Tel Aviv University

Faculty of Engineering, School of Electrical Engineering

RF Circuits and Antennas

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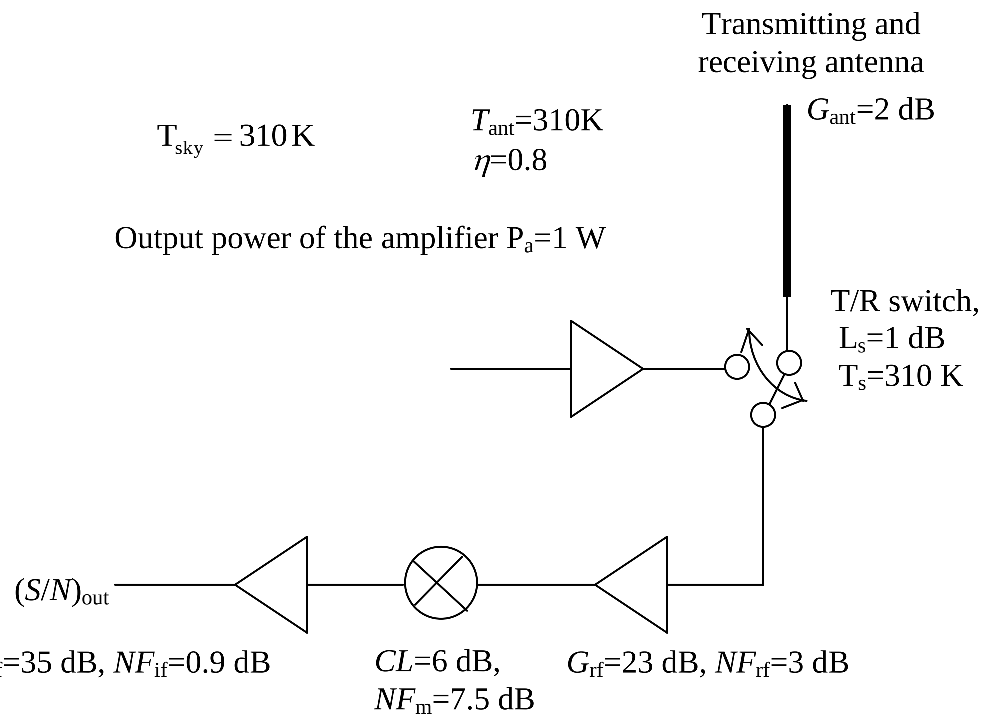
Recitation #2

1. A radio link transmitter has a power of 10 W. If the antenna has a gain of 30 dB, and we assume a worst-case side-lobe level of –16 dB (below the main beam), what are the power densities in the main beam and side-lobe regions of the antenna, at a distance of 30 m?
2. A plane wave with a power density of $2\times10^{-6}\ \mathrm{W/m^2}$ is incident in the main beam direction of an antenna having a gain of 12 dB. If the frequency is 8 GHz, what is the power delivered to a matched load at the antenna terminals?

Problem #3.

Consider a microwave transmit-receive system depicted below:



a. Determine the equivalent noise temperature and noise figure of the receiver system (including the noise temperature of the antenna).

b. What is the maximum communication range (in km) between two identical transmit and receive systems if it's bandwidth $BW = 10\,MHz$, RF frequency $f = 1GHz$ and $(S/N)_{out} \geq 8dB$?

c. By how many dB should the antenna gain be increased in order to double the maximum communication range?

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Problem #1

The power density in the main beam direction is equal to:

$$S = \frac{P_{in} G}{4\pi r^2},$$
$$G = 10^3,\ P_{in} = 10\,\mathrm{W}, r = 30\,\mathrm{m}.$$
$$S = \frac{10 \cdot 10^3}{4\pi \cdot 30^2} = 0.884 \frac{W}{m^2}.$$

In the side lobe region:

$$G_{sl} = (30 - 16) = 14\,\mathrm{dB} = 25.12,$$
$$S_{sl} = \frac{10 \cdot 25.12}{4\pi \cdot 30^2} = 0.022 \frac{\mathrm{W}}{\mathrm{m}^2}.$$

Problem #2

$$A_e = \frac{G\lambda^2}{4\pi},$$
$$P_{rec} = A_e S = \frac{G\lambda^2}{4\pi} S,$$
$$G = 12\,\mathrm{dB} = 10^{1.2} = 15.85,$$
$$\lambda = \frac{300}{8} = 37.5\,\mathrm{mm} = 0.0375\,\mathrm{m},$$
$$\mathrm{S} = 2 \times 10^{-6} \frac{\mathrm{W}}{\mathrm{m}^2},\ P_{rec} = \frac{15.85 \cdot (0.0375)^2}{4\pi} \cdot 2 \times 10^{-6} = 3.55 \times 10^{-9}\,\mathrm{W}.$$

