Exercises on Countability and Uncountability

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Which of the following sets are countable and which are uncountable?

(i) The set of complex numbers
(ii) The set of partitions of $\mathbb{N}$ into finite pieces
(iii) The set of partitions of $\mathbb{N}$ into finitely many pieces
(iv) The set $\mathbf{Q} \to \mathbb{R}$ of functions from the rationals to the reals
(v) The set of functions $f : \mathbb{N} \to \mathbb{N}$ s.t $f(n) = 0$ for all but finitely many n
(vi) The set of functions $f : \mathbb{N} \to \mathbb{N}$ s.t $f(n) = 0$ or 1 for all but finitely many n
(vii) The set of functions $f : \mathbb{N} \to \mathbb{N}$ s.t $f(n) = n$ for all but finitely many n
(viii) The set of functions $f : \mathbb{N} \to \mathbb{N}$ s.t $(\forall n)(f(n) \leq n)$
(ix) The set of subsets of $\mathbb{N}$ with finite complement ("cofinite")