The Optimal design of Climate Agreements
Inequality and incentives for carbon policy

WORK IN PROGRESS

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Capital theory

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Introduction

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- Trade-off between intensive margins and extensive margin :
  - Climate club with a small number of countries with (i) a tax higher/closer to the *first-best*, (ii) large emissions reductions but (iii) potentially large carbon "leakage" effects
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- Trade-off between intensive margins and extensive margin :
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  - Extensive climate agreements with a large number of countries but lower optimal tax to accommodate participation constraints
- Develop an Integrated Assessment Model (IAM) with heterogeneous countries to :
  - Evaluate the welfare costs of global warming
  - Solve optimal carbon policy of climate clubs
  - Analyze the strategic implications of joining climate agreements
  - Design to optimal size of the climate club

Literature

- Climate change & optimal carbon taxation
  - RA model : Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
  - HA model : Krusell Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
  - Spatial models : Cruz, Rossi-Hansberg (2022, 2023)

  $\Rightarrow$ *Optimal and constrained policy with heterogeneous countries*
- Unilateral vs. climate club policies :
  - Climate clubs : Nordhaus (2015), Non-cooperative taxation : Chari, Kehoe (1990), Suboptimal policy : Hassler, Krusell, Olovsson (2019)
  - Trade policy : Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2021)

  $\Rightarrow$ *Climate cooperation and optimal design of climate club*
- Optimal policy in heterogeneous agents models
  - Policy with limited instruments : Diamond (1973), Davila, Walther (2022)
  - Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) ...

  $\Rightarrow$ *Application to climate and carbon taxation policy*

Model – Household

- Deterministic Neoclassical economy, in continuous time
  - countries $i \in \mathbb{I}$, heterogeneous in productivity z_i/wealth w_{it}, temperature τ_{it}, energy cost/reserves $\mathcal{R}_{it}$
  - In each country, 4 agents : (i) household, (ii) homogeneous good firm, (iii) fossil and (iv) renewable energy producers.

- Representative household problem in each country i :

$$\mathcal{V}_{i0} = \max_{\{c_{it}, k_{it}, b_{it}\}} \int_0^\infty e^{-\rho t} \, u(c_{it}) dt$$

- Dynamics of wealth of country i, $w_{it} = b_{it} + k_{it}$ More details

$$\dot{w}_{it} := \dot{k}_{it} + \dot{b}_{it} = w_{it}\ell_{it} + \pi^f_{it} + r_t b_{it} + (r_t - \delta)k_{it} - c_{it} + \mathrm{t}^{ls}_{it}$$

  - Labor income $w_{it}\ell_{it}$ from homogeneous good firm, profit π^f_{it} from fossil firm

Model – Representative Firm

- Competitive homogeneous good producer in country i

$$\max_{k_{it}, e^f_{it}, e^r_{it}} \mathcal{D}^y(\tau_{it}) z_i f(k_{it}, e^f_{it}, e^r_{it}) - w_{it}\ell_{it} - r_t k_{it} - (q^f_t + \mathrm{t}^f_{it}) e^f_{it} - (q^r_t + \mathrm{t}^r_{it}) e^r_{it}$$

  - Energy mix with fossil e^f_{it} – emitting carbon – subject to price q^f_t and tax/subsidy t^f_{it}. Similarly "clean" renewable e^r_t, at price q^r_{it} and tax t^r_{it}.
  - Climate externality : effect of temperature on damage/TFP, $\mathcal{D}^y_i(\tau) \in (0, 1)$

Model – Energy markets

- Competitive fossil fuels energy producer :
  - Extraction of fossil energy e^x_{it} depleting reserves $\mathcal{R}_{it}$ $\Rightarrow$ Hotelling problem

$$\pi^f_{it} = \max_{e^x_{it}} q^f_t e^x_{it} - \mathcal{C}^f(e^x_{it}, \mathcal{R}_{it})$$

$$\dot{\mathcal{R}}_{it} = -e^x_{it} \qquad\qquad \mathcal{R}_{i0} = \bar{\mathcal{R}}_i$$

  - Fossil energy traded in international markets :

$$\sum_{\mathbb{I}} e^f_{it} = \sum_{\mathbb{I}} e^x_{it}$$

    - Unique fossil price q^f_t clearing the market More details

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- Renewable energy in each country i with exogenous price q^r_{it}

Climate system

- Fossil energy input e_t^f causes climate externality

$$\mathcal{E}_t = \sum_{\mathbb{I}} e_{it}^f$$

- Cumulative GHG in atmosphere $\mathcal{S}_t$ increases temperature

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

- Country's local temperature :

$$\tau_{it} = \bar{\tau}_{i0} + \Delta_i \, \mathcal{S}_t$$

  - Linear model : Climate sensitivity/pattern scaling factor Δ_i, Carbon exit from atmosphere δ_s

Model – Solution

- Step 0 : Competitive equilibrium / Business as usual : No policy $\mathrm{t}_{it} = 0$
- Step 1 : First Best, All instruments available $\{\mathrm{t}^{f}_{it}, \mathrm{t}^{r}_{it}, \mathrm{t}^{ls}_{it}\}_{it}$ including transfers across countries
- Step 2 : Second best, Optimal (Ramsey) policy for a given climate club $\mathbb{J}$
- Step 3 : Countries decide whether to join the climate club : participation constraints
- Step 4 : Optimal design of size $\mathbb{J}$ and countries $j \in \mathbb{J}$ in the climate agreement

