

# The Optimal design of Climate Agreements

## Inequality and incentives for carbon policy

WORK IN PROGRESS

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*Capital theory*

November 2023

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- ▶ Trade-off between intensive margins and extensive margin :
  - Climate club with a small number of countries with (i) a tax higher/closer to the *first-best*, (ii) large emissions reductions but (iii) potentially large carbon “leakage” effects
  - Extensive climate agreements with a large number of countries but lower optimal tax to accommodate participation constraints

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  - Extensive climate agreements with a large number of countries but lower optimal tax to accommodate participation constraints
- ▶ Develop an Integrated Assessment Model (IAM) with heterogeneous countries to :
  - Evaluate the welfare costs of global warming
  - Solve optimal carbon policy of climate clubs
  - Analyze the strategic implications of joining climate agreements
  - Design to optimal size of the climate club

# Literature

- ▶ Climate change & optimal carbon taxation
  - RA model : Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
  - HA model : Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
  - Spatial models : Cruz, Rossi-Hansberg (2022, 2023)

⇒ *Optimal and constrained policy with heterogeneous countries*
- ▶ Unilateral vs. climate club policies :
  - Climate clubs : Nordhaus (2015), Non-cooperative taxation : Chari, Kehoe (1990), Suboptimal policy : Hassler, Krusell, Olovsson (2019)
  - Trade policy : Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021)

⇒ *Climate cooperation and optimal design of climate club*
- ▶ Optimal policy in heterogeneous agents models
  - Policy with limited instruments : Diamond (1973), Davila, Walther (2022)
  - Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) ...

⇒ *Application to climate and carbon taxation policy*



## Model – Household

- ▶ Deterministic Neoclassical economy, in continuous time
  - countries  $i \in \mathbb{I}$ , heterogeneous in productivity  $z_i$ /wealth  $w_{it}$ , temperature  $\tau_{it}$ , energy cost/reserves  $\mathcal{R}_{it}$
  - In each country, 4 agents : (i) household, (ii) homogeneous good firm, (iii) fossil and (iv) renewable energy producers.
- ▶ Representative household problem in each country  $i$  :

$$\mathcal{V}_{i0} = \max_{\{c_{it}, k_{it}, b_{it}\}} \int_0^{\infty} e^{-\rho t} u(c_{it}) dt$$

- ▶ Dynamics of wealth of country  $i$ ,  $\dot{w}_{it} = \dot{b}_{it} + \dot{k}_{it}$  [More details](#)

$$\dot{w}_{it} := \dot{k}_{it} + \dot{b}_{it} = w_{it}\ell_{it} + \pi_{it}^f + r_t b_{it} + (r_t - \delta)k_{it} - c_{it} + \mathbf{t}_{it}^{ls}$$

- Labor income  $w_{it}\ell_{it}$  from homogeneous good firm, profit  $\pi_{it}^f$  from fossil firm

## Model – Representative Firm

- Competitive homogeneous good producer in country  $i$

$$\max_{k_{it}, e_{it}^f, e_{it}^r} \mathcal{D}^y(\tau_{it}) z_i f(k_{it}, e_{it}^f, e_{it}^r) - w_{it} \ell_{it} - r_t k_{it} - (q_t^f + \mathfrak{t}_{it}^f) e_{it}^f - (q_t^r + \mathfrak{t}_{it}^r) e_{it}^r$$

- Energy mix with fossil  $e_{it}^f$  – emitting carbon – subject to price  $q_t^f$  and tax/subsidy  $\mathfrak{t}_{it}^f$ . Similarly “clean” renewable  $e_{it}^r$ , at price  $q_{it}^r$  and tax  $\mathfrak{t}_{it}^r$ .
- Climate externality : effect of temperature on damage/TFP,  $\mathcal{D}_i^y(\tau) \in (0, 1)$

## Model – Energy markets

► Competitive fossil fuels energy producer :

- Extraction of fossil energy  $e_{it}^x$  depleting reserves  $\mathcal{R}_{it} \Rightarrow$  Hotelling problem

$$\pi_{it}^f = \max_{e_{it}^x} q_t^f e_{it}^x - \mathcal{C}^f(e_{it}^x, \mathcal{R}_{it})$$

$$\dot{\mathcal{R}}_{it} = -e_{it}^x \quad \mathcal{R}_{i0} = \bar{\mathcal{R}}_i$$

- Fossil energy traded in international markets :

$$\sum_{\mathbb{I}} e_{it}^f = \sum_{\mathbb{I}} e_{it}^x$$

- Unique fossil price  $q_t^f$  clearing the market [More details](#)

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► Renewable energy in each country  $i$  with exogenous price  $q_{it}^r$

## Climate system

- ▶ Fossil energy input  $e_t^f$  causes climate externality

$$\mathcal{E}_t = \sum_{\mathbb{I}} e_{it}^f$$

- ▶ Cumulative GHG in atmosphere  $\mathcal{S}_t$  increases temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- ▶ Country's local temperature :

$$\tau_{it} = \bar{\tau}_{i0} + \Delta_i \mathcal{S}_t$$

- Linear model : Climate sensitivity/pattern scaling factor  $\Delta_i$ , Carbon exit from atmosphere  $\delta_s$

## Model – Solution

- ▶ Step 0 : Competitive equilibrium / Business as usual : No policy  $t_{it} = 0$
- ▶ Step 1 : First Best, All instruments available  $\{t_{it}^f, t_{it}^r, t_{it}^{ls}\}_{it}$  including transfers across countries
- ▶ Step 2 : Second best, Optimal (Ramsey) policy for a given climate club  $\mathbb{J}$
- ▶ Step 3 : Countries decide whether to join the climate club : participation constraints
- ▶ Step 4 : Optimal design of size  $\mathbb{J}$  and countries  $j \in \mathbb{J}$  in the climate agreement

