The Inequality of Climate Change

WORK IN PROGRESS

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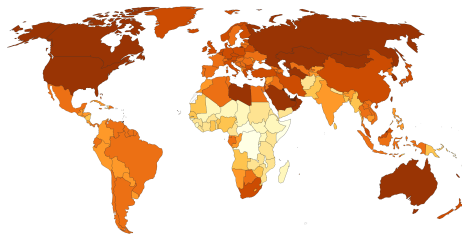
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Introduction – Motivation

Global warming is caused by greenhouse gas emissions (GHG) generated by human economic activity :

- **Unequal causes :** Developed economies account for over 65% of cumulative GHG emissions ($\sim 25\%$ each for the EU and the US)
- **Unequal consequences :** Increase in temperatures disproportionately affects developing countries where the climate is already warm
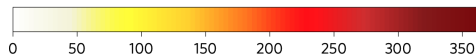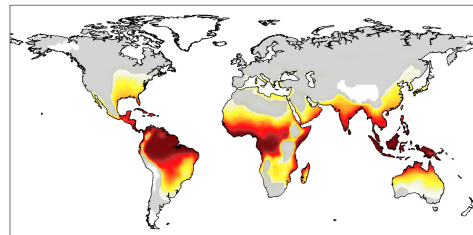
Per capita CO_2 emissions, 2021

Carbon dioxide (CO_2) emissions from fossil fuels and industry[1]. Land use change is not included.

RCP 8.5

No data   0 t   0.1 t   0.2 t   0.5 t   1 t   2 t   5 t   10 t   20 t

0   50   100   150   200   250   300   350

Number of days per year above deadly threshold

Introduction – this project

▶ Marginal damages of climate & temperature varies across countries

▶ What is the optimal taxation of energy in the presence of climate externality *and* inequality ?

- In a context, where fossil fuels taxation and climate policy redistributes across countries

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- ▶ Develop a simple and flexible model of climate economics
  - Standard IAM model with heterogeneous regions
  - Normative implications : Ramsey policy + possibility to study uncertainty

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▶ Marginal damages of climate & temperature varies across countries

▶ What is the optimal taxation of energy in the presence of climate externality *and* inequality ?
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▶ Develop a simple and flexible model of climate economics
  - Standard IAM model with heterogeneous regions
  - Normative implications : Ramsey policy + possibility to study uncertainty

○ Evaluate the heterogeneous welfare costs of global warming

○ Provide analytical formulas and a numerical methodology to compute the cost of carbon
  - Heterogeneity increases the welfare cost of carbon

○ Solve world optimal carbon policy with heterogeneous regions
  - Does the optimal carbon tax coincide with the social cost of carbon ?
  ⇒ Maybe not, depending on transfer policy : need to adjust for inequality level
  - What are the welfare gains of suboptimal policies ?

Toy model

▶ Consider two countries $i = N, S$, (North/South)
  – HH consuming good c_i and producing with energy e_i and productivity z_i
  – Energy producer with profit $\pi(E)$ owned by country i with share θ_i

▶ Household problem :

$$\mathcal{V}_i = \max_{c_i, e_i} U(c_i)$$

$$c_i + q^e e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) \qquad [\lambda_i]$$

▶ Subject to damage $\mathcal{D}_i(\mathcal{S})$ and climate externalities :

$$\mathcal{S} = \mathcal{S}_0 + \overbrace{\xi_N e_N + \xi_S e_S}^{=\text{GHG emissions}}$$

▶ And consuming energy in a single energy market with price q^e

$$E = e_N + e_S \qquad\qquad q^e = c'(E) \qquad\qquad \pi(E) = q^e E - c(E)$$

Toy model – Competitive equilibrium

▶ Three dimensions of heterogeneity :

1. Different levels of productivity z_i : $z_N > z_S$
2. Different climate damage $\gamma_i = -\frac{\mathcal{D}'_i(\mathcal{S})}{\mathcal{S}\mathcal{D}_i(\mathcal{S})}$, $\gamma_S > \gamma_N$
3. Different energy rent θ_i : $\theta_N > \theta_S$
⇒ Yields heterogeneity in consumption $c_N > c_S$

▶ Competitive equilibrium Result :

• Marginal Product of Energy = Energy Cost

$$MPe_i = q^e = c'(E) \qquad with \qquad MPe_i := \mathcal{D}_i(\mathcal{S})z_iF'(e_i)$$

• Inequality

$$\lambda_i = U'(c_i) \qquad\qquad c_i = \mathcal{D}_i(\mathcal{S})z_iF(e_i) + \theta_i\pi(E) - q^e e_i$$

Toy model – First Best and Decentralization

▶ Comparison with Social planner with full transfers (First Best)

$$\mathbb{W} = \max_{\{c_i, e_i\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$\sum_{i=N,S} c_i + c(E) = \sum_{i=N,S} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad [\lambda]$$

$$\mathcal{S} := \mathcal{S}_0 + \xi_N e_N + \xi_S e_S \qquad E := e_N + e_S$$

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- Marginal Product of Energy = Energy Cost + Social Cost of Carbon

$$MPe_i = c'(E) + \xi_i \underbrace{\overline{SCC}}_{=\mathbf{t}^e} \qquad with \qquad \overline{SCC} := -\sum_{i=N,S} \mathcal{D}_i'(\mathcal{S}) z_i F(e_i)$$

- Redistribution

$$\omega_S U'(c_S) = \omega_N U'(c_N) = \lambda$$

- Decentralization, needs to redistribute with *with lump-sum transfers $T_S = -T_N$*

$$\Rightarrow \qquad c_i + (q^e + \mathbf{t}^e) e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + \theta_i \pi(E) + T_i$$

- Are lump-sum transfers feasible?

Toy model – Second Best - Ramsey Problem

▶ Assume now that *lump-sum transfers across countries* are prohibited

- Allow for carbon tax $\mathbf{t}_i^e$ and lump-sum rebate $T_i = \mathbf{t}_i^e e_i$

$$\mathcal{W} = \max_{\{c_i, e_i\}_i} \sum_{i=N,S} \omega_i U(c_i)$$

$$s.t \qquad c_i + (q^e + \mathbf{t}_i^e)e_i = \mathcal{D}_i(\mathcal{S})z_i F(e_i) + \theta_i \pi(E) + T^i \qquad [\phi_i]$$

$$\mathcal{S} := \mathcal{S}_0 + \xi_N e_N + \xi_S e_S \qquad E := e_N + e_S \qquad q^e = c'(E)$$

- Ramsey policy result :
  - Planner's marginal value of wealth

$$\phi_i = \omega_i U'(c_i)$$

  - Energy decision :

$$\phi_i \big[MPe_i - c'(E) \big] + \xi_i \underbrace{\sum_j \phi_j \mathcal{D}_j'(\mathcal{S})z_j F(e_j)}_{\propto -SCC} + \pi'(E) \underbrace{\sum_j \phi_j \theta_j}_{= \text{ rent redistribution}} - c''(E) \underbrace{\sum_j \phi_j^k e_j}_{= \text{ cost redistribution}} = 0$$