Model – Equilibrium

- Equilibrium
  - Given, initial conditions $\{w_{i0}, \tau_{i0}, \mathcal{R}_{i0}, \mathcal{S}_{i0}\}$ and country-specific policies $\{\mathrm{t}^f_{it}, \mathrm{t}^r_{it}, \mathrm{t}^{ls}_{it}\}$, a **competitive equilibrium** is a continuum of sequences of states $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$, controls $\{\mathbf{c}\} = \{c_{it}, b_{it}, k_{it}, e^f_{it}, e^r_{it}, e^x_{it}\}_{it}$ and price sequences $\{\mathbf{q}\} = \{r^\star_t, q^f_t, q^r_t\}$ such that :
    - Households choose policies $\{c_{it}, b_{it}\}_{it}$ to max utility s.t. budget constraint, giving $\dot{w}_{it}$
    - Firm choose policies $\{k_{it}, e^f_{it}, e^r_{it}\}_{it}$ to max profit
    - Fossil and renewables firms extract/produce $\{e^x_{it}, \bar{e}^r_{it}\}_{it}$ to max static profit, yielding $\dot{\mathcal{R}}_t$
    - Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t\}_t$, & $\{\tau_{it}\}_{it}$.
    - Prices $\{r^\star_t, q^f_t, q^r_{it}\}$ adjust to clear the markets : $\sum_{\mathbb{I}} e^x_{it} = \sum_{\mathbb{I}} e^f_{it}$ and $e^r_{it} = \bar{e}^r_{it}$, and $\sum_{\mathbb{I}} b_{it} = 0$, with bonds $b_{it} = w_{it} - k_{it}$
  - Pontryagin Max. Principle : costates $\{\psi\} = \{\lambda^w_{it}, \psi^\tau_{it}, \psi^s_{it}\} \Rightarrow$ system of coupled ODEs

More details

Step 0 : Competitive equilibrium

- Key objects :
  - Marginal value of wealth $\lambda_{it}^{w} = u'(c_{it})$
  - Marginal value of carbon ψ_{it}^{S} for country i
  - "Local social cost of carbon" (LCC) for region i :

$$LCC_{it} := -\frac{\partial \mathcal{V}_{it}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{it}/\partial c_{it}} = -\frac{\psi_{it}^{S}}{\lambda_{it}^{w}}$$

    - Stationary equilibrium closed-form formula, analogous to GHKT (2014) Here

Step 1 : First-Best, Optimal policy with transfers

- First-Best, Maximizing welfare of the Social Planner :

$$\mathcal{W}_0 = \max_{\{\mathbf{t},\boldsymbol{x},\boldsymbol{c},\boldsymbol{q}\}_{it}} \sum_{\mathbb{I}} \int_0^{\infty} e^{-\rho t}\, \omega_i\, u(c_{it})\, dt = \sum_{\mathbb{I}} \mathcal{W}_{i0}$$

  - Full set of instruments $\mathbf{t} = \{\mathrm{t}^f_{it}, \mathrm{t}^r_{it}, \mathrm{t}^{ls}_{it}\}$, including transfers *across countries*

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  - Full set of instruments $\mathbf{t} = \{\mathrm{t}^f_{it}, \mathrm{t}^r_{it}, \mathrm{t}^{ls}_{it}\}$, including transfers *across countries*
  - Key objects : Local vs. Global Social Cost of Carbon :

$$SCC^{fb}_t := -\frac{\partial \mathcal{W}_t / \partial \mathcal{S}_t}{\partial \mathcal{W}_t / \partial c_t} = -\frac{\psi^S_t}{\lambda^w_t} = -\frac{\sum_{\mathbb{I}} \psi^S_{it} di}{\frac{1}{I}\sum_{\mathbb{I}} \lambda^w_{it} di} \qquad LCC_{it} := -\frac{\partial \mathcal{W}_{it} / \partial \mathcal{S}_t}{\partial \mathcal{W}_{it} / \partial c_{it}} = -\frac{\psi^S_{it}}{\lambda^w_{it}}$$

Step 1 : First-Best, Optimal policy with transfers

- *Proposition 1 :* Optimal carbon tax :

$$\mathrm{t}_t^S = SCC_t^{fb}$$

  - Result as in Representative Agent economy, c.f. Nordhaus DICE (1996), GHKT (2014)

$$SCC_t^{fb} = -\frac{\psi_t^S}{\lambda_t^w} = -\sum_{\mathbb{I}} \frac{\psi_{it}^S}{\lambda_{it}^w} = \sum_{\mathbb{I}} LCC_{it}$$

- Lump-sum transfers redistribute across countries, s.t.

$$\omega_i u'(c_{it}) = \lambda_{it}^w = \lambda_t^w = \lambda_{jt}^w = \omega_j u'(c_{jt}) \ \ \forall i,j \in \mathbb{I}$$

  - Imply cross-countries lump-sum transfers $\exists\, i \ \ s.t.\ T_i \geq 0$ or $\exists\, j \ \ s.t.\ T_j \leq 0$
  - There exist Pareto weights $\{\omega_i\}$ shutting down redistribution $T_i = 0$, e.g. $\omega_i = 1/u'(c_{it})$

