# The Inequality of Climate Change & Optimal Energy policy

WORK IN PROGRESS

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*JMP Proposal*

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  - Normative implications : Optimal Ramsey policy for carbon taxation
  - Provide a numerical methodology and a quantitative model

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- Develop a simple and flexible model of climate economics
    - Standard IAM model with heterogeneous regions
    - Normative implications : Optimal Ramsey policy for carbon taxation
    - Provide a numerical methodology and a quantitative model
- ◦ Evaluate the heterogeneous welfare costs of global warming
    - Damages of climate & temperature varies across countries

    ⇒ Inequality increases the welfare cost of carbon
- ◦ Solve world optimal carbon policy with heterogeneous regions
    - Does the optimal carbon tax coincide with the social cost of carbon ?

    ⇒ Depends on transfer policy : need to adjust the tax for inequality level

# Toy model

- Consider two countries $i = N, S$, (North/South)
  - Household consumes good $c_i$, produced by a rep. firm with energy $e_i$ and productivity $z_i$
  - Energy producers extract energy $e_i^x$ selling it at price $q^e$
- Country-$i$ planner problem :

$$\mathcal{V}_i = \max_{c_i, e_i, e_i^x} U(c_i)$$
$$c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - q^e e_i + e_i^x q^e - c_i(e_i^x)$$
$$e_N + e_S = e_N^x + e_S^x$$

- Climate damage $\mathcal{D}_i(\mathcal{S})$ with $\mathcal{S} = \overbrace{\xi(e_N + e_S)}^{=\text{GHG emissions}}$
- Competitive equilibrium Result :
  - Energy decision :

$$MPe_i = q^e = c_i'(e_i^x) \qquad with \qquad MPe_i := \mathcal{D}_i(\mathcal{S}) z_i F'(e_i)$$

# Toy model – First Best and Decentralization

- Comparison with Social planner with full transfers (First Best)

$$\mathbb{W} = \max_{\{c_i, e_i, e_i^x\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$\sum_{i=N,S} c_i + c_i(e_i^x) = \sum_{i=N,S} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad \& \qquad \sum_{i=N,S} e_i = \sum_{i=N,S} e_i^x$$

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- *Proposition 1 :* First-Best and decentralization :
  - Redistribution with *lump-sum transfers / tax* $T_S \geq 0$ & $T_N \leq 0$

$$\omega_S U'(c_S) = \omega_N U'(c_N)$$
$$\Rightarrow \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) - (q^e + \mathbf{t}^e) e_i + e_i^x q^e - c_i(e_i^x) + T_i$$

  - *Common* energy tax : $\mathbf{t}_i^e \equiv \mathbf{t}^e = \xi\, \overline{SCC}$

$$MPe_i = c'(e_i^x) + \underbrace{\xi \overline{SCC}}_{=\mathbf{t}^e} \qquad \textit{with} \qquad \overline{SCC} := -\sum_{i=N,S} \mathcal{D}_i'(\mathcal{S}) z_i F(e_i)$$

# Toy model – Optimal energy policy without transfers

- Assume now that *lump-sum transfers across countries* are prohibited
  - Ramsey policy, allowing country-specific carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$

- *Proposition 2 :* Second-Best without transfers :
  - Inequality because of lack of redistribution

$$\omega_S U'(c_S) \neq \omega_N U'(c_N)$$

  - Energy tax integrate redistributive concerns : $\mathbf{t}_i^e \neq \mathbf{t}^e := \xi\, SCC$

$$MPe_i = c_i'(e_i^x) + \mathbf{t}_i^e$$

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[\xi\, SCC + c''(\bar{E})\, SCE\right]$$

# Social Cost of Carbon (SCC) with inequality

- The Energy taxation integrates three motives :

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[\xi\, SCC + c''(\bar{E})\, SCE\right] \qquad \Rightarrow \qquad \mathbf{t}_S^e \leq \mathbf{t}_N^e \quad \underline{i.f.f.} \quad \omega_S U'(c_S) \geq \omega_N U'(c_N)$$

