Homework 3

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October 2023

1.1

- $$\frac{\delta \mathbf{W}(\mathbf{x};\mathbf{p})}{\delta \mathbf{p}} = \begin{bmatrix} \frac{\delta \mathbf{W}_x(\mathbf{x};\mathbf{p})}{\delta \mathbf{p}_1} & \dots & \frac{\delta \mathbf{W}_x(\mathbf{x};\mathbf{p})}{\delta \mathbf{p}_n} \\ \frac{\delta \mathbf{W}_y(\mathbf{x};\mathbf{p})}{\delta \mathbf{p}_1} & \dots & \frac{\delta \mathbf{W}_y(\mathbf{x};\mathbf{p})}{\delta \mathbf{p}_n} \end{bmatrix}$$

- $$\underset{\Delta \mathbf{p}}{argmin} \quad \|\mathbf{I}_{t+1}(\mathbf{x}' + \Delta \mathbf{p}) - \mathbf{I}_t(\mathbf{x})\|_2^2$$
  $$\underset{\Delta \mathbf{p}}{argmin} \quad \|\mathbf{I}_{t+1}(\mathbf{x}') + \nabla \mathbf{I}_{t+1}\frac{\delta \mathbf{W}}{\delta \mathbf{p}}\Delta \mathbf{p} - \mathbf{I}_t(\mathbf{x})\|_2^2$$
  $$\underset{\Delta \mathbf{p}}{argmin} \quad \|\nabla \mathbf{I}_{t+1}\frac{\delta \mathbf{W}}{\delta \mathbf{p}}\Delta \mathbf{p} + (\mathbf{I}_{t+1}(\mathbf{x}') - \mathbf{I}_t(\mathbf{x}))\|_2^2$$
  $$\mathbf{A} = \nabla \mathbf{I}_{t+1}\frac{\delta \mathbf{W}}{\delta \mathbf{p}} \qquad \mathbf{b} = (\mathbf{I}_{t+1}(\mathbf{x}') - \mathbf{I}_t(\mathbf{x}))$$

- $\mathbf{A}^T\mathbf{A}$ must be full rank for a unique solution to $\Delta \mathbf{p}$ to exist.

