# Regime Switching for Dynamic EquiCorrelation

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**Abstract**

We introduce a regime-switching dynamic, inspired by the Markov Switching Conditional Correlation model of Pelletier (2006), into the Dynamic Equicorrelation (DECO) model proposed by Engle and Kelly (2012). The DECO model decomposes the correlation matrix into blocks, within which conditional correlations are equal. Correlations between blocks can either follow a common structure or be block-specific. Our Regime Switching Dynamic Equicorrelation (RSDECO) model is therefore well-suited for modeling large conditional correlations in the presence of level shifts. Extensive simulations demonstrate that RS-DECO effectively reproduces the true correlation levels for a large number of variables. Analyzing daily correlations between commodity, stock, and bond returns from 01/04/2000 to 12/29/2022 reveals significant changes in correlation patterns over time. The first major and significant regime shift occurs around September 15, 2008. Furthermore, the dynamics of correlations between the three asset classes appear to have become unstable after 2020.

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*Keywords*: Dynamic Correlations; Large cross-sections; Regime Switching.

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# 1 Introduction

Institutional investors routinely monitor hundreds or even thousands of assets, requiring accurate estimates of their returns' volatilities and correlations to make informed investment decisions. These correlations are essential components of portfolio management, asset pricing, and risk management. Engle (2002) Dynamic Conditional Correlation (hence DCC) model is one of the most popular multivariate GARCH models. This model suffers from *curse of dimensionality* as the number of parameters increases much faster than the number of returns.[1] Estimation is possible for a limited number of returns, even if constraints are imposed on the parameters of the model.

Alessi et al. (2009), De Nard et al. (2021), and more recently Trucíos et al. (2023) utilize factor models to estimate high-dimensional covariance matrices. However, a notable challenge with this approach is the identification of factors. In contrast, Engle et al. (2019) propose a robustified DCC model, which combines the original DCC framework with shrinkage methods to produce large covariance matrices that are less prone to estimation errors. In some applications, such as portfolio selection, it can be more practical to estimate the inverse of the correlation matrix (precision matrix) directly. The model by Lee et al. (2021) integrates the precision matrix estimation methods of Fan and Lv (2016) with the DCC approach of Engle (2002), providing a hybrid solution tailored for such scenarios.

More relevant to this paper, another line of research focuses on decomposing complex numerical problems into a series of simpler, more manageable sub-problems (Palandri (2009), Engle (2009a,b), and Archakov et al. (2024)). The Dynamic Equicorrelation (DECO) model introduced by Engle and Kelly (2012) offers a compelling approach to tackling high-dimensional challenges. This model assumes that all correlations are equal to a single common value, referred to as the equicorrelation, which evolves over time. The DECO can be extended to a $B$-Blocks DECO model, with block-specific equicorrelations, and the specific inter-block equicorrelations for each pair of blocks. Bauwens and Xu

[1]See for instance Bauwens et al. (2006) and De Almeida et al. (2018) for a literature review

(2023) recently extended the DCC and the DECO models by introducing measures of realized variances and correlations estimated from intraday data.

Several papers have proposed multivariate GARCH with regime switching based on observable variables (Tse and Tsui (2002), Audrino and Trojani (2011), and Silvennoinen and Teräsvirta (2009)). Since the threshold is unknown in practice, the question of how to determine it remains a major challenge. Pelletier (2006) suggests using a Markov switching model for the correlation. The value of the current correlation depends on the underlying regime, and the conditional covariance varies according to changes in the conditional variances and the correlation.

The innovation of this paper is to add a regime switching dynamic to the DECO model in the spirit of Pelletier (2006). In our model, called Regime Switching Dynamic Equicorrelation (RSDECO), equicorrelations are constant within each regime but vary between regimes by a *first order* Markov process. Our model introduces discrete breaks into the modelling of conditional correlation to capture a finite number of switching regimes. A series of Monte Carlo simulations highlights the performance of our approach. The RS-DECO model appears to perform well in reproducing the true correlation and also in dealing with the high dimensional problem.

As an empirical application, we estimate conditional correlations using a sample of 24 daily commodity returns and 4 US equity returns from 4 January 2000 to 29 December 2022. First, we find two dynamic correlation regimes with a significant regime shift on 15 September 2008, coinciding exactly with the Lehman bankruptcy date. The second regime is associated with a dramatic increase in the correlation between seemingly unrelated commodities.

However, the simple RSDECO version detects a regime shift at the end of 2005, well before the financial crisis, associated with the process of commodity financialization. Second, our results show that correlations have returned to their pre-crisis levels by the end of 2013, indicating the temporary nature of the impact of the financial crisis on commodity correlation trends. Finally, a more detailed analysis of commodity group correlations from Block-RSDECO reveals heterogeneity among commodity group co-movements.

The paper is organized as follows. In section 2, we develop the RSDECO model and the estimation method. Section 3 presents the results of the simulation study. Section 4 presents our empirical application and Section 5 concludes.

# 2 The model

## 2.1 The regime switching DECO

We consider the vector of $r_t = (r_{1t}, .., r_{kt})'$, of size $k \times 1$ assets returns such as $r_t|I_{t-1} \sim NID(\mu_t, H_t)$ where $I_t = \{r_s, s \leq t\}$ is the information set at time t. The conditional covariance matrix $H_t$ is decomposed as:

$$H_t = D_t R_t D_t$$

where $D_t$ is a $k \times k$ diagonal matrix containing the conditional standard deviations. In (Engle, 2002) classical DCC(1,1) model, the dynamics of $R_t$ is modelled as:

$$\begin{aligned} R_t &= Q_t^{*-1/2} Q_t Q_t^{*-1/2} \\ Q_t &= C + A'\epsilon_{t-1}\epsilon_{t-1}^{'} A + B' Q_{t-1} B \end{aligned}$$

where $\epsilon_t = D_t^{-1}(r_t - \mu)$ is the vector of standardized residuals, $Q_t$ the quasi correlation matrix, and $Q_t^*$ a diagonal matrix with the main diagonal of $Q_t$

The Dynamic EquiCorrelation Model (DECO) of Engle and Kelly (2012) is based on the assumption that all correlations are equal to the equicorrelation which varies over time. The conditional correlation matrix is therefore written as:

$$R_t = (1 - \rho_t) I_k + \rho_t J_k \tag{1}$$

where $\rho_t$ refers to the equicorrelation, $I_k$ is a $k-$ dimensional identity matrix and $J_k$ is a $k \times k$ matrix of ones. The estimation of the equicorrelation $\rho_t$ is model-free. In the DECO model, $\rho_t$ is estimated as the average of the conditional correlations from a CC model:

$$\rho_t = \frac{1}{k(k-1)}(\iota' R_t^{CC} \iota - k) \tag{2}$$

where $\iota$ is a $k \times 1$ vector of ones. In this case, we face the same dimensionality issue when $k$ increases.

In the spirit of Pelletier (2006), we adopt a regime-switching structure for the correlation process in the DECO model (RSDECO) and assume that the dynamic of the equicorrelation is characterized by $N$ regimes. The transition between regimes is modelled by a first-order Markov chain. The equicorrelation at date $t$ follows:

$$\rho_t = \sum_{n=1}^{N} \mathbb{1}_{\{s_t=n\}} \rho^n, s_t \in \{1, ..., N\} \tag{3}$$

where $\{s_t\}_{t\in\mathbb{N}}$ is a sequence following a first-order Markov chain with $N$ states. $\rho^n$ is a conditional equicorrelation corresponding to regime $n$. The unobservable realisation of the regime $\{s_t\}_{t\in\mathbb{N}}$ is determined by the transition probability matrix $P$ whose elements are $p_{ij} = P(s_t = j|s_{t-1} = i), i, j \in 1, ..., N$. The resulting state-dependent correlation matrix at date $t$ is therefore:

$$R_t = \begin{bmatrix} 1 & & & & \\ & \ddots & & \rho_t & \\ & & 1 & & \\ & \rho_t & & \ddots & \\ & & & & 1 \end{bmatrix} \tag{4}$$

Following Pelletier (2006), we include the identification constraint: $\rho^1 > \rho^2 > ... > \rho^N$ to deal with the relabelling switching problem. Engle and Kelly (2012) points out that the equicorrelation must verify the condition $\rho^n \in ]\frac{-1}{k-1}, 1[$ to ensure that $R_t$ is positive definite[2].

[2]It is noteworthy that as the number of variables $k$ becomes more significant, the lower bound, $\frac{-1}{k-1}$, decreases and approaches zero. This would prevent the detection of negative correlations when we include a significant number of series in the analysis.

## 2.2 Block RSDECO

As in DECO, we decompose the correlation matrix $R_t$ into blocks of equicorrelations to obtain a more flexible and realistic model of conditional correlation. A block characterised by a single equicorrelation can be associated with a particular asset class, while the correlation between blocks estimates the correlation between different asset classes. We make the simplifying assumption that both intra-block and inter-block correlations follow the same first-order Markov chain. This block decomposition appears to significantly improve both the estimation and the explanatory power of our model. Taking into account both economic assumptions and practical considerations, we present two ways of introducing block structure.

### 2.2.1 Reduced-Block RSDECO

As a first step, we assume that, for each state $s_t = 1, ..., N$, each block has its equicorrelation, while the equicorrelation is the same between all blocks. This formulation is called *Reduced Block*-RSDECO (hereafter *RB*-RSDECO). Let $B$ be the number of blocks, in this configuration the total number of correlations to be estimated is $(B + 1)N$. The conditional correlation matrix $R_t$ is written as:

$$R_t = \sum_{n=1}^{N} \mathbb{1}_{\{s_t = i\}} R_n, \ \ s_t = 1, ..., n \tag{5}$$

Each state $n = 1, ..., N$ is characterized by its conditional correlation matrix $R_n$:

$$R_n = \begin{bmatrix} \boxed{\begin{matrix} 1 & & \rho_1^n \\ & \ddots & \\ \rho_1^n & & 1 \end{matrix}} & & & \rho_{inter}^n \\ & \boxed{\begin{matrix} 1 & & \rho_2^n \\ & \ddots & \\ \rho_2^n & & 1 \end{matrix}} & & \\ & & \ddots & \\ \rho_{inter}^n & & & \boxed{\begin{matrix} 1 & & \rho_B^n \\ & \ddots & \\ \rho_B^n & & 1 \end{matrix}} \end{bmatrix} \tag{6}$$

$\rho_i^n, i = 1, ..., B$ is equicorrelation for block $i$ in state $n$ while $\rho_{inter}^n$ is the correlation between blocks supposed to be the same between all blocks. The correlation matrices are such as $R_n \neq R_{n'}$ for $n \neq n'$. The distribution of the process defined by $\{s_t\}_{t \in \mathbb{N}}$ is determined by a transition matrix $P$ as previously.

