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1. $(\forall x)(\forall y)(\texttt{has-forgiven}(x, y) \to \texttt{is-a-saint}(x))$
2. $(\forall x)(\texttt{is-in-the-logic-class}(x) \to (\exists y)(\texttt{is-in-the-history-class}(y) \wedge \neg(\texttt{is-cleverer-than}(x, y));$
3. $\neg L(m, m) \wedge (\forall x)(x \neq m \to L(x, m))$
4. $(\exists x)(B(x) \wedge S(j, x)) \wedge (\exists x)(B(x) \wedge S(r, x));$
5. $(\exists x)(B(x) \wedge S(j, x) \wedge S(r, x));$
6. $(\forall x)(L(x, m)) \wedge (\forall x)(L(b.x) \to x = m);$

And now the examples from Lewis Carroll:

1. $\texttt{Man}(\texttt{Socrates})$; $(\forall x)(\texttt{Man}(x) \to \texttt{Mortal}(x))$. Therefore $\texttt{Mortal}(\texttt{Socrates})$.
2. $\texttt{Cow}(\texttt{Daisy})$; $(\forall x)(\texttt{Cow}(x) \to \texttt{eats-grass}(x))$. Therefore $\texttt{eats-grass}(\texttt{Daisy})$.
   $\texttt{Cow}(\texttt{Daisy})$; $(\forall x)(\texttt{Cow}(x) \to \texttt{Mad}(x))$. Therefore $\texttt{mad}(\texttt{Daisy})$.
3. $(\forall x)(\texttt{thief}(x) \to \neg(\texttt{honest}(x))$; $(\exists x)(\neg(\texttt{honest}(x) \wedge \texttt{found-out}(x))$. Therefore $(\exists x)(\texttt{thief}(x) \wedge \texttt{found-out}(x))$
4. $\neg(\exists x)(\texttt{muffin}(x) \wedge \texttt{wholesome}(x))$; $(\forall x)(\texttt{Puffy}(x) \to \neg(\texttt{wholesome}(x))$. Therefore $(\forall x)(\texttt{Muffin}(x) \to \texttt{puffy}(x))$
5. $(\forall x)(\texttt{proud-of-tail}(x) \to \texttt{peacock}(x))$; $(\exists x)(\texttt{proud-of-tail}(x) \wedge \neg(\texttt{can-sing}(x)))$; $(\exists x)(\texttt{peacock}(x) \wedge \neg \texttt{can-sing}(x)))$.
6. $\texttt{Relieves}(\texttt{Warmth}, \texttt{Pain})$; $(\forall x)(\texttt{useful-in-toothache}(x) \to \texttt{relieves}(x, \texttt{pain})$. Therefore $\texttt{Useful-in-toothache}(\texttt{warmth})$
7. $(\forall x)(\texttt{wise}(x) \to \texttt{walks-on-feet}(x))$; $(\forall x)(\neg(\texttt{wise}(x)) \to \texttt{walks-on-hands}(x))$; $(\forall x)(\neg(\texttt{walks-on-feet}(x) \wedge \texttt{walks-on-hands}(x)))$

   You might want to try to capture that fact that `walks-on-feet` and `walks-on-hands` share some structure, and have a two-place relation `walks-on`. Then i think you will also want `feet-of` and `hands-of`, so you would end up with

   $(\forall x)(\texttt{wise}(x) \to (\forall y)(\texttt{feet-of}(x, y) \to \texttt{walks-on}(x, y)))$

   and of course

   $(\forall x)(\neg\texttt{wise}(x) \to (\forall y)(\texttt{hands-of}(x, y) \to \texttt{walks-on}(x, y)))$

   You might feel that the following are equally good formalisations:

   $(\forall x)(\texttt{wise}(x) \to (\exists y)(\exists z)(\texttt{feet-of}(x, y) \wedge \texttt{feet-of}(x, z) \wedge \neg(y = z) \wedge \texttt{walks-on}(x, y) \wedge \texttt{walks-on}(x, z)))$ ... and the same for unwise men and hands.

8. $(\forall x)(\texttt{fossil}(x) \to \neg\texttt{can-be-crossed-in-love}(x))$; $(\forall x)(\texttt{oyster}(x) \to \texttt{can-be-crossed-in-love}(x))$; therefore $(\forall x)(\texttt{oyster}(x) \to \neg\texttt{fossil}(x))$
9. $(\forall x)(\texttt{anxious-to-learn}(x) \to \texttt{works-hard}(x))$; $(\exists x)(\texttt{student}(x) \wedge \texttt{works-hard}(x))$; therefore $(\exists x)(\texttt{student}(x) \wedge \texttt{anxious-to-learn}(x))$
10. $(\forall y)(\texttt{song(y)} \to \texttt{his}(x) \to \texttt{last-an-hour}(y))$; $(\forall x)(\texttt{song}(x) \wedge \texttt{last-an-hour}(x) \to \texttt{tedious}(x))$; therefore $(\forall z)(\texttt{song}(z) \wedge \texttt{his}(z) \to \neg\texttt{tedious}(z))$.
11. $(\exists x)(\texttt{lesson}(x) \wedge \texttt{tedious}(x))$; $(\forall z)(\texttt{difficult}(z) \to \texttt{merits-attention}(z))$. therefore $(\exists x)(\texttt{lesson}(x) \wedge \texttt{merits-attention}(x))$.
12. $(\forall y)(\texttt{human}(y) \to \texttt{mammal}(y))$; $(\forall y)(\texttt{mammal}(y) \to \texttt{warmblooded}(y))$. Therefore $(\forall z)(\texttt{human}(z) \to \texttt{warmblooded}(z)))$.