University Of Portsmouth
BSc (Hons) Computer Science
First Year

Architecture and Operating Systems - Maths

M30943
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20 Credits

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Contents

S.1. Introduction To Module

27-09-2022 10:00 Zhaojie Zoom

The goal of this module is to: help with maths involved in programming; prepare for other technical units (including: databases, functional programming; discrete maths; theoretical computer science; computer vision; and more); and to gain confidence.
The maths element (this element) is 30% of the Architecture and Operating Systems module. The two components are independent until the final grade for the module is produced.

Assessments

There are two components to the assessments for this element of the module.

Online Tests

Throughout the year, there are 7 online tests which will be completed through the Pearson MyMathsLab. Overall, these tests will equal 15% of the overall module score. Each test should last for 20 minutes and has a practice test available which can be accessed anytime in the year, and as many times as you wish. The real test can only be attempted once. After you submit the test, you will get an instant score.
Calculators are permitted, however it may not always be advisable to use one.

End Of Year Test

At the end of the year, there will be an end of year test. This is equal to 15% of the overall module score.

Staff & Support

Jhaojie is the main lecturer for the module. He will be assisted by Bryan.
Outside of class, Xia and Kirsten are available from the academic tutors office to help where needed. They can be booked through Moodle.
A MathsCafe runs during term-time for drop in Maths support. It is held at the following times

- Monday 12:00-14:00, LG learning and teaching space;
- Tuesday 12:00-14:00, LG learning and teaching space;
- Wednesday, 12:00-14:00, Zoom;
- Wednesday 14:00-16:00, Library 0.36;
- Thursday 12:00-14:00, LG learning and teaching space;
- Friday 12:00-14:00, LG learning and teaching space.

When writing working out for questions, full steps should be written down. This allows errors to be found and corrected.
Take a pen and paper to practical classes.

S.2. Basic Numeracy and Basic Algebra

27-09-22 10:20 Zhaojie Zoom

Negative Numbers

Subtracting a negative number is equivalent to adding a positive number. This can be seen in the following example.

$$
\begin{aligned}
2-(-5) &= \\
2+5 &= \\
&= 7
\end{aligned}
$$

The result of multiplying or dividing two numbers of the same sign is always positive.
The result of multiplying of dividing two numbers of opposing signs is always negative.

BIDMAS

The order in which to carry out operations in complex mathematical expressions is defined by the following priority list

1 Brackets

2 Indices

3 Division

3 Multiplication

4 Addition

4 Subtraction

Fractions

The names of different components of a fraction are as follows: $fraction = \frac{numerator}{denominator} = \frac{p}{q}$

Addition & Subtraction of Fractions

To add or subtract two fractions, their denominator needs to be the same. Then the addition/subtraction is performed just to the numerator. The fraction is usually then simplified.

Multiplication of Fractions

To multiply two fractions together: first, multiply the numerators together then multiply the denominators together.

Division of Fractions

To divide one fraction by another, multiply the first fraction by the inverse (reciprocal) of the second fraction. Simplify where necessary.

Simplification Of Fractions

A fraction is in its simplest form where there are no factors other than one to both the numerator and the denominator.

Algebra

The use of letters in maths is called Algebra. It defines the rules of how to manipulate with symbols.

Addition & Subtraction of like terms

Term

Either a single umber or variable, or the product of several and/or variables, for example $3y$.

Constant

A term without a symbol, for example, 2.

Like terms are multiples of the same variables; they can be added/ subtracted.

Multi-variable simplification

$$\begin{aligned} 24y^2 + 7x + 12xy - 4x - 5y^2 + 3xy = \\ 19y^2 + 15xy + 3x = \end{aligned}$$

Multiplication algebraic expressions

The fundamental concept behind multiplication of terms is to multiply the numbers and multiply the variables (using the rules for multiplication of indices if possible), taking into account the sign rules where multiplying terms with different signs.