  - A measure of inequality : $\frac{\omega_i U'(c_i)}{\frac{1}{2}\sum_j \omega_j U'(c_j)} \lesseqgtr 1$

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$$SCC = -\mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}_j'(\mathcal{S}) z_j F(e_j)\right) - \mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)] \gtrapprox -\mathbb{E}_j[\mathcal{D}_j'(\mathcal{S}) z_j F(e_j)]$$

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  - Social cost of energy (SCE) accounts for redistribution btw importer and exporter

$$SCE = \mathbb{C}\mathrm{ov}_j\left(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, e_j - e_j^x\right) \lessapprox 0 \qquad\qquad c''(\bar{E}) := \left(\sum_i \frac{1}{c_i''(e_i^x)}\right)^{-1} \geq 0$$

# Toy model – Optimal policy : no transfers & participation constraint

- Assume that lump-sum transfers are prohibited & countries can exit climate agreements
  - Ramsey policy, allowing country-specific carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$
  - Participation constraint, with $\bar{c}_i$ autarky level of consumption

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- *Proposition 3 :* Second-Best without transfers & participation constraints
  - Participation incentive change our measure of inequality

$$\widetilde{\omega}_N U'(c_N) \lesseqgtr \widetilde{\omega}_S U'(c_S) \qquad \widetilde{\omega}_N = \omega_N + \nu_N \geq \omega_N$$

  - Energy tax integrate both redistributive and participation concerns : $\mathbf{t}_i^e \neq \mathbf{t}^e := \xi\, SCC$

$$\mathbf{t}_i^e = \frac{\frac{1}{2}\sum_j \widetilde{\omega}_j U'(c_j)}{\widetilde{\omega}_i U'(c_i)} \left[\xi\, SCC + c''(\bar{E})\, SCE\right] \qquad \Rightarrow \qquad \mathbf{t}_N^e \lesseqgtr \mathbf{t}_S^e$$

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- Without such instruments, energy taxes are country $i$-specific & accounts for 3 motives :
  - $\mathbf{t}^e_i = \frac{\overline{\omega U'(c)}}{\omega_i U'(c_i)} [\xi\, SCC + c''(E)\, SCE]$
  - Adjusted for Pareto weights $\omega_i$ and marg. utility of consumption $U'(c_i)$
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# Model – Representative Household

- Deterministic Neoclassical economy, in continuous time
  - heterogeneous countries $i \in \mathbb{I}$
  - In each country, 4 agents : (i) representative household, (ii) homogeneous good firm, (iii) fossil and (iv) renewable energy producers.

- Representative household problem in each country $i$ :

$$\mathcal{V}_{it_0} = \max_{\{c_{it}, k_{it}, b_{it}\}} \int_{t_0}^{\infty} e^{-\rho t}\, u_i(c_{it}, \tau_{it}) dt$$

- Dynamics of wealth of country $i$, $w_{it} = b_{it} + k_{it}$ More details

$$\dot{w}_{it} := \dot{k}_{it} + \dot{b}_{it} = y_{it} + \pi^f_{it} + \pi^r_{it} + r^\star_t b_{it} + (r^\star_t - \bar{\delta}) k_{it} - c_{it} + \mathbf{t}^{ls}_{it}$$

  - Labor income $y_{it}$ from homogeneous good firm.
  - All the lower-case variables are expressed per unit of efficient labor $y_{it} = Y_{it}/(L_{it}A_{it})$

# Model – Representative Firm

- Competitive homogeneous good producer in country $i$

$$\max_{k_{it}, e^f_{it}, e^r_{it}} \mathcal{D}^y(\tau_{it}) z_i f(k_{it}, e^f_{it}, e^r_{it}) - r^\star_t k_{it} - (q^f_t + \mathbf{t}^f_{it}) e^f_{it} - (q^r_t + \mathbf{t}^r_{it}) e^r_{it} - y_{it}$$