At each date $t$, the conditional correlation matrix $R_t$ is therefore equal to:

$$R_t = \begin{bmatrix} \boxed{\begin{matrix} 1 & & \rho_{1,t} \\ & \ddots & \\ \rho_{1,t} & & 1 \end{matrix}} & & & \rho_{inter,t} \\ & \boxed{\begin{matrix} 1 & & \rho_{2,t} \\ & \ddots & \\ \rho_{2,t} & & 1 \end{matrix}} & & \\ & & \ddots & \\ \rho_{inter,t} & & & \boxed{\begin{matrix} 1 & & \rho_{B,t} \\ & \ddots & \\ \rho_{B,t} & & 1 \end{matrix}} \end{bmatrix} \tag{7}$$

where $\rho_{i,t} = \sum_{n=1}^{N} \mathbb{1}_{\{s_t = n\}} \rho_i^n$ and $\rho_{inter,t} = \sum_{n=1}^{N} \mathbb{1}_{\{s_t = n\}} \rho_{inter}^n$ are respectively equicorrelation for block $i \in 1, ...B$ and the equicorrelation between blocks at date $t$.

As for the simple RSDECO, identification constraints are required. For a block $B$ of size $n_B$, the equicorrelation parameters $\rho_B^n$, $n = 1, ..., N$, must satisfy the identification constraint $\rho_B^1 > \rho_B^2 > ... > \rho_B^N$ for $n = 1, ..., N$. $R_t$ matrix is a positive definite if and only if $\rho_B^n \in ]\frac{-1}{k_B-1}, 1[$.

The constraints on the inter-block equicorrelation are slightly different due to the estimation procedure. As shown in the Estimation subsection, a $B$-block RSDECO is estimated by maximizing the sum of the likelihoods of all possible pairs of $B$-blocks that are estimated from the two-block RSDECO likelihood. As in Engle and Kelly (2012), the equicorrelation in regime $n$ between two blocks $i$ and $j$ of size $k_i$ and $k_j$ respectively, denoted by $\rho_{ij}^n$, must be checked:

$$\rho_{ij}^n \in (-l_{ij}^n, l_{ij}^n) \tag{8}$$

with

$$l_{ij}^n = \sqrt{\frac{(\rho_i^n(k_i-1)+1)(\rho_j^n(k_j-1)+1)}{k_i k_j}} \tag{9}$$

for $n = 1, ..., N$. To respect the positive definite constraint, the inter-block equicorrelation $\rho_{inter,t}^n$ in regime $n$ must satisfy the relation (9) for every pair, which is equivalent to having:

$$l_{inter}^n = min(l_{ij}^n) \tag{10}$$

for $i, j = 1, ..., B$ and $i \neq j$. This constraint may seem strong, but it keeps the model parsimonious as the number of groups increases.

### 2.2.2 Full-block RSDECO

A more complete way to express Block-RSDECO is to relax the assumption of a common inter-block equicorrelation $\rho_t^{inter}$. We assume specific equicorrelation for each pair of blocks. Within this formulation, called *Full Block*-RSDECO (hereafter *FB*-RSDECO), the conditional correlation matrices for each state $n = 1, ..., N$ has the following specification:

$$R^n = \left[\begin{array}{c|c|c|c} \begin{matrix} 1 & & \rho_1^n \\ & \ddots & \\ \rho_1^n & & 1 \end{matrix} & \rho_{12}^n & \cdots & \rho_{1B}^n \\ \hline \rho_{21}^n & \begin{matrix} 1 & & \rho_2^n \\ & \ddots & \\ \rho_2^n & & 1 \end{matrix} & \cdots & \rho_{2B}^n \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \rho_{B1}^n & \rho_{B2}^n & \cdots & \begin{matrix} 1 & & \rho_B^n \\ & \ddots & \\ \rho_B^n & & 1 \end{matrix} \end{array}\right] \quad n = 1, ..., N$$

The Correlation matrix $R_t$ is now equal to:

$$R_t = \left[\begin{array}{c|c|c|c} \begin{matrix} 1 & & \rho_{1,t} \\ & \ddots & \\ \rho_{1,t} & & 1 \end{matrix} & \rho_{12,t} & \cdots & \rho_{1B,t} \\ \hline \rho_{21,t} & \begin{matrix} 1 & & \rho_{2,t} \\ & \ddots & \\ \rho_{2,t} & & 1 \end{matrix} & \cdots & \rho_{2B,t} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \rho_{B1,t} & \rho_{B2,t} & \cdots & \begin{matrix} 1 & & \rho_{B,t} \\ & \ddots & \\ \rho_{B,t} & & 1 \end{matrix} \end{array}\right]$$

where $\rho_{i,t} = \sum_{n=1}^{N} \mathbb{1}_{\{s_t=n\}} \rho_i^n$ and $\rho_{ij,t} = \sum_{n=1}^{N} \mathbb{1}_{\{s_t=n\}} \rho_{ij}^n$ are respectively equicorrelation for block $i \in 1, ..., B$ and the equicorrelation between blocks $i$ and $j$ at date $t$.

As shown in Table 1, because the *FB*-RSDECO structure is more complete and accurate than the *RB*-RSDECO structure, it requires more equicorrelations to be estimated.

Block equicorrelations are bounded as shown in equation (8), while inter-block equicorrelations are bounded as shown in equation (9).

Table 1: Number of estimated correlations.

| Model | Number of correlations | Bounds |
|---|---|---|
| RSDECO | $N$ | $\left(\frac{-1}{k-1}, 1\right)$ |
| *RB*-RSDECO | $B(N+1)$ | $\left(\frac{-1}{k_i-1}, 1\right)$ (Block $i$)<br>$min(l^n_{ij})$ (inter-Block) |
| *FB*-RSDECO | $(B + \frac{B(B-1)}{2})N$ | $\left(\frac{-1}{k_i-1}, 1\right)$ (Block $i$)<br>$(l^n_{ij})$ (inter-Block) |

## 2.3 Estimation

Our model can be estimated using the two-step procedure of Engle and Sheppard (2001). $\Theta = (\varphi, \theta)$ is the parameter space of the RSDECO, where $\varphi$ are the parameters of the univariate volatility model and $\theta$ are the parameters of the correlation model. Assuming Gaussian innovations, the first step involves estimating the $k$ univariate GARCH and corresponds to the volatility component of the log-likelihood:

$$\ell_v(\varphi|\mathbf{r}_t) = -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_t|) + \mathbf{r}_t' D_t^{-2}\mathbf{r}_t) \tag{11}$$

These variance estimates of $r_t$ are used to construct the matrix $D_t$:

$$D_t = \text{diag}\{h_{i,t}^{1/2}\} \tag{12}$$

Then the standardized residuals are given by:

$$\epsilon_t = D_t^{-1}\mathbf{r}_t \tag{13}$$

In the second step, the correlation component is estimated by maximising the likelihood conditional on the volatility parameters estimated in the first step:

$$\ell_c(\theta|\mathbf{r}_t, \varphi) = -\frac{1}{2}\sum_{t=1}^{T}(\log(|R_t|) + \epsilon_t' R_t^{-1}\epsilon_t) \tag{14}$$

The *reduced* and *full* block RSDECO are estimated using the same strategy as for Engle and Kelly (2012). A simple way to estimate the $B$ block likelihood is to sum the estimated $B(B-1)/2$ two-block likelihoods.

$$\ell_c(\theta|\mathbf{r}_t, \varphi) = \sum_{i=1}^{B}\sum_{j=i+1}^{B} \ell_{ij,c}(\theta|\mathbf{r}_t, \varphi) \tag{15}$$

where $\ell_{ij,c}$ is the two-block likelihood of the submodel consisting of blocks $i$ and $j$.

As explained above, the transition from one regime to another is done using the *first-order* Markov chain. Since the Markov chain $s_t$ is an unobservable regime variable, we will need to infer the state of these Markov chains through a filtration step (see Hamilton (1989, 1990, 1994)). Let $\xi_{jt}$ be the probability of being in regime $j$. The inference about the state of the Markov chain is then defined by

$$\hat{\xi}_{t|t} = \frac{(\hat{\xi}_{t|t-1} \odot \eta_t)}{\mathbf{1}'(\hat{\xi}_{t|t-1} \odot \eta_t)} \tag{16}$$

where $\odot$ denotes element-by-element multiplication. $\hat{\xi}_{t|t}$ contains the probability to be in each regime at time $t$ given the observations set up to $t$. Assuming a *first-order* Markov process, the probability at time $t+1$ is conditional only on information included in $\hat{\xi}_{t|t}$:

$$\hat{\xi}_{t+1|t} = P \times \hat{\xi}_{t|t} \tag{17}$$

Smoothed probabilities can be computed from the regular Markov-Switching representation. These filtered probabilities refer to inferences on the state conditional on all the information set of the sample. Computation is done using the filter of Kim (1994) given by:

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \{P'[\hat{\xi}_{t+1|T} \oslash \hat{\xi}_{t+1|t}]\} \tag{18}$$

where $\oslash$ denotes element-by-element division.

The conditional correlation matrix is:

$$\hat{R}_{t|T} = \sum_{n=1}^{N} \hat{R}_{s_t=n} \hat{\xi}_{s_t=n,t|T}$$

Determining the number of identifiable regimes required for the Markov process to characterise the observed process is a particular problem for our RSDECO model, as it is for the RSDC model of Pelletier (2006). As explained in Krolzig (1997), testing procedures suffer from non-standard asymptotic distributions of the likelihood ratio test statistic due to the presence of nuisance parameters under the null hypothesis. Alternatively, and following Pelletier (2006), the number of regimes is determined in an ad hoc manner. In other words, we have to estimate models by incrementally increasing the number of regimes and then selecting the appropriate number of regimes using the resulting quality for the additional regime as a criterion. If, during the additional regime, the smoothed probabilities and correlations are represented only by a few blips that cannot be explained by an event study, they look more like outliers than effective regimes. This may be an indication of the appropriate number of regimes.