Social Cost of Carbon with inequality

▶ Measure of inequality

$$\widehat{\phi}_i = \frac{\phi_i}{\overline{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{2}\sum_j \omega_j U'(c_j)} \lessgtr 1 \qquad\qquad \overline{\phi} = \tfrac{1}{2}\big(\omega_N U'(c_N) + \omega_S U'(c_S)\big)$$

▶ The SCC is exacerbated by heterogeneity

$$\begin{aligned}
SCC &= -\sum_j \widehat{\phi}_j \mathcal{D}'_j(\mathcal{S}) y_j \\
&= -\mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \mathcal{D}'_j(\mathcal{S}) y_j\Big) - \mathbb{E}_j[\mathcal{D}'_j(\mathcal{S}) y_j] > -\mathbb{E}_j[\mathcal{D}'_j(\mathcal{S}) y_j] = \overline{SCC}
\end{aligned}$$

• Why ? Low-income countries tend to be warmer/more vulnerable to climate change

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• Why ? Low-income countries tend to be warmer/more vulnerable to climate change

▶ For the social value of rent (exporters) and energy cost (importer), it's the contrary !

$$SVR = \mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, \theta_j \pi_j'(E)\Big) + \pi'(E) < \pi'(E)$$
$$SCE = \mathbb{C}\mathrm{ov}_j\Big(\frac{\omega_j U'(c_j)}{\frac{1}{2}\sum_j \omega_j U'(c_j)}, e_j\, c''(E)\Big) + c''(E) < c''(E) = \pi'(E)$$
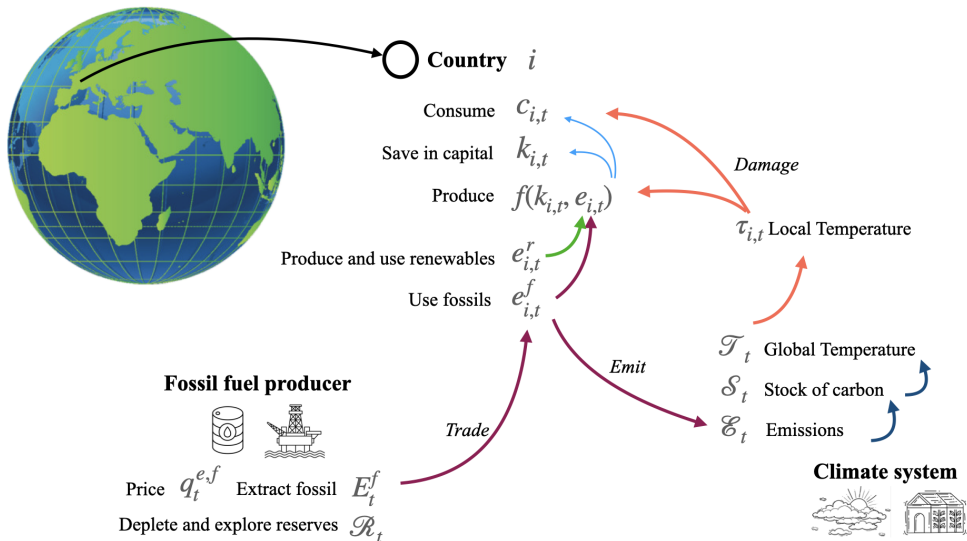
Optimal energy policy

▶ Energy taxation :

$$MPe_i = c'(E) + \xi_i \mathbf{t}_i^e$$

$$\mathbf{t}_i^e = \frac{\frac{1}{2} \sum_j \omega_j U'(c_j)}{\omega_i U'(c_i)} \left[SCC - SVR + SCE \right]$$

▶ Four motives with a single tax and lump-sum rebate
  - Distribution : Tax is lower for poorer countries $\omega_S U'(c_S) > \omega_N U'(c_N) \Rightarrow \mathbf{t}_S^e < \mathbf{t}_N^e$
  - Optimal tax level : Depends on
    – Distribution of climate damage in *SCC*
    – Distribution of energy rent in *SVR*
    – Distribution of energy spending in *SCE*

Summary of the quantitative dynamic model

Country i

Consume $c_{i,t}$

Save in capital $k_{i,t}$

Produce $f(k_{i,t}, e_{i,t})$

Produce and use renewables $e_{i,t}^r$

Use fossils $e_{i,t}^f$

Damage

$\tau_{i,t}$ Local Temperature

Fossil fuel producer

Emit

$\mathcal{T}_t$ Global Temperature

$\mathcal{S}_t$ Stock of carbon

$\mathcal{E}_t$ Emissions

Trade

Climate system

Price $q_t^{e,f}$ Extract fossil E_t^f

Deplete and explore reserves $\mathcal{R}_t$

Summary of the Model Environment

1. Households in individual countries $i \in \mathbb{I}$ consuming c_{it} [Household/Firms] [HH Solution]
   - Markets : borrow/save on world bond markets b_{it} / invest in productive capital k_{it}
   - Energy spending : fossil energy e_{it}^{f} and renewable e_{it}^{r}
   - Taxation, fossil, $\mathbf{t}_{it}^{f}$ and renewable $\mathbf{t}_{it}^{r}$
2. Energy markets [Energy]
   - Representative (Competitive) Fossil Fuel producer making profit $\pi_{t}^{f}(q_{t}^{f}, E_{t}^{f}, \mathcal{R}_{t})$
   - Extended Hotelling problem : Extraction E_{t}^{f} vs. Exploration $\mathcal{I}_{t}$
   - Redistribute share θ_{i} to household of country i
   - Renewables with price q_{t}^{r}
3. Climate system [Climate]
   - Linear dynamics : emissions $\mathcal{E}_{t}$ to atm. carbon $\mathcal{S}_{t}$ to temperature $\mathcal{T}$, cf Dietz Venmans (19)
   - Damage function on TFP (c.f. DICE) $\mathcal{D}_{i}(\tau_{it})$ and utility $u(c_{it}, \tau_{it}) = U\big(\mathcal{D}_{i}(\tau_{it})c_{it}\big)$, U CRRA

▶ Heterogeneity :

1. Productivity z_i
2. Population p_i
3. Temperature scaling Δ_i
4. Fossil energy rent θ_i
5. Carbon intensity of energy mix ξ_i
6. Local damage γ_i
7. Capital stock k_{it}
8. Local temperature τ_{it}
$\Rightarrow$ Yield inequality in consumption c_{it}

Model – Solution

► Main model equations and equilibrium [Equilibrium]

    1. Household problem $\mathcal{V}_i(w_{it_0}, \tau_{it_0}) = \max_{\{c,k,e^f,e^r\}} \int e^{-\rho t} u(c_{it}, \tau_{it}) dt$

$$\dot{w}_{it} = r_t^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{it}^{ls}$$

