University Of Portsmouth
BSc (Hons) Computer Science
First Year

Architecture and Operating Systems - Maths

M30943
September 2022 - May 2023
20 Credits

Thomas Boxall
up2108121@myport.ac.uk

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Introduction To Module

27-09-2022 10:00 Zhaojie Zoom

The goal of this module is to: help with maths involved in programming; prepare for other technical units (including: databases, functional programming; discrete maths; theoretical computer science; computer vision; and more); and to gain confidence.
The maths element (this element) is 30% of the Architecture and Operating Systems module. The two components are independent until the final grade for the module is produced.

Assessments

There are two components to the assessments for this element of the module.

Online Tests

Throughout the year, there are 7 online tests which will be completed through the Pearson MyMathsLab. Overall, these tests will equal 15% of the overall module score. Each test should last for 20 minutes and has a practice test available which can be accessed anytime in the year, and as many times as you wish. The real test can only be attempted once. After you submit the test, you will get an instant score.
Calculators are permitted, however it may not always be advisable to use one.

End Of Year Test

At the end of the year, there will be an end of year test. This is equal to 15% of the overall module score.

Staff & Support

Jhaojie is the main lecturer for the module. He will be assisted by Bryan.
Outside of class, Xia and Kirsten are available from the academic tutors office to help where needed. They can be booked through Moodle.
A MathsCafe runs during term-time for drop in Maths support. It is held at the following times

- Monday 12:00-14:00, LG learning and teaching space;
- Tuesday 12:00-14:00, LG learning and teaching space;
- Wednesday, 12:00-14:00, Zoom;
- Wednesday 14:00-16:00, Library 0.36;
- Thursday 12:00-14:00, LG learning and teaching space;
- Friday 12:00-14:00, LG learning and teaching space.

When writing working out for questions, full steps should be written down. This allows errors to be found and corrected.
Take a pen and paper to practical classes.

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Basic Numeracy and Basic Algebra

27-09-22 10:20 Zhaojie Zoom

Negative Numbers

Subtracting a negative number is equivalent to adding a positive number. This can be seen in the following example.

$$
\begin{aligned}
2-(-5) &= \\
2+5 &= \\
&= 7
\end{aligned}
$$

The result of multiplying or dividing two numbers of the same sign is always positive.
The result of multiplying of dividing two numbers of opposing signs is always negative.

BIDMAS

The order in which to carry out operations in complex mathematical expressions is defined by the following priority list

1 Brackets

2 Indices

3 Division

3 Multiplication

4 Addition

4 Subtraction

Fractions

The names of different components of a fraction are as follows: $fraction = \frac{numerator}{denominator} = \frac{p}{q}$

Addition & Subtraction of Fractions

To add or subtract two fractions, their denominator needs to be the same. Then the addition/subtraction is performed just to the numerator. The fraction is usually then simplified.

Multiplication of Fractions

To multiply two fractions together: first, multiply the numerators together then multiply the denominators together.

Division of Fractions

To divide one fraction by another, multiply the first fraction by the inverse (reciprocal) of the second fraction. Simplify where necessary.

Simplification Of Fractions

A fraction is in its simplest form where there are no factors other than one to both the numerator and the denominator.

Algebra

The use of letters in maths is called Algebra. It defines the rules of how to manipulate with symbols.

Addition & Subtraction of like terms

Term

Either a single umber or variable, or the product of several and/or variables, for example $3y$.

Constant

A term without a symbol, for example, 2.

Like terms are multiples of the same variables; they can be added/ subtracted.

Multi-variable simplification

$$\begin{aligned} 24y^2 + 7x + 12xy - 4x - 5y^2 + 3xy = \\ 19y^2 + 15xy + 3x = \end{aligned}$$

Multiplication algebraic expressions

The fundamental concept behind multiplication of terms is to multiply the numbers and multiply the variables (using the rules for multiplication of indices if possible), taking into account the sign rules where multiplying terms with different signs.

Multiplying algebra example

$(2a)(6ab^2) = 12a^2b^2$

Expressions

Removing Brackets

In the expression $a(b + c)$, a is multiplies by all the bracketed terms to give $ab + ac$.
In the expression $(a+b)(c+d)$, $(a+b)$ is multiplied by the other pair of brackets as individual terms. Giving the answer as $ac + ad + bc + bd$
This principle along with the principle of simplifying algebra can be used to remove brackets fromm more complex expressions.