1.2 & 1.3

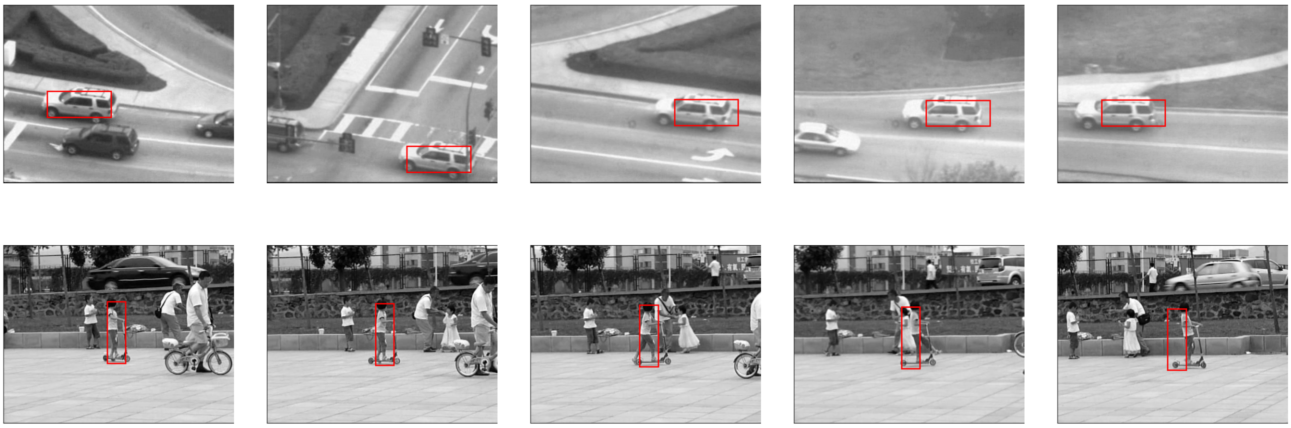

Table 1: No Correction Tracking

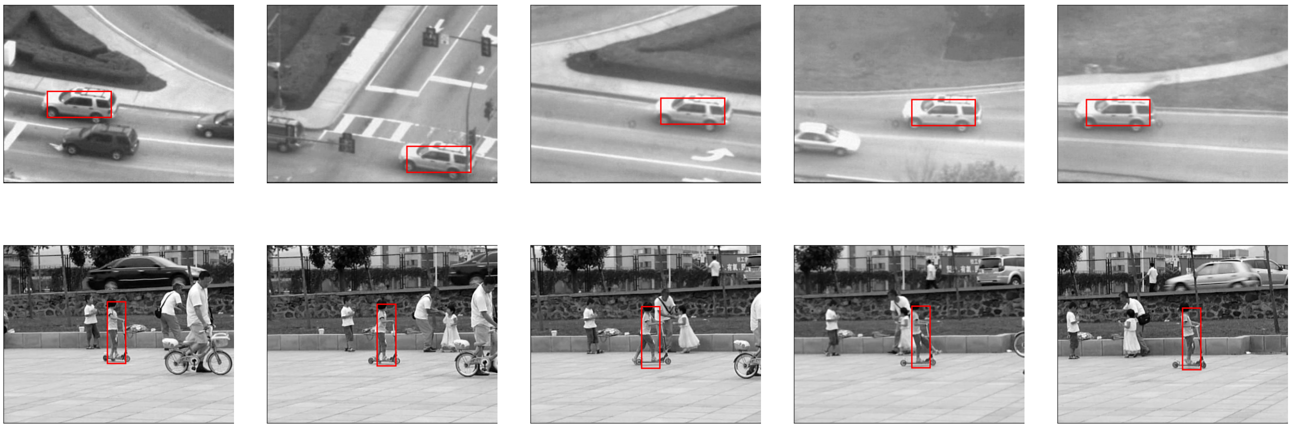

Table 2: Template Correction Tracking

2

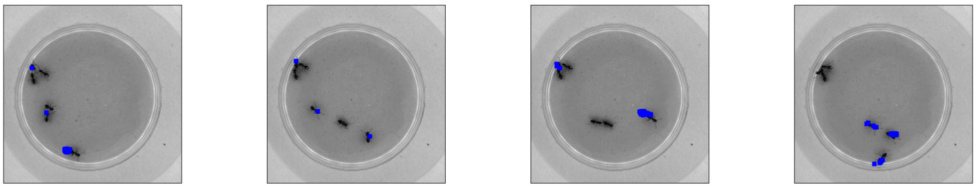

Table 3: Ants: Left to right frames 30, 60, 90, 120

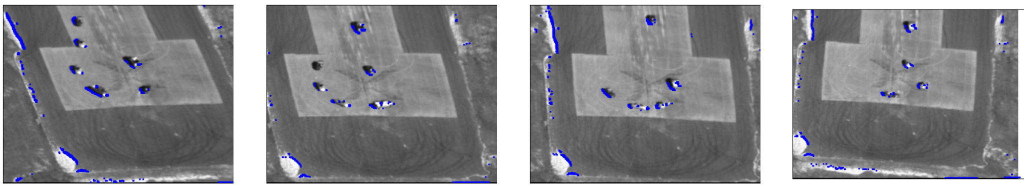

Table 4: Airplane: Left to right frames 30, 60, 90, 120

3

Inverse affine composition is faster than forward composition because the inverse uses the jacobian and the gradient of the template. These do not change each iteration, so they can be calculated once for each frame. This is different from the forward affine where the gradient of the warped template must be calculated each time we calculate Δp for each frame.

	Forward Affine	Reverse Affine
Aerial	260.8675355911255 sec	68.64155340194702 sec
Ant	53.520527362823486 sec	47.01314401626587 sec

Table 5: Duration

When comparing the computation time for the inverse affine and the forward affine methods, there was a significant increase in speed for the aerial images and a small increase in computation speed for the ant images. This is likely due the aerial images moving more, thus requiring more iterations to calculate Δp. Hence there is a more significant speed up when using the inverse affine method for the aerial images.

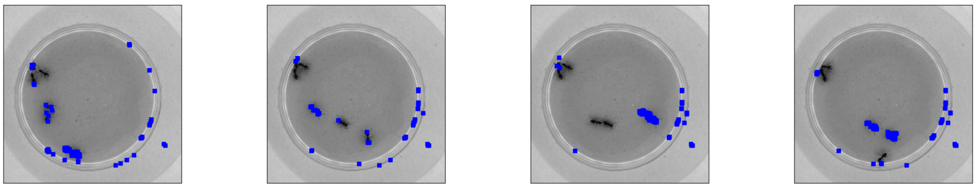

Table 6: Inverse Affine Ants: Left to right frames 30, 60, 90, 120

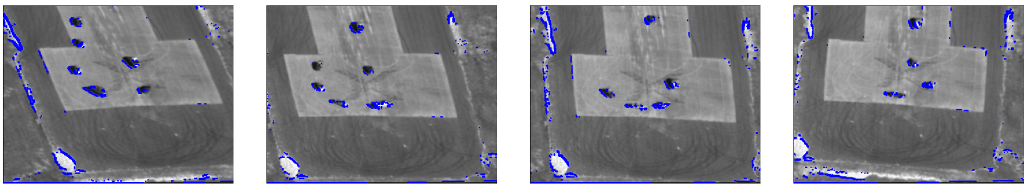

Table 7: Inverse Affine Airplane: Left to right frames 30, 60, 90, 120