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2. Energy Markets

$$E_t^f = \int_{\mathbb{I}} e_{it}^f \, di \qquad q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \begin{aligned} \dot{\mathcal{R}}_t &= -E_t^f + \delta_R \mathcal{I}_t \\ \pi_t^f &= q_t^{e,f} E_t^f - \nu(E_t^f, \mathcal{R}_t) - \mu(\mathcal{I}_t, \mathcal{R}_t) \end{aligned}$$

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3. Climate system

$$\mathcal{E}_t = \int_{\mathbb{I}} \xi_i \, e_{it}^f \, di \qquad \dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t \qquad \dot{\tau}_{it} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \bar{\tau}_{it_0})\big)$$

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4. Household Decisions, consumption/saving, and energy

$$\dot{c}_{it} = c_{it} \frac{1}{\eta}(r_t^\star - \rho) \qquad MPk_{it} - \bar{\delta} = r_t^\star \qquad MPe_{it} = \mathcal{Q}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r)$$

$$\Rightarrow \qquad e_{it}^f = \mathcal{Q}_{q^f}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r) e_{it} \qquad e_{it}^r = \mathcal{Q}_{q^r}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r) e_{it}$$

Optimal policy

▶ Social planner, First best with a full set of instruments :

- Solves world's inequality, using lump-sum transfers such that
  $\lambda_t = \omega_i u'(c_{it}) = \omega_j u'(c_{jt}) \; \forall i, j \in \mathbb{I}$
- Pigouvian tax in RA economy with $\mathbf{t}_t^f = -\frac{\lambda_t^s}{\lambda_t^k} =: SCC_t$, c.f. GHKT (2014)
- Imply cross-countries lump-sum transfers

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▶ Second best / Ramsey planner :

- Doesn't have access to redistribution / lump-sum transfers
- Can only use region-i-specific distortive energy taxes : $\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r\}$
- Redistribute lump sum the tax revenues : $\mathbf{t}_{it}^{ls} = \mathbf{t}_{it}^f e_{it}^f + \mathbf{t}_{it}^r e_{it}^r$

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▶ Questions :

- Is the level of energy tax regions specific ?
  - $\Rightarrow$ Yes ! depends on the distribution of wealth/consumption
- What is the level of the Pigouvian tax ?
  - $\Rightarrow$ $\propto$ Welfare cost/climate damage : "social costate" for carbon $\mathcal{S}$, i.e. ψ^S
  - $\Rightarrow$ Inequality/Heterogeneity in damage change the *level* of this tax

The Ramsey Problem

▶ Consider a Social Planner that care about aggregate welfare :

$$\mathcal{W}_{t_0} = \max_{\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, c_{it}, e^f_{it}, e^r_{it}, k_{it}, \lambda^k_{it}, \tau_{it}, \mathcal{S}_t, \mathcal{R}_t, \mathcal{I}_t, \lambda^{\mathcal{R}}_t\}_{it}} \int_{t_0}^{\infty} \int_{\mathbb{I}} e^{-\bar{\rho}t}\, \omega_i\, u(c_{it}, \tau_{it})\, di\, dt$$

subject to

- Optimality conditions of households, for c_i, e^f_i, e^r_i and k_i
- Optimality conditions of the Fossil firm, for E^f, $\mathcal{I}$ and $\mathcal{R}$
- Optimality condition of the renewable firm, for e^r_i
- Climate and temperature dynamics τ_i and $\mathcal{S}$
- Given Pareto weights ω_i

$\Rightarrow$ Large scale system of ODE $\boxed{\text{More details - Hamiltonian}}$

- A Ramsey plan is a set $\{\mathbf{t}^f_{it}, \mathbf{t}^r_{it}, \mathbf{t}^{ls}_{it}\}_{it}$ s.t. the competitive equilibrium is maximizing welfare
- States $\{w_{it}, \tau_{it}, c_{it}, k_{it}, e^f_{it}, e^r_{it}, E^f_t, \mathcal{E}_t, \mathcal{S}_t, \mathcal{R}_t\}$
- Costates $\{\psi^w_{it}, \psi^\tau_{it}, \psi^{\mathcal{S}}_{it}, \psi^{\mathcal{R}}_t\}$

The Ramsey Problem – Solution 1

- Measure of inequality : given by the shadow value of wealth

$$\widehat{\psi}_{it}^{w} = \frac{\psi_{it}^{w}}{\overline{\psi}_{t}^{w}} = \frac{\omega_i u_c(c_i, \tau_{it}) p_i}{\int_{j \in \mathbb{I}} \omega_j u_c(c_{jt}, \tau_{jt}) p_j dj} \lessgtr 1 \qquad \text{low } z_i, k_i / \text{high } \tau_{it} \ \Rightarrow \ \text{low } c_i, \text{ high } \psi_{it}^{w} \propto \omega_i u'(c_i) p_i$$

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- SCC : Shadow values of carbon $\psi_{it}^{\mathcal{S}}$ give measures for local/global cost of carbon SCC

$$\psi_t^{\mathcal{S}} = \frac{\partial \mathcal{W}_t}{\partial \mathcal{S}_t} = \int_{j \in \mathbb{I}} \psi_{jt}^{\mathcal{S}} \, dj \qquad\qquad LSCC_{it}^{sp} := -\frac{\psi_{it}^{\mathcal{S}}}{\psi_{it}^w}$$

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- Expression for the social cost of carbon SCC_t

$$SCC_t = -\frac{\psi_t^{\mathcal{S}}}{\overline{\psi}_t^w} = \mathbb{C}\text{ov}_j\left(\widehat{\psi}_{it}^w, LSCC_{j,t}^{sp}\right) + \mathbb{E}_j[LSCC_{j,t}^{sp}] > \mathbb{E}_j[LSCC_{j,t}^{sp}] = \overline{SCC}_t$$

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$$\psi_t^{\mathcal{S}} = \frac{\partial \mathcal{W}_t}{\partial \mathcal{S}_t} = \int_{j \in \mathbb{I}} \psi_{jt}^{\mathcal{S}} dj \qquad\qquad LSCC_{it}^{sp} := -\frac{\psi_{it}^{\mathcal{S}}}{\psi_{it}^w}$$

- Expression for the social cost of carbon SCC_t

$$SCC_t = -\frac{\psi_t^{\mathcal{S}}}{\overline{\psi}_t^w} = \mathbb{C}\text{ov}_j\left(\widehat{\psi}_{it}^w, LSCC_{j,t}^{sp}\right) + \mathbb{E}_j[LSCC_{j,t}^{sp}] > \mathbb{E}_j[LSCC_{j,t}^{sp}] = \overline{SCC}_t$$

- SVF : Energy taxation redistribution

$$SVF_t = \frac{\phi_t^{Ef}}{\overline{\psi}_t^k} = \int_{\mathbb{I}} \widehat{\psi}_{jt}^k e_{jt}^f dj - \partial_{q^f} \pi^f \int_{\mathbb{I}} \widehat{\psi}_{jt}^k \theta_j dj = \mathbb{C}\text{ov}_j\left(\widehat{\psi}_{jt}^k, e_{jt}^f\right) - E_t^f \, \mathbb{C}\text{ov}_j\left(\widehat{\psi}_{jt}^k, \theta_{jt}^f\right)$$

