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Part b(i)

Use the hint: e is surjective so $\phi = e(n_0)$ for some n_0 in N. Now $\phi(n) = f(e(n)(n))$ holds for all $n \in N$, so in particular for n_0 whence

$$\phi(n_0) = f(\underline{e(n_0)}(n_0)).$$

But $e(n_0)$ (underlined) is ϕ so we can simplify the displayed equation to

$$\phi(n_0) = f(\phi(n_0))$$

... which is to say that $\phi(n_0)$ is a fixed point for f.

Part b(ii)

If D has two elements or more then clearly D has a permutation that has no fixed point. Take f to be such a permutation to obtain a contradiction.