# 3 Simulation study

In this section, we perform a simulation exercise to demonstrate the performance of our model. The results of our RSDECO model are then compared with those of DECO using simulated correlation data. The simulations are carried out in an environment where the true correlations are known. The data generated then correspond to the standardised residuals after the de-garching step. The idea is to avoid the search step, which is useless for our experiments. Under this assumption, for each simulated standardised residual we generate data as follows:

$$\varepsilon_t = \eta_t R_t^{0.5} \tag{19}$$

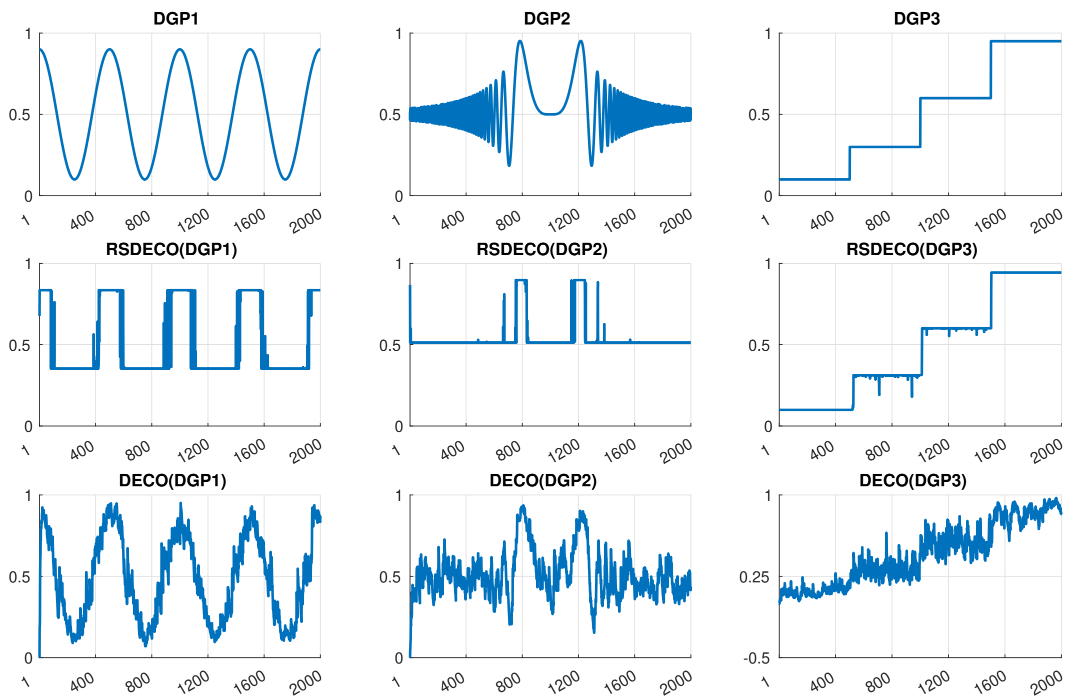
where $\eta_t$ are normally distributed random numbers. We generate five artificial equicorrelation data sets using a variety of models:

- DGP 1: $R_t^{sim} = 0.5 + 0.4\cos(2\pi t/500)$.
- DGP 2: $R_t^{sim} = 0.5 + \sin(s^4)/(1 + \sqrt{|s^4|})$, with $s = 5 - t/200$.
- DGP 3: $R_t^{sim} = \begin{cases} 0.1 \text{ if } t \in [1; 500] \\ 0.3 \text{ if } t \in [501; 1000] \\ 0.6 \text{ if } t \in [1001; 1500] \\ 0.95 \text{ if } t \in [1501; 2000] \end{cases}$
- DGP 4: a 2-Block DECO with DCC dynamics with two regimes.
  $R_t^{sim} = \begin{cases} \rho_{1,1,t} = -0.1,\ \rho_{2,2,t} = 0.3,\ \rho_{1,2,t} = -0.1 \text{ if } t \in [750, 1250] \\ \rho_{1,1,t} = 0.8,\ \rho_{2,2,t} = 0.4,\ \rho_{1,2,t} = -0.3 \text{ otherwise} \end{cases}$
- DGP 5: a 3-Block DECO with DCC dynamics with two regimes.
  $R_t^{sim} = \begin{cases} \left.\begin{array}{l} \rho_{1,1,t} = 0.9,\ \rho_{2,2,t} = 0.7,\ \rho_{3,3,t} = 0.5 \\ \rho_{1,2,t} = 0.3,\ \rho_{1,3,t} = 0.1,\ \rho_{2,3,t} = 0.3 \end{array}\right\} \text{ if } t \in [750, 1250] \\ \left.\begin{array}{l} \rho_{1,1,t} = 0.2,\ \rho_{2,2,t} = 0.2,\ \rho_{3,3,t} = 0.2 \\ \rho_{1,2,t} = 0.15,\ \rho_{1,3,t} = 0.1,\ \rho_{2,3,t} = 0.1 \end{array}\right\} \text{ otherwise} \end{cases}$

Each simulation contains $T = 2000$ observations. However, the number of series $K$ increases from $K = 50$ to $K = 400$. Using sufficiently large samples allows us to test whether our model does not suffer from the high dimensional problem. While DGP 1 and 2 are based on trigonometric functions and were chosen to reproduce smooth regime changes, DGP 3 is based on an RSDECO dynamic with four regimes. We then introduce blocks when generating DGP 4 and 5, which are respectively a 2-block DECO with DCC dynamics with two regimes and a 3-block DECO with DCC dynamics with two regimes. The correlation value for each of the DGPs 4 and 5 is chosen so that the condition in Equation (8) is satisfied, which guarantees that $R_t$ is positive definite. DGP 5 is used to compare the B-block RSDECO and the B-block DECO in their *full* and *reduced* forms.

Our RSDECO model is compared with the DECO model proposed by Engle and Kelly (2012). Two different error measures are used for this purpose, namely the Mean Squared Error (MSE) and the Mean Absolute Deviation (MAD). [3] Figure 1 illustrates simulated and estimated correlations using the DECO and RSDECO models. Comparing the two model estimates suggests that both models appear to perform well in reproducing the true correlation, with a natural relative advantage for the RSDECO model in capturing regime switching. The overall result of the error measure is satisfactory. As shown in Table 2, both MAE and MAD are quite weak, indicating that the estimated correlations from either RSDECO or DECO are very close to the true values. The MAD statistic tends to favour RSDECO when the DPG is either DGP 2 or DGP 3, regardless of the number of series. However, the results for the MAE are mixed. In conclusion, the error measure statistics do not provide conclusive evidence in favour of one model over the other but show that the RSDECO model tends to perform better when the data-generating process is subject to structural breaks as well as switching regimes. It is worth noting that the same qualitative result is obtained for a different number of series, suggesting that both RSDECO and DECO deal well with the high dimensional problem.

Figure 1: Simulated results for DGP 1-DGP 3




[3] $MAE = \frac{1}{T}\sum_{t=1}^{T}(\hat{R}_t - R_t)^2$ and $MAD = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{R}_t - R_t\right|$.

Figure 2: Simulated results for DGP 4

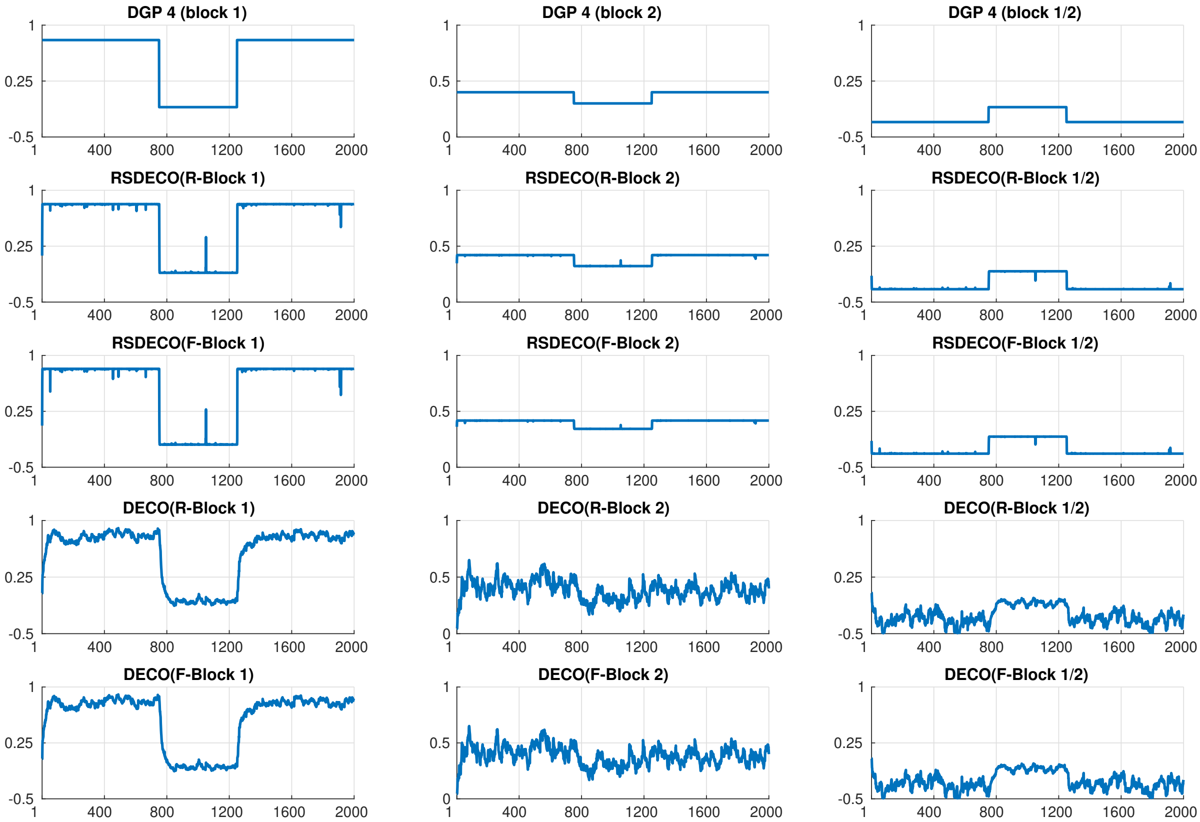
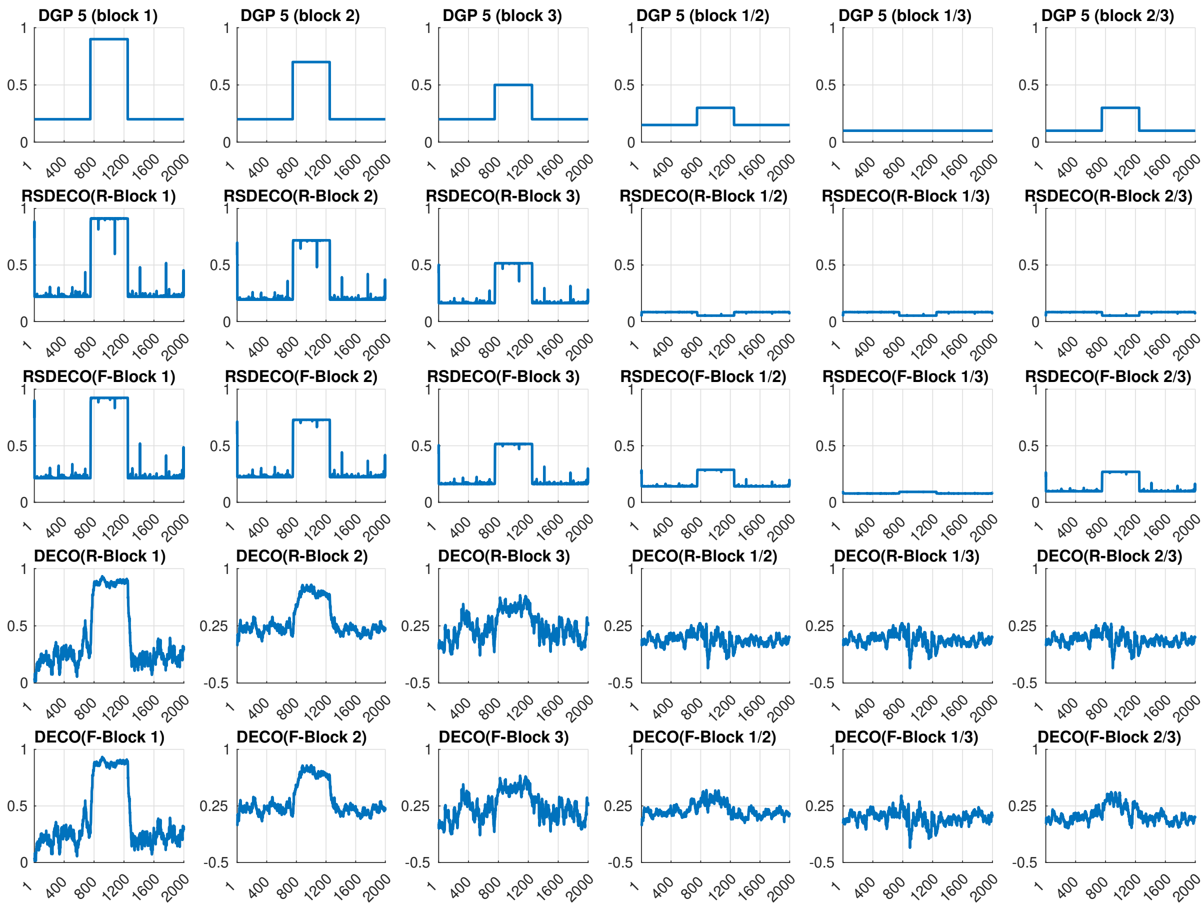



