The Heterogeneous causes and effects of Climate Change

WORK IN PROGRESS

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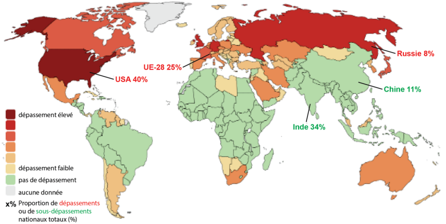
Economic Dynamics

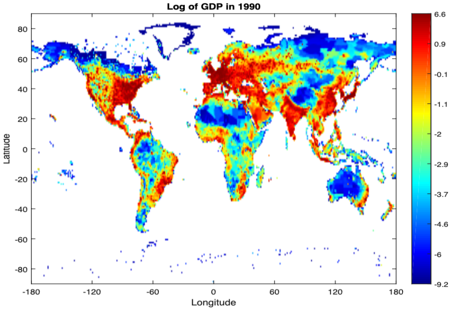
April 2022

Introduction – Motivation

- The cumulative GHG emissions are mostly in developed countries / advanced economies ...

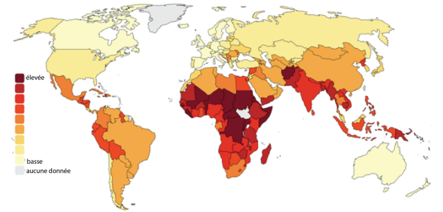


Source : Lancet planetary Health - Quantifying national responsibility for climate breakdown: an equality-based attribution approach for carbon dioxide emissions in excess of the planetary boundary - Jason Hickel 2020

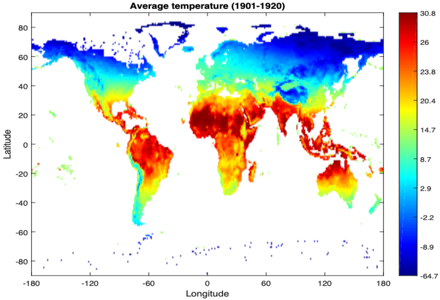


Introduction – Motivation

- ... while the impact will affect mostly developing economies, where temperature is already very high



Source : Notre Dame Global Adaptation Initiative



Introduction

- In presence heterogeneity, no country has an incentive to internalize the climate externality
  - Is the equilibrium without policy intervention bound to collapse ?
- What is the optimal policy against climate change ?
  - Should countries contribute according to their present/past emissions they have caused ?
  - Or the damage they are contributing ? (standard Pigouvian tax)
  - Can the social planner allocation be attained with tax instruments ?
- Policy based on carbon taxes
  - Different taxes per region ?
  - Or uniform taxes and transfers
  - Is a country better off joining the world ”social planner policy” or deviating ?

Introduction

- Integrated assessment models rely on strong assumptions
  - ... about the economy (NCG model)
  - ... about the climate system (may not be consistent w/ climate models)
  - ... about the damage function (in DICE, only affecting TFP)
- IAMs also tend to be computational/untractable
  - Nordhaus' Multi-regions DICE (2016), Cai, Lontzek, Judd (2019), Kotlikoff, Kubler, Polbin, Scheidegger (2021), Hassler, Krusell, Olovsson, Smith (2019-2021), Cruz, Rossi-Hansberg (2021)
- This project :
  - Theoretical investigation on the importance of heterogeneity
  - Design of the social planner allocation in comparison to the competitive equilibrium allocation
  - Sensitivity analysis on which dimension of heterogeneity matters the most, and uncertainty about the models

Model

- Neoclassical economy, in continuous time
  - countries/regions $i \in \mathcal{I}$: exante heterogeneous in dimensions $\underline{s}$
  - heterogeneous expost in dimensions $\overline{s}$ (c.f. next slide), total state $s = \{\underline{s}, \overline{s}\}$

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  - For now s_i is productivity z_i and/or population N_i
    - Productivity grow at rate μ and population grow at rate n

