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Spreads in the Merton model

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Credit Risk

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1 Inputs

Consider the following parameters for the Merton model:

- Risk-free rate = 0.05
- Asset volatility (σ) = 0.2
- Face value of debt (F) = 100

2 Questions

Ex. 1 — Assume that the Assets Value (V_0) is 200. Plot the default spread for maturities up to 10 years (start at 0.2 years).

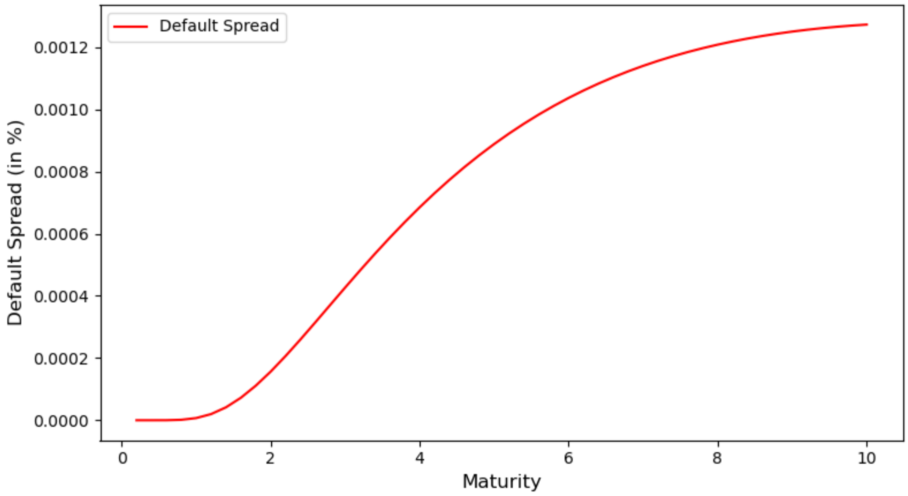

FIGURE 1: Default Spreads for $V_0 = 200$

Ex. 2 — Assume that the Assets Value (V_0) is 95. Plot the default spread for maturities up to 10 years (start at 0.2 years).

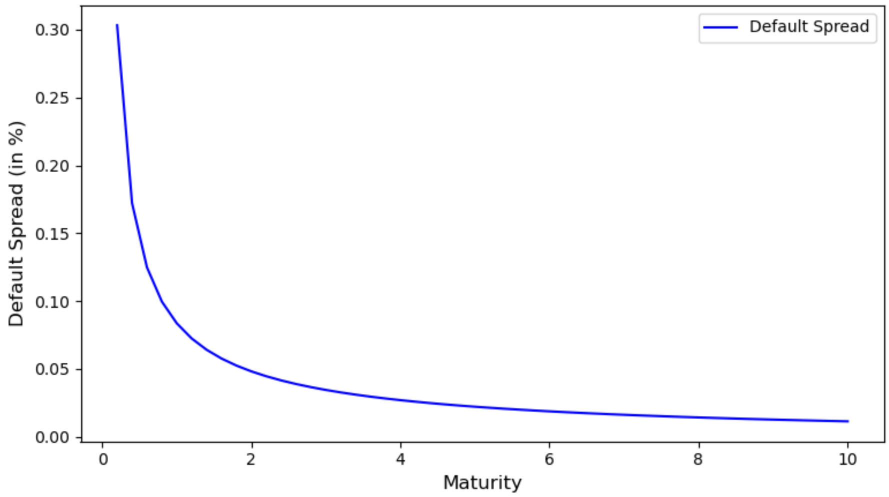

FIGURE 2: Default Spreads for $V_0 = 95$

Merton developed a structural model based on the Black Scholes (1973) option pricing model. This model can both be used for equity valuation and credit risk management [1]. In the Merton model, it is assumed that the total value of the firm (V_0) follows a geometric Brownian motion process, which can be defined as

$$dV = \mu V dt + \sigma_V V dW$$

where V is the total value of the firm's assets (random variable), μ is the expected continuous return of V, σv is the firm value volatility, and dW is the Gauss-Wiener process.

As mentioned, the Merton model uses the Black and Scholes model of options in which the firm's equity value follows the stipulated process for a call option. A call option on the underlying assets has the same properties as a caller has, namely, a demand on the assets after reaching the strike price of the option. Formally, this can be expressed as:

$$E_T = \begin{cases} A_T - K & \text{if } A_T > K \\ 0 & \text{if } A_T \leq K \end{cases}$$

In this case, the exercise price of the option equals the book value of the firm's obligations. If the value of the assets is insufficient to cover the firm's obligations, then shareholders with a call option do not exercise their option and leave the firm to their creditors [2]. Thus,

$$E_T = max(A_T - K, 0).$$

This is exactly the payoff of a call option on A_T with a strike price K and maturity T. Therefore, the Black-Scholes option pricing methods can be applied, assuming the asset value follows a GBM. Using the formula for the price of a call option, the value of equity at time 0 is

$$Call(V_0, F, r, T, \sigma) = V_0 \cdot N(d_1) - F \cdot e^{-rT} \cdot N(d_2)$$

where

- $\mathrm{d}_1 = \frac{ln(\frac{V}{F}) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}$
- $\mathrm{d}_2 = d_1 - \sigma\sqrt{T}$

