

COMPUTER SCIENCE TRIPOS Part IA 2011

Paper 2 Q 6

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(b)

$$z \in \bigcup_{n \in \mathbb{N}} f(x_n)$$

$$(\exists n \in \mathbb{N})(z \in f(x_n))$$

$$(\exists n \in \mathbb{N})(z \in \{b : (\exists X \subseteq x_n)(\langle X, b \rangle \in F)\})$$

$$(\exists n \in \mathbb{N})(\exists X \subseteq x_n)(\langle X, z \rangle \in F)$$

and we want this to imply

$$z \in f(\bigcup_{n \in \mathbb{N}} x_n)$$

$$(\exists X \subseteq \bigcup_{n \in \mathbb{N}} x_n)(\langle X, z \rangle \in F)$$

which is immediate

(c)

$$x \in f(\bigcup_{n \in \mathbb{N}} x_n)$$

iff

$$(\exists X \subseteq \bigcup_{n \in \mathbb{N}} x_n)(\langle X, x \rangle \in F)$$

Now X is finite, so there is a finite n_0 s.t. $X \subseteq \bigcup_{n \leq n_0} x_n$. So

$$x \in f(\bigcup_{n \leq n_0} x_n)$$

so

$$x \in f\left(\bigcup_{n \in \mathbb{N}} x_n\right)$$