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- Households owns the representative firm
  - No trade/no migration/no agglomeration economies
  - Maximize :

$$U_{i,t_0}(s) = \int_{t_0}^{\infty} e^{-(\rho-n)t} u_i(c_t, T_t) dt$$

Model and dynamics

- Utility of HH vs. social planner

$$U_{i,t_0}(s) = \int_{t_0}^{\infty} e^{-(\rho-n)t} u_i(c_t, T_t) dt \qquad W_{t_0} = \int_{\mathcal{I}} \omega_i U_{i,t_0} di = \int_{\mathbb{S}} \omega(s) U_{i,t_0}(s) g(s) ds$$

  - Distribution over states $g(s)$
  - Some states $\bar{s}$ vary over time : capital k_i and temperature T_i

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  - Distribution over states $g(s)$
  - Some states $\bar{s}$ vary over time : capital k_i and temperature T_i
- Dynamics of capital :

$$\dot{k}_t = \mathcal{D}(T_t) f(k_t, e_t) - (n + \mu + \delta) k_t - c_t - \nu(e_t)$$

- Two choices :
  - c consumption, e energy/emissions, Production fct $f(k,e) = z\, k^{\alpha} e^{\varepsilon}$
  - Fossil energy is produced subject to a cost $\nu(e) = \bar{\nu} e^{\nu}$
  - Exhaustible resource with a world stock R
  - Damage $\mathcal{D}_i(T_t)$ affect country's PPF
  - (Future) include abatement/clean energy

Energy and externality

- Energy/emission is a choice and cause two types of externality
  - Decrease the world stock of resources R_t (à la Hotelling)

$$\dot{R}_t = -\int_{\mathcal{I}} e_{i,t} di = -\int_{\mathbb{S}} e_t(s) g_t(s) ds$$

  - Change the world climate $\mathcal{T}$: analytical formula :

$$\begin{aligned}\dot{\mathcal{S}}_t &= \int_{\mathbb{S}} \xi \; e_t(s) g_t(s) ds - \delta_s \mathcal{S}_t \\ \dot{\mathcal{T}}_t &= \varepsilon(\chi S_t - T_t)\end{aligned}$$

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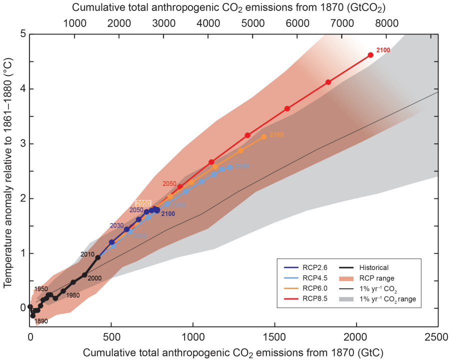
$$\dot{\mathcal{T}}_t = \varepsilon(\chi S_t - T_t)$$

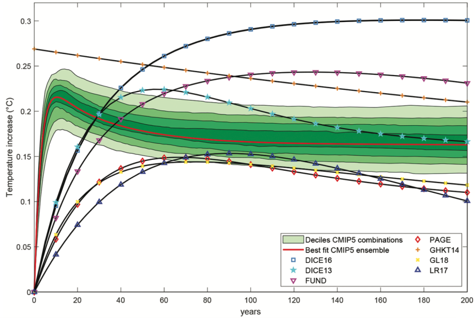
    – ε is the inverse of persistence, so if $\varepsilon \to \infty$, we obtain a linear model :

$$\mathcal{T}_t = \bar{\mathcal{T}} + \chi S_t = \bar{\mathcal{T}} + \chi \int_{t_0}^{t} \int_{\mathcal{I}} \xi e_i di \, d\tau \Big|_{GtC}$$

- The externality depends on the policy $e(s)$ as function of state s
  - Naturally countries richer/more productive/with a larger population use more energy !

