Abstracts for NF75

March 25, 2012

Roy Cook: Logicism, Separation, and Complement

One of the outstanding problems plaguing the Scottish logicist foundational program in the philosophy of mathematics has been providing an account of set theory. This paper provides the beginnings of such an account. In particular, it will be shown that any account of acceptable abstraction principles – that is, any solution to the *Bad Company Problem* – will entail that the *Axiom of Complement*:

$$Complement : (\forall x)(\exists y)(\forall z)(z \in y \leftrightarrow z \notin x)$$

holds of the universe of sets, and as a result that the *Axiom of Separation*:

$$Separation : (\forall x)(\forall Y)(\exists z)(\forall w)(w \in z \leftrightarrow (w \in x \wedge Y(w)))$$

fails. The resulting set theory has much in common with systems based on WVO Quine's *New Foundations* (in particular, with the system(s) obtained via Church-Oswald models).

Edoardo Rivello: Beneš' model: an old result revisited

Beneš 1954, was the first published attempt to prove the consistency of NF via a partial model of Hailperin's finite axiomatization. In my talk, I offer an analysis of Beneš' proof in a De Giorgi-style setting for set theory. This approach leads to an abstract version of Beneš' theorem that emphasizes the *invariant* content of the axioms proved to be consistent, in a sense of *invariance* that the talk intends to rigorously state and to help to clarify. In the second part, time permitting, some tentative speculation will be made about possible developments of the topic in two directions: (1) what set theories can be proved to be consistent via Beneš-like constructions? and (2) how we can elaborate on Beneš model to get a consistency proof for full NF?

Marcel Crabbé: Consistent extensions of $\mathbf{NF}_3$

Abstract:

NF_3 is the first order theory associated with the theory of family of sets, TT_3, in the same way NF is the mate of its type theory.

Old results about consistent extensions of NF_3 will be reviewed. We will not only consider fragments of NF which are extensions of NF_3, but also consistent extensions of NF_3 contradicting NF.

Zachiri McKenzie: Automorphisms of structures that are well-behaved on a linear order

This research was motivated by question of Randall Holmes who asked if it is possible to have a model $\mathcal{M}$ of ZFC admitting an automorphism $j : \mathcal{M} \longrightarrow \mathcal{M}$ such that

(i) $j(n) \geq n$ for all $n \in \omega^{\mathcal{M}}$,

(ii) $j(\alpha) < \alpha$ for some $\alpha \in \omega_1^{\mathcal{M}}$.

I answer this question positively by proving the following general theorem that links the existence of models with the desired automorphism to the existence of non-standard models with standard parts:

Theorem. *Let* $\mathcal{L} = \langle <, \bar{a}, \bar{b}, \ldots \rangle$ *be a signature. If* $\mathcal{M} = \langle M^{\mathcal{M}}, <^{\mathcal{M}}, \bar{a}^{\mathcal{M}}, \bar{b}^{\mathcal{M}}, \ldots \rangle$ *is an* $\mathcal{L}$*-structure such that*

(i) $\langle \{x \in M^{\mathcal{M}} \mid \mathcal{M} \models x < \bar{a}\}, <^{\mathcal{M}} \rangle \cong \langle \alpha, \in \rangle$ *for an ordinal* $\alpha \geq \omega$,

(ii) $\langle \{x \in M^{\mathcal{M}} \mid \mathcal{M} \models x < \bar{b}\}, <^{\mathcal{M}} \rangle$ *is ill-founded.*

Then there is an $\mathcal{L}$*-structure* $\mathcal{N} \models \mathrm{Th}_{\mathcal{L}}(\mathcal{M})$ *admitting an automorphism* $j : \mathcal{N} \longrightarrow \mathcal{N}$ *such that*

(i) $j(x) \geq x$ *for all* $x \in \bar{a}^{\mathcal{N}}$,

(ii) $j(x) < x$ *for some* $x \in \bar{b}^{\mathcal{N}}$.

Coupled with the Barwise Compactness Theorem this allows us to prove:

Theorem. *Let* $\mathcal{L} = \langle \in, \bar{\alpha}, \bar{\beta} \rangle$*. If* $\langle M, \in \rangle$ *is a transitive model of* ZFC *and* $\alpha, \beta \in M$ *are ordinals such that* $\alpha \geq \omega$ *and* $\beta > \alpha$ *is admissible, then there is an* $\mathcal{L}$*-structure* $\mathcal{N} \models \mathrm{Th}_{\mathcal{L}}(\langle M, \in, \alpha, \beta \rangle)$ *admitting and automorphism* $j : \mathcal{N} \longrightarrow \mathcal{N}$ *such that*

(i) $j(x) \geq x$ *for all* $x \in \bar{\alpha}^{\mathcal{N}}$,

(ii) $j(x) < x$ *for some* $x \in \bar{\beta}^{\mathcal{N}}$.

I will finish by discussing how this result can be used to build models of NFU + $\mathrm{AxCount}_{\leq}$.

Sergei Tupailo: Consistency of Strictly Impredicative NF

and a little more...

We present the following our contribution into the consistency problem of **NF** *(Journal of Symbolic Logic, v. 75, n. 4, 1326–1338, 2010)*:

An instance of Stratified Comprehension

$$\forall x_1 \ldots \forall x_n \exists y \forall x \, (x \in y \leftrightarrow \phi(x, x_1, \ldots, x_n))$$

is called *strictly impredicative* iff, under stratification, x receives the minimal type. Using the technology of forcing, we prove that the fragment of **NF**, we called **NFSI**, based on strictly impredicative Stratified Comprehension is consistent. A crucial part in this proof, namely showing genericity of a certain symmetric filter, is due to Robert Solovay.

As a bonus, our interpretation also satisfies some instances of Stratified Comprehension which are *not* strictly impredicative. Since, at this conference, you may hear from other people how to have different models of **NFSI** per se, in the talk I will try to show in detail how my 2010 model satisfies some instances of $\mathbf{NF_3}$ which are *not* a part of **NFSI**.

Apparently, consistency of the described **NF** subsystems was not known before.