Removing brackets from a more-complex expression

$$\begin{aligned}(x+6)(x-3) &= x(x+6)+(-3)(x+6)\\ &= x^2+6x-3x-18\\ &= x^2+3x-18\end{aligned}$$

Substitution

Where letters are replaced by actual numerical values.

Simple Linear Equations

Equations state that two quantities, usually one is known and one is not, are equal. We can use this information to solve the equation - to work out what the unknown quantity is.
A linear equation comes in the form of $ax+b=c$ where a, b and c are given numbers and x is an unknown quantity.

Solve $4x+8=0$ for x

We can start by removing one of the known values, by subtracting 8 from both sides. This results in $4x=-8$ We can then divide both sides by 4 to get $x=-2$, which is our solution.

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Everyday Maths

03-10-22 10:00 Zhaojie Zoom

Percentage

A percentage is a fraction where the denominator is 100. Percent corresponds to "per 100".

Ratio

Ratios are used to compare two or more quantities. The symbol used is : (a colon).
To simplify ratios, divide both parts of the ratio by the hightest common factor.

Average

The average of a set of numbers, sometimes known as a mean, can be calculated using the following formula:

$$\text{average} = \frac{\text{sum of a set of values}}{\text{number of values}}$$

Probability

Probabilities express how likely something is to happen. They are expressed as a decimal number between 0 and 1. A probability of 1 means the event must happen and a probability of 0 means the event will never happen.

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Powers and Logarithms

11-10-22 10:00 Zhaojie Zoom

Powers

2^4 rads as 2 to the power of 4 and it means

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

In the example above, 2 is the base and 4 is the index (or power).

Special Cases of Powers

$x^0 = 1$ This will be true for all cases except for where $x = 0$, in this case $x^0 = undefined$.
$x^1 = 1$ This is true for all values of x.

Laws Of Indices

There are three laws of indices.

1. $a^n \times a^m = a^{n+m}$ (when multiplying, add the indices)
2. $\frac{a^n}{a^m} = a^{n-m}$ (when dividing, subtract the indices)
3. $(a^n)^m = a^{n \times m}$ (when raising one power to another, multiply the indices).

Negative Powers

With negative powers, there is a general rule

$$a^{-n} = \frac{1}{a^n}$$

Fractional Powers

Where a and n are positive numbers, the general rule indices

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Simplify the following expression

$$\begin{aligned}\sqrt{\frac{72a^{12}b^7c^2}{2a^2b^3c^{-10}}} &= \\ &= \sqrt{36a^{10}b^4c^{12}} \\ &= (36a^{10}b^4c^{12})^{\frac{1}{2}} \\ &= 6a^5b^2c^6\end{aligned}$$

Logarithms

Logarithm

A logarithm determines how many times a certain number must be multiplied by itself to reach another number.

The general rule for logarithms is shown below, this is applicable where $a > 1$.

$$y = a^x$$

$$\log_a y = x$$

Base of a logarithm

The most commonly used bases are

- 10 ($\log_{10}$)
- 2 ($\log_2$)
- natural logarithm e ($\log_e$ or ln)

First Law Of Logs

$$\log_a x + \log_a y = \log_a xy$$

All bases must be the same.

Second Law Of Logs

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

All bases must be the same.

Third Law Of Logs

$$n \log_a x = \log_a x^n$$

This law applies of n is an integer, fractional, positive or negative.

Example

Simplify

$$
\begin{aligned}
\log_2 y - 3\log_2 2y + 2\log_2 4y = \\
&= \log_2 y - \log_2 (2y)^3 + \log_2 (4y)^2 \\
&= \log_2 y - \log_2 8y^3 + \log_2 16y^2 \\
&= \log_2 \left(\frac{y \times 16y^2}{8y^3} \right) \\
&= \log_2 2 \\
&= 1
\end{aligned}
$$

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Functions and Sequences

19-10-22 10:00 Zhaojie Zoom

Functions

Function

A function is a rule that recieves an input and produces an output. A function can only produce a single output for any given input.