Temperature dynamics





Linear temperature model - IPCC report / Dietz, van der Ploeg, Rezai, Venmans (2021)

Temperature dynamics

- Emissions of all countries change the world climate $\mathcal{T}$ through cumulative emission

$$\dot{\mathcal{S}}_t = \int_{\mathbb{S}} \xi \, e_t(s) g_t(s) ds - \delta_s \mathcal{S}_t$$

$$\dot{\mathcal{T}}_t = \varepsilon(\chi S_t - T_t)$$

- Temperature in country i potentially affected with sensitivity Δ_i

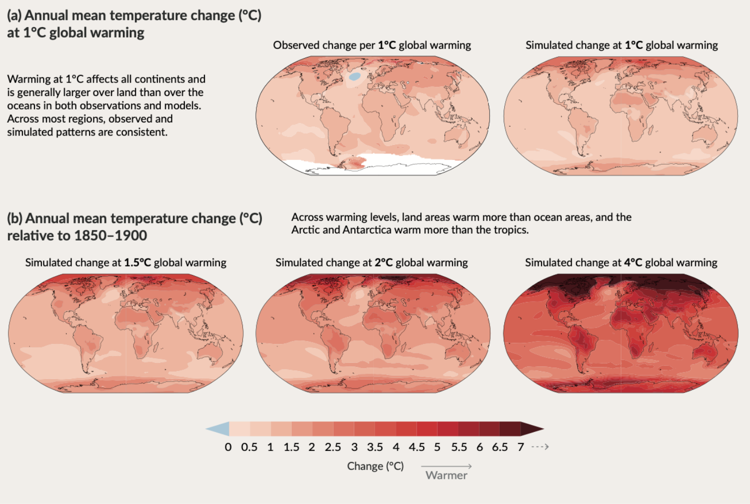
$$\dot{T}_t = \Delta_i \dot{\mathcal{T}}_t$$

- Possibility of a more complex climate block :
  - Add 5 or 6 more state variables !

$$\dot{\mathbf{J}}_t = \Phi \mathbf{J}_t + \rho^e \int_{\mathbb{S}} \xi e(s) g(s) ds$$

$$F_t = \mathcal{F} \mathbf{J}_t \qquad \dot{\mathcal{T}}_t = \Phi^T \mathcal{T} + \eta F_t$$

Temperature dynamics



Damage functions

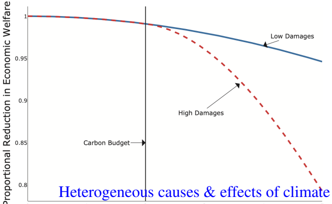
- Climate change has two effects :
  - Affect household utility function $u(c, T)$

$$u(c_t, T_t) = \mathcal{D}^u(T_t)\frac{c_t^{1-\eta}}{1-\eta} \qquad \mathcal{D}^u(T) = \exp(-\phi(T - T^\star)^2)$$

  - Affect firm productivity $Z_t = \mathcal{D}(T_t)z$ as a shifter – as in Nordhaus DICE-2016

$$\mathcal{D}^y(T) = \exp(-\gamma(T - T^\star)^2)$$

    - Deviation (positive/negative) from "ideal" temperature $T^\star = 15^\circ C$
    - Damage sensitivity γ_i and ϕ_i can also be heterogeneous and uncertain



Summing up

- System of coupled ODEs - PMP :
  - State variables $s = (z, k, R, S, T)$ and two controls (c, e)

$$\dot{k}_t = \mathcal{D}(T_t) z_t f(k_t, e_t) - (n + \mu + \delta) k_t - c_t - \nu(e_t)$$

$$\dot{R}_t = -E_t \qquad E_t = e^{(n+\mu)t} \int_{\mathbb{S}} e_t(s) g_t(s) ds$$

$$\dot{T}_t = \Delta_i \varepsilon (\chi S_t - T_t) \qquad \dot{\mathcal{S}}_t = \xi E_t - \delta^s \mathcal{S}_t$$

$$s_0 = (z_{0,i}, k_{0,i}, R_0, T_{0,i})$$

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  - Hamiltonian :

$$\mathcal{H}(s, c, e, \{\lambda\}) = u(c, T) + \lambda^k \dot{k} + \lambda^R \dot{R} + \lambda^S \dot{S} + \lambda^T \dot{T}$$

