ILLINOIS
Gies College of Business

Modern Portfolio Theory

Dr. Sung Won Kim

ILLINOIS
Gies College of Business

Modern Portfolio Theory

Introduction

Dr. Sung Won Kim

Introduction to Modern Portfolio Theory (MPT)

Introduced by Harry Markowitz in 1952

The investment theory is based on the risk aversion of individuals

The goal is to maximize expected returns based on a given level of risk

The theory emphasizes risk as an inherent part of higher returns

MPT attempts to analyze the interrelationship between different investments

ILLINOIS
Gies College of Business

Modern Portfolio Theory

Expected Returns

Dr. Sung Won Kim

Expected Returns of one Security

The amount of profit/loss an investor can anticipate on an investment given the investment's historical returns

It is calculated as the historical average of an assets returns.

However, it does not take risk into account.

INSERT Example of expect returns one security

Expected Returns of a Portfolio

Expected return for an investment portfolio is the weighted average of the expected return of each of its components

Weight of a security is the ratio of the investment in the security to the total investment in the portfolio

Expected Return Calculation

The formula for the expected return on a portfolio is given by

$$E(r) = (w_a * r_a) + (w_b * r_b)$$

where,

w is the weight of a security

r is the return of the security

$\Sigma w_i = 1$

Expected Returns of a Portfolio

$$\begin{aligned}\text{Expected return} = {} & (\text{weight of A})\times(\text{expected return of A}) \\ & + (\text{weight of B})\times(\text{expected return of B}) \\ & + (\text{weight of C})\times(\text{expected return of C})\end{aligned}$$

Expected Return Example

Let us consider an investment in a two asset portfolio.

- Asset A – 12% return with 20% standard deviation
- Asset B – 16% return with 35% standard deviation

Given $10,000, let us invest $6000 in Asset A and $4000 in Asset B.

Expected Return Calculation

The expected return is calculated as follows:

0.6 0.4

$$E(r) = \underline{\left(\frac{6}{10}\right)} * \underline{12} + \underline{\left(\frac{4}{10}\right)} * \underline{16} = 7.2 + 6.4$$

$$\frac{6000}{10,000}$$

$$\boxed{\boldsymbol{E(r) = 13.6\%}}$$

Add example using Real data

ILLINOIS
Gies College of Business

Modern Portfolio Theory

Risk of a Security

Dr. Sung Won Kim

Risk of a Security

Risk represents the chance that the actual return of an investment will be different than expected.

Risk is synonymous with volatility—the greater the portfolio volatility, the greater the risk.

Volatility refers to the amount of risk or uncertainty related to the size of changes in the value of a security.

The most common measures of volatility of a security’s return are variance and standard deviation.

Risk of security example

Find two graphs one not volatile and one very volitle.

Volatility of a Portfolio

Volatility of a portfolio can be calculated by first determining the following parameters:

- The variance of expected return of a security
- The covariance of a portfolio of securities

Observations indicate that the variance of a portfolio decreases as the number of portfolio assets increases – diversification.

Standard Deviation

The standard deviation of returns is calculated as the square root of the sum of squares of the difference between each return data point and the mean of returns divided by the number of returns considered.

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{\sum(\mathrm{r} - \mathrm{E(r)})^2}{\mathrm{n}}}$$

The variance of a portfolio is the square of the standard deviation of the portfolio (i.e., $V_P = \sigma_P^2$).

Correlation and Covariance

The correlation indicates the strength of the relationship between the returns of two assets.

$$\text{Correlation: } \rho_{a,b} = \frac{\sum(r_{a,i}-\overline{r_a})(r_{b,i}-\overline{r_b})}{\sqrt{\sum(r_{a,i}-\overline{r_a})^2 \sum(r_{b,i}-\overline{r_b})^2}}$$

Covariance is a measure of the joint variability of two random variables.

$$\text{Covariance: } \boxed{Cov(a,b) = \rho_{a,b} * \sigma_a * \sigma_b}$$

$-1 \le \rho \le 1$

Volatility Calculations Example

Let us consider an investment in a two asset portfolio

- Asset A – 12% return with 20% standard deviation
- Asset B – 16% return with 35% standard deviation

The correlation between the returns on the two assets is 0.6.

Given $10,000, let us invest $6000 in Asset A and $4000 in Asset B.

Volatility Calculations

The volatility of the portfolio is given by:

$$V_P = (w_a^2 * \sigma_a^2) + (w_b^2 * \sigma_b^2) + (2 * \rho_{a,b} * \sigma_a * \sigma_b * w_a * w_b)$$

$$= (0.6^2)(0.2^2) + (0.4^2) * (0.35^2) + 2(0.6)(0.2)(0.35)(0.6)(0.4)$$

$$= 0.0144 + 0.0196 + 0.02016 = 0.05416 \Rightarrow \boldsymbol{\sigma_P^2 = 0.05416}$$

