

KMV-Merton equations

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1 Questions

Ex. 1 — Consider the two KMV equations:

$$S = V_0 N(d_1) - F e^{-iT} N(d_2)$$

and

$$\sigma_S = N(d_1) \frac{V_0}{S} \sigma$$

The total value of a company's equity is $S = 100$ million dollars and the volatility of equity is $\sigma_S = 0.25$. The total debt of $F = 70$ million dollars will have to be paid in $T = 2$ years. The risk-free rate is $i = 3\%$.

Solve the two equations for the value of the assets (V) and the volatility of the assets (σ).

Remark on scaling. For better numerical results, write the first equation as

$$1 = \frac{V_0}{S} N(d_1) - \frac{F}{S} e^{-iT} N(d_2)$$

Hint: There are several alternative ways of solving this.

- Alternative 1: Use `fsolve()` in Matlab.
- Alternative 2: What we have is $F(x,y)=0$ and $G(x,y)=0$, so use any numerical optimizer — like the solver in Excel or `fminsearch()` in Matlab — to find (x, y) that minimize $F^2 + G^2$.