209/309 second test

October 14, 2008

Complete the details of the following definition:

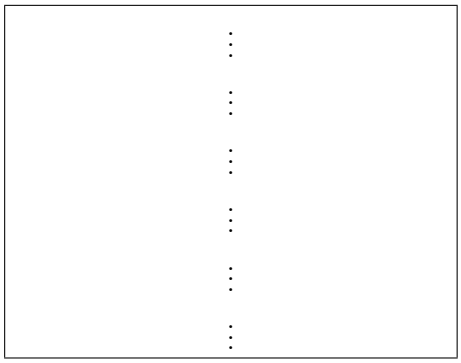
- $W \models A \wedge B$ iff ...
- $W \models A \vee B$ iff ...
- $W \models A \rightarrow B$ iff ...
- $W \models \neg A$ iff ...
- $W \models (\exists x)A(x)$ iff ...
- $W \models (\forall x)A(x)$ iff ...

1 Propositional logic

When depicting a model in the style of those over the page be sure to decorate each world only with the atomic propositions true in it—as is done there.

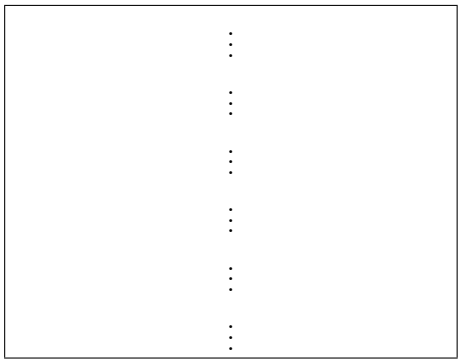
1. Find a countermodel for $(A \rightarrow B) \vee (B \rightarrow A)$;
2. Find a countermodel for $\neg\neg A \vee \neg A$;
3. Find a countermodel for $((A \rightarrow B) \rightarrow A) \rightarrow A$;
4. Find a model that satisfies $\neg(A \wedge B)$ but does not satisfy $\neg A \vee \neg B$;
5. Find a model that satisfies $\neg\neg A \rightarrow A$ but does not satisfy $\neg\neg A \vee \neg A$;
6. Find a model that satisfies $\neg\neg A \vee \neg A$ but does not satisfy $A \vee \neg A$;
7. Find a model that satisfies $(A \rightarrow B) \rightarrow B$ but does not satisfy $A \vee B$.
8. Find a model that satisfies $(A \rightarrow B) \rightarrow B$ but does not satisfy $(B \rightarrow A) \rightarrow A$;
9. Find a model that satisfies $A \rightarrow (B \vee C)$ but doesn't satisfy $(A \rightarrow B) \vee (A \rightarrow C)$);

10. Consider the following model



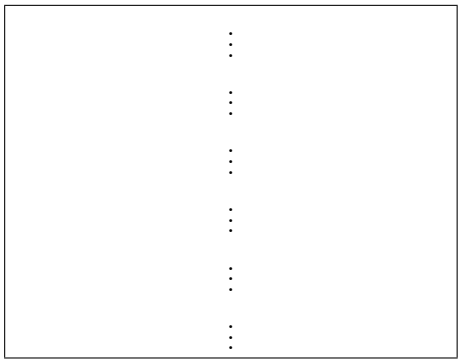
Does it satisfy
(i) $A \vee \neg A$?
(ii) $B \vee \neg B$?
(iii) $\neg\neg(A \vee \neg A)$?
(iv) $\neg\neg(B \vee \neg B)$?
(v) $\neg\neg A \vee \neg A$?
(vi) $((A \to B) \to A) \to A$
(vii) $(A \to B) \vee (B \to A)$

11. Consider the following model



Does it satisfy
(i) $A \vee \neg A$?
(ii) $B \vee \neg B$?
(iii) $\neg\neg(A \vee \neg A)$?
(iv) $\neg\neg(B \vee \neg B)$?
(v) $\neg\neg A \vee \neg A$?
(vi) $((A \to B) \to A) \to A$
(vii) $(A \to B) \vee (B \to A)$

12. Consider the following model



Does it satisfy
(i) $A \vee \neg A$?
(ii) $B \vee \neg B$?
(iii) $\neg\neg(A \vee \neg A)$?
(iv) $\neg\neg(B \vee \neg B)$?
(v) $\neg\neg A \vee \neg A$?
(vi) $((A \to B) \to A) \to A$
(vii) $(A \to B) \vee (B \to A)$

2 Predicate logic

1. Find a model that satisfies $\neg\neg(\exists x)(F(x)$ but does not satisfy $(\exists x)\neg\neg F(x))$;
2. Find a countermodel for $(\exists x)(\forall y)(F(y) \to F(x))$;
3. Find a countermodel for $(\exists x)F(x)) \vee (\forall y)\neg F(y)$;
4. Find a model that satisfies $\neg(\forall x)(F(x))$ but does not satisfy $(\exists x)(\neg F(x))$;
5. Find a model that satisfies $A \to (\exists x)(F(x)$ but does not satisfy $(\exists x)(A \to F(x)$