Radio Frequency Circuits & Antenna

Thomas Glezer
Tel Aviv University

—

Homework: 7

May 24, 2022

1 Derive the radar equation for the bistatic case, where the transmit and receive antennas have gains G_t and G_r, and are at distances R_t and R_r from the target, respectively.

$$S = \frac{P_t G_t}{4\pi R_t^2}, \quad S_r = \frac{P_t G_t}{4\pi R_t^2} \cdot \frac{\sigma}{4\pi R_r^2}$$
$$\therefore P_r = S_r A_r = S_r \frac{\lambda^2 G_r}{4\pi} = \frac{P_t G_t}{4\pi R_t^2} \cdot \frac{\sigma}{4\pi R_r^2} \cdot \frac{\lambda^2 G_r}{4\pi}$$

2 A stationary Doppler radar system illuminates a target moving at a radial velocity $\mathcal{V}_r$ (positive in the direction towards the radar) as shown below:

a) Find the minimum pass band of the filter $[f_1, f_2]$, so that the target is detected in the velocity range 100÷700 m/s.

$$f_{d,min} = \frac{2v_{min} f_{rf}}{c} = \frac{2 \cdot 100}{3 \cdot 10^8} \cdot 3 \cdot 10^9 = 2[KHz]$$
$$f_{d,max} = \frac{2v_{max} f_{rf}}{c} = \frac{2 \cdot 700}{3 \cdot 10^8} \cdot 3 \cdot 10^9 = 14[KHz]$$
$$3[GHz] + 2[Hz] \leq f_d \leq 3[GHz] + 14[GHz]$$
$$f = f_{LO} - f_d$$
$$f_1 = f_{LO} - (f_{RF} + f_{d,max})$$
$$f_2 = f_{LO} - (f_{RF} + f_{d,min})$$
$$9.986[MHz] \leq f_d \leq 9.998[MHz]$$
$$B = f_2 - f_1 = 12[KHz]$$

b) Find the maximum range for detection of the target by the radar assuming that detection requires a 12 dB signal-to-noise ratio at the output of the filter.

$$G_{RF} = 18[dB] \rightarrow 10^{1.8} = 63.10$$
$$N_{RF} = 2.3[db] \rightarrow 10^{0.23} = 1.7$$
$$T_{RF} = (N_{RF} - 1)T_0 = (1.7 - 1)290 = 202.5[K]$$
$$CL = 5.7[dB] \rightarrow 10^{0.57} = 3.715$$
$$NF_m = 6.7[dB] \rightarrow 10^{0.67} = 4.677$$
$$T_m = (NF_m)T_0 = 1066[K]$$
$$G_{if} = 23[dB] \rightarrow 10^{2.3} = 199.5$$
$$NF_{it} = 0.9[dB] \rightarrow 10^{0.09} = 1.23$$
$$T_{it} = (N_{it} - 1)T_0 = (1.23 - 1)290 = 66.78[K]$$
$$IL = 0.5[dB] \rightarrow 10^{0.05} = 1.122$$
$$T_{filter} = (IL - 1)T_p = 0.122 \cdot 300 = 36.61[K]$$

$$\begin{aligned}
T_{cos} &= T_{rf} + \frac{T_m}{G_{rf}} + \frac{T_{it}CL}{G_{rf}} + \frac{T_{filter}CL}{G_{rf}G_{if}} \\
&= 202.5 + \frac{1066}{63.1} + \frac{66.78 \cdot 3.715}{63.1} + \frac{36.61 \cdot 3.715}{63.1 \cdot 199.5} = 223.3[K] \\
T_A &= \eta T_b + (1-\eta)T_{ant} = 0.8 \cdot 150 + 0.2 \cdot 310 = 182[K] \\
T_{sys} &= T_A + T_{cos} = 182 + 223.3 = 405.3[K]
\end{aligned}$$

$$P_r = \frac{P_t G_a^2 \lambda^2 \sigma}{(4\pi)^3 R^4}, \quad S_{out} = P_r G_{sys}, \quad N_{out} = KBT_{sys}G_{sys}$$