  - Costates : $\widetilde{\rho} = \rho - n + \mu(\eta - 1)$

$$\begin{array}{lcr} -\dot{\lambda}_t + \widetilde{\rho}\lambda_t^k = \partial_k \mathcal{H}(\cdot) & \Rightarrow & \dot{\lambda}_t^k = -\lambda_t^k (r_t - \widetilde{\rho}) \\ -\dot{\lambda}_t^R + \widetilde{\rho}\lambda_t^R = \partial_R \mathcal{H}(\cdot) = 0 & \Rightarrow & \dot{\lambda}_t^R = \widetilde{\rho}\lambda_t^R \\ -\dot{\lambda}_t^S + \widetilde{\rho}\lambda_t^S = \partial_S \mathcal{H}(\cdot) & \& & -\dot{\lambda}_t^T + \widetilde{\rho}\lambda_t^T = \partial_T \mathcal{H}(\cdot) \end{array}$$

Marginal value of temperature

- Marginal values of the climate variables : λ^S and λ^T

$$\dot{\lambda}_t^T = \lambda_t^T(\widetilde{\rho} + \Delta\varepsilon) + \overbrace{\gamma(T_t - T^\star)\mathcal{D}^y(T_t)}^{\partial_T \mathcal{D}^y} f(k_t, e_t)\lambda_t^k + \overbrace{\phi(T_t - T^\star)\mathcal{D}^u(T_t)}^{\partial_T \mathcal{D}^u} u(c_t)$$

$$\dot{\lambda}_t^S = \lambda^S{}_t(\widetilde{\rho} - \delta^s) - \Delta\varepsilon\chi\lambda_t^T$$

- Marg. value of carbon λ^S in location w/ state s increases with :
  - Temperature $T - T^\star$
  - Damage sensitivity to temperature for TFP γ and utility ϕ
  - The development level $f(k, e)$ and $u(c)$

Marginal value of temperature

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  - Temperature $T - T^\star$
  - Damage sensitivity to temperature for TFP γ and utility ϕ
  - The development level $f(k, e)$ and $u(c)$
- The cost of carbon can be defined SCC closed form

$$CC_{i,t} = \frac{\partial U_{i,t}/\partial \mathcal{S}_{i,t}}{\partial U_{i,t}/\partial c_{i,t}} = \frac{\lambda_t^S(s)}{\lambda_t^k(s)}$$

  - Ratio of marg. value of carbon/marg. value of consumption !
  - Can integrate over time t & state-space i : $SCC_t = \int_{\mathbb{S}} \frac{\lambda_t^S(s)}{\lambda_t^k(s)} g(s) ds$
  - Uncertainty SCC with uncertainty

Choice of energy and emission

- Different choices of emissions depending on the level of externality !
  - Small economies ? Additional $e_t(s)$ don't affect $E_t = \int_{\mathbb{S}} e_t(s) g_t(s) ds$
    - 1. Only private tradeoff : marg. product of energy = marg cost

$$\mathcal{D}(T_t) z_t \, \partial_e f(k_t, e_t) = \nu'(e_t)$$

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$$\mathcal{D}(T_t) z_t \, \partial_e f(k_t, e_t) = \nu'(e_t)$$

    - 2. Private tradeoff + energy monopolist : MPE = MC + Hotelling rent

$$\mathcal{D}(T_t) z_t \, \partial_e f(k_t, e_t) = \nu'(e_t) + \frac{\lambda_t^R}{\lambda_t^k(s)}$$

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  - Large economies ? $e(s)$ does affect $E_t = \sum_{\mathbb{S}} \xi e(s)$
    - 3. Internalize scarcity and externality (for yourself, state s !)

$$\mathcal{D}(T_t) z_t \, \partial_e f(k_t, e_t) = \nu'(e_t) + \frac{\lambda_t^R}{\lambda_t^k(s)} + \frac{\xi \lambda_t^S(s)}{\lambda_t^k(s)}$$

