

# Intro to ACF and PACF

FORECASTING USING ARIMA MODELS IN PYTHON

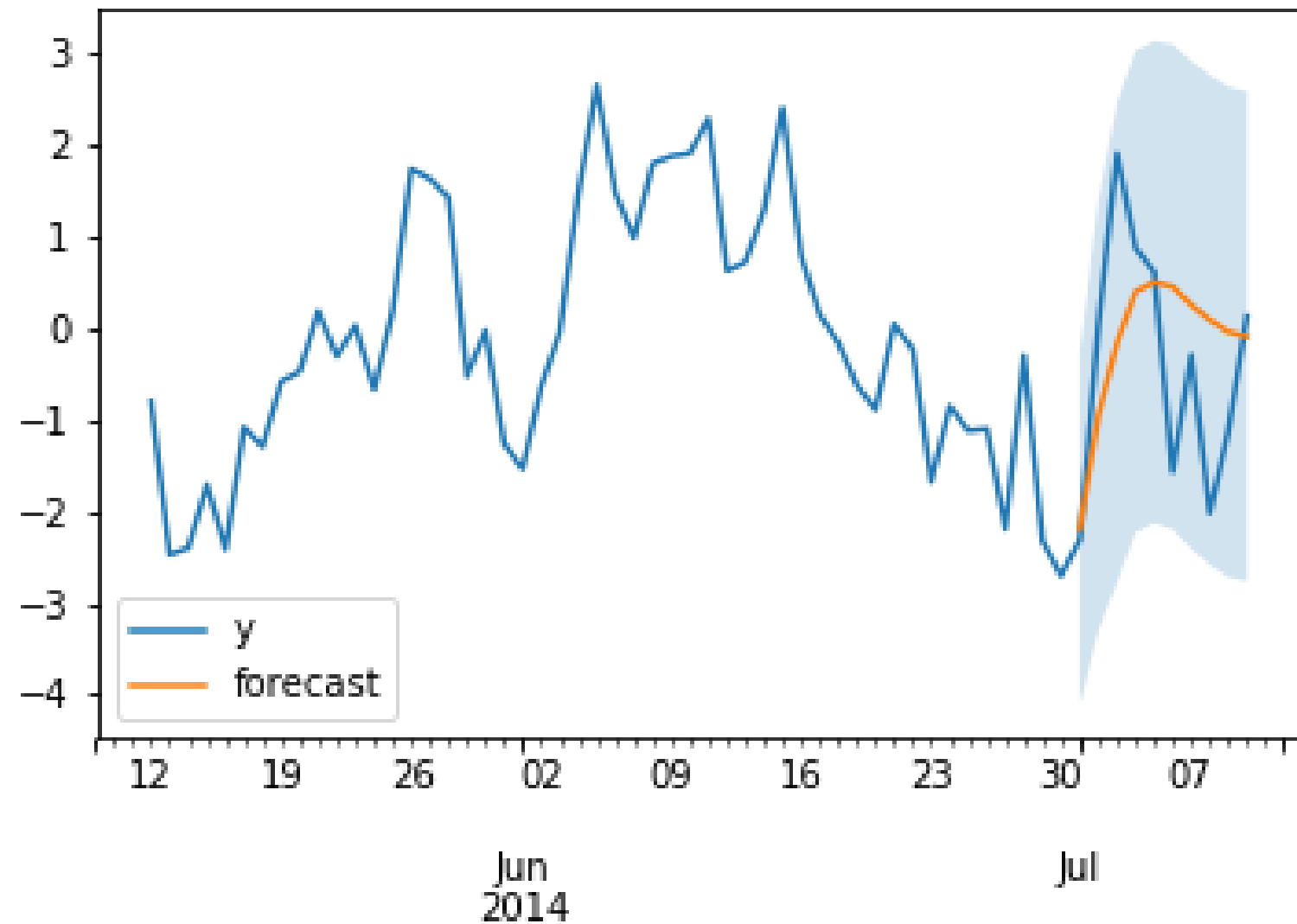


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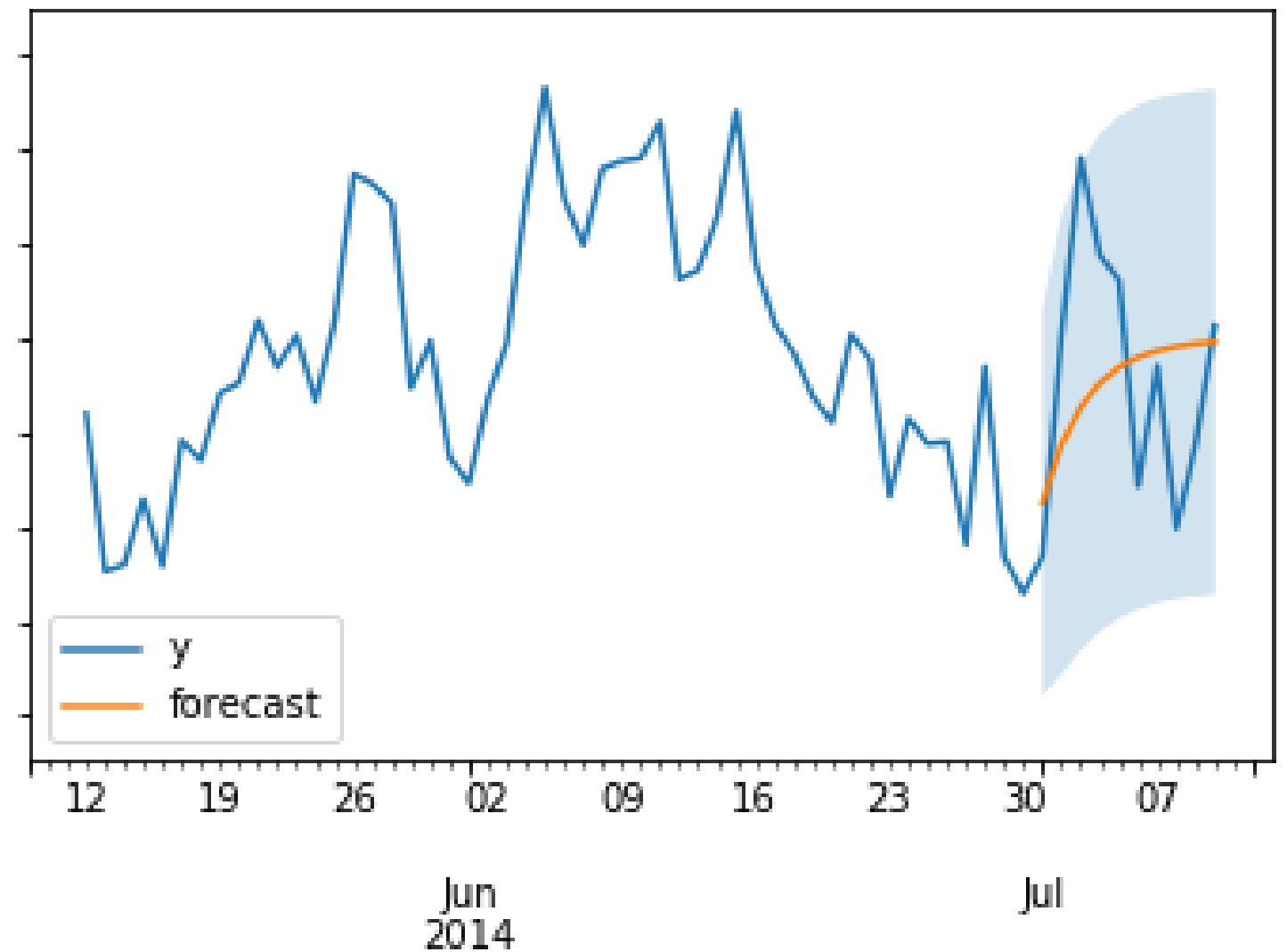
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# Motivation

