterogeneous impacts
- Redistributive effects of policy
  - Pigouvian tax that covers aggregate marginal damages
  - Can account for inequality both for heterogeneous welfare costs of climate and redistributive effects of energy price, for importers and exporters
- Study suboptimal policies
  - If carbon taxes are unfeasible : renewable subsidy ?
- Future plans
  - Dynamics on the capacity of renewable ?
  - Endogenous growth in TFP/energy saving technology
    Learning-by-doing : positive externality ?
  - Uncertainty (simple tree ?)

Appendices

Numerical Applications

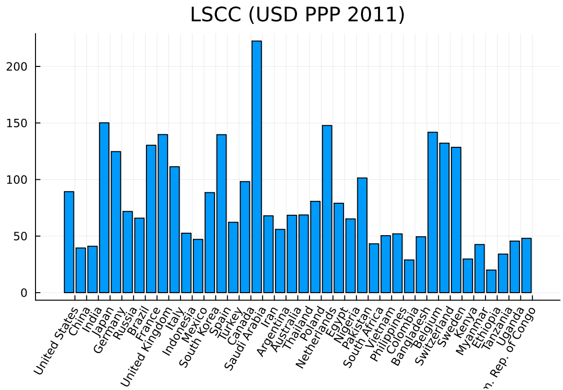
Numerical Application

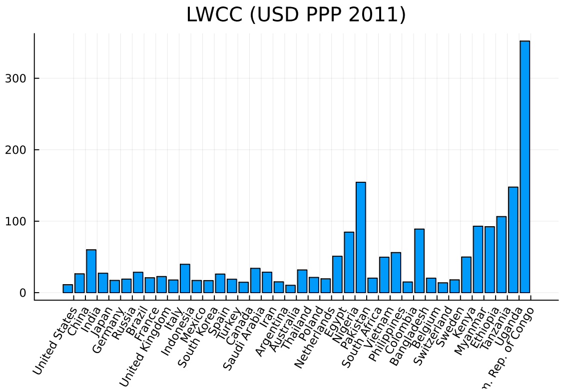
- Data : 40 countries
- Temperature (of the *largest city*), GDP, energy, population
- Calibrate z to match the distribution of output per capita at steady state



Local Cost of Carbon

- Difference $LSCC_i = \psi^{S}_{it} / \psi^{k}_{it}$ and $LWCC_{it} = \widehat{\psi}^{k}_{it} LSCC_{it} = \psi^{S}_{it} / \bar{\psi}^{k}_{it}$





Model 1

- Neoclassical economy, in continuous time Back
  - countries/regions $i \in \mathbb{I}$: ex-ante heterogeneous : productivity z_i and more
  - ex-post heterogeneity in capital and temperature $\{k_i, \tau_i\}$
- Representative household problem in each country i :

$$\mathcal{V}_{i,t_0} = \max_{\{c_{it}, e^f_{it}, e^r_{it}\}} \int_{t_0}^{\infty} e^{-\rho t}\, u(c_{it}, \tau_{it}) dt$$

- Dynamics of wealth of country i, More details with wealth $w_{it} = b_{it} + k_{it}$:

$$\dot{w}_{it} = r^{\star} w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r^{\star}_t) k_{it} + \theta_i \pi^f_t + \pi^r_{it} - (q^f_t + \mathbf{t}^f_{it}) e^f_{it} - (q^r_t + \mathbf{t}^r_{it}) e^r_{it} - c_{it} + \mathbf{t}^{ls}_{it}$$

  - Damage $\mathcal{D}^y(\tau_{it})$ affect country's production and consumption $u(\cdot, \tau_{it})$
  - Energy mix : $e_{it} = \mathscr{E}(e^f_{it}, e^r_{it} | \sigma_e)$ with fossil e^f_{it} – emitting carbon – vs. renewable e^r_t
  - Energy rents redistributed : share θ_i for fossils / fully for local renew. firm.
  - Prices, fossil q^f_t and non-carbon q^r_t (c.f. next slides)

