Nova School of Business and Economics

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KMV-Merton equations

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Ex. 1 — Consider the two KMV equations:

$$S = V_0 N(d_1) - Fe^{iT} N(d_2)$$

and

$$\sigma_S = N(d_1)\frac{V_0}{S}\sigma$$

The total value of company's equity is S = 100 million dollars and the volatility of equity is $\sigma_S = 0.25$. The total debt of F = 70 million dollars will have to be paid in T = 2 years. The risk-free rate is i = 3%. Solve the two equations for the value of the assets (V) and the volatility of the assets (σ).

The KMV model was established by Keaholfer, McQuown and Vasicek in 1974 and is founded on assumptions of Mertons bond pricing model. In 2002, it was bought by Moody's Analytics, and is now maintained and developed on a continuous basis by Moody's KMV, a division of Moody's Analytics [1]. In comparison to J.P. Morgan's $CreditMetrics^{TM}$, KMV provides a "rating" model rather than a "Value at Risk due to credit" model [2]. KMV's approach is also known as the structural approach of pricing credit risk as it includes the asset-liability structure of a company to calculate EDF (Expected default frequency) [2]. The EDF is a firm specific, forward-looking measure of actual probability of default. The main difficulty, as well as in other structural models, is how to assign dynamics to the company value, which is an unobserved process [3].

In general, three steps are required to derive the actual probabilities of default:

1. Estimation of the market value and volatility of the firm's asset
2. Calculation of the distance to default, an index measure of default risk
3. Scaling of the distance to default to actual probabilities of default using a default database

In this paper though, the focus only lies on the first of the above mentioned points - the estimation of the firm's assets and volatility. Given the data of the firm as provided in the task, it is possible to arrange a system of two equations and two unknowns, which can be represented as:

$$1 = \frac{V_0}{S} N(d_1) - \frac{F}{S} e^{-iT} N(d_2)$$

In order to estimate the parameters of the two unknows, V_0 and σ, one needs to equate both KMV equations to zero, such that:

$$\frac{V_0}{S} N(d_1) - \frac{F}{S} e^{-iT} N(d_2) - 1 = 0 \qquad (1)$$

$$N(d_1)\frac{V_0}{S}\sigma - \sigma_S = 0 \qquad (2)$$

The first step is to calculate the value of d1 and d2. Once the values are computed, one can estimate N(d1) and N(d2). Finally, in the last step, the group computed the sum of squared errors from the two equations above and minimized them subject to V_0 and σ using the Solver optimization function in Excel. The estimated result for the value of assets equals to $165.92 million and for the volatility, a value of 15.07% was computed.

In Table 1, a summary of all estimated values can be seen. For a detailed overview of the calculations, please refer to the attached Excel.

Parameter	Value
d1	4.44
d2	4.23
σ_S	25.00%
Value of Assets (in $ Mio.)	165.92
Volatility of Assets	15.07%

Table 1: Estimated results

Auxiliary

All materials (Excel and pdf) for this assignment are available on the author's Github account.

References

[1] Pasquale Cirillo. *Default Probabilities - The KMV Model*. https://courses.edx.org/c4x/DelftX/TW3421x/asset/Week5_PD2_3.pdf. Accessed on 2021-04-15.

[2] Florian Rehm and Markus Rudolf. "KMV Credit Risk Modeling". In: *Risk Management* (2000), pp. 141–154. DOI: https://doi.org/10.1007/978-3-662-04008-9_8.

[3] Tomas Kliestik, Maria Misankova, and Katarina Kocisova. "Calculation of distance to default". In: *Procedia Economics and Finance* 23 (2015), pp. 238–243. DOI: http://core.ac.uk/download/pdf/82141934.pdf.