Radio Frequency Circuits & Antenna

Thomas Glezer
Tel Aviv University

—

Homework: 8

May 25, 2022

1 A four-port network is characterized by the scattering matrix shown below:

$$\underline{\underline{\mathbf{S}}} = \begin{pmatrix} 0.17\exp(j90^\circ) & 0.63\exp(-j45^\circ) & 0.68\exp(-j45^\circ) & 0 \\ 0.63\exp(-j45^\circ) & 0 & 0 & 0.56\exp(j45^\circ) \\ 0.68\exp(-j45^\circ) & 0 & 0.24\exp(j38^\circ) & 0.38\exp(j45^\circ) \\ 0 & 0.56\exp(j45^\circ) & 0.38\exp(-j45^\circ) & 0.32\exp(-j32^\circ) \end{pmatrix} \tag{1}$$

a) Is this network loss-less?

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 0.17^2 + 0.63^2 + 0.68^2 = 0.8882 < 1 \tag{2}$$

Therefore, the network isn't lossless.

b) Is this network reciprocal?

$$S_{34} \neq S_{43} \tag{3}$$

Therefore, the network isn't reciprocal.

c) What is the return loss at port 1 when all other ports are matched?

$$RL = -20\log|S_{11}| = -20\log 0.17 = 15.39[dB] \tag{4}$$

d) What is the insertion loss and phase between ports 2 and 4, when all other ports are matched?

$$IL_{24} = -20\log|S_{24}| = -20\log 0.56 = 5.04[dB], \quad \varphi = 45^\circ \tag{5}$$

e) What is the reflection coefficient seen at port 1 if a short circuit is placed at the terminal plane of port 3, and all other ports are matched?

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 \tag{6}$$
$$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4 \tag{7}$$
$$a_2 = a_4 = 0, a_3 = \rho b_3, \quad \rho = \frac{Z_l - Z_0}{Z_l + Z_0} \tag{8}$$

$$b_1 = S_{11}a_1 + S_{13}a_3 \tag{9}$$
$$b_3 = S_{31}a_1 + S_{33}a_3 \tag{10}$$
$$b_3 = S_{31}a_1 + S_{33}\rho b_3 \tag{11}$$
$$b_3(1 - S_{33}\rho) = S_{31}a_1 \tag{12}$$
$$a_3 = \rho b_3 = \frac{\rho S_{31}}{1 - S_{33}\rho}a_1 \tag{13}$$

Plugging (13) into (9):

$$b_1 = \left(S_{11} + \frac{\rho S_{31}}{1 - S_{33}\rho}\right)a_1 \tag{14}$$

$$\tag{15}$$

When we short circuit $Z_l \to \infty, \rho \to -1$, then the reflection seen at port-1 is:

$$\frac{b_1}{a_1} = S_{11} - \frac{S_{31}}{1 + S_{33}\rho} \tag{16}$$
$$= 0.17j - \frac{0.68^2 \exp(-90°)}{1.1891 + 0.1475j} \tag{17}$$
$$= 0.17j + \frac{0.4624}{1.1891 + 0.1475j} \cdot \frac{1.1891 - 0.1475j}{1.1891 - 0.1475j} \tag{18}$$
$$= 0.17j + \frac{0.4624(1.1891 - 0.1475j)}{1.1891^2 + 0.1475^2} \tag{19}$$
$$= 0.17j + 0.383 - 0.0475j \tag{20}$$
$$= 0.383 + 0.1225j \tag{21}$$
$$= 0.402\angle 17.74 \tag{22}$$

2 A four-port network is characterized by the scattering matrix shown below:

$$\underline{\underline{\mathbf{S}}} = \begin{pmatrix} 0.17\angle 139.2° & 0.39\angle 8.02° & 0.67\angle 91.07° & 0.60\angle -130.8° \\ 0.39\angle 8.02° & 0.17\angle 139.2° & 0.67\angle 91.07° & 0.60\angle 49.2° \\ 0.67\angle 91.07° & 0.67\angle 91.07° & 0.30\angle -30.58° & 0.01\angle 73.4° \\ 0.60\angle -130.8° & 0.60\angle 49.2° & 0.01\angle 73.4° & 0.52\angle 104.8° \end{pmatrix} \tag{23}$$

The reference plane at port 3 is shifted outward at a distance of $\lambda/5$ and the reference plane at port 2 is shifted inward at a distance of $\lambda/6$ (λ is the wave length in the transmission line). Find the new scattering matrix of the network.

$$\underline{\underline{\tilde{\mathbf{S}}}} = \begin{pmatrix} 0.17\angle 139.2° & 0.39\angle 68.02° & 0.67\angle 19.07° & 0.60\angle -130.8° \\ 0.39\angle 68.02° & 0.17\angle 259.7° & 0.67\angle 79.07° & 0.60\angle 109.2° \\ 0.67\angle 19.07° & 0.67\angle 79.07° & 0.30\angle -174.58° & 0.01\angle 1.4° \\ 0.60\angle -130.8° & 0.60\angle 109.2° & 0.01\angle 1.4° & 0.52\angle 104.8° \end{pmatrix} \tag{24}$$