ARMA(3,0) Dynamic Forecast



ARMA(1,1) Dynamic Forecast



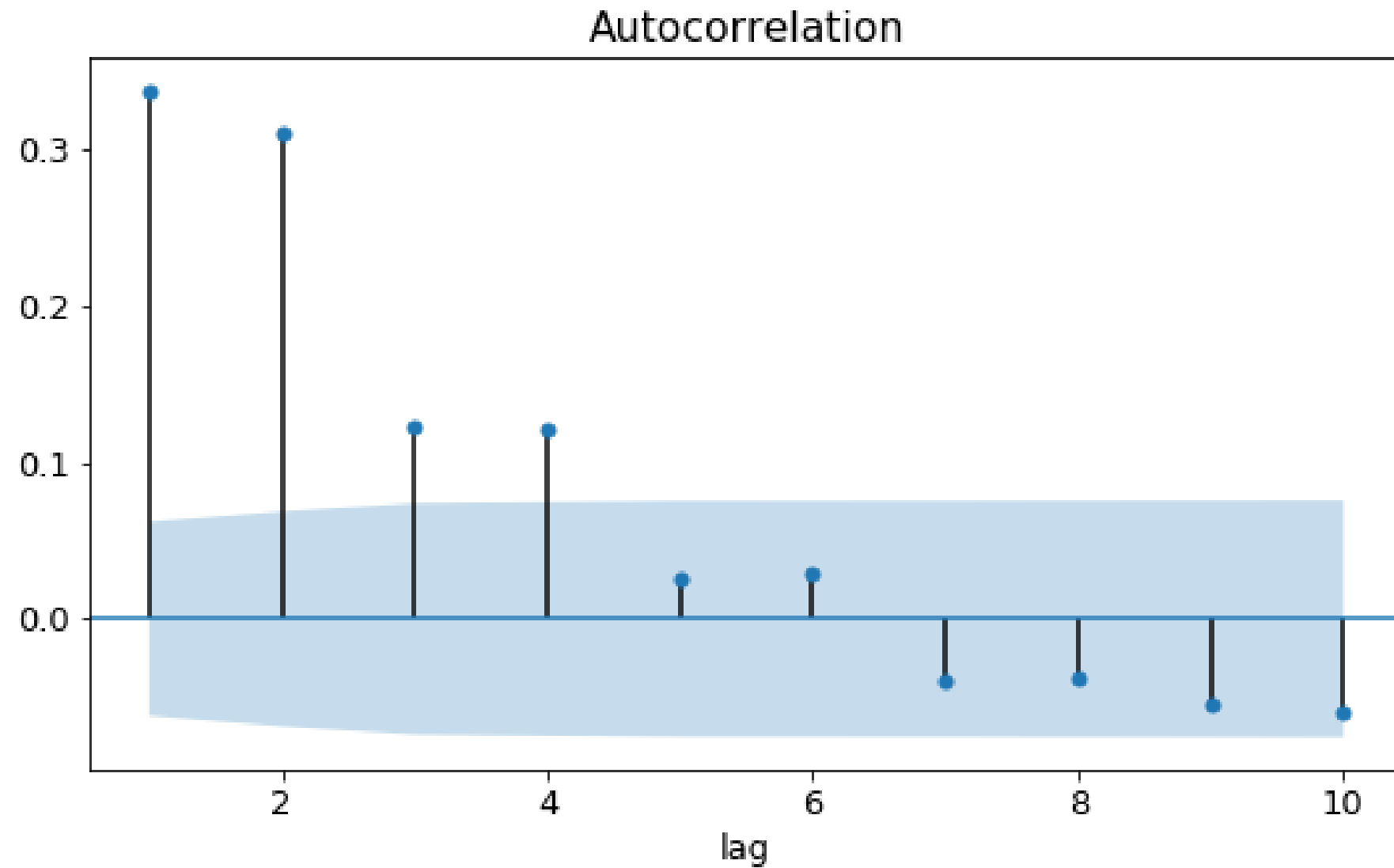
# ACF and PACF

- ACF - Autocorrelation Function
- PACF - Partial autocorrelation function

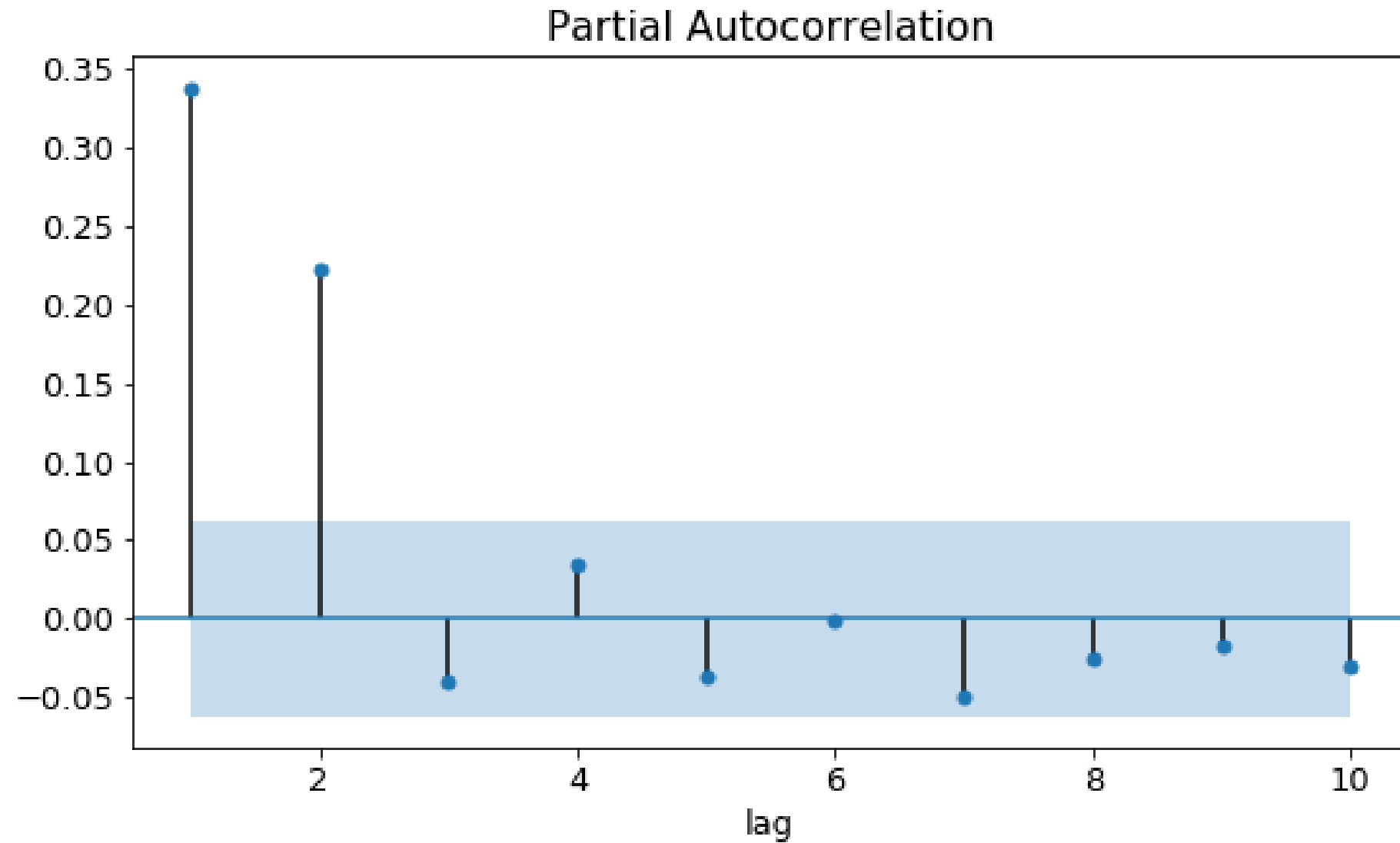
# What is the ACF

- lag-1 autocorrelation  $\rightarrow \text{corr}(y_t, y_{t-1})$
- lag-2 autocorrelation  $\rightarrow \text{corr}(y_t, y_{t-2})$
- ...
- lag-n autocorrelation  $\rightarrow \text{corr}(y_t, y_{t-n})$

# What is the ACF



# What is the PACF



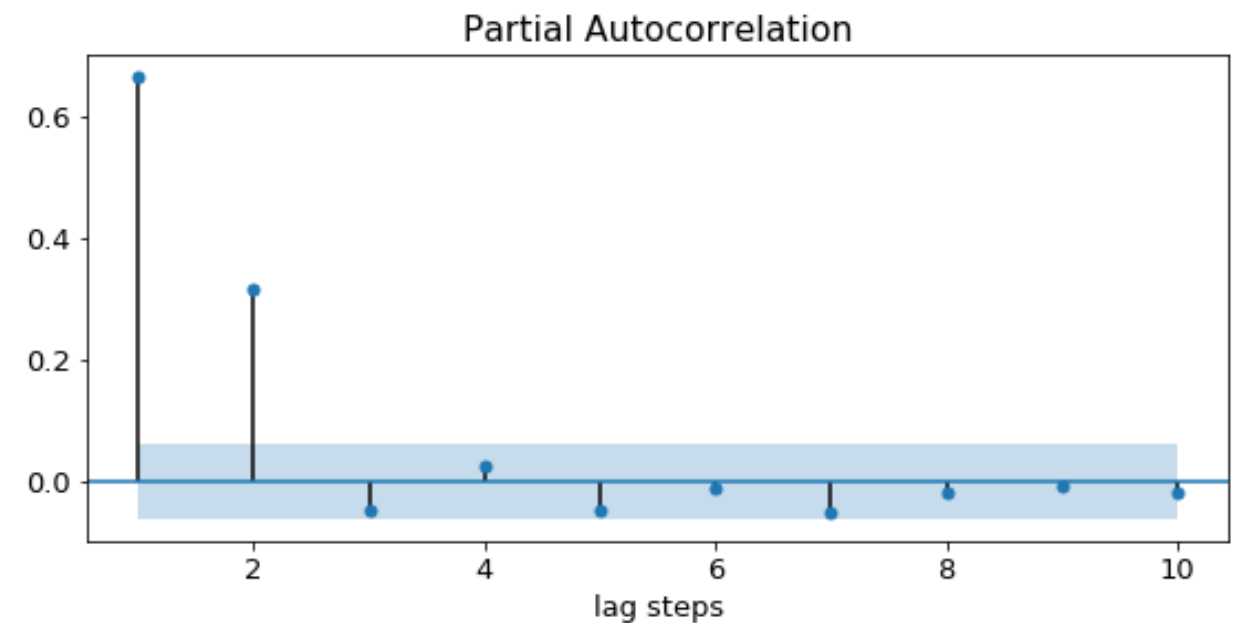
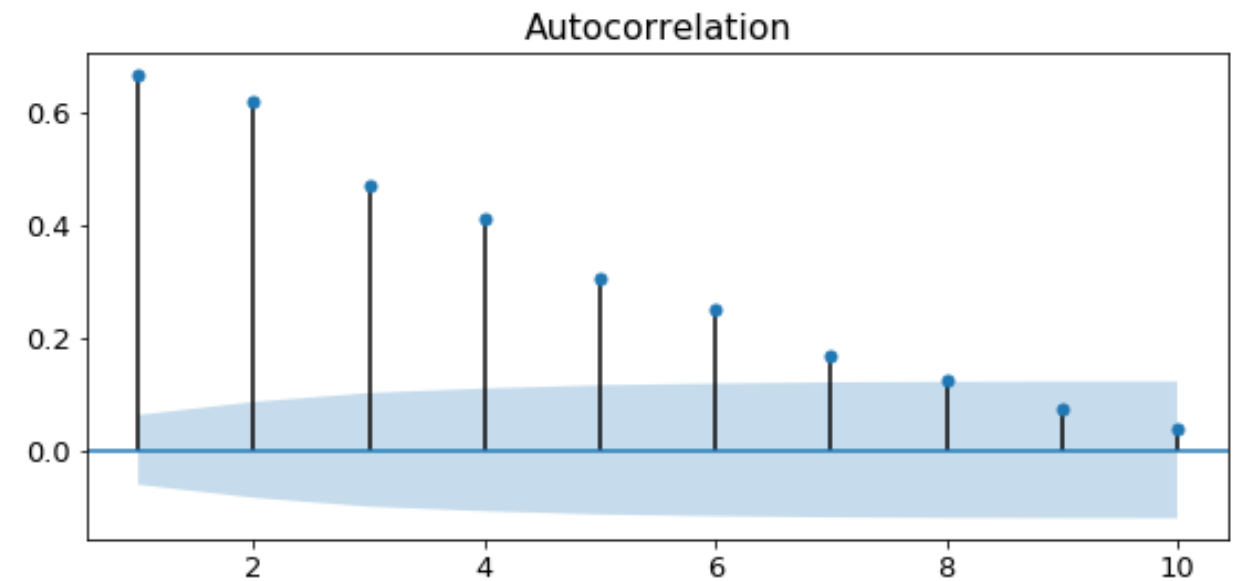
# Using ACF and PACF to choose model order

AR(p)

ACF      Tails off

PACF      Cuts off after lag p

- AR(2) model →



# Using ACF and PACF to choose model order

MA(q)

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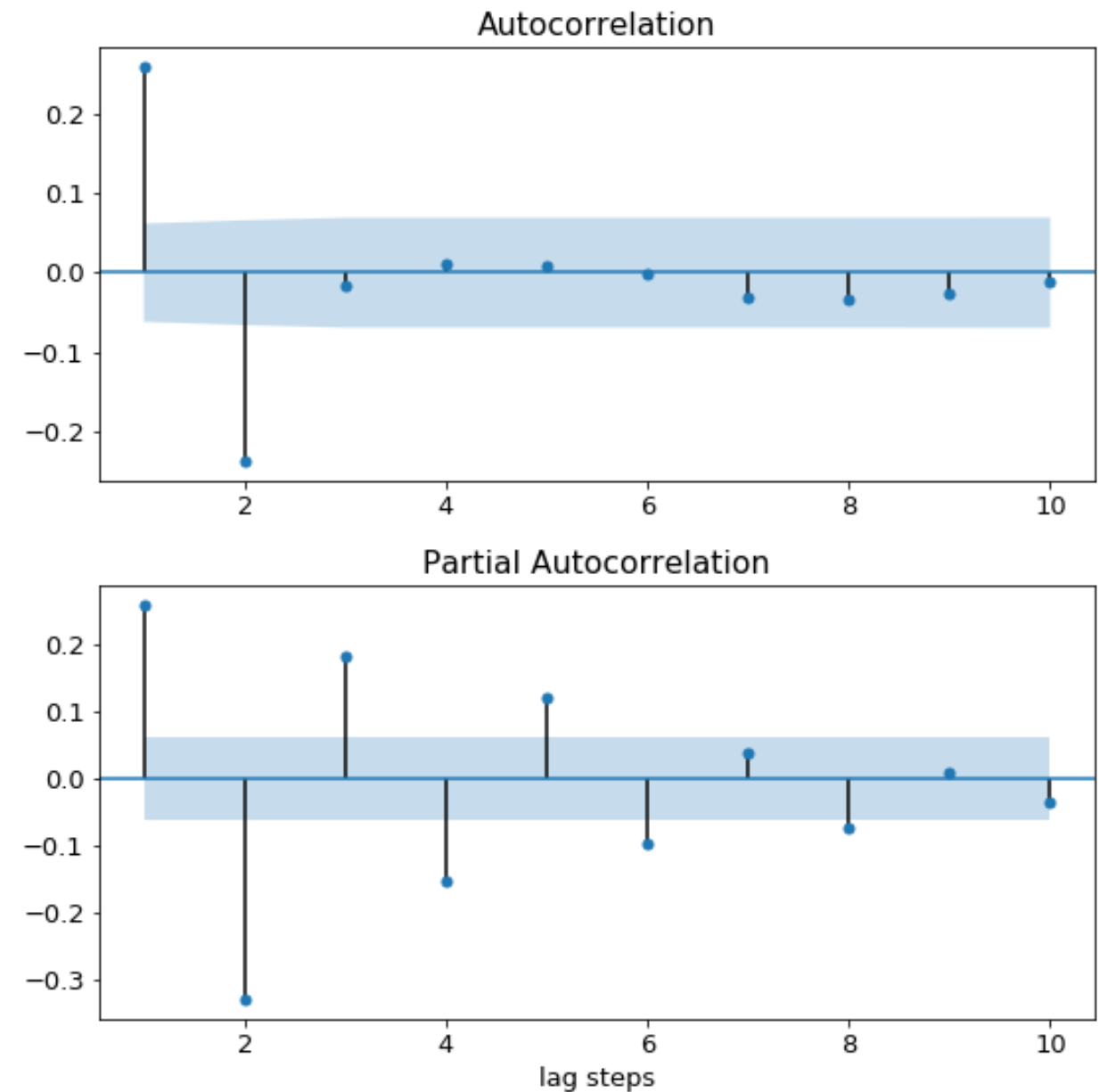
ACF	Cuts off after lag q
-----	----------------------

