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Second Year

Discrete Mathematics and Functional Programming (`DMAFP`)

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Thomas Boxall
`up2108121@myport.ac.uk`

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Lecture - Sets

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1.1 Introduction

Sets underpin maths and Computer Science. A set is a collection of objects, which are called the elements (also known as members of the set). For example, a set of the numbers 1, 3, 8; or the collection of students in a class born in March. There are two characteristics of sets:

1. There are no repeated occurrences of elements
2. There is no particular order of the elements

1.2 Set Notation

The elements of a set are enclosed in braces with their names being denoted by a *letter*, for example:

$$A = \{1, 2, 3\}, \quad C = \{Portsmouth, Brighton, London\}$$

There are two ways that we can describe the members of a set. We can *list the elements* which is mainly used for finite sets, for example:

$$A = \{3, 6, 9, 12\}$$

We can *specify a property* that all the elements in the set have in common. The '|' character is read 'such that', sometimes ':' is used in it's place. For example:

$$B = \{x | x \text{ is a multiple of 3 and } 0 < x < 15\}$$

We can also use *three dots* to informally denote a sequence of elements that we don't wish to write down, for example:

$$C = \{1, \ldots, 10\}$$

1.2.1 Sets of Numbers

There are some reserved letters to denote specific sets of numbers in maths. These are shown below:

- $\mathbb{N}$ (or N) is used for the set of natural numbers (integers >= 0). $\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$
- $\mathbb{Z}$ (or Z) is used for the set of integers. $\mathbb{N} = \{\ldots, -1, 0, 1, \ldots\}$
- $\mathbb{Q}$ (or Q) is used for the set of rational numbers (number which can be expressed as a quotient or fraction). $\mathbb{Q} = \{0, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$
- $\mathbb{R}$ (or R) is used for the set of real numbers. $\mathbb{R} = \{\ldots, -1, 0, \frac{1}{2}, \ldots\}$

1.2.2 Elements of a Set

We can use the $\in$ symbol to denote if an element is a member of a given set. For example, if x is a member of S - then we can say:

$$x \in S$$

The symbol $\notin$ denotes an element is not a member of a given set. For example, if y is **not** a member of S - then we can say:

$$y \notin S$$

1.2.3 Many Ways to Say The Same Thing

There are several ways of describing the same set, for example for the set S of *odd integers*:

$$\begin{aligned} S &= \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\} \\ &= \{x | x \text{ is an odd integer }\} \\ &= \{x | x = 2k + 1 \text{ for some integer } k\} \\ &= \{x | x = 2k + 1 \text{ for some } k \in \mathbb{Z}\} \\ &= \{2k + 1 | k \in \mathbb{Z}\} \end{aligned}$$

The phrase "for some [integers K]", means "for all [integers k]"

1.2.4 Empty Sets

Where a set has *no elements*, it is called an empty set or null set. It's denoted with the $\emptyset$ symbol, for example:

$$\emptyset = \{\}$$

1.2.5 Finite & Infinite Sets

If the number of elements in the set is fixed (for example when counting the elements at a fixed rate for a set amount of time), then the set is *finite*. If the set X is finite, then we call $|X|$ the *cardinality* of X therefore:

$$|X| = \text{number of elements in } X$$

If the counting never stops then X is an infinite set.

1.2.6 Subsets

A subset is where one set's elements are entirely present in another set. There are three conditions we need to know about:

- $A \subseteq B$: A is a subset of B therefore every element in A is also in B.
- $A \nsubseteq B$: A is not a subset of B.
- $A \subset B$: A is a proper subset of B, therefore B has at least one additional element which is not in A.

Part II

Functional Programming