The Optimal Design of Climate Agreements
Inequality, Trade, and Incentives for carbon policy

WORK IN PROGRESS

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Introduction

▶ Fighting climate change requires implementing ambitious carbon reduction policies

 • However, one of the sources of international inaction is the "free-riding problem"

 • Implementing taxation of carbon and fossil fuels is costly for individual countries

 • Moreover, such climate policy redistributes across countries through
   (i) energy markets (ii) change in climate, and (iii) reallocation of activity through trade

▶ One proposal to solve climate inaction is the idea of "climate club" – Nordhaus (2015)
 • Climate coalitions taxing carbon are inherently "unstable"
 • Trade sanctions need to be imposed on non-participants to sustain a "club" and reduce
   emissions meaningfully

⇒ How can we design an optimal climate agreement that implements the optimal energy
   taxation in the presence of inequality and policy constraints?

 • Climate agreement boils down to a carbon price, a tariff rate and a set of countries

Introduction – this project

- ▶ Trade-off between intensive margin and extensive margin:
  - – Agreement with a small set of countries, high tax, large emissions reductions for members
  - – Extensive agreements with a large number of countries but lower optimal tax to accommodate participation constraints
  - • Trade tariffs/sanctions for non-members are crucial for the stability of the agreement

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▶ Build an Integrated Assessment Model (IAM) with heterogeneous countries & trade to:
  ○ Evaluate the welfare costs of global warming and solve optimal carbon policy
  ○ Analyze the strategic implications of joining climate agreements
  ○ Design the optimal size of the climate club

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▶ Preview of the results:

  • For "Coalitional Nash" climate agreement – the world optimal policy is sustainable: high carbon tax, high tariffs, participation of the entire world

Literature

- ► Climate change & optimal carbon taxation
    - RA model: Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
    - HA model: Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
    - Spatial models: Cruz, Rossi-Hansberg (2022, 2023) among others
    - ⇒ *Optimal and constrained policy with heterogeneous countries & trade*
- ► Unilateral vs. climate club policies:
    - Climate clubs: Nordhaus (2015), Non-cooperative taxation: Chari, Kehoe (1990), Suboptimal policy: Hassler, Krusell, Olovsson (2019)
    - Trade policy: Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021), Hsiao (2022), Costinot, Donalson, Vogel, Werning (2015), Adao, Costinot (2022), Antras, Fort, Gutierrez, Tintelnot (2022)
    - ⇒ *Climate cooperation and optimal design of climate club*
- ► Optimal policy in heterogeneous agents models
    - Policy with limited instruments: Diamond (1973), Davila, Walther (2022)
    - Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) . . .
    - ⇒ *Application to climate and carbon taxation policy*

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Literature

- ▶ Nordhaus (2015)
  - Examine "stable climate coalitions" (club imposing carbon tax) in a simple model
  - Abstract from General Equilibrium and distributional effects
  - Results: Penalty tariffs necessary to enforce a climate club

- ▶ Farrokhi, Lashkaripour (2021)
  - Study and characterize the optimal trade policy with climate externality
  - General static trade model. Results: unilateral tariffs not effective
  - Sequential search for one stable climate club if EU or US join.

- ▶ Main contribution:
  - Search for the *optimal* climate agreement
  - GE on good and energy market and redistribution effects are first-order
  - Cost of climate change is endogenous to policy (damages are non-linear)
  - Possibility of analyzing other distributional policies (transfers, *loss and damage funds*)
  - General framework for analyzing macrodynamics

Model – Household

▶ Deterministic Neoclassical economy, static (for today)

  – countries $i \in \mathbb{I}$, heterogeneous in productivity z_i, temperature T_i, energy extraction cost $\mathcal{C}_i$
  – In each country, 3 agents:
    (i) Household, (ii) final good firm, (iii) (fossil) energy producer

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1. Representative household problem in each country i:

$$\mathcal{V}_i = \max_{c_{ij}} u(c_i) \qquad \mathbb{P}_i c_i = w_i + \pi_i^f + \mathsf{t}_i^{ls} \qquad c_i = \left(\sum_j a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$

$$c_{ij} \quad \text{at price} \quad \overbrace{(1+\mathsf{t}_{ij}^b)}^{\text{tariff}} \overbrace{\tau_{ij}}^{\text{iceberg cost}} \mathsf{p}_j$$

$$c_i \quad \text{at price} \quad \mathbb{P}_i = \left(\sum_j a_{ij}[(1+\mathsf{t}_{ij}^b)\tau_{ij}\mathsf{p}_j]^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

- Labor income w_i from final good firm (labor normalized to 1), profit π_i^f from fossil firm

Model – Firms

2. Competitive final good producer in country i

$$\max_{e_i^f} \; p_i \, \mathcal{D}(T_i) \, z_i f(e_i^f, e_i^r) - w_i - (q^f + t_i^f) e_i^f$$

- Productivity / TFP residual z_i $\Rightarrow$ creates inequalities across countries
- Fossil energy demand per unit of labor e_i^f, emitting carbon
  Fossil energy price q^f and tax/subsidy t^f
- Renewable energy e_i^r at constant price q^r
- Climate externality on temperature T_i
  - Damage affect productivity: $\mathcal{D}(T) \in (0, 1]$

Model – Energy markets & Emissions

▶ Competitive fossil fuels energy producer:

- Supply fossil energy e_i^x by extraction at cost $\mathcal{C}_i^f(e_i^x)$

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x)\mathbb{P}_i$$

- Energy traded in international markets, at price q^f

$$E = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

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▶ Climate system

- Fossil energy e^f releases GHGs in the atmosphere

$$\mathcal{E} = \sum_{\mathbb{I}} e_i^f$$

- Country i's local temperature:

$$T_i = \bar{T}_{i0} + \Delta_i \mathcal{E}$$

  – Linear model + Climate sensitivity/pattern scaling factor Δ_i

Model – Equilibrium

- Given policies $\{t_i^f, t_{ij}^b, t_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_{ij}, e_i^f, e_i^x\}_{ij}$, states $\{T_i\}_i$ and prices $\{p_i, w_i\}_i, q^f$ such that:

○ Households choose $\{c_{ij}\}_{ij}$ to max. utility s.t. budget constraint

○ Firm choose inputs $\{e_i^f, e_i^r\}_i$ to max. profit

○ Fossil firms extract/produce $\{e_i^x\}_i$ to max. profit. + Elastic renewable supply $\{e_i^r\}$

○ Emissions $\mathcal{E}$ affects climate $\{T_i\}_i$.