In maths, function are written as follows

$$f(x) = x + 3$$

We can use any different letters we want.

Calculating output when given an input

A function f is defined by $f(x) = 3x + 1$. Calculate the output when the input is 4.

$$f(4) = 3 \times 4 + 1 = 13$$

Composite Functions

A composite function is where the output of one function feeds directly into the input of another function. This can be expressed as follows

$$g(f(x))$$

Composite Function eample

Given $f(x) = x^2$ and $g(x) = x + 1$. Find a value of the composite function $f(g(x))$ and $g(f(x))$ for $x = 3$.

$$f(g(3)) = f(3 + 1) = f(4 = 4^2 = 16)$$

$$g(f(3)) = g(3^2) = g(9) = 9 + 1 = 10$$

Sequences

Sequence

A sequence is a set of number written down in a specific order. Each element in the sequence is called a term.

There are two types of sequence, finite and infinite sequence. Finite sequences have a fixed number of elements and infinite sequences can go on forever.

Sequence Notation

We use subscript notation to refer to different terms in the sequence. The first term in the sequence can be called x_1, the second x_2 and so on.

Recurrence Relation

A recurrence relation is an equation that recursively defines a sequence. One or more initial terms are given and each further term of the sequence is defined as a function of the preceding terms. For example

$$F_n = f_{n-1} + F_{n-2} \geq 2$$

$$F_0 = 0, F_1 = 1$$

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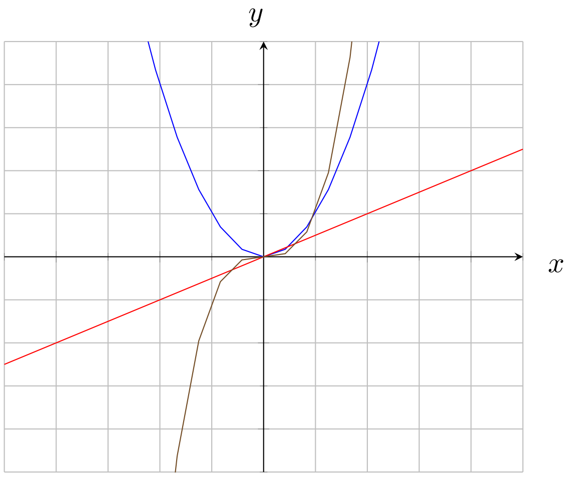
Graphs of Functions/ Straight Lines

24-01-23 10:00 Zhaojie Zoom

It can be useful to represent functions pictorially, this is usually done by means of graphs of functions.
These graphs have coordinate systems whereby: there are horizontal (x) and vertical (y) axis; an origin $(0,0)$; and coordinates are given in the form (x,y) refer to a specific point on the graph. Coordinates can be both positive and negative.



x^2 | $\frac{x}{2}$ | x^3

Polynomial Functions

A polynomial of degree n is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1}$$

where the a_i are real numbers (called the coefficient of the polynomial).
The polynomial degree can be found by seeing what the highest power of x there is in the function. For example

$$f(x) = 4x^3 - 3x^2 + 2$$

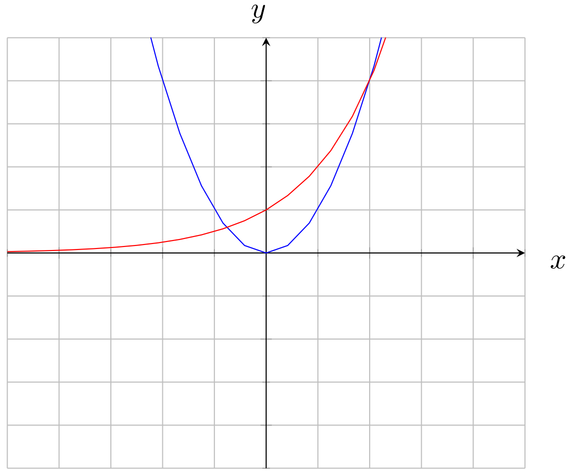
is a polynomial of degree 3.
Functions in which the power of x is not an integer, are not polynomial functions.