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PACF	Tails off
------	-----------

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- MA(2) model →





# Using ACF and PACF to choose model order

ARMA(p,q)

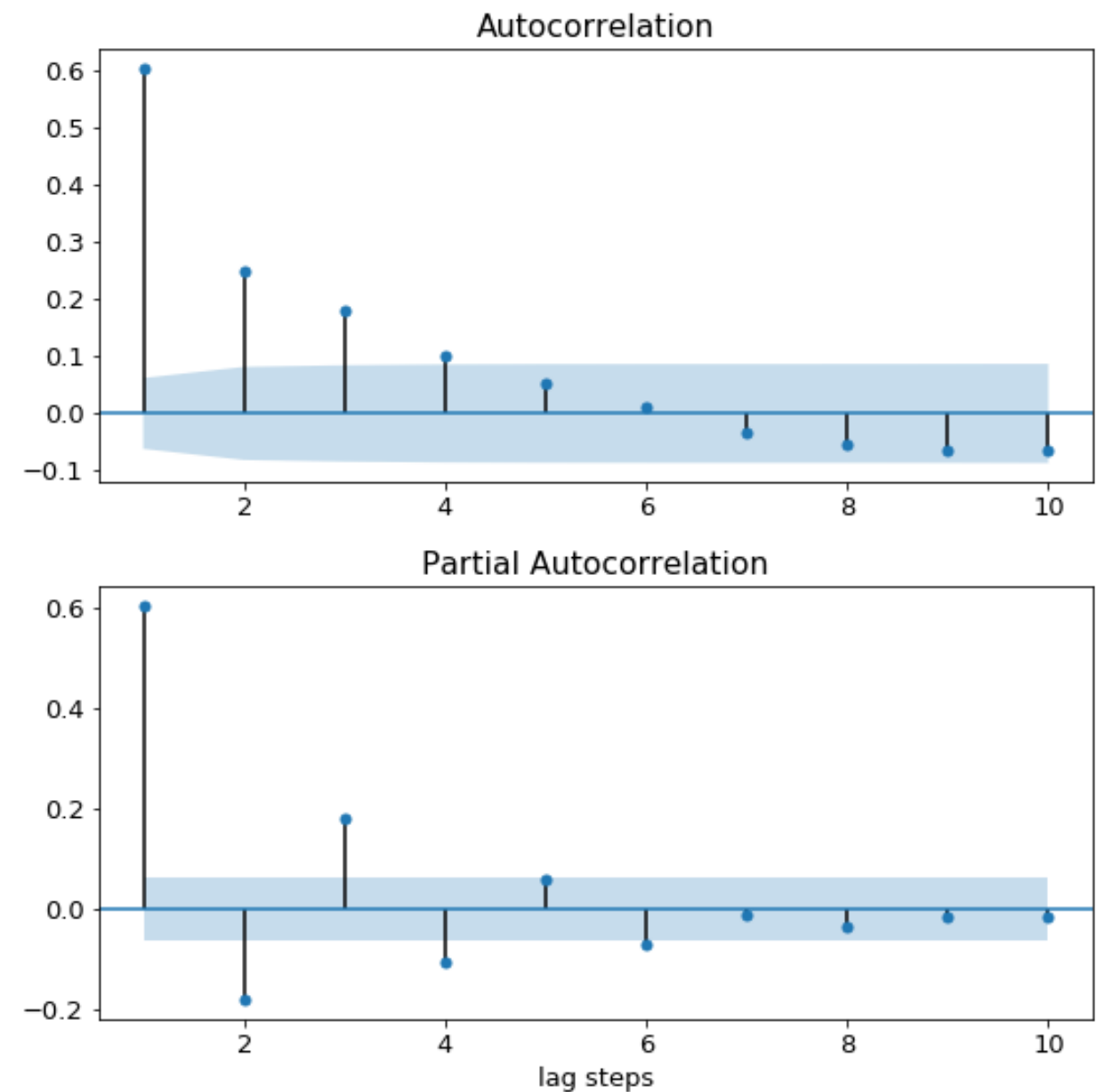
---

ACF	Tails off
-----	-----------

---

PACF	Tails off
------	-----------

---



# Using ACF and PACF to choose model order

	AR(p)	MA(q)	ARMA(p,q)
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

# Implementation in Python

```
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

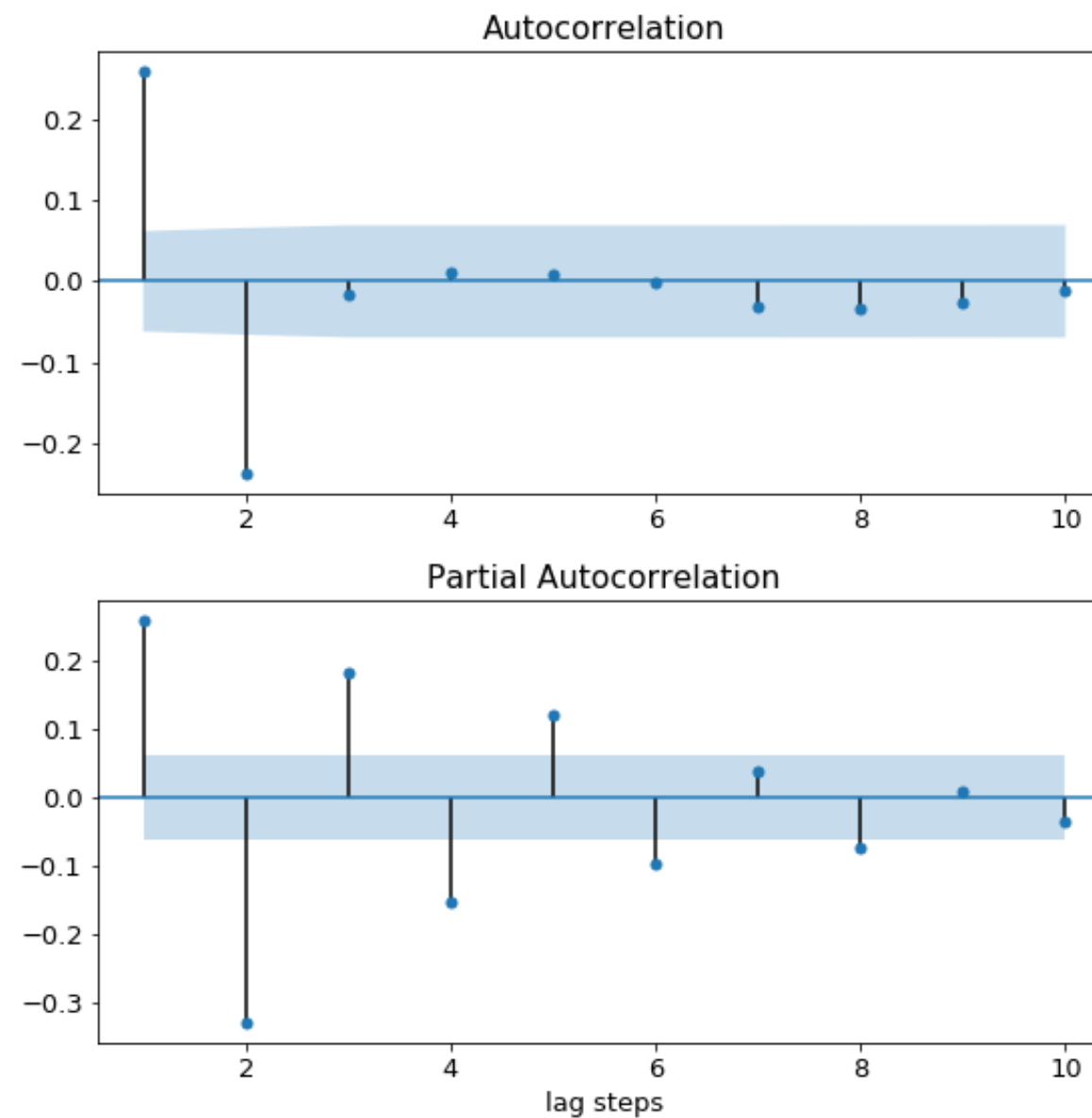
```
# Create figure
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(8, 8))

# Make ACF plot
plot_acf(df, lags=10, zero=False, ax=ax1)

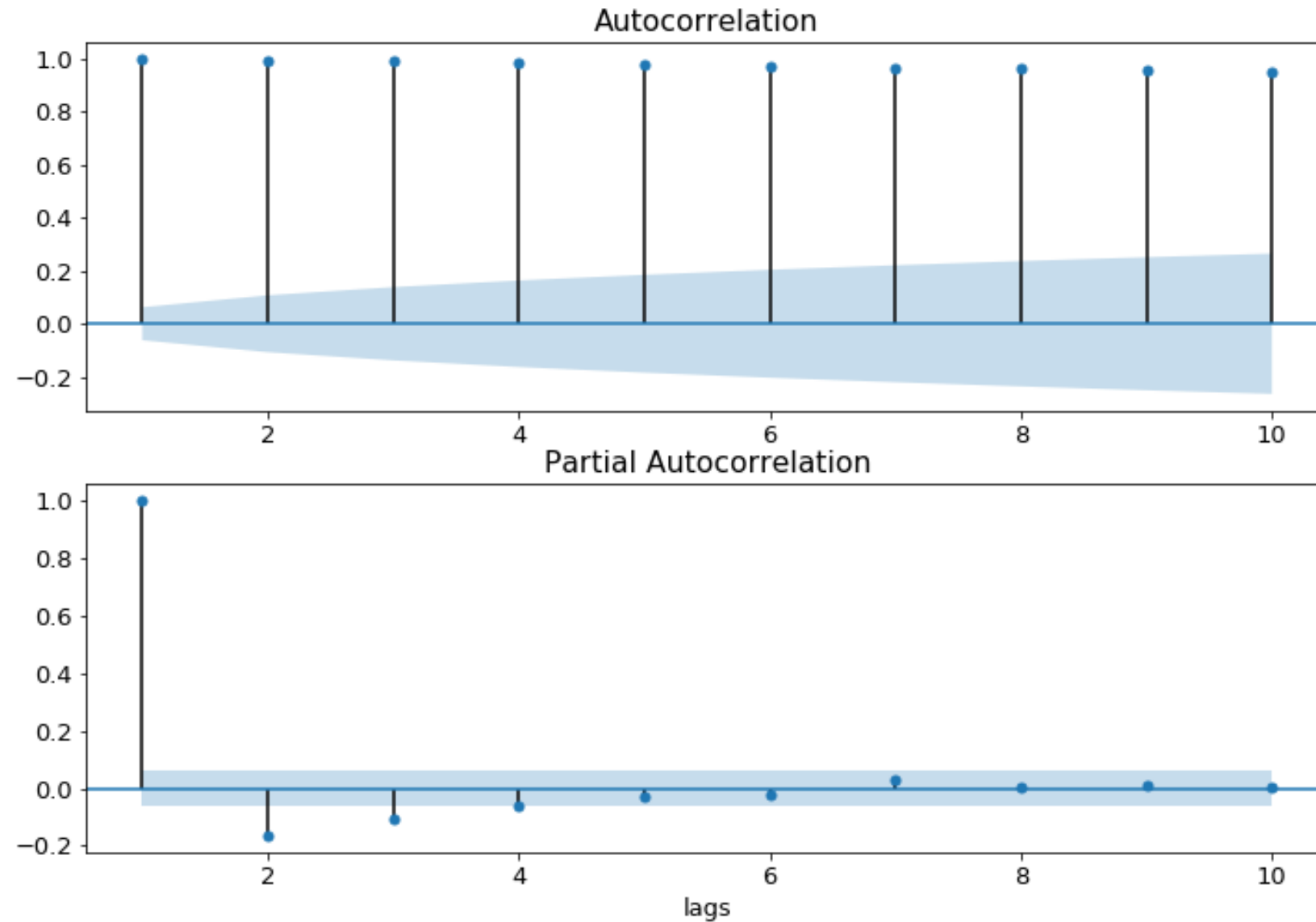
# Make PACF plot
plot_pacf(df, lags=10, zero=False, ax=ax2)