Multiplying algebra example

$(2a)(6ab^2) = 12a^2b^2$

Expressions

Removing Brackets

In the expression $a(b + c)$, a is multiplies by all the bracketed terms to give $ab + ac$.
In the expression $(a + b)(c + d)$, $(a + b)$ is multiplied by the other pair of brackets as individual terms. Giving the answer as $ac + ad + bc + bd$
This principle along with the principle of simplifying algebra can be used to remove brackets fromm more complex expressions.

Removing brackets from a more-complex expression

$$\begin{aligned} (x + 6)(x - 3) &= x(x + 6) + (-3)(x + 6) \\ &= x^2 + 6x - 3x - 18 \\ &= x^2 + 3x - 18 \end{aligned}$$

Substitution

Where letters are replaced by actual numerical values.

Simple Linear Equations

Equations state that two quantities, usually one is known and one is not, are equal. We can use this information to solve the equation - to work out what the unknown quantity is.
A linear equation comes in the form of $ax + b = c$ where a, b and c are given numbers and x is an unknown quantity.

Solve $4x + 8 = 0$ for x

We can start by removing one of the known values, by subtracting 8 from both sides. This results in $4x = -8$ We can then divide both sides by 4 to get $x = -2$, which is our solution.

BASIC NUMERACY & ALGEBRA WORKSHEET 1

02-10-22 Worksheet

1. Calculate:

$$\begin{aligned}\frac{18}{6} \times 3 - 18 + 2 \times (-4) = & \\ &= \frac{18}{6} \times 3 - 18 - 8 \\ &= 3 \times 3 - 18 - 8 \\ &= 9 - 18 - 8 \\ &= -9 - 8 \\ &= 17\end{aligned}$$

2. Calculate:

$$\begin{aligned}\frac{12}{6} + \frac{12}{-4} + (-2) \times (-1) = & \\ &= \frac{12}{6} + \frac{12}{-4} + 2 \\ &= 2 - 3 + 2 \\ &= 1\end{aligned}$$

3. Calculate:

$$\begin{aligned}(-3) \times (-1) - 2 - 6 + 8 = & \\ &= 3 - -4 + 8 \\ &= 3 + 4 + 8 \\ &= 15\end{aligned}$$

4. Calculate:

$$\begin{aligned}(3^2 \times 2 - 2) \div 2^3 = & \\ &= (9 \times 2 - 2) \div 2^3 \\ &= (18 - 2) \div 2^3 \\ &= (16) \div 2^3 \\ &= 16 \div 8 \\ &= 2\end{aligned}$$

5. Solve $2x - 8 = 3x - 7$ for x.

$$\begin{aligned}2x - 8 &= 3x - 7 \\ 2x - 1 &= 3x \\ -1 &= 1x\end{aligned}$$

6. Solve $4x - 8 = 2(x + 3) + 10$ for x.

$$\begin{aligned}4x - 8 &= 2x + 6 + 10 \\ 4x - 8 &= 2x + 16 \\ 4x &= 2x + 24 \\ 2x &= 24 \\ x &= 12\end{aligned}$$

7. Evaluate the expression $\frac{5}{3} - \frac{1}{3} \times \frac{9}{2}$ and write in its simplest form.

$$\begin{aligned}\frac{5}{3} - \frac{1}{3} \times \frac{9}{2} &= \\ &= \frac{5}{3} - \frac{9}{6} \\ &= \frac{30}{18} - \frac{27}{18} \\ &= \frac{3}{18} \\ &= \frac{1}{6}\end{aligned}$$

8. Evaluate the expression $\frac{7}{2} - \frac{1}{4} + \frac{6}{8}$ and write it in its simplest form.

$$\begin{aligned}\frac{7}{2} - \frac{1}{4} + \frac{6}{8} &= \\ &= \left(\frac{7}{2} - \frac{1}{4}\right) + \frac{6}{8} \\ &= \left(\frac{28}{8} - \frac{2}{8}\right) + \frac{6}{8} \\ &= \left(\frac{28}{8}\right) + \frac{6}{8} \\ &= \frac{32}{8} \\ &= \frac{16}{4} \\ &= \frac{4}{1}\end{aligned}$$