The Ramsey Problem – Solution 2

▶ Optimal policy for fossil energy, FOC of Ramsey planner :

$$\Rightarrow \quad \widehat{\psi_{it}^w} \, \mathbf{t}_{it}^f = \xi_i \, SCC_t + SVF_t \, \mathcal{C}_{EE}^f \qquad \& \qquad \mathbf{t}_{it}^r = 0$$

▶ Pigouvian tax :
- Integrate several redistribution motives : Climate SCC_t & fossil price redistribution SVF
- **Depends** on country's consumption level $\widehat{\psi_{it}^w}$: lower tax on poorer/high $\widehat{\psi_{it}^w}$ countries

▶ Welfare costs of suboptimal taxes :
- Depends on the curvature of the production function $f(k, e)$ and substitution e_{it}^f and e_{it}^r
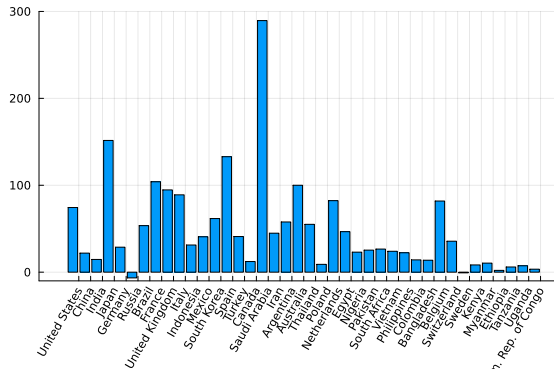
Numerical Application

▶ Data : 40 countries
▶ Temperature (of the *largest city*), GDP, energy, population
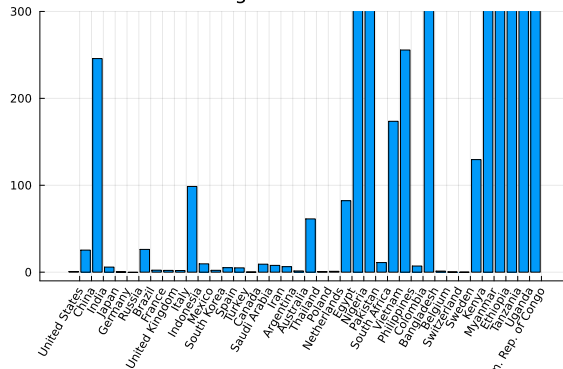▶ Calibrate z to match the distribution of output per capita at steady state

CO2 Emissions (Tons of CO2 per capita, 2011)
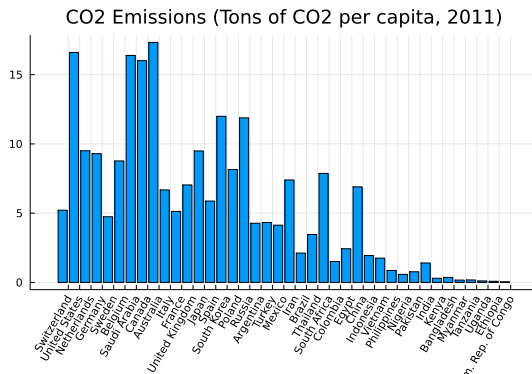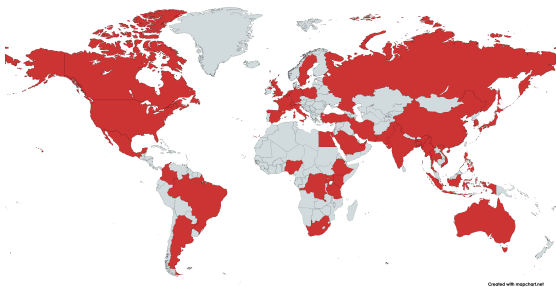
Local Cost of Carbon

▶ Difference $LSCC_i = \psi_{it}^{\mathcal{S}}/\psi_{it}^w$ and $LWCC_{it} = \widehat{\psi}_{it}^w LSCC_{it} = \psi_{it}^{\mathcal{S}}/\bar{\psi}_t^k$
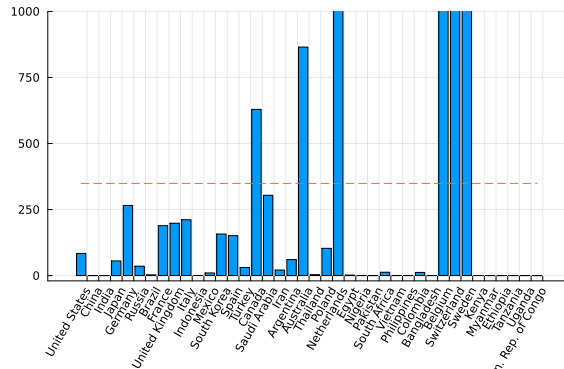

LSCC in 2100 | Reweighted LSCC in 2100

Social Cost of Carbon and Carbon Tax

▶ Difference $LSCC_i = \psi_{it}^{\mathcal{S}}/\psi_{it}^{w}$ and $\mathbf{t}_{it} = (1/\widehat{\psi}_{it}^{w})SCC$
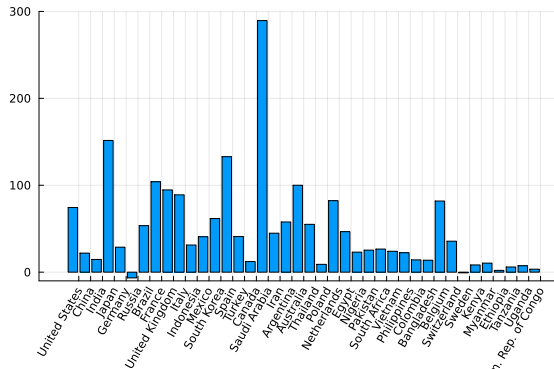
Conclusion

- ▶ Climate change has redistributive effects & heterogeneous impacts
- ▶ Redistributive effects of policy

  - Pigouvian tax that covers aggregate marginal damages
  - Can account for inequality both for heterogeneous welfare costs of climate and redistributive effects of energy price, for importers and exporters

- ▶ Study suboptimal policies
  - If carbon taxes are unfeasible : renewable subsidy ?
- ▶ Future plans
  - Dynamics on the capacity of renewable ?
  - Endogenous growth in TFP/energy saving technology
    Learning-by-doing : positive externality ?
  - Uncertainty : HA model with aggregate risk hard to solve

Appendices

Numerical Applications

Numerical Application

- ▶ Data : 40 countries
- ▶ Temperature (of the *largest city*), GDP, energy, population
- ▶ Calibrate z to match the distribution of output per capita at steady state