Exponential Functions

Exponential functions are functions in which the base is always a positive number other than 1 where the variable x is the power, for example

$$g(x) = 2^x$$

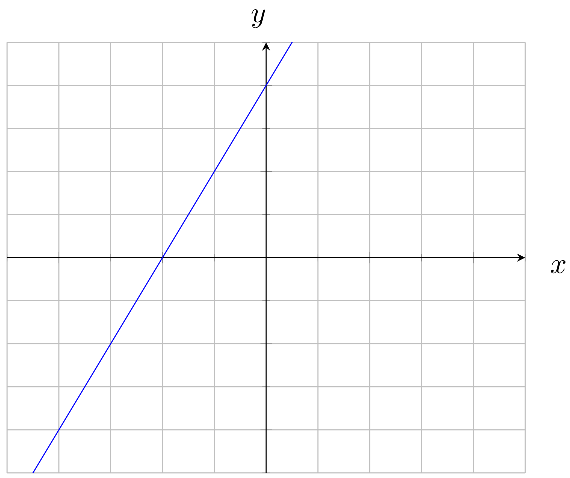
Exponential functions and polynomial functions plot completely different graphs.



x^2 | 2^x

Linear Functions

Generally, when referring to linear functions, they will be in the form $y = mx + x$. This is a polynomial function of maximum degree 1. m and c are real constants.

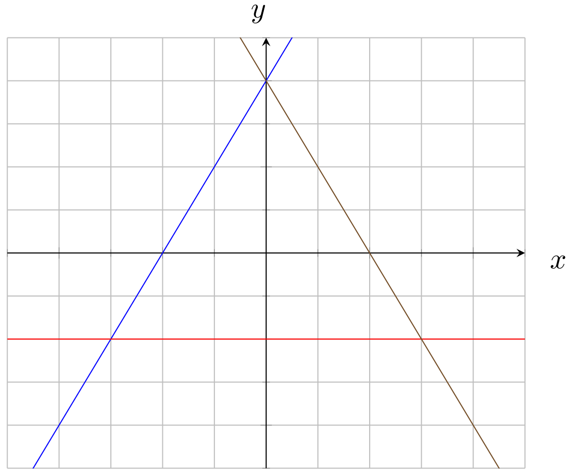


$2x + 4$

The graph of the function $f(x) = mx + x$ is always a straight line.

Gradient Of the Straight Line

The value of gradient (m in the equation $y = mx + c$) determines the steepness of the straight line. If m is positive then the line will rise as we move from left to right; if m is negative the line will fall and if m is 0, the line will be horizontal.



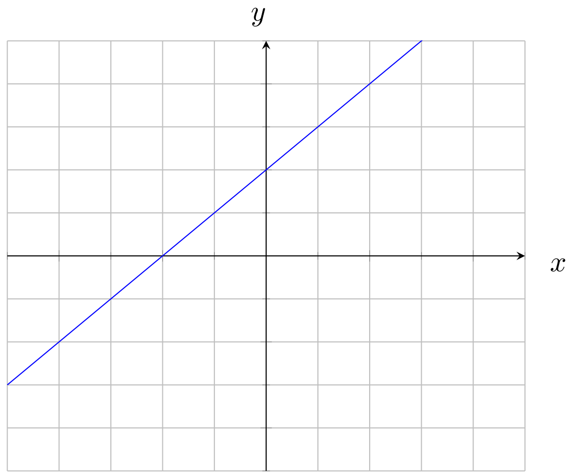
$2x + 4$ | -2 | $-2x + 4$

Parallel lines have the same gradient (same value of m). Lines parallel to the x-axis have equations of the form $y = k$ where k is a constant. Lines parallel to the y-axis have equations $x = k$ where k is a constant.
To calculate gradient, you need to choose any two points on the line, look for points where the line crosses a grid mark on the paper as this makes it easier; then find the y-difference (Δy) and find the x-difference (Δx) then apply the following formula.

$$\frac{\Delta y}{\Delta x} = m$$

y intercept

The point at which the line on a graph intercepts the vertical axis is called the y intercept.
In the equation $y = mx + c$, c is the value of the y intercept.
In the graph below, the y value at the intercept is 2.