Model 2 – Energy markets

- Fossil fuels energy producer :
  - Extended-Hotelling problem (depleting reserves with stock effects and exploration)

$$\max_{\{E_t^f, \mathcal{I}_t\}_t} \int_0^\infty e^{-\rho t} \pi_t^f(q_t^f, E_t^f, \mathcal{R}_t) dt \qquad \text{with } \pi_t(E_t^f, \mathcal{R}_t) = q_t^{e,f} E_t^f - \mathcal{C}^f(E_t^f, \mathcal{R}_t) - \mathcal{C}^i(\mathcal{I}_t, \mathcal{R}_t)$$

$$s.t. \qquad E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta_R \mathcal{I}_t$$

  - Optimal pricing with finite-resources rents More details

$$q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \mathcal{C}_{\mathcal{I}}^i(\mathcal{I}_t, \mathcal{R}_t) = \delta_R \lambda_t^R$$

Model 2 – Energy markets

- Fossil fuels energy producer :
  - Extended-Hotelling problem (depleting reserves with stock effects and exploration)

$$\max_{\{E_t^f, \mathcal{I}_t\}_t} \int_0^\infty e^{-\rho t} \pi_t^f(q_t^f, E_t^f, \mathcal{R}_t) dt \qquad \text{with } \pi_t(E_t^f, \mathcal{R}_t) = q_t^{e,f} E_t^f - \mathcal{C}^f(E_t^f, \mathcal{R}_t) - \mathcal{C}^i(\mathcal{I}_t, \mathcal{R}_t)$$

$$s.t. \qquad E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta_R \mathcal{I}_t$$

  - Optimal pricing with finite-resources rents More details

$$q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \mathcal{C}_{\mathcal{I}}^i(\mathcal{I}_t, \mathcal{R}_t) = \delta_R \lambda_t^R$$

- Renewable energy as a substitute technology *for each country* i (Static problem)

$$\pi_{it}^r = \max_{\{e_{it}^r\}} q_{it}^r e_{it}^r - \mathcal{C}^r(e_{it}^r) \qquad \Rightarrow \qquad q_{it}^r = \mathcal{C}_E^r(e_t^r)$$

Back

Model 3 - Climate model :

- Fossil energy input e_t^f causes climate externality

$$\mathcal{E}_t = \int_{\mathbb{I}} \xi_i \, e_{it}^f \, di$$

- World climate – cumulative GHG in atmosphere $\mathcal{S}_t$ leads to increase in temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- Impact of climate on country's local temperature :

$$\dot{\tau}_{i,t} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{i,t} - \bar{\tau}_{i,t_0})\big)$$

  - Simple model : Climate sensitivity to carbon χ, Climate reaction/inertia ζ, Carbon content of fossils ξ, Country scaling factor Δ_i, Carbon exit for atmosphere δ_s
  - Back

Model 4 – Household Solution

- Household solves a consumption/saving/energy decision, as in the NCG More details
  - Using Pontryagin (PMP), we obtain a system of coupled ODEs More details

Model 4 – Household Solution

- Household solves a consumption/saving/energy decision, as in the NCG More details
  - Using Pontryagin (PMP), we obtain a system of coupled ODEs More details
  - Consumption/Saving Euler equation (financial integration) :

$$\dot{c}_{it} = c_{it}\frac{1}{\eta}(r_t^\star - \rho) \qquad MPk_{it} - \bar{\delta} = r_t^\star$$

  - Energy decisions :
    Static demand for the two sources of energy : fossil e_{it}^f and renewable $e_{i,t}^r$ for every i, taking prices $\{q^f, q^r\}$ as given

$$MPe_{it} = \mathcal{Q}(q_{i,t}^f + \mathbf{t}_{it}^f, q_{i,t}^r + \mathbf{t}_{it}^r)$$

$$\Rightarrow \qquad e_{i,t}^f = \mathcal{Q}_{q^f}(q_{i,t}^f + \mathbf{t}_{it}^f, q_{i,t}^r + \mathbf{t}_{it}^r)e_{it} \qquad e_{i,t}^r = \mathcal{Q}_{q^r}(q_{i,t}^f + \mathbf{t}_{it}^f, q_{i,t}^r + \mathbf{t}_{it}^r)e_{i,t}$$

  - with $MPk_{it} = \mathcal{D}(\tau_{it})z_{i,t}f_{k,it}$ and $MPe_{it} = \mathcal{D}^y(\tau_{i,t})z_{i,t}f_e(k_{i,t}, e_{i,t})$, and $\mathcal{Q}(\cdot)$ are aggregators functions (e.g. CES) and $\mathcal{Q}_{q^f}(\cdot)$ demand for fossil.