plt.show()
```

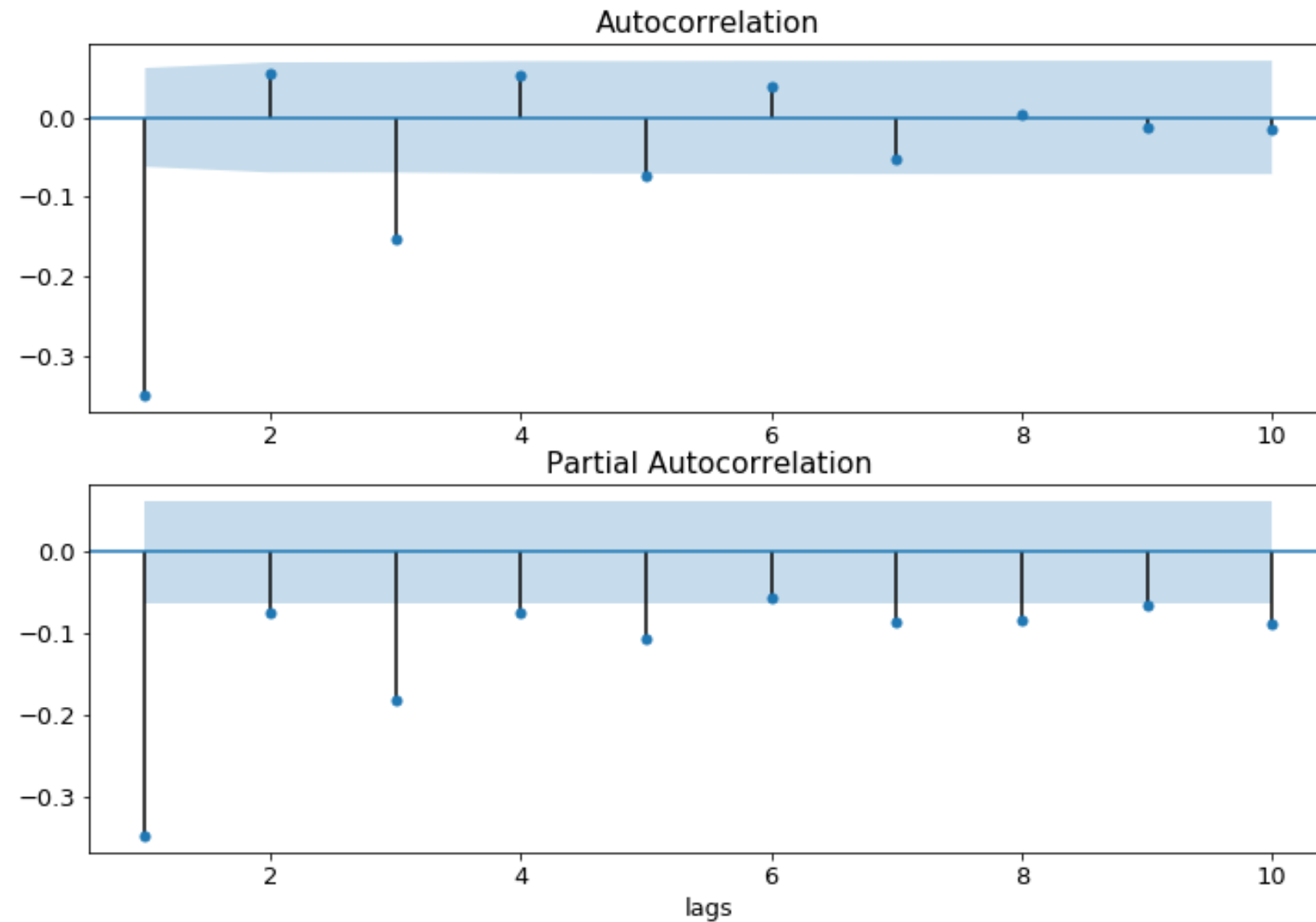
# Implementation in Python



# Over/under differencing and ACF and PACF



# Over/under differencing and ACF and PACF



# Let's practice!

FORECASTING USING ARIMA MODELS IN PYTHON

# AIC and BIC

FORECASTING USING ARIMA MODELS IN PYTHON



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# AIC - Akaike information criterion

- Lower AIC indicates a better model
- AIC likes to choose simple models with lower order

# BIC - Bayesian information criterion

- Very similar to AIC
- Lower BIC indicates a better model
- BIC likes to choose simple models with lower order

# AIC vs BIC

- BIC favors simpler models than AIC
- AIC is better at choosing predictive models
- BIC is better at choosing good explanatory model

# AIC and BIC in statsmodels

```
# Create model
model = SARIMAX(df, order=(1,0,1))

# Fit model
results = model.fit()

# Print fit summary
print(results.summary())
```

## Statespace Model Results

```
=====
Dep. Variable:          y      No. Observations:          1000
Model:                SARIMAX(2, 0, 0)  Log Likelihood      -1399.704
Date:                Fri, 10 May 2019    AIC                 2805.407
Time:                01:06:11           BIC                 2820.131
Sample:              01-01-2013          HQIC                2811.003
                  - 09-27-2015
Covariance Type:      opg
```

# AIC and BIC in statsmodels

```
# Create model
model = SARIMAX(df, order=(1,0,1))

# Fit model
results = model.fit()

# Print AIC and BIC
print('AIC:', results.aic)
print('BIC:', results.bic)
```

AIC: 2806.36

BIC: 2821.09

# Searching over AIC and BIC

```
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):
        # Fit model
        model = SARIMAX(df, order=(p,0,q))
        results = model.fit()

        # print the model order and the AIC/BIC values
        print(p, q, results.aic, results.bic)
```

```
0 0 2900.13 2905.04
0 1 2828.70 2838.52
0 2 2806.69 2821.42
1 0 2810.25 2820.06
1 1 2806.37 2821.09
1 2 2807.52 2827.15
...
```

# Searching over AIC and BIC

```
order_aic_bic = []
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):
        # Fit model
        model = SARIMAX(df, order=(p, 0, q))
        results = model.fit()

        # Add order and scores to list
        order_aic_bic.append((p, q, results.aic, results.bic))

# Make DataFrame of model order and AIC/BIC scores
order_df = pd.DataFrame(order_aic_bic, columns=['p', 'q', 'aic', 'bic'])
```

# Searching over AIC and BIC

```
# Sort by AIC
print(order_df.sort_values('aic'))
```

	p	q	aic	bic
7	2	1	2804.54	2824.17
6	2	0	2805.41	2820.13
4	1	1	2806.37	2821.09
2	0	2	2806.69	2821.42
...				

```
# Sort by BIC
print(order_df.sort_values('bic'))
```

	p	q	aic	bic
3	1	0	2810.25	2820.06
6	2	0	2805.41	2820.13
4	1	1	2806.37	2821.09
2	0	2	2806.69	2821.42
...				



# Non-stationary model orders

```
# Fit model  
model = SARIMAX(df, order=(2,0,1))  
results = model.fit()
```

```
ValueError: Non-stationary starting autoregressive parameters  
found with `enforce_stationarity` set to True.
```

# When certain orders don't work

```
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):

        # Fit model
        model = SARIMAX(df, order=(p, 0, q))
        results = model.fit()

        # Print the model order and the AIC/BIC values
        print(p, q, results.aic, results.bic)
```

# When certain orders don't work

```
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):
        try:
            # Fit model
            model = SARIMAX(df, order=(p,0,q))
            results = model.fit()

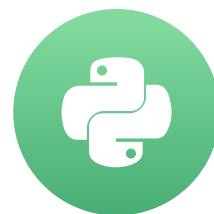
            # Print the model order and the AIC/BIC values
            print(p, q, results.aic, results.bic)
        except:
            # Print AIC and BIC as None when fails
            print(p, q, None, None)
```

# Let's practice!

FORECASTING USING ARIMA MODELS IN PYTHON

# Model diagnostics

FORECASTING USING ARIMA MODELS IN PYTHON



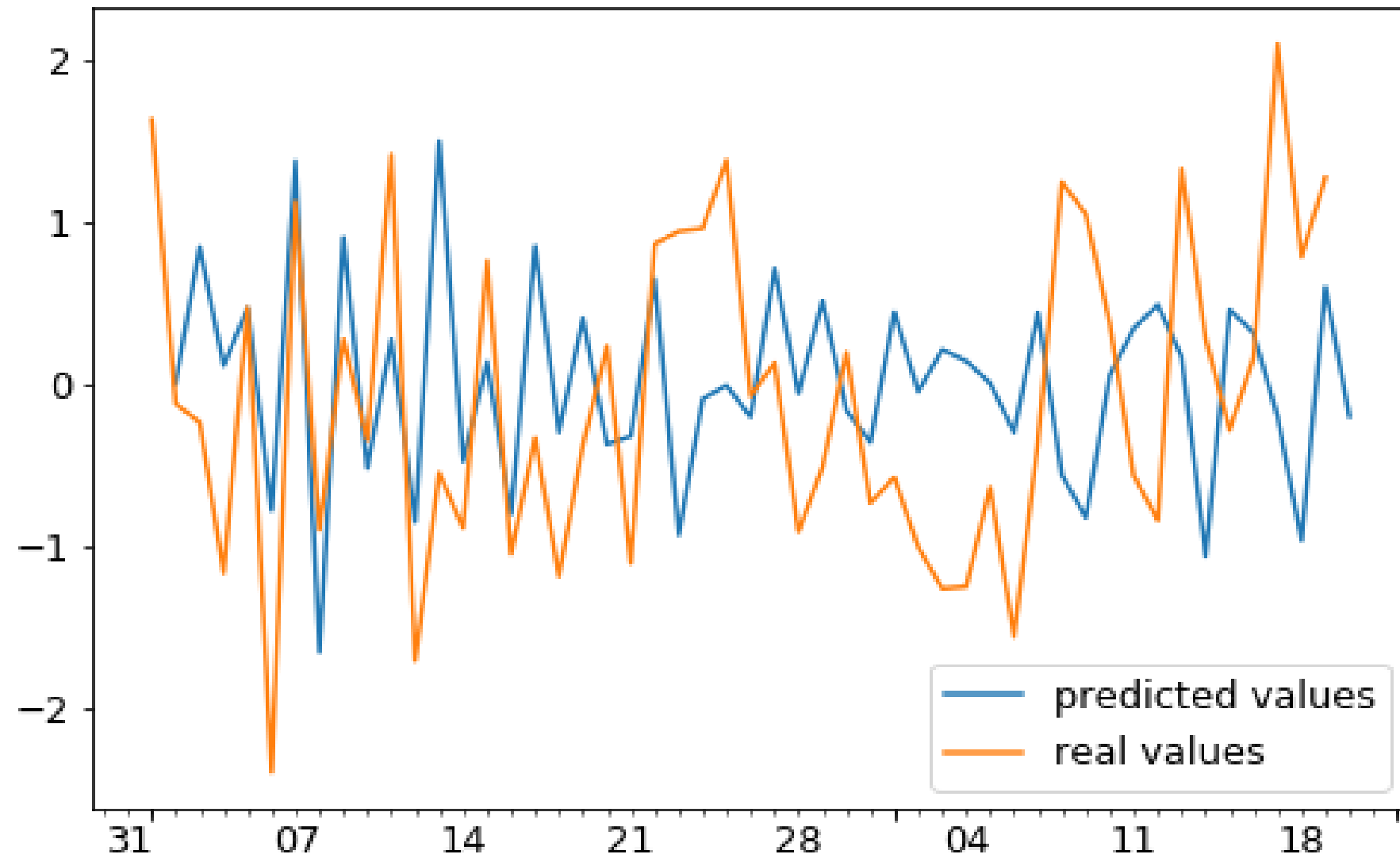
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# Introduction to model diagnostics

- How good is the final model?

# Residuals



# Residuals

```
# Fit model
model = SARIMAX(df, order=(p,d,q))
results = model.fit()