# Model – Equilibrium

## ► Equilibrium

- Given, initial conditions  $\{w_{i0}, \tau_{i0}, \mathcal{R}_{i0}, \mathcal{S}_{i0}\}$  and country-specific policies  $\{t_{it}^f, t_{it}^r, t_{it}^{ls}\}$ , a **competitive equilibrium** is a continuum of sequences of states  $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$ , controls  $\{\mathbf{c}\} = \{c_{it}, b_{it}, k_{it}, e_{it}^f, e_{it}^r, e_{it}^x\}_{it}$  and price sequences  $\{\mathbf{q}\} = \{r_t^*, q_t^f, q_t^r\}$  such that :
  - Households choose policies  $\{c_{it}, b_{it}\}_{it}$  to max utility s.t. budget constraint, giving  $\dot{w}_{it}$
  - Firm choose policies  $\{k_{it}, e_{it}^f, e_{it}^r\}_{it}$  to max profit
  - Fossil and renewables firms extract/produce  $\{e_{it}^x, \bar{e}_{it}^r\}_{it}$  to max static profit, yielding  $\dot{\mathcal{R}}_t$
  - Emissions  $\mathcal{E}_t$  affects climate  $\{\mathcal{S}_t\}_t$ , &  $\{\tau_{it}\}_{it}$ .
  - Prices  $\{r_t^*, q_t^f, q_t^r\}$  adjust to clear the markets :  $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$  and  $e_{it}^r = \bar{e}_{it}^r$ , and  $\sum_{\mathbb{I}} b_{it} = 0$ , with bonds  $b_{it} = w_{it} - k_{it}$
- Pontryagin Max. Principle : costates  $\{\psi\} = \{\lambda_{it}^w, \psi_{it}^\tau, \psi_{it}^s\} \Rightarrow$  system of coupled ODEs

More details

## Step 0 : Competitive equilibrium

► Key objects :

- Marginal value of wealth  $\lambda_{it}^w = u'(c_{it})$
- Marginal value of carbon  $\psi_{it}^S$  for country  $i$
- “Local social cost of carbon” (LCC) for region  $i$  :

$$LCC_{it} := -\frac{\partial \mathcal{V}_{it} / \partial S_t}{\partial \mathcal{V}_{it} / \partial c_{it}} = -\frac{\psi_{it}^S}{\lambda_{it}^w}$$

- Stationary equilibrium closed-form formula, analogous to GHKT (2014) [Here](#)



## Step 1 : First-Best, Optimal policy with transfers

- First-Best, Maximizing welfare of the Social Planner :

$$\mathcal{W}_0 = \max_{\{\mathbf{t}, \mathbf{x}, \mathbf{c}, \mathbf{q}\}_{it}} \sum_{\mathbb{I}} \int_0^{\infty} e^{-\rho t} \omega_i u(c_{it}) dt = \sum_{\mathbb{I}} \mathcal{W}_{i0}$$

- Full set of instruments  $\mathbf{t} = \{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r, \mathbf{t}_{it}^{ls}\}$ , including transfers *across countries*

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- Full set of instruments  $\mathbf{t} = \{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r, \mathbf{t}_{it}^{ls}\}$ , including transfers *across countries*
- Key objects : Local vs. Global Social Cost of Carbon :

$$SCC_t^{fb} := -\frac{\partial \mathcal{W}_t / \partial \mathcal{S}_t}{\partial \mathcal{W}_t / \partial c_t} = -\frac{\psi_t^S}{\lambda_t^w} = -\frac{\sum_{\mathbb{I}} \psi_{it}^S di}{\frac{1}{I} \sum_{\mathbb{I}} \lambda_{it}^w di}$$

$$LCC_{it} := -\frac{\partial \mathcal{W}_{it} / \partial \mathcal{S}_t}{\partial \mathcal{W}_{it} / \partial c_{it}} = -\frac{\psi_{it}^S}{\lambda_{it}^w}$$

## Step 1 : First-Best, Optimal policy with transfers

- Proposition 1 : Optimal carbon tax :

$$t_t^S = SCC_t^{fb}$$

- Result as in Representative Agent economy, c.f. Nordhaus DICE (1996), GHKT (2014)

$$SCC_t^{fb} = -\frac{\psi_t^S}{\lambda_t^w} = -\sum_{\mathbb{I}} \frac{\psi_{it}^S}{\lambda_{it}^w} = \sum_{\mathbb{I}} LCC_{it}$$

- Lump-sum transfers redistribute across countries, s.t.

$$\omega_i u'(c_{it}) = \lambda_{it}^w = \lambda_t^w = \lambda_{jt}^w = \omega_j u'(c_{jt}) \quad \forall i, j \in \mathbb{I}$$

- Imply cross-countries lump-sum transfers  $\exists i \text{ s.t. } T_i \geq 0$  or  $\exists j \text{ s.t. } T_j \leq 0$
- There exist Pareto weights  $\{\omega_i\}$  shutting down redistribution  $T_i = 0$ , e.g.  $\omega_i = 1/u'(c_{it})$

## Step 2 : Ramsey policy with limited transfers

- Second best without access to lump-sum transfers : choice of a carbon tax  $\{t_t^f, t_t^r\}$

- Tax receipts redistributed lump-sum :  $t_{it}^{ls} = t_t^f e_{it}^f + t_t^r e_{it}^r$
- Inequality across regions :

$$\hat{\lambda}_{it}^w = \frac{\lambda_{it}^w}{\lambda_t^w} = \frac{\omega_i u'(c_{it})}{\frac{1}{I} \sum_{j \in \mathbb{I}} \omega_j u'(c_{jt})} \leq 1$$