9. Simplify.

(i) $3(-7y)$
$=-21y$

(ii) $(2y)(y^2)$
$=3y^3$

(iii) $3y^2(-2y)$
$=6y^3$

(iv) $(-2y)(-2y)$
$=-4y^2$

10. Expand the following expression

$$\begin{aligned}2x(4x - \frac{1}{2}) - 4x(5 - \frac{x}{2}) &= \\ &= ...\end{aligned}$$

11. Expand the following expression

$$\begin{aligned}6(4x + \frac{3}{2}) - 4x(5 + \frac{x}{2} + x^2) &= \\ &= ...\end{aligned}$$

12. Expand the following expression

$$\begin{aligned}(x+3)(2x-6)+2 &= \\ &= (2x^2-6x+6x-18)+2 \\ &= 2x-16\end{aligned}$$

13. Expand and fully simplify the following expression

$$\begin{aligned}(2y-3)^2 &= \\ &= (2y-3)(2y-3) \\ &= 4y^2-6y-6y+9 \\ &= 4y^2-12y+9\end{aligned}$$

14. Simplify the following expression.

$$\begin{aligned}3x(x-y^2)+3(x^2+1)-7x^2y &= \\ &= 3x^2-3xy^2+3x^2+3-7x^y \\ &= 6x^2-3xy^2+3-7x^2y\end{aligned}$$

S.3. Everyday Maths

03-10-22 10:00 Zhaojie Zoom

Percentage

A percentage is a fraction where the denominator is 100. Percent corresponds to ”per 100”.

Ratio

Ratios are used to compare two or more quantities. The symbol used is : (a colon).
To simplify ratios, divide both parts of the ratio by the hightest common factor.

Average

The average of a set of numbers, sometimes known as a mean, can be calculated using the following formula:

$$\text{average} = \frac{\text{sum of a set of values}}{\text{number of values}}$$

Probability

Probabilities express how likely something is to happen. They are expressed as a decimal number between 0 and 1. A probability of 1 means the event must happen and a probability of 0 means the event will never happen.

WORKSHEET 2

07-10-22 Worksheet

1. **VAT at 17.5% is added to a telephone bill of £80. What is the total amount to be paid?**

$$80 \times 1.175 = 94$$

2. **A sweater is bought in a sale for £20 after a 15% reduction. What was the original price of the sweater?**

$$\begin{aligned} \pounds 20 &= 85\% \\ \therefore \pounds 0.253 &= 1\% \\ \therefore \pounds 23.50 &= 100\% \end{aligned}$$

3. **A flat is bought for £180000. Each year the price rises by 5%. How much is the flat worth after 3 years?**

$$180000 \times 10.5^3 = \pounds 208372.50$$

4. **John bought a sports car for £18500. Two years later, he sold the car for £14000. Work out the percentage loss.**

$$\begin{aligned} \frac{\text{change}}{\text{original}} \times 100 &= \text{percent diff} \\ \frac{18500 - 14000}{18500} \times 100 &= 24.3\% \end{aligned}$$

5. **A car was bought for £12000. Each year it depreciated in value by 5%. Work out the value of the car after 3 years.**

$$12000 \times 0.95^3 = \pounds 10288.50$$

6. **On April 1st a train journey increases in price by 5%. Six months later it increases in prce again by 2%. Work out the final price of a ticket that originally cost £40.**

$$\begin{aligned} 40 \times 1.05 &= 42 \\ 42 \times 1.02 &= \pounds 42.84 \end{aligned}$$

7. **Toothpaste is sold in three different sized tubes.**

   - **50ml = £1.24,**
   - **75ml = £1.96,**
   - **100ml = £2.42.**

   Which of the tubes of toothpaste gives best value for money?

$$\begin{aligned} 50ml &= 0.0248 \\ 75ml &= 0.02613 \\ 100ml &= 0.0242 \leftarrow \text{this is cheapest} \end{aligned}$$

8. **Express the following percentages as fractions:**

   (a) **50%**
   (b) **75%**

(c) 36%
(d) 100%
(e) 12.5%

(a) $\frac{1}{2}$
(b) $\frac{3}{4}$
(c) $\frac{9}{25}$
(d) $\frac{1}{1} = 1$
(e) $\frac{1}{8}$

9. **A photograph of length 27cm is to be enlarger in the ratio 3:7. What is the length of the enlarger photograph?**
...

