$$\vec{A} = \mu I_m l \frac{e^{-jkr}}{4\pi r} \hat{z} \tag{1}$$

$$\hat{\boldsymbol{z}} = \cos\theta \cdot \hat{\boldsymbol{r}} - \sin\theta \cdot \hat{\boldsymbol{\theta}} \tag{2}$$

Thus, plugging $(2) \rightarrow (1)$

$$\vec{A} = \mu I_m l \frac{e^{-jkr}}{4\pi r} [\cos \theta \cdot \hat{r} - \sin \theta \cdot \hat{\theta}]$$
 (3)

$$\vec{\boldsymbol{H}} = \boldsymbol{\nabla} \times \vec{\boldsymbol{A}_T} \tag{4}$$

$$\vec{\boldsymbol{H}}_r = 0 \tag{5}$$

$$\vec{\boldsymbol{H}}_{\theta} = \frac{\mu I_m l}{4\pi} \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \varphi} - \frac{\partial (r \cdot A_{\varphi})^0}{\partial r} \right) = 0 \tag{6}$$

$$\vec{\boldsymbol{H}}_{\varphi} = \frac{\mu I_m l}{4\pi} \frac{1}{r} \left(\frac{\partial (r \cdot A_{\theta})}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \tag{7}$$

$$= \frac{\mu I_m l}{4\pi} \frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{e^{-jkr}}{r} (-\sin \theta) \right) - \frac{\partial}{\partial \theta} \left(\frac{e^{-jkr}}{r} (\cos \theta) \right) \right) \tag{8}$$

$$= \frac{\mu I_m l}{4\pi} \frac{1}{r} \left(\frac{\partial}{\partial r} \left(e^{-jkr} (-\sin \theta) \right) - \frac{\partial}{\partial \theta} \left(\frac{e^{-jkr}}{r} (\cos \theta) \right) \right)$$
(9)

$$= \frac{\mu I_m l}{4\pi} \frac{1}{r} \left(jk(e^{-jkr} \sin \theta) - \left(\frac{e^{-jkr}}{r} (-\sin \theta) \right) \right)$$
 (10)

$$\vec{\boldsymbol{H}}_{\varphi} = \mu I_m l \frac{e^{-jkr}}{4\pi r} (jk + \frac{1}{r}) \sin \theta \cdot \hat{\boldsymbol{\varphi}}$$
(11)

$$\vec{E} = \frac{1}{j\omega\varepsilon} \nabla \times \vec{H} \tag{12}$$

$$\vec{E}_r = \frac{\mu I_m l}{j\omega\varepsilon} \frac{1}{4\pi} \frac{1}{r\sin\theta} \left(\frac{\partial (H_\varphi \sin\theta)}{\partial \theta} - \frac{\partial \mathcal{U}_\theta}{\partial \varphi} \right)^0$$
(13)

$$= \frac{\mu I_m l}{j\omega\varepsilon} \frac{1}{4\pi} \frac{1}{r\sin\theta} \left(\frac{\partial}{\partial\theta} \left(\frac{e^{-jkr}}{r} (jk + 1/r)\sin^2(\theta) \right) \right)$$
(14)

$$= \frac{\mu I_m l}{j\omega\varepsilon} \frac{e^{-jkr}}{4\pi r^2} (jk + 1/r) \frac{1}{\sin\theta} 2\cos\theta\sin\theta \tag{15}$$

$$\vec{E}_r = 2 \frac{\mu I_m l}{j\omega\varepsilon} \frac{e^{-jkr}}{4\pi r^2} (jk + 1/r) \cos\theta \cdot \hat{r}$$
(16)