⇒ ceteris paribus, poorer countries have higher  $\hat{\lambda}_{it}^w$

- Social Cost of Carbon integrates these inequalities :

$$SCC_t^{sb} = \sum_{i \in \mathbb{I}} \hat{\lambda}_{it}^w LCC_{it}$$

$$SCC_t^{sb} = \sum_{i \in \mathbb{I}} LCC_{it} + \text{Cov}_i(\hat{\lambda}_{it}^w, LCC_{it})$$

## Step 2 : Ramsey Problem – Optimal Carbon & Energy Policy

- Taxing fossil energy has additional redistributive effects :
  - Lower eq. fossil fuels price benefit importers and hurt exporters
  - New measure : Social Cost of Fossil (SCF)

$$SCF_t^{sb} := \frac{\partial \mathcal{W}_t / \partial E_t^f}{\partial \mathcal{W}_t / \partial c_t} = \mathcal{C}_{EE}^f \mathbb{Cov}_i \left( \hat{\lambda}_{it}^w, e_{it}^f - e_{it}^x \right) \quad \mathcal{C}_{EE}^f = \left( \sum_{i \in \mathbb{I}} (\mathcal{C}_{i, e^x e^x}^f)^{-1} \right)^{-1}$$

- with  $\mathcal{C}_{EE}^f$  and  $\mathcal{C}_{i, e^x e^x}^f \propto$  fossil energy supply elasticity

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- ▶ Proposition 2 : Optimal fossil energy tax :

$$\Rightarrow \quad t_t^f = SCC_t^{sb} + SCF_t^{sb}$$

- Social cost of carbon :  $SCC_t^{sb} = \sum_{\mathbb{I}} \hat{\lambda}_{it}^w LCC_{it}$

- ▶ Tax on enewable energy  $e_t^r$ , no externality + constant return to scale :  $t_t^r = 0$

## Step 3 : Ramsey Problem with participation constraints

- ▶ Assume countries can exit climate agreements + lump-sum transfers prohibited
  - Participation constraint, with  $\bar{c}_i$  autarky consumption (no trade in energy/assets)

$$u(c_{it}) \geq u(\bar{c}_{it}) \quad [\nu_{it}]$$

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- ▶ Proposition 3 : Second-Best without transfers & participation constraints
  - Participation incentive change our measure of inequality

$$\tilde{\lambda}_{it}^w = \frac{\omega_i u'(c_{it}) + \nu_{it} u'(c_{it})}{\frac{1}{I} \sum_{j \in \mathbb{I}} (\omega_j + \nu_{jt}) u'(c_{jt})} \neq \hat{\lambda}_{it}^w$$

- Optimal fossil energy tax :

$$\Rightarrow \quad \mathbf{t}_t^f = \textcolor{green}{SCC}_t^{pc} + \textcolor{red}{SCF}_t^{pc}$$

$$= \sum_{i \in \mathbb{I}} \tilde{\lambda}_{it}^w \textcolor{green}{LCC}_{it} + \mathcal{C}_{EE}^f \sum_{i \in \mathbb{I}} \tilde{\lambda}_{it}^w (\textcolor{red}{e}_{it}^f - \textcolor{red}{e}_{it}^x)$$



## Step 4 : the Design of a Climate agreement

- Climate Agreement planner maximizes :

$$\mathcal{W}_0(\mathbb{J}) = \max_{\{\mathbf{t}, \dots\}_{it}} \frac{1}{\mathbb{J}} \sum_{\mathbb{J}} \int_0^{\infty} e^{-\rho t} \omega_i u(c_{it}) dt$$

$$s.t. \quad u(c_{it}) \geq u(\bar{c}_i) \quad \forall t, i \in \mathbb{J}$$

- Choice of countries  $\mathbb{J} \subset \mathbb{I}$  to maximize welfare

- Other countries  $\mathbb{I} \setminus \mathbb{J}$  in autarky : own bond  $\tilde{r}$ /energy  $\tilde{q}^f$  market
- Alternative : Optimal trade tax/tariffs  $\Rightarrow$  *work in progress*

- Adding country  $j$  to  $\mathbb{J}$

- Changes the optimal tax :

$$\mathbf{t}_t^f(\mathbb{J}) = \textcolor{green}{SCC}_t^{ca}(\mathbb{J}) + \textcolor{red}{SCF}_t^{ca}(\mathbb{J}) = \sum_{i \in \mathbb{J}} \tilde{\lambda}_{it}^w \textcolor{green}{LCC}_{it} + \mathcal{C}_{EE}^f \sum_{i \in \mathbb{J}} \tilde{\lambda}_{it}^w (\textcolor{red}{e}_{it}^f - \textcolor{red}{e}_{it}^x)$$

- Change the equilibrium on energy markets :

$$\text{price } q_t^f \quad s.t. \quad \sum_{j \in \mathbb{J}} \textcolor{red}{e}_{it}^f = \sum_{j \in \mathbb{J}} e_{it}^f$$

## Step 4 : the Design of a Climate agreement

- ▶ Tradeoff extensive/intensive margin
- ▶ Reduction in emissions  $\mathcal{E} = \sum_{i \in \mathbb{I}} e_i^f$  depends both on :
  - The level of tax  $t^f$ , since high  $t^f \Leftrightarrow$  large change in emissions  $\Delta \mathcal{E}(\mathbb{J})$
  - The *number* of countries  $\mathbb{J}$  in a stable coalition