10. **A recipe for 12 people uses 500g of plain flour. How much flour is needed for 18 people?**

$$500 \times 1.5 = 750g$$

11. **If 15 oranges costs £1.80, how much will 23 identical oranges cost?**

$$1.8 \div 15 = 0.12$$
$$0.12 \times 23 = £2.76$$

12. **It took 4 people 7 days to build a fence. At the same rate how many people do we need if we want to build the fence in just 2 day?**

$$4 \times 7 = 28\text{pepoles worth of work}$$
$$28 \div 2 = 14\text{people}$$

13. **An inheritance is divided between three people in the ratio 4:9:2. If the lease amount received is £2300, calculate how much the other two people received.**

$$2300 = 2 \therefore 1 = 1150$$
$$1150 \times 4 = 4600(4)$$
$$1150 \times 9 = 10350(9)$$

14. **Make is taking the NetFun unit at the School of Computing. His mark for the coursework was 26 (50% of overall mark). Make wants to have the final mark at least 70. Figure out whether it is l possible and if yes, calculate his lowest possible mark for the exam (50% of overall mark). Answer the same questions for the final mark at least 60.**
...

15. **Tony walks 24km in 6 hours. (a) What is his average speed in km/h? (b) If he had taken 2 hours longer, what would have been his average speed?**

$$(a)\ 4km/h \quad (b)\ 3km/h$$

16. **A marathon race was completed by 5 participants in the times given below: 2.7 hr, 8.3 hr, 3.5 hr, 5.1 hr, 4.9 hr. What is the average race time for this marathon?**

$$\frac{1.7 + 8.3 + 3.5 + 5.1 + 4.9}{5} = 4.9$$

17. [*] **Two cyclists, initially 30 miles apart, travel towards each other. Cyclist A goes at 14 mph, and cyclist B at 16 mph. A fly flies back and forth between their noses at 30 mph. How far does the fly fly?**
...

18. [*] **Driving through roadworks on a motorway, the sign said 'speed limit 50; average speed calculated'. I noticed that for 12 minutes I was going at 60 mph. For how long do I have to go at 30 to be legal?**
...

19. **A painter wants to draw a big rectangle with sides 3 and 5 metres. He wants to use 3 different colours: blue, green, and yellow. A part with blue colour will cover 30% of the total area of rectangle. The rest area will be painted equally by green and yellow colour. The cost of blue colour is £2.20 (per square metre), green colour is £2.50 (per square metre), yellow colour is £3 (per square metre). How much the painter will pay for the colours?**

$$\begin{aligned} 3 \times 5 &= 15m^2 \\ 15 \div 100 &= 0.15 \\ 0.15 \times 30 &= 4.5 \\ 4.5 \times 2.20 &= £9.90 \\ \\ 0.15 \times 35 &= 5.25 \\ 5.25 \times 2.5 &= £13.13 \\ 5.25 \times 3 &= £15.75 \\ \\ 9.9 + 13.13 + 15.75 &= £38.78 \end{aligned}$$

20. **A die is thrown. Find the probability of**
    (a) **obtaining a score less than 6**
    (b) **obtaining a score more than 6**
    (c) **obtaining an even score less than 5**
    (d) **obtaining an even score less than 2**

    (a) $\frac{5}{6}$
    (b) $\frac{0}{6} = 0$
    (c) $\frac{2}{6} = \frac{1}{3}$
    (d) $\frac{0}{6} = 0$

21. **A box contians 16 red blocks, 20 blue blocks, 24 orange blocks and 10 black blocks. A block is picked at random. Calculate the probability that the block is**
    (a) **black**
    (b) **orange**
    (c) **blue or red**
    (d) **green**
    (e) **not orange**

$16 + 20 + 24 + 10 = 70$

(a) $\frac{10}{70} = \frac{1}{7}$

(b) $\frac{24}{70}$

(c) $\frac{20 + 16}{70} = \frac{36}{70}$

(d) $\frac{0}{70} = 0$

(e) $\frac{70 - 24}{70} = \frac{46}{70}$

S.4. Powers and Logarithms

11-10-22 10:00 Zhaojie Zoom

Powers

2^4 rads as 2 to the power of 4 and it means

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

In the example above, 2 is the base and 4 is the index (or power).