Problem #3

a) The gains and the noise figures of the elements in linear scale are

$$L_s = 1\,\mathrm{dB} = 10^{0.1} = 1.26,$$
$$G_{RF} = 23\,\mathrm{dB} = 10^{2.3} = 199.53,$$
$$N_{RF} = 3\,\mathrm{dB} = 10^{0.3} = 2.0,$$
$$CL = 6\,\mathrm{dB} = 10^{0.6} = 3.98 \quad \text{conversion loss},$$
$$NF_m = 7.5\,\mathrm{dB} = 10^{0.75} = 5.62,$$
$$G_{if} = 35\,\mathrm{dB} = 10^{3.5} = 3162.3,$$
$$NF_{if} = 0.9\,\mathrm{dB} = 10^{0.09} = 1.23.$$

Now we can calculate the noise temperature of the elements:

$$T_{es} = (L_s - 1)T_s = (1.26 - 1) \times 310 = 80.6\,\text{K},$$
$$T_{RF} = (NF_{RF} - 1)T_0 = (2 - 1) \times 290 = 290\,\text{K},$$
$$T_m = (NF_m - 1)T_0 = (5.62 - 1) \times 290 = 1339.8\text{K},$$
$$T_{if} = (NF_{if} - 1)T_0 = (1.23 - 1) \times 290 = 66.7\,\text{K}.$$

The equivalent noise temperature of the receiver can be found as

$$T_{rec} = T_{es} + T_{RF} \cdot L_s + \frac{T_m \cdot L_s}{G_{RF}} + \frac{T_{if} \cdot CL \cdot L_s}{G_{rf}} = 80.6 + 290 \cdot 1.26 + \frac{1339.8 \cdot 1.26}{199.53} + \frac{66.7 \cdot 1.26 \cdot 3.98}{199.53} =$$
$$= 456.14\,\text{K}.$$

The noise temperature of the antenna is given by

$$T_A = \eta T_{sky} + (1 - \eta)T_{ant} = 0.8 \cdot 310 + (1 - 0.8) \cdot 310 = 310\,\text{K}.$$

The noise power at the antenna terminals is:

$$N_i = kBT_A.$$

The output noise power is

$$N_{out} = kBG_{sys}T_A + kBG_{sys}T_{rec} = kBG_{sys}(T_A + T_{rec}) = kBG_{sys}T_{sys},$$
$$G_{sys} = -L_s + G_{RF} - CL + G_{if},\; T_{sys} = T_A + T_{rec}.$$

So, the equivalent noise temperature and noise figure of the receiver system (including the noise temperature of the antenna) are

$$T_{sys} = T_A + T_{rec} = 456.14 + 310 = 766.14\,\text{K}$$
$$NF_{sys} = 1 + \frac{T_{sys}}{T_0} = 1 + \frac{766.14}{290} = 3.64 = 5.61\text{dB}.$$

b) If P_r is the received signal power at the antenna terminals, then the output signal power is

$$S_{out} = P_r G_{sys}.$$

From the Friis power transmission equation we can find P_r

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2.$$

The output signal-to-noise ratio is

$$\left(\frac{S}{N}\right)_{out} = \frac{P_r G_{sys}}{kBT_{sys}G_{sys}} = \frac{P_r}{kBT_{sys}} = \frac{P_t G_t G_r}{kBT_{sys}(4\pi)^2} \cdot \left(\frac{\lambda}{R}\right)^2$$

It is necessary that

$$\left(\frac{S}{N}\right)_{out} > \left(\frac{S}{N}\right)_{\min} = 8\,\text{dB}.$$

From this demand we can extract the ratio R/λ

$$\left(\frac{R}{\lambda}\right)^2 < \frac{P_t G_t G_r}{kBT_{sys}(4\pi)^2 (S/N)_{\min}}.$$

The power delivered to the input of transmitting antenna is L_s times less than output power of the amplifier P_a=1 W:

$$P_t = \frac{P_a}{L_s} = \frac{1\,\text{W}}{1.26} = 0.794\,\text{W}.$$

Now we can calculate the communication range:

$$G_t = G_r = 2\,\text{dB}=10^{0.2} = 1.585,\ B = 10^7\ \text{Hz},\ \left(\frac{S}{N}\right)_{\min} = 8\,\text{dB}=10^{0.8} = 6.31,$$

$$\left(\frac{R}{\lambda}\right) < \frac{0.794 \cdot (1.585)^2}{1.38\times 10^{-23} \times 10^7 \cdot 766.14 \times (4\pi)^2 \times 6.31}, \Rightarrow \frac{R}{\lambda} < 1.38\times 10^5,$$

$$f = 1\text{GHz},\ \lambda = \frac{300}{f\ (\text{in GHz})} = \frac{300}{1} = 300\,\text{mm}=0.3\,\text{m},$$

$$R < 41.3\times 10^3\ \text{m},\quad R < 41.3\,\text{km}.$$

c) As we can see from the developed formula for R/λ the maximum communication range is proportional to the gain of transmitting/receiving antenna. So, in order to double the maximum communication range the gain of antenna should be increased by two times or in other words by three dB.