Step 2 : Ramsey policy with limited transfers

- Second best without access to lump-sum transfers : choice of a carbon tax $\{\mathrm{t}_t^f, \mathrm{t}_t^r\}$
  - Tax receipts redistributed lump-sum : $\mathrm{t}_{it}^{ls} = \mathrm{t}_t^f e_{it}^f + \mathrm{t}_t^r e_{it}^r$
  - Inequality across regions :

$$\widehat{\lambda}_{it}^w = \frac{\lambda_{it}^w}{\lambda_t^w} = \frac{\omega_i u'(c_{it})}{\frac{1}{I}\sum_{\mathbb{I}} \omega_j u'(c_{jt})} \lessgtr 1$$

$\Rightarrow$ ceteris paribus, poorer countries have higher $\widehat{\lambda}_{it}^w$

  - Social Cost of Carbon integrates these inequalities :

$$SCC_t^{sb} = \sum_{\mathbb{I}} \widehat{\lambda}_{it}^w \, LCC_{it}$$

$$SCC_t^{sb} = \sum_{\mathbb{I}} LCC_{it} + \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_{it}^w, LCC_{it}\right)$$

Step 2 : Ramsey Problem – Optimal Carbon & Energy Policy

- Taxing fossil energy has additional redistributive effects :
  - Lower eq. fossil fuels price benefit importers and hurt exporters
  - New measure : Social Cost of Fossil (SCF)

$$SCF_t^{sb} := \frac{\partial \mathcal{W}_t / \partial E_t^f}{\partial \mathcal{W}_t / \partial c_t} = \mathcal{C}_{EE}^f \, \mathbb{C}\mathrm{ov}_i \left(\widehat{\lambda}_{it}^w , e_{it}^f - e_{it}^x \right) \qquad \mathcal{C}_{EE}^f = \left(\sum_{i \in \mathbb{I}} \left(\mathcal{C}_{i,e^x e^x}^f \right)^{-1} \right)^{-1}$$

– with $\mathcal{C}_{EE}^f$ and $\mathcal{C}_{i,e^x e^x}^f \propto$ fossil energy supply elasticity

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- *Proposition 2 :* Optimal fossil energy tax :

$$\Rightarrow \qquad \mathrm{t}_t^f = SCC_t^{sb} + SCF_t^{sb}$$

  – Social cost of carbon : $SCC_t^{sb} = \sum_{\mathbb{I}} \widehat{\lambda}_{it}^w LCC_{it}$

- Tax on enewable energy e_t^r, no externality + constant return to scale : $\mathrm{t}_{it}^r = 0$

Step 3 : Ramsey Problem with participation constraints

- Assume countries can exit climate agreements + lump-sum transfers prohibited
  - Participation constraint, with $\bar{c}_i$ autarky consumption (no trade in energy/assets)

$$u(c_{it}) \geq u(\bar{c}_{it}) \qquad [\nu_{it}]$$

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- *Proposition 3 :* Second-Best without transfers & participation constraints
  - Participation incentive change our measure of inequality

$$\widetilde{\lambda}^w_{it} = \frac{\omega_i u'(c_{it}) + \nu_{it} u'(c_{it})}{\frac{1}{I}\sum_{\mathbb{I}}(\omega_j + \nu_{jt}) u'(c_{jt})} \neq \widehat{\lambda}^w_{it}$$

  - Optimal fossil energy tax :

$$\Rightarrow \qquad \mathrm{t}^f_t = SCC^{pc}_t + SCF^{pc}_t$$

$$= \sum_{i \in \mathbb{I}} \widetilde{\lambda}^w_{it} \, LCC_{it} + \mathcal{C}^f_{EE} \sum_{i \in \mathbb{I}} \widetilde{\lambda}^w_{it} (e^f_{it} - e^x_{it})$$

Step 4 : the Design of a Climate agreement

- Climate Agreement planner maximizes :

$$\mathcal{W}_0(\mathbb{J}) = \max_{\{\mathbf{t},\ldots\}_{it}} \frac{1}{\mathbb{J}} \sum_{\mathbb{J}} \int_0^\infty e^{-\rho t}\, \omega_i\, u(c_{it})\, dt$$

$$s.t. \qquad u(c_{it}) \geq u(\bar{c}_i) \qquad\qquad \forall t, i \in \mathbb{J}$$

- Choice of countries $\mathbb{J} \subset \mathbb{I}$ to maximize welfare
  - Other countries $\mathbb{I} \backslash \mathbb{J}$ in autarky : own bond $\tilde{r}$/energy $\tilde{q}^f$ market
  - Alternative : Optimal trade tax/tariffs $\Rightarrow$ *work in progress*
- Adding country j to $\mathbb{J}$
  - Changes the optimal tax :

$$\mathrm{t}_t^f(\mathbb{J}) = SCC_t^{ca}(\mathbb{J}) + SCF_t^{ca}(\mathbb{J}) = \sum_{i\in\mathbb{J}} \widetilde{\lambda}_{it}^w\, LCC_{it} + \mathcal{C}_{EE}^f \sum_{i\in\mathbb{J}} \widetilde{\lambda}_{it}^w (e_{it}^f - e_{it}^x)$$