  - Energy mix with fossil $e^f_{it}$ – emitting carbon – subject to price $q^f_t$ and tax/subsidy $\mathbf{t}^f_{it}$. Similarly "clean" renewable $e^r_t$, at price $q^r_{it}$ and tax $\mathbf{t}^r_{it}$.
  - No international trade in goods and Labor is immobile

# Model – Energy markets

- Competitive fossil fuels energy producer :
  - Static problem (for now) extract energy $e^x_{it}$ depleting reserves $\mathcal{R}_{it}$

$$\pi^f_{it} = \max_{e^x_{it}} q^f_t e^x_{it} - \mathcal{C}^f_i(e^x_{it}, \mathcal{R}_{it})$$

$$\dot{\mathcal{R}}_{it} = -e^x_{it} \qquad \mathcal{R}_{it_0} = \mathcal{R}_{i0} \qquad \mathcal{R}_{it} \geq 0$$

  - Fossil energy traded in international markets :

$$\int_{\mathbb{I}} e^f_{it} p_i \, di = \int_{\mathbb{I}} e^x_{it} \, di$$

  - Optimal extraction

$$q^f_t = \mathcal{C}^f_e(e^x_{it}, \mathcal{R}_{it})$$

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  - Optimal extraction

$$q^f_t = \mathcal{C}^f_e(e^x_{it}, \mathcal{R}_{it})$$

- Renewable energy as a substitute technology in each country $i$ (Static problem for now)

$$\pi^r_{it} = \max_{\{\bar{e}^r_{it}\}} q^r_{it}\bar{e}^r_{it} - \mathcal{C}^r_i(\bar{e}^r_{it}) \qquad \Rightarrow \qquad q^r_{it} = \mathcal{C}^r_e(\bar{e}^r_t) = z^r_{it}$$

# Climate system

- Fossil energy input $e_t^f$ causes climate externality

$$\mathcal{E}_t = \xi \int_{\mathbb{I}} e_{it}^f p_i \, di$$

- Cumulative GHG in atmosphere $\mathcal{S}_t$ increases temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- Country's local temperature :

$$\dot{\tau}_{it} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \bar{\tau}_{it_0})\big)$$

  - Linear model : Climate sensitivity to carbon $\chi$, Climate reaction/inertia $\zeta$, Carbon content of fossils $\xi$, Country $i$ linear pattern scaling factor $\Delta_i$, Carbon exit from atmosphere $\delta_s$

# Model – Equilibrium

- Equilibrium
  - Given, initial conditions $\{w_0, \tau_0\}$ and country-specific policies $\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, \mathbf{t}^{ls}_{it}\}$, a competitive equilibrium is a continuum of sequences of states $\{w_{it}, \tau_{it}\}_{it}$ and $\{\mathcal{S}_t, \mathcal{T}_t, \mathcal{R}_t\}_t$ and policies $\{c_{it}, b_{it}, k_{it}, e^f_{it}, e^r_{it}, e^x_{it}\}_{it}$ and price sequences $\{r^\star_t, q^f_t, q^r_t\}$ such that :

  - Households choose policies $\{c_{it}, b_{it}\}_{it}$ to max utility s.t. budget constraint, giving $\dot{w}_{it}$
  - Firm choose policies $\{k_{it}, e^f_{it}, e^r_{it}\}_{it}$ to max profit
  - Fossil and renewables firms extract/produce $\{e^x_{it}, \bar{e}^r_{it}\}_{it}$ to max static profit, yielding $\dot{\mathcal{R}}_t$
  - Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{it}\}_{it}$.
  - Prices $\{r^\star_t, q^f_t, q^r_{it}\}$ adjust to clear the markets : $\int_{\mathbb{I}} e^x_{it} di = \int_{\mathbb{I}} e^f_{it} di$ and $e^r_{it} = \bar{e}^r_{it}$, and $\int_{i\in\mathbb{I}} b_{it} di = 0$, with bonds $b_{it} = w_{it} - k_{it}$