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  - The *number* of countries  $\mathbb{J}$  in a stable coalition
- ▶ Naive approach :
  - Combinatorial problem :  $\mathcal{P}(\mathbb{I})$  with  $2^{|\mathbb{I}|}$  choices

$$\max_{\mathbb{J} \in \mathcal{P}(\mathbb{I})} W_0(\mathbb{J})$$

- Search for complementarity

$$\Delta W(\mathbb{J}', j) := W(\mathbb{J}' \cup j) - W(\mathbb{J}') > \Delta W(\mathbb{J}, j) \quad \text{when } \mathbb{J}' \supset \mathbb{J} \quad \text{for all } j \in \mathbb{I}$$

- Choice of countries  $\mathbb{J}$  yields optimal tax  $t^f(\mathbb{J})$

## Step 4 : the Design of a Climate agreement

### ► Tradeoff extensive/intensive margin

### ► Alternative approach :

- From the level of the tax  $\mathfrak{t}^f(\mathbb{J})$  imposed on club  $\mathbb{J}$ , we can deduce the number of countries  $\widetilde{\mathbb{J}}$  with binding participation constraints

$$\widetilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\bar{c}_i) \quad \forall i \in \widetilde{\mathbb{J}}$$

- Find a fixed point of function  $\widetilde{\mathbb{J}} = f(\mathbb{J}, \mathfrak{t}^f)$
- Sequential approach :
  - Start from  $\mathbb{J} = \mathbb{I}$
  - Search for  $\mathfrak{t}^f$  that yield  $\mathbb{J} = f(\mathbb{J}, \mathfrak{t}^f)$
  - If  $Im(f(\mathbb{J}, \mathfrak{t}^f)) \subsetneq \mathbb{J}$  remove countries one-by-one.
  - Repeat (2-3) until convergence or unraveling

## Quantification and numerical method

### ► Quantification [More details](#)

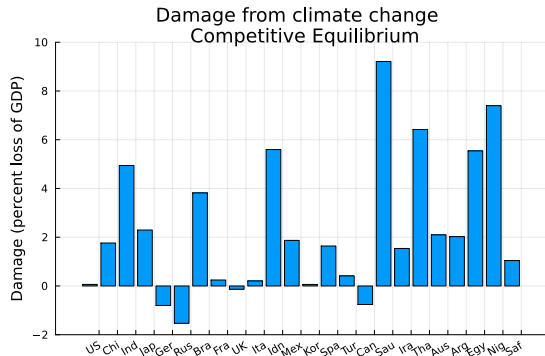
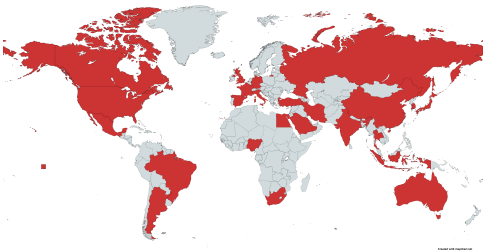
- Production  $\bar{y} = zf(k, e^f, e^r)$  with Nested CES capital/energy  $\sigma_y < 0$  and fossil/renewable  $\sigma_e > 1$ . Calibrate parameters to match GDP / energy shares data.
- Quadratic damage as in Nordhaus DICE  $y = \mathcal{D}_i(\tau)\bar{y}$  with  $\mathcal{D}_i(\tau) = e^{-\gamma(\tau-\tau_i)^2}$
- Energy parameters to match production/reserves

### ► Numerical method [More details](#)

- Sequential approach : rely on Pontryagin Maximum Principle
- Can simulate models with arbitrary numbers of dimensions of heterogeneity

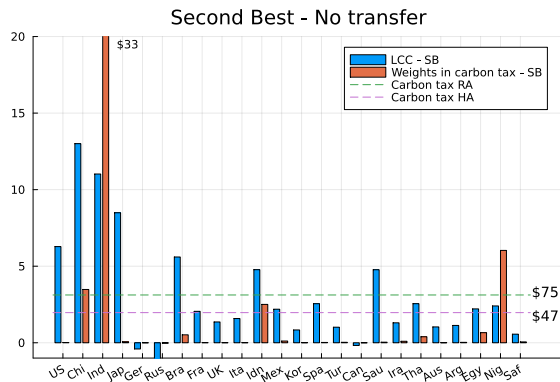
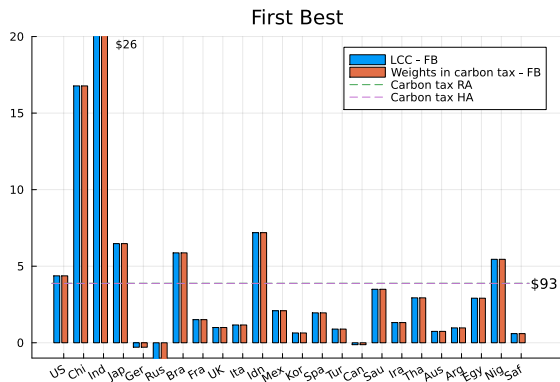
## Numerical Application – Competitive equilibrium

- Data : 24 countries, (G20+4 large countries)



# Local Cost of Carbon & Carbon Tax – First and Second Best

► Difference  $LCC_i = \frac{\psi_i^S}{\lambda_i^w}$  vs.  $\hat{\lambda}_i^w LCC_i = \frac{\psi_i^S}{\lambda_i^w}$  since  $SCC = \sum_{\mathbb{I}} \hat{\lambda}_i^w LCC_i$



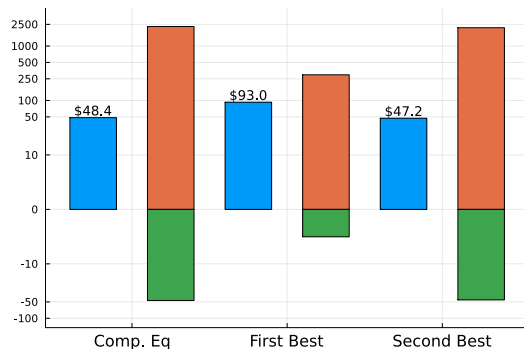
## Comparison - Value of wealth vs. Social Cost of Carbon