Choice of energy and emission

- Different choices of emissions depending on the level of externality !
  - Social planner allocation ? Choose the policies of all $s \in \mathbb{S}$:
    - 4. Internalize aggregate effect of scarcity and externality :

$$\mathcal{D}(T_t) z_t \, \partial_e f(k_t, e_t) = \nu'(e_t) + \frac{\lambda_t^R}{\lambda_t^k(s)} + \underbrace{\frac{\xi}{\omega(s)\lambda_t^k(s)} \int_{\mathbb{S}} \omega(\tilde{s}) \lambda_t^S(\tilde{s}) g(\tilde{s}) d\tilde{s}_t}_{=\text{Carbon tax } \tau^e(s)}$$

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  - Static carbon tax : internalize damage *weighted* by $\lambda_t^k = u'(c_t)$! (assume Pareto weights $\omega(s) = 1$)

$$\tau^e(s) = \frac{1}{\lambda_t^k(s)}\int_{\mathbb{S}}\lambda_t^S(\tilde{s})g(\tilde{s})d\tilde{s}_t \neq \int_{\mathbb{S}}\frac{\lambda_t^S(\tilde{s})}{\lambda_t^k(\tilde{s})}g(\tilde{s})d\tilde{s}_t = SCC_t$$

  - Future : Joint choice of e, e^{fossil} and $e^{\text{renew.}}$ and/or ξ

Competitive equilibrium vs. Social planner

- Social planner : choose all controls $\{c, e\}$

$$W_{t_0} = \max_{\{c(s),e(s)\}_{s\in\mathbb{S}}} \int_{\mathbb{S}} \omega(s) U_{i,t_0}(s) g(s) ds$$

- In this environment, different sources of externality :
  1. Choice of energy/emissions s given the states s : ***Static***
     - Externality of the finite resource problem
     - Damage due to temperature
  2. Evolution of state and cstate $s = (z, k, R, S, T)/\lambda$: ***Dynamic***
     - Dynamic problem :
       state matters for optimal choice and for future emission !
     - More productive/capital-rich countries emit more :
     - Climate change should affect the marginal value of z and k
  3. Social planner corrects these two features
     - First : c.f. previous slides, Second : see the next slide !

Competitive equilibrium vs. Social planner

- Competitive equilibrium : state $s = (z, k, R, S, T)$ and costate λ evolution

$$-\dot{\lambda}_t + \widetilde{\rho}\lambda_t = \partial_s \mathcal{H}(s, \lambda)$$

- Social planner allocation :
  - Take into account the distribution $g(s)$ in the costate dynamics :
  - Second term : effect of shifting the distribution $g(s)$ by one unit of s
  - Impact of $\bar{E}_t = \int_{\mathbb{S}} e_t(s) g_t(s) ds$

$$-\dot{\lambda}_t + \widetilde{\rho}\lambda_t = \partial_s \mathcal{H}(s, \lambda(s)) + \partial_s e(s) \int_{\mathbb{S}} \partial_{\bar{E}} \mathcal{H}(\tilde{s}, \lambda(\tilde{s})) g(\tilde{s}) d\tilde{s}$$

  - Marginal effect of state s on choice of energy $e(s)$ through $\partial_s e(s)$
  - Marginal value of state λ_t increases with the externality of $\bar{E}$! !

$$\partial_{\bar{E}} \mathcal{H}(s, \lambda(s)) = \lambda_t^R + \xi \lambda_t^S(\tilde{s})$$

Competitive equilibrium vs. Social planner : Example

- Competitive equilibrium, state : capital k and costate λ^k

$$\dot{\lambda}_t^k = (\widetilde{\rho} - r_t)\lambda_t^k$$

- Social planner allocation :
  - Impact of $\bar{E}_t = \int_{\mathbb{S}} e_t(s) g_t(s) ds$

$$\dot{\lambda}_t^k = (\widetilde{\rho} - r_t)\lambda_t - \frac{\partial e(s)}{\partial k} \int_{\mathbb{S}} \left(\lambda_t^R + \xi \lambda_t^S(\tilde{s})\right) g(\tilde{s}) d\tilde{s}$$