# Assign residuals to variable
residuals = results.resid
```

```
2013-01-23    1.013129
2013-01-24    0.114055
2013-01-25    0.430698
2013-01-26   -1.247046
2013-01-27   -0.499565
...          ...
```



# Mean absolute error

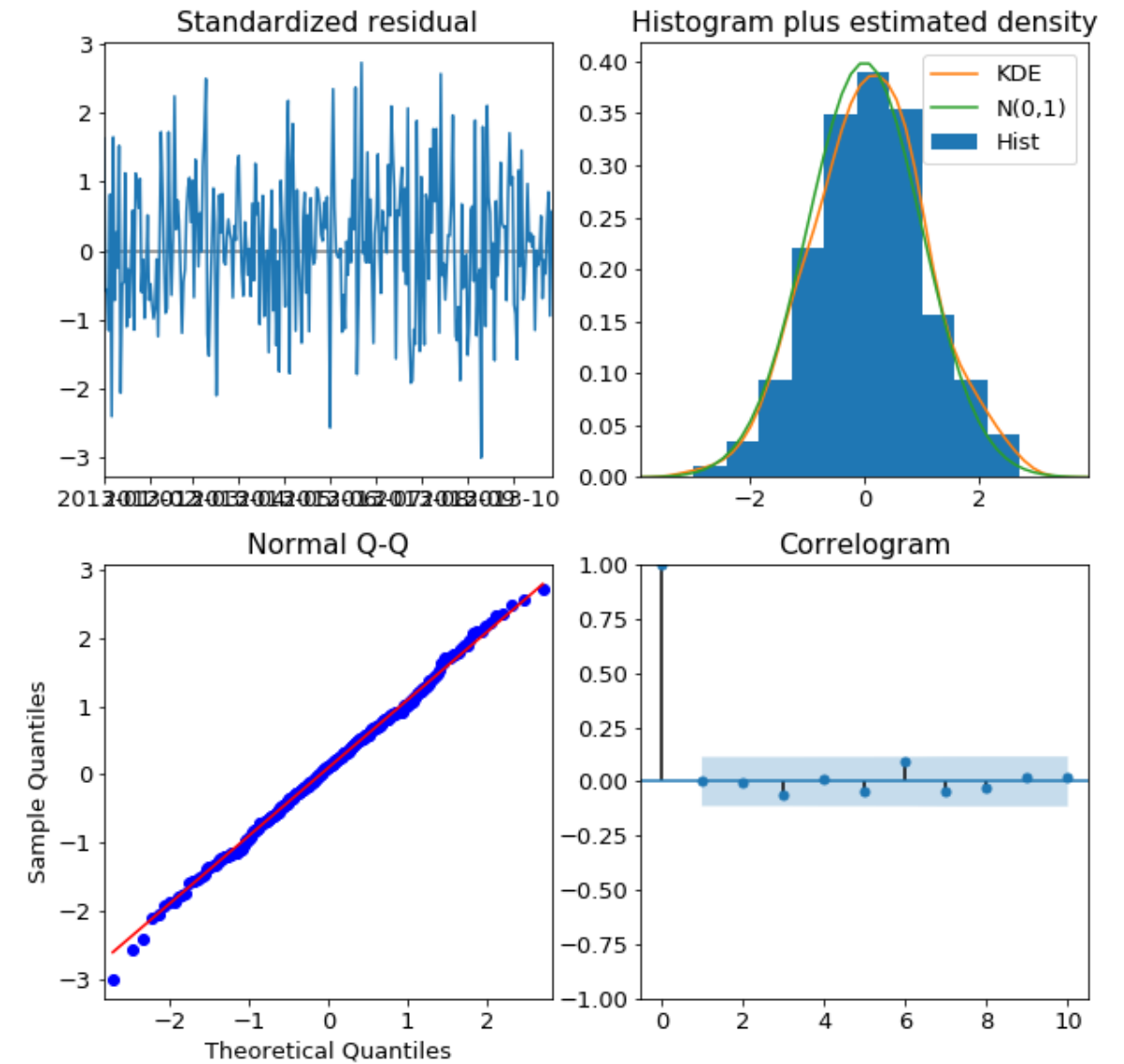
How far are the predictions from the real values?