CO2 Emissions (Tons of CO2 per capita, 2011)
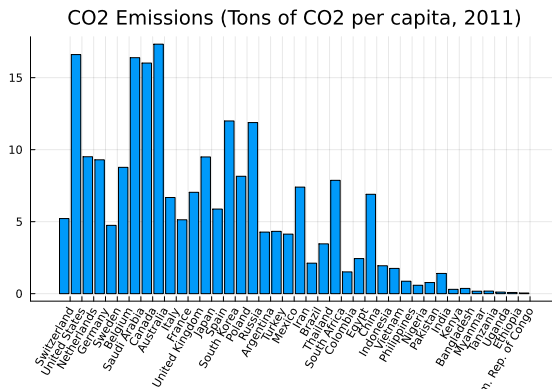
Model 1

► Neoclassical economy, in continuous time (Back)

  – countries/regions $i \in \mathbb{I}$: ex-ante heterogeneous : productivity z_i and more

  – ex-post heterogeneity in capital and temperature $\{k_i, \tau_i\}$

► Representative household problem in each country i :

$$\mathcal{V}_{it_0} = \max_{\{c_{it}, e_{it}^f, e_{it}^r\}} \int_{t_0}^{\infty} e^{-\rho t} u(c_{it}, \tau_{it}) dt$$

► Dynamics of wealth of country i, (More details) with wealth $w_{it} = b_{it} + k_{it}$:

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_t^r + \mathbf{t}_{it}^r) e_{it}^r - c_{it} + \mathbf{t}_{it}^{ls}$$

  • Damage $\mathcal{D}^y(\tau_{it})$ affect country's production and consumption $u(\cdot, \tau_{it})$
  • Energy mix : $e_{it} = \mathcal{E}(e_{it}^f, e_{it}^r | \sigma_e)$ with fossil e_{it}^f – emitting carbon – vs. renewable e_t^r
  • Energy rents redistributed : share θ_i for fossils / fully for local renew. firm.
  • Prices, fossil q_t^f and non-carbon q_t^r (c.f. next slides)

Model 2 – Energy markets

► Fossil fuels energy producer :

- Extended-Hotelling problem (depleting reserves with stock effects and exploration)

$$\max_{\{E_t^f, \mathcal{I}_t\}_t} \int_0^\infty e^{-\rho t} \pi_t^f(q_t^f, E_t^f, \mathcal{R}_t) dt \qquad \text{with } \pi_t(E_t^f, \mathcal{R}_t) = q_t^{e,f} E_t^f - \mathcal{C}^f(E_t^f, \mathcal{R}_t) - \mathcal{C}^i(\mathcal{I}_t, \mathcal{R}_t)$$

$$s.t. \qquad E_t^f = \int_{\mathbb{I}} e_{it} \, di \qquad \dot{\mathcal{R}}_t = -E_t^f + \delta_R \mathcal{I}_t$$

- Optimal pricing with finite-resources rents (More details)

$$q_t^{e,f} = \mathcal{C}_E^f(E_t^f, \mathcal{R}_t) + \lambda_t^R \qquad \qquad \mathcal{C}_{\mathcal{I}}^i(\mathcal{I}_t, \mathcal{R}_t) = \delta_R \lambda_t^R$$

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▶ Renewable energy as a substitute technology *for each country i* (Static problem)

$$\pi_{it}^r = \max_{\{e_{it}^r\}} q_{it}^r e_{it}^r - \mathcal{C}^r(e_{it}^r) \qquad \Rightarrow \qquad q_{it}^r = \mathcal{C}_E^r(e_t^r)$$

(Back)

Model 3 - Climate model :

▶ Fossil energy input e_t^f causes climate externality

$$\mathcal{E}_t = \int_{\mathbb{I}} \xi_i \, e_{it}^f \, di$$

▶ World climate – cumulative GHG in atmosphere $\mathcal{S}_t$ leads to increase in temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

▶ Impact of climate on country's local temperature :

$$\dot{\tau}_{it} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \bar{\tau}_{it_0})\big)$$

- Simple model : Climate sensitivity to carbon χ, Climate reaction/inertia ζ, Carbon content of fossils ξ, Country scaling factor Δ_i, Carbon exit for atmosphere δ_s
- Back

Model 4 – Household Solution

▶ Household solves a consumption/saving/energy decision, as in the NCG (More details)

• Using Pontryagin (PMP), we obtain a system of coupled ODEs (More details)

Model 4 – Household Solution

▶ Household solves a consumption/saving/energy decision, as in the NCG `More details`

- Using Pontryagin (PMP), we obtain a system of coupled ODEs `More details`
- Consumption/Saving Euler equation (financial integration) :

$$\dot{c}_{it} = c_{it}\frac{1}{\eta}(r_t^\star - \rho) \qquad\qquad MPk_{it} - \overline{\delta} = r_t^\star$$

- Energy decisions :
  Static demand for the two sources of energy : fossil e_{it}^f and renewable e_{it}^r for every i, taking prices $\{q^f, q^r\}$ as given

$$MPe_{it} = \mathcal{Q}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r)$$
$$\Rightarrow \qquad e_{it}^f = \mathcal{Q}_{q^f}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r)e_{it} \qquad\qquad e_{it}^r = \mathcal{Q}_{q^r}(q_t^f + \mathbf{t}_{it}^f, q_{it}^r + \mathbf{t}_{it}^r)e_{it}$$

- with $MPk_{it} = \mathcal{D}(\tau_{it})z_i f_{k,it}$ and $MPe_{it} = \mathcal{D}^y(\tau_{it})z_i f_e(k_{it}, e_{it})$, and $\mathcal{Q}(\cdot)$ are aggregators functions (e.g. CES) and $\mathcal{Q}_{q^f}(\cdot)$ demand for fossil.

Model – Equilibrium

► Three types of interactions (Equilibrium)

- On climate (externality) + heterogeneous effects of temperatures
- On bonds markets + capital constraints
- On energy market + redistribution effects of energy rent
- No bilateral flows (eq. doesn't exist with continuum and trade or migration)

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▶ Equilibrium

- Given, initial conditions $\{k_0, \tau_0\}$ and country-specific policies $\{\mathbf{t}_{it}^f, \mathbf{t}_{it}^r, \mathbf{t}_{it}^{ls}\}$, a competitive equilibrium is a continuum of sequences of states $\{k_{it}, \tau_{it}\}_{it}$ and $\{\mathcal{S}_t, \mathcal{T}_t, \mathcal{R}_t\}_t$ and policies $\{c_{it}, e_{it}^f, e_{it}^r\}_{it}$ and $\{E_t^f, \mathcal{E}_t, \mathcal{I}_t\}_t$, and price sequences $\{q_t^f, q_t^r\}$ such that :

  ○ Households choose policies $\{c_{it}, e_{it}^f, e_{it}^r\}_{it}$ to max utility s.t. budget constraint, giving $\dot{k}_{it}$
  ○ Renewable energy firm produce $\{e_{it}^r\}$ to max static profit
  ○ Fossil fuel firm extract and explore $\{E_t^f, \mathcal{I}_t\}$ to max profit, yielding $\dot{\mathcal{R}}_t$
  ○ Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t, \mathcal{T}_t\}_t$, & $\{\tau_{it}\}_{it}$.
  ○ Prices $\{q_t^f, q_{it}^r, r_t^\star\}$ adjust to clear the markets : $E_t^f = \int_{\mathbb{I}} e_{it}^f di$ and $e_t^r = e_{it}^r$, and $\int_{i \in \mathbb{I}} b_{it} di = 0$