    - Marginal value of state λ_t decreases with the externality of $\bar{E}$
    - Consume more today because more capital in the future affects the choice of energy $e(s)$ through $\partial_s e(\tilde{s})$

$$\frac{\partial e}{\partial k} = \left(\frac{\mathcal{D}(T_t)\, z\, \epsilon}{\bar{\nu}}\right)^{\frac{1}{\nu - \epsilon}} \frac{\alpha}{\nu - \epsilon}\, k^{\frac{\alpha}{\nu - \epsilon} - 1}$$

Carbon taxation

- Impose a tax on energy, denote resulting welfare $W_{t_0}^{tax}$

$$\tau^e \xi = SCC = \int_{\mathbb{S}} \frac{\lambda_t^S}{\lambda_t^k} g(\tilde{s}) d\tilde{s}_t \qquad \text{or} \qquad \tau^e(s)\xi = \frac{1}{\lambda_t^k} \int_{\mathbb{S}} \lambda_t^S(\tilde{s}) g(\tilde{s}) d\tilde{s}_t$$

- Is carbon taxation a way to reach the first best ?

$$W_{t_0}^{tax} \gtrless W_{t_0}^{SP}$$

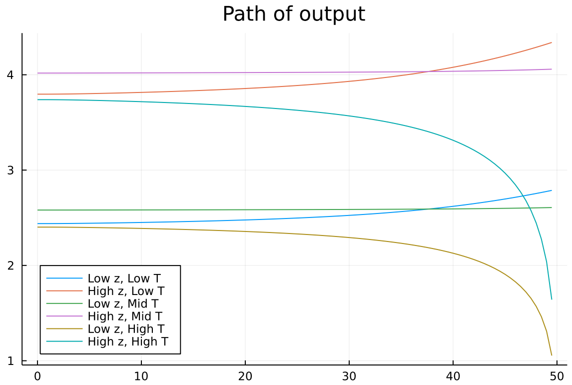
  - No ! Dynamic externalities still present !
- Will countries join the policy/social planner allocation ? Only if :

$$U_{t_0}^{SP} > U_{i,t_0}^{tax} > U_{i,t_0}^{CE}$$

  - No coordination ! Some countries benefit from climate change !
- Can tax τ be made "more" country-specific / more progressive ? + Correlated with current/past emissions ?

Toy example - simulation

- ”Business as usual” scenario : no internalization
- 6 countries : productivity $z \in \{z_\ell, z_h\}$ & temperature $T_t \in \{T_\ell, T_m, T_h\}$
- Compare to one country with $z = \bar{z}$ and $T_t = \bar{T}$



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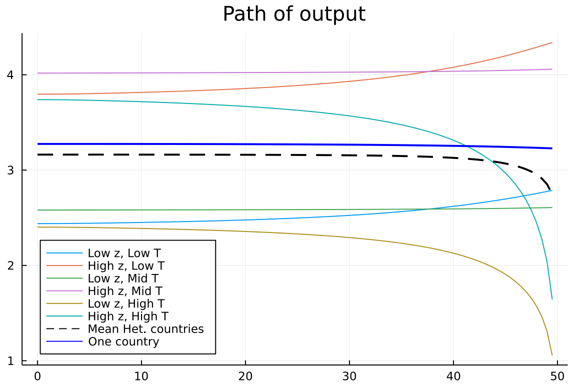


Conclusion

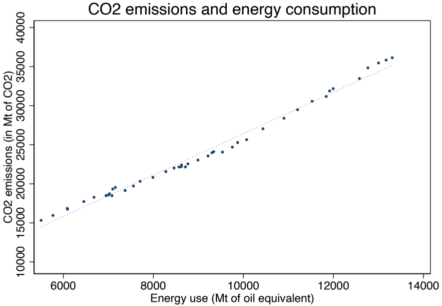
- Climate change is induced by externality
  - Energy/Emission choice doesn't include the impact on other countries
  - Cause strengthened by heterogeneity in wealth (capital/productivity)
  - Effect strengthened by heterogeneity in impact (temperature/damage)

