

A Model Tripos Question for Set Theory and Logic

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A **circular order**¹ is a ternary relation $R(x, y, z)$, whose typical example is the relation that holds between points x , y and z on the unit circle if, starting from x and moving clockwise, one encounters y before z .

(1) *Write down a set of axioms for circular orders.*

A group is **circularly-orderable** if it has a circular ordering that interacts in the obvious way with the multiplication of the group. The typical example is the additive group of integers-mod- p .

(2) *Write down a set of axioms for circularly orderable groups.*

(3) *Prove that a group is circularly orderable iff all its finitely generated subgroups are circularly orderable.*

(4) *Is the multiplicative group of (nonzero) integers mod p (p prime) circularly ordered?*

¹See Edward V. Huntington 'Inter-relations among the four principle types of order' Transactions of the American Mathematical Society **38** (1935) pp 1–9.