- Social Cost of Carbon can be decomposed :

$$SCC_t := -\frac{\partial \mathcal{W}_t / \partial \mathcal{S}_t}{\partial \mathcal{W}_t / \partial c_t} = -\frac{\psi_t^S}{\lambda_t^w} = -\frac{\sum_{\mathbb{I}} \psi_{it}^S di}{\frac{1}{I} \sum_{\mathbb{I}} \lambda_{it}^w di}$$

- Here plot that decomposition :

$$\log(SCC_t) = \log(-\psi_t^S) - \log(\lambda_t^w)$$





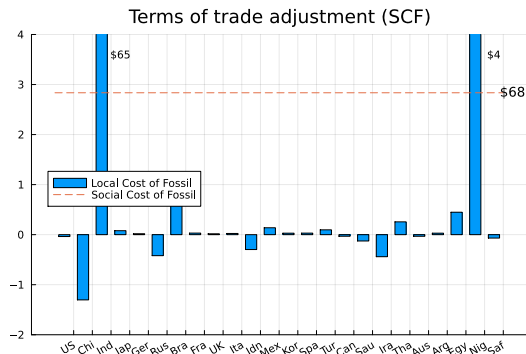
# Local Cost of Fossil and Terms of Trade Adjustment

- Social Cost of Fossil Energy :

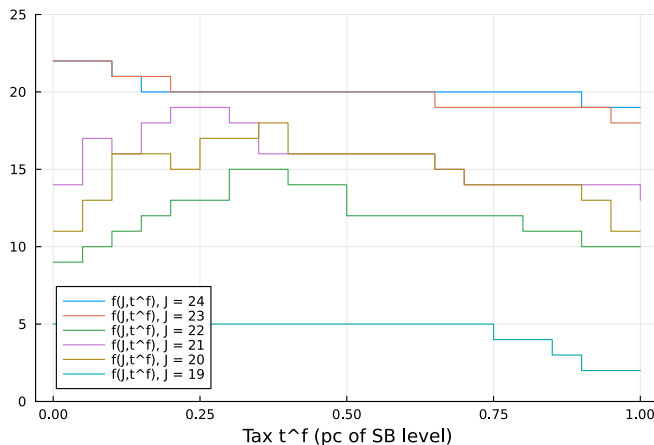
$$SCF_t = c_{EE}^f \sum_{\mathbb{I}} \hat{\lambda}_{it}^w (e_{it}^f - e_{it}^x) \quad c_{EE}^{f-1} = \sum_{\mathbb{I}} (c_{i,e^x}^f$$

- Here plotting local cost of fossil :

$$LCF_{it} = \hat{\lambda}_{it}^w (e_{it}^f - e_{it}^x)$$



# Climate club design and unraveling



- Plot of  $f(j, t^f) = \tilde{J}$ : for a club of size  $j$  and tax  $t^f$ ,  $\tilde{J}$  countries willing to participate
- Removing China (23  $\rightarrow$  22) and the US (20  $\rightarrow$  19) causes unraveling

## Climate club design and unraveling

- ▶ Unraveling caused by lack of complementarity in this economy
- ▶ Trade in good (c.f. Nordhaus 2014)
  - Possibility to include tradable at a single price  $p$  and non-tradable only consumed domestically
  - Optimal tariff/trade tax – e.g. increasing in the number of countries outside the club – can change incentives and induce complementarity
- ▶ Long-run effects of climate
  - Benefit of increasing the size of the club for curbing emissions and future damages
- ▶ Dynamics in energy markets
  - Scarcity / fossil price increasing vs. Renewable getting cheaper change energy dependence
- ▶ Accounting for dynamic participation constraints
  - If countries can join/exit later, may reinforce/dampen the benefits of joining today

## Conclusion

- ▶ Climate change has redistributive effects & heterogeneous impacts
- ▶ Optimal carbon policy takes into account inequality and redistribution
  - Depends on the availability of transfer mechanisms
  - Pigouvian tax & Social Cost of Carbon change with inequality and terms-of-trade effects
- ▶ Climate agreement design accounts jointly for
  - The optimal choice of countries participating
  - The level of tax, chosen for correcting externality & respecting participation constraints

# Appendices

## More details – Capital market

- In each country, the agent can save in two assets, capital  $k_{it}$  and bonds  $b_{it}$  :

$$\begin{cases} \dot{k}_{it} &= \mathcal{D}_i^y(\tau_{it})z_{if}(k_{it}, e_{it}) - (\delta + n + \bar{g})k_{it} + \iota_{it} \\ \dot{b}_{it} &= r^*b_{it} + \theta_i\pi_t^f + \pi_{it}^r - (q_t^f + \mathfrak{t}_{it}^f)e_{it}^f - (q_t^r + \mathfrak{t}_{it}^r)e_{it}^r - \iota_{it} - c_{it} + \mathfrak{t}_{it}^{ls} \\ b_{it} &\geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting  $\iota_{it}$  and defining wealth  $w_{it} = k_{it} + b_{it}$ , we obtain the main equation

$$\begin{aligned} \dot{w}_{it} &= r^*w_{it} + \mathcal{D}_i^y(\tau_{it})z_{if}(k_{it}, e_{it}) - (\bar{\delta} + r_t^*)k_{it} + \theta_i\pi_t^f + \pi_{it}^r - (q_t^f + \mathfrak{t}_{it}^f)e_{it}^f - (q_t^r + \mathfrak{t}_{it}^r)e_{it}^r - c_{it} + \mathfrak{t}_{it}^{ls} \\ k_{it} &\leq \frac{1}{1 - \vartheta}w_{it} \end{aligned}$$