Model – Equilibrium

- Three types of interactions Equilibrium
  - On climate (externality) + heterogeneous effects of temperatures
  - On bonds markets + capital constraints
  - On energy market + redistribution effects of energy rent
  - No bilateral flows (eq. doesn't exist with continuum and trade or migration)

Model – Equilibrium

- Three types of interactions Equilibrium
  - On climate (externality) + heterogeneous effects of temperatures
  - On bonds markets + capital constraints
  - On energy market + redistribution effects of energy rent
  - No bilateral flows (eq. doesn't exist with continuum and trade or migration)
- Equilibrium
  - Given, initial conditions $\{k_0, \tau_0\}$ and country-specific policies $\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, \mathbf{t}^{ls}_{it}\}$, a competitive equilibrium is a continuum of sequences of states $\{k_{it}, \tau_{it}\}_{i,t}$ and $\{\mathcal{S}_t, \mathcal{T}_t, \mathcal{R}_t\}_t$ and policies $\{c_{i,t}, e^f_{i,t}, e^r_{i,t}\}_{i,t}$ and $\{E^f_t, \mathcal{E}_t, \mathcal{I}_t\}_t$, and price sequences $\{q^f_t, q^r_t\}$ such that :

  - Households choose policies $\{c_{i,t}, e^f_{i,t}, e^r_{i,t}\}_{i,t}$ to max utility s.t. budget constraint, giving $\dot{k}_{it}$
  - Renewable energy firm produce $\{e^r_{it}\}$ to max static profit
  - Fossil fuel firm extract and explore $\{E^f_t, \mathcal{I}_t\}$ to max profit, yielding $\dot{\mathcal{R}}_t$
  - Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{i,t}\}_{i,t}$.
  - Prices $\{q^f_t, q^r_{it}, r^\star_t\}$ adjust to clear the markets : $E^f_t = \int_{\mathbb{I}} e^f_{it} di$ and $e^r_{it} = e^r_{it}$, and $\int_{i \in \mathbb{I}} b_{it} di = 0$

Impact of increase in temperature

- Using Damage fct $\mathcal{D}^y(\tau_{i,t}) = e^{-\frac{1}{2}\gamma_i(\tau_{i,t}-\tau_i^\star)^2}$ and $u(c,\tau) = u(\mathcal{D}^u(\tau_{i,t})c)$, w/ $u(\hat{c}) = \frac{c^{1-\eta}}{1-\eta}$
- Marginal values of the climate variables : $\lambda^S_{i,t}$ and $\lambda^\tau_{i,t}$

$$\dot{\lambda}^\tau_{i,t} = \lambda^\tau_{i,t}(\rho+\zeta) + \overbrace{\gamma_i(\tau_{i,t}-\tau_i^\star)\mathcal{D}^y(\tau_{i,t})}^{-\partial_\tau \mathcal{D}^y(\tau_{i,t})} f(k_{i,t},e_{i,t})\lambda^k_{i,t} + \overbrace{\phi_i(\tau_{i,t}-\tau_i^\star)\mathcal{D}^u(\tau_{i,t})^{1-\eta}c_{i,t}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}^S_{i,t} = \lambda^S_{i,t}(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{i,t}$$

- Costate $\lambda^S_{i,t}$: marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{i,t}-\tau_i^\star$ & damage sensitivity of TFP γ_i and utility ϕ_i
  - Development level $f(k_{i,t},e_{i,t})$ and $c_{i,t}$
  - Climate params : χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed

Local Social cost of carbon

- The marginal "externality damage" or "local social cost of carbon" (SCC) for region i :

$$LSCC_{i,t} := -\frac{\partial \mathcal{V}_{i,t}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{i,t}/\partial c_{i,t}} = -\frac{\lambda^S_{i,t}}{\lambda^k_{i,t}}$$

  - Ratio of marg. cost of carbon vs. the marg. value of consumption/capital
  - Theorem : ***Stationary LSCC*** :
    When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ, ϕ, temperature, and output, consumption.

$$LSCC_{i,t} \equiv \frac{\chi\,\Delta_i}{\widetilde{\rho} + \delta^s}\left(\tau_{i,\infty} - \tau_i^\star\right)\left[\gamma_i\, y_{i,\infty} + \phi_i\, c_{i,\infty}\right]$$