Table 2: Error measures of correlation estimates $\underset{(MAD)}{MSE}$

| | DGP 1 | DGP 2 | DGP 3 | DGP 4 | DGP 5 |
|---|---|---|---|---|---|
| 50 series: | | | | | |
| RSDECO | 0.023 (0.124) | 0.033 (0.073) | 0.034 (0.009) | 0.045 (0.055) | 0.045 (0.055) |
| DECO | 0.006 (0.060) | 0.018 (0.087) | 0.023 (0.054) | 0.028 (0.044) | 0.028 (0.044) |
| 100 series: | | | | | |
| RSDECO | 0.021 (0.119) | 0.032 (0.076) | 0.032 (0.005) | 0.042 (0.051) | 0.042 (0.051) |
| DECO | 0.006 (0.061) | 0.017 (0.087) | 0.022 (0.050) | 0.028 (0.045) | 0.028 (0.045) |
| 200 series: | | | | | |
| RSDECO | 0.024 (0.125) | 0.034 (0.075) | 0.034 (0.003) | 0.046 (0.056) | 0.046 (0.056) |
| DECO | 0.005 (0.057) | 0.015 (0.078) | 0.020 (0.052) | 0.025 (0.043) | 0.025 (0.043) |
| 400 series: | | | | | |
| RSDECO | 0.023 (0.124) | 0.032 (0.071) | 0.033 (0.002) | 0.044 (0.051) | 0.044 (0.051) |
| DECO | 0.018 (0.117) | 0.028 (0.080) | 0.036 (0.080) | 0.042 (0.044) | 0.042 (0.044) |

MAE: Mean Absolute Error. In brackets, MAD: Mean Absolute Deviation

Figure 3: Simulated results for DGP 5




# 4 Empirical application

We use the three specifications of the RSDECO model to estimate conditional correlations between commodity futures, stock indices and bonds. The advantage of using the RSDECO, and especially its Block versions, to deal with this issue is not only to detect structural changes in correlation dynamics but also to be able to provide an accurate measure of both intra and inter-commodity, stock and bond-group correlations. The common strategy in previous papers in measuring intra and inter-commodity group correlations is respectively to either calculate the average of all pairwise correlations or to use indices. In the case of the RSDECO, inter and intra-commodity group correlations are estimated measures.

## 4.1 Data

The data set is made up of 24 US daily commodity futures, 4 US equity indices (The MSCI, the Standard & Poors 500, the NASDAQ and the RUSSELL 200), and the one-year, five-year and ten-year US holding bond returns[4] covering the period from January 4th, 2000 to December 29th, 2022. The commodity futures contracts belong mainly to the Goldman Sachs Commodity Index (GSCI), which is the most popular commodity index. Our data originated from Bloomberg which constructs and provides continuous price series from futures data. Commodities can be classified into 5 groups, namely Energy, Industrial Metals, Precious Metals, Agriculture and Livestock. The full list of commodity futures series with the group classification can be found in Appendix A. It updates the appendix in Charlot et al. (2016), who studied 32 commodity futures between 2000 and 2015. All series are then transformed into first logarithmic differences.

Tables 4, 5 and 6 report summary statistics respectively for the commodity futures returns, the equity index returns and the bond returns. We take futures returns (changes in log prices multiplied by 100) and collateralize them with the daily 3-month US Treasury Bill (T-bill) secondary market rate. Figure 14 in Appendix A shows all data series returns used for this exercise.

## 4.2 Empirical Results

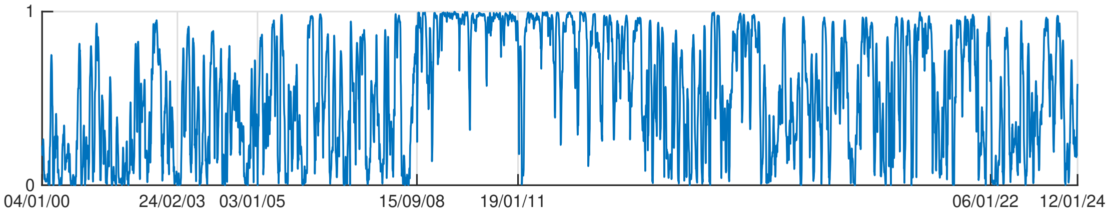
We separate our data into three blocks corresponding to commodities, stocks and bonds and estimate Markov-switching models with two regimes. Figure 4 displays the smoothed probabilities for the simple RSDECO (sub-figure 4a), the Reduced Block RSDECO (sub-figure 4b) and the Full Block RSDECO (sub-figure 4c). All of these figures show the same changes in regimes. The first major change happens towards September 15, 2008 when we can observe a swift range in regime. It is noteworthy that, before the financial crisis regime, smoothed probabilities show a slight increase, confirming the view that commodity financialisation began impacting the correlation between commodity and financial assets

[4]See in appendix C for the computation of these returns.

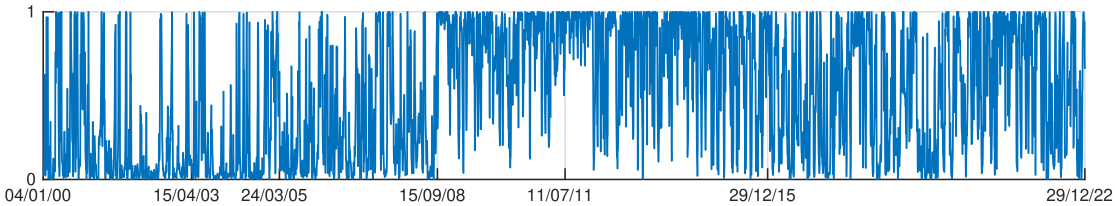
starting from 2004 (Tang and Xiong, 2012; Charlot et al., 2016, among others).[5] Finally, at the end of the sample, we observe the beginning of a return to the first regime.

Figure 4: Smoothed Probabilities.

(a) RSDECO




(b) 3-Blocks Reduced Form RSDECO




(c) (b) 3-Blocks Full Block RSDECO

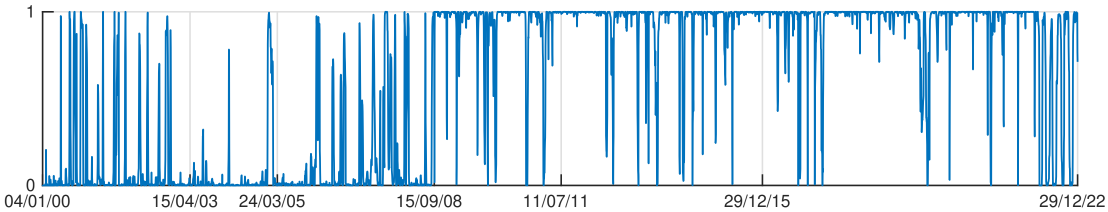
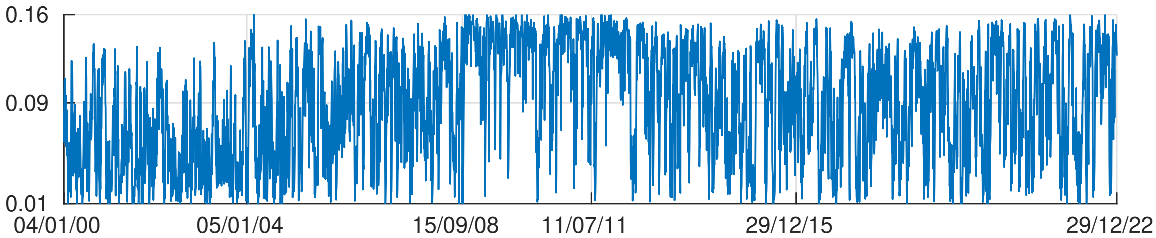



Figure 5: Correlations for simple RSDECO.




Figures 5, 6 and 13 display corresponding estimated equicorrelations for the simple RSDECO, the Reduced Block RSDECO and the Full Block RSDECO models, respectively.

We focus on equicorrelations given by the Full Block RSDECO. Interblock equicorrelations between the three asset classes changed sign after the financial crisis in 2008.

[5]This is also in line with theoretical findings of Basak and Pavlova (2016) that all commodity and commodity-equity correlations rise with financialization.

The equity-bond and the commodity-bond correlations which were negligible before 2008 become negative after this date. We observe the opposite pattern for the commodity-equity correlation which becomes more positive after 2008. It is worth noting that all intra-correlations increase after 2008 but with varying magnitudes reflecting heterogeneity across blocks. During the low correlation regime, intra-correlations for different blocks range on average between 5% and 20%. During the financial crisis regime, while commodity intra-correlation rises to only about 15%, equity and bond intra-correlations reach about 70% and 40%, respectively. Both intra and commodity-equity estimated correlations are consistent with the "loss spiral" view (Brunnermeier and Pedersen (2009); Kyle and Xiong (2001)) that when the market collapses, investors, especially if they are leveraged, sell assets to raise liquidity, causing unrelated asset returns to fall, increasing correlation between them.

As for commodity and equity-bond inter-correlations, results reflect the "flight to quality" phenomena. Vayanos (2004) has shown that when risk-averse investors experience a negative wealth effect in times of financial crisis, it reduces their risk-bearing capacity and leads them to sell risky assets to buy risk-free assets such as government bonds. During the financial crisis regime correlations between commodity and bond returns dropped from about 0% to about -30%, and that between equity and bond returns from 0% to about -40%.

The literature on the relationship between oil prices and stock prices can also help to understand the variations in the sign of the equity-commodity correlation. Killian and Park. (2009) shows that the impact of an oil price shock on stock prices depends on the origin of this shock. Before 2008, an increase in commodity prices could be perceived as an increase in expected inflation. Alquist et al. (2020) explain the positive equity-commodity correlation after 2008 by a reduction in the inflation premium. In a deflationary context, an increase in commodity prices is less prone to raise inflation expectations. We can observe that regime changes become more frequent after 2020. This could be related to rising uncertainty in the financial market in the context of higher inflation and geopolitical uncertainty.

Figure 6: Correlations: 3-Block-RSDECO (Reduced Form).