- Two polar cases :
- $\vartheta \rightarrow 0$ , full autarky (no trade), and  $w_{it} = k_{it}$
  - $\vartheta \rightarrow 1$ , full financial integration :

$$k_{it} \quad s.t. \quad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it})z_{if}(k_{it}, e_{it}) - (\delta + n + \bar{g})k_{it} = r_t^*$$

## Impact of increase in temperature

- Marginal values of the climate variables :  $\lambda_{it}^s$  and  $\lambda_{it}^\tau$

$$\dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho + \zeta) + \overbrace{\gamma_i(\tau_{it} - \tau_i^*)\mathcal{D}^y(\tau_{it})f(k_{it}, e_{it})\lambda_{it}^k}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} + \overbrace{\phi_i(\tau_{it} - \tau_i^*)\mathcal{D}^u(\tau_{it})^{1-\eta}c_{it}^{1-\eta}}^{\partial_\tau u(c, \tau)}$$

$$\dot{\lambda}_{it}^s = \lambda_{it}^s(\rho + \delta^s) - \zeta \chi \Delta_i \lambda_{it}^\tau$$

- Costate  $\lambda_{it}^s$  : marg. cost of 1Mt carbon in atmosphere, for country  $i$ . Increases with :
- Temperature gaps  $\tau_{it} - \tau_i^*$  & damage sensitivity of TFP  $\gamma_i^y$  and utility  $\gamma_i^u$
  - Development level  $f(k_{it}, e_{it})$  and  $c_{it}$
  - Climate params :  $\chi$  climate sensitivity,  $\Delta_i$  “catching up” of  $\tau_i$  and  $\zeta$  reaction speed
  - [back](#)

## Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature  $\Leftrightarrow$  solving ODE

$$\dot{\lambda}_{it}^{\tau} = \lambda_t^{\tau}(\tilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^*)\mathcal{D}^y(\tau)f(k, e)\lambda_t^k + \phi(\tau - \tau^*)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}_t^S = \lambda_t^S(\tilde{\rho} + \delta^S) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda_{it}^{\tau}$$

- Solving for  $\lambda_t^{\tau}$  and  $\lambda_t^S$ , in stationary equilibrium  $\dot{\lambda}_t^S = \dot{\lambda}_t^{\tau} = 0$

$$\lambda_{it}^{\tau} = - \int_t^{\infty} e^{-(\tilde{\rho} + \zeta)u} (\tau_u - \tau^*) \left( \gamma \mathcal{D}^y(\tau_u) y_{\tau} \lambda_u^k + \phi \mathcal{D}^u(\tau_u) u(c_u) \right) du$$

$$\lambda_{it}^{\tau} = - \frac{1}{\tilde{\rho} + \Delta\zeta} (\tau_{\infty} - \tau^*) \left( \gamma \mathcal{D}^y(\tau_{\infty}) y_{\infty} \lambda_{\infty}^k + \phi \mathcal{D}^u(\tau_{\infty}) u(c_{\infty}) \right)$$

$$\lambda_t^S = - \int_t^{\infty} e^{-(\tilde{\rho} + \delta^S)u} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,u}^{\tau} dj du$$

$$= \frac{1}{\tilde{\rho} + \delta^S} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,\infty}^{\tau}$$

$$= - \frac{\chi}{\tilde{\rho} + \delta^S} \frac{\zeta}{\tilde{\rho} + \zeta} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left( \gamma \mathcal{D}^y(\tau_{j,\infty}) y_{\infty} \lambda_{j,\infty}^k + \phi \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

$$\lambda_t^S \xrightarrow{\zeta \rightarrow \infty} - \frac{\chi}{\tilde{\rho} + \delta^S} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left( \gamma \mathcal{D}^y(\tau_{j,\infty}) y_{j,\infty} \lambda_{j,\infty}^k + \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$



## Cost of carbon / Marginal value of temperature

### ► *Proposition (Stationary LSCC) :*

When  $t \rightarrow \infty$  and for a BGP with  $\mathcal{E}_t = \delta_s \mathcal{S}_t$  and  $\tau_t \rightarrow \tau_\infty$ , the LSCC is *proportional* to climate sensitivity  $\chi$ , **marg. damage**  $\gamma_i^y$ ,  $\gamma_i^u$ , **temperature**, and **output, consumption**.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^*) \left( \gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \right)$$

- Stationary equilibrium :  $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
- Fast temperature adjustment  $\zeta \rightarrow \infty$
- [Back](#)

## Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta \chi \xi \omega \int_{t_0}^T e^{(n+\bar{g})t - \delta_s(T-t)} q_t^f - \sigma_e \int_{j \in \mathbb{I}} (z_j z_{j,t}^e \mathcal{D}(\tau_{j,t}))^{\sigma-1} y_{j,t} q_{j,t}^{\sigma_e - \sigma} dj dt$$

- Geographical factors determining warming  $\Delta_i$
- Climate sensitivity  $\chi$  & carbon exit from atmosphere  $\delta_s$
- Growth of population  $n$ , aggregate productivity  $\bar{g}$
- Deviation of output from trend  $y_i$  & relative TFP  $z_j$
- Directed technical change  $z_t^e$ , elasticity of energy in output  $\sigma$
- Fossil energy price  $q^{ef}$  and Hotelling rent  $g^{qf} \approx \dot{\lambda}_t^R / \lambda_t^R = \rho$
- Change in energy mix, renewable share  $\omega$ , price  $q_t^r$  & elasticity of source  $\sigma_e$