    - More general formula : Here, Proof : Here + What determine temperatures ? Details Temperature

More details – Capital market

- In each countries, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} & = \mathcal{D}_i^y(\tau_{it}) z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g}) k_{it} + \iota_{it} \\ \dot{b}_{it} & = r^\star b_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - \iota_{it} - c_{it} + \mathbf{t}_{it}^{ls} \\ b_{it} & \geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{i}^{l}$$

$$k_{it} \leq \frac{1}{1 - \vartheta} w_{it}$$

- Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it}) z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r_t^\star$$

back

More details – Energy market

- Fossil fuel producer : price the Hotelling rent with the maximum principle :

$$\mathcal{H}^m(\mathcal{R}_t, \lambda_t^R, E_t, \mathcal{I}_t^e) = \pi_t(E_t^f, \mathcal{I}_t^f, \mathcal{R}_t) + \lambda_t^R(\delta^R \mathcal{I}_t^e - E_t)$$

- Rent λ_t^R grows with interest ρ and with the marginal gain of increasing reserves

$$\begin{aligned}
\dot{\lambda}_t^R &= \rho\lambda_t^R - \partial_R \mathcal{C}(E_t^f, \mathcal{I}_t^f, \mathcal{R}_t) \\
&= \rho\lambda_t^R + \frac{\bar{\nu}\nu}{1+\nu}\left(\frac{E_t^\star}{R_t}\right)^{1+\nu} + \frac{\bar{\mu}\mu}{1+\mu}\left(\frac{I_t^\star}{R_t}\right)^{1+\mu} \\
\dot{\lambda}_t^R &= \rho\lambda_t^R + \frac{\bar{\nu}^{-1/\nu}\nu}{1+\nu}\left(q^{e,f} - \lambda_t^R\right)^{1+1/\nu} + \frac{\bar{\mu}^{-1/\mu}\mu}{1+\mu}\left(\delta^R\lambda_t^R\right)^{1+1/\mu}
\end{aligned}$$

- Because of decreasing return to scale and Hotelling rents : profits are > 0

$$\pi_t(E_t^f, \mathcal{R}_t, \lambda_t^R) = \frac{1+\nu-\bar{\nu}}{1+\nu}\left(\frac{E_t^f}{\mathcal{R}_t}\right)^{1+\nu}\mathcal{R}_t + \lambda_t^R E_t^f - \frac{\bar{\mu}^{-1/\mu}}{1+\mu}\left(\delta^r\lambda_t^R\right)^{1+\frac{1}{\mu}}$$

back

More details – PMP – Competitive equilibrium

- Household problem : State variables $s_{i,t} = (k_i, \tau_i, z_i, p_i, \Delta_i)$ Back
- Pontryagin Maximum Principle

$$\mathcal{H}^{hh}(s,\{c\},\{e^f\},\{e^r\},\{\lambda\}) = u(c_i,\tau_i) + \lambda^k_{i,t}\Big(\mathcal{D}(\tau_{it})f(k_t,e_t) - (n+\bar{g}+\delta)k_t - q^f_t e^f_{it} - q^r_{it}e^r_{it} - c_t\Big) + \lambda^\tau_{i,t}\zeta\Big(\Delta_i\,\chi\,\mathcal{S}_t - (\tau_{it}-\tau_{i0})\Big) + \lambda^S_{i,t}\Big(\mathcal{E}_t - \delta^s\mathcal{S}_t\Big)$$

$$[c_t] \qquad u'(c_{it}) = \lambda^k_{i,t}$$

$$[e^f_t] \qquad MPe^f_{it} = \mathcal{D}(\tau_{i,t})z\,\partial_e f(k_{i,t},e_{i,t})\Big(\frac{e^f_{i,t}}{\omega e_{i,t}}\Big)^{-\frac{1}{\sigma_e}} = q^f_t$$

$$[e^r_t] \qquad MPe^r_{it} = \mathcal{D}(\tau_{i,t})z\,\partial_e f(k_{i,t},e_{i,t})\Big(\frac{e^r_{i,t}}{(1-\omega)e_{i,t}}\Big)^{-\frac{1}{\sigma_e}} = q^r_{it}$$

$$[k_t] \qquad \dot{\lambda}^k_t = -\lambda^k_t\big(\mathcal{D}(\tau_{i,t})\partial_k f(k_{i,t},e_{i,t}) - \delta - \bar{g} - n - \rho\big)$$