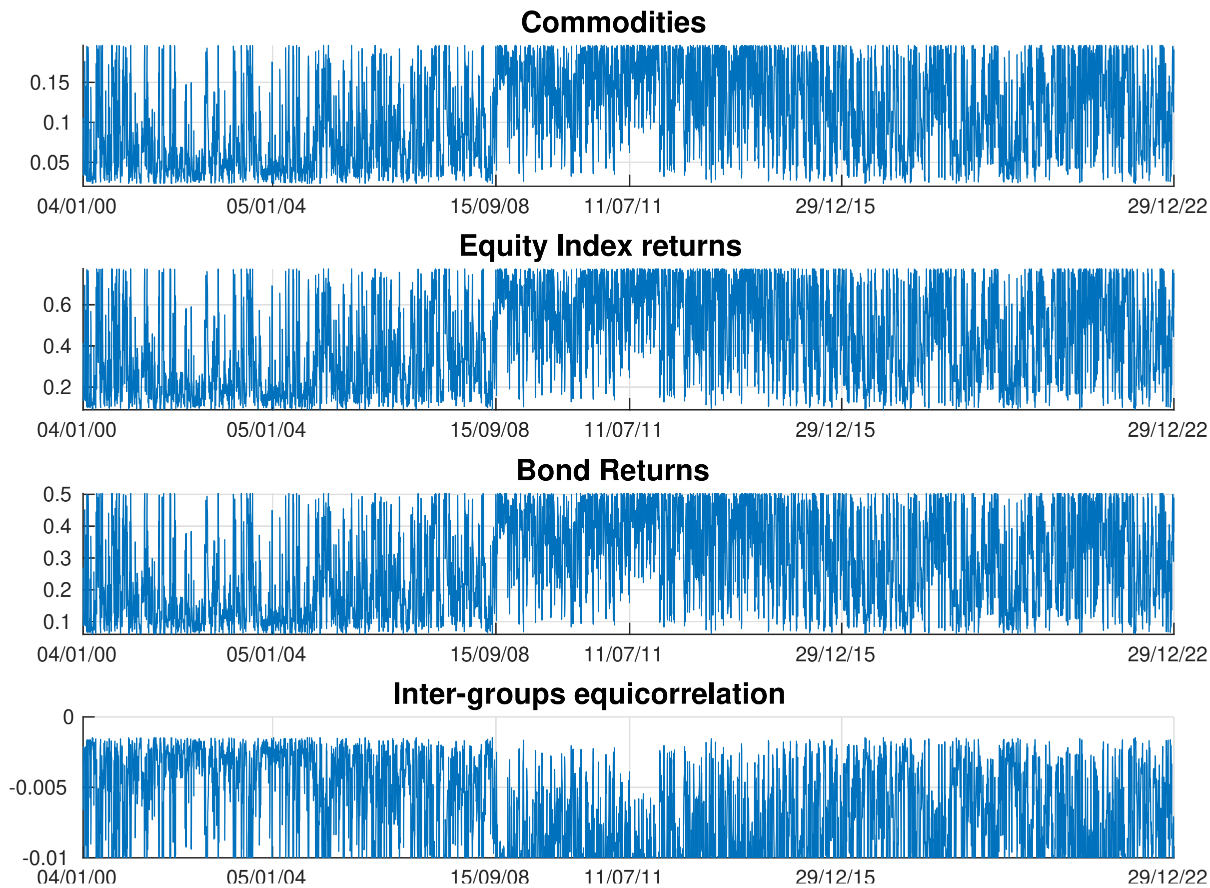


Figure 7: Correlations: 3-Block-RSDECO (Full Block).

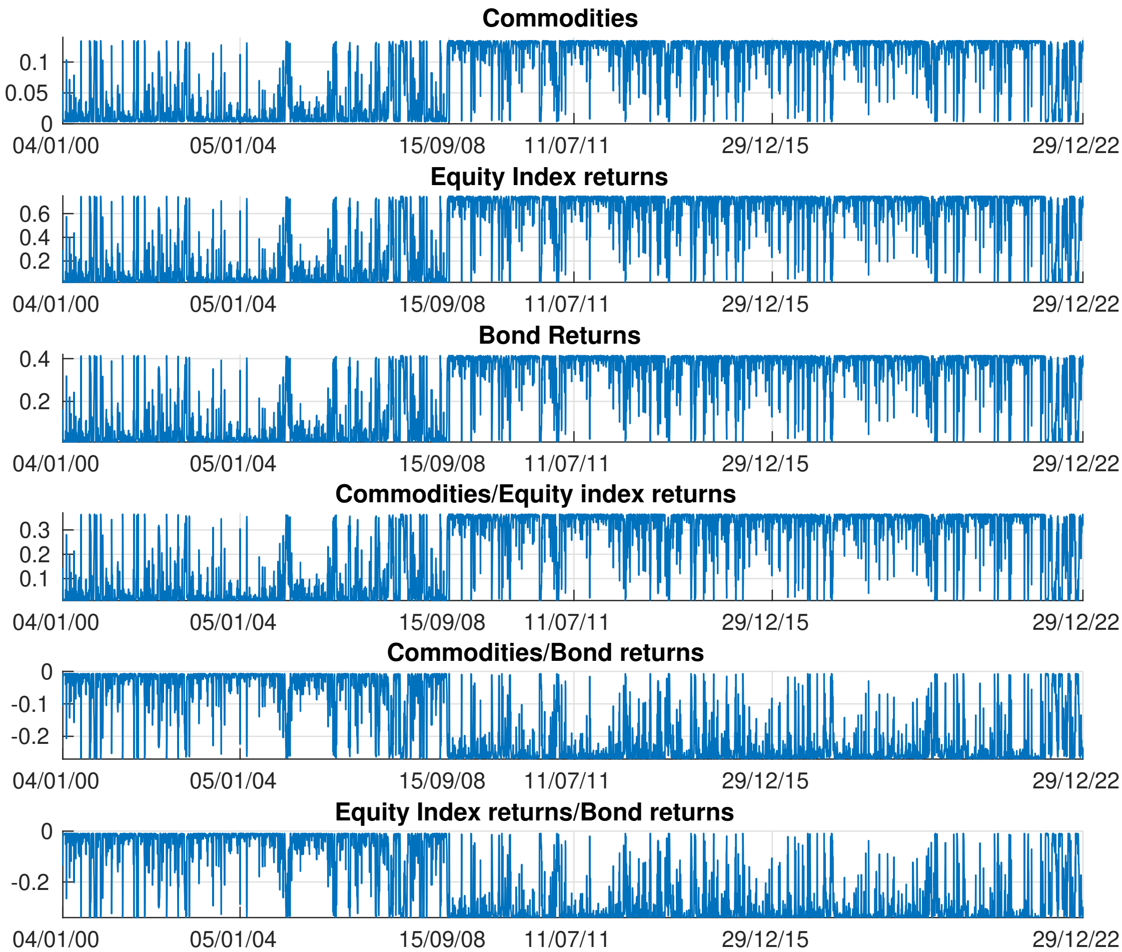


Figure 8: Correlations: 7-Block-RSDECO (Full Block).

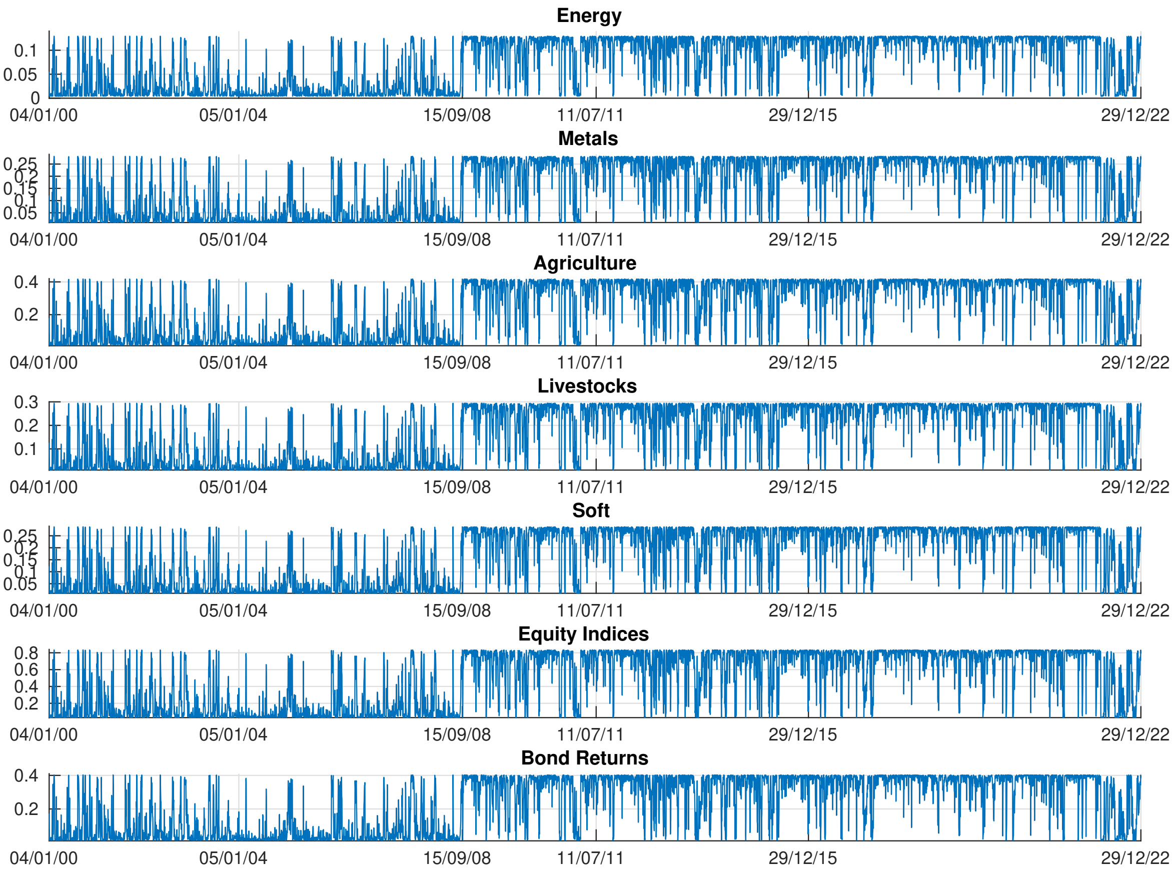


Figure 9: Correlations: 7-Block-RSDECO (Full Block).