○ Government budget clear $\sum_i t_i^{ls} = \sum_i t_i^f e_i^f + \sum_{i,j} t_{ij}^b c_{ij} \tau_{ij} p_j$

○ Prices $\{p_i, w_i, q^f\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_{it}^x = \sum_{\mathbb{I}} e_{it}^f$ and for each good

$$y_i := \mathcal{D}(T_i)\, z_i f(e_i^f) = \sum_{k \in \mathbb{I}} \tau_{ki} c_{ki} + \sum_{k \in \mathbb{I}} g_{ki}$$

with g_{ki} net export of good i to pay for energy in k
In expenditure, with import shares $s_{ij} = \frac{c_{ij} \tau_{ij} p_j}{c_i \mathbb{P}_i}$, it yields $\quad p_i y_i = \sum_{k \in \mathbb{I}} \frac{s_{ki}}{1 + t_{ki}^b} \left(p_k y_k + \widetilde{t}_k^{ls} \right)$

Model – Extensions & Dynamics

▶ Quantitative model (today)
  • Use capital as well to produce at interest $r = \rho$ (BGP)

▶ Dynamics, extensions:
  1. Energy market:
     – Fossil energy extraction/exploration reserves $\Rightarrow$ Hotelling problem
  2. Households
     – Consumption / saving in bonds and in capital $\Rightarrow$ Euler equation, Keynes-Ramsey rule
     – International markets to borrow/lend bonds
  3. Climate system with inertia: standard IAMs
  4. Population dynamics
  5. (Exogenous) growth: TFP change and Energy-augmenting Directed TC.

Benchmark: Optimal world policy – Summary of results

▶ Consider a social planner maximizing world's welfare:

- Choose a single carbon tax $\mathfrak{t}^f$ for the world

$$\mathcal{W} = \max_{\{\mathbf{t},\mathbf{c},\mathbf{e}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

▶ Summary:

- Optimal climate policy depends on the availability of redistribution instruments (i.e. lump-sum transfers $\mathfrak{t}_i^{ls}$ across countries)
- Without redistribution motives, optimal Pigouvian carbon tax: $\mathfrak{t}^f = SCC$
- Otherwise, optimal carbon tax should account for
  (i) inequality and local climate damage, (ii) energy supply elasticities, (iii) energy terms-of-trade redistribution effects, (iv) energy demand distortions

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▶ Details:
  - ***Competitive equilibrium*** Details eq 0
  - ***First-Best***, with unlimited instruments Details eq 1
  - ***Second-best***, Ramsey policy with limited instruments Details eq 2

Ramsey Problem with participation constraints

► Consider that countries can "exit" climate agreement.

► *Definition* A climate agreement is a set $\{\mathbb{J}, t^f, t^b\}$, with $\mathbb{J} \subseteq \mathbb{I}$ countries and a C.E. $\{\boldsymbol{c}, \boldsymbol{e}, \boldsymbol{q}\}$ such that:
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f on fossil energy
  - Countries can leave:
    If j exits the agreement, club members $i \in \mathbb{J}$ pay uniform tariffs $t_{ij}^b = t^b$ on goods from j.
    They still trade with club members in energy at price q^f. They obtain $\hat{c}_j$

  - Exit decision:
    Subcoalition exit: only $\hat{\mathbb{J}}$ stay in the agreement, "Coalitional-Nash" / "Core"

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► Participation constraints:

$$u(c_i) \geq u(\hat{c}_i(\hat{\mathbb{J}})) \qquad [\nu_i] \qquad \forall \, \hat{\mathbb{J}} \subseteq \mathbb{J} \backslash \{i\}$$

Stable climate agreements

▶ Consider a climate agreement $\{\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b\}$

- It is a Nash equilibrium if it is stable to unilateral deviation, $\hat{\mathbb{J}} = \mathbb{J}\backslash\{i\}$

- It is a coalitional Nash eq. (or "core") $\mathbb{C}(\mathfrak{t}^f, \mathfrak{t}^b)$ if it robust to deviation of sub-coalitions
  $\hat{\mathbb{J}} \subseteq \mathbb{J}\backslash\{i\}$

    – i.e. no subcoalition would be better off than in the current aggreement $\mathbb{J}$

    – note: the "core" $\mathbb{C}(\mathfrak{t}^f, \mathfrak{t}^b)$ can be empty

▶ Objective: search for the optimal climate agreement such that:

$$\max_{\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b} \mathcal{W}(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b) \qquad s.t. \qquad \mathbb{J} \in \mathbb{C}(\mathfrak{t}^f, \mathfrak{t}^b)$$

▶ Welfare, for coalition $\mathbb{J}$, weighting all countries $i \in \mathbb{I}$

$$\mathcal{W}(\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b) = \sum_{i \in \mathbb{I}} \omega_i\, u(c_i) \qquad s.t. \qquad \{\boldsymbol{c}, \boldsymbol{e}, \boldsymbol{q}\}_i \quad \text{is a C.E. for agreement} \quad \{\mathbb{J}, \mathfrak{t}^f, \mathfrak{t}^b\}$$

Approach: joint solution of policy and coalition

▶ Choosing policy $\{t^f, t^b\}$ at the same time as the set of countries $\mathbb{J}$

  • From one agreement $\{\mathbb{J}, t^f, t^b\}$, one can deduce the set $\widetilde{\mathbb{J}}$ of countries with binding participation constraints

$$\widetilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\hat{c}_i) \quad \forall i \in \widetilde{\mathbb{J}} \quad \& \quad \forall \hat{\mathbb{J}} \subseteq \mathbb{J} \backslash \{i\}$$

  • Search for the couple $\{t^f, t^b\}$ such that $\mathbb{J} = \widetilde{\mathbb{J}}$
    – Difficult (but feasible)
    – Can be extended to dynamic settings:
      choose a path of $\{t^f_t, t^b_t\}$ instead of a path of combinations of clubs

  • Today: Brute force solution with a small number of countries

  • Heuristics for an algorithm:
    – Start from the world agreement $\{\mathbb{I}, t^{f\star}, t^{b\star}\}$
    – For $\{\mathbb{J}, t^f, t^b\}$, deduce the country with binding participation constraints $\widetilde{\mathbb{J}} = f(\mathbb{J}, t^f, t^b)$
    – Search for a fixed point $\mathbb{J} = f(\mathbb{J}, t^f, t^b)$