Impact of increase in temperature

▶ Using Damage fct $\mathcal{D}^y(\tau_{it}) = e^{-\frac{1}{2}\gamma_i(\tau_{it}-\tau_i^\star)^2}$ and $u(c,\tau) = u(\mathcal{D}^u(\tau_{it})c)$, w/ $u(\hat{c}) = \frac{c^{1-\eta}}{1-\eta}$

▶ Marginal values of the climate variables : λ_{it}^S and λ_{it}^τ

$$\dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho+\zeta) + \overbrace{\gamma_i(\tau_{it}-\tau_i^\star)\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})}f(k_{it},e_{it})\lambda_{it}^k + \overbrace{\phi_i(\tau_{it}-\tau_i^\star)\mathcal{D}^u(\tau_{it})^{1-\eta}c_{it}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}_{it}^S = \lambda_{it}^S(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda_{it}^\tau$$

▶ Costate λ_{it}^S : marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{it} - \tau_i^\star$ & damage sensitivity of TFP γ_i and utility ϕ_i
  - Development level $f(k_{it},e_{it})$ and c_{it}
  - Climate params : χ climate sensitivity, Δ_i "catching up" of τ_i and ζ reaction speed
  - back

Local Social cost of carbon

▶ The marginal "externality damage" or "local social cost of carbon" (SCC) for region i :

$$LSCC_{it} := -\frac{\partial \mathcal{V}_{it}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{it}/\partial c_{it}} = -\frac{\lambda_{it}^S}{\lambda_{it}^k}$$

- Ratio of marg. cost of carbon vs. the marg. value of consumption/capital

- Theorem : **Stationary LSCC** :
  When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ, ϕ, temperature, and output, consumption.

$$LSCC_{it} \equiv \frac{\chi \, \Delta_i}{\widetilde{\rho} + \delta^s} \, (\tau_{i,\infty} - \tau_i^\star) \big[\gamma_i \, y_{i,\infty} + \phi_i \, c_{i,\infty} \big]$$

  – More general formula : (Here) , Proof : (Here) + What determine temperatures ? (Details Temperature)

More details – Capital market

▶ In each countries, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} &= \mathcal{D}_i^y(\tau_{it})z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g})k_{it} + \iota_{it} \\ \dot{b}_{it} &= r^\star b_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f)e_{it}^f - (q_t^r + \mathbf{t}_{it}^r)e_{it}^r - \iota_{it} - c_{it} + \mathbf{t}_{it}^{ls} \\ b_{it} &\geq -\vartheta k_{it} \end{cases}$$

▶ Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it})z_{it}f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star)k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathbf{t}_{it}^f)e_{it}^f - (q_t^r + \mathbf{t}_{it}^r)e_{it}^r - c_{it} + \mathbf{t}_i^l$$

$$k_{it} \leq \frac{1}{1-\vartheta} w_{it}$$

▶ Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it})z_i\partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r_t^\star$$

back

More details – Energy market

▶ Fossil fuel producer : price the Hotelling rent with the maximum principle :

$$\mathcal{H}^m(\mathcal{R}_t, \lambda_t^R, E_t, \mathcal{I}_t^e) = \pi_t(E_t^f, \mathcal{I}_t^f, \mathcal{R}_t) + \lambda_t^R(\delta^R \mathcal{I}_t^e - E_t)$$

▶ Rent λ_t^R grows with interest ρ and with the marginal gain of increasing reserves

$$\dot{\lambda}_t^R = \rho \lambda_t^R - \partial_R \mathcal{C}(E_t^f, \mathcal{I}_t^f, \mathcal{R}_t)$$
$$= \rho \lambda_t^R + \frac{\bar{\nu}\nu}{1+\nu}\Big(\frac{E_t^\star}{R_t}\Big)^{1+\nu} + \frac{\bar{\mu}\mu}{1+\mu}\Big(\frac{I_t^\star}{R_t}\Big)^{1+\mu}$$
$$\dot{\lambda}_t^R = \rho \lambda_t^R + \frac{\bar{\nu}^{-1/\nu}\nu}{1+\nu}\big(q^{e,f} - \lambda_t^R\big)^{1+1/\nu} + \frac{\bar{\mu}^{-1/\mu}\mu}{1+\mu}\big(\delta^R \lambda_t^R\big)^{1+1/\mu}$$

▶ Because of decreasing return to scale and Hotelling rents : profits are > 0

$$\pi_t(E_t^f, \mathcal{R}_t, \lambda_t^R) = \frac{1+\nu-\bar{\nu}}{1+\nu}\Big(\frac{E_t^f}{\mathcal{R}_t}\Big)^{1+\nu}\mathcal{R}_t + \lambda_t^R E_t^f - \frac{\bar{\mu}^{-1/\mu}}{1+\mu}\big(\delta^r \lambda_t^R\big)^{1+\frac{1}{\mu}}$$

back

More details – PMP – Competitive equilibrium

▶ Household problem : State variables $s_{it} = (k_i, \tau_i, z_i, p_i, \Delta_i)$ Back

▶ Pontryagin Maximum Principle

$$\mathcal{H}^{hh}(s, \{c\}, \{e^f\}, \{e^r\}, \{\lambda\}) = u(c_i, \tau_i) + \lambda_{it}^k \Big(\mathcal{D}(\tau_{it})f(k_t, e_t) - (n + \bar{g} + \delta)k_t - q_t^f e_{it}^f - q_{it}^r e_{it}^r - c_t\Big)$$
$$+ \lambda_{it}^\tau \zeta \Big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{i0})\Big) + \lambda_{it}^S \Big(\mathcal{E}_t - \delta^s \mathcal{S}_t\Big)$$

$$[c_t] \qquad u'(c_{it}) = \lambda_{it}^k$$

$$[e_t^f] \qquad MPe_{it}^f = \mathcal{D}(\tau_{it})z\,\partial_e f(k_{it}, e_{it})\Big(\frac{e_{it}^f}{\omega e_{it}}\Big)^{-\frac{1}{\sigma_e}} = q_t^f$$

$$[e_t^r] \qquad MPe_{it}^r = \mathcal{D}(\tau_{it})z\,\partial_e f(k_{it}, e_{it})\Big(\frac{e_{it}^r}{(1-\omega)e_{it}}\Big)^{-\frac{1}{\sigma_e}} = q_{it}^r$$

$$[k_t] \qquad \dot{\lambda}_t^k = -\lambda_t^k\big(\mathcal{D}(\tau_{it})\partial_k f(k_{it}, e_{it}) - \delta - \bar{g} - n - \rho\big)$$

