Extract from review A

• Two remarks on style: firstly, the author resorts to the language of computer science: cardinals are "implemented" (actually I thought that was quite good) but referring to the V_α hierarchy as "a recursive data type" seems to add nothing to our understanding, and would just be obfuscatory to some. This brings in a spurious connection. Secondly, it would be best to limit statements of the kind "it is deeply unimportant that ..." put in for rehtorical effect.

• Although the book is supposed to be introductory, there are many remarks or comments that are dropped in which are too arcane for the unaided student reader or even the non-specialist (whether postgraduate or undergraduate).

• Exemplifying this, but also bringing up another matter: there is a collection of remarks taken up with defunct set theories: even if we allow NF a living status here, the other references (ii) and (iv) on p31 are either not of present day concern or are too minor, too localised, to be of concern in a general introduction (no one will have any idea as to what Hinnions school says or does).

• We see no reason to argue from the fact that we have notions of *type* in mathematics that we should intepret those notions in a stratified set theory (or indeed one which is also typed).

• The Chapter on the Axiom of Replacement does not really give justice to the main reason for its acceptance: namely that given a set $A = \{......\}$ then *replacing* each element of A with another object yields still a set. In the book it is thought of as simply the range of a function on a set is a set. True, but that is more sophisticated than the "replacement" just mentioned. The author says that (6.1.4) against replacement, one has the function f that sends n to $\mathcal{P}^n(\mathbb{N})$. Then taking the union $\bigcup\{\mathcal{P}^n(\mathbb{N}) : n \in \mathbb{N}\}$ we have a set of big size thus breaking the "limitation of size" principle that Axiom of Replacement is supposed to endorse. However this is a) only with Replacement as a Function, and moreover b) only in combination with Union does one arrive at a big-sized set. It is argument against them *jointly* but not singly.

In 6.2 it is said that he suspects set theorists of bad faith: "the real reason why set theorists adopt the axiom because that enbles them to do the things they want to do". This is false. Mathematicians themselves use replacement (in harmless instances) everytime they make up a definition, eg of the form, the n-fold Cartesian product of a set X for "all n". Moreover the use of Replacement indeed allows one to *get started*: without it we have no way of picking any representative of an isomorphism type for wellorders. or indeed of finding von Neumann ordinals isomorpic to the given wellordering.

6.3.1 Paragraph 2: "*it is an obscure consequence of the second incompleteness theorem that...*" this is an obscure comment and should be elaborated.

The argument from normal forms on restricted quantifiers (6.3.3): there is nothing false here, but it is a weak argument. I have never heard this argued for replacement: it brings in an irrelevant syntactic dimension to an argument about set existence.

• I liked the first part of the Chapter 8 on Axiom of Choice, but I think some of the space would be better used, introducing more variants of AC (Tychonoff, Maximal Ideals...) and showing how the mathematical fields under consideration use AC - if after all this is an introductory book. Although most of the book avoids any detailed mathematics of the interrelationship of the various axioms, an exception appears to be made for the negations of Foundation and Choice being consistent with the other axioms, and where there is use made of Bernays-Rieger models. Id suggest the discussion under 8.7 seems too fast to be useful.

• Omissions: the discussion of *axiom of determinacy* is thin: “it caused a few flutters amongst platonists”, “it could not be ignored, it was too interesting for that”. This is unscientific language. “Certain large cardinal hypotheses imply that it is true in a natural substructure of the universe. That way they get the best of both worlds”. This sounds close to disrespect (although I assume it was not so intended): moreover it ignores the axiom of projective determinacy, consistent with full choice that gives a very “complete” theory of the hereditarily countable sets. This should be covered if the book is to be considered a modern treatment.

• Chapter on Independence Proofs seems like a missed opportunity. The statement that “*each axiomatic set theory is geared to a particular aspect of mathematics...*” just does not seem right either historically or otherwise. Even the definition of Independence appears wrong (Φ is independent of T, if $T \nvdash \Phi$, $T \nvdash \neg\Phi$ — time should be spent properly stating these matters in an introductory volume).

“*Everything in* $H_{\aleph_1}$ *is countable and therefore wellordered* ”? This is false without countable choice in V, so it should be made clear this is the prevailing presumption.

