Abstract of a talk on **Generalized logic and quantum probabilities**
Meeting Room 14, Centre for Mathematical Sciences
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Bivalent propositional logic is generalized by allowing truth-value "gaps". Although there are only two primitives in logic, true and false, this means we have three valuation relations: a proposition is true, false, or is undecided in any valuation, and the generalized "global" logic is not bivalent but 3-valued. Unlike Lukasiewicz however we distinguish two separate senses of "not", each characterized by one of the classical "Laws of Thought". Negation, $\neg$, expresses an opposite truth-value satisfying the Law of Double Negation, $\neg\neg p \equiv p$. Denial, ~, expresses a failure of truth and satisfies Law of Excluded Middle, $p \vee \sim p = 1$. The generalized logic is represented by a "global algebra" which is a distributive lattice with separate orthogonal and complement operations. This algebra is "locally Boolean" in the sense that every element has an ortho-complement with respect to a subalgebra of the system, i.e. the orthogonal of any element is also a complement relative to this subalgebra. Only when the logic is bivalent will the algebra be fully Boolean, where the orthogonal and complement coincide.

An analogue of Stone's theorem associates with this algebra a "truth-system", a system of maximal filters representing the maximal valuations of the logic that find each proposition true. This truth-system is a Boolean field of sets even though the algebra representing generalized logic is not – only in bivalent logic do these two structures of representative algebra and truth-system coincide. The truth-system provides a new foundation for modalities that is simpler, more general and more intuitive than Kripke's rules based on "possible worlds". It is also a probability space for the generalized logic and so the modal logic can be refined by introducing Kolmogorov probability measures over this field that generate "degrees of truth", providing a formal foundation for fuzzy logic and sets. New strongly conditional probabilities are introduced on the new foundations that in a generalized logic lacking bivalent valuations have all the "quantum" peculiarities – irreducible statistics, measurement dependence and the failure to satisfy Bell's inequalities. This suggests a logical interpretation of quantum mechanics that regards the peculiarities as essentially logical but is completely opposed to "quantum logic", the notion that these theories use a logic that is non-distributive. The argument of Birkhoff and von Neumann for quantum logic is rejected here.

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