Special Cases of Powers

$x^0 = 1$ This will be true for all cases except for where $x = 0$, in this case $x^0 = undefined$.
$x^1 = 1$ This is true for all values of x.

Laws Of Indices

There are three laws of indices.

1. $a^n \times a^m = a^{n+m}$ (when multiplying, add the indices)
2. $\dfrac{a^n}{a^m} = a^{n-m}$ (when dividing, subtract the indices)
3. $(a^n)^m = a^{n\times m}$ (when raising one power to another, multiply the indices).

Negative Powers

With negative powers, there is a general rule

$$a^{-n} = \frac{1}{a^n}$$

Fractional Powers

Where a and n are positive numbers, the general rule indices

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Simplify the following expression

$$\begin{aligned}\sqrt{\frac{72a^{12}b^7c^2}{2a^2b^3c^{-10}}} &= \\ &= \sqrt{36a^{10}b^4c^{12}} \\ &= (36a^{10}b^4c^{12})^{\frac{1}{2}} \\ &= 6a^5b^2c^6\end{aligned}$$

Logarithms

Logarithm

A logarithm determines how many times a certain number must be multiplied by itself to reach another number.

The general rule for logarithms is shown below, this is applicable where $a > 1$.

$$y = a^x$$

$$\log_a y = x$$

Base of a logarithm

The most commonly used bases are

- 10 ($\log_{10}$)
- 2 ($\log_2$)
- natural logarithm e ($\log_e$ or ln)

First Law Of Logs

$$\log_a x + \log_a y = \log_a xy$$

All bases must be the same.

Second Law Of Logs

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

All bases must be the same.

Third Law Of Logs

$$n \log_a x = \log_a x^n$$

This law applies of n is an integer, fractional, positive or negative.

Example

Simplify

$$\begin{aligned} \log_2 y - 3\log_2 2y + 2\log_2 4y = & \\ &= \log_2 y - \log_2 (2y)^3 + \log_2 (4y)^2 \\ &= \log_2 y - \log_2 8y^3 + \log_2 16y^2 \\ &= \log_2 \left(\frac{y \times 16y^2}{8y^3} \right) \\ &= \log_2 2 \\ &= 1 \end{aligned}$$

S.5. Functions and Sequences

19-10-22 10:00 Zhaojie Zoom

Functions

Function

A function is a rule that recieves an input and produces an output. A function can only produce a single output for any given input.

In maths, function are written as follows

$$f(x) = x + 3$$

We can use any different letters we want.

Calculating output when given an input

A function f is defined by $f(x) = 3x + 1$. Calculate the output when the input is 4.

$$f(4) = 3 \times 4 + 1 = 13$$

Composite Functions

A composite function is where the output of one function feeds directly into the input of another function. This can be expressed as follows

$$g(f(x))$$

Composite Function eample

Given $f(x) = x^2$ and $g(x) = x + 1$. Find a value of the composite function $f(g(x))$ and $g(f(x))$ for $x = 3$.

$$f(g(3)) = f(3 + 1) = f(4 = 4^2 = 16)$$

$$g(f(3)) = g(3^2) = g(9) = 9 + 1 = 10$$

Sequences

Sequence

A sequence is a set of number written down in a specific order. Each element in the sequence is called a term.

There are two types of sequence, finite and infinite sequence. Finite sequences have a fixed number of elements and infinite sequences can go on forever.

Sequence Notation

We use subscript notation to refer to different terms in the sequence. The first term in the sequence can be called x_1, the second x_2 and so on.

Recurrence Relation

A recurrence relation is an equation that recursively defines a sequence. One or more initial terms are given and each further term of the sequence is defined as a function of the preceding terms. For example

$$F_n = f_{n-1} + F_{n-2} \geq 2$$

$$F_0 = 0, F_1 = 1$$