Leverage the combinatorial discrete choice literature

▶ Trade Literature on sourcing decisions:

– Combinatorial discrete choice , c.f. Antras, Fort, Tintelnot (2017), Jia (2008), Arkolakis, Eckert, Shi (2023), Alfaro-Ureña, Castro-Vincenzi, Fanelli, Morales (2024)

▶ Search for complementarity, given **one** policy $\{t^f, t^b\}$

• Key concept: Single crossing difference in choice - from below (SCD-C):

$$\Delta \mathcal{W}(\mathbb{J}', j) := \mathcal{W}(\mathbb{J}' \cup j) - \mathcal{W}(\mathbb{J}') > \Delta \mathcal{W}(\mathbb{J}, j) \qquad \text{when } \mathbb{J}' \supseteq \mathbb{J} \qquad \text{for all } j \in \mathbb{I}$$

• Adding an extra member j is increasingly profitable with the size of the club $\mathbb{J}$

• Can use a "squeezing procedure" to bypass the combinatorial problem

▶ Search optimal combination for **all** policies $\{t^f, t^b\}$?

• Single crossing difference in type (SCD-T):

$$\text{Define} \quad \Lambda_i(\mathbb{J}) = \{t^f, t^b \in \mathbb{R}_+^2 \mid \Delta \mathcal{W}(\mathbb{J}, i) > 0\}$$

• Welfare function has SCD-T if $\Lambda_i(\mathbb{J})$ is a connected set

Quantification

- ▶ Quadratic damage as in Nordhaus DICE $y = \mathcal{D}_i(T)\bar{y}$ with $\mathcal{D}_i(T) = e^{-\gamma_i(T-T_{i0})^2}$

- ▶ Energy parameters to match production/reserves,
  Isoelastic cost function $\mathcal{C}_i(e_i^x) = \bar{\nu}_i(e_i^x/\mathcal{R}_i)^{1+\nu}\mathcal{R}_i$

- ▶ Armington model, distance τ_{ij} and preferences a_{ij} to match import shares $s_{ij} = \frac{c_{ij}\tau_{ij}p_j}{c_i\mathbb{P}_i}$

- ▶ Production $\bar{y} = zf(\ell_i, k_i, e_i^f, e_i^r)$
  - Nested CES energy vs. labor-capital Cobb-Douglas bundle $k_i^\alpha \ell_i^{1-\alpha}$ (elasticity $\sigma_y < 1$),
    and fossil/renewable $\sigma_e > 1$, $CES(e_i^f, e_i^r)$
  - TFP, and DTC, z_i, z_i^e, calibrated to match GDP / energy shares data.

- ▶ Details More details

Welfare and Pareto weights

- Welfare:

$$\mathcal{W}(\mathbb{J}) = \sum_{i \in \mathbb{I}} \omega_i \, u(c_i)$$

- Pareto weights ω_i:

$$\omega_i = \frac{1}{u'(\bar{c}_i)}$$

for $\bar{c}_i$ consumption in initial equilibrium "without climate change", i.e. year $= 2000$
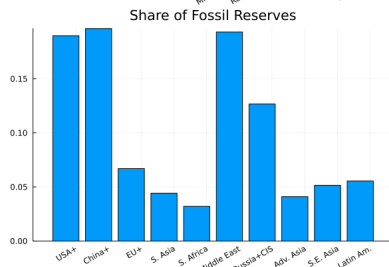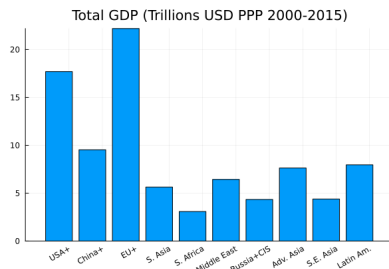
- Imply no redistribution motive in $t = 2000$

$$\omega_i u'(\bar{c}_i) = \omega_j u'(\bar{c}_j) \qquad \forall i, j \, \in \mathbb{I}$$

- Climate change, taxation, and climate agreement (tax + tariffs) have redistributive effects $\Rightarrow$ change distribution of c_i.
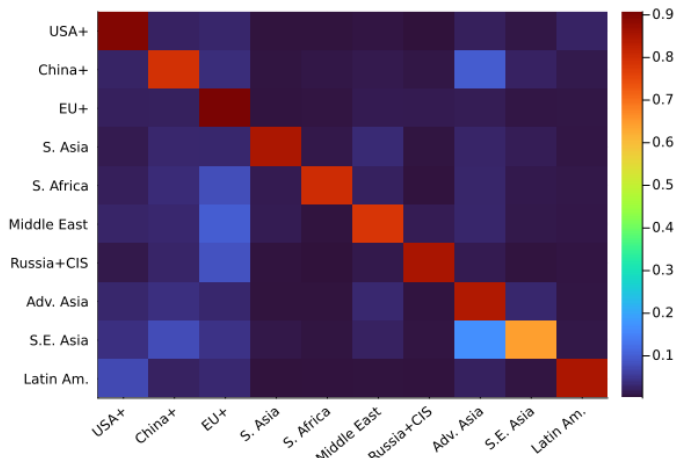

Pareto weights, competitive eq. 2000
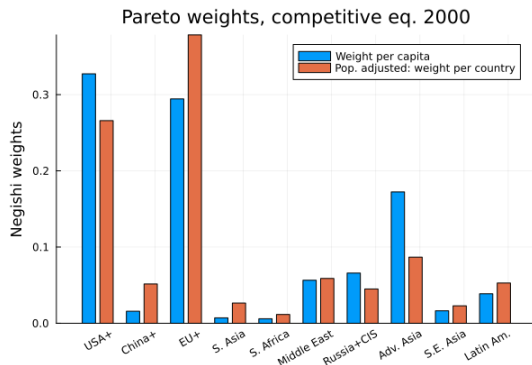
Numerical Application - Sample of "10 regions"

Total GDP (Trillions USD PPP 2000-2015)

Share of Fossil Reserves

▶ Data on trade shares $s_{ij} = \frac{c_{ij}\tau_{ij}\mathbb{p}_j}{c_i\mathbb{P}_i}$, 10 regions, Average 2000-2015

