Radio Frequency Circuits & Antenna

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Homework: 4

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1 For a z-directed $\lambda/2$ dipole placed symmetrically at the origin, determine:

a) The vector effective length: Assuming a cosinusoidal behavior:
We can define $L = \lambda/2$, i.e. $kL = \pi$:

$$I(z) = I(0)\cos(kz), \quad |z| < L/2$$

recall:

$$\underline{\mathbf{A}}(\underline{\mathbf{r}}) = \hat{\underline{\mathbf{z}}}\mu \int_{-L/2}^{L/2} dz' I(z') \frac{e^{-jk|\underline{\mathbf{r}}-\underline{\mathbf{r}}'|}}{4\pi|\underline{\mathbf{r}}-\underline{\mathbf{r}}'|} \approx \hat{\underline{\mathbf{z}}}\mu \frac{e^{-jkr}}{4\pi r} \int_{-L/2}^{L/2} dz' I(z') e^{+jk\hat{\underline{\mathbf{r}}}\cdot\underline{\mathbf{r}}'}$$
$$\underline{\mathbf{A}}(\underline{\mathbf{r}}) \approx \hat{\underline{\mathbf{z}}}\mu \frac{e^{-jkr}}{4\pi r} \int_{-L/2}^{L/2} dz' I(z') e^{+jkz'\cos(\theta)}$$

Then using the above relation we get:

$$\begin{aligned}
\underline{\mathbf{A}}(\underline{\mathbf{r}}) &= \hat{\underline{\mathbf{z}}}\mu g(r) \int_{-L/2}^{L/2} dz' I(0)\cos(kz') e^{+jkz'\cos(\theta)} \\
&= \hat{\underline{\mathbf{z}}}\mu g(r) I(0) \int_{-L/2}^{L/2} dz' \frac{e^{+jkz'} + e^{-jkz'}}{2} e^{+jkz'\cos(\theta)} \\
&= \cdots \int_{-L/2}^{L/2} dz' 0.5(e^{jkz'(\cos(\theta)+1)} + e^{jkz'(\cos(\theta)-1)}) \\
&= \cdots \frac{e^{\frac{-jkL\cos(\theta)}{2}} \cdot ((e^{jkL\cos(\theta)} + 1)\sin(kL/2) + j\cos(\theta)\cos(kL/2)(e^{jkL\cos(\theta)} - 1))}{(\cos^2(\theta)j^2 + 1)k} \\
&= \cdots \frac{e^{\frac{-j\pi\cos(\theta)}{2}} \cdot ((e^{j\pi\cos(\theta)} + 1)\cancelto{1}{\sin(\pi/2)} + j\cos(\theta)\cancelto{0}{\cos(\pi/2)}(e^{j\pi\cos(\theta)} - 1))}{(1 - \cos^2(\theta))k} \\
&= \cdots \frac{e^{\frac{-j\pi\cos(\theta)}{2}} \cdot (e^{j\pi\cos(\theta)} + 1)}{\sin^2(\theta)k} \\
&= \cdots \frac{e^{\frac{j\pi\cos(\theta)}{2}} + e^{\frac{-j\pi\cos(\theta)}{2}}}{\sin^2(\theta)k}
\end{aligned}$$

$$\underline{\mathbf{A}}(\underline{\mathbf{r}}) = \hat{\underline{\mathbf{z}}}\frac{\mu}{k}2g(r)I(0)\frac{\cos[\pi/2 \cdot \cos(\theta)]}{\sin^2(\theta)}$$

Recall:

$$\underline{\mathbf{E}} = \underline{\hat{\theta}}jk\eta(I_0 d)g(r)\sin(\theta)$$

Thus:

$$\underline{\mathbf{E}} = \underline{\hat{\theta}}jk\eta g(r)I(0)\frac{2\cos[\pi/2 \cdot \cos(\theta)]}{k\sin(\theta)}$$

And following from definition we had:

$$\vec{\mathbf{h}}(\theta,\varphi) = \frac{1}{I_{in}}\iiint \vec{\mathbf{J}}(\vec{\mathbf{r}}')e^{j\vec{\mathbf{k}}\vec{\mathbf{r}}'}dV$$

Solved previously:

$$\vec{\mathbf{h}}(\theta,\varphi) \approx \frac{2}{k}\frac{\cos[\pi/2 \cdot \cos(\theta)]}{\sin(\theta)}\underline{\hat{\theta}}$$

b) The maximum value (in magnitude) of the vector effective length

$$\begin{aligned}\max\vec{\mathbf{h}}(\theta,\varphi) &= \frac{2}{k}\max\frac{\cos[\pi/2 \cdot \cos(\theta)]}{\sin(\theta)}\\ &= \frac{2}{k} = \frac{2}{2\pi/\lambda} = \frac{\lambda}{\pi}\end{aligned}$$

c) Maximum open-circuit output voltage when a uniform plane wave with an eletric field as given below is incident broadside on the dipole.