```
mae = np.mean(np.abs(residuals))
```

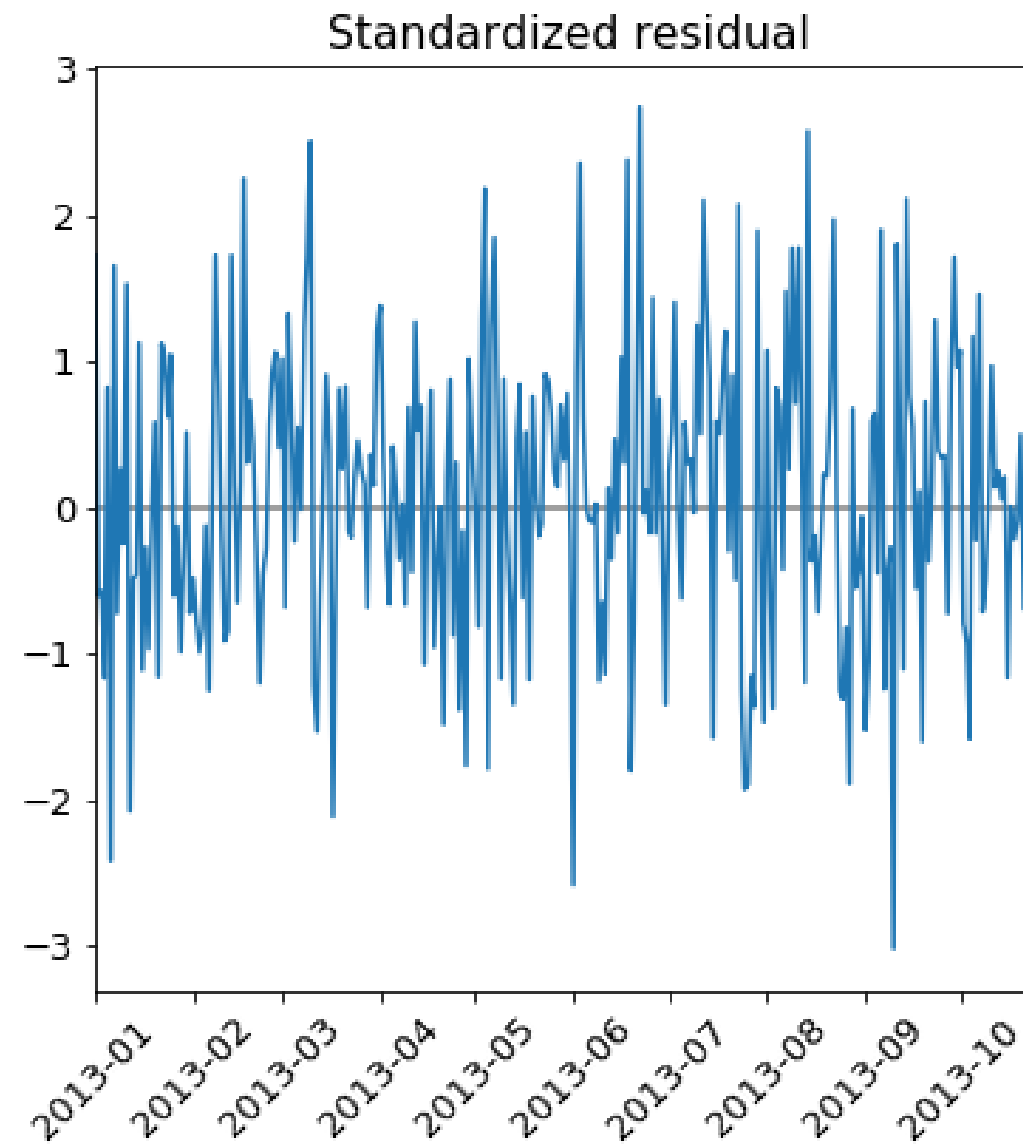
# Plot diagnostics

If the model fits well the residuals will be white Gaussian noise

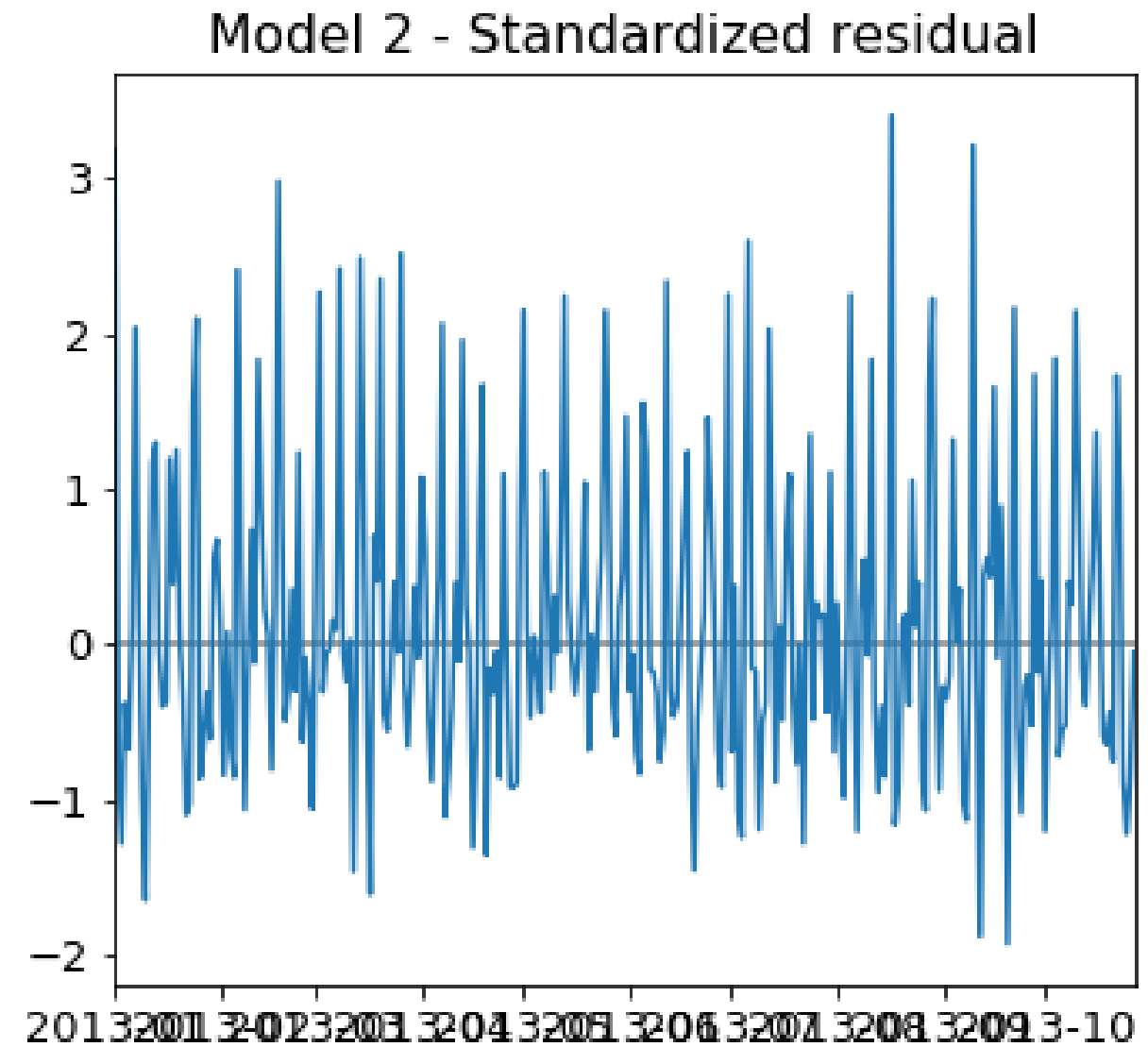
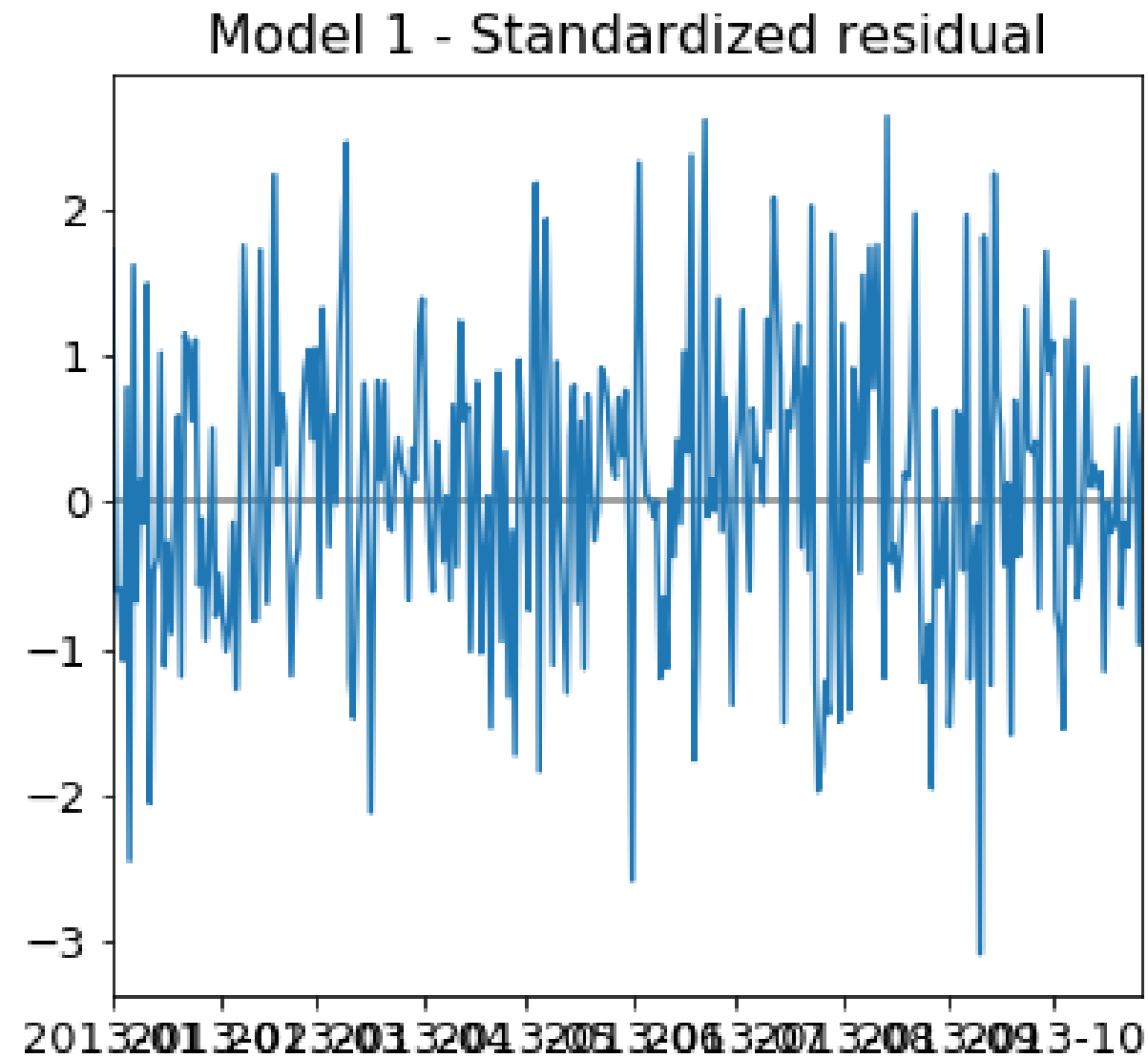
```
# Create the 4 diagnostics plots
results.plot_diagnostics()
plt.show()
```



# Residuals plot

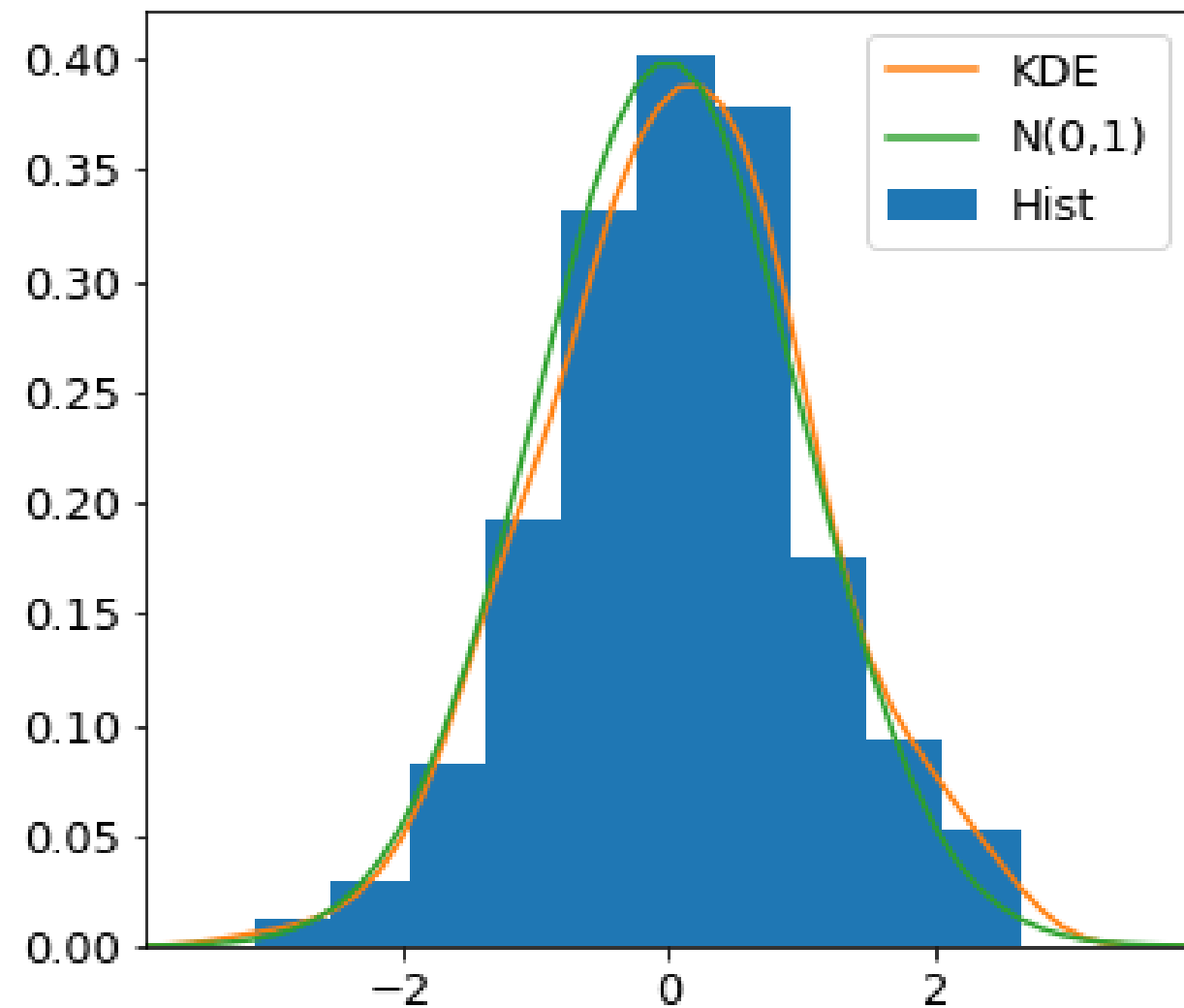