Cost of Climate Change

▶ Trade reallocation, change in shares s_{ij} due to climate change

Welfare loss, Climate change, 2050 vs. 2000
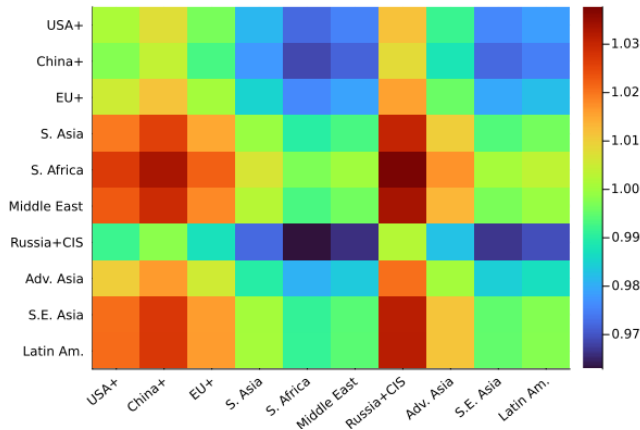
Gains from trade

▶ Welfare loss of i from increasing trade tariffs of all countries $\mathbb{I}\backslash\{i\}$ on variety from country i

- Heterogeneous effects depending energy exporter status
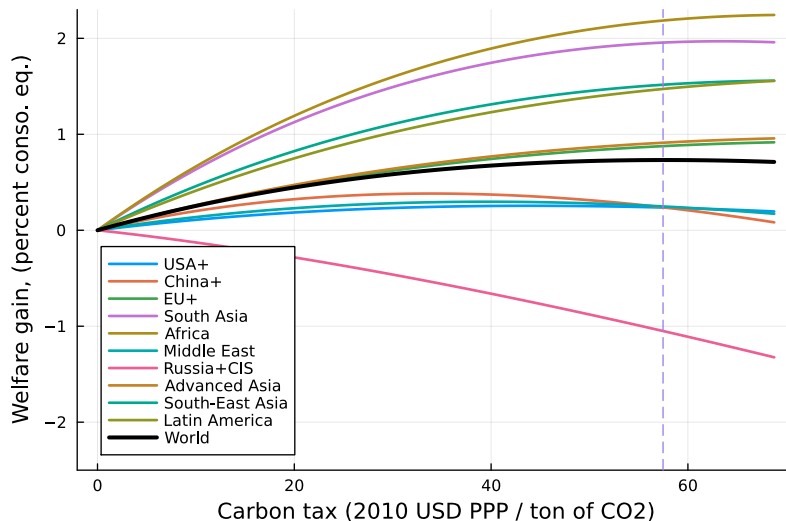
- Fossil exporters benefit from constant energy rent
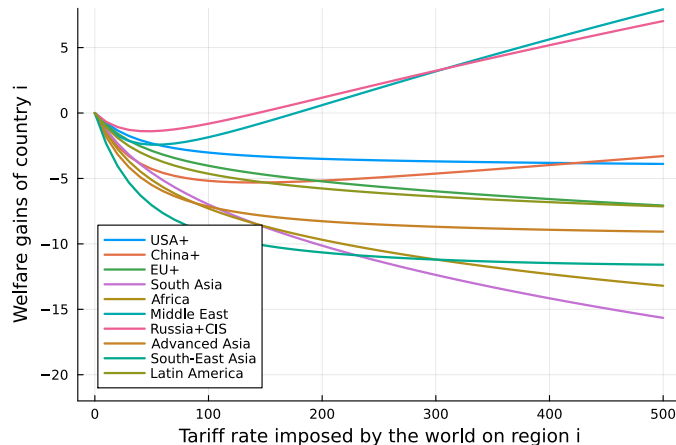
Gains from cooperation – Second Best

- Optimal carbon tax, Second Best: $\sim \$55/tCO_2 \ (\sim \$205/tC)$

- Reduce fossil fuels / CO_2 emissions by 22% compared to Business as Usual (BAU)

- Welfare difference between World Second-Best Policy and BAU (Comp. Eq.)
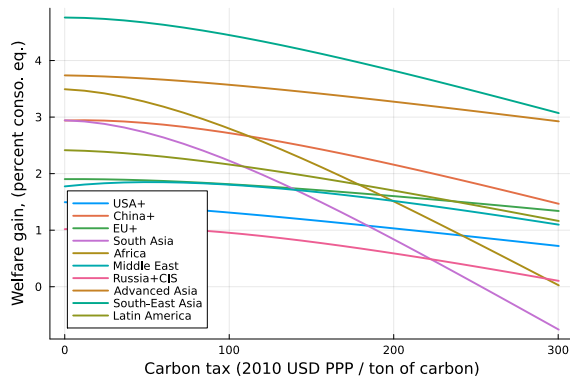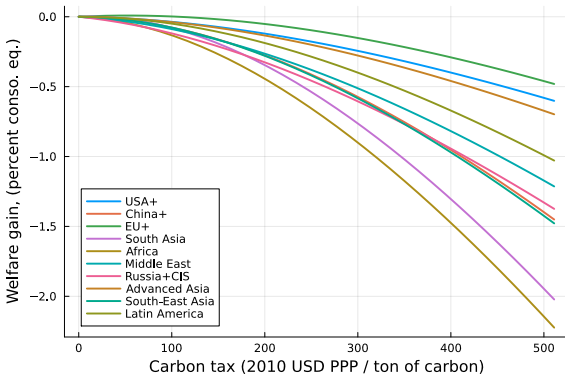

Welfare loss, Second Best vs. Comp. Eq.
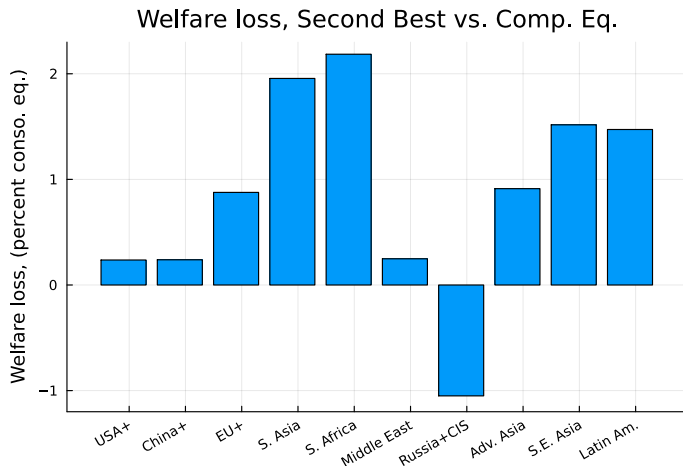
Gains from cooperation – Second Best – Tax variation