- Fossil Energy Monopoly :

$$\mathcal{H}^m(\mathcal{R}_t,\lambda^R_t,E^f_t,\mathcal{I}_t) = \pi_t(E^f_t,\mathcal{I}_t,\mathcal{R}_t) + \lambda^R_t(\delta^R\mathcal{I}_t - E^f_t)$$

$$[\mathcal{R}_t] \qquad \dot{\lambda}^R_t = \rho\lambda^R_t + \frac{\bar{\nu}\nu}{1+\nu}\Big(\frac{E^\star_t}{R_t}\Big)^{1+\nu} + \frac{\bar{\mu}\mu}{1+\mu}\Big(\frac{I^\star_t}{R_t}\Big)^{1+\mu}$$

$$[E^f_t] \qquad q^{e,f}_t = \nu_E(E,\mathcal{R}) + \lambda^R_t = \bar{\nu}\Big(\frac{E_t}{\mathcal{R}_t}\Big)^\nu + \lambda^R_t$$

$$[\mathcal{I}_t] \qquad \lambda^R_t\delta^R = \mu_I(I_t,R_t) = \bar{\mu}\Big(\frac{\mathcal{I}_t}{\mathcal{R}_t}\Big)^\mu \qquad \mathcal{I}_t = R_t\Big(\frac{\lambda^R_t\delta}{\bar{\mu}}\Big)^{1/\mu}$$

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}^{\mathcal{T}}_{i,t} = \lambda^{\mathcal{T}}_t(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\lambda^k_t + \phi(\tau - \tau^\star)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}^S_t = \lambda^S{}_t(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda^{\mathcal{T}}_{i,t}$$

- Solving for $\lambda^{\mathcal{T}}_t$ and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^S_t = \dot{\lambda}^{\mathcal{T}}_t = 0$

$$\lambda^{\mathcal{T}}_{i,t} = -\int_t^\infty e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_u)y_{\mathcal{T}}\lambda^k_u + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda^{\mathcal{T}}_{i,t} = -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_\infty)y_\infty\lambda^k_\infty + \phi\mathcal{D}^u(\tau_\infty)u(c_\infty)\Big)$$

$$\begin{aligned}
\lambda^S_t &= -\int_t^\infty e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,u}dj\,du \\
&= \frac{1}{\widetilde{\rho} + \delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,\infty} \\
&= -\frac{\chi}{\widetilde{\rho} + \delta^s}\frac{\zeta}{\widetilde{\rho} + \zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_\infty\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj \\
\lambda^S_t &\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho} + \delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

Cost of carbon / Marginal value of temperature

- Closed form solution for CC :
  - In stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - no internalization of externality (business as usual)

$$LSCC_{i,t} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s}(\tau_\infty - \tau^\star)\Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty\Big)$$

- Heterogeneity + uncertainty about models Back

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega \int_{t_0}^{T} e^{(n+\bar{g})t-\delta_s(T-t)} q_t^{f\ -\sigma_e} \int_{j\in\mathbb{I}} \left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e-\sigma} dj\, dt$$

  - Geographical factors determining warming Δ_i
  - Climate sensitivity χ & carbon exit from atmosphere δ_s
  - Growth of population n, aggregate productivity $\bar{g}$
  - Deviation of output from trend y_i & relative TFP z_j
  - Directed technical change z_t^e, elasticity of energy in output σ
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}_t^R/\lambda_t^R = \rho$
  - Change in energy mix, renewable share ω, price q_t^r & elasticity of source σ_e

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \widetilde{\gamma}\right) + (\sigma_e - \sigma)(1-\omega)g^{q^r} - (\sigma_e(1-\omega) + \sigma\omega)g^{q^f}$$