ILLINOIS
Gies College of Business

Modern Portfolio Theory

Efficient Frontier

Dr. Sung Won Kim

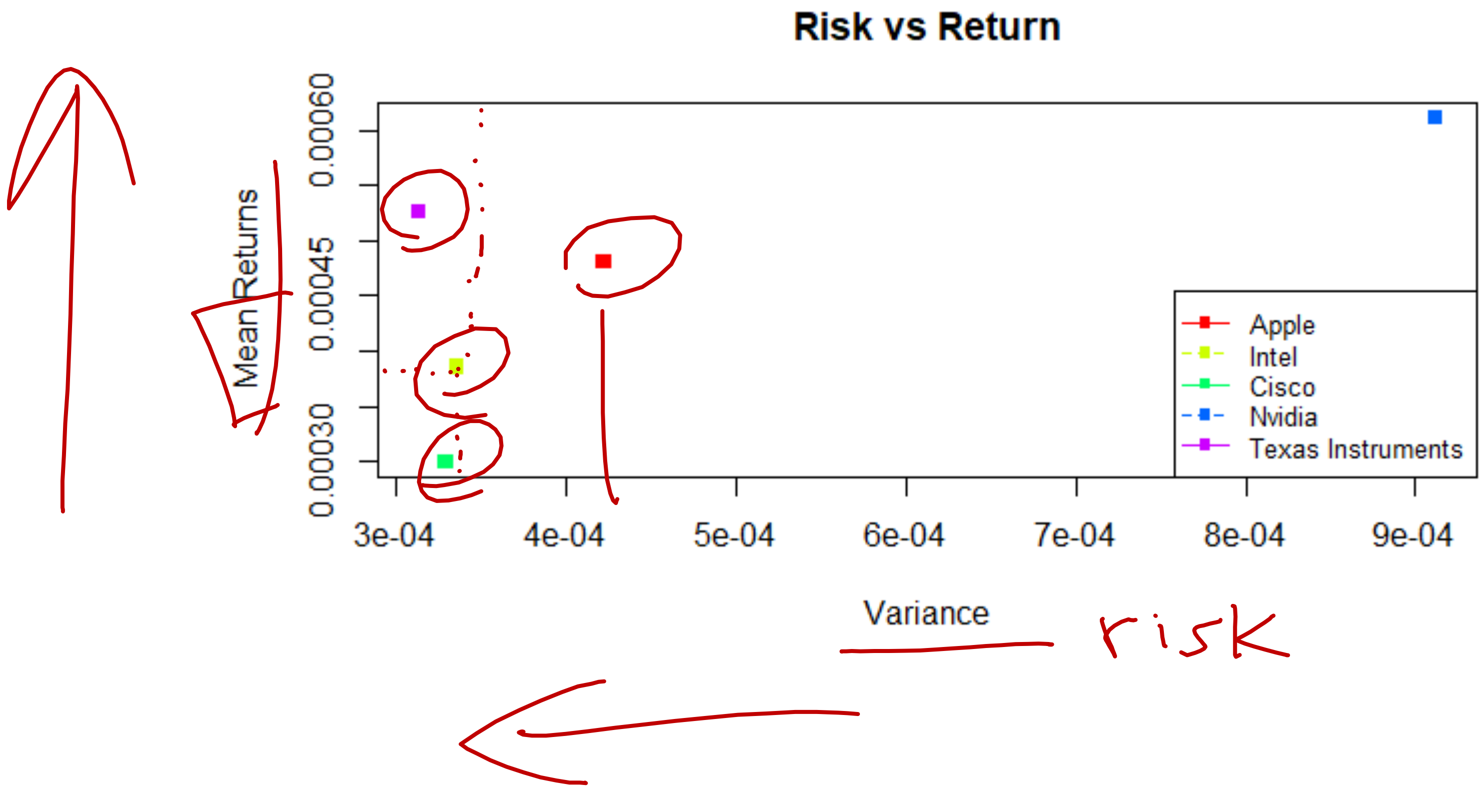
Risk-Return Trade-Off

Basic principle that the riskier the investment, the greater the required potential return

If investors are willing to bear risk, then they expect to earn a risk premium.

The risk-return trade-off refers to the possibility of higher return on investments but does not guarantee higher returns.

Risk-Return Trade-Off



Sharpe Ratio

Sharpe ratio is the measure of risk-adjusted return of a financial portfolio:

$$\text{Sharpe ratio} = \frac{E(r_p) - r_f}{\sigma_p}$$

$$\frac{E(r_i)}{\sigma_i}$$

$E(r_p)$ - Expected return of a portfolio

r_f - Risk-free return

σ_p - Standard deviation of the portfolio returns

The ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk.

The greater a portfolio's Sharpe ratio, the better its risk-adjusted performance.

Sharpe Ratio Calculation

Let us consider the example used to calculate the portfolio parameters.

We have calculated the expected return ($E(r) \ = \ 13.5\%$) and variance of the portfolio ($\sigma_P^2 = 0.05416$).

Let's assume the risk free rate (r_f) to be 0%.

The Sharpe ratio is calculated as:

$$\text{Sharpe ratio} = \frac{0.135 \ - \ 0.000}{\sqrt{0.05416}} = \frac{0.135}{0.233} = \boxed{\mathbf{0.58}}$$

Portfolio Weights

Portfolio weight is the percentage composition of a particular holding in a portfolio.

The degree of risk reduction depends, among other things, upon the weight of the assets in a portfolio.

In this class, we assume only positive weights in all assets (i.e., long-only portfolios, or mutual funds).