- Approximations at  $T \equiv$  Generalized Kaya (or  $I = PAT$ ) identity [More details](#)

$$\frac{\dot{\tau}_T}{\tau_T} \propto n + \bar{g}^y - (1 - \sigma)(g^{z^e} - \tilde{\gamma}) + (\sigma_e - \sigma)(1 - \omega)g^{q^r} - (\sigma_e(1 - \omega) + \sigma\omega)g^{q^f}$$

## Equilibrium – Mean Field Games

- Uniqueness : Lasry-Lions monotonicity on the function :

$$\int_{\mathbb{I}} [f(t, x, \mu', \alpha) - f(t, x, \mu, \alpha)] d(\mu' - \mu)(x) \geq 0 \quad \mu, \mu' \in \mathcal{P}(\mathbb{X})$$

- Work in progress : checking such conditions along the transition

$$\sum_{i \in \mathbb{I}} (u(c^{\star}_{(w, \tau, p')}) - u(c^{\star}_{(w, \tau, p)})) [p'(w, \tau) - p(w, \tau)] \geq 0$$

with  $p'(w, \tau)$  empirical distribution  $p'(w, \tau) = \frac{1}{\mathbb{I}} \sum_{i \in \mathbb{I}} \delta_{\{x_i = (w, \tau)\}} \equiv$  population distribution !

- Mean Field approximation & Carmona Delarue (2013)

- Mean-Field is an  $\varepsilon$ -equilibrium of the  $N$ -player game when  $N \rightarrow \infty$
- Require symmetry and invariance under permutation

Back

## Sequential solution method

► Summary of the model :

- ODEs for states  $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates  $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^\tau, \lambda_{it}^s\}_{it}$
- Non-linear equations (FOCs) for household controls  $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$  and static demands for energy/capital  $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$  and static supplies  $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$ .
- Market clearing as equation for prices  $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness [More details](#)

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- Market clearing as equation for prices  $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness [More details](#)

### ► Global Numerical solution :

- Discretize agents (countries) space  $i \in \mathbb{I}$  with  $M$  and time-space  $t \in [t_0, t_T]$  with  $T$  periods
- Express as a large vector  $\mathbf{y} = \{\mathbf{x}, \boldsymbol{\lambda}, \mathbf{c}, \mathbf{q}\}$  in a large non-linear function

$$F(\mathbf{y}) = \mathbf{0}$$

- Solve for the large system with  $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$  unknowns and  $N$  equations with gradient-descent – Newton-Raphson methods.

## Sequential method : Pros and Cons

### ► Why use a sequential approach ?

- *Global approach* : Only need to follow the trajectories for  $i$  agents :
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :  
*Productivity*  $z_i$  *Population*  $p_i$ , *Temperature scaling*  $\Delta_i$ , *Fossil energy cost*  $\bar{\nu}_i$ , *Energy mix*  $\epsilon_i, \omega_i, z_i^r$ ,  
*Local damage*  $\gamma_i^y, \gamma_i^u, \tau_i^*$ , *Directed Technical Change*  $z_i^e$
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :  
*For now* : *Wealth*  $w_{it}$ , *temperature*  $\tau_{it}$ , *reserves*  $\mathcal{R}_{it}$ , *Carbon*  $S_t$   
*Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)*
  - Newton method & Non-linear solvers very efficient

### ► Why not :

- Numerical constraint to solve a large system of ODEs and non-linear equations :  
 ⇒ Constraint on  $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ , so either  $M$  or  $T$  can't be too large
- Relying on numerical solvers/structure of the problem can be opaque

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## Quantification – Firms

- Production function  $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \varepsilon(e^f, e^r)) = \left[ (1 - \epsilon_i)^{\frac{1}{\sigma_y}} k^{\alpha \frac{\sigma_y - 1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} (z_i^e \varepsilon_i(e^f, e^r))^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[ \omega_i^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1 - \omega_i)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

- Calibrate TFP  $z_i$  to match  $y_i = GDP_i$  per capita in 2011 (PPP).
- Today :  $\omega_i = \bar{\omega} = 85\%$  and  $\epsilon_i = \bar{\epsilon} = 12\%$  for all  $i$
- Future :  $(z_i^e, \omega_i, \epsilon_i)$  to match Energy/GDP  $(e_i^f + e_i^r)/y_i$  and energy mix  $(e_i^f, e_i^r)$

- Damage functions in production function  $y$  :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm, y} (\tau - \tau_i^*)^2}$$

- Asymmetry in damage to match empirics with  $\gamma^y = \gamma^{+, y} \mathbb{1}_{\{\tau > \tau_i^*\}} + \gamma^{-, y} \mathbb{1}_{\{\tau < \tau_i^*\}}$
- Today  $\gamma_i^{\pm, y} = \bar{\gamma}^{\pm, y}$  &  $\tau_i^* = \bar{\alpha} \tau_{it0} + (1 - \bar{\alpha}) \tau^*$

## Quantification – Energy markets

- ▶ Fossil production  $e_{it}^x$  and reserve  $\mathcal{R}_{it}$ 
  - Cost  $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i} \left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i} \mathcal{R}$
  - Now :  $\bar{\nu}_i$  to match extraction data  $e_i^x$ ,  $\mathcal{R}_{it}$  calibrated to *proven reserves* data from BP.  
 $\nu_i = \nu = 1$  quadratic extraction cost.
  - Future : Choose  $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$  to match marginal cost  $\mathcal{C}_e$  & extraction data  $e_i^x$  (BP, IEA)



## Quantification – Energy markets

### ► Fossil production $e_{it}^x$ and reserve $\mathcal{R}_{it}$

- Cost  $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i} \left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i} \mathcal{R}$
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 $\nu_i = \nu = 1$  quadratic extraction cost.
- Future : Choose  $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$  to match marginal cost  $\mathcal{C}_e$  & extraction data  $e_i^x$  (BP, IEA)

### ► Renewable : Production $\bar{e}_{it}^r$ and price $q_{it}^r$

- Now :  $q_{it}^r = z^r e^{-g_r t}$ , with  $g_r$  growth rate in renewable energy price decreases.
- Future : Choose  $z_i^r$  to match the energy mix  $(e_i^f, e_i^r)$

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## Quantification – Future Extensions :

### ► Damage parameters :

- $\gamma_i^{\pm,y}$  depends on daily temperature distribution  $\tau \sim \mathcal{T}_i(\bar{T}, \sigma^T)$  following Rudik et al. (2022)
- Use Climate Lab's (Greenstone et al) estimates for damage  $\gamma_i$  ?