Back

More details – PMP – Ramsey Optimal Allocation

- Hamiltonian :

$$\begin{aligned}
\mathcal{H}^{sp}(s,\{c\},\{e^f\},\{e^r\},\{\lambda\},\{\psi\}) = \int_{\mathbb{I}} \omega_i u(c_i,\tau_i) p_i di \\
+ \psi^k_{i,t}\Big(\mathcal{D}(\tau_{it}) f(k_{it},e_{it}) - (n+\bar{g}+\delta)k_t + \theta_i \pi(E^f_t,\mathcal{I}_t,\mathcal{R}_t) + \pi^r_{it}(e^r_{it}) - (q^f_t + \mathbf{t}^f_{it})e^f_{it} - (q^r_{it} + \mathbf{t}^r_{it})e^r_{it} - c_t + \mathbf{t}^{ls}_t\Big) \\
+ \psi^S_t\Big(\mathcal{E}_t - \delta^s \mathcal{S}_t\Big) + \psi^\tau_{it}\zeta\Big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{i0})\Big) + \psi^{\mathcal{R}}_{it}\Big(- E^f_t + \delta^R \mathcal{I}_t\Big) \\
+ \psi^{\lambda k}_{i,t}\Big(\lambda^k_t(\rho - r_t)\Big) + \psi^{\lambda R}_t\Big(\rho\lambda^R_t + \mathcal{C}^f_{\mathcal{R}}(E^f_t,\mathcal{I}_t,\mathcal{R}_t)\Big) + \phi^c_{it}\big(u_c(c_i,\tau_{it}) - \lambda^k_{it}\big) \\
+ \phi^{ef}_{it}\Big(e^f_{it} - \mathcal{Q}_{q^f}(q^f_t + \mathbf{t}^f_{it}, q^r_t + \mathbf{t}^r_{it})e_{it}\Big) + \phi^{er}_{it}\Big(e^r_{it} - \mathcal{Q}_{q^r}(q^f_t + \mathbf{t}^f_{it}, q^r_t + \mathbf{t}^r_{it})e_{it}\Big) \\
+ \phi^e_{it}\Big(f_e(k_{it},e_{it}) - \mathcal{Q}(q^f_t + \mathbf{t}^f_{it}, q^r_t + \mathbf{t}^r_{it})\Big) + \phi^{Ef}_t\Big(q^f_t - \mathcal{C}^f_E(\cdot) - \lambda^{\mathcal{R}}_t\Big) + \phi^{Er}_{it}\Big(q^r_{it} - \mathcal{C}^r_e(\cdot)\Big) + \phi^{\mathcal{I}f}_t\big(\delta\lambda^{\mathcal{R}}_t - \mathcal{C}^f_{\mathcal{I}}(\cdot)\big)
\end{aligned}$$

- Back

Ramsey Optimal Allocation - FOCs

- FOCs w.r.t. $\{c_{it}, e_{it}, e^f_{it}, e^r_{it}, \mathcal{I}_t\}$, prices $\{q^f_t, q^r_{it}\}$ and taxes, denoting $\tilde{q}_{it} = q_t + \mathbf{t}_{it}$

$$[c_{it}] \qquad \psi^k_{it} = \underbrace{\omega_i u_c(c_i, \tau_{it}) p_i}_{=\text{direct effect}} + \underbrace{\phi^c_{it} u_{cc}(c_i, \tau_{it})}_{=\text{effect on savings}}$$

$$[e_{it}] \qquad \psi^k_{it} f_{e,it} + \phi^e_{it} f_{ee,it} - \phi^{ef}_{it} \mathcal{Q}_{q^f} - \phi^{er}_{it} \mathcal{Q}_{q^r} = 0 \qquad \Rightarrow \qquad \phi^e_{it} = \frac{1}{f_{ee,it}} \left(\phi^{ef}_{it} \mathcal{Q}_{q^f} + \phi^{er}_{it} \mathcal{Q}_{q^r} - \psi^k_{it} f_{e,it} \right)$$

$$[e^f_{it}] \qquad \phi^{ef}_{it} = \psi^k_{it} \tilde{q}^f_t - \psi^k_{it} \mathbf{t}^f_{it} - \xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) \qquad [e^r_{it}] \qquad \phi^{er}_{it} = \psi^k_{it} \tilde{q}^r_t - \psi^k_{it} \mathbf{t}^r_{it} + \phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot)$$

$$[\tilde{q}^f_{it}] \qquad \phi^e_{it} \mathcal{Q}_{q^f} + \phi^{ef}_{it} \mathcal{Q}_{q^f q^f} + \phi^{er}_{it} \mathcal{Q}_{q^r q^f} = 0$$