Efficient Frontier

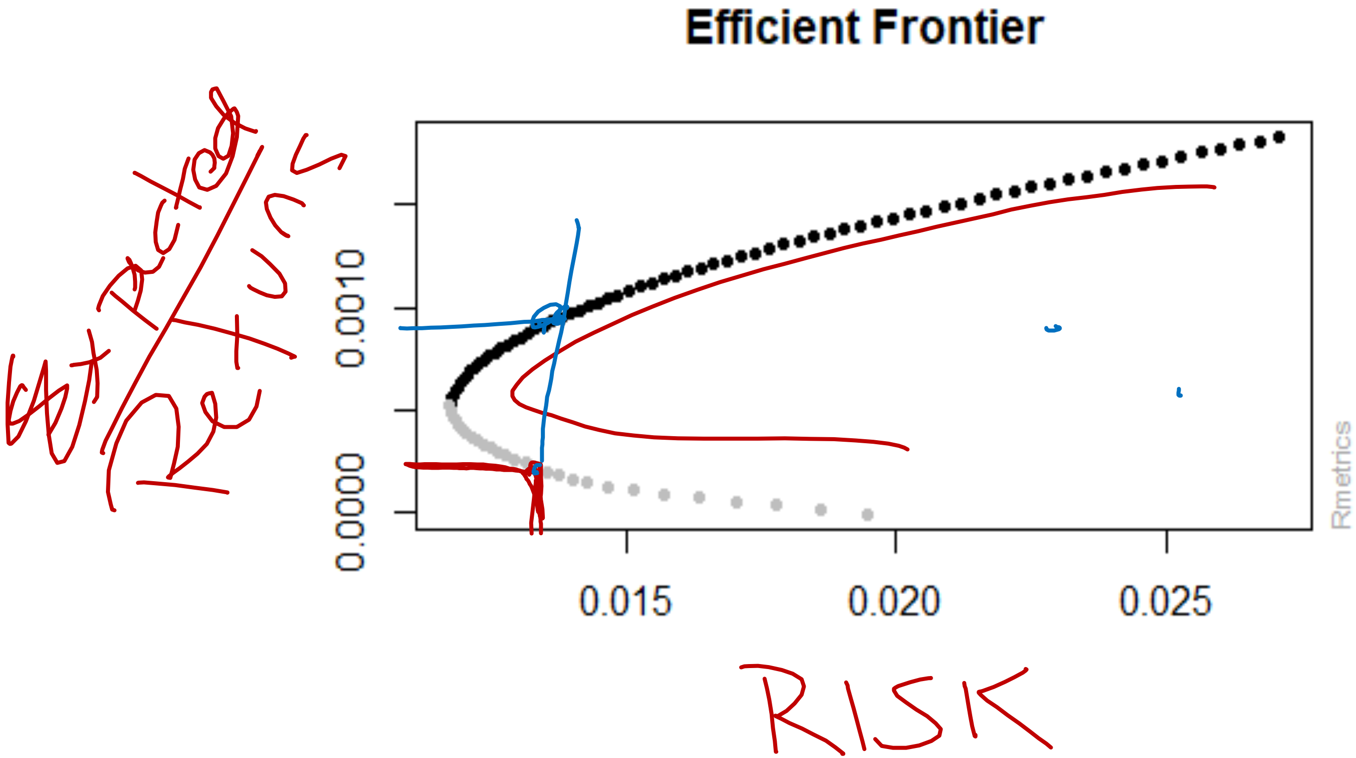
Represents the best combination of expected returns and risk.

The efficient frontier is the lowest risk for a given expected return or the highest return for a given level of risk.

Portfolios below the efficient frontier are not desirable.

Usually depicted in graphic form as a curve on a graph comparing risk against the expected return of a portfolio

Efficient Frontier – Tech Stocks

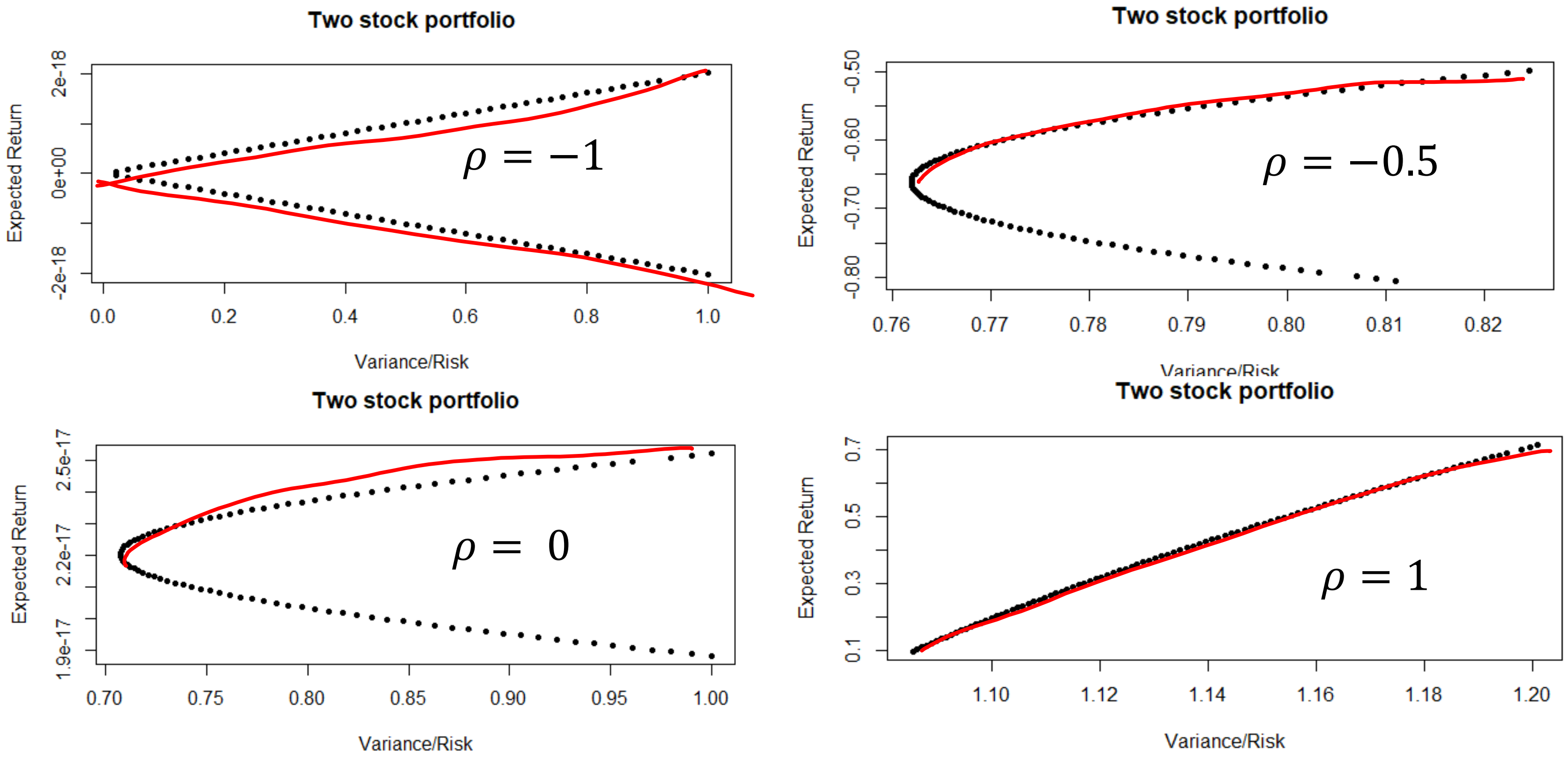


Recall

The volatility of the portfolio is given by:

$$V_P = (w_a^2 * \sigma_a^2) + (w_b^2 * \sigma_b^2) + (2 * \rho_{a,b} * \sigma_a * \sigma_b * w_a * w_b)$$

Shape of the Frontier



ILLINOIS
Gies College of Business

Modern Portfolio Theory

Minimum Variance Portfolio

Dr. Sung Won Kim

Minimum Variance Portfolio

This indicates a portfolio, comprised of risky assets, with the least amount of variance (risk).

