FREQUENCY DEPENDENCE OF THERMAL EXCITATION OF MICRO-MECHANICAL RESONATORS

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INTRODUCTION

Thermo-mechanical excitation of micro-mechanical resonators is very attractive from technological point of view. Cantilever beams can be excited electro-thermally, by heating one side of the beam. The resulting temperature distribution in the beam is characterised with a thermal diffusion length δ and the excitation effectivity depending on the ratio of the diffusion length with the beam thickness: δ/h. The influence of the material properties, the frequency and the geometry on the effectivity is investigated. The aim of the research is to obtain design rules which cân be used for designing thermal excitated resonant sensors.

MODEL

To obtain a harmonic heat generation term with frequency ω, an a.c. voltage $U_{AC} = U_{ac}\cdot\cos\omega t$ is superimposed on a d.c. voltage U_{DC}:

$$Q(t) = \frac{1}{R}\cdot\left[(U_{DC}^2+0.5U_{ac}^2)+2U_{DC}U_{ac}\cos\omega t+0.5U_{ac}^2\cos 2\omega t\right]$$

The heat flow in the beam-y direction is described with a one-dimensional diffusion equation:

$$\frac{\partial\theta}{\partial t} = \alpha\cdot\frac{\partial^2\theta}{\partial y^2}$$

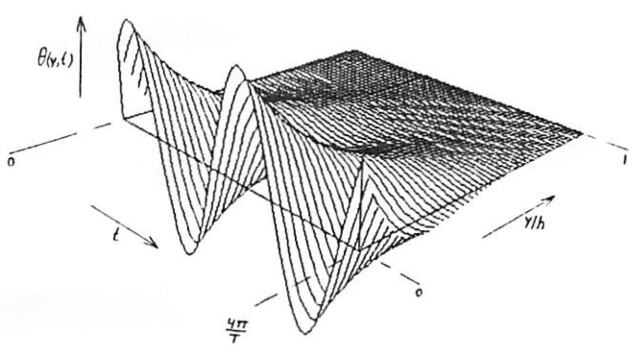
with α thermal diffusion constant. The (dynamic part of the) temperature distribution is given by ($P_o = 2U_{DC}U_{ac}/R$):

$$\theta(y,t) = \left[\frac{e^{-\sigma h}}{2\sinh(\sigma h)}\cdot e^{\sigma y} + \frac{e^{-\sigma h}}{2\sinh(\sigma h)}\cdot e^{-\sigma y}\right]\cdot\frac{P_o}{\sigma k}\cdot e^{i\omega t}$$

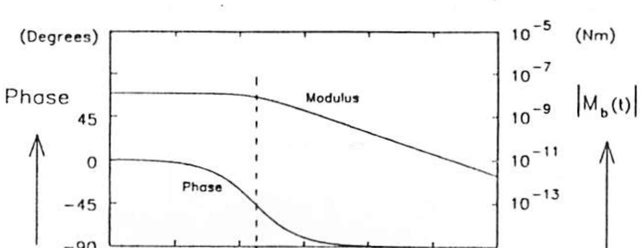
with $\sigma = (1+i)/\delta$ and $\delta = (2\alpha/\omega)^{1/2}$. The thermally induced bending moment $M_b(t)$ is proportional to the first moment of the temperature distribution over the beam thickness:

$$M_b(t) = P_o\beta\cdot EI\cdot 12/k\cdot\frac{1}{\gamma^2}\left(\frac{1}{2}+\frac{1}{\gamma}-\frac{2}{\gamma(1+e^{-\gamma})}\right)$$

with β the thermal expansion coefficient, EI the beam bending stiffness, k the thermal conductivity and $\gamma = h\cdot\sigma$.



Temperature distribution as function of time and position.

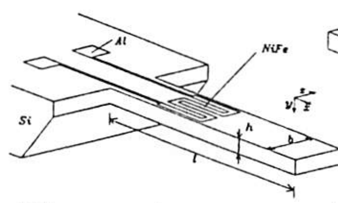


Modulus and phase of mechanical bending moment M_b as function of the frequency. The turn over frequency $F_t = 6/\pi h^2$.

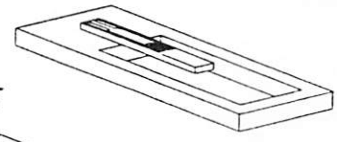
EXPERIMENTS

Beam vibrations were detected using a Gain Phase analyzer in combination with a Michelson interferometer set up. Absolute beam vibration amplitudes were measured using a Mach-Zehnder Heterodyne interferometer.

samples



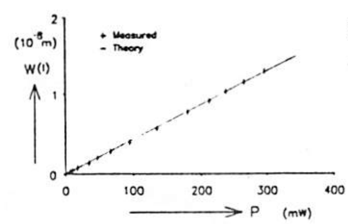
Silicon support (etched structure)



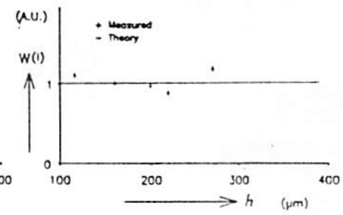
Brass support (beam glued to substrate)

RESULTS

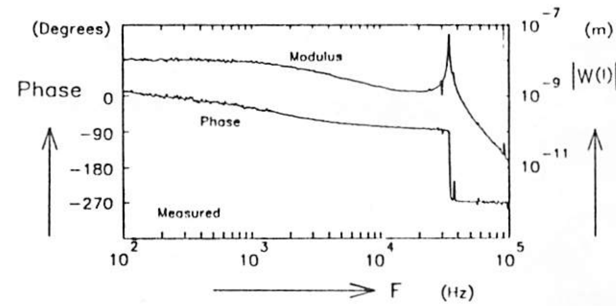
linearity



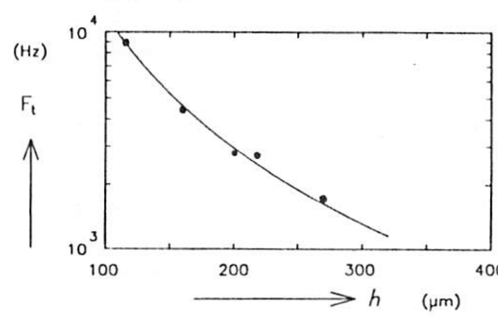
beam vibration amplitude



transfer function



turn-over frequency



CONCLUSIONS

The theory predicts a characteristic turn over frequency $F_t$in the thermally induced mechanical moment. For frequencies below F_t, the moment is constant and in-phase with the generated heat. Above F_t, the moment has a phase of $-\pi/2$ and is proportional to ω^{-1}. This is of importance for the design of the beams as well as for the design of the electronic feedback circuit in a practical oscillator.
The (quasi static) measured electro-thermal excitation of cantilever beams is linear with the dissipated power in the resistor. The results for the absolute beam displacement W(l) and for the turn over frequency F_t as a function of the beam thickness agree well with the theory.