# Residuals plot

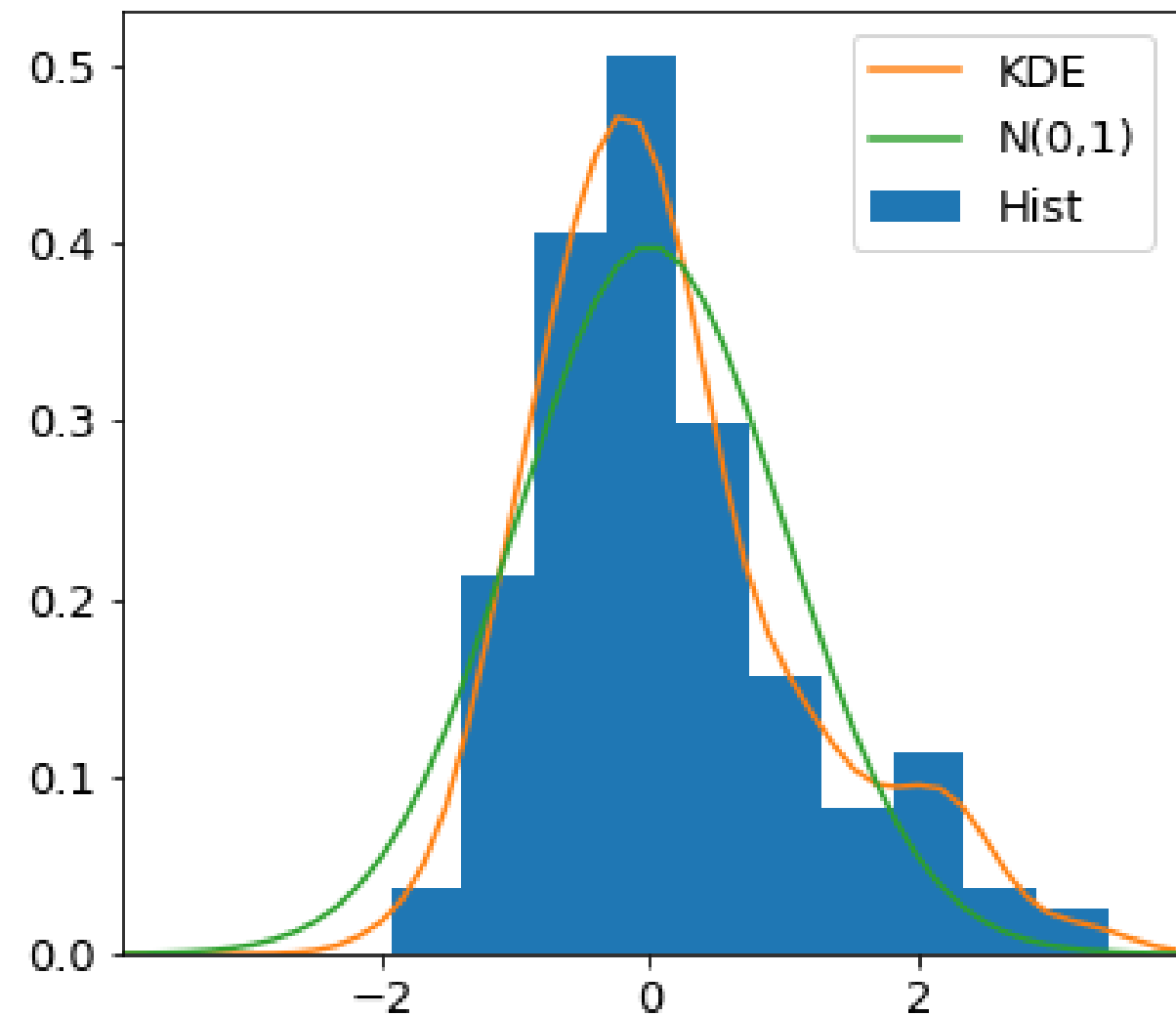


# Histogram plus estimated density

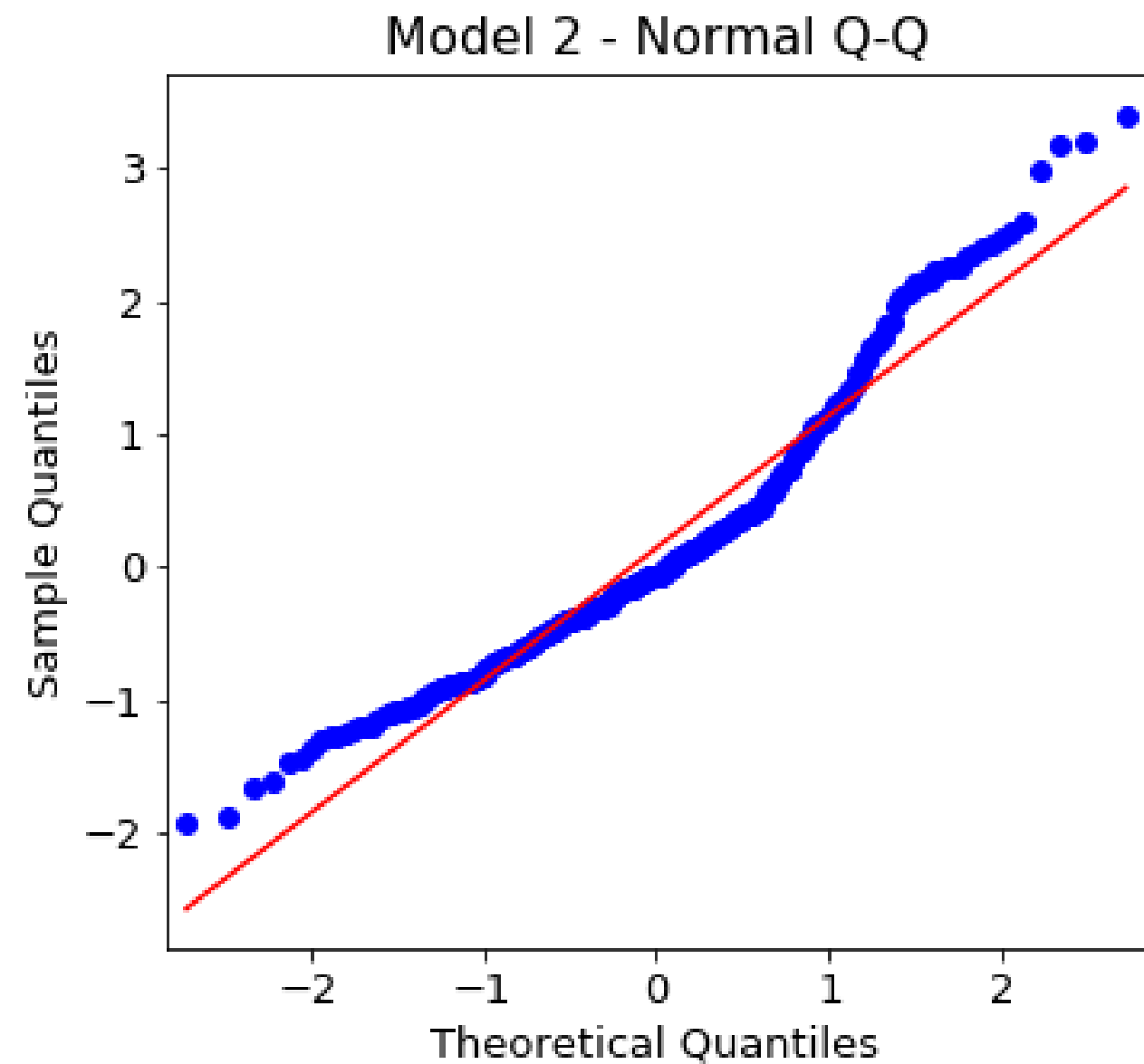
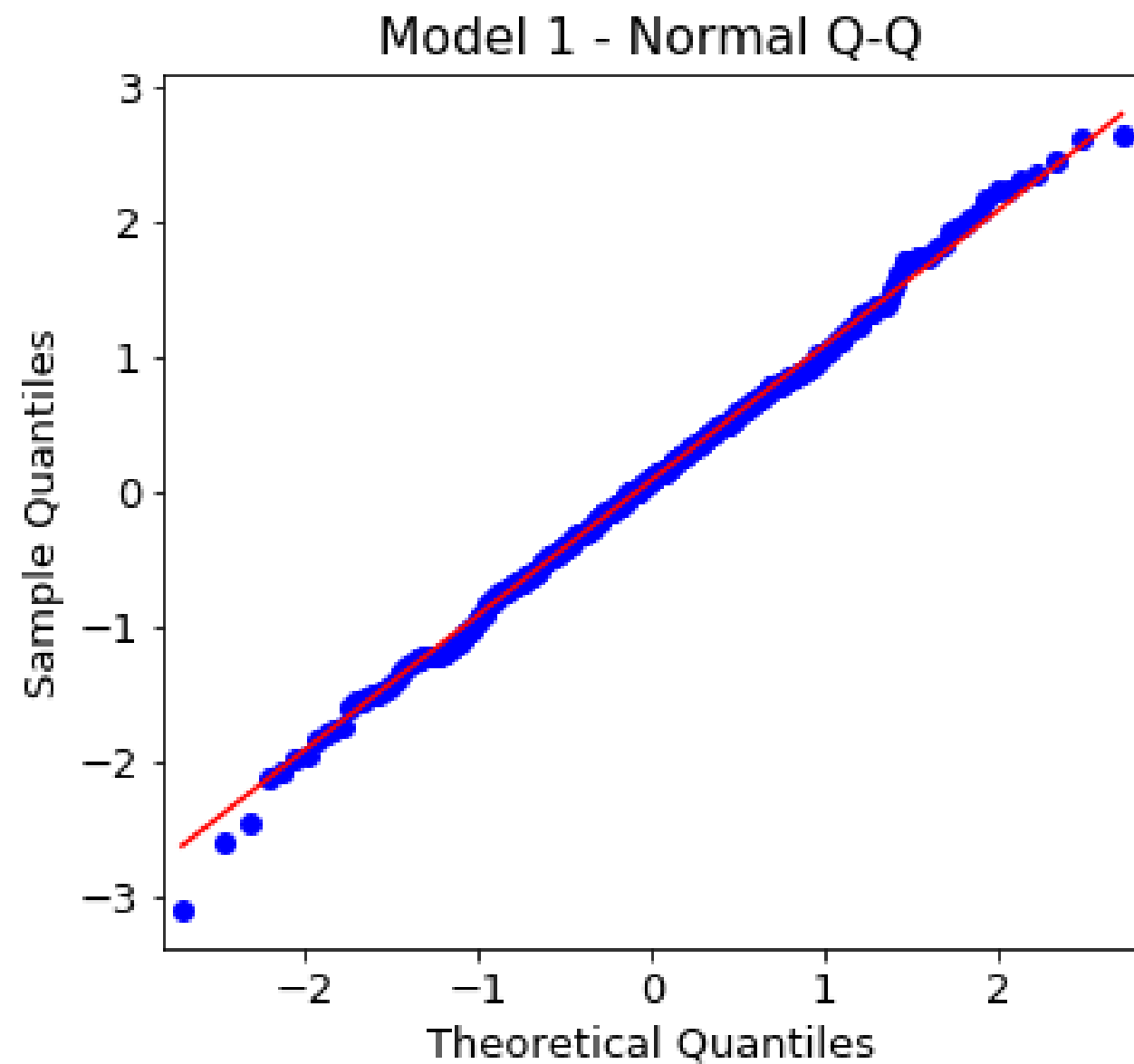
Model 1 - Histogram plus estimated density



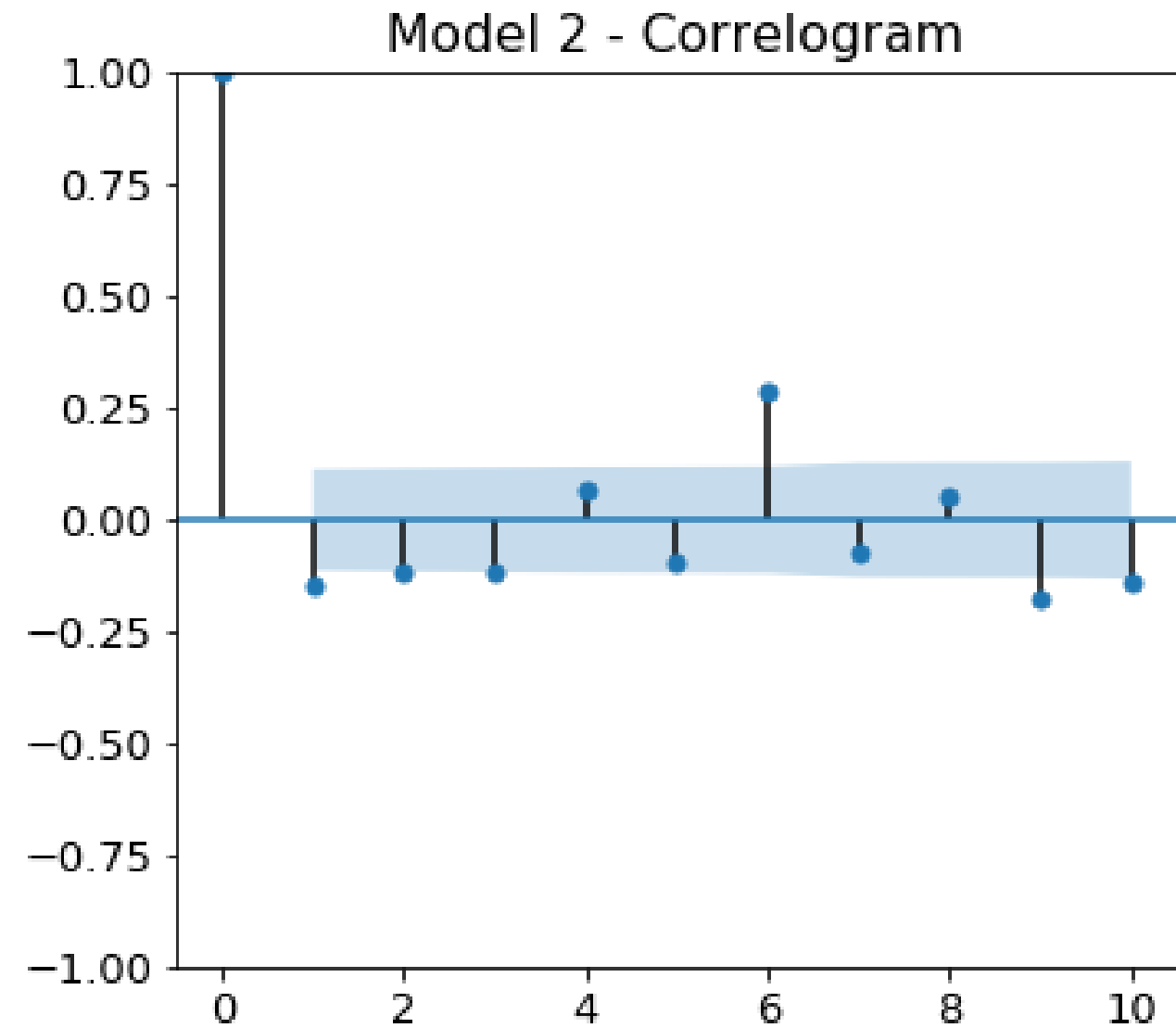
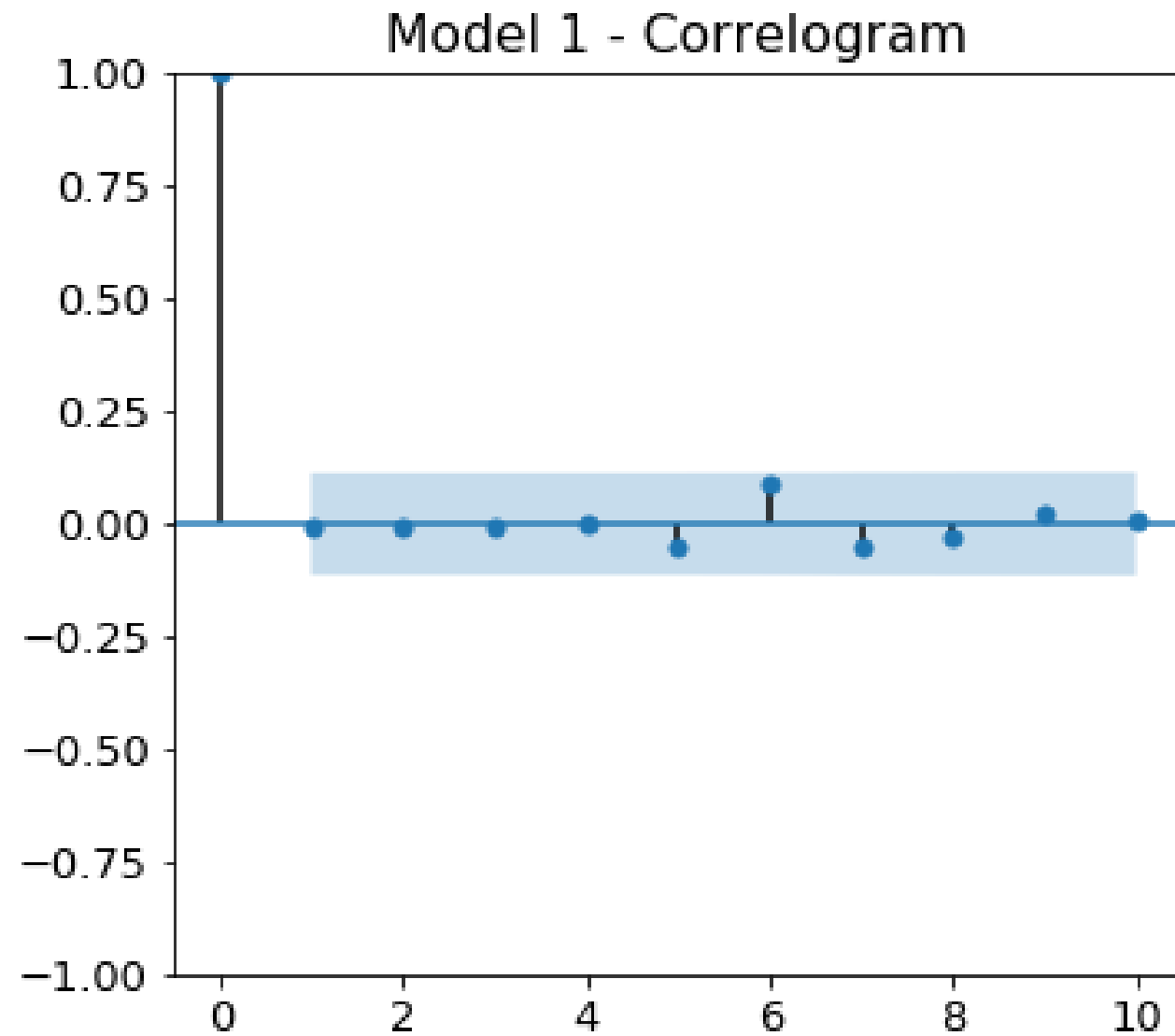
Model 2 - Histogram plus estimated density



# Normal Q-Q



# Correlogram



# Summary statistics