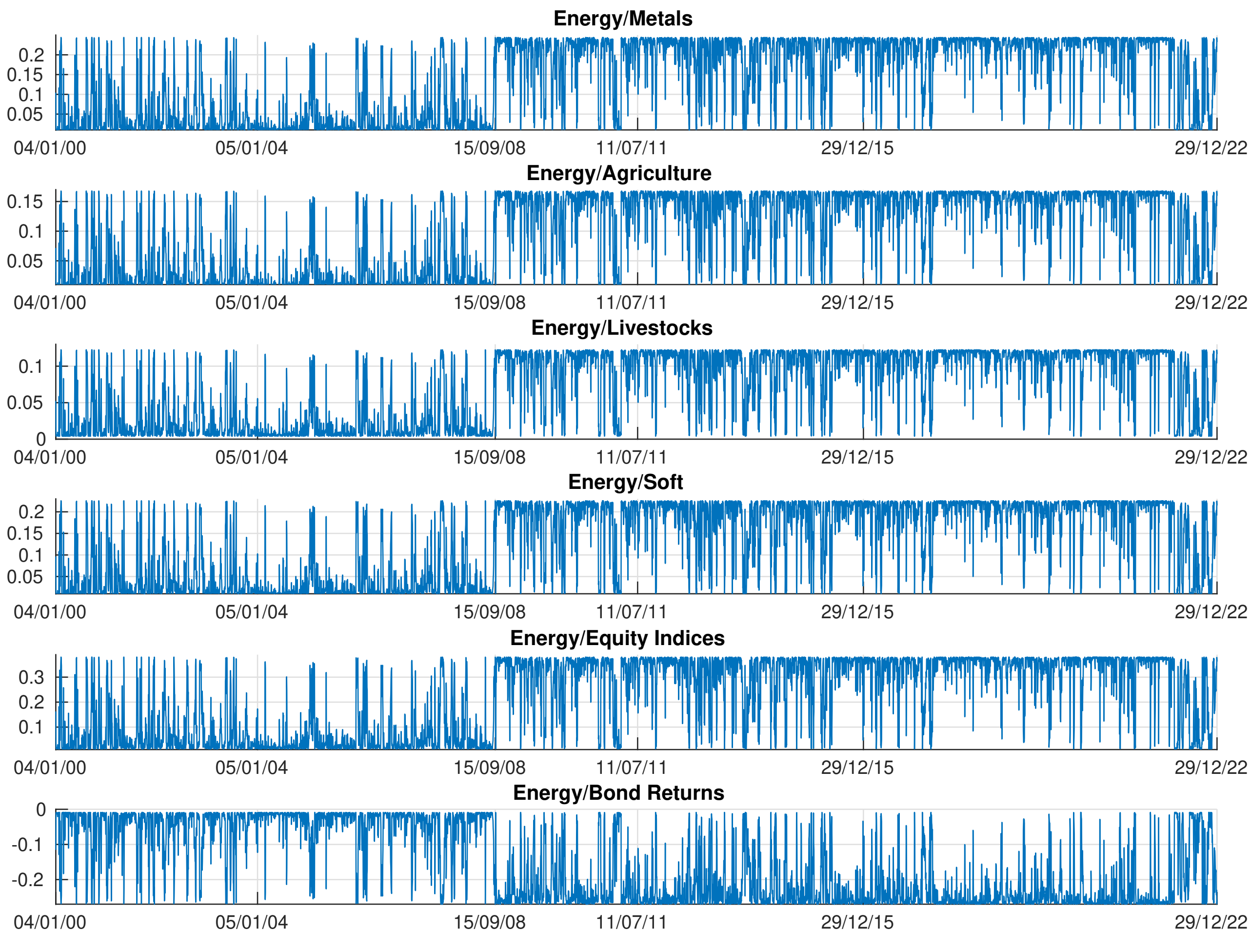


Figure 10: Correlations: 7-Block-RSDECO (Full Block).

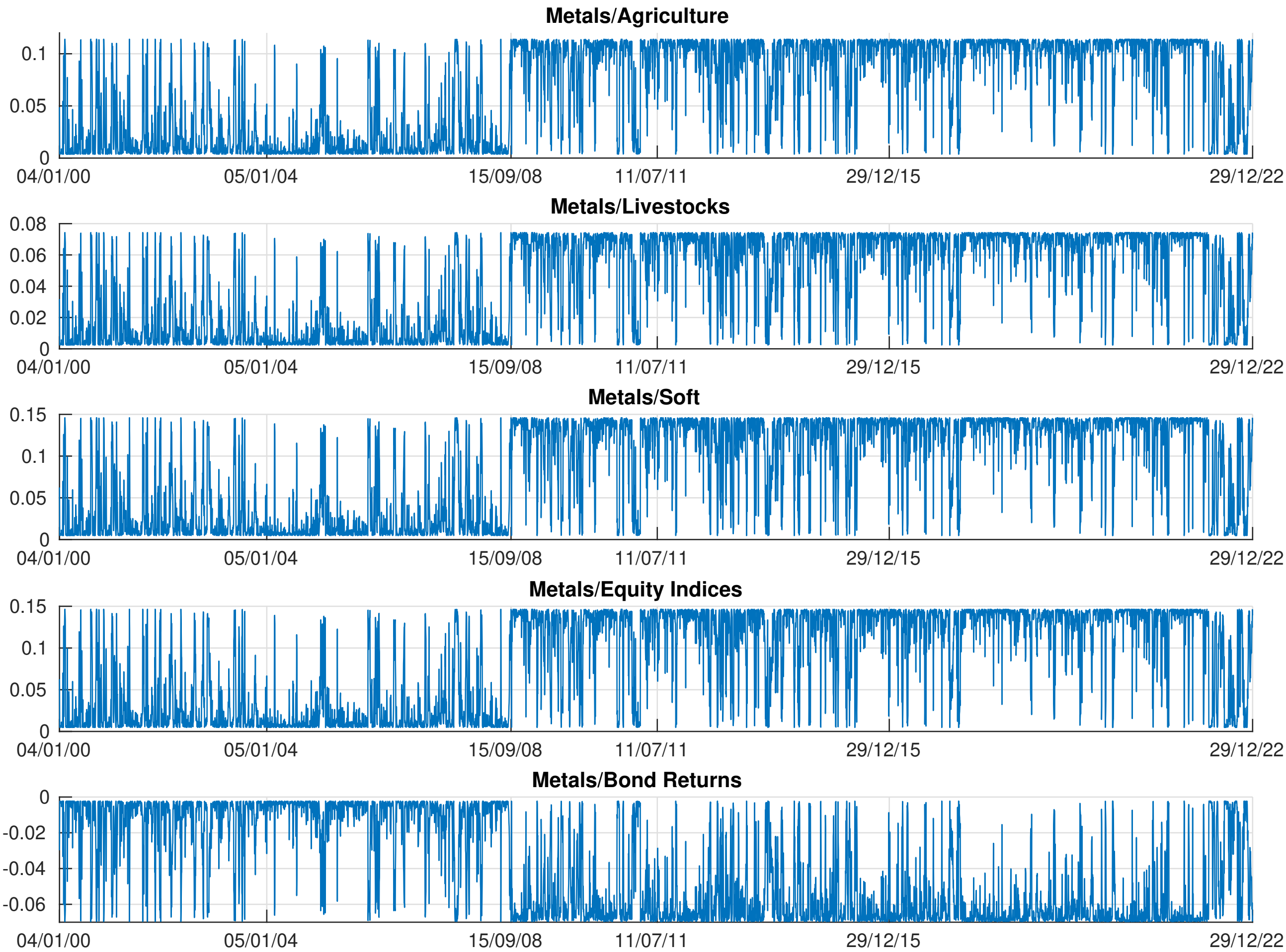


Figure 11: Correlations: 7-Block-RSDECO (Full Block).

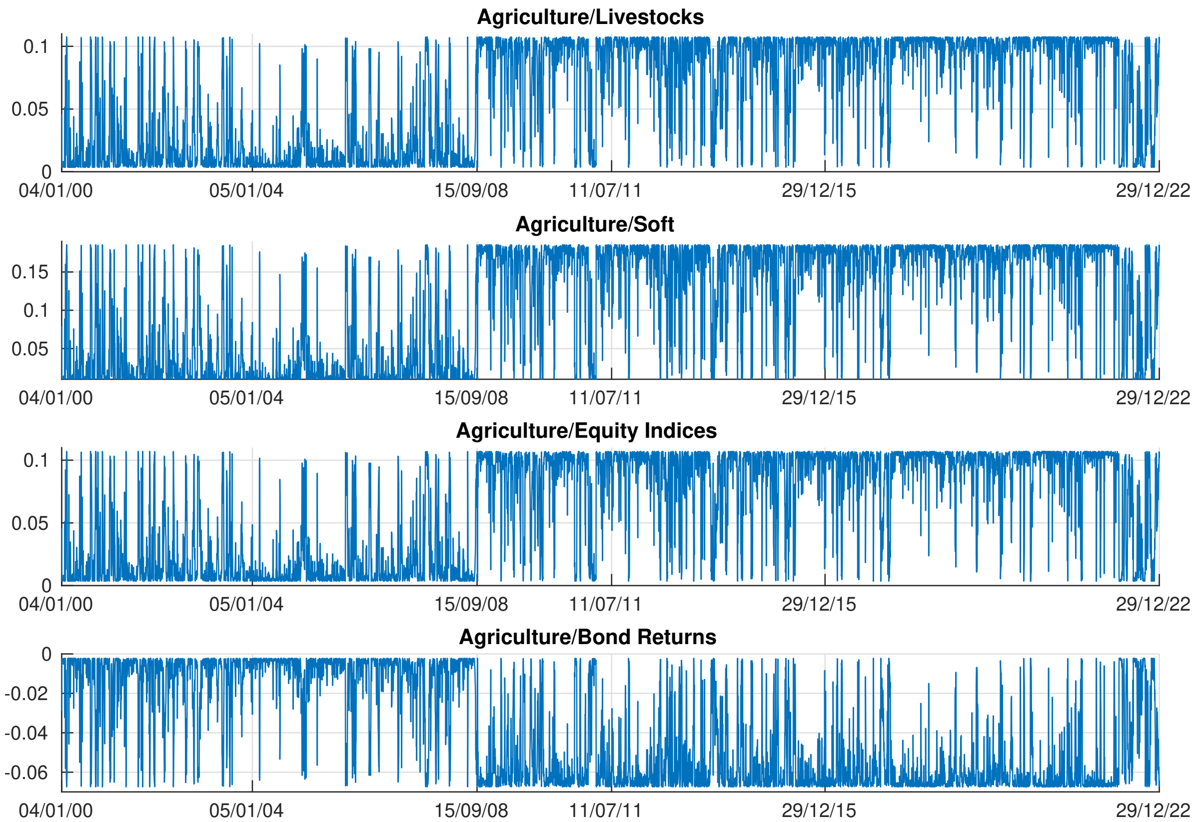


Figure 12: Correlations: 7-Block-RSDECO (Full Block).