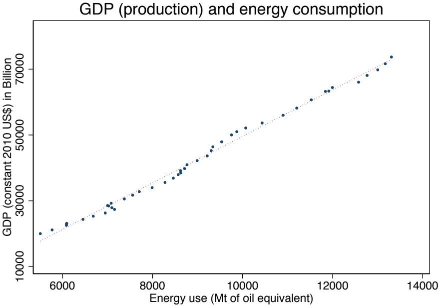
- Social planner allocation correct for these different dimensions
  - Both Static correction $\equiv$ modified Pigouvian carbon taxation
  - And dynamic : through the marginal value of states

- Future plans :
  - Simulation of the three equilibria $CE/tax/SP$
  - Match the distribution of $y = z f(k, e)$ and T to the data
  - Social cost of carbon including heterogeneity and model uncertainty

Motivation

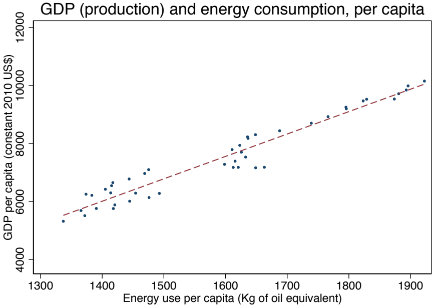
- CO_2 emissions correlate linearly with energy use
- Energy use (85% from fossils) correlates with output/growth

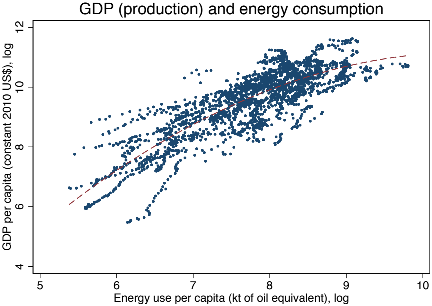




Introduction – Motivation

- Also true per capita and for the trajectory of individual countries





Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}_t^T = \lambda_t^T(\widetilde{\rho} + \Delta\varepsilon) + \gamma(T - T^\star)\mathcal{D}^y(T)f(k,e)\lambda_t^k + \phi(T - T^\star)\mathcal{D}^u(T)u(c)$$
$$\dot{\lambda}_t^S = {\lambda^S}_t(\widetilde{\rho} - \delta^s) - \Delta\varepsilon\chi\lambda_t^T$$

- Solving for λ_t^T and λ_t^S, in stationary equilibrium $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$

$$\begin{aligned}
{\lambda^S}_t &= \int_t^\infty e^{-(\widetilde{\rho}-\delta^s)\tau}\Delta\varepsilon\chi\lambda_\tau^T d\tau \\
\lambda_t^T &= \int_t^\infty e^{-(\widetilde{\rho}+\Delta\varepsilon)\tau}(T_\tau - T^\star)\Big(\gamma\mathcal{D}^y(T_\tau)y_\tau\lambda_\tau^k + \phi\mathcal{D}^u(T_\tau)u(c_\tau)\Big)d\tau \\
\lambda_t^T &= \frac{1}{\widetilde{\rho} + \Delta\varepsilon}(T - T^\star)\Big(\gamma\mathcal{D}^y(T_\infty)y_\infty\lambda_\infty^k + \phi\mathcal{D}^u(T_\infty)u(c_\infty)\Big) \\
\lambda_t^S &= \frac{1}{\widetilde{\rho} - \delta^s}\Delta\varepsilon\chi\lambda_\infty^T \\
&= \frac{\Delta\chi}{\widetilde{\rho} - \delta^s}\frac{\varepsilon}{\widetilde{\rho} + \Delta\varepsilon}(T - T^\star)\Big(\gamma\mathcal{D}^y(T_\infty)y_\infty\lambda_\infty^k + \phi\mathcal{D}^u(T_\infty)u(c_\infty)\Big) \\
\lambda_t^S &\xrightarrow[\varepsilon\to\infty]{} \frac{\Delta\chi}{\widetilde{\rho} - \delta^s}(T - T^\star)\Big(\gamma\mathcal{D}^y(T_\infty)y_\infty\lambda_\infty^k + \mathcal{D}^u(T_\infty)u(c_\infty)\Big)
\end{aligned}$$