Free riding problem ... solved with trade tariffs

▶ Welfare difference between World Second-Best Policy and Unilateral deviation
  – Recover Nordhaus (2015) result

Taxes combination can recover any climate coalition

► Choice of any couples $(t^f, t^b) \in \mathbb{R}^2_+$ allow to enforce any coalitions (any number of countries)

► Trade penalties change the country's outside options, ruling out unilateral deviations

⇒ One can reproduce the second-best: full-cooperation, high-tax and maximum welfare
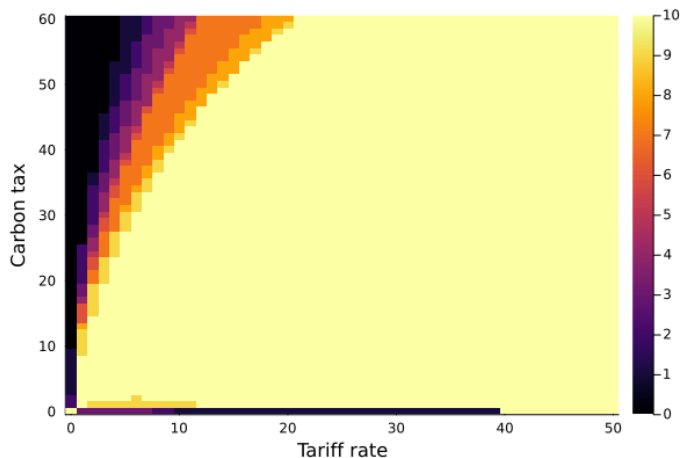
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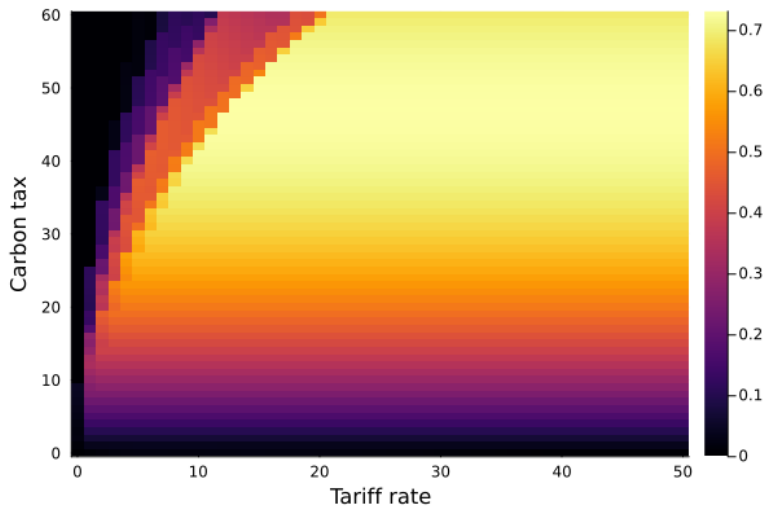
Comparative statics - 2075 (2.1°) instead of 2025 (1.35°)
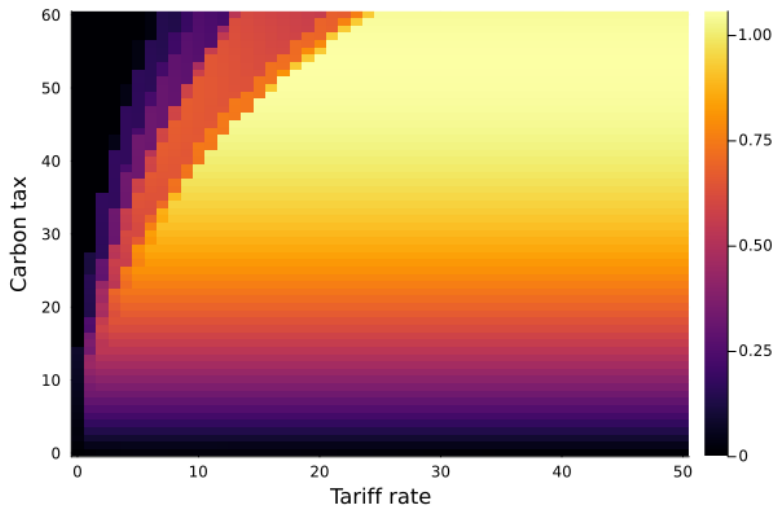
Coalition Nash equilibrium – Optimal agreement is the Second-Best

▶ With flexible t^f, t^b, the "agreement designer" can reproduce the world's optimal policy
  • Carbon taxation corrects externality, accounting for terms-of-trade / redistribution effects
  • Trade tariffs serve as penalties to enforce the stability of the club

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▶ Same mechanisms with conditional transfers (loss and damages funds – COP27)
  - Ensure stability (change one side of the participation constraint)
  - How to decide/rationalize who "deserves" the funds? the poorest? the most vulnerable? or the countries with the highest outside options?
  - Coase type of arguments: harder to bargain on I-instruments (c.f. Weitzman 2014)

▶ Obvious conclusion?
  - With enough instruments, easy to reach full coordination
  - In practice, coordination failure to implement binding agreements

Conclusion

▶ In this project, I solve for the optimal climate policy
  • Accounting for inequality as it depends on the availability of transfer mechanisms
  • Redistributing through GE effects on energy and good markets ⇒ terms-of-trade effects

▶ Climate agreement design jointly solves for:
  • The optimal choice of countries participating
  • The carbon tax level, both for correcting externality & respecting participation constraints

▶ Can reproduce any coalition with arbitrary trade tariffs or conditional transfers
  • Can achieve the second-best, world climate agreement and largest emission reductions
  • Objective to extend this to dynamic settings where the tradeoffs are less obvious/more realistic.