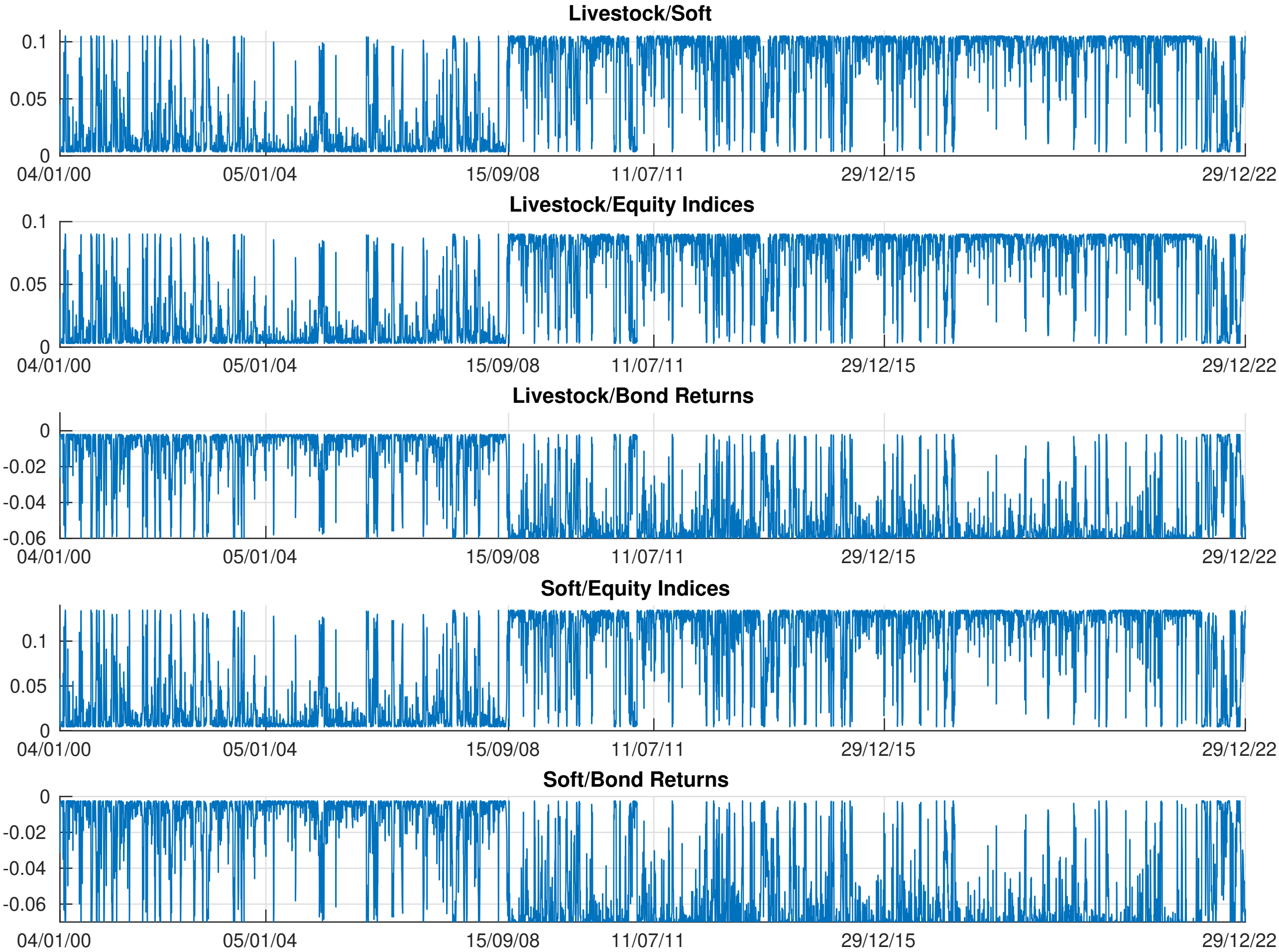
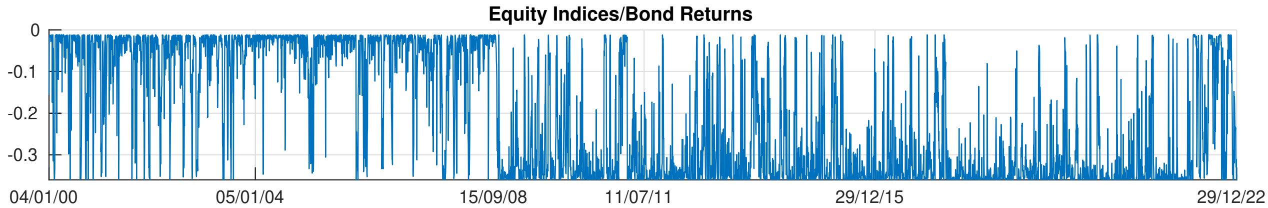



Figure 13: Correlations: 7-Block-RSDECO (Full Block).



# 5 Conclusion

In this paper we have extended the Engle and Kelly (2012) DECO model with a Markov switching model. Our model is therefore able to estimate conditional correlations for a large number of assets, while taking into account level shifts of these correlations. From an econometric point of view, our model could be extended in several directions. A first possible extension would be to consider independent regime-switching for each block. This would allow us to uncover different degrees of persistence between regimes. Another possible extension would be to consider time-varying transition probabilities as in Diebold et al. (1994) and Filardo (1994). The literature on stock-bond and stock-commodity correlations has shown the importance of macroeconomic or financial variables in shaping these correlations.

Given the practical interest of our RS-DECO, it would be interesting to evaluate its performance in financial applications such as portfolio selection or risk management. Recently, Moura and Ruiz (2020) compared the performance of several DCC-type models and stochastic volatility-type models to select mean-variance and minimum-variance portfolios with up to 1000 assets. The authors confirm previous results of Engle et al. (2019) and De Nard et al. (2021), who find that DCC-type models give satisfactory results when the objective is to select the minimum variance portfolio. It would be interesting to compare the performance of the RS-DECO with the results obtained in these works.

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# Appendix

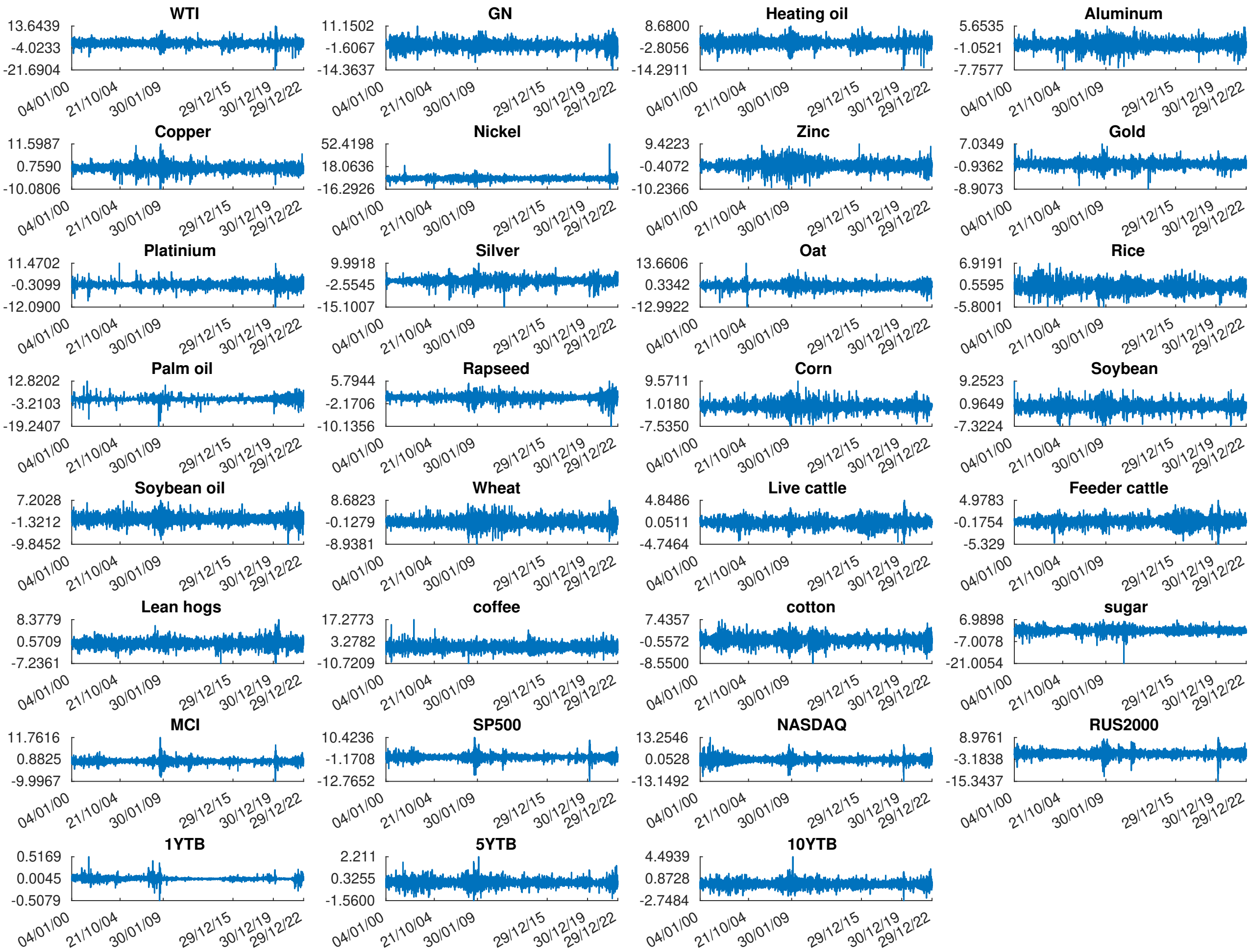
# A Dataset

Table 3: Commodity futures data

| Commodities | Exchange | Bloomberg Ticker | Contracts | GSCI (2023 RPDW) |
|---|---|---|---|---|
| **Energy** | | | | 61.46% |
| WTI crude oil | NYM | CL1-CL15 | Every month | 21.82% |
| Heating oil | NYM | HO1-HO15 | Every month | 4.62% |
| Natural gas | NYM | NG1-NG15 | Every month | 4.71% |
| **Industrial Metals** | | | | 10.57% |
| Aluminum | LME | LMAHDS (03 and 15) | Every month | 3.8% |
| Copper | LME | LMCADS (03 and 15) | Every month | 4.34% |
| Nickel | LME | LMNIDS (03 and 15) | Every month | 0.98% |
| Zinc | LME | LMZSDS (03 and 15) | Every month | 0.95% |
| **Precious Metals** | | | | 4.12% |
| Gold | CMX | GC1-GC12 | Mar, May, Jul, Sep, Dec | 3.73% |
| Silver | CMX | SI1-SI13 | Mar, May, Jul, Sep, Dec | 0.38% |
| Platinum | NYM | PL1-PL5 | Jan, Apr, Jul, Oct | 0 |
| **Agriculture** | | | | 17.97% |
| **1- Grain** | | | | 14.08% |
| Corn | CBT | C1-C11 | Jan, Mar, May, Jul, Aug, Nov | 5.65% |
| Soybeans | CBT | S1-S7 | Jan, Mar, May, Jul, Aug, Sep, Oct, Dec | 3.60% |
| Rice | CBT | RR1-RR7 | Mar, May, Jul, Sep, Nov, Jan | 0 |
| Oats | CME | O1-O5 | Mar, May, Jul, Sep, Dec | 0 |
| Soybean oil | CBT | BO1-BO10 | Jan, Mar, May, Jul, Aug, Sep, Oct, Dec | 0 |
| Palm oil | MDE | KO1-KO10 | Jan, Mar, May, Jul, Aug, Sep, Nov | 0 |
| Rapseed | MDE | IJ1-IJ7 | Mar, May, Jul, Sep, Dec | 0 |
| Wheat | CBT | W 1-W 10 | Mar, May, Jul, Sep, Dec | 3.30% |
| **2- Soft** | | | | 3.89% |
| Sugar | ICE | SB1-SB8 | Mar, May, Jul, Oct | 1.45% |
| Coffee | ICE | KC1-KC8 | Mar, May, Jul, Sep, Dec | 0.93% |
| Cotton | ICE | CT1-CT10 | Mar, May, Jul, Oct, Dec | 1.26% |
| **Livestock** | | | | 5.85% |
| Feeder cattle | CME | FC1-FC8 | Jan, Mar, Apr, May, Aug, Sep, Oct | 1.04% |
| Live cattle | CME | LC1-LC7 | Feb, Apr, Jun, Aug, Oct, Dec | 2.98% |
| Lean hogs | CME | LH1-LH9 | Feb, Apr, Jun, Aug, Oct, Dec | 1.82% |

This table gives details of the 21 commodity futures contracts used in our study to calculate the daily log returns from January 04, 2000 to December 29, 2022. Commodity futures contracts are generic contracts which refer to as constructing continuous futures prices series from futures active contract data. For instance, CL1 is the WTI Generic 1st contract, based on a 1-month WTI contract. Exchange-traded commodity futures markets are given in the second column. The third and fourth column contain maturity months of futures contracts and their Bloomberg tickers and the last four columns report the market value weights of each individual commodity within the major commodity indices, principally Standard and Poor's Goldman Sachs Commodity Index (GSCI) and Dow Jones UBS Commodity Index (DJ-UBS)as well as Rogers International Commodity index (RICI) and Thomson Reuters/Jefferis CRB Index (CRB), as of 2023. As commodity futures expires every one to three months, indices need to specify a rolling rule to transfer weights of the futures from the expiring period's contract to the next available contract. The rollover schedules of the GSCI, DJ-UBS, RJ/CRB indices are business days 5-9, 6-10 and 1-4 respectively. For the RICI index, it runs from the day prior to the last RICI business day of the month to the first RICI business day of the following month (for more details see RICI handbook). Industrial metals are traded on LME and daily settlement prices are quoted for the futures contracts to a fixed maturity period of 3- and 15-months.