```
print(results.summary())
```

```
...
=====
Ljung-Box (Q):                32.10    Jarque-Bera (JB):                0.02
Prob(Q):                      0.81     Prob(JB):                     0.99
Heteroskedasticity (H):        1.28     Skew:                         -0.02
Prob(H) (two-sided):           0.21     Kurtosis:                     2.98
=====
```

- `Prob(Q)` - p-value for null hypothesis that residuals are uncorrelated
- `Prob(JB)` - p-value for null hypothesis that residuals are normal



# Let's practice!

FORECASTING USING ARIMA MODELS IN PYTHON

# Box-Jenkins method

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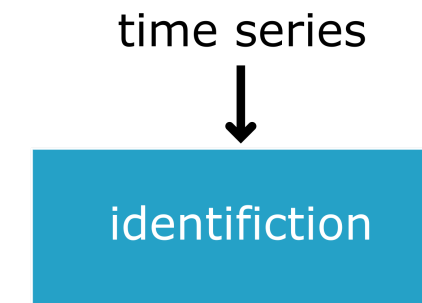
# The Box-Jenkins method

From raw data → production model

- identification
- estimation
- model diagnostics

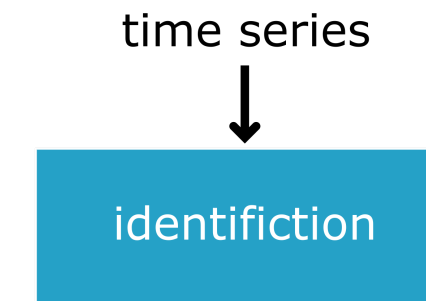
# Identification

- Is the time series stationary?
- What differencing will make it stationary?
- What transforms will make it stationary?
- What values of  $p$  and  $q$  are most promising?



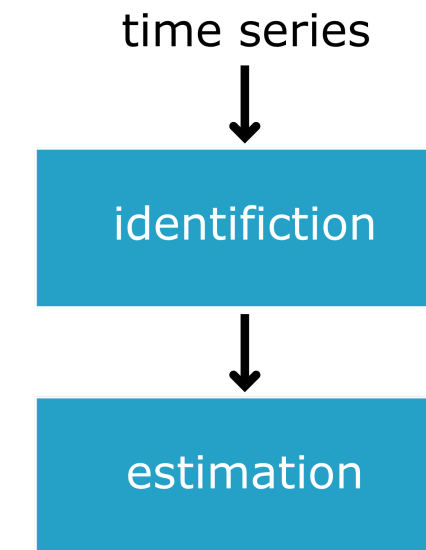
# Identification tools

- Plot the time series
  - `df.plot()`
- Use augmented Dicky-Fuller test
  - `adfuller()`
- Use transforms and/or differencing
  - `df.diff()` , `np.log()` , `np.sqrt()`
- Plot ACF/PACF
  - `plot_acf()` , `plot_pacf()`



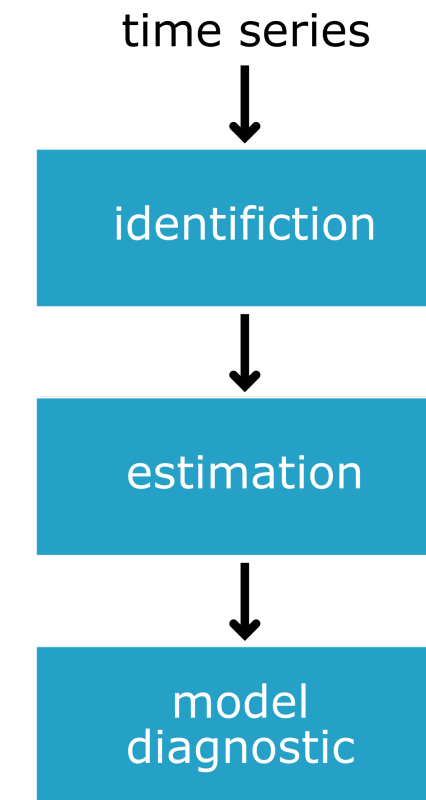
# Estimation

- Use the data to train the model coefficients
- Done for us using `model.fit()`
- Choose between models using AIC and BIC
  - `results.aic` , `results.bic`

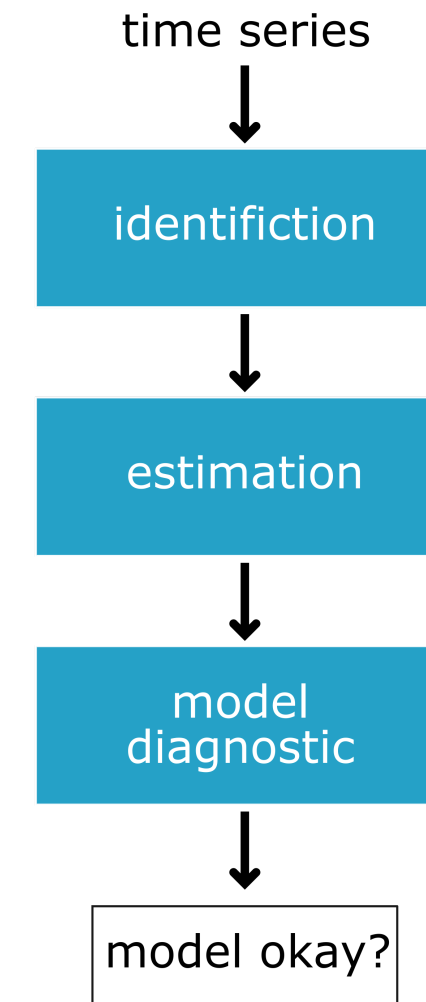


# Model diagnostics

- Are the residuals uncorrelated
- Are residuals normally distributed
  - `results.plot_diagnostics()`
  - `results.summary()`



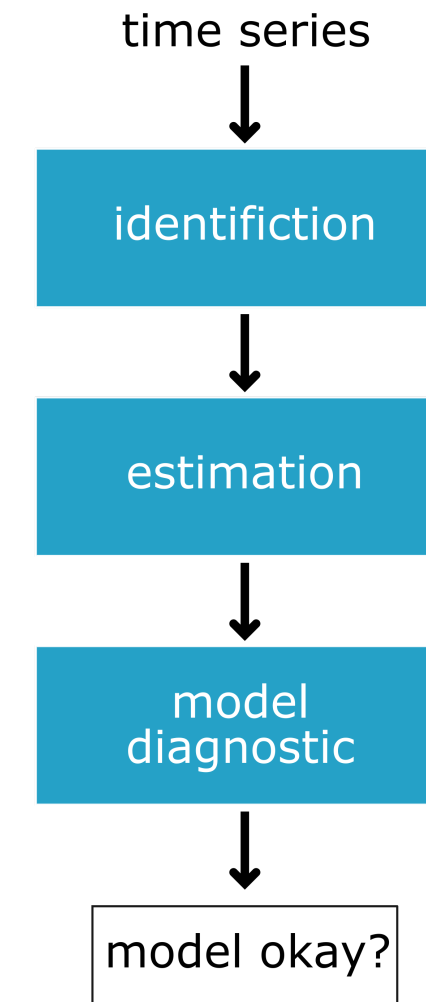
# Decision





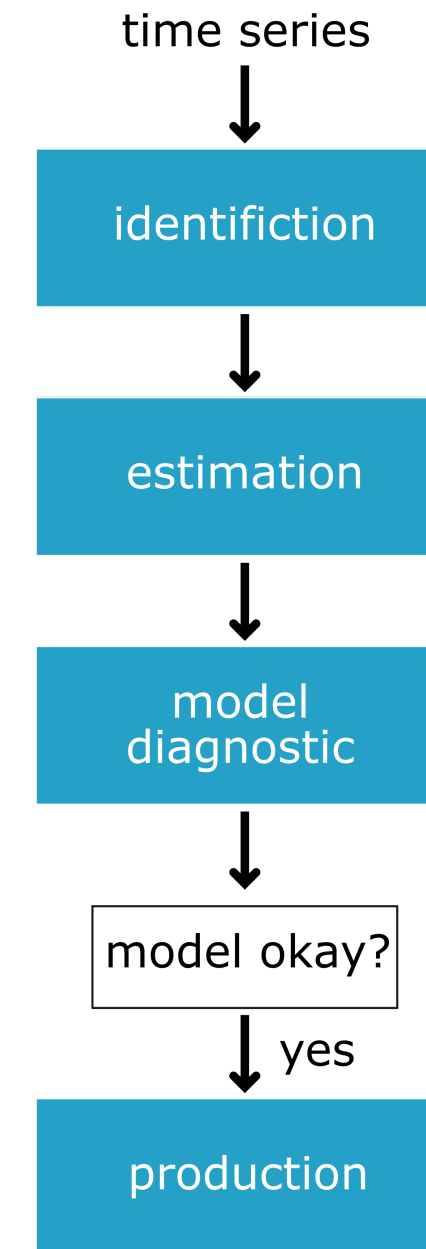
# Repeat

- We go through the process again with more information
- Find a better model

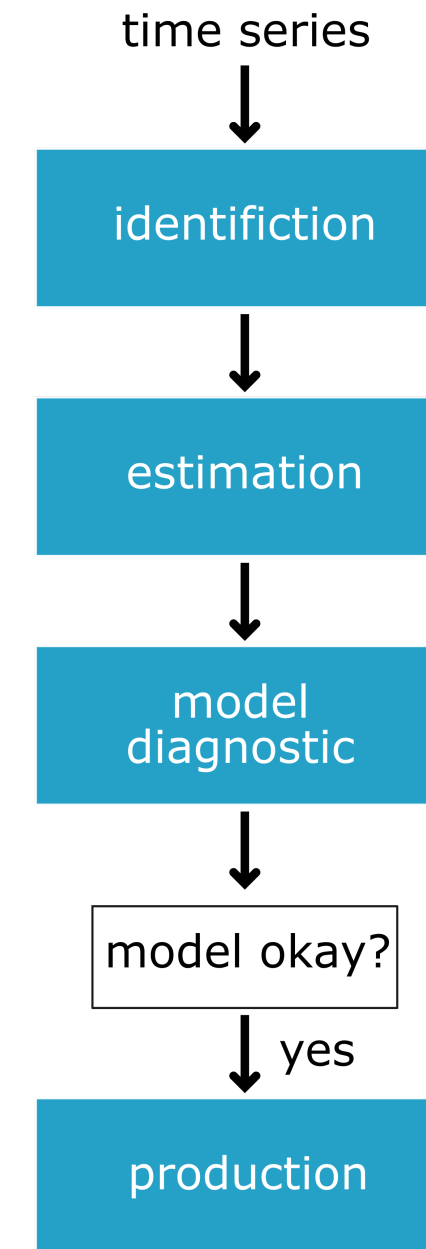


# Production

- Ready to make forecasts
  - `results.get_forecast()`



# Box-Jenkins



# Let's practice!

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