  - Change the equilibrium on energy markets :

$$\text{price } q_t^f \qquad s.t. \qquad \sum_{j\in\mathbb{J}} e_{it}^f = \sum_{j\in\mathbb{J}} e_{it}^f$$

Step 4 : the Design of a Climate agreement

- Tradeoff extensive/intensive margin
- Reduction in emissions $\mathcal{E} = \sum_{i \in \mathbb{I}} e_i^f$ depends both on :
  - The level of tax $\mathfrak{t}^f$, since high $\mathfrak{t}^f \Leftrightarrow$ large change in emissions $\Delta\mathcal{E}(\mathbb{J})$
  - The *number* of countries $\mathbb{J}$ in a stable coalition

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  - The *number* of countries $\mathbb{J}$ in a stable coalition
- Naive approach :
  - Combinatorial problem : $\mathcal{P}(\mathbb{I})$ with $2^{|\mathbb{I}|}$ choices

$$\max_{\mathbb{J} \in \mathcal{P}(\mathbb{I})} \mathcal{W}_0(\mathbb{J})$$

  - Search for complementarity

$$\Delta W(\mathbb{J}', j) := W(\mathbb{J}' \cup j) - W(\mathbb{J}') > \Delta W(\mathbb{J}, j) \qquad \text{when } \mathbb{J}' \supset \mathbb{J} \qquad \text{for all } j \in \mathbb{I}$$

  - Choice of countries $\mathbb{J}$ yields optimal tax $\mathfrak{t}^f(\mathbb{J})$

Step 4 : the Design of a Climate agreement

- Tradeoff extensive/intensive margin
- Alternative approach :
  - From the level of the tax $\mathrm{t}^f(\mathbb{J})$ imposed on club $\mathbb{J}$, we can deduce the number of countries $\widetilde{\mathbb{J}}$ with binding participation constraints

$$\widetilde{\mathbb{J}} \quad s.t. \quad u(c_i) \geq u(\bar{c}_i) \quad \forall i \in \widetilde{\mathbb{J}}$$

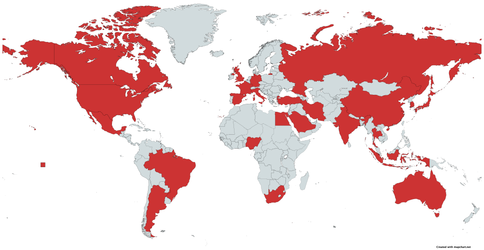
  - Find a fixed point of function $\widetilde{\mathbb{J}} = f(\mathbb{J}, \mathrm{t}^f)$
  - Sequential approach :
    - Start from $\mathbb{J} = \mathbb{I}$
    - Search for t^f that yield $\mathbb{J} = f(\mathbb{J}, \mathrm{t}^f)$
    - If $Im(f(\mathbb{J}, \mathrm{t}^f) \subsetneq \mathbb{J}$ remove countries one-by-one.
    - Repeat (2-3) until convergence or unraveling

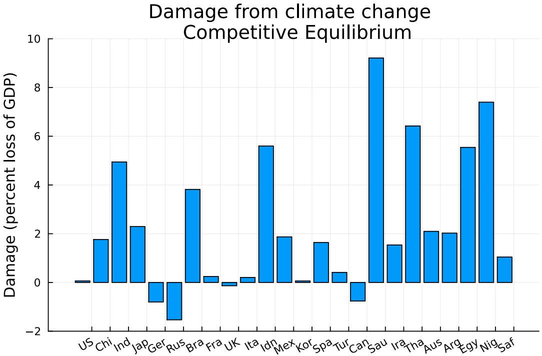
Quantification and numerical method

- Quantification More details
  - Production $\bar{y} = zf(k, e^f, e^r)$ with Nested CES capital/energy $\sigma_y < 0$ and fossil/renewable $\sigma_e > 1$. Calibrate parameters to match GDP / energy shares data.
  - Quadratic damage as in Nordhaus DICE $y = \mathcal{D}_i(\tau)\bar{y}$ with $\mathcal{D}_i(\tau) = e^{-\gamma(\tau-\tau_i)^2}$
  - Energy parameters to match production/reserves
- Numerical method More details
  - Sequential approach : rely on Pontryagin Maximum Principle
  - Can simulate models with an arbitrary number of dimensions of heterogeneity

Numerical Application – Competitive equilibrium

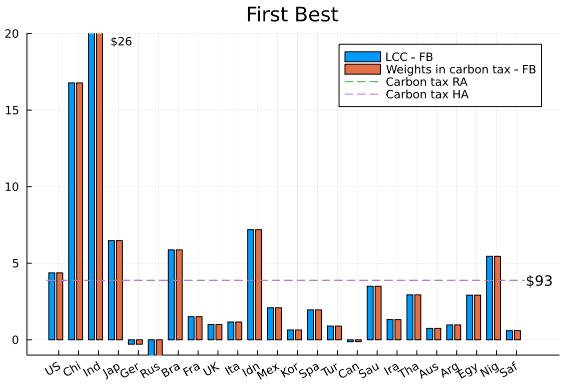
- Data : 24 countries, (G20+4 large countries)

