

## 1. Quine's set theory NF .....

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### Description of Essay

The set theory revealed to the world in Quine [1937] was a bit of a backwater for a very long time, largely because of unanswered questions about its consistency. Recently Randall Holmes has come up with an alleged consistency proof. The general view among the *cognoscenti* is that it is more likely correct than not; nobody actually *disbelieves* it, and there are people who have checked it and believe it to be correct. Given that, it would asking too much of a Part III essay to cover it. However an essay on NF focussed on setting out the requisite historical and technical background required would not only be feasible but would also be an instructive exercise for the student and helpful to any subsequent reader wishing to master the proof when it eventually appears.

Such an essay should cover the following. Specker's results connecting NF and Simply Typed Set Theory; Holmes' work on tangled types and tangled cardinals arising from Jensen's proof of  $\text{Con}(\text{NFU})$ ; Rosser's counting axiom and the refutation of AC, including a brief treatment of the relevant parts of cardinal arithmetic without choice (cardinal trees, amorphous cardinals). That much would be core, but probably not on its own enough for an essay. A study of the Rieger-Bernays permutation method would be a useful (but not essential) preparation for Holmes' proof, and in any case the method is a topic of central importance in the study of NF; a treatment of this material can be easily given and the exercise is instructive. Fraenkel-Mostowski permutation methods are relevant but that is an essay topic on its own account. Church-Oswald models for set theories with a universal set are another topic that could be covered, useful and interesting but less central to the consistency project—though germane to NF. Another interesting topic (under investigation in Cambridge) not directly relevant to the consistency proofs but worthy of treatment is the failure of cartesian-closedness in the category of sets according to NF, and more generally a category-theoretic treatment of the world of NF sets.

The only comprehensive reference at this stage is [1], but there is a wealth of other material linked from <http://math.boisestate.edu/~holmes/holmes/nf.html> and the setter will be happy to supply more detailed information on demand.

### Relevant Courses

*Essential:* Part II Logic and Set Theory or equivalent.

## References

- [1] Forster, T.E., Set Theory with a Universal set. Oxford Logic Guides. OUP 1992, second edition 1995.
- [2] Holmes, M. R. Elementary set theory with a universal set. volume 10 of the Cahiers du Centre de logique, Academia, Louvain-la-Neuve (Belgium), 241 pages, ISBN 2-87209-488-1. available online at <http://www.logic-center.be/cahiers/cahiersangl.html>