COMPUTER SCIENCE TRIPOS PART 1A
2014 Paper 2 Question 8

Thomas Forster

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(a)

Look at this long and hard, and DON'T PANIC. Eventually you will come to the conclusion that X must be nonempty, so the predicate P simply says "is nonzero". There is only one mark for this, so you should expect to do it within about a minute. It's just a comprehension exercise. ("Can you read this syntax?")

For four marks you need to be able to say that $A \times \emptyset$ is always $\emptyset$, *whatever A is*. If you didn't know that then you hadn't read my Queen Mary notes, and you clearly need to. Go and stand in the corner.

(b)

This is quite a useful exercise, because it tests two things. One is your ability to reason abstractly about arbitrary sets (itself a novel and rebarbative experience) and the second is your ability to not panic when presented with nasty looking formulæ but instead to actually *read them carefully and slowly*—rather than to free-associate.

$\sharp$ takes functions to functions. (They've written it '$(\cdot)^{\sharp}$' rather than '$\sharp$' beco's they want to write '$f^{\sharp} \dots$' instead of '$\sharp(f) \dots$'.) Read the displayed formula slowly to see what sort of things are arguments of $\sharp$ and what sort of things are values. Once you've grasped that, ask yourself: "What sort of thing might a function $(A \to \mathcal{P}(B)) \to (\mathcal{P}(A) \to \mathcal{P}(B))$ actually *do...*?" You can spend a minute or so thinking about that. In fact there is only one sensible function of that type, and the definition that follows defines that very function.