$$
\begin{prooftree}
\AxiomC{$b \in a \;\vdash\; b \in a$}
\AxiomC{$a \in b \;\vdash\; a \in b$}
\AxiomC{$b \in a \;\vdash\; b \in a$}
\RightLabel{$\neg$ L}
\UnaryInfC{$b \in a, b \notin a \;\vdash$}
\RightLabel{$\to$ L}
\BinaryInfC{$b \in a, a \in b, a \in b \to b \notin a \;\vdash$}
\RightLabel{$\forall$ L}
\UnaryInfC{$b \in a, a \in b, (\forall z)(z \in b \to b \notin z) \;\vdash$}
\RightLabel{$\to$ L}
\BinaryInfC{$a \in b, b \in a, b \in a \to (\forall z)(z \in b \to b \notin z), (\forall z)(z \in b \to b \notin z) \to b \in a \;\vdash$}
\RightLabel{$\wedge$ L$_1$}
\UnaryInfC{$a \in b, b \in a, b \in a \longleftrightarrow (\forall z)(z \in b \to b \notin z) \;\vdash$}
\RightLabel{$\neg$ R}
\UnaryInfC{$b \in a, b \in a \longleftrightarrow (\forall z)(z \in b \to b \notin z) \;\vdash\; a \notin b$}
\RightLabel{$\to$ R}
\UnaryInfC{$b \in a \longleftrightarrow (\forall z)(z \in b \to b \notin z) \;\vdash\; b \in a \to a \notin b$}
\RightLabel{$\forall$ L}
\UnaryInfC{$(\forall y)(y \in a \longleftrightarrow (\forall z)(z \in y \to y \notin z)) \;\vdash\; b \in a \to a \notin b$}
\RightLabel{$\forall$ R}
\UnaryInfC{$(\forall y)(y \in a \longleftrightarrow (\forall z)(z \in y \to y \notin z)) \;\vdash\; (\forall z)(z \in a \to a \notin z)$}
\RightLabel{$\to$ L}
\BinaryInfC{$(\forall y)(y \in a \longleftrightarrow (\forall z))(z \in y \to y \notin z)), a \in a \to (\forall z)(z \in a \to a \notin z), (\forall z)(z \in a \to a \notin z) \to a \in a \;\vdash$}
\RightLabel{$\wedge$ L$_1$}
\UnaryInfC{$(\forall y)(y \in a \longleftrightarrow (\forall z)(z \in y \to y \notin z)), a \in a \longleftrightarrow (\forall z)(z \in a \to a \notin z) \;\vdash$}
\RightLabel{$\forall$ L}
\UnaryInfC{$(\forall y)(y \in a \longleftrightarrow (\forall z)(z \in y \to y \notin z)), (\forall y)(y \in a \longleftrightarrow (\forall z)(z \in y \to y \notin z)) \;\vdash$}
\RightLabel{contraction-L}
\UnaryInfC{$(\forall y)(y \in a \longleftrightarrow (\forall z)(z \in y \to y \notin z)) \;\vdash$}
\RightLabel{$\exists$ L}
\UnaryInfC{$(\exists x)(\forall y)(y \in x \longleftrightarrow (\forall z)(z \in y \to y \notin z)) \;\vdash$}
\RightLabel{(1)}
\end{prooftree}
$$

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\begin{prooftree}
\AxiomC{$a \in a \;\vdash\; a \in a$}
\AxiomC{$a \in a \;\vdash\; a \in a$}
\AxiomC{$a \in a \;\vdash\; a \in a$}
\RightLabel{$\neg$ L}
\UnaryInfC{$a \in a, a \notin a \;\vdash$}
\RightLabel{$\to$ L}
\BinaryInfC{$a \in a, a \in b \to a \notin a \;\vdash$}
\RightLabel{$\forall$ L}
\UnaryInfC{$(\forall z)(z \in a \to a \notin z), a \in a \;\vdash$}
\RightLabel{$\to$ L}
\BinaryInfC{$a \in a \to (\forall z)(z \in a \to a \notin z), a \in a \;\vdash$}
\end{prooftree}
$$