Lecture March 8th

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$(\forall x)(F(x)) \wedge (\forall x)(G(x))$ pretty obviously the same as $(\forall x)(F(x) \wedge G(x))$

$(\exists x)(F(x)) \vee (\exists x)(G(x))$ less obviously the same as $(\forall x)(F(x) \vee G(x))$, but it is possible to persuade yourself that it is the same by reasoning along the following lines.

> "If there is a thing that is F then this thing (whatever it is) is also F-or-G, so there is a thing which is F-or-G. By the same token "If there is a thing that is G then this thing (whatever it is) is also F-or-G, so there is a thing which is F-or-G. So I can deduce that there is a thing which is F-or-G both from the news that there is a thing which is F, and also, separately, from the news that there is something which is G. So I can deduce it from their disjunction."

(An aside: it's obvious that $(\exists x)(F(x)) \wedge (\exists x)(G(x))$ is not the same as $(\forall x)(F(x) \wedge G(x))$. Clearly from the news that there is a thing that is a frog and thing that is not a frog I cannot infer that there is a thing that is both a frog and a non-frog)

Consider the example

> Everyone has a friend who lives in a ziggurrat or who has measles.

$$(\forall x)(\exists y)(F(x, y) \wedge (Z(y) \vee M(y)))$$

$$(\forall x)[(\exists y)(F(x, y) \wedge Z(y)) \vee (\exists y)(F(x, y) \wedge M(y))]$$

These might not *look* equivalent, but they are!

Concealment

> Jenny is a mother.

If we have $M(x, y)$ a two-place relation of x-is-mother-of-y then we have to write $(\exists x)(M(j, x))$

> No lecturer goes to any lectures

How do I say "x is a lecturer" if the only remotely relevant predicate letter I have is the three-place (x, y, z) (from the notes) which says "x lectures y for (course) z"? A lecturer is a thing that lectures someone for something. So "x is a lecturer" must be

$$(\exists y)(\exists z)(T(x, y, z))$$

Scope

God will spare the city if there is even one righteous man in it

$F(x)$: x is a righteous man in the city; P: God will not fry the city.
Both the following are correct!

$$(\exists x)(F(x)) \to P$$

$$(\forall x)(F(x) \to P)$$

It's very disconcerting that one has a '$\exists$' and the other a '$\forall$'. The way to cope with this to remind yourself to keep an eye on *scope*: look where the brackets open and close. In the top line the principal connective is the '$\to$'. In the second it is the '$\forall$'.

Function letters

When you have a binary relation that can hold between a thing and precisely one other thing then you can use a function letter: mother-of, father of . . . . You have precisely one of each of these. So we can write '$m(x)$' for "mother-of-x". Now you have to be careful! '$m(x)$' looks like '$M(x)$', but they are quite different syntactically. '$M(x)$' expresses a **proposition**; '$m(x)$' points to a thing. '$M(x)$' is the kind of expression you can join together with similar expressions by means of '$\to$', '$\wedge$' and so on; '$m(x)$' is the kind of thing you can put inside a predicate letter:

$Z(m(x))$: x's mum lives in a ziggurrat.

Observe that if we write '$(\forall x)(\forall y)(\ldots)$' (so we are saying that something holds for all x and for all y) there is no implication that the x and y are distinct. If we write 'boyfriend-of x' as '$b(x)$' then we can capture

No two girls share a boyfriend

by

$$(\forall x)(\forall y)(b(x) = b(y) \to x = y)$$

and the 'x' and the 'y' definitely have to be able to point to the same thing!
$(\forall x)(\forall y)(f(x) \neq m(y))$