The minimum variance portfolio can be pointed out on the efficient frontier (which we will see in our R code).

The optimal minimum variance portfolio will decrease overall volatility with each investment added to it, even if the individual investments are volatile in nature.

Capital Allocation Line (CAL)

In constructing portfolios, investors often combine risky assets with risk-free assets to reduce risks.

A complete portfolio is defined as a combination of a risky asset portfolio, with return R_p, and the risk-free asset, with return R_{f}.

The optimal risky portfolio is found at the point where the CAL is tangent to the efficient frontier.

CAL (cont'd)

$y = mx + b$

The expected return of a complete portfolio is given as:

$$\boxed{E(r_c) = r_f + S_P \sigma_c}$$ → Equation for CAL

r_c is the expected return of the complete portfolio of the risk-free asset and risky portfolio.

S_P is the Sharpe ratio of the risky portfolio.

σ_c is the standard deviation of the complete portfolio, which is given by $\sigma_c = w_P * \sigma_P + w_f * \sigma_f$. w_P is the allocation in the risky portfolio and w_f in the risk free asset.

$$\frac{E(r) - r_f}{\sigma}$$

Since $\sigma_f = 0$, we have $\boxed{\boldsymbol{\sigma_c = w_P * \sigma_P}}$

Tangency Portfolio

The "**tangency** portfolio" is the point on the efficient frontier that has the highest Sharpe ratio.

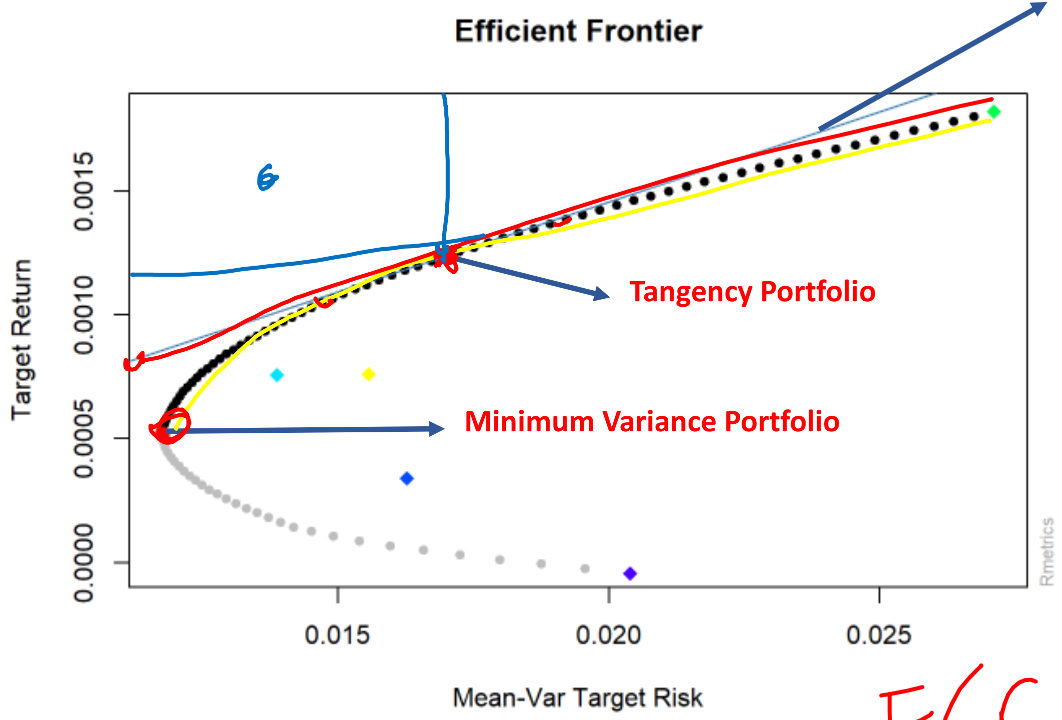
The point of tangency between the capital allocation line (CAL) and the efficient frontier

CAL makes up a combination of risk-free assets and risky portfolio.

At the point of tangency, the investor fully invests in the risky portfolio.