# Calibration – Household

- Household utility $u_i(c,\tau) = U\big(\mathcal{D}_i^u(\tau)c\big)$ with CRRA $U(\tilde{c}) = \frac{\tilde{c}^{1-\eta}}{1-\eta}$
- Damage functions in utility $u$ or production function $y$ :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm,y}(\tau-\tau_i^\star)^2}$$

and similarly for $\mathcal{D}_i^u(\tau)$, with $\gamma^{\pm,y} = \gamma^{\oplus,y}\mathbb{1}_{\{\tau>\tau_i^\star\}} + \gamma^{\ominus,y}\mathbb{1}_{\{\tau<\tau_i^\star\}}$

  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $\tau_i^\star = \bar{\alpha}\tau_{it_0} + (1-\bar{\alpha})\tau^\star$.
  - Future : $\gamma_i^{\pm,y}$ depends on daily temperature distribution $\tau \sim \mathcal{T}_i(\bar{T},\sigma^T)$ following Rudik et al. (2022)

# Calibration – Firms

- Production function $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \varepsilon(e^f, e^r)) = \left[(1-\epsilon)^{\frac{1}{\sigma_y}} k^{\alpha \frac{\sigma_y - 1}{\sigma_y}} + \epsilon^{\frac{1}{\sigma_y}} \left(z_i^e \, \varepsilon(e^f, e^r)\right)^{\frac{\sigma_y - 1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y - 1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\omega)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e - 1}}$$

  - Calibrate TFP $z_i$ to match $y_i = GDP_i$ per capita in 2011 (PPP).
  - Now : $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all $i$
  - Future : $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix $(e_i^f, e_i^r)$

# Calibration – Energy markets

- Fossil production $e^x_{it}$ and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now : $\bar{\nu}_i = \bar{\nu}$ and $\nu_i = \nu$ and $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction level data $e^x_i$ (BP, IEA)
  - Extension : Divide fossils into oil/gas/coal, and match the production/cost/reserves data across countries + use a dynamic model (extraction/exploration Hotelling problem).

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  - Extension : Divide fossils into oil/gas/coal, and match the production/cost/reserves data across countries + use a dynamic model (extraction/exploration Hotelling problem).
- Renewable : Production $\bar{e}^r_{it}$ and price $q^r_{it}$
  - Now : $q^r_{it} = z^r e^{-g_r t}$, with $g_r$ growth rate in renewable energy price decreases.
  - Future : Choose $z^r_i$ to match the energy mix $(e^f_i, e^r_i)$
  - Extension : make the problem dynamic with capacity $C^r_{it}$

# Model Solution

- Household consumption/saving problem
  - Using Pontryagin Max. Principle : states $\{x\} = \{w_{it}, \tau_{it}\}$, controls $\{c\} = \{c_{it}, b_{it}, k_{it}\}$ and costates $\{\lambda\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\} \Rightarrow$ system of coupled ODEs.

$$\mathcal{H}^{hh}(\{x\}, \{c\}, \{\lambda\}) = u(c_i, \tau_i) + \lambda^w_{it}\dot{w}_{it} + \lambda^\tau_{it}\dot{\tau}_{it} + \lambda^S_{it}\dot{S}_t$$

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  - ODE for the costate for wealth $\lambda^w_{it} = u_c(c_{it}, \tau_{it}) \ \Rightarrow$ Euler equation
  - The "local social cost of carbon" (SCC) for region $i$ :

$$LSCC_{it} := -\frac{\partial \mathcal{V}_{it}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{it}/\partial c_{it}} = -\frac{\lambda^S_{it}}{\lambda^w_{it}}$$

    - ODEs for Costates : temperature $\lambda^\tau_{it}$ and carbon $\lambda^S_{it}$, More details
    - Stationary equilibrium closed-form formula, analogous to GHKT (2014) Here

# Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$

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  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q_t^f, r_t^\star\}_t$
- Global Numerical solution :
  - Discretize agents (countries) space $i \in \mathbb{I}$ with $M$ and time-space $t \in [t_0, t_T]$ with $T$ periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and $N$ equations with gradient-descent – Newton-Raphson methods.