3 A four-port network is characterized by the scattering matrix shown below:

$$\underline{\underline{\mathbf{S}}} = \begin{pmatrix} 0.3\angle -30° & 0 & 0 & 0.8\angle 0° \\ 0 & 0.7\angle -30° & 0.7\angle -45° & 0 \\ 0 & 0.7\angle -45° & 0.7\angle -30° & 0 \\ 0.8\angle 0° & 0 & 0 & 0.3\angle -30° \end{pmatrix} \tag{25}$$

If ports 3 and 4 are connected by a lossless matched transmission line with an electrical length of 60° , find the resulting insertion loss and phase between ports 1 and 2.

$$a_2 = 0, \quad a_3 = Tb_4, \quad a_4 = Tb_3, \quad T = 1\angle -60° \tag{26}$$

$$b_1 = S_{11}a_1 + S_{14}Tb_4 \tag{27}$$
$$b_2 = S_{23}Tb_4 \tag{28}$$
$$b_3 = S_{33}Tb_4 \tag{29}$$
$$b_4 = S_{41}a_1 + S_{44}Tb_3 = S_{41}a_1 + S_{33}S_{44}T^2 b_4 \tag{30}$$
$$b_4 = \frac{S_{41}}{1 - S_{33}S_{44}T^2}a_1 \tag{31}$$
$$b_2/a_1 = \frac{S_23S_{41}T}{1 - S_{33}S_{44}T^2} = \frac{0.7\angle-45^\circ \cdot 0.8 \cdot 1\angle-60^\circ}{1 - 0.7\angle-30^\circ \cdot 0.3\angle-30^\circ \cdot 1\angle-120^\circ} \tag{32}$$
$$= \frac{0.56\angle-105^\circ}{1 - 0.21\angle-180^\circ} = 0.463\angle-105^\circ \tag{33}$$

$$IL = -20\log(0.463) = 6.69[dB], \quad \varphi = \arg(b_2/a_1) = -105^\circ \tag{34}$$

4 A two-port network is characterized by the scattering matrix shown below:

$$\underline{\underline{S}} = \begin{pmatrix} \frac{1+j}{2} & \frac{1+j}{2} \\ \frac{1-j}{2} & \frac{1+j}{2} \end{pmatrix} \tag{35}$$

Is the network lossless?

$$S_{11}S_{12}^* + S_{21}S_{22}^* = 1/4[(1+j)(1-j) + (1-j)(1-j)] = 1/4[1+1+1-2j-1] = 1/2(1-j) \neq 0$$

Therefore, the network isn't lossless.

5 Two identical reciprocal and lossless two-port networks are connected by a lossless transmission line of length l (see Figure 1). The propagation constant of the line is β and characteristic impedance is Z_0. Every two-port network is represented as shunt element $Y = jb$. The characteristic impedance of the ports is Z_0.

a) Find the scattering matrix element S_{11} of the system consisting of the two two-ports networks and the transmission line.

The shunt element matrix can be presented like:

$$\underline{\underline{S}} = \begin{pmatrix} 1 & 0 \\ jb & 1 \end{pmatrix} \tag{36}$$

The transmission line section matrix can be presented like:

$$\underline{\underline{S}} = \begin{pmatrix} \cos\theta & jZ_0\sin\theta \\ jY_0\sin\theta & \cos\theta \end{pmatrix}, \quad \theta = \beta l \tag{37}$$

The ABCD matrix of the equivalent circuit can be find in the next way:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ jb & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & jZ_0 \sin\theta \\ jY_0 \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ jb & 1 \end{pmatrix} \tag{38}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ jb & 1 \end{pmatrix} \begin{pmatrix} \cos\theta - bZ_0 \sin\theta & jZ_0 \sin\theta \\ j[Y_0 \sin\theta + b\cos\theta] & \cos\theta \end{pmatrix} \tag{39}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos\theta - bZ_0 \sin\theta & jZ_0 \sin\theta \\ j[Y_0 \sin\theta + 2b\cos\theta - Z_0 b^2 \sin\theta] & \cos\theta - bZ_0 \sin\theta \end{pmatrix} \tag{40}$$

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \tag{41}$$

$$S_{11} = \frac{\cos\theta - Z_0 b \sin\theta + j(\sin\theta - 2bZ_0\cos\theta + (Z_0 b)^2 \sin\theta - \sin\theta) - \cos\theta + Z_0 b \sin\theta}{2(\cos\theta - Z_0 b \sin\theta) + j[2\sin\theta + 2bZ_0 \cos\theta - (Z_0 b)^2 \sin\theta]} \tag{42}$$

$$S_{11} = \frac{j((Z_0 b)^2 \sin\theta - 2bZ_0 \cos\theta)}{2(\cos\theta - Z_0 b \sin\theta) + j[2\sin\theta + 2bZ_0 \cos\theta - (Z_0 b)^2 \sin\theta]} \tag{43}$$

b) Find the length l of the transmission line section for which $|S_{11}| = 0$ and express it using Z_0, b, β.

$$S_{11} = 0 \tag{44}$$

$$(Z_0 b)^2 \sin\theta - 2bZ_0 \cos\theta = 0 \tag{45}$$

$$Z_0 b \sin\theta - 2\cos\theta = 0 \tag{46}$$

$$\tan\theta = \frac{2}{Z_0 b} \tag{47}$$

$$l = 1/\beta \arctan\frac{2}{Z_0 b} \tag{48}$$