Appendices

Step 0: Competitive equilibrium & Trade

▶ Each household in country *i* maximize utility and firms maximize profit

▶ Standard trade model results:

- Consumption and trade:

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i \mathbb{P}_i} = a_{ij} \frac{(\tau_{ij}(1+\mathrm{t}_{ij}^b)\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}(\tau_{ik}(1+\mathrm{t}_{ik}^b)\mathrm{p}_k)^{1-\theta}} \qquad \& \qquad \mathbb{P}_i = \Big(\sum_j a_{ij}(\tau_{ij}\mathrm{p}_j)^{1-\theta} \Big)^{\frac{1}{1-\theta}}$$

- Energy consumption doesn't internalize climate damage:

$$\mathrm{p}_i MPe_i = q^e$$

- Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

- "Local Social Cost of Carbon", for region *i*

$$LCC_i = \frac{\partial \mathcal{W}_i/\partial \mathcal{E}}{\partial \mathcal{W}_i/\partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i)z_i f(e_i^f)\frac{\mathrm{p}_i}{\mathbb{P}_i} \qquad (> 0 \text{ if heat causes losses})$$

back

Step 1: World First-best policy

▶ Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- Full array of instruments: cross-countries lump-sum transfers t_i^{ls}, individual carbon taxes t_i^f on energy e_i^f, bilateral tariffs t_{ij}^b
- Budget constraint: $\sum_i t_i^{ls} = \sum_i t_i^f e_i^f + \sum_{i,j} t_{ij}^b c_{ij} \tau_{ij} p_j$

▶ Maximize welfare subject to
- Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1: World First-best policy

▶ Social planner results:
  • Consumption:
$$\omega_i u'(c_i) = \Big[\sum_j a_{ij}(\tau_{ij}\omega_j\mu_j)^{1-\theta} \Big]^{\frac{1}{1-\theta}}$$

  • Energy use:
$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

  • Social cost of carbon:
$$SCC = -\frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}'_j(T_j)\bar{y}_j}{\frac{1}{I}\sum_j \omega_j\mu_j}$$

back

Step 2: World optimal Ramsey policy

▶ Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

- One single instrument: uniform carbon tax t^f on energy e_i^f
- Rebate tax lump-sum to HHs $t_i^{ls} = t^f e_i^f$

▶ Ramsey policy: Primal approach, maximize welfare subject to
- Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
- Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
- Trade-off faced by the planner:
  - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

back

Step 2: World optimal Ramsey policy

▶ The planner takes into account
  - (i) the marginal value of wealth λ_i
  - (ii) the shadow value of good i, from market clearing, μ_i:

$$\text{w/o trade} \qquad \omega_i u'(c_i) = \omega_i \lambda_i$$

$$\text{vs. w/ trade in goods:} \qquad \omega_i u'(c_i) = \Big(\sum_{j \in \mathbb{I}} a_{ij}(\tau_{ij} \mathsf{p}_j)^{1-\theta} \Big[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij}(1 - s_{ij}) \Big]^{1-\theta} \Big)^{\frac{1}{1-\theta}}$$

▶ Relative welfare weights, representing inequality

$$\text{w/o trade:} \qquad \widehat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \lessgtr 1 \qquad \Rightarrow \qquad \begin{array}{l}\text{ceteris paribus, poorer} \\ \text{countries have higher } \widehat{\lambda}_i\end{array}$$

$$\text{vs. w/ trade:} \qquad \widehat{\lambda}_i = \frac{\omega_i(\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i(\lambda_i + \mu_i)} \lessgtr 1$$

Step 2: Optimal policy – Social Cost of Carbon

▶ Key objects: Local vs. Global Social Cost of Carbon:

- Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i

- "Local social cost of carbon" (LCC) for region i:

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e_i^f) \mathsf{p}_i \qquad (> 0 \text{ if heat causes losses})$$

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- Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W}/\partial \mathcal{E}}{\partial \mathcal{W}/\partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

- Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i \big(\widehat{\lambda}_i, LCC_i\big)$$

Step 2: Optimal policy – Other motives

▶ Taxing fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy

▶ New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \mathcal{C}_{EE}^{f} \, \mathbb{C}\text{ov}_i\left(\widehat{\lambda}_i, e_i^f - e_i^x\right) \; - \; \mathbb{C}\text{ov}_i\left(\widehat{\lambda}_i, \frac{q^f(1 - s_i^f)}{\sigma}\right)$$

○ Params: $\mathcal{C}_{EE}^{f}$ agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

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○ Params: $\mathcal{C}_{EE}^{f}$ agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

Step 2: Optimal policy – Other motives

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○ Params: $\mathcal{C}_{EE}^{f}$ agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

▶ *Proposition 2:* Optimal fossil energy tax:

$$\Rightarrow \quad \mathfrak{t}^f = SCC + SVF$$

    – Social cost of carbon: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i$

back

Step 3: Ramsey Problem with participation constraints

- ▶ Consider that countries can "exit" climate agreement.
- ▶ For a climate "club" of $\mathbb{J} \subset \mathbb{I}$ countries:
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $t^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f

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▶ Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

▶ Welfare:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{\mathbb{J}} \omega_i \, u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i \, u(c_i)$$

Step 3: Ramsey Problem with participation constraints

► Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

► *Proposition 3.1:* Second-Best social valuation with participation constraints
  • Participation incentives change our measure of inequality

w/ trade:
$$\omega_i(1+\nu_i)u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij}(\tau_{ij}\mathrm{p}_j)^{1-\theta} \left[\omega_i \tilde{\lambda}_i + \omega_j \tilde{\mu}_j + \tilde{\eta}_{ij}(1-s_{ij}) \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \qquad \hat{\tilde{\lambda}}_i = \frac{\omega_i(\tilde{\lambda}_i+\tilde{\mu}_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_i(\tilde{\lambda}_i+\tilde{\mu}_i)} \neq \hat{\lambda}_i$$

vs. w/o trade
$$\hat{\tilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J}\sum_{\mathbb{J}}\omega_j(1+\nu_j)u'(c_j)} \neq \hat{\lambda}_i$$

  • Similarly, the "effective Pareto weights" are $\alpha\omega_i$ for countries outside the club $i \notin \mathbb{J}$ and $\omega_i(\alpha - \nu_i)$ for retaliation policy on $i \in \mathbb{J}$

Step 3: Participation constraints & Optimal policy

▶ *Proposition 3.2:* Second-Best taxes:

- Taxation with imperfect instruments:
    - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
    - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu \sigma}{q^f (1 - s^f)}$
      with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.