Efficient Frontier – Tech Stocks

Capital Allocation Line

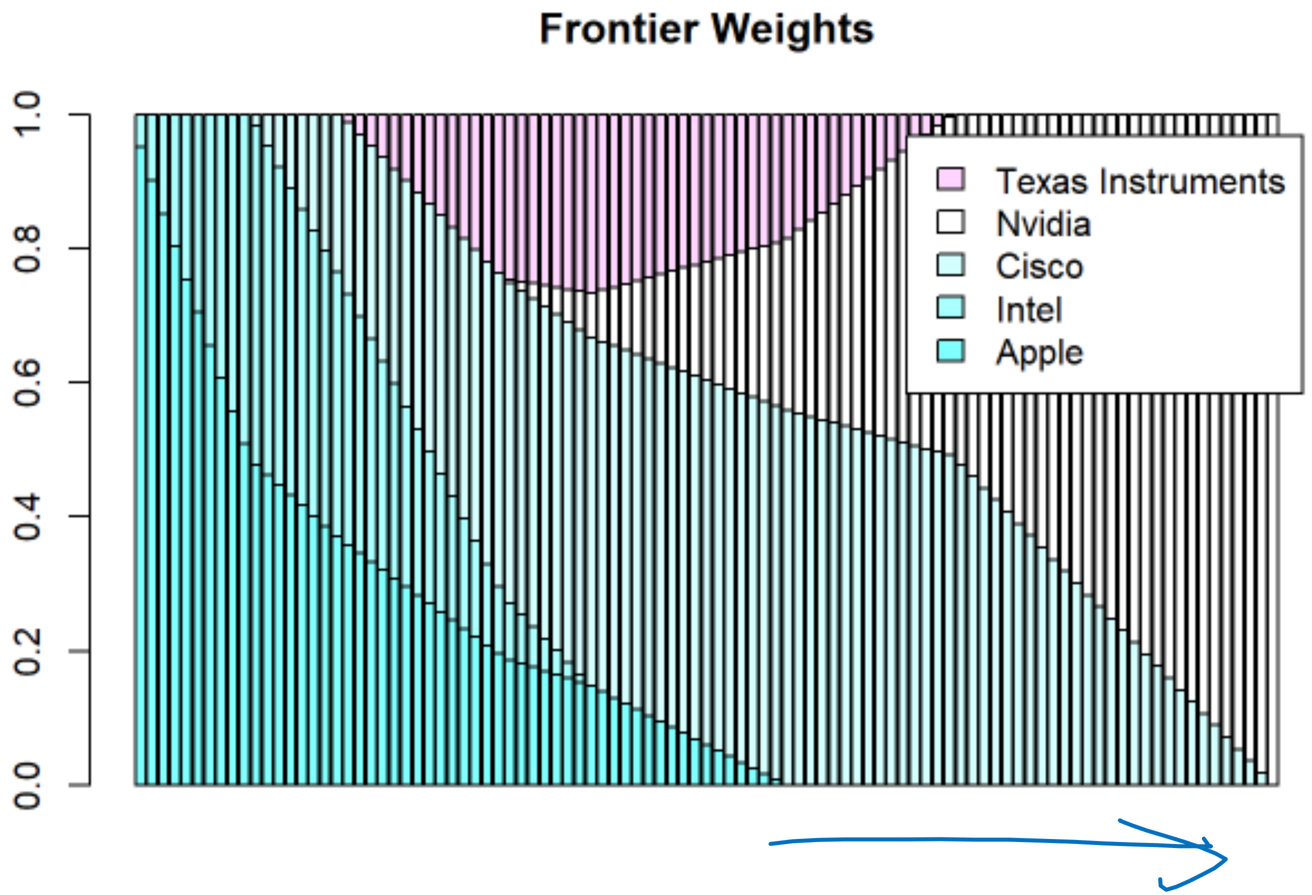


Asset Weights

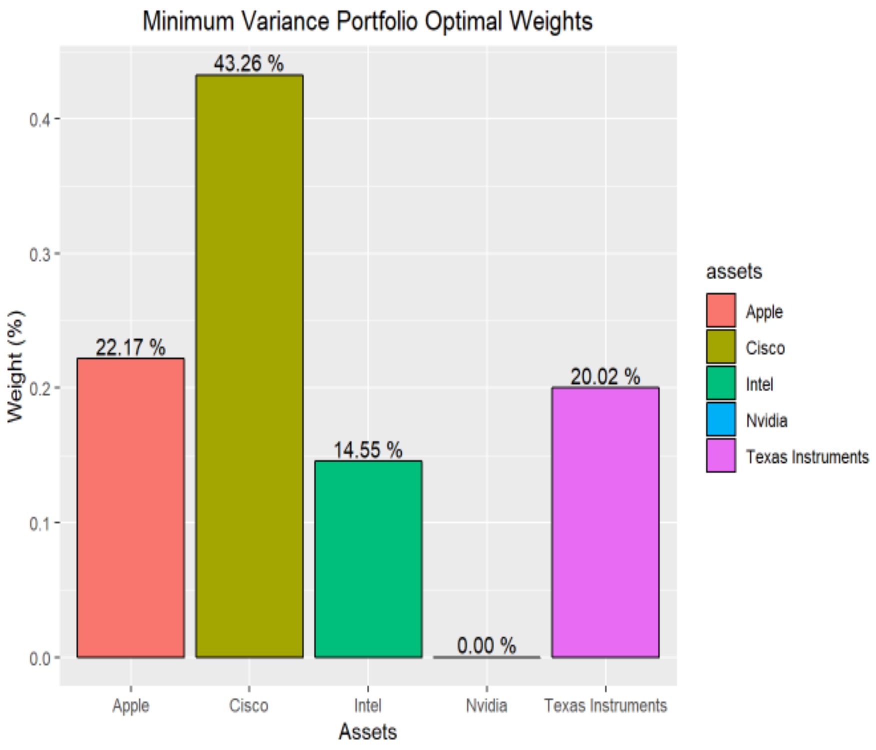
	Minimum Variance Portfolio	Tangency Portfolio
Apple	0.2217163	0.00000000
Intel	0.1455053	0.00000000
Cisco	0.4325933	0.50630443
Nvidia	0.0000000	0.45027452
Texas Instruments	0.2001851	0.04342105

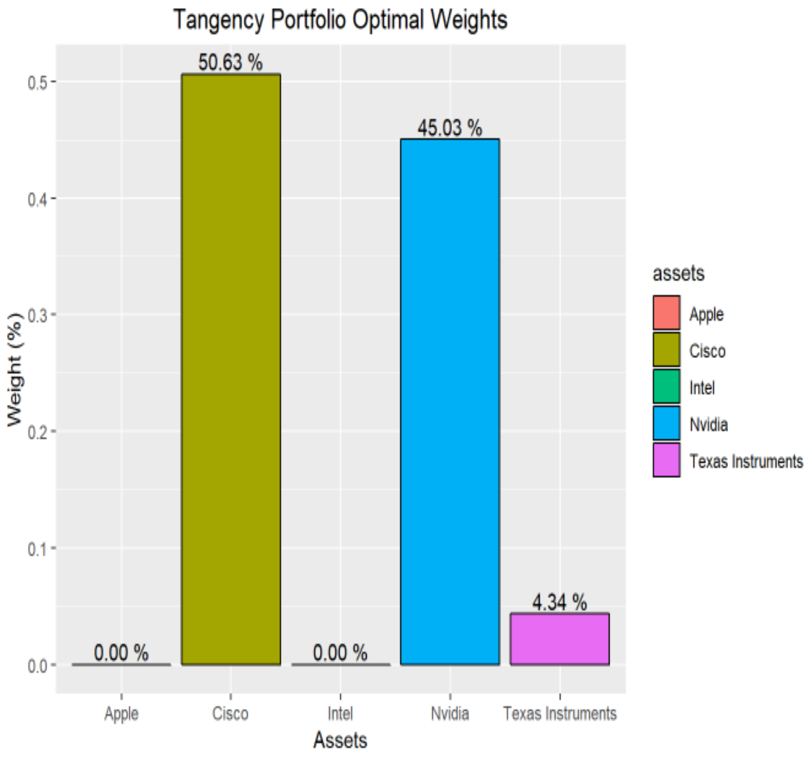
$$E(r_c) = \boxed{r_f} + \boxed{S_p}\sigma_c$$