### ► Fossil Energy markets :

- Divide fossils  $e_{it}^f / e_{it}^x$  into oil/gas/coal
- Match the production/cost/reserves data across countries
- Use a dynamic model : extraction/exploration a la Hotelling

### ► Renewables Energy markets :

- Make the problem dynamic with investment in capacity  $C_{it}^r$

### ► Population dynamics

- Match UN forecast for growth rate / fertility

# Calibration

TABLE – Baseline calibration (★ = subject to future changes)

<i>Technology &amp; Energy markets</i>			
$\alpha$	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
$\epsilon$	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
$\sigma$	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
$\omega$	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
$\sigma_e$	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
$\delta$	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	0.01★	Long run TFP growth	Conservative estimate for growth
$g_e$	0.01★	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
$g_r$	-0.01★	Long run renewable price decrease	Conservative / Match price fall in R.E.
$\nu$	2★	Extraction elasticity of fossil energy	Cubic extraction cost
<i>Preferences &amp; Time horizon</i>			
$\rho$	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
$\eta$	2.5	Risk aversion	
$n$	0.01★	Long run population growth	Conservative estimate for growth
$\omega_i$	1	Pareto weights	Uniforms / Utilitarian Social Planner
$T$	90	Time horizon	Horizon 2100 years since 2010

# Calibration

TABLE – Baseline calibration (★ = subject to future changes)

<i>Climate parameters</i>			
$\xi$	0.81	Emission factor	Conversion 1 <i>MTOE</i> $\Rightarrow$ 1 <i>MT CO<sub>2</sub></i>
$\zeta$	0.3	Inverse climate persistence / inertia	Sluggishness of temperature $\sim$ 11–15 years
$\chi$	2.1/1e6	Climate sensitivity	Pulse experiment : 100 <i>GtC</i> $\equiv$ 0.21°C medium-term warming
$\delta_s$	0.0014	Carbon exit from atmosphere	Pulse experiment : 100 <i>GtC</i> $\equiv$ 0.16°C long-term warming
$\gamma^{\oplus}$	0.00234★	Damage sensitivity	Nordhaus' DICE
$\gamma^{\ominus}$	$0.2 \times \gamma^{\oplus}$ ★	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
$\alpha^{\tau}$	0.2★	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
$\tau^{\star}$	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
<i>Parameters calibrated to match data</i>			
$p_i$		Population	Data – World Bank 2011
$z_i$		TFP	To match GDP Data – World Bank 2011
$\tau_i$		Local Temperature	To match temperature of largest city
$\mathcal{R}_i$		Local Fossil reserves	To match data from BP Energy review

## Step 4 : the Design of a Climate agreement

- Welfare effect : 1st order :

$$\delta(\mathbb{J}, j) = \mathcal{W}_{t_0}(\mathbb{J} \cup \{j\}) - \mathcal{W}_{t_0}(\mathbb{J}) = \omega_j u(c_{jt}) + \sum_{i \in \mathbb{J}} \Delta \mathcal{W}_i$$

$$\begin{aligned} \Delta \mathcal{W}_i \approx d\mathcal{W}_i = & \lambda_i^w (1 - \theta_i) \underbrace{(\epsilon_i^e + (1 - \epsilon_i^e) \alpha \sigma_i^{k/e})}_{\text{production } f(k,e)} \underbrace{(-\omega_i^f \sigma_i^f + (1 - \omega_i^f) \sigma_i^{r/f})}_{\text{energy use } \varepsilon(e^f, e^r)} \left( \underbrace{\frac{t^f}{q^f + t^f} \frac{dt^f}{t^f}}_{\text{tax change}} + \underbrace{\frac{q^f}{q^f + t^f} \frac{dq^f}{q^f}}_{\text{GE effect}} \right) \\ & + \lambda_i^w \underbrace{\theta_i (1 + \frac{1}{\nu_i})}_{\text{fossil rent/supply}} \underbrace{\frac{q^f}{q^f + t^f} \frac{dq^f}{q^f}}_{\text{GE effect}} + \underbrace{\psi_i^S}_{\tau_i \text{ damage}} \left[ \underbrace{\chi \sum_{j \in \mathbb{I}} \varepsilon_j \sigma_j^f}_{\text{climate sens}^{ty}} \right] \left( \underbrace{\frac{t^f}{q^f + t^f} \frac{dt^f}{t^f}}_{\text{tax change}} + \underbrace{\frac{q^f}{q^f + t^f} \frac{dq^f}{q^f}}_{\text{GE effect}} \right) \end{aligned}$$

- **Direct effect** on energy use on production and substitutability with renewable  
 cost-share  $\epsilon_e$ , fossil-share  $\omega_i$ , elasticity  $\sigma_j^f$  & capital-energy cross elast<sup>ty</sup>.  $\sigma_{k,e}$ , fossil-renewable cross elast<sup>ty</sup>.  $\sigma_i^{r/f}$
- **Indirect effect** through energy market fossil rent  $\theta_i$ , supply elasticity  $\nu_i$
- **Indirect climate effect** of a reduction in world emissions