Thus, the payoff to debt holders is then

$$D_0 = V_0 - E_0 = V_0 - Call(V_0, F, r, T, \sigma)$$

An alternative (more intuitive) expression for the value of debt is:

$$D_0 = F \cdot e^{-rT} - Put(V_0, F, r, T, \sigma)$$

By using the fundamental insight of the Merton model as described above, we get that the payoff of a defaultable corporate bond can be replicated by a combination of a long position in a riskless zero-coupon bond and a short position in a put option written on the value of the firm (V_0) and strike price (K) equal to the face value of debt (F). In equilibrium, this strategy must produce a return equal to the risk-free rate (r), i.e.,

$$B_0 = Fe^{-r \cdot T} - P_0$$

The default or credit spread equals the difference between the risk free rate, r, and the bond yield, $Y(\tau)$, which can formally be presented as

$$\text{default spread (s)} = Y(\tau) - r$$
$$\text{default spread (s)} = -\frac{1}{\tau} ln \left(N(d_2) + \frac{V_0}{Fe^{-r\tau}}(-d_1) \right)$$

The time-dependent behaviors of the credit or default spread depends on whether d $\geq$ 1 or d $<$ 1 (see Figure 3), where d can be defined as [3]

$$d = \frac{Fe^{-r\tau}}{V_0} = \text{“quasi” debt-to-asset ratio}$$

- As time approaches maturity, the credit spread always tends to zero when d $\leq$ 1 but tends toward infinity when d $>$ 1
- At times far from maturity, the credit spread has low value for all values of d since sufficient time has been allowed for the firm value to have a higher potential to grow beyond F

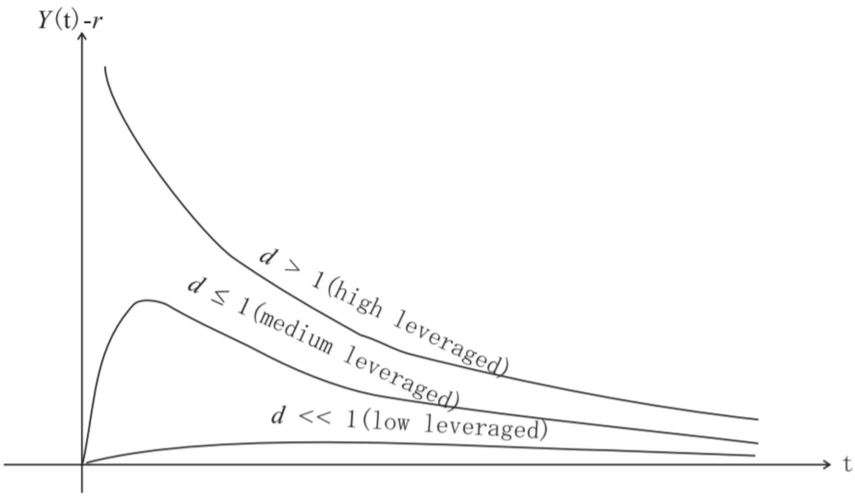

Figure 3: Credit Spreads dependent on d

By applying the default spread equation as defined above, the group got the spreads (s) for different maturities T = 0.20, 0.40, 0.60, . . . 9.60, 9.80, 10, which can be seen in Figure 1 and Figure 2.

Ex. 3 — Comment on the different shapes of these two plots. In particular, for each plot give an intuitive and short explanation for the following:

1. Why do the spreads increase/decrease?

   In the first scenario, where $V_0 = 200$ and F = 100, the firm is in solid financial conditions, with smaller spreads for smaller maturities. However, as the time to maturity increases, the risk that the firm will experience financial distress also increases, which again must be reflected in an increase in the credit spread.

   In the second scenario, where $V_0 = 95$ and F = 100, the firm seems to be insolvent right from the beginning - thus, the default spreads (s) are higher than compared to the first scenario. As maturity increases, the probability that the firm will recover from its current situation increases. As a result, the company will be able to meet its fixed debt payment and therefore, the spread will decrease for future maturities.

2. Do short-term spreads (up to around 1 year) seem reasonable? Why do they look like they do?

   In the first scenario, where $V_0 = 200$ and F = 100, the firm seems to be in a solvent financial condition. Thus, for small maturities (up to 1 year), the company will most likely be able to repay the value of debt, and this must also be reflected in smaller spreads (here: almost zero), which indeed is observable in Figure 1.

   In the second scenario, where $V_0 = 95$ and F = 100, the firm seems to be insolvent right from the beginning. For small maturities (up to 1 year) the company will most likely default on the promised payments and the default spread (s) must increase accordingly. This behavior results in overly exaggerated short-term spreads that no longer reflect a premium over and above the risk-free rate, but merely signal that the firm will default. One additional reason, why the graph looks the way it does, is by computing the *"quasi" debt-to-asset ratio d* as defined above. By plugging in the values, one gets for d = 1.00129, which according to Figure 3 means that the firm has a high leverage, which is also in-line with the plotted Figure 2 (downward sloping spread-curve).

Auxiliary

All materials (Python code and pdf) for this assignment are available on the author's Github account.

References

[1] Breaking Down Finance. *Merton Credit Risk Model.* https://breakingdownfinance.com/finance-topics/risk-management/merton-model/. Accessed on 2021-04-16. 2019.

[2] João Gabriel de Moraes Souza. *Credit Risk - Estimating Bank Default Models.* https://lamfo-unb.github.io/2020/05/26/Credit-Risk-Estimating-Bank-Default-Models/. Accessed on 2021-04-16. May 2020.

[3] Rating Express. *Credit valuation model: Merton's firm value model.* https://www.ratingexpress.net/content/MArton.pdf. Accessed on 2021-04-16.