Frontier Weights



Portfolio Optimal Weights – Tech Stocks





Further Analysis

Portfolios with provision for short positions can be generated.

Other constraints such as risk targets, returns targets, or alpha targets can be set using certain functions.

The benefits of diversification can be studied by varying the number of stocks in Portfolio 3 from 5 to 10 or more.

ILLINOIS
Gies College of Business

Modern Portfolio Theory

Diversification

Dr. Sung Won Kim

Diversification

Diversification is the process of mixing a variety of investments in a portfolio with the intention to reduce overall risk.

Diversification strives to smooth out unsystematic risk events in a portfolio, so the positive performance of some investments neutralizes the negative performance of others.

Diversification limits portfolio risk but can also mitigate performance, at least in the short term.

Example: Diversification Benefits

We will construct two portfolios:

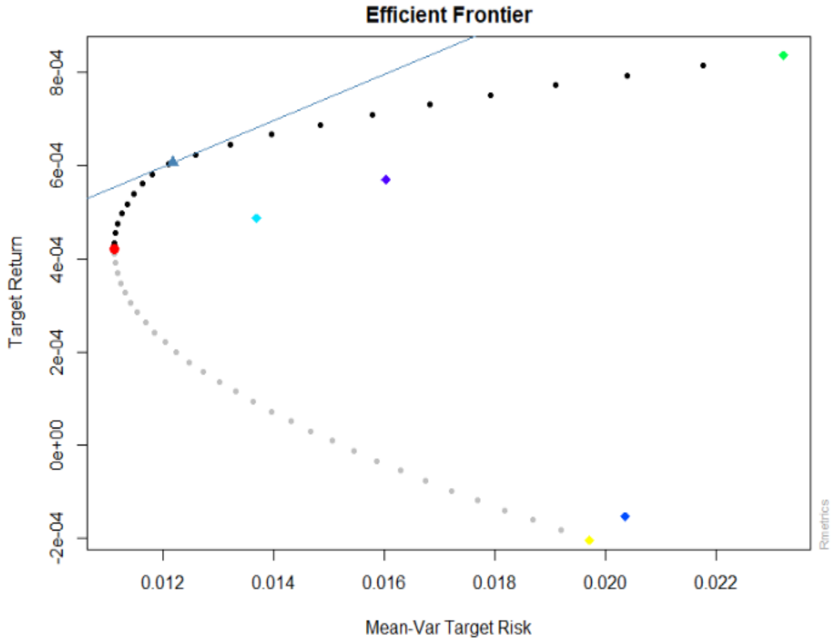
Portfolio A: 5 random stocks

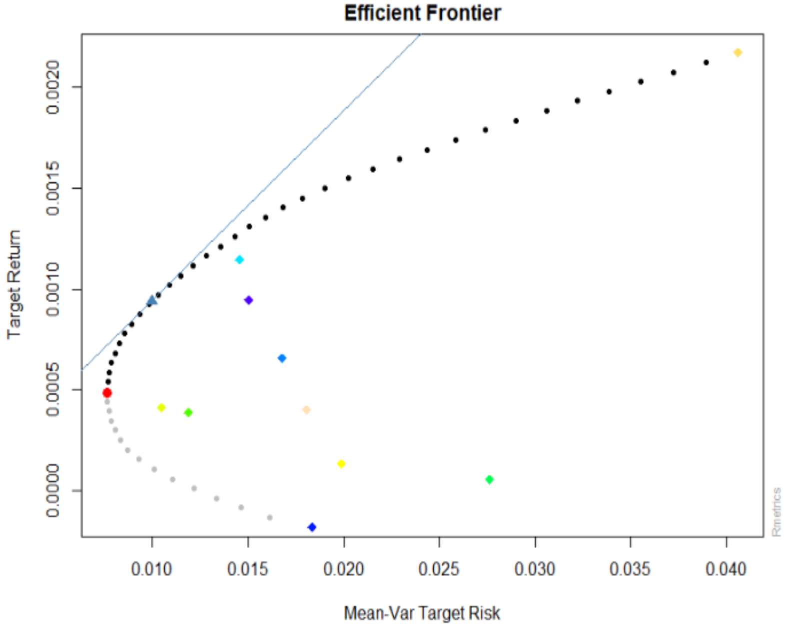
Portfolio B: 10 random stock

According to the MPT, we should observe a lower risk on portfolio B.

Correlations of assets drive diversification.

Portfolio A vs. Portfolio B





	Tangency Portfolio	Min Var Portfolio
mean	0.1652294	0.1117540
VaR	0.3000456	0.2869651

	Tangency Portfolio	Min Var Portfolio
mean	0.2665076	0.1297272
VaR	0.2561280	0.1921751

Observation

We can see that the risk decreases significantly when the number of stocks in the portfolio is increased.

Studies have shown that using 30 stocks in a portfolio provides the most benefits of diversification.

From the formula for the variance of a portfolio, we can see that the variance is directly related to the correlation between the assets – the lower the correlation, the lower the variance.

