1)

$\pi \leftrightarrow z \vee y$ 2)

$y \leftrightarrow b \wedge w$

$z \leftrightarrow a \wedge x$

$w \leftrightarrow \neg a$

$x \leftrightarrow \neg b$

The logical equivalent of the circuit is

$\phi = (a \wedge \neg b) \vee (b \wedge \neg a)$

The Tseytin transformation would be

$\psi =$

$(\neg \pi \vee (z \vee y)) \wedge (\neg(z \vee y) \vee \pi) \wedge ((\neg y \vee (b \wedge w)) \wedge$
$(y \vee \neg(b \wedge w)) \wedge (z \vee \neg(a \wedge x)) \wedge (\neg z \vee (a \wedge x))$
$\wedge (w \vee a) \wedge (\neg a \vee \neg w) \wedge (x \vee b) \wedge (\neg x \vee \neg b)$ 3)

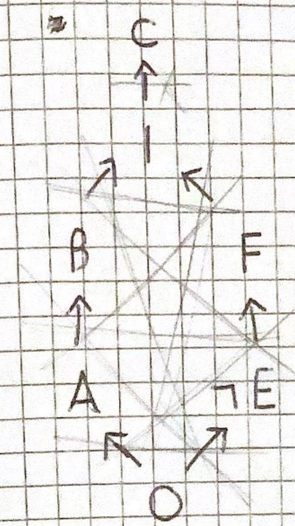
$\psi = (\neg \pi \vee z \vee y) \wedge (\neg z \vee \pi) \wedge (\neg y \vee \pi) \wedge (\neg y \vee b) \wedge (\neg y \vee w)$
$\wedge (y \vee \neg b \vee \neg w) \wedge (z \vee \neg a \vee \neg x) \wedge (\neg z \vee a) \wedge (\neg z \vee x)$
$\wedge (w \vee a) \wedge (\neg a \vee \neg w) \wedge (x \vee b) \wedge (\neg x \vee \neg b)$

in CNF

2) $E \vee F = \neg E \to F$

$\neg A \vee B = A \to B$

$C = C \vee 0 = \neg C \to 0 = 1 \to C$



9 satisfying assignments

$\neg E \wedge F \wedge A \wedge B \wedge C$

$\neg E \wedge F \wedge \neg A \wedge B \wedge C$

$\neg E \wedge F \wedge \neg A \wedge \neg B \wedge C$

$E \wedge F \wedge A \wedge B \wedge C$

$E \wedge F \wedge \neg A \wedge B \wedge C$

$E \wedge F \wedge \neg A \wedge \neg B \wedge C$

$E \wedge \neg F \wedge A \wedge B \wedge C$

$E \wedge \neg F \wedge \neg A \wedge B \wedge C$

$E \wedge \neg F \wedge \neg A \wedge \neg B \wedge C$

3) A Boolean Algebra is a set B containing elements 0 and 1, together with operations $\wedge$, $\vee$, and $\neg$ that satisfy the following axioms:

$(a \vee b) \vee c = a \vee (b \vee c)$

$(a \wedge b) \wedge c = a \wedge (b \wedge c)$

$a \vee b = b \vee a$

$a \wedge b = b \wedge a$

$a \vee (a \wedge b) = a$

$a \wedge (a \vee b) = a$

$a \vee 0 = a$

$a \wedge 1 = a$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \vee \neg a = 1$

$a \wedge \neg a = 0$

3)

```
twoTrue :: TwoBool
twoTrue _ _ = True

twoFalse :: TwoBool
twoFalse _ _ = False

twoNot :: TwoBool -> TwoBool
twoNot x a b = not (x a b)

(&&) :: TwoBool -> TwoBool -> TwoBool
(x && y) a b = (x a b) && (y a b)

(||) :: TwoBool -> TwoBool -> TwoBool
(x || y) a b = (x a b) || (y a b)
```

The element 0 of F is the function $\perp$ that takes an input and returns $\{0\}$

The element 1 of F is the function $\top$ that takes an input and returns $\{1\}$

The ~~element~~ $\wedge$ operation on F is defined such that given two functions $\star$ and $*$ in F that take inputs a, b and c, d respectively:

$$\star \wedge * = (a \star b) \wedge (c * d)$$

so it will return the conjunction of the results of $\star$ and $*$

The $\vee$ operation on F is defined in a similar manner:

$$\star \vee * = (a \star b) \vee (c * d)$$

such that it returns the disjunction of the results of $\star$ and $*$

The $\neg$ operation on F is defined as

$$\neg \star = \neg (a \star b)$$

where $\star$ is a function taking inputs a and b and the $\neg \star$ is the negation of the result of applying $\star$.

Therefore since the set $B = \{0, 1\}$ with $\wedge$, $\vee$ and $\neg$ is a non-trivial boolean algebra and the functions $\wedge$, $\vee$ and $\neg$ defined on F take as an input only 0 or 1 and the 0 and 1 elements of F take as an input only 0 or 1 the resulting set ~~of~~ F is also a boolean algebra by extension of B