Cost of carbon / Marginal value of temperature

- Closed form solution for CC :
  - In stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\varepsilon \to \infty$
  - no internalization of externality (business as usual)

$$CC_t(s) \equiv \frac{\Delta\chi}{\widetilde{\rho} - \delta^s}(T_t - T^\star)\left(\gamma\mathcal{D}^y(T_\infty)y_\infty + \phi\mathcal{D}^u(T_\infty)\frac{c_\infty}{1-\eta}\right)$$

- Heterogeneity + uncertainty about models

Uncertainty about models :

- In our model, we rely strongly on model specification :
  - Parameters θ of models :
    - Climate system and damages : $(\chi, \varepsilon, \delta^s, \gamma, \phi)$
    - Economic model : $\xi, \epsilon, \nu^f, \mu, n, \eta$ or extended : $\omega, \sigma^e, \sigma^f, \nu^{\text{renew}}, z^{\text{renew}}$
    - Models with probability weight $\pi(\theta)$
  - Social cost of carbon, weighted for model uncertainty :

$$SCC_t(\theta) = \int_{\mathbb{S}} \frac{\lambda_t^S(s,\theta)}{\lambda_t^k(s,\theta)} g(s)ds$$

$$S\bar{C}C_t = \int_{\Theta} SCC_t(\theta)\pi(\theta)d\theta = \int_{\Theta}\int_{\mathbb{S}} \frac{\lambda_t^S(s,\theta)}{\lambda_t^k(s,\theta)} g(s)ds\pi(\theta)d\theta$$

  - Counterfactual computation of SCC
    - Representative country / no uncertainty $\frac{\lambda_t^S}{\lambda_t^k}$
    - With heterogeneous regions / no uncertainty $SCC_t(\bar{\theta})$
    - No heterogeneity / model uncertainty $\int_{\Theta} \frac{\lambda_t^S(\bar{s},\theta)}{\lambda_t^k(\bar{s},\theta)}\pi(\theta)d\theta$
    - With heterogeneous regions / with model uncertainty $S\bar{C}C_t$
  - back

Sequential vs. recursive method

- Previous treatment : sequential approach :
  - Solve a system of ODE (because no uncertainty here) :
    - Monte Carlo for state s
    - Shooting algorithm for states s and costate λ^s
    - Can handle many states $s = (z, k, R, S, T)$ or $s = (z, N, k, \gamma/\phi, R, F, T)$
- Alternative treatment : recursive approach :
  - Solve a system of (S)PDE : Mean Field Game system
    - Hamilton Jacobi Bellman equation
    - Kolmogorov Forward/Fokker Plank equation
  - Master equation ?

Recursive approach

- Solve a system of PDE : Mean Field Game system
  - State Dynamics :

$$\dot{k}(s,c,e) = \exp(-\gamma T^2) z f(k,e) - (n+\mu+\delta)k - c - \nu(e)$$
$$\dot{T}(s,c,e) = \bar{\tau}_t \tau(T) \int_{\mathbb{S}} e(\tilde{s}) g(\tilde{s}) d\tilde{s}$$

  - Hamilton Jacobi Bellman equation

$$-\partial_t v(s) + (\rho - n) v(s) = \max_{c,e} u(c,T) + \partial_k v(s) \dot{k}(s,c,e) + \partial_R v(s) \dot{R} + \partial_T v(s) \dot{T}(s,c,e)$$

  - Kolmogorov Forward/Fokker Plank equation

$$\partial_t g(s) = -\partial_k \left[g(s) \dot{k}(s,c,e) \right] - \partial_R \left[g(s) \dot{R} \right] - \partial_T \left[g(s) \dot{T}(s,c,e) \right]$$