## Radio Frequency Circuits & Antenna

Thomas Glezer Tel Aviv University

Homework: 6

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## Problem 1

0.1 Determine the equivalent noise temperature and noise figure of the receiver system (including the noise temperature of the antenna).

$$F_T = 1 + T_{tot}/T_0 \tag{1}$$

$$G_{rf} = 25[dB] \to 316.2 \quad \land \quad F_{rf} = 3[dB] \to 2$$
 (2)

$$L = 7[dB] \rightarrow 5 \quad \land \quad F_m = 8[dB] \rightarrow 6.31$$
 (3)

$$G_{if} = 40[dB] \to 10000 \quad \land \quad F_{if} = 1[dB] \to 1.26$$
 (4)

$$T_{rf} = (F_{rf} - 1)T_0 = (2 - 1)300 = 300[K]$$
(5)

$$T_m = (CL - 1)T_0 = (5 - 1)300 = 1200[K]$$
(6)

$$T_{if} = (F_{if} - 1)T_0 = (1.26 - 1)300 = 77.6[K]$$
 (7)

$$T_{tot} = T_{rf} + \frac{T_m}{G_{rf}} + \frac{T_{if}CL}{G_{rf}} \tag{8}$$

$$= 300 + 1200/316.2 + 77.6 \cdot 5/316.2 \tag{9}$$

$$T_{tot} = 305.02[K] (10)$$

$$\therefore F_T = 1 + 305.02/300 = 2.0167 \to 3.05[dB] \tag{11}$$

0.2 Determine the noise power at the output of receiver system if the bandwidth BW = 100 MHz.

$$N_{out} = KBG_{sys}(T_A + T_{tot}) (12)$$

$$T_A = \eta T_{sky} + (1 - \eta)T_{ant} = 0.8 \cdot 100 + (1 - 0.8) \cdot 300 = 140[K]$$
 (13)

$$G_{sys} = G_{rf} + G_{if} - CL = 25 + 40 - 8 = 57[dB] \rightarrow 501187$$
 (14)

Thus, combining (13) & (14) into (12):

$$N_{out} = 1.38 \cdot 10^{-23} \cdot 100 \cdot 10^3 \cdot 501187 \cdot (140 + 305.02) \tag{15}$$

$$N_{out} = 3.078 \cdot 10^{-10} \to -95.11[dB] \tag{16}$$

0.3 The receiving antenna is a round plate with a radius a. Suppose that the effective area of the antenna is equal to 60% of the physical area of antenna aperture. Calculate the minimal value of a that provides  $(S/N)_{out} \geq 20[dB]$ .

$$S = \frac{P_t \cdot G}{4\pi r^2} = \frac{100 \cdot 10^3}{4\pi (36000 \cdot 10^3)^2} = 6.14 \cdot 10^{-12} [W/M^2]$$
 (17)

$$A_{phys} = \frac{\lambda^2}{4\pi} \cdot \text{round plate area} = \frac{300/14}{4\pi} 4\pi a^2 = 21.42 \cdot a^2$$
 (18)

$$A_{eff} = e_a A_{phys} G_{sys} = 0.6 \cdot 21.42 \cdot a^2 \cdot 501187 = 6443835a^2$$
 (19)

$$P_{rec} = A_{eff}S = 3.956 \cdot a^2 \cdot 10^{-5} \to -44.02 + 20 \log a$$
 (20)

$$-44.02 + 20\log a - (-95.11) \ge 20 \tag{21}$$

$$20\log a \ge -31.09\tag{22}$$

$$\log a \ge -1.55\tag{23}$$

$$a = 27.9[mm] \tag{24}$$

0.4 The second satellite has the same parameters as the first satellite and it transmits a signal that we wouldn't like to receive and so for us it is considered a noise. The gain of the receiving antenna in the direction of the second satellite is X dB lower than in the direction of the first satellite. Calculate the minimal value of X that guarantees a degradation of signal to noise ratio at the output of the receiver system by not more than 1 dB.

$$P_{noise} = -44.02 + 20\log a + X \tag{25}$$

$$(S/N)_{normal} = 51.09 + 20\log a \tag{26}$$

$$51.09 + 20\log a - (-44.02 + 20\log a + X) \le 1 \tag{27}$$

$$X + 7.07 \le 1 \tag{28}$$

$$X \le -6.07$$
 (29)

## Problem 2

0.5 Find the gain  $G_{a1}$  of relay receiving antenna to obtain a 15 dB signal-to-noise ratio at the relay output marked as point 1 in the circle.

Calculating for noise:

$$N_{out} = KBG_{sys}(T_A + T_{tot}) (30)$$

$$T_A = \eta T_{sky} + (1 - \eta)T_{ant} = 0.9 \cdot 150 + (1 - 0.9) \cdot 300 = 165[K]$$
 (31)

$$G_{sys} = G_1 + G_2 - CL = 30 + 30 - 7 = 53[dB] \rightarrow 199526$$
 (32)

$$G_1 = 30[dB] \to 1000 \quad \land \quad F_1 = 2[dB] \to 1.58$$
 (33)

$$CL = 7[dB] \rightarrow 5 \quad \land \quad F_m = 8[dB] \rightarrow 6.31$$
 (34)

$$G_2 = 30[dB] \to 1000 \quad \land \quad F_2 = 2[dB] \to 1.58$$
 (35)

$$T_1 = (F_1 - 1)T_0 = (1.58 - 1)300 = 174[K]$$
(36)

$$T_m = (CL - 1)T_0 = (5 - 1)300 = 1200[K]$$
(37)

$$T_2 = (F_2 - 1)T_0 = (1.58 - 1)300 = 174[K]$$
 (38)

$$T_{tot} = T_1 + \frac{T_m}{G_1} + \frac{T_2 CL}{G_1} \tag{39}$$

$$= 174 + 1200/1000 + 174 \cdot 5/1000 \tag{40}$$

$$T_{tot} = 176.07[K] (41)$$

Thus, combining (31), (32) and (41) into (30):

$$N_{out} = 1.38 \cdot 10^{-23} \cdot 100 \cdot 10^3 \cdot 199526 \cdot (165 + 176.07)$$
 (42)

$$N_{out} = 9.4 \cdot 10^{-11} \to -100.27[dB] \tag{43}$$

## 0.6 Find the gain $G_r$ of the receiving antenna to obtain a 10 dB signal-to-noise ratio at the receiver output.