$$G_{sys} = \frac{G_{rf}G_{it}}{CL \cdot IL}, \quad \frac{S_{out}}{N_{out}} = \frac{P_r}{KBT_{sys}} > 10^{12/10}, \quad \lambda = c/f = \frac{3 \cdot 10^8}{3 \cdot 10^9} = 0.1[M]$$

$$\frac{P_t G_a^2 \lambda^2 \sigma}{(4\pi)^3 KBT_{sys} \cdot R^4} > 10^{1.2}$$

$$\begin{aligned}
R^4 &< \frac{20 \cdot 10^3 \cdot 10^{4.2} \cdot 0.1^2 \cdot 0.1}{(4\pi)^3 \cdot 1.38 \cdot 10^{-23} \cdot 12 \cdot 10^3 \cdot 405.3 \cdot 10^{1.2}} \\
R^4 &< 9.4746 \cdot 10^{20} \\
\therefore R &< 1.754 \cdot 10^5[m]
\end{aligned}$$

c) The antenna's main lobe also illuminates a ground area with a total RCS of 10000 m^2 at the same range as the target. Find the minimum attenuation of the filter (in dB) at a frequency of 10 MHz so that the target is detected within a range of 50 km. Note: IL = 0.5 dB indicates the loss within the pass band.

$$S_{out} = P_r G_{sys}, \quad P_r = \frac{P_t G_a^2 \lambda^2 \sigma}{(4\pi)^3 R_1^4}, \quad R_1 = 50[Km], \quad G_{sys} = \frac{G_{rf}G_{it}}{CL \cdot IL}$$

$$N_{gr} = P_r' G_{sys}', \quad P_r' = \frac{P_t G_a^2 \lambda^2 \sigma'}{(4\pi)^3 R_1^4} = P_r \frac{\sigma'}{\sigma}, \quad \sigma' = 10^4[m^2], \quad G_{sys}' = \frac{G_{rf}G_{it}}{CL \cdot IL'} = G_{sys}\frac{IL}{IL'}$$

$$N_{gr} = P_r' G_{sys}' = P_r \frac{\sigma'}{\sigma}\frac{IL}{IL'} = S_{out}\frac{\sigma'}{\sigma}\frac{IL}{IL'}, \quad q = \frac{\sigma'}{\sigma}\frac{IL}{IL'}$$

$$N_{gr} = S_{out} q$$

$$N_t = N_{out} + N_{gr} = KBT_{sys}G_{sys} + qP_rG_{sys}$$

$$\begin{aligned}
\frac{N_t}{S_{out}} &< 10^{-1.2} \\
\frac{(KBT_{sys} + qP_r)G_{sys}}{P_r G_{sys}} &< 10^{-1.2} \\
\frac{KBT_{sys}}{P_r} + q &< 10^{-1.2}
\end{aligned}$$

$$\begin{aligned}
KBT_{sys} &= 1.38 \cdot 10^{-23} 12 \cdot 10^3 \cdot 405.3 = 6.71 \cdot 10^{-17}[W] \\
P_r = \frac{P_t G_a^2 \lambda^2 \sigma}{(4\pi)^3 R_1^4} &= \frac{20 \cdot 10^3 \cdot 10^8 \cdot 0.1^2 \cdot 0.1}{(4\pi)^3 (50 \cdot 10^3)^4} = 1.61 \cdot 10^{-13}[W] \\
\frac{KBT_{sys}}{P_r} &= 4.17 \cdot 10^{-4}
\end{aligned}$$

$$\begin{aligned}
4.17 \cdot 10^{-4} + q &< 10^{-1.2} \\
q &< 0.0631 - 4.17 \cdot 10^{-4} \\
q &< 0.0627
\end{aligned}$$

$$\begin{aligned}
q &= \frac{\sigma'}{\sigma} \frac{IL}{IL'} \\
IL' &= \frac{\sigma'}{\sigma} \frac{IL}{q} \\
IL' &> \frac{10^4}{0.1} \frac{10^{0.5/10}}{0.0627} = 1.79 \cdot 10^6
\end{aligned}$$

$$IL' > 62.5[dB]$$