- Optimal fossil energy tax $t^f(\mathbb{J})$:

$$\Rightarrow \quad t^f(\mathbb{J}) = SCC + SVF$$

$$= \frac{1}{1 - \vartheta_{\mathbb{J}^c}} \sum_{i \in \mathbb{I}} \widetilde{\lambda}_i \, LCC_i + \frac{1}{1 - \vartheta_{\mathbb{J}^c}} \mathcal{C}_{EE}^f \sum_{i \in \mathbb{I}} \widetilde{\lambda}_i (e_i^f - e_i^x) - \sum_{i \in \mathbb{J}} \widetilde{\lambda}_i \, \frac{q^f (1 - s_i^f)}{\sigma}$$

- Optimal tariffs/export taxes $t^{b,r}(\mathbb{J})$ and $t^b(\mathbb{J})$: In search for a closed-form expression
  As of now, only opaque system of equations (fixed point w/ demand/multipliers)

back

Countries' incentives – Model w/o trade in goods

▶ Experiment: Model with trade in energy but not in "goods"
  • Start from the equilibrium where carbon tax $t^f(\mathbb{J}) = 0$,
      ⇒ country i is indifferent to join the club $\mathbb{J}$ or not
  • Linear approximation around that point ⇒ small changes in carbon tax dt^f

Countries' incentives – Model w/o trade in goods

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- Start from the equilibrium where carbon tax $t^f(\mathbb{J}) = 0$,
  $\Rightarrow$ country i is indifferent to join the club $\mathbb{J}$ or not

- Linear approximation around that point $\Rightarrow$ small changes in carbon tax dt^f

- Change in welfare if $i \in \mathbb{J}$ vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})} = -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta\, y_i \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) \ + \ \pi_i \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

- Difference in the GE effect on energy markets, for $\sigma \approx 1$

$$dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}} = -\Big(E_{\mathbb{J}} - E_{\mathbb{J}\setminus\{i\}}\Big)\frac{\sigma\, dt^f}{q^f(1-s^f)}\frac{1}{1 + \frac{\nu\sigma}{(1-s^f)}}$$

○ Params: σ energy demand elast^y, s^f energy cost share, ν energy supply elas^y, Climate damage γ_i and curv. δ

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Countries' incentives – Armington Model with trade in goods

▶ Trade in energy and goods *à la* Armington, Linear approx. around $t^f \approx 0$ and $t^b \approx 0$

• Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i \in \mathbb{J}}}{u'(c_i^{i \in \mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i \notin \mathbb{J}}}{u'(c_i^{i \notin \mathbb{J}})c_i} = -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}})$$

$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i \in \mathbb{J}} - dE_{i \notin \mathbb{J}})$$

$$+ \eta_i^y \left(\frac{d p_i}{p_i} \Big|_{i \in \mathbb{J}} - \frac{d p_i}{p_i} \Big|_{i \notin \mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j \in \mathbb{I}} s_{ij} \left(\frac{d p_j}{p_j} \Big|_{i \in \mathbb{J}} - \frac{d p_j}{p_j} \Big|_{i \notin \mathbb{J}} \right)$$

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$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i dt^f - \gamma_i (T_i - T_{i0})^\delta \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

$$- e_i \frac{q^f \nu}{E_{\mathbb{I}}} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E} (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

$$+ \eta_i^y \left(\frac{dp_i}{p_i} \Big|_{i\in\mathbb{J}} - \frac{dp_i}{p_i} \Big|_{i\notin\mathbb{J}} \right) - s_{i\mathbb{J}^c} dt^b - \sum_{j\in\mathbb{I}} s_{ij} \left(\frac{dp_j}{p_j} \Big|_{i\in\mathbb{J}} - \frac{dp_j}{p_j} \Big|_{i\notin\mathbb{J}} \right)$$

• GE effect on energy markets, same as before
• GE effect on goods markets, equilibrium on expenditure $v_i = p_i c_i$, for $\theta \approx 1$

$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki} \left(\frac{dv_k}{v_k} - \theta \frac{dt_{ki}^b}{1+t_{ki}^b} \right) \qquad \alpha_{ki} = \frac{c_{ki}\tau_{ki}p_i}{\sum_\ell c_{k\ell}\tau_{k\ell}p_\ell} \frac{v_k}{v_i} = s_{ki} \frac{v_k}{(1+t_{ki}^b)v_i}$$

– Params: σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i p_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Countries' incentives – Armington Model with trade in goods

▶ Trade in energy and goods *à la* Armington, Linear approx. around $\mathfrak{t}^f \approx 0$ and $\mathfrak{t}^b \approx 0$

• Change in welfare if $i \in \mathbb{J}$, vs. $i \notin \mathbb{J}$

$$\frac{d\mathcal{W}_{i|i\in\mathbb{J}}}{u'(c_i^{i\in\mathbb{J}})c_i} - \frac{d\mathcal{W}_{i|i\notin\mathbb{J}}}{u'(c_i^{i\notin\mathbb{J}})c_i} = -e_i d\mathfrak{t}^f - \gamma_i(T_i - T_{i0})^\delta \, \eta_i^y \Delta_i (dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

$$- e_i \frac{q^f \nu}{E_\mathbb{J}}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}}) + \eta_i^f \frac{(1+\nu)}{E}(dE_{i\in\mathbb{J}} - dE_{i\notin\mathbb{J}})$$

$$+ \eta_i^y \left(\frac{d\mathsf{p}_i}{\mathsf{p}_i}\big|_{i\in\mathbb{J}} - \frac{d\mathsf{p}_i}{\mathsf{p}_i}\big|_{i\notin\mathbb{J}}\right) - s_{i\mathbb{J}^c} d\mathfrak{t}^b - \sum_{j\in\mathbb{I}} s_{ij}\left(\frac{d\mathsf{p}_j}{\mathsf{p}_j}\big|_{i\in\mathbb{J}} - \frac{d\mathsf{p}_j}{\mathsf{p}_j}\big|_{i\notin\mathbb{J}}\right)$$

• GE effect on energy markets, same as before
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$$\frac{dv_i}{v_i} = \sum_{k\in\mathbb{I}} \mathcal{P}_k \alpha_{ki}\left(\frac{dv_k}{v_k} - \theta\frac{d\mathfrak{t}_{ki}^b}{1+\mathfrak{t}_{ki}^b}\right) \qquad \alpha_{ki} = \frac{c_{ki}\tau_{ki}\mathbb{p}_i}{\sum_\ell c_{k\ell}\tau_{k\ell}\mathbb{p}_\ell}\frac{v_k}{v_i} = s_{ki}\frac{v_k}{(1+\mathfrak{t}_{ki}^b)v_i}$$

– Params: σ energy demand elasticity, s^f energy cost share, ν energy supply elasticity, share of output y in income $\eta_i^y = \frac{y_i \mathbb{p}_i}{v_i}$, fossil rent share $\eta_i^f = \frac{\pi_i}{v_i}$

Complementarity in coalition formation – Model w/o trade in goods

▶ Is marginal gain $\Delta \mathcal{W}(\mathbb{J}, j) := \mathcal{W}(\mathbb{J} \cup j) - \mathcal{W}(\mathbb{J})$ "growing" in $\mathbb{J}$?