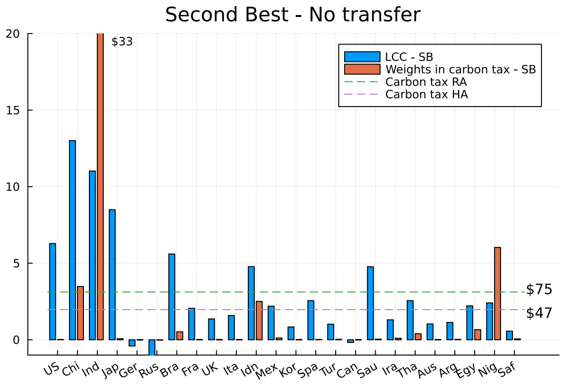




Local Cost of Carbon & Carbon Tax – First and Second Best

- Difference $LCC_i = \frac{\psi_i^{\mathcal{S}}}{\lambda_i^w}$ vs. $\widehat{\lambda}_i^w LCC_i = \frac{\psi_i^{\mathcal{S}}}{\bar{\lambda}^w}$ since $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i^w \, LCC_i$





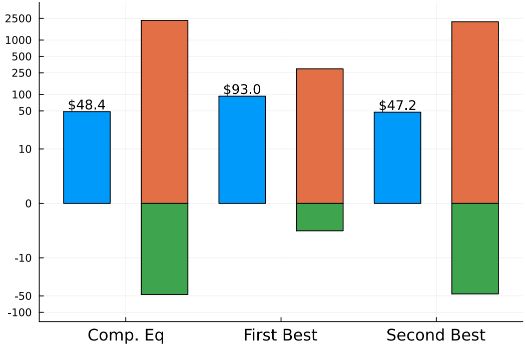
Comparison - Value of wealth vs. Social Cost of Carbon

- Social Cost of Carbon can be decomposed :

$$SCC_t := -\frac{\partial \mathcal{W}_t / \partial \mathcal{S}_t}{\partial \mathcal{W}_t / \partial c_t} = -\frac{\psi_t^S}{\lambda_t^w} = -\frac{\sum_{\mathbb{I}} \psi_{it}^S di}{\frac{1}{I}\sum_{\mathbb{I}} \lambda_{it}^w di}$$

- Here plot that decomposition :

$$\log(SCC_t) = \log(-\psi_t^S) - \log(\lambda_t^w)$$



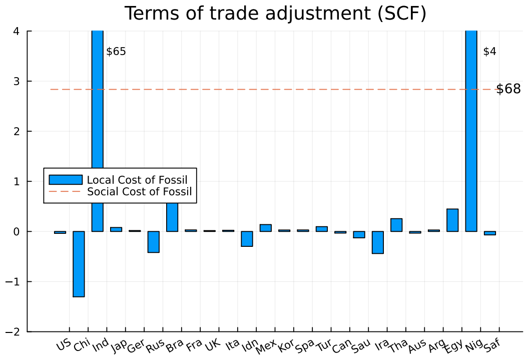
Local Cost of Fossil and Terms of Trade Adjustment

- Social Cost of Fossil Energy :

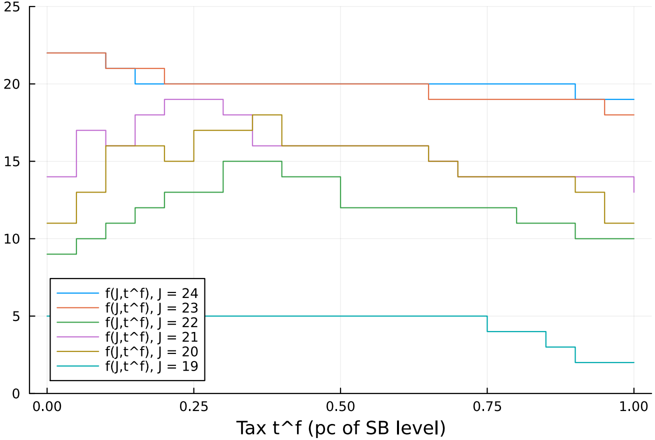
$$SCF_t = \mathcal{C}^f_{EE} \sum_{\mathbb{I}} \widehat{\lambda}^w_{it} \left(e^f_{it} - e^x_{it} \right) \qquad \mathcal{C}^{f\ -1}_{EE} = \sum_{\mathbb{I}} (\mathcal{C}^f_{i,e^x e^x}$$

- Here plotting local cost of fossil :

$$LCF_{it} = \widehat{\lambda}^w_{it} (e^f_{it} - e^x_{it})$$



Climate club design and unraveling



- Plot of fct $f(\mathbb{J}, t^f) = \widetilde{\mathbb{J}}$: for a club of size $\mathbb{J}$ and tax t^f, $\widetilde{\mathbb{J}}$ countries willing to participate
- Removing China (23 $\rightarrow$ 22) and the US (20 $\rightarrow$ 19) causes unraveling

Conclusion

- Climate change has redistributive effects & heterogeneous impacts
- Optimal carbon policy takes into account inequality and redistribution
    - Depends on the availability of transfer mechanisms
    - Pigouvian tax & Social Cost of Carbon change with inequality and terms-of-trade effects
- Climate agreement design accounts jointly for
    - The optimal choice of countries participating
    - The level of tax, chosen for correcting externality & respecting participation constraints

