Exercises for Logic-for-Linguists

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Exercise 1 *Let P abbreviate "I bought a lottery ticket" and Q "I won the jackpot". To what natural English sentences do the following formulæ correspond?*

1. $\neg P$;
2. $P \vee Q$;
3. $P \wedge Q$;
4. $P \to Q$;
5. $\neg P \to \neg Q$;
6. $\neg P \vee (P \wedge Q)$.

Exercise 2 *for second week*

What are the principal connectives and the immediate subformulæ of the formulæ below?

1. $(P \to Q) \vee (Q \to P)$
2. $(P \to Q) \to \neg(Q \to P)$
3. $P \to (P \vee Q)$
4. $P \to (Q \vee P)$
5. $P \to (P \wedge Q)$
6. $P \vee \neg P$
7. $\neg(A \vee \neg(A \wedge B))$
8. $(A \vee B) \wedge (\neg A \vee \neg B)$
9. $P \to (Q \to P)$
10. $(P \longleftrightarrow Q) \wedge (P \vee Q)$
11. $(P \longleftrightarrow Q) \longleftrightarrow (Q \longleftrightarrow P)$

12. $A \vee (B \wedge (C \vee D))$;

13. $\neg(P \vee Q)$

14. $A \to [(A \to C) \to ((B \to C) \to C)]$

15. $B \to [(A \to C) \to ((B \to C) \to C)]$

16. $(A \vee B) \to [(A \to C) \to ((B \to C) \to C)]$.

Exercise 3 *for second week*

In the following table

(1)	$A \wedge A$	A
(2)	$A \vee A$	A
(3)	$\neg(A \vee B)$	$(\neg A) \wedge (\neg B)$
(4)	$\neg(A \wedge B)$	$(\neg A) \vee (\neg B)$
(5)	$A \vee (B \vee C)$	$(A \vee B) \vee C$
(6)	$A \wedge (B \wedge C)$	$(A \wedge B) \wedge C$
(7)	$A \vee (B \wedge C)$	$(A \vee B) \wedge (A \vee C)$

we find that, in each line, the two formulæ are logically equivalent.

Write out truth-tables to prove it.

Exercise 4 *Abbreviate "Jack arrives late for lunch" etc etc., to single letters, and use these abbreviations to* formalise *the above arguments. (To keep things simple you can ignore the tenses!)*

1. *If Jill arrives late for lunch, she will be cross with Jack. Jack will arrive late. Therefore Jill will be cross with Jack.*

2. *If Jill arrives late for lunch, Jack will be cross with her. Jill will arrive late. Therefore Jill will be cross with Jack.*

3. *If Jill arrives late for lunch, Jack will be cross with her. Jack will arrive late. Therefore Jill will be cross with Jack.*

4. *If Jack arrives late for lunch, Jill will be cross with him. Jack will arrive late. Therefore Jill will be cross with Jack.*

Exercise 5 *(for third week) Formalise in propositional logic, using propositional letters of your choice.*

Do we need to say anything about 'only if'?

1. *If George is guilty he'll be reluctant to answer questions; George is reluctant to answer questions. Therefore George is guilty.*

2. *If George is broke he won't be able to buy lunch; George is broke. Therefore George will not be able to buy lunch.*

3. *Assuming that the lectures are dull, if the text is not readable then Alfred will not pass.*

4. *If Logic is difficult Alfred will pass only if he concentrates.*
5. *If Alfred studies, then he receives good marks. If he does not study, then he enjoys college. If he does not receive good marks then he does not enjoy college.*

   [Deduce from this that Alfred receives good marks]
6. *If Herbert can take the flat only if he divorces his wife then he should think twice. If Herbert keeps Fido, then he cannot take the flat. Herbert's wife insists on keeping Fido. If Herbert does not keep Fido then he will divorce his wife—provided that she insists on keeping Fido. [Deduce: Herbert should think twice]*
7. *If Herbert grows rich, then he can take the flat. If he divorces his wife he will not receive his inheritance. Herbert will grow rich if he receives his inheritance. Herbert can take the flat only if he divorces his wife.*
8. *If God exists then He is omnipotent. If God exists then he is omniscient. If God exists then he is benevolent. If God can prevent evil then—if He knows that evil exists—then He is not benevolent if he does not prevent it. If God is omnipotent, then He can prevent evil. If God is omniscient then He knows that evil exists if it does indeed exist. Evil does not exist if God prevents it. Evil exists. [Deduce that God does not exist]*

Exercise 6 *Write out truth-tables for all the formulæ in exercise 2.*

Exercise 7 *(for third week) Identify the principal connective of each formula below. In each pair of formulæ, say whether they are (i) logically equivalent or are (ii) negations of each other or (iii) neither.*

$(\neg A \wedge \neg B)$;	$\neg(A \vee B)$
$(\neg A \vee \neg B)$;	$\neg(A \vee B)$
$(\neg A \wedge \neg B)$;	$\neg(A \wedge B)$
$(\neg A \vee \neg B)$;	$\neg(A \wedge B)$