# Sequential method : Pros and Cons

- Why use a sequential approach ?
  - *Global approach* : *Only* need to follow the trajectories for $i$ agents :
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
    *Productivity $z_i$ Population $p_i$, Temperature scaling $\Delta_i$, Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$, Local damage $\gamma_i^y, \gamma_i^u, \tau_i^\star$, Directed Technical Change $z_i^e$*
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
    *For now : Wealth $w_{it}$, temperature $\tau_{it}$, reserves $\mathcal{R}_{it}$, Carbon $\mathcal{S}_t$*
    *Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)*
  - Newton method & Non-linear solvers very efficient

- Why not :
  - Numerical constraint to solve a large system of ODEs and non-linear equations :
  
  $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either $M$ or $T$ can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

# Competitive equilibrium

- Simulation for $M =$ countries and $T = 20$.
- Result of CE to show here

# Optimal policy

- Social planner, First best with a full set of instruments :
  - Lump-sum transfers to solve inequality, s.t.

$$\lambda_t = \omega_i u'(c_{it}) = \omega_j u'(c_{jt}) \;\; \forall i, j \in \mathbb{I}$$

  - Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^S}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)
  - Imply cross-countries lump-sum transfers $\exists\, i \;\; s.t.\; T_i > 0$ and $\exists\, j \;\; s.t.\; T_j < 0$

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- Second best without access to lump-sum transfers
  - Only region-$i$-specific distortive energy taxes : $\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r\}$. Tax receipts redistributed lump-sum : $\mathbf{t}_{it}^{ls} = \mathbf{t}_{it}^f e_{it}^f + \mathbf{t}_{it}^r e_{it}^r$
  - Welfare of the Ramsey planner :

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t}, \boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}_{it}} \int_{t_0}^{\infty} \int_{\mathbb{I}} e^{-\bar{\rho} t} \, \omega_i \, u(c_{it}, \tau_{it}) p_i \, di \, dt$$

# The Ramsey Problem – Optimal Energy Policy

- Optimal Pigouvian tax for fossil energy :

$$\Rightarrow \quad \widehat{\psi}^{w}_{it}\, \mathbf{t}^{f}_{it} = p_i \xi\, SCC_t + SCF_t\, \mathcal{C}^{f}_{EE} \qquad \& \qquad \mathbf{t}^{r}_{it} = 0$$

  - Integrate several redistribution motives :

$$\widehat{\psi}^{w}_{it} = \frac{\psi^{w}_{it}}{\overline{\psi}^{w}_{t}} = \frac{\omega_i u_c(c_{it}, \tau_{it}) p_i}{\int_{j \in \mathbb{I}} \omega_j u_c(c_{jt}, \tau_{jt}) p_j dj} \lesseqgtr 1$$

$\Rightarrow$ lower tax on poorer/high $\widehat{\psi}^{w}_{it}$ countries

  - Level depends on $SCC_t$ & fossil price $SCF_t$

$$SCC_t = \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}^{w}_{it}, LSCC_{jt}\right) + \mathbb{E}_j[LSCC_{jt}]$$

$$SCF_t = \mathbb{C}\mathrm{ov}_j\left(\widehat{\psi}^{w}_{it}, e^{f}_{jt} - e^{x}_{jt}\right) \qquad \mathcal{C}^{f}_{EE} = \left(\int_{j \in \mathbb{I}} \frac{1}{\mathcal{C}^{f}_{j,e^x e^x}} dj\right)^{-1}$$

# Optimal Energy tax

- Numerical results for $M =$ countries and $T = 20$.
- Result of Optimal Policy to show here