Appendices

More details – Capital market

- In each countries, the agent can save in two assets, capital k_{it} and bonds b_{it} :

$$\begin{cases} \dot{k}_{it} & = \mathcal{D}_i^y(\tau_{it}) z_i f(k_{it}, e_{it}) - (\delta + n + \bar{g}) k_{it} + \iota_{it} \\ \dot{b}_{it} & = r^\star b_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathrm{t}_{it}^f) e_{it}^f - (q_t^r + \mathrm{t}_{it}^r) e_{it}^r - \iota_{it} - c_{it} + \mathrm{t}_{it}^{ls} \\ b_{it} & \geq -\vartheta k_{it} \end{cases}$$

- Combining, substituting ι_{it} and defining wealth $w_{it} = k_{it} + b_{it}$, we obtain the main equation

$$\dot{w}_{it} = r^\star w_{it} + \mathcal{D}^y(\tau_{it}) z_{it} f(k_{it}, e_{it}) - (\bar{\delta} + r_t^\star) k_{it} + \theta_i \pi_t^f + \pi_{it}^r - (q_t^f + \mathrm{t}_{it}^f) e_{it}^f - (q_t^r + \mathrm{t}_{it}^r) e_{it}^r - c_{it} + \mathrm{t}_{it}^{ls}$$

$$k_{it} \leq \frac{1}{1-\vartheta} w_{it}$$

- Two polar cases :
  - $\vartheta \to 0$, full autarky (no trade), and $w_{it} = k_{it}$
  - $\vartheta \to 1$, full financial integration :

$$k_{it} \qquad s.t. \qquad MPk_{it} - \bar{\delta} = \mathcal{D}_i^y(\tau_{it}) z_i \partial_k f(k_{it}, e_{it}) - (\delta + n + \bar{g}) = r_t^\star$$

back

Impact of increase in temperature

- Marginal values of the climate variables : λ^s_{it} and λ^τ_{it}

$$\dot{\lambda}^\tau_{it} = \lambda^\tau_{it}(\rho+\zeta) + \overbrace{\gamma_i(\tau_{it}-\tau^\star_i)\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it},e_{it})\lambda^k_{it} + \overbrace{\phi_i(\tau_{it}-\tau^\star_i)\mathcal{D}^u(\tau_{it})^{1-\eta}c^{1-\eta}_{it}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}^S_{it} = \lambda^S_{it}(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{it}$$

- Costate λ^S_{it} : marg. cost of 1Mt carbon in atmosphere, for country i. Increases with :
  - Temperature gaps $\tau_{it}-\tau^\star_i$ & damage sensitivity of TFP γ^y_i and utility γ^u_i
  - Development level $f(k_{it},e_{it})$ and c_{it}
  - Climate params : χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
  - back

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}^{\mathcal{T}}_{it} = \lambda^{\mathcal{T}}_t(\tilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\lambda^k_t + \phi(\tau - \tau^\star)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}^{S}_{t} = \lambda^{S}{}_t(\tilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta\chi\lambda^{\mathcal{T}}_{it}$$

- Solving for $\lambda^{\mathcal{T}}_t$ and $\lambda^{\mathcal{S}}_t$, in stationary equilibrium $\dot{\lambda}^{S}_t = \dot{\lambda}^{\mathcal{T}}_t = 0$

$$\lambda^{\mathcal{T}}_{it} = -\int_t^\infty e^{-(\tilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_u)y_{\mathcal{T}}\lambda^k_u + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda^{\mathcal{T}}_{it} = -\frac{1}{\tilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_\infty)y_\infty\lambda^k_\infty + \phi\mathcal{D}^u(\tau_\infty)u(c_\infty)\Big)$$

$$\begin{aligned}
\lambda^S_t &= -\int_t^\infty e^{-(\tilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,u}dj\,du \\
&= \frac{1}{\tilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda^{\mathcal{T}}_{j,\infty} \\
&= -\frac{\chi}{\tilde{\rho}+\delta^s}\frac{\zeta}{\tilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_\infty\lambda^k_{j,\infty} + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

$$\lambda^S_t \xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\tilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda^k_{j,\infty} + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj$$

Cost of carbon / Marginal value of temperature

- ***Proposition (Stationary LSCC)*** :
  When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LSCC is *proportional* to climate sensitivity χ, marg. damage γ_i^y, γ_i^u, temperature, and output, consumption.

$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^\star) \Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \Big)$$

  - Stationary equilibrium : $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - Back

Social cost of carbon & temperature

- Cost of carbon depends only on final temperatures and path of emissions :

$$\tau_T - \tau_{t_0} = \Delta\,\chi\,\xi\,\omega \int_{t_0}^{T} e^{(n+\bar{g})t - \delta_s(T-t)}\, q_t^{f\ -\sigma_e} \int_{j\in\mathbb{I}} \left(z_j z^e_{j,t} \mathcal{D}(\tau_{j,t})\right)^{\sigma-1} y_{j,t}\, q_{j,t}^{\sigma_e-\sigma}\, dj\, dt$$