Figure 14: Commodity and financial data series returns



# B Descriptive statistics

Table 4: Descriptive statistics: commodity futures returns.

| | Min | Max | Mean | S. dev. | Med. | Kurtosis | Skewness | JB test * | ADF test** |
|---|---|---|---|---|---|---|---|---|---|
| WTI | -21.6904 | 13.6439 | 2.2219e-02 | 1.9904 | 0.1192 | 10.8189 | -0.6670 | 15076.0992 (1.0000e−03) | 1.0000 (1.0000e−03) |
| GN | -14.3637 | 11.1502 | 1.0279e-02 | 2.0672 | 0.0239 | 5.5543 | -0.1375 | 1581.5100 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Heating oil | -14.2911 | 8.6800 | 2.8039e-02 | 1.8059 | 0.0762 | 6.4589 | -0.3419 | 2978.9720 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Aluminum | -7.7577 | 5.6535 | 6.9598e-03 | 1.2440 | 0.0000 | 5.4992 | -0.2089 | 1538.5355 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Copper | -10.0806 | 11.5987 | 2.5801e-02 | 1.5555 | 0.0000 | 7.7011 | -0.0897 | 5303.5272 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Nickel | -16.2926 | 52.4198 | 2.3403e-02 | 2.3308 | 0.0000 | 51.1747 | 1.9009 | 559585.1770 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Zinc | -10.2366 | 9.4223 | 1.5618e-02 | 1.7123 | 0.0000 | 6.0051 | -0.1494 | 2185.3337 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Gold | -8.9073 | 7.0349 | 3.2415e-02 | 1.0070 | 0.0348 | 8.4228 | -0.3262 | 7148.6942 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Platinium | -12.0900 | 11.4702 | 1.6939e-02 | 1.3857 | 0.0701 | 9.2824 | -0.2116 | 9500.6308 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Silver | -15.1007 | 9.9918 | 2.6521e-02 | 1.5414 | 0.0644 | 9.8036 | -0.7569 | 11641.1972 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Oat | -12.9922 | 13.6605 | 1.9385e-02 | 1.5386 | 0.0228 | 8.1289 | -0.1908 | 6338.3436 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Rice | -5.8001 | 6.9191 | 1.8876e-02 | 1.2003 | 0.0000 | 5.4020 | 0.1510 | 1404.3388 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Palm oil | -19.2407 | 12.8202 | 2.1976e-02 | 1.3035 | 0.0000 | 26.6896 | -1.0372 | 135508.3855 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Rapseed | -10.1356 | 5.7944 | 2.0671e-02 | 0.9123 | 0.0350 | 10.5672 | -0.6777 | 14161.7165 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Corn | -7.5350 | 9.5711 | 1.7713e-02 | 1.3866 | 0.0000 | 6.2335 | 0.0052 | 2505.4609 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Soybean | -7.3224 | 9.2523 | 2.1165e-02 | 1.3675 | 0.0523 | 6.2740 | -0.2372 | 2622.4492 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Soybean oil | -9.8452 | 7.2028 | 2.3065e-02 | 1.4068 | -0.0083 | 5.7662 | -0.0685 | 1838.0989 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Wheat | -8.9381 | 8.6823 | 1.8101e-02 | 1.5343 | -0.0300 | 5.5158 | 0.0888 | 1524.2499 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Live cattle | -4.7464 | 4.8486 | 1.4073e-02 | 0.7768 | 0.0292 | 6.4936 | -0.3403 | 3035.6416 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Feeder cattle | -5.3290 | 4.9783 | 1.4350e-02 | 0.8436 | 0.0267 | 6.2842 | -0.2493 | 2644.2221 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Lean hogs | -7.2361 | 8.3779 | 8.9693e-03 | 1.1676 | 0.0555 | 6.4239 | -0.1792 | 2839.9350 (1.0000e−03) | 1.0000 (1.0000e−03) |
| coffee | -10.7209 | 17.2773 | 5.2532e-03 | 1.9275 | 0.0000 | 6.4981 | 0.3192 | 3029.9027 (1.0000e−03) | 1.0000 (1.0000e−03) |
| cotton | -8.5500 | 7.4357 | 6.7928e-03 | 1.3701 | 0.0299 | 5.7047 | -0.1237 | 1767.5880 (1.0000e−03) | 1.0000 (1.0000e−03) |
| sugar | -21.0054 | 6.9898 | 1.8428e-02 | 1.4775 | 0.0000 | 12.6677 | -0.7362 | 22915.8633 (1.0000e−03) | 1.0000 (1.0000e−03) |
| Engle DCC test **[a] | (195.6000) (0.0000e+00) | | | | | | | | |

* In brackets, critical values for the tests. ** Augmented Dickey-Fuller test.

[a]* In brackets, p-values for the tests. ** With 5 lags.

This table reports summary statistics for the 21 daily collateralized commodity futures returns from January 04, 2000 to January 15, 2024. We take futures returns (changes in log prices multiplied by 100) and collateralize them with the daily 3-month US Treasury Bill (T-bill) secondary market rate. Details about commodity futures contracts and sources are provided in Appendix A.

Table 5: Descriptive statistics: equity index returns.

| | Min | Max | Mean | S. dev. | Med. | Kurtosis | Skewness | JB test * | ADF test** |
|---|---|---|---|---|---|---|---|---|---|
| MSCI | -9.9967 | 11.7616 | 1.0430e-02 | 1.0439 | 0.0595 | 14.8696 | -0.4924 | 33992.3383 (1.0000$e-$03) | 1.0000 (1.0000$e-$03) |
| SP500 | -12.7652 | 10.4236 | 1.7550e-02 | 1.2558 | 0.0570 | 13.1231 | -0.4093 | 24716.6558 (1.0000$e-$03) | 1.0000 (1.0000$e-$03) |
| NASDAQ | -13.1492 | 13.2546 | 1.7158e-02 | 1.6036 | 0.0881 | 8.8796 | -0.1725 | 8312.1643 (1.0000$e-$03) | 1.0000 (1.0000$e-$03) |
| RUS2000 | -15.3437 | 8.9761 | 2.2663e-02 | 1.5665 | 0.0820 | 9.6830 | -0.5389 | 10980.4489 (1.0000$e-$03) | 1.0000 (1.0000$e-$03) |
| Engle DCC test **[a] | (542.7586) (0.0000$e$+00) | | | | | | | | |

* In brackets, critical values for the tests. ** Augmented Dickey-Fuller test.

[a]* In brackets, p-values for the tests. ** With 5 lags.

This table reports summary statistics for four equity indices belonging the financial asset block. Details about equity indices and sources are provided in Appendix A.

Table 6: Descriptive statistics: bond returns.

| | Min | Max | Mean | S. dev. | Med. | Kurtosis | Skewness | JB test * | ADF test** |
|---|---|---|---|---|---|---|---|---|---|
| 1YTB | -0.5079 | 0.5169 | 7.3040e-03 | 0.0393 | 0.0030 | 24.9285 | 0.4585 | 115427.4471 (1.0000$e-$03) | 1.0000 (1.0000$e-$03) |
| 5YTB | -1.5600 | 2.2110 | 1.2477e-02 | 0.2770 | 0.0074 | 6.4973 | 0.0362 | 2932.1111 (1.0000$e-$03) | 1.0000 (1.0000$e-$03) |
| 10YTB | -2.7483 | 4.4939 | 1.7047e-02 | 0.4885 | 0.0133 | 5.9106 | 0.0435 | 2031.8698 (1.0000$e-$03) | 1.0000 (1.0000$e-$03) |

* In brackets, critical values for the tests. ** Augmented Dickey-Fuller test.

This table reports summary statistics for three bond returns belonging the financial asset block. Details about data and sources are provided in Appendix A.

# C Computation of holding bond returns

We expose the method of Jones et al. (1998) to calculate the bond returns. We use the "daily constant maturity interest rate series" from the Federal Reserve Bank of St. Louis FRED economic data. The U.S. Treasury bonds have seminannual coupon payments and the coupon on the hypothetical bonds is half the stated coupon yield. Then the price of the bond at the beginning of the holding period is equal to its face value. We calculate an end-of-period price on this bond using the next day's yield augmented with the accrued interest rate:

$$P_{n-\#hd,t+1} = \sum_{i=1}^{2n-1} \frac{\frac{1}{2}y_{nt}}{\left(1+\frac{1}{2}y_{n,t+1}\right)^i + \left(1+\frac{1}{2}y_{n,t+1}\right)^{2n}} \frac{1+\frac{1}{2}y_{nt}}{} + \frac{\#\text{holding days}}{365} y_{nt}$$

where $P_{n-\#hd,t+1}$ is the end-of-period price of the bond, $n$ is the number of years the bond is referring to, $t$ is the time, and $y_{nt}$ is the yield of an $n$-period bond at time $t$. The $\#hd$-return is calculated as;

$$r_{t+1} = P_{n-\#hd,t+1} - 1$$

The excess returns are calculated using the three-month interest rate as the risk-free rate that accrues over the holding period,which varies between one and five days because of weekends and holidays:

$$r^e_{t+1} = r_{t+1} - \frac{\#\text{holding days}}{365} y_{3mo,t}$$

The stock and commodity indices are obtained from datastream. The returns on these indices are calculated as:

$$r_{index,t+1} = \frac{P_{index,t+1} - P_{index,t}}{P_{index,t}}$$

The risk-free rate that accrues over the holding period is substracted to obtain the excess returns:

$$r^e_{index,t+1} = r_{index,t+1} - -\frac{\#\text{holding days}}{365} y_{3mo,t}$$