Half-filling families

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A family $\mathcal{A}$ of finite subsets of a ground set X is called *half-filling* if for every finite set $E \subset X$ there exists $A \in \mathcal{A}$ such that

$$|A \cap E| \geq \frac{1}{2}|E|.$$

The following Ramsey-type problem is still open: given a hereditary, half-filling family $\mathcal{A}$ of finite subsets of ω_1, is there an infinite set $U \subset \omega_1$ such that all finite subsets of U belong to $\mathcal{A}$? It is not difficult to show that a 'no' answer would have the following consequence: for every countable ordinal α there is a minimal compact, hereditary, half-filling family on $\mathbb{N}$ with ordinal rank at least α. We prove that this consequence is in fact true.