  - Geographical factors determining warming Δ_i
  - Climate sensitivity χ & carbon exit from atmosphere δ_s
  - Growth of population n, aggregate productivity $\bar{g}$
  - Deviation of output from trend y_i & relative TFP z_j
  - Directed technical change z^e_t, elasticity of energy in output σ
  - Fossil energy price $q^{e,f}$ and Hotelling rent $g^{q^f} \approx \dot{\lambda}^R_t/\lambda^R_t = \rho$
  - Change in energy mix, renewable share ω, price q^r_t & elasticity of source σ_e

- Approximations at $T \equiv$ Generalized Kaya (or $I = PAT$) identity More details

$$\frac{\dot{\tau}_T}{\tau_T} \propto\ n + \bar{g}^y - (1-\sigma)\left(g^{z^e} - \widetilde{\gamma}\right) + (\sigma_e - \sigma)(1-\omega) g^{q^r} - (\sigma_e(1-\omega) + \sigma\omega) g^{q^f}$$

Back

Equilibrium – Mean Field Games

- Uniqueness : Lasry-Lions monotonicity on the function :

$$\int_{\mathbb{I}} [f(t,x,\mu',\alpha) - f(t,x,\mu,\alpha)] d(\mu' - \mu)(x) \geq 0 \qquad \mu, \mu' \in \mathcal{P}(\mathbb{X})$$

  - Work in progress : checking such conditions along the transition

$$\sum_{i \in \mathbb{I}} \left(u(c^{\star}(w,\tau,p')) - u(c^{\star}(w,\tau,p)) \right) [p'(w,\tau) - p'(w,\tau)] \geq 0$$

  with $p'(w,\tau)$ empirical distribution $p'(w,\tau) = \frac{1}{\mathbb{I}} \sum_{i \in \mathbb{I}} \delta_{\{x_i = (w,\tau)\}} \equiv$ population distribution !

- Mean Field approximation & Carmona Delarue (2013)
  - Mean-Field is an ε-equilibrium of the N-player game when $N \to \infty$
  - Require symmetry and invariance under permutation
  - Back

Sequential solution method

- Summary of the model :
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^s_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness More details

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  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness More details
- Global Numerical solution :
  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Sequential method : Pros and Cons

- Why use a sequential approach ?
  - *Global approach* : *Only* need to follow the trajectories for i agents :
  - Arbitrary (!) number of dimension of *ex-ante* heterogeneity :
    Productivity z_i Population p_i, Temperature scaling Δ_i, Fossil energy cost $\bar{\nu}_i$, Energy mix $\epsilon_i, \omega_i, z_i^r$, Local damage $\gamma_i^y, \gamma_i^u, \tau_i^\star$, Directed Technical Change z_i^e
  - Potentially large dimensions of *ex-post* heterogeneity and aggregate state variables :
    For now : Wealth w_{it}, temperature τ_{it}, reserves $\mathcal{R}_{it}$, Carbon $\mathcal{S}_t$
    Extension with a large climate system as a proof of concept (e.g. Cai, Lontzek, Judd, 2013)
  - Newton method & Non-linear solvers very efficient
- Why not :
  - Numerical constraint to solve a large system of ODEs and non-linear equations :

  $\Rightarrow$ Constraint on $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$, so either M or T can't be too large
  - Relying on numerical solvers/structure of the problem can be opaque

back

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(\tau_i) z_i f(k, \varepsilon(e^f, e^r))$

$$f_i(k, \varepsilon(e^f, e^r)) = \left[(1-\epsilon_i)^{\frac{1}{\sigma_y}} k^{\alpha\frac{\sigma_y-1}{\sigma_y}} + \epsilon_i^{\frac{1}{\sigma_y}} \left(z_i^e\ \varepsilon_i(e^f, e^r)\right)^{\frac{\sigma_y-1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y-1}}$$

$$\varepsilon(e^f, e^r) = \left[\omega_i^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e-1}{\sigma_e}} + (1-\omega_i)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e-1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e-1}}$$

  - Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2011 (PPP).
  - Today : $\omega_i = \bar{\omega} = 85\%$ and $\epsilon_i = \bar{\epsilon} = 12\%$ for all i
  - Future : $(z_i^e, \omega_i, \epsilon_i)$ to match Energy/GDP $(e_i^f + e_i^r)/y_i$ and energy mix (e_i^f, e_i^r)

- Damage functions in production function y :

$$\mathcal{D}_i^y(\tau) = e^{-\gamma_i^{\pm,y}(\tau-\tau_i^\star)^2}$$

  - Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y}\mathbb{1}_{\{\tau>\tau_i^\star\}} + \gamma^{-,y}\mathbb{1}_{\{\tau<\tau_i^\star\}}$
  - Today $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $\tau_i^\star = \bar{\alpha}\tau_{it_0} + (1-\bar{\alpha})\tau^\star$

Quantification – Energy markets

- Fossil production e^x_{it} and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now : $\bar{\nu}_i$ to match extraction data e^x_i, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. $\nu_i = \nu = 1$ quadratic extraction cost.
  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e^x_i (BP, IEA)

Quantification – Energy markets

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  - Future : Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e^x_i (BP, IEA)
- Renewable : Production $\bar{e}^r_{it}$ and price q^r_{it}
  - Now : $q^r_{it} = z^r e^{-g_r t}$, with g_r growth rate in renewable energy price decreases.
  - Future : Choose z^r_i to match the energy mix (e^f_i, e^r_i)

back

Quantification – Future Extensions :