- Linear approximation for small $\{t^f, t^b\}$

$$\Delta \mathcal{W}(\mathbb{J}, j) = -\omega_j u'(c_j) e_j dt^f + \Big[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) \Delta_i \gamma_i (T_i - T_{i0})^\delta y_i \Big] \frac{\sigma \, e_j dt^f}{q^f (1 - s^f + \nu\sigma)}$$

$$+ \Big[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) e_i \Big] \frac{1}{1 + \frac{1-s^f}{\nu\sigma}} \frac{e_j dt^f}{E_{\mathbb{I}}} - \Big[\sum_{i \in \mathbb{I}} \omega_i u'(c_i) \pi_i \Big] \frac{(1+\nu)}{E_{\mathbb{I}}} \frac{\sigma \, e_j dt^f}{q^f (1 - s^f + \nu\sigma)}$$

- Free-riding problem: $\Delta \mathcal{W}(\mathbb{J}, j)$ could be negative

- If $\Delta \mathcal{W}(\mathbb{J}, j) > 0$, what effects does $\mathbb{J}$ have on marginal gain?
  - Marginal climate benefit decreases in $\mathbb{J}$, since temperature T_i declines!
  - G.E. effect on energy price: $E_{\mathbb{I}}$, q and π^f decreases with $\mathbb{J}$, effect on demand ambiguous
  - Similar formula for the case with trade tariffs: Work in progress.

Quantification – Firms

▶ Production function $y_i = \mathcal{D}_i^y(T_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \ell, \varepsilon(e^f, e^r)) = \left[(1 - \epsilon_i)^{\frac{1}{\sigma_y}} \left(k^\alpha \ell^{1-\alpha} \right)^{\frac{\sigma_y - 1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} \left(z_i^e \, \varepsilon_i(e^f, e^r) \right)^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega_i^{\frac{1}{\sigma_e}} \left(e^f \right)^{\frac{\sigma_e - 1}{\sigma_e}} + (1 - \omega_i)^{\frac{1}{\sigma_e}} \left(e^r \right)^{\frac{\sigma_e - 1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

- Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
- Today: $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
- Future: $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

▶ Damage functions in production function y:

$$\mathcal{D}_i^y(T) = e^{-\gamma_i^{\pm, y}(T - T_i^\star)^2}$$

- Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+, y} \mathbb{1}_{\{T > T_i^\star\}} + \gamma^{-, y} \mathbb{1}_{\{T < T_i^\star\}}$
- Today $\gamma_i^{\pm, y} = \bar{\gamma}^{\pm, y}$ & $T_i^\star = \bar{\alpha} T_{it_0} + (1 - \bar{\alpha}) T^\star$

Quantification – Energy markets

▶ Fossil production e_{it}^x and reserve $\mathcal{R}_{it}$

- Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$

- Now: $\bar{\nu}_i$ to match extraction data e_i^x, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. $\nu_i = \nu = 1$ quadratic extraction cost.

- Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e_i^x (BP, IEA)

Quantification – Energy markets

- ▶ Fossil production e_{it}^x and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now: $\bar{\nu}_i$ to match extraction data e_i^x, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. $\nu_i = \nu = 1$ quadratic extraction cost.
  - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e_i^x (BP, IEA)

- ▶ Renewable: Production $\bar{e}_{it}^r$ and price q_{it}^r
  - Now: $q_{it}^r = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
  - Future: Choose z_i^r to match the energy mix (e_i^f, e_i^r)

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Quantification – Future Extensions:

▶ Damage parameters:
  • $\gamma_i^{\pm,y}$ depends on daily temperature distribution $T \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
  • Use Climate Lab's (Greenstone et al) estimates for damage γ_i ?

▶ Fossil Energy markets:
  • Divide fossils e_{it}^f / e_{it}^x into oil/gas/coal
  • Match the production/cost/reserves data across countries
  • Use a dynamic model: extraction/exploration a la Hotelling

▶ Renewables Energy markets:
  • Make the problem dynamic with investment in capacity C_{it}^r

▶ Population dynamics
  • Match UN forecast for growth rate / fertility

Calibration

Table: Baseline calibration ($\star$ = subject to future changes)

		Technology & Energy markets	
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
g_e	$0.01^\star$	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	$-0.01^\star$	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	$2^\star$	Extraction elasticity of fossil energy	Cubic extraction cost
		Preferences & Time horizon	
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	$0.01^\star$	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

Table: Baseline calibration ($\star$ = subject to future changes)

Climate parameters			
ξ	0.81	Emission factor	Conversion $1\ MTOE \Rightarrow 1\ MT\ CO_2$
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature $\sim 11-15$ years
χ	2.1/1e6	Climate sensitivity	Pulse experiment: $100\ GtC \equiv 0.21°C$ medium-term warming
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment: $100\ GtC \equiv 0.16°C$ long-term warming
$\gamma^{\oplus}$	0.00234$\star$	Damage sensitivity	Nordhaus' DICE
$\gamma^{\ominus}$	$0.2 \times \gamma^{\oplus}$ $\star$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^T	0.2$\star$	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
$T^\star$	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
Parameters calibrated to match data			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
T_i		Local Temperature	To match temperature of largest city
$\mathcal{R}_i$		Local Fossil reserves	To match data from BP Energy review

Sequential solution method

▶ Summary of the model:
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, T_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^T, \lambda_t^{\mathcal{S}}, \lambda_{it}^{\mathcal{R}}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q_t^f, r_t^\star\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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▶ Global Numerical solution:

- Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
- Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

- Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method: Pros and Cons

▶ Why use a sequential approach?

- *Global approach*: *Only* need to follow the trajectories for *i* agents:

– Arbitrary (!) number of dimension of *ex-ante* heterogeneity:
  *Productivity z_i Population p_i, Temperature scaling Δ_i, Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$,
  Local damage $\gamma_i^y, \gamma_i^u, T_i^\star$, Directed Technical Change z_i^e*

– Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables:
  For now: Wealth w_{it}, temperature T_{it}, reserves $\mathcal{R}_{it}$, Carbon $\mathcal{S}_t$
  Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)

– Newton method & Non-linear solvers very efficient

▶ Why not:

- Numerical constraint to solve a large system of ODEs and non-linear equations:
⇒ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large

- Relying on numerical solvers/structure of the problem can be opaque

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