# Conclusion & Future plans

- 

# Appendices

# More details – Capital market

- In each countries, the agent can save in two assets, capital $k_{it}$ and bonds $b_{it}$ :

$$\begin{cases} \dot{k}_{it} & = \mathcal{D}^y_i(\tau_{it}) z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g}) k_{it} + \iota_{it} \\ \dot{b}_{it} & = r^\star b_{it} + \theta_i \pi^f_t + \pi^r_{it} - (q^f_t + \mathbf{t}^f_{it}) e^f_{it} - (q^r_t + \mathbf{t}^r_{it}) e^r_{it} - \iota_{it} - c_{it} + \mathbf{t}^{ls}_{it} \\ b_{it} & \geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting $\iota_{it}$ and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r^\star_t) k_{it} + \theta_i \pi^f_t + \pi^r_{it} - (q^f_t + \mathbf{t}^f_{it}) e^f_{it} - (q^r_t + \mathbf{t}^r_{it}) e^r_{it} - c_{it} + \mathbf{t}$$

$$k_{it} \leq \frac{1}{1-\vartheta} w_{it}$$

- Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}^y_i(\tau_{it}) z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r^\star_t$$

back

# Impact of increase in temperature

- Marginal values of the climate variables : $\lambda^s_{it}$ and $\lambda^\tau_{it}$

$$\dot{\lambda}^\tau_{it} = \lambda^\tau_{it}(\rho+\zeta) + \overbrace{\gamma_i(\tau_{it}-\tau^\star_i)\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it},e_{it})\lambda^k_{it} + \overbrace{\phi_i(\tau_{it}-\tau^\star_i)\mathcal{D}^u(\tau_{it})^{1-\eta} c_{it}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}^S_{it} = \lambda^S_{it}(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{it}$$

- Costate $\lambda^S_{it}$ : marg. cost of $1Mt$ carbon in atmosphere, for country $i$. Increases with :
  - Temperature gaps $\tau_{it}-\tau^\star_i$ & damage sensitivity of TFP $\gamma^y_i$ and utility $\gamma^u_i$
  - Development level $f(k_{it},e_{it})$ and $c_{it}$
  - Climate params : $\chi$ climate sensitivity, $\Delta_i$ "catching up" of $\tau_i$ and $\zeta$ reaction speed
  - back

# Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}^{\mathcal{T}}_{it} = \lambda^{\mathcal{T}}_t(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\lambda^k_t + \phi(\tau - \tau^\star)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}^S_t = \lambda^S{}_t(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda^{\mathcal{T}}_{it}$$

- Solving for $\lambda^{\mathcal{T}}_t$ and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^S_t = \dot{\lambda}^{\mathcal{T}}_t = 0$

$$\lambda^{\mathcal{T}}_{it} = -\int_t^\infty e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_u)y_\tau\lambda^k_u + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda^{\mathcal{T}}_{it} = -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_\infty)y_\infty\lambda^k_\infty + \phi\mathcal{D}^u(\tau_\infty)u(c_\infty)\Big)$$

$$\begin{aligned}
\lambda^S_t &= -\int_t^\infty e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,u}dj\,du \\
&= \frac{1}{\widetilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,\infty} \\
&= -\frac{\chi}{\widetilde{\rho}+\delta^s}\frac{\zeta}{\widetilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_\infty\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj \\
\lambda^S_t &\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

# Cost of carbon / Marginal value of temperature

- ***Proposition (Stationary LSCC)*** :
  When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity $\chi$, marg. damage $\gamma_i^y$, $\gamma_i^u$, temperature, and output, consumption.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^\star) \Big( \gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \Big)$$

  - Stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - Back

# Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega \int_{t_0}^{T} e^{(n+\bar{g})t-\delta_s(T-t)}\, q_t^{f\ -\sigma_e} \int_{j\in\mathbb{I}} \left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e-\sigma}\, dj\, dt$$

  - Geographical factors determining warming $\Delta_i$
  - Climate sensitivity $\chi$ & carbon exit from atmosphere $\delta_s$
  - Growth of population $n$, aggregate productivity $\bar{g}$
  - Deviation of output from trend $y_i$ & relative TFP $z_j$
  - Directed technical change $z_t^e$, elasticity of energy in output $\sigma$
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}_t^R/\lambda_t^R = \rho$
  - Change in energy mix, renewable share $\omega$, price $q_t^r$ & elasticity of source $\sigma_e$

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto\; n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \tilde{\gamma}\right) + (\sigma_e - \sigma)(1-\omega)g^{q^r} - \left(\sigma_e(1-\omega) + \sigma\omega\right)g^{q^f}$$

Back