- Damage parameters :
  - $\gamma_i^{\pm,y}$ depends on daily temperature distribution $\tau \sim \mathcal{T}_i(\bar{T}, \sigma^T)$ following Rudik et al. (2022)
  - Use Climate Lab's (Greenstone et al) estimates for damage γ_i ?
- Fossil Energy markets :
  - Divide fossils e_{it}^f / e_{it}^x into oil/gas/coal
  - Match the production/cost/reserves data across countries
  - Use a dynamic model : extraction/exploration a la Hotelling
- Renewables Energy markets :
  - Make the problem dynamic with investment in capacity C_{it}^r
- Population dynamics
  - Match UN forecast for growth rate / fertility

Calibration

TABLE – Baseline calibration ($\star$ = subject to future changes)

Technology & Energy markets			
α	0.35	Capital share in $f(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $f(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2020)
ω	0.8	Fossil energy share in $e(\cdot)$	Fossil/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
g_e	$0.01^\star$	Long run energy directed technical change	Conservative / Acemoglu et al (2012)
g_r	$-0.01^\star$	Long run renewable price decrease	Conservative / Match price fall in R.E.
ν	$2^\star$	Extraction elasticity of fossil energy	Cubic extraction cost
Preferences & Time horizon			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	2.5	Risk aversion	
n	$0.01^\star$	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
T	90	Time horizon	Horizon 2100 years since 2010

Calibration

TABLE – Baseline calibration ($\star$ = subject to future changes)

Climate parameters			
ξ	0.81	Emission factor	Conversion 1 *MTOE* $\Rightarrow$ 1 *MT* CO_2
ζ	0.3	Inverse climate persistence / inertia	Sluggishness of temperature $\sim 11-15$ years
χ	$2.1/1e6$	Climate sensitivity	Pulse experiment : 100 $GtC \equiv 0.21^\circ C$ medium-term warmin
δ_s	0.0014	Carbon exit from atmosphere	Pulse experiment : 100 $GtC \equiv 0.16^\circ C$ long-term warming
$\gamma^\oplus$	$0.00234^\star$	Damage sensitivity	Nordhaus' DICE
$\gamma^\ominus$	$0.2 \times \gamma^{\oplus\ \star}$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^τ	$0.2^\star$	Weight historical climate for optimal temp.	Marginal damage decorrelated with initial temp.
$\tau^\star$	15.5	Optimal yearly temperature	Average spring temperature / Developed economies
Parameters calibrated to match data			
p_i		Population	Data – World Bank 2011
z_i		TFP	To match GDP Data – World Bank 2011
τ_i		Local Temperature	To match temperature of largest city
$\mathcal{R}_i$		Local Fossil reserves	To match data from BP Energy review

Step 4 : the Design of a Climate agreement

- Welfare effect : 1st order :

$$\delta(\mathbb{J},j) = \mathcal{W}_{t_0}(\mathbb{J} \cup \{j\}) - \mathcal{W}_{t_0}(\mathbb{J}) = \omega_j u(c_{jt}) + \sum_{i \in \mathbb{J}} \Delta \mathcal{W}_i$$

$$\Delta \mathcal{W}_i \approx d\mathcal{W}_i = \lambda_i^w (1-\theta_i) \Big(\underbrace{\epsilon_i^e + (1-\epsilon_i^e)\alpha\sigma_i^{k/e}}_{\text{production } f(k,e)} \Big) \Big(\underbrace{-\omega_i^f \sigma_i^f + (1-\omega_i^f)\sigma_i^{r/f}}_{\text{energy use } \varepsilon(e^f,e^r)} \Big) \Big(\underbrace{\frac{\mathrm{t}^f}{q^f+\mathrm{t}^f} \frac{d\mathrm{t}^f}{\mathrm{t}^f}}_{\text{tax change}} + \underbrace{\frac{q^f}{q^f+\mathrm{t}^f} \frac{dq^f}{q^f}}_{\text{GE effect}} \Big)$$

$$+ \lambda_i^w \underbrace{\theta_i \big(1 + \frac{1}{\nu_i}\big)}_{\text{fossil rent/supply}} \underbrace{\frac{q^f}{q^f+\mathrm{t}^f} \frac{dq^f}{q^f}}_{\text{GE effect}} + \underbrace{\psi_i^S}_{\tau_i \text{ damage}} \Big[\underbrace{\chi}_{\substack{\text{climate} \\ \text{sens}^{ty}}} \sum_{j \in \mathbb{I}} \varepsilon_i \sigma_j^f \Big] \Big(\underbrace{\frac{\mathrm{t}^f}{q^f+\mathrm{t}^f} \frac{d\mathrm{t}^f}{\mathrm{t}^f}}_{\text{tax change}} + \underbrace{\frac{q^f}{q^f+\mathrm{t}^f} \frac{dq^f}{q^f}}_{\text{GE effect}} \Big)$$

- Direct effect on energy use on production and substitutability with renewable
  cost-share ϵ_e, fossil-share ω_i, elasticity σ_j^f & capital-energy cross elast^{ty}. $\sigma_{k,e}$, fossil-renewable cross elast^{ty}. $\sigma_i^{r/f}$
- Indirect effect through energy market fossil rent θ_i, supply elasticity ν_i
- Indirect climate effect of a reduction in world emissions