Recall

The volatility of the portfolio is given by:

$$V_P = (w_a^2 * \sigma_a^2) + (w_b^2 * \sigma_b^2) + (2 * \rho_{a,b} * \sigma_a * \sigma_b * w_a * w_b)$$

Dataset Selection

Portfolio 1: five tech hardware stocks

- Apple Inc
- Intel Corp
- Cisco Systems
- Nvidia Corp
- Texas Instruments

Portfolio 2: five stocks from different sectors

- Microsoft Corp
- Johnson & Johnson
- JP Morgan Chase & Co
- Exxon Mobil Corp
- Home Depot

Portfolio 3: *n* number of random stocks across sectors

R Packages Used

fPortfolios – generates portfolios

Quantmod – obtains historical stock price data

ggplot2 – plots graphs

batchGetSymbols – downloads current components of S&P500 index from wikipedia

timeSeries – converts numerical data to time series data

fPortfolio Package

The package is specially geared toward portfolio optimization

Key functions from the package used during the class are–

portfolioFrontier – generates portfolios (set at 100) for the given stock returns data

getWeights – extracts the weights of each stock in all the generated portfolios

minvariancePortfolio – returns the portfolio with the minimal risk on the efficient frontier

tangencyPortfolio – returns the portfolio with the highest return/risk ratio on the efficient frontier

ILLINOIS
Gies College of Business

Introduction to Algorithmic Trading

Introduction

Dr. Sung Won Kim

What Is Algorithmic Trading

Algorithmic trading is a style that utilizes a computer’s ability to process data quickly and react faster.

This approach is almost always based on hard data rather than forecasts or opinions.

The main objective of algorithmic trading is not necessarily to maximize profits but rather to control execution costs and market risk

Algorithmic Trading Strategies

These are some of the major algorithmic trading strategies:

- Mean Reversion
- Momentum Strategy
- Statistical Arbitrage
- Trend Following Strategy

Mean Reversion

Mean reversion strategies exploit the tendency of many asset prices to revert to the mean after periods where they are oversold or overbought.

Mean reversion strategies try to capitalize on extreme changes in the price of a security, assuming that it will revert to its previous state.

Relative Strength Index (RSI) is the most commonly used indicator for mean reverting strategies.

Momentum Strategy

Momentum strategies are based on the assumption that the price of financial instruments has some inertia.

Momentum trading is generally used to capture strong moves in short time frames.

Momentum is the acceleration in a stock's price that can be due to earnings, sentiment, news, greed, or fear.

Momentum strategies are a **particular case** of trend-following strategies.

Statistical Arbitrage

This strategy identifies pricing discrepancies between securities through mathematical modelling techniques.

This strategy is market neutral as it involves a long position and also a short position in different securities.

The end objective of such strategies is to generate alpha (higher than normal profits) for the trading firms.

Trend Following Strategy

The goal of this strategy is to buy an asset when its price trend goes up, and sell when its trend goes down, expecting price movements to continue.

Trading decisions are made based on technical analysis, market patterns, and indicators.

The most commonly used indicator is the **Moving Average,** which we will study in this class.

Data – Advanced Auto Parts Stock

ILLINOIS
Gies College of Business

Introduction to Algorithmic Trading

Trend Following Strategy

Dr. Sung Won Kim

Basic Trading Strategy

The strategy used in this study is called a trend-following strategy.

It works on the basis of historical average price movements of the asset.

We will compute what is known as a Simple Moving Average for two time periods – the average price over the n previous days.

If the smaller time period line crosses the longer time period line for below – we have a buy, and a sell if it crosses from above.