▶ Fossil Energy Monopoly :

$$\mathcal{H}^m(\mathcal{R}_t, \lambda_t^R, E_t^f, \mathcal{I}_t) = \pi_t(E_t^f, \mathcal{I}_t, \mathcal{R}_t) + \lambda_t^R(\delta^R \mathcal{I}_t - E_t^f)$$

$$[\mathcal{R}_t] \qquad \dot{\lambda}_t^R = \rho\lambda_t^R + \frac{\bar{\nu}\nu}{1+\nu}\Big(\frac{E_t^\star}{R_t}\Big)^{1+\nu} + \frac{\bar{\mu}\mu}{1+\mu}\Big(\frac{I_t^\star}{R_t}\Big)^{1+\mu}$$

$$[E_t^f] \qquad q_t^{e,f} = \nu_E(E, \mathcal{R}) + \lambda_t^R = \bar{\nu}\Big(\frac{E_t}{\mathcal{R}_t}\Big)^\nu + \lambda_t^R$$

$$[\mathcal{I}_t] \qquad \lambda_t^R \delta^R = \mu_I(I_t, R_t) = \bar{\mu}\Big(\frac{\mathcal{I}_t}{\mathcal{R}_t}\Big)^\mu \qquad \mathcal{I}_t = R_t\Big(\frac{\lambda_t^R \delta}{\bar{\mu}}\Big)^{1/\mu}$$

Cost of carbon / Marginal value of temperature

▶ Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}_{it}^{\tau} = \lambda_t^{\tau}(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^{\star})\mathcal{D}^y(\tau)f(k,e)\lambda_t^k + \phi(\tau - \tau^{\star})\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}_t^{\mathcal{S}} = \lambda_t^{\mathcal{S}}(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta\chi\lambda_{it}^{\tau}$$

▶ Solving for λ_t^{τ} and $\lambda_t^{\mathcal{S}}$, in stationary equilibrium $\dot{\lambda}_t^{\mathcal{S}} = \dot{\lambda}_t^{\tau} = 0$

$$\lambda_{it}^{\tau} = -\int_t^{\infty} e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^{\star})\Big(\gamma\mathcal{D}^y(\tau_u)y_{\tau}\lambda_u^k + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda_{it}^{\tau} = -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_{\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^y(\tau_{\infty})y_{\infty}\lambda_{\infty}^k + \phi\mathcal{D}^u(\tau_{\infty})u(c_{\infty})\Big)$$

$$\lambda_t^{\mathcal{S}} = -\int_t^{\infty} e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda_{j,u}^{\tau}dj\,du$$

$$= \frac{1}{\widetilde{\rho} + \delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda_{j,\infty}^{\tau}$$

$$= -\frac{\chi}{\widetilde{\rho} + \delta^s}\frac{\zeta}{\widetilde{\rho} + \zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{\infty}\lambda_{j,\infty}^k + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj$$

$$\lambda_t^{\mathcal{S}} \xrightarrow{\zeta \to \infty} -\frac{\chi}{\widetilde{\rho} + \delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda_{j,\infty}^k + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj$$

Cost of carbon / Marginal value of temperature

▶ Closed form solution for CC :
- In stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
- Fast temperature adjustment $\zeta \to \infty$
- no internalization of externality (business as usual)

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s}(\tau_\infty - \tau^\star)\left(\gamma \mathcal{D}^y(\tau_\infty)y_\infty + \phi \mathcal{D}^u(\tau_\infty)c_\infty\right)$$

▶ Heterogeneity + uncertainty about models Back

Social cost of carbon & temperature

▶ Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega \int_{t_0}^{T} e^{(n+\bar{g})t - \delta_s(T-t)} q_t^{f\,-\sigma_e} \int_{j\in\mathbb{I}} \left(z_j z_{j,t}^e \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e-\sigma} dj\, dt$$

- Geographical factors determining warming Δ_i
- Climate sensitivity χ & carbon exit from atmosphere δ_s
- Growth of population n, aggregate productivity $\bar{g}$
- Deviation of output from trend y_i & relative TFP z_j
- Directed technical change z_t^e, elasticity of energy in output σ
- Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \lambda_t^R / \lambda_t^R = \rho$
- Change in energy mix, renewable share ω, price q_t^r & elasticity of source σ_e

▶ Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto n + \bar{g}^y - (1-\sigma)\big(g^{z^e} - \widetilde{\gamma}\big) + (\sigma_e - \sigma)(1-\omega)g^{q^r} - (\sigma_e(1-\omega) + \sigma\omega)g^{q^f}$$

Back

More details – PMP – Ramsey Optimal Allocation

▶ Hamiltonian :

$$\mathcal{H}^{sp}(s, \{c\}, \{e^f\}, \{e^r\}, \{\lambda\}, \{\psi\}) = \int_{\mathbb{I}} \omega_i u(c_i, \tau_i) p_i di$$

$$+ \psi_{it}^k \Big(\mathcal{D}(\tau_{it}) f(k_{it}, e_{it}) - (n + \bar{g} + \delta) k_t + \theta_i \pi(E_t^f, \mathcal{I}_t, \mathcal{R}_t) + \pi_{it}^r(e_{it}^r) - (q_t^f + \mathbf{t}_{it}^f) e_{it}^f - (q_{it}^r + \mathbf{t}_{it}^r) e_{it}^r - c_t + \mathbf{t}_t^{ls} \Big)$$

$$+ \psi_t^{\mathcal{S}} \Big(\mathcal{E}_t - \delta^s \mathcal{S}_t \Big) + \psi_{it}^{\tau} \zeta \Big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{i0}) \Big) + \psi_{it}^{\mathcal{R}} \Big(-E_t^f + \delta^R \mathcal{I}_t \Big)$$

$$+ \psi_{it}^{\lambda k} \Big(\lambda_t^k (\rho - r_t) \Big) + \psi_{it}^{\lambda R} \Big(\rho \lambda_t^R + \mathcal{C}_{\mathcal{R}}^f(E_t^f, \mathcal{I}_t, \mathcal{R}_t) \Big) + \phi_{it}^c \big(u_c(c_i, \tau_{it}) - \lambda_{it}^k \big)$$

$$+ \phi_{it}^{ef} \Big(e_{it}^f - \mathscr{Q}_{q^f} \big(q_t^f + \mathbf{t}_{it}^f, q_t^r + \mathbf{t}_{it}^r \big) e_{it} \Big) + \phi_{it}^{er} \Big(e_{it}^r - \mathscr{Q}_{q^r} \big(q_t^f + \mathbf{t}_{it}^f, q_t^r + \mathbf{t}_{it}^r \big) e_{it} \Big)$$

$$+ \phi_{it}^e \Big(f_e(k_{it}, e_{it}) - \mathscr{Q} \big(q_t^f + \mathbf{t}_{it}^f, q_t^r + \mathbf{t}_{it}^r \big) \Big) + \phi_t^{Ef} \big(q_t^f - \mathcal{C}_E^f(\cdot) - \lambda_t^{\mathcal{R}} \big) + \phi_{it}^{Er} \big(q_{it}^r - \mathcal{C}_e^r(\cdot) \big) + \phi_t^{\mathcal{I}f} \big(\delta \lambda_t^{\mathcal{R}} - \mathcal{C}_{\mathcal{I}}^f(\cdot) \big)$$