$x + 2$

Finding the Equation of a Straight Line

A striaght line passes through (7,1) and (-3,2). Find its equation

Gradient can be found from

$$m = \frac{1-2}{7-(-3)} = -\frac{1}{10} = -0.1$$

Hence $y = -0.1x + c$
Substitute values in for $x = 7$ and $y = 7$ into $y = 0.1x + c$ we find c.

$$1 = -0.1 \times 7 + c \therefore c = 1.7$$

We can use the second point to verify this.

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Matrices

2023-01-31 10:00 Zhaojie Zoom

A matrix is a set of numbers/ variables arranged in the form of a rectangle and enclosed in curved brackets. For example

$$A = \begin{pmatrix} 3 & -2 & 2 & 9 \\ 2 & -1 & 0 & 0 \\ 1 & 2 & 1 & 5 \end{pmatrix} \text{ or } B = \begin{pmatrix} 3 \\ 2 \\ 1 \\ -3 \end{pmatrix}$$

Each entry in a matrix is known as an element. The size of a matrix is given by its number of rows $\times$ number of columns (in that order), for example the size of A is 3×4 and the size of B is 4×1.
The elements of a matrix can be referred to by their row and column number. The elements of the matrix

$$A = \begin{pmatrix} 3 & -2 \\ 2 & -1 \\ 1 & 2 \end{pmatrix}$$

can be referred using elements

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

The main diagonal of a matrix (A) is the collection of elements a_{ij} where $i = j$

Addition & Subtraction of Matrices

Certain combinations of matrices can be added, subtracted and multiplied (but never divided). If two matrices have the same size, they can be added or subtracted by simply adding or subtracting the corresponding elements. Examples of addition and subtraction can be seen below.

$$\begin{pmatrix} 3 & -2 \\ 2 & -1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 4 & -5 \\ -2 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 3+4 & -2-5 \\ 2-2 & -1+1 \\ 1+0 & 2+2 \end{pmatrix} = \begin{pmatrix} 7 & -7 \\ 0 & 0 \\ 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} 4 & -5 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 3-4 & -2+5 \\ 2+2 & -1-1 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 4 & -2 \end{pmatrix}$$

Multiplication of Matrices

Two matrices can be multiplied together if the number of columns in the first matrix is the same as the number of rows in the second matrix.
If A has size $p \times q$ and B has size $q \times s$ then AB has size $p \times s$.
The element in row i and column j of AB is the result of the 'multiplication' of the row i by column j, for example

$$A = \begin{pmatrix} 3 & 2 \\ 2 & -1 \\ -2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 1 & -1 & 0 \\ 1 & -1 & 1 & -2 \end{pmatrix}$$

Then the element in row 1 and column 1 of AB is

$$[3, 2] \times [4, 1] = 3 \times 4 + 2 \times 1 = 14$$

The element in row 2 and column 3 of AB is

$$[2, -1] \times [-1, 1] = 2 \times (-1) + (-1) \times 1 = -3$$

To multiply a matrix by a number, multiply each element by that number individually.

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Basics of Logic

2023-02-07 10:00 Zhaojie Zoom

Symbolic logic has provided the theoretical basis for many areas of computer science such as: digital logic circuit designs; relational database theory; programming; algorithms; and expert systems. Symbolic logic is also seen in everyday life.

Propositions

Proposition

A statement (declarative sentence) that is either *true* or *false*, but not both.

The true/ false value is called the *truth value* of the proposition.
'London is the capital of the UK'; '2 + 3 = 7'; and 'There are 8 days in a week' are all examples of propositions however 'Come here'; 'take two aspirins'; and 'Do you speak German?' are not.

Notation

A proposition is represented by a single letter, a proposition variable (similarly to sets and variables in algebra, etc).

Logical Connectives

Proposition can be combined with logical connectives to obtain compound statements (logical expressions). Three of the most important connectives are shown below.

Name	Connective	Symbol
negation	not	$\neg$
conjunction	and	$\wedge$
disjunction	or	$\vee$

The truth value of a compound statement (for example p or q) depends only on

- the truth values of the statements being combined; and
- on the types of connectives being used.

