1. **WRITING PROJECT. Report on Path Independence.** Make a list of the main ideas and facts on path independence and dependence in this section. Then work this list into a report. Explain the definitions and the practical usefulness of the theorems, with illustrative examples of your own. No proofs.

2. **On Example 4.** Does the situation in Example 4 of the text change if you take the domain $0 < \sqrt{x^2 + y^2} < 3/2$?

3–9 PATH INDEPENDENT INTEGRALS

Show that the form under the integral sign is exact in the plane (Probs. 3–4) or in space (Probs. 5–9) and evaluate the integral. Show the details of your work.

3. $\int_{(\pi/2,\,\pi)}^{(\pi,\,0)} (\frac{1}{2}\cos\frac{1}{2}x\cos 2y\,dx - 2\sin\frac{1}{2}x\sin 2y\,dy)$

4. $\int_{(4,\,0)}^{(6,\,1)} e^{4y}(2x\,dx + 4x^2\,dy)$

5. $\int_{(0,\,0,\,\pi)}^{(2,\,1/2,\,\pi/2)} e^{xy}(y\sin z\,dx + x\sin z\,dy + \cos z\,dz)$

6. $\int_{(0,\,0,\,0)}^{(1,\,1,\,0)} e^{x^2+y^2+z^2}(x\,dx + y\,dy + z\,dz)$

7. $\int_{(0,\,2,\,3)}^{(1,\,1,\,1)} (yz\sinh xz\,dx + \cosh xz\,dy + xy\sinh xz\,dz)$

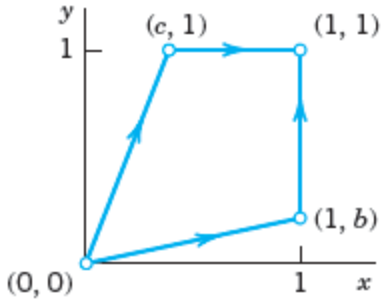
8. $\int_{(5,\,3,\,\pi)}^{(3,\,\pi,\,3)} (\cos yz\,dx - xz\sin yz\,dy - xy\sin yz\,dz)$

9. $\int_{(0,\,1,\,0)}^{(1,\,0,\,1)} (e^x\cosh y\,dx + (e^x\sinh y + e^z\cosh y)\,dy + e^z\sinh y\,dz)$

10. **PROJECT. Path Dependence.** **(a)** Show that $I = \int_C (x^2y\,dx + 2xy^2\,dy)$ is path dependent in the xy-plane.

(b) Integrate from (0, 0) along the straight-line segment to $(1, b)$, $0 \leqq b \leqq 1$, and then vertically up to (1, 1); see the figure. For which b is I maximum? What is its maximum value?

(c) Integrate I from (0, 0) along the straight-line segment to $(c, 1)$, $0 \leqq c \leqq 1$, and then horizontally to (1, 1). For $c = 1$, do you get the same value as for $b = 1$ in (b)? For which c is I maximum? What is its maximum value?



Project 10. Path Dependence

11. On Example 4. Show that in Example 4 of the text, $\mathbf{F} = \text{grad}\,(\arctan\,(y/x))$. Give examples of domains in which the integral is path independent.

12. CAS EXPERIMENT. Extension of Project 10. Integrate $x^2y\,dx + 2xy^2\,dy$ over various circles through the points (0, 0) and (1, 1). Find experimentally the smallest value of the integral and the approximate location of the center of the circle.

13–19 PATH INDEPENDENCE?

Check, and if independent, integrate from (0, 0, 0) to (a, b, c).

13. $2e^{x^2}(x\cos 2y\,dx - \sin 2y\,dy)$

14. $(\sinh xy)\,(z\,dx - x\,dz)$

15. $x^2y\,dx - 4xy^2\,dy + 8z^2x\,dz$

16. $e^y\,dx + (xe^y - e^z)\,dy - ye^z\,dz$

17. $4y\,dx + z\,dy + (y - 2z)\,dz$

18. $(\cos xy)(yz\,dx + xz\,dy) - 2\sin xy\,dz$

19. $(\cos\,(x^2 + 2y^2 + z^2))\,(2x\,dx + 4y\,dy + 2z\,dz)$

20. Path Dependence. Construct three simple examples in each of which two equations (6′) are satisfied, but the third is not.