More seriously: there was scope here for a discussion on “new axioms/hypotheses” how Cohen/Gödel arguments changed our view of status of eg the Continuum Hypothesis. This is another omission, but presumably an intended one? presumably the author wishes to stick just to discussing the ZFC axioms and none other?

Extract from review B

This is a welcome addition to the literature on the foundations of set theory. Discussion of the axioms is not a fashionable pursuit; when present, say in a textbook, it is usually a justification of the line adopted, not a discussion of alternatives, both actual and historical, or of the limits or lacunae of the axiomatization proposed.

Here for each axiom, especially replacement and choice, some history and some philosophy throw light on the meaning and function of the axioms, and they are really illuminating.

The book presupposes a knowledge of the basic theory, is a companion to such introductory books as Enderton’s, Elements of set theory, or Vaught’s, or even more advanced ones.

Audience should be composed of mathematics and philosophy graduate students (the author mentions also computer science students, but I think this is wishful thinking), because issues addressed are of different and complementary kinds; some more palatable to philosophically minded students (“Intension and extension”, “What is a mathematical Object”), others more of a mathematical character (“Set Pictures”, the use of larger sets, inductive definitions, reflection principles). Possibly there exists somebody able to appreciate both. But also scholars will benefit from reading and meditating this book

I don’t think that any part should be subtracted; rather, one would like more, but then it would loose its freshness and agility.

Extract from review C

The proposed book fits into a new niche: a short, informal text about the motivations of the axioms of set theory.

Audience: for someone versed in set theory, there are interesting views presented, but several important issues left out or treated off-handedly. For someone not versed in set theory, circularities of presentation as well as specialized asides will be confusing. One is left uncertain and a bit stranded as to the relationship of the axioms of set theory to its history and current practice.

I would advise an adequate, head-on treatment of three issues: truth, infinity, and logic. On truth, the author sometimes discusses what is true, but does not say that he would mean by it adequately. On infinity, the author assumes familiarity, as if the battle has been won, when in fact historically one of the Cantor's main incentives was to subsume the actual infinite and its apparent problems in his development of set theory. On logic, the author assumes first-order logic is already the working logic of set theory and the reader, when in fact Zermelo's definite property for his Assonderungsaxiom was hardly clear—and the development of first-order logic postdates set theory–and the reader's understanding of the logical framework may be fleeting.

These issues are prominent in the discussion of the replacement axiom. What is the meaning of asking whether it is true? Cantor had the alephs, but without replacement, one cannot get to the von Neumann omega-sub-omega rendition. The author in 6.1.4. argues against the Beth series of ever higher power sets having a supremum, but that is the very nature of the Cantorian infinite. Later, he argues for Replacement in terms of quantifier pushing, but this seems very odd and quite a posterori.

Of more specific issues, I found the discussion on the worries about circularity rambling. Most mathematicians and even students are aware of the permeation in mathematics of set-theoretic ideas and constructs, and this is not accounted for as a natural reason for getting at set-theoretic foundations.

Just before 4.1.1, it is declared that ordinals are not sets. This is very confusing, but supposedly having to do with the cumulative hierarchy motivation of set theory. First, this motivation should be adequately referenced (e.g. Wang and Parsons are not mentioned), but more, the traditional way is to proceed informally with a notion of stage and only later formalize to ordinals. To the new reader, having the cumulative hierarchy informally and formally, having ordinals in and out of set theory is confusing. One should follow the usual account: Cantor had ordinal numbers outside of set theory, but they were quite precise, and von Neumann later brought them into set theory as his ordinals. The iterative conception is only an informal picture, and the von Neumann cumulative hierarchy is a formal rendition.

In 5.1 the axiom of extensionality is unstated. In 5.3 the axiom of infinity is unstated– again, infinity seems to be assumed as familiar here.

Very specific points: on page 8, one can define rank(x) as the least α such that x is a subset of V_α; the odd footnote about $\alpha + 1$ is not necessary. On page 13, the empty set is motivated operationally in one way, but surely it is better motivated operationally as notation for having empty intersection, or mutually exclusive events in probability. On page 18, the reader asks, what is the Burali-Forti paradox? On page 53, the number of urelements and height of the model is straight from Zermelo 1930, and should be so accredited; also, the use of replacement here is no longer first-order, but some meta principle.