$$\Rightarrow \qquad \left(\frac{\mathcal{Q}^2_{q^f}}{f_{ee,it}} + \mathcal{Q}_{q^f q^f} \right) \left[-\xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it} \mathbf{t}^f_{it} \right] + \left(\frac{\mathcal{Q}_{q^f} \mathcal{Q}_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^r q^f} \right) \left[\phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot) - \psi^k_{it} \mathbf{t}^f_{it} \right]$$

$$[\tilde{q}^r_{it}] \qquad \phi^e_{it} \mathcal{Q}_{q^r} + \phi^{ef}_{it} \mathcal{Q}_{q^f q^r} + \phi^{er}_{it} \mathcal{Q}_{q^r q^r} = 0$$

$$\Rightarrow \qquad \left(\frac{\mathcal{Q}_{q^f} \mathcal{Q}_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^f q^r} \right) \left[-\xi \psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it} \mathbf{t}^f_{it} \right] + \left(\frac{\mathcal{Q}^2_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^r q^r} \right) \left[\phi^{Er}_{it} \mathcal{C}^r_{e^r e^r}(\cdot) - \psi^k_{it} \mathbf{t}^f_{it} \right] = 0$$

$$[q^f_{it}] \qquad \phi^{Ef}_t = \int_{\mathbb{I}} \psi^k_{jt} e^f_{jt} dj - \partial_{q^f} \pi^f(\cdot) \int_{\mathbb{I}} \theta_j \psi^k_{jt} dj \qquad [q^r_{it}] \qquad \phi^{Er}_{it} = \psi^k_{it} e^r_{it} - \psi^k_{it} \partial^r_q \pi^r_{it} = 0$$

$$[\mathcal{I}_t] \qquad \delta\, \psi^{\mathcal{R}}_t + \partial^2_{\mathcal{R}\mathcal{I}} \mathcal{C}(\cdot)\, \psi^{\lambda,\mathcal{R}}_t - \phi^{\mathcal{I}}_t \partial^2_{\mathcal{I}\mathcal{I}} \mathcal{C}(\cdot) = 0$$

- Back

Ramsey Optimal Allocation - FOCs

- Backward equations for planner's costates

$$[k_i] \qquad \dot{\psi}^k_{it} = \psi^k_{it}(\tilde{\rho} - r_{it}) + \psi^{\lambda k}_{it}\lambda^k_{it}\partial_k MPk_i + \frac{f_{ek,it}}{f_{ee,it}}\left[- \xi\psi^S_t p_i + \phi^{Ef}_t \mathcal{C}^f_{EE}(\cdot) - \psi^k_{it}\mathbf{t}^f_{it}\right]$$

$$[\mathcal{S}_i] \qquad \dot{\psi}^{\mathcal{S}}_t = (\tilde{\rho} + \delta^s)\psi^{\mathcal{S}}_t - \int_{\mathbb{I}} \Delta_j \zeta\chi\psi^{\tau}_{jt}\, dj$$

$$[\tau_i] \qquad \dot{\psi}^{\tau}_t = (\tilde{\rho} + \zeta)\psi^{\tau}_t - \Big(\omega_i u_{\tau}(c_{it}, \tau_{it}) + \psi^k_{it}\mathcal{D}'(\tau_{it})f(k_{it}, e_{it}) + \phi^c_{it}u_{c,\tau}(c_{it}, \tau_{it}) + \mathcal{D}'(\tau_{it})f_e\phi^e_{it}\Big)$$

$$[\mathcal{R}] \qquad \dot{\psi}^{\mathcal{R}}_t = \psi^{\mathcal{R}}_t\Big(\tilde{\rho} - \partial^2_{\mathcal{R}\mathcal{R}}\mathcal{C}(\cdot)\Big) - \phi^{Ef}_t\partial^2_{\mathcal{R}E}\mathcal{C}(\cdot)$$

$$[\lambda^k_i] \qquad \dot{\psi}^{\lambda,k}_t = \psi^{\lambda,k}_t\big[\tilde{\rho} - (\rho - r_{i,t})\big] + \phi^c_{i,t}$$

$$[\lambda^{\mathcal{R}}_i] \qquad \dot{\psi}^{\lambda,\mathcal{R}}_t = \psi^{\lambda,\mathcal{R}}_t(\tilde{\rho} - \rho) + \phi^{Ef}_t - \delta\phi^{\mathcal{I}f}_t$$

- Back