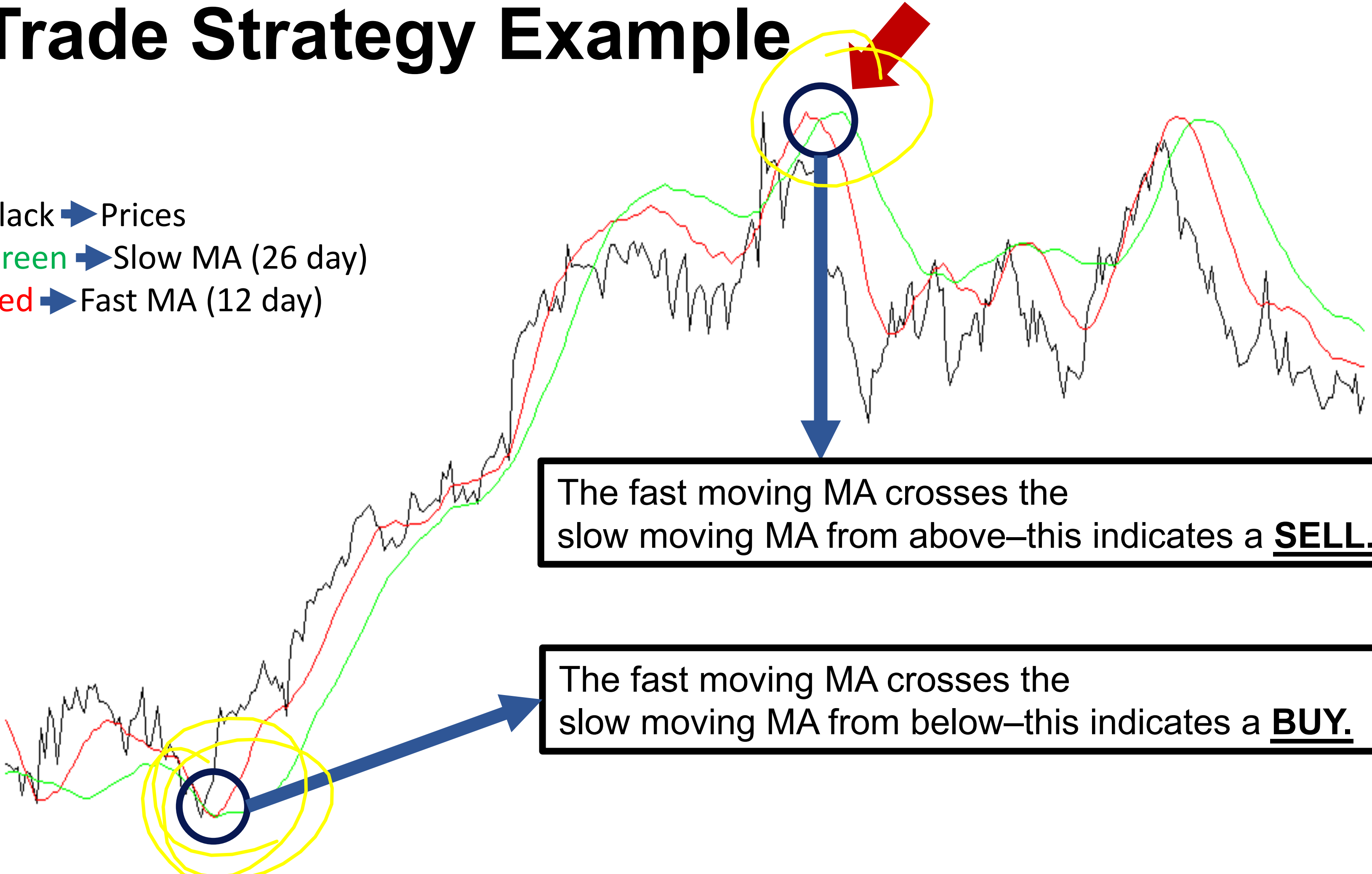
Signal Generator

The trade signal is generated as follows:

- If the fast moving average crosses the slow moving average from below – buy, else sell/hold.
- The position once bought/sold will be held until there is change is signal direction.
- When the signal goes from sell to buy, the sold assets are bought back and more assets are bought.

Trade Strategy Example

Black ➔ Prices
Green ➔ Slow MA (26 day)
Red ➔ Fast MA (12 day)



The fast moving MA crosses the slow moving MA from above–this indicates a **SELL.**

The fast moving MA crosses the slow moving MA from below–this indicates a **BUY.**

ILLINOIS
Gies College of Business

Introduction to Algorithmic Trading

Backtesting

Dr. Sung Won Kim

Backtesting a Trading Strategy

Backtesting is a key component of developing an effective trading strategy.

We reconstruct, using historical data, the trades that would have occurred using the rules established in the trading strategy.

The result we obtained offer statistics to gauge the effectiveness of the strategy.

Backtesting the Trend-Following Strategy

To backtest the moving average strategy, we chose the stock price data for Advanced Auto Parts.Inc (AAP) and carry out the following steps:

- We obtain the simple moving averages for 12 days and 26 days.
- We convert the stock price data to returns.
- We check the position of the 12-day MA (fast) with respect to the 26-day MA (slow).
- If the fast moving average is above the slow moving average, it is a buy, and sell if it is below.

Backtesting the Trend-Following Strategy

We denote a long position as +1 and a short position as -1.

Now, we have the actual returns based on historical data and the position initiated by the strategy.

To obtain the performance of the algorithm, we multiply the position initiated on date with the returns for that day.

A positive return on a long day is profitable and so is a short on a negative return day.

In any other scenario, we end up with a loss day.

Backtesting the Trend-Following Strategy

Executing trades as such, we will end with the daily returns on each trade.

The average of the daily returns will give us the expected daily return from this strategy.

We can study the volatility of the returns and the Sharpe ratio to get a better understanding of the performance of the strategy.

Results

For the given backtest data set of Advanced Auto Parts Stock returns from 2010 to present, we have the following parameters:

Expected Return – 13.5%

Standard Deviation – 1.86%

Sharpe Ratio – 6.2

Looking at the results above, we can see that the strategy has performed considerably well with a very high-risk adjusted return.

Results Table

	AAPPrices	AAPRet	SMA12	SMA26	UD	Trade	Position	AlgoRet
2010-02-09	40.59709	0.0184518342	39.69536	39.29896	1	BUY	1	0.0184518342
2010-02-10	41.33931	0.0181174266	39.84429	39.37221	1	BUY	1	0.0181174266
2010-02-11	41.55416	0.0051837979	40.00706	39.46273	1	BUY	1	0.0051837979
2010-02-12	41.80806	0.0060914838	40.18773	39.54987	1	BUY	1	0.0060914838
2010-02-16	41.63229	-0.0042130762	40.42048	39.63063	1	BUY	1	-0.0042130762
2010-02-17	41.87643	0.0058472623	40.69963	39.71477	1	BUY	1	0.0058472623
2010-02-18	39.21032	-0.0657833589	40.67196	39.71139	1	BUY	1	-0.0657833589
2010-02-19	39.06381	-0.0037433613	40.57511	39.72866	1	BUY	1	-0.0037433613
2010-02-22	39.29820	0.0059821767	40.49780	39.73430	1	BUY	1	0.0059821767
2010-02-23	39.27869	-0.0004966346	40.44002	39.75909	1	BUY	1	-0.0004966346
2010-02-24	39.51307	0.0059495229	40.41886	39.80229	1	BUY	1	0.0059495229
2010-02-25	39.77674	0.0066506647	40.41235	39.84849	1	BUY	1	0.0066506647
2010-02-26	39.84510	0.0017172930	40.34968	39.91009	1	BUY	1	0.0017172930
2010-03-01	40.33340	0.0121803977	40.26586	39.98784	1	BUY	1	0.0121803977
2010-03-02	40.37246	0.0009680090	40.16738	40.04794	1	BUY	1	0.0009680090
2010-03-03	40.51895	0.0036217729	40.05995	40.08512	0	SELL	-1	-0.0036217729
2010-03-04	40.80217	0.0069655497	39.99078	40.13132	0	SELL	-1	-0.0069655497

ILLINOIS
Gies College of Business

Introduction to Algorithmic Trading

Conclusion

Dr. Sung Won Kim

Conclusion

The basic trend-following strategy was considerably profitable on an annualized basis.

While this was a simple strategy, we can also employ more advanced predictive models to predict the movement of the stock prices.

Coupling multiple strategies is also a good way to minimize bad loss days.