$$\mathbf{E}^{inc}(\theta = 90^\circ) = 10^{-3}\hat{\theta}[V/\lambda]$$

$$
\begin{aligned}
V_{o.c.} &= \max \vec{\mathbf{h}}(\theta, \varphi) \cdot \vec{\mathbf{E}} \\
&= \frac{\lambda}{\pi} \hat{\theta} 10^{-3} \hat{\theta} [V/\lambda] \\
&= \frac{10^{-3}}{\pi} [V] = 3.18 \cdot 10^{-4} [V]
\end{aligned}
$$

2 The input impedance of a $\lambda/2$ dipole assuming that the input (feed) terminals are at the center of the dipole, is equal to $73 + 42.5[\Omega]$. Assuming the dipole is lossless find:

a) Input impedance assuming that the input (feed) terminals have been shifted to a point on the dipole which is located $\lambda/8$ away from either end point of the dipole:

$$I_e(z') = \hat{\mathbf{z}} I_0 \sin[k(l/2 - |z'|)], \quad |z'| < l/2$$

At infinitesimal distance:

$$I_{in} = I_0 \sin[kl/2] = I_0 \sin[\frac{2\pi}{2\lambda} \lambda/2] = I_0$$

At $\lambda/8$ distance:

$$
\begin{aligned}
I'_{in} &= I_0 \sin[k(l/2 - |z'|)]_{z'=l/2-\lambda/8} = I_0 \sin[k \cdot \lambda/2] \\
&= I_0 \sin[\frac{2\pi}{\lambda} \lambda/8] = I_0 \sin[\pi/4] = I_0 \cdot \sqrt{2}/2 = I_0/\sqrt{2}
\end{aligned}
$$

$$z'/z = (I_{in}/I'_{in})^2 = (1/1/\sqrt{2})^2 = 2$$

b) Capacitive or inductive reactance that must be placed parallel to the new input terminals of part (a) so that the antenna becomes resonant (make the total input impedance real):

$$Y' = z'^{-1} = 1/(2z') = 1/(146 + j85) = \frac{1}{168.94\angle 30.207^\circ}$$

$$\begin{aligned} &= 0.005919(\cos(30.207) - j\sin(30.207)) \\ &= 5.12 \cdot 10^{-3} - j2.978 \cdot 10^{-3}[\Omega^{-1}] \end{aligned}$$

$$\begin{aligned} \tilde{Y} &= j2.978 \cdot 10^{-3} \\ \tilde{Z} &= 1/\tilde{Y} = -j/(2.978 \cdot 10^{-3}) = -j335.8[\Omega] \end{aligned}$$

$$Y_{in}^t = \tilde{Y} + Y' = 5.12 \cdot 10^{-3}[\Omega^{-1}]$$

$$Z_{in}^t = 1/Y_{in}^t = 195.3[\Omega]$$

c) VSWR of the new input terminals when the resonant dipole of part (b) is connected to a $300[\Omega]$ transmission line:

$$|\Gamma| = \left|\frac{Z_0 - Z_{in}^t}{Z_0 + Z_{in}^t}\right| = \frac{300 - 195.3}{300 + 195.3} = 0.211$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.211}{1 - 0.211} = 1.536$$

3 A uniform array of 20 isotropic elements is placed along the z-axis with $\lambda/2$ spacing between the adjacent elements. Calculate a progressive phase shift β (in radians) for the following array types (main beam radiation directions):

$$kd\cos(\theta_0) + \beta = 0$$

$$\beta = -kd\cos(\theta_0) = -\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos(\theta_0) = -\frac{\pi}{2}\cos(\theta_0)$$

a) Broadside : $\beta = 0$

b) End-fire with maximum at $\theta = 0°$: $\beta = -\pi/2$

c) End-fire with maximum at $\theta = 180°$: $\beta = \pi/2$

d) Phase-array with maximum radiation at $\theta = 50°$: $\beta = -\pi/2\cos(50°) \approx -1.01$

4 Find the maximum distance between the elements in a linear scanning array to suppress grating lobes if the array is designed to scan to the maximum angles of:

A classic solution to this problem is given by:

$$d_{\max} = \frac{\lambda}{1 + |\cos(\theta_0)|}$$

a) $\theta = 0°$: $\lambda/2 = 0.5\lambda$

b) $\theta = 30°$: $\lambda/(1 + \sqrt{3}/2) = 0.536\lambda$

c) $\theta = 45°$: $\lambda/(1+\sqrt{2}/2) = 0.586\lambda$

d) $\theta = 60°$: $\lambda/(1+1/2) = 0.\bar{6}\lambda$

e) $\theta = 135°$: $\lambda/(1+\sqrt{2}/2) = 0.586\lambda$