Negation

If p is a statement, then the negation of p is the statement not p, denoted by $\neg p$.

p	$\neg p$
T	F
F	T

Conjunction

If p and q are statements, the conjunction of p and q is the compound statement 'p and q', denoted by $p \wedge q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

If p and q are statements, the disjunction of p and q is the compound statement 'either p or q or both' or simply p or q, denoted by $p \vee q$.

p	q	$p \wedge q$
T	T	T
T	F	T
F	T	T
F	F	F

Hierarchy of Evaluation

The truth value of more complicated compound statements can be evaluated step-by-step.

1. Brackets
2. $\neg$
3. $\vee$, $\wedge$(equal priority, work left to right)

Logical Equivalence

Two logical expressions are said to be logically equivalent, $\equiv$, if they have identical truth values for each possible value of their statement variables.

Laws

These expressions can be simplified in the same way as seen in the Computer side of Architecture and Operating Systems. See those notes for a breakdown of the different methods. NB. they use different symbols than that used in this set of notes.

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Basics of Sets

2023-02-14 10:00 Zhaojie Zoom

Set

A collection of objects, called the elemts of the set.

Sets provide a convenient language for describing many of the concepts in Computer Science. A letter many be used for the name of a set (similar to a variable in algebra).

Set Notation

Elements of a set are enclosed by curly braces { and }. Sets do not have repeated elements and there is not a particular order of elements.
$\{1, 2, 3, 4\}$ denotes the set whose elements are 1, 2, 3, and 4. This is the same as $\{2, 4, 3, 1\}$.

Members of Sets

If A is the set, the notation $x \in A$ means that x is an element of A. $x \notin A$ means that x is not an element of A.

Listing vs Sequence of Elements

Commonly, we list the elements in a set for finite sets, for example

$$A = \{3, 6, 9, 12\}$$

For large, sets we can use ... to denote as sequence of elements. For example, the two sets below are the same, just represented differently

$$B = \{1, 2, \ldots, 10\}$$
$$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Specifying a property

Sets can be represented by specifying a property that the elements of the set have in common. For example, the set below contains all integers larger than 0 and smaller than 9.

$$B = \{z | z \text{ is an integer and } 0 < z < 9\}$$

B could be rewritten as

$$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

Size of the Set

Cardinality

The size of the set. If we have set A, the *cardinality* of A is represented by $|A|$.

For example, the size of the set

$$G = \{4, 6, 8\}$$

is three as it contains three elements, $|G| = 3$

Finite

The number of elements in the set is fixed.

Empty Set

A set without any elements. This is denoted by either $\varnothing$ or $\{\}$.

Note that $\{\varnothing\}$ is not an empty set, it contains one element - an empty set.

Infinite Set

The number of elements in a set is infinite. For example

$$\{x|x > 2012\}$$

Subsets

Subset

All elements of set A are also elements of set B. Therefore we can say that A is a subset of B.

The subset notation for the example above would be $A \subseteq B$.
If A is not a subset of B, then we can write $A \nsubseteq B$.
If $A \subseteq B$ and there is at least one element in B that is not in A, then A is called a *proper* subset of B and has the notation $A \subset B$.

Equality of Sets

Two sets are equal if they have exactly the same elements. This is written as $A = B$.

Operations on Sets

Operations on sets can be performed to give new sets.
Venn Diagrams can be used to represent these operations pictorially.

Intersection

Intersection

All the elements of one set A that also belong to B, but no other elements.

$$A \cap B = \{x|x \in A \text{ and } x \in B\}$$

For example, if $A = \{a, b, c\}$ and $B = \{c, d\}$, then $A \cap B = \{c\}$.

Disjoint

If $A \cap B = \varnothing$ then the sets A and B are disjoint (which means they have no element in common).

Union

Union

All the elements combined from multiple sets (remembering the rules about set membership).

For example, if $A = \{a, b, c\}$ and $B = \{c, d\}$ then $A \cup B = \{a, b, c, d\}$.

Difference

Difference

The difference of two sets A and B is the set of elements which are in A but not in B.

$$A \backslash B = \{x | x \in A \text{ and } x \notin B\}$$

NB. In some resources $A - B$ is used instead of $A \backslash B$.
For example, if $A = \{a, b, c\}$ and $B = \{c, d\}$ then $A \backslash B = \{a, b\}$ and $B \backslash A = \{d\}$.
In general $A \backslash B \neq B \backslash A$.