▶ Back

Ramsey Optimal Allocation - FOCs

▶ FOCs w.r.t. $\{c_{it}, e_{it}, e_{it}^f, e_{it}^r, \mathcal{I}_t\}$, prices $\{q_t^f, q_{it}^r\}$ and taxes, denoting $\tilde{q}_{it} = q_t + \mathbf{t}_{it}$

$[c_{it}]$
$$\psi_{it}^k = \underbrace{\omega_i u_c(c_i, \tau_{it}) p_i}_{=\text{direct effect}} + \underbrace{\phi_{it}^c u_{cc}(c_i, \tau_{it})}_{=\text{effect on savings}}$$

$[e_{it}]$
$$\psi_{it}^k f_{e,it} + \phi_{it}^e f_{ee,it} - \phi_{it}^{ef} \mathcal{Q}_{q^f} - \phi_{it}^{er} \mathcal{Q}_{q^r} = 0 \quad \Rightarrow \quad \phi_{it}^e = \frac{1}{f_{ee,it}} \big(\phi_{it}^{ef} \mathcal{Q}_{q^f} + \phi_{it}^{er} \mathcal{Q}_{q^r} - \psi_{it}^k f_{e,it} \big)$$

$[e_{it}^f]$ $\quad \phi_{it}^{ef} = \psi_{it}^k \tilde{q}_t^f - \psi_{it}^k \mathbf{t}_{it}^f - \xi \psi_t^S p_i + \phi_t^{Ef} \mathcal{C}_{EE}^f(\cdot)$ $\qquad [e_{it}^r]$ $\qquad \phi_{it}^{er} = \psi_{it}^k \tilde{q}_t^r - \psi_{it}^k \mathbf{t}_{it}^r + \phi_{it}^{Er} \mathcal{C}_{e^r e^r}^r(\cdot)$

$[\tilde{q}_{it}^f]$ $\quad \phi_{it}^e \mathcal{Q}_{q^f} + \phi_{it}^{ef} \mathcal{Q}_{q^f q^f} + \phi_{it}^{er} \mathcal{Q}_{q^r q^f} = 0$

$$\Rightarrow \quad \Big(\frac{\mathcal{Q}_{q^f}^2}{f_{ee,it}} + \mathcal{Q}_{q^f q^f} \Big) \big[- \xi \psi_t^S p_i + \phi_t^{Ef} \mathcal{C}_{EE}^f(\cdot) - \psi_{it}^k \mathbf{t}_{it}^f \big] + \Big(\frac{\mathcal{Q}_{q^f} \mathcal{Q}_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^r q^f} \Big) \big[\phi_{it}^{Er} \mathcal{C}_{e^r e^r}^r(\cdot) - \psi_{it}^k \mathbf{t}_{it}^f \big]$$

$[\tilde{q}_{it}^r]$ $\quad \phi_{it}^e \mathcal{Q}_{q^r} + \phi_{it}^{ef} \mathcal{Q}_{q^f q^r} + \phi_{it}^{er} \mathcal{Q}_{q^r q^r} = 0$

$$\Rightarrow \quad \Big(\frac{\mathcal{Q}_{q^f} \mathcal{Q}_{q^r}}{f_{ee,it}} + \mathcal{Q}_{q^f q^r} \Big) \big[- \xi \psi_t^S p_i + \phi_t^{Ef} \mathcal{C}_{EE}^f(\cdot) - \psi_{it}^k \mathbf{t}_{it}^f \big] + \Big(\frac{\mathcal{Q}_{q^r}^2}{f_{ee,it}} + \mathcal{Q}_{q^r q^r} \Big) \big[\phi_{it}^{Er} \mathcal{C}_{e^r e^r}^r(\cdot) - \psi_{it}^k \mathbf{t}_{it}^f \big] = 0$$

$[q_t^f]$ $\quad \phi_t^{Ef} = \int_{\mathbb{I}} \psi_{jt}^k e_{jt}^f dj - \partial_{q^f} \pi^f(\cdot) \int_{\mathbb{I}} \theta_j \psi_{jt}^k dj$ $\qquad [q_{it}^r]$ $\qquad \phi_{it}^{Er} = \psi_{it}^k e_t^r - \psi_{it}^k \partial_q^r \pi_{it}^r = 0$

$[\mathcal{I}_t]$ $\quad \delta \psi_t^{\mathcal{R}} + \partial_{\mathcal{R}\mathcal{I}}^2 \mathcal{C}(\cdot) \psi_t^{\lambda,\mathcal{R}} - \phi_t^{\mathcal{I}} \partial_{\mathcal{I}\mathcal{I}}^2 \mathcal{C}(\cdot) = 0$

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Ramsey Optimal Allocation - FOCs

▶ Backward equations for planner's costates

$$[k_i] \qquad \dot{\psi}_{it}^k = \psi_{it}^k(\tilde{\rho} - r_{it}) + \psi_{it}^{\lambda k}\lambda_{it}^k \partial_k MPk_i + \frac{f_{ek,it}}{f_{ee,it}}\big[-\xi\psi_t^S p_i + \phi_t^{Ef}\mathcal{C}_{EE}^f(\cdot) - \psi_{it}^k \mathbf{t}_{it}^f\big]$$

$$[\mathcal{S}_i] \qquad \dot{\psi}_t^{\mathcal{S}} = (\tilde{\rho} + \delta^s)\psi_t^{\mathcal{S}} - \int_{\mathbb{I}} \Delta_j \zeta \chi \psi_{jt}^{\tau} dj$$

$$[\tau_i] \qquad \dot{\psi}_t^{\tau} = (\tilde{\rho} + \zeta)\psi_t^{\tau} - \Big(\omega_i u_\tau(c_{it}, \tau_{it}) + \psi_{it}^k \mathcal{D}'(\tau_{it})f(k_{it}, e_{it}) + \phi_{it}^c u_{c,\tau}(c_{it}, \tau_{it}) + \mathcal{D}'(\tau_{it})f_e\phi_{it}^e\Big)$$

$$[\mathcal{R}] \qquad \dot{\psi}_t^{\mathcal{R}} = \psi_t^{\mathcal{R}}\big(\tilde{\rho} - \partial_{\mathcal{R}\mathcal{R}}^2\mathcal{C}(\cdot)\big) - \phi_t^{Ef}\partial_{\mathcal{R}E}^2\mathcal{C}(\cdot)$$

$$[\lambda_i^k] \qquad \dot{\psi}_t^{\lambda,k} = \psi_t^{\lambda,k}\big[\tilde{\rho} - (\rho - r_{it})\big] + \phi_{it}^c$$

$$[\lambda_i^{\mathcal{R}}] \qquad \dot{\psi}_t^{\lambda,\mathcal{R}} = \psi_t^{\lambda,\mathcal{R}}(\tilde{\rho} - \rho) + \phi_t^{